One Sector Growth Model with Uncertainty

Consider a stochastic infinite horizon problem where a social planner seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ \mu \ \ln c_t \ + \ (1-\mu) \ \ln (1-l_t)$$

subject to:

$$c_t + \frac{\phi}{2}(k_{t+1} - k_t)^2 + k_{t+1} - (1-\delta)k_t = e^{z_t} \ k_t^{\theta} \ l_t^{1-\theta}$$

where $c_t \equiv$ consumption at time t, $k_t \equiv$ capital stock at time t, $l_t \equiv$ labor supplied at time t, and z follows an AR(1) process:

$$z_t = \rho z_{t-1} + \epsilon_t; \qquad \epsilon_t \sim N(0, \eta^2) i.i.d.$$

Assume the following parameter values:

- **Preferences:** $\mu = 0.34, \qquad \beta = 0.99$
- **Technology:** $\theta = 0.36, \quad \delta = 0.025, \quad \phi = 0.025/25, \quad \rho = 0.9, \quad \eta = 0.01$

1. Discretize the productivity process into finite state Markov Chain and the endogenous state k into finite grid points. Then, use value function iteration method to obtain value and policy functions.
2. Report the value and policy functions as 3-D graphs.
3. Numerically compute the stationary distribution of the capital stock in this economy and display the plot of this distribution.
4. Report the standard deviation of GDP, Consumption, Investment, and Employment. Also report the autocorrelation of output and the correlation of output with each of consumption, investment and employment.
5. Now, set ϕ = 0. Compare the results with ($\phi \neq 0$) and without adjustment costs ($\phi = 0$). What changes and why?