

## Homework 2: Random Coefficients

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### Question 1:

When  $BB$  acquires  $BB2$  through a buy-out, the price for remaining local breweries in the market that used to be served by  $BB2$  before the buy out is likely to increase. This is because the consumers prefer local breweries and with the buyout the competition in local breweries in that market decreased.

On the other hand, the price of corporate breweries is likely to decrease in this market because there are now more corporate breweries serving this market (i.e., increased competition).

### Question 2:

Yes, the home market price of  $BB2$  could fall even if the product characteristics remain unchanged. This is because consumers experience disutility from the brand being owned by a corporate brewery as they may perceive it as less authentic or less local. This shift in perception could reduce demand for  $BB2$  in the home market, which will cause reduction in price. By our assumption that there are 2 other corporate breweries ( $AB$  and  $MC$ ) serving this market, we do not have to worry about the case where the buyout would result in monopoly of  $BB$  in that market. Moreover, once the corporate brewery acquires  $BB2$ , the corporate brewery might use its network to expand the market for  $BB2$  – it can be made available in all the markets, which might also reduce price in home market.

The assumption that product characteristics remain unchanged after the acquisition may or may not hold in reality. The corporate brewery might choose to maintain the existing product characteristics to preserve brand loyalty among local consumers. Alternatively, it may decide to modify the product characteristics to better align with the preferences of a broader consumer base, especially now that the product is available in all markets. If we assume that the local brand that the corporate brewery acquired had good characteristics which the people from other markets (that was not served originally by the local brewery) would also prefer, then the product characteristics would likely not change.

### Question 3:

It is reasonable to assume that corporate breweries are more efficient than local breweries and operate with lower marginal costs. In this case, the acquired local brewery ( $BB2$ ) gains greater flexibility to reduce prices due to these cost efficiencies. By adopting  $BB$ 's cost structure,  $BB2$  can more easily adjust prices downward to counter the reduced demand in

the home market caused by consumer disutility from corporate ownership. This strengthens the argument made in Question 2 that the home market price would likely fall.

#### Question 4:

Data was generated using the parameter values suggested in the question. There are 196 ( $14 \times 14$ ) markets. The value of  $r$  was set to 4 and the local breweries were allocated in a way that there are exactly two local beers available in each of the 196 markets. Alcohol percentage and Bitterness were drawn from a uniform distribution between 0 and 1 independently for each brewery. These, in addition to the dummy variables representing whether the brewery is local, corporate or subsidiary, are considered the observed product characteristics ( $x_j$ ). The unobservable characteristics ( $\xi_j$ ) was drawn from  $N(0,1)$ . The cost shifters ( $w_{jt}$  and  $\omega_{jt}$ ) are drawn from  $N(0,1)$  and the marginal cost was calculated using the equation given in the question. I set the value of  $\alpha = 1$  for data generation.

In this set up, the choice probability  $s_j$  for each market  $t$  can be written as:

$$s_j = \int \frac{e^{u_{ij}}}{1 + \sum_j' e^{u_{ij'}}} dF(\nu_i) = \frac{1}{N} \sum_{i=1}^N \frac{e^{u_{ij}}}{1 + \sum_j' e^{u_{ij'}}}$$

where

$$u_{ij} = x_j \beta - \alpha p_j + \xi_j + \sum_k \beta_1^k x^k \nu_i^k$$

The analytical expression for the derivative of the integral above with respect to price are:

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha(1 - s_j) & \text{if } j = k, \\ \alpha s_j s_k & \text{if } j \neq k \end{cases}$$

With these, the FOCs are:

1. For firms without subsidiary (local and  $BB$ ):

$$p_j = c_j + \frac{1}{\alpha(1 - s_j)}$$

2. For firms with subsidiary ( $AB - AB2$ ,  $MC - MC2$ ):

$$p_j = c_j + \frac{1 + \alpha(p_{owned\_other} - c_{owned\_other}) s_{owned\_other}}{\alpha(1 - s_j)}$$

Then the matrix of simulated own and cross price derivatives of market shares implied by true model is (I choose one market to show these):

	$local_1$	$local_{61}$	$AB$	$AB2$	$MC$	$MC2$	$BB$
$local_1$	-0.1800	0.0525	0.0074	0.0372	0.0231	0.0089	0.0292
$local_{61}$	0.0525	-0.1732	0.0070	0.0353	0.0219	0.0085	0.0277
$AB$	0.0074	0.0070	-0.0304	0.0050	0.0031	0.0012	0.0039
$AB2$	0.0372	0.0353	0.0050	-0.1332	0.0155	0.0060	0.0196
$MC$	0.0231	0.0219	0.0031	0.0155	-0.0885	0.0037	0.0122
$MC2$	0.0089	0.0085	0.0012	0.0060	0.0037	-0.0366	0.0047
$BB$	0.0292	0.0277	0.0039	0.0196	0.0122	0.0047	-0.1088

Table 1: True (simulated) cross and own price derivative for market 1

### Question 5: Estimation: Nested logit Specification:

After we obtain data using random coefficients, we estimated the model using a nested logit specification where the nests are the outside good, local brewery, and corporate brewery. The model was estimated by 2SLS using the excluded cost shifters ( $w_{jt}$ ) and alcohol percentage of other firms within group (nest) for each market as instrument for price and within-group shares. The regression estimates are:

$\beta^{local}$	1.70 (0.1768)
$\beta^{subsidiary}$	1.01 (0.4785)
$\beta^{corporate}$	1.186 (0.381)
$\beta^{alcohol}$	-0.479 (0.2794)
$\beta^{bitterness}$	2.347 (0.9844)
$\alpha$	-0.841 (0.2355)
$\sigma$	0.235 (0.3221)

Table 2: Estimated of Nested Logit model

### Question 6:

The nested logit model is mis-specified in this context because it oversimplifies substitution patterns and consumer preferences. It allows preferences to be correlated within groups

(nests), making products within the same group closer substitutes. However, it assumes symmetric cross-price elasticities within groups, which can be overly restrictive and unrealistic in capturing true substitution patterns. This can lead to a biased estimation of cross-price elasticities – cross-price elasticities within nests may be overestimated and those outside nests may be underestimated.

### Question 7:

The analytical expression for the own and cross price derivative of market shares for nested logit is:

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\frac{\alpha}{1-\sigma} s_j (1 - \sigma s_{j|g} - (1 - \sigma) s_j) & \text{if } j = k, \\ \alpha s_j s_k (1 + \frac{\sigma}{1-\sigma} \frac{s_{j|g}}{s_j}) & \text{if } j \neq k \text{ and } j, k \in \text{same group/nest}, \\ \alpha s_j s_k & \text{if } j \neq k \text{ and } j, k \in \text{different group/nest} \end{cases}$$

Then the matrix of estimated own and cross price derivatives of market shares implied by estimated nested logit model is (I choose one market to show these):

	<i>local</i> <sub>1</sub>	<i>local</i> <sub>61</sub>	<i>AB</i>	<i>AB2</i>	<i>MC</i>	<i>MC2</i>	<i>BB</i>
<i>local</i> <sub>1</sub>	-0.1809	0.0737	0.0062	0.0313	0.0194	0.0075	0.0246
<i>local</i> <sub>61</sub>	0.0737	-0.1753	0.0059	0.0297	0.0184	0.0071	0.0233
<i>AB</i>	0.0062	0.0059	-0.0332	0.0070	0.0044	0.0017	0.0055
<i>AB2</i>	0.0313	0.0297	0.0070	-0.1385	0.0220	0.0085	0.0278
<i>MC</i>	0.0194	0.0184	0.0044	0.0220	-0.0943	0.0053	0.0172
<i>MC2</i>	0.0075	0.0071	0.0017	0.0085	0.0053	-0.0397	0.0067
<i>BB</i>	0.0246	0.0233	0.0055	0.0278	0.0172	0.0067	-0.1147

Table 3: Cross and Own price derivative implied by estimated nested logit for market 1

If we compare these results with the results in 4, we can see that the within nest derivatives are over-estimated and the out-of-group derivatives are underestimated. This is as expected as discussed in Question 6. The bias of share derivatives is not large, meaning that the mis-specification is not overly problematic in our case.

### Question 8:

Suppose the local brewery in the 1<sup>st</sup> market (*local*<sub>1</sub>) was acquired by the corporate brewery *BB*, turning *local*<sub>1</sub> into *BB2*, a subsidiary that is now available in all markets. I present a summary of the prices for both *BB* and *BB2* across all markets, as well as specifically in market 1, which was the original location of *BB2*. The post-buyout prices are simulated

using the nested logit model, while the pre-buyout prices are derived from the random coefficients model in Question 4. These prices are partially comparable, as the product characteristics remain the same as in Question 4. The only change is that we expanded *BB2*'s market presence to all 196 markets and generate product characteristics and marginal costs for these new markets.

The analytical expression for the first order conditions for nested logit model are:

1. For firms without subsidiary (locals):

$$p_j = c_j + \frac{1 - \sigma}{\alpha(1 - \sigma s_{j|g} - (1 - \sigma)s_j)}$$

2. For firms with subsidiary (all corporate after buyout):

$$p_j = c_j + \frac{(1 - \sigma)\{1 + \alpha s_{OO}(p_{OO} - c_{OO})(1 + \frac{\sigma s_{j|g}}{(1 - \sigma)s_j})\}}{\alpha(1 - \sigma s_{j|g} - (1 - \sigma)s_j)}$$

where *OO* in sub-script means other firms owned by the same corporate brewery, *AB-AB2*, *BB-BB2* and *MC-MC2*,  $s_{j|g}$  is the within group market share of firm *j*, groups or nest are  $g \in \{outside, local, corporate\}$ .

Post-buyout				
	No Adoption of BB's Cost		Adoption of BB's Cost	
	All market	Market 1	All Market	Market 1
<i>BB</i>	2.107 (0.129)	2.028 (-)	2.135 (0.130)	2.053 (-)
<i>BB2</i>	2.304 (0.145)	2.179 (-)	2.131 (0.117)	2.038 (-)
Pre-buyout				
	All Market		Market 1	
<i>BB</i>	1.984 (0.123)		1.851 (-)	
<i>BB2 (Local<sub>1</sub>)</i>	2.292 (0.080)		2.168 (-)	

Table 4: Post- and Pre- buyout prices of *BB* and *BB2* – mean and (standard deviation)

When comparing the post-buyout prices for *BB* and *BB2*, we observe two scenarios: one where *BB2* adopts *BB*'s cost structure and one where it does not. Without adopting *BB*'s cost structure, the average market price of *BB2* is higher, which is expected since *BB2* has higher marginal costs. However, once *BB2* adopts *BB*'s cost structure, its marginal cost

decreases, leading to a lower price. After adopting  $BB$ 's cost structure, the average price of both firms becomes nearly identical.

Looking at the pre-buyout prices, we see that the price of  $BB2$  (or  $local_1$ ) is higher than that of  $BB$ . This is expected, as before the buyout, consumers preferred  $BB2$  due to its status as a local brewery.

*Note: The python codes for these simulations are submitted separately as Jupyter notebook file: **Dahal RandomCoef\_HW2.ipynb**.*