Final Computational Component

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1 Value and Policy Functions:

The 3-D graphs of Value and Policy functions when the productivity process is discretized into 11 Markov chains and the endogenous state K was discretized into 400 grid points are shown in Figure 1-3 for different values of ϕ.

We used the value function iteration method to obtain these functions. In this method, we started with an initial guess of the value function and used the computer to iterate over and try to converge to the accurate value function i.e. finding a value function is a fixed point problem that was solved numerically using a computer.

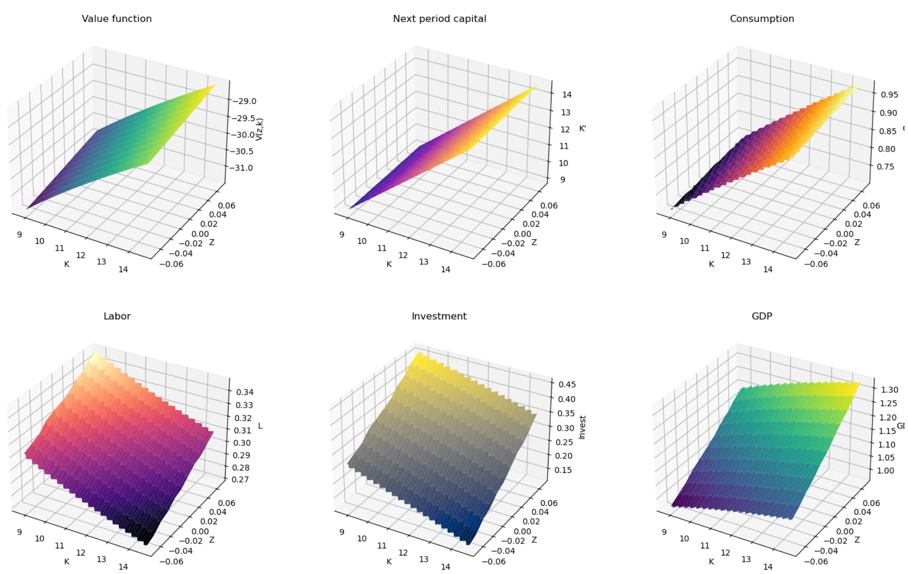

Figure 1: Value and Policy Functions: $\phi = 0$, nz = 11, nk = 400

The value function, policy function for capital, and policy function for consumption show an upward slope for both current capital and productivity shock for all values of ϕ used. This is as expected since an increase in current capital

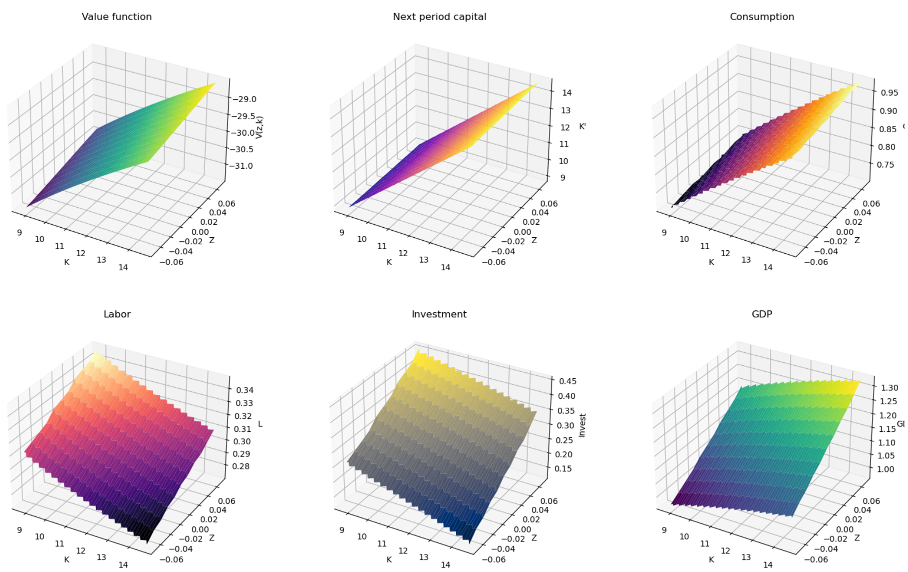

Figure 2: Value and Policy Functions: $\phi = 0.025$, nz = 11, nk = 400

will cause the value of the lifetime utility (represented by the value function) to increase - the higher the capital to begin with, the higher will be the lifetime utility. Similarly, higher capital to begin with also lead to higher next-period capital and higher consumption today (as shown by the respective policy functions). Moreover, if there is a positive productivity shock to the production process, the higher the value of such productivity shock, the higher will be the lifetime utility (represented by the value function), consumption, and next-period capital (represented by the respective policy functions).

The interesting thing to observe is that the policy function for labor and investment shows a downward trend with the increase in current capital when there are no or very little adjustment costs ($\phi = 0$ or $phi = 0.025$). However, they show an increasing trend with the increase in current capital when there is a very high adjustment cost ($\phi = 25$).

The reason behind this is that when there is no or very little adjustment cost involved, if the current capital increases, the person will feel richer and thus does not want to work more, and not want to invest more - because if k is high they want to decrease capital to reach stationary distribution. In other words, they will be able to maintain the same level of next period capital while working less.

However, if there is a very high adjustment cost involved, and as seen already the next-period capital is increasing in current capital (from policy function of capital), to maintain such capital accumulation, the person has to work harder, thus increasing the labor. Similarly, due to such high adjustment cost, the investment required to maintain the higher next-period capital given higher

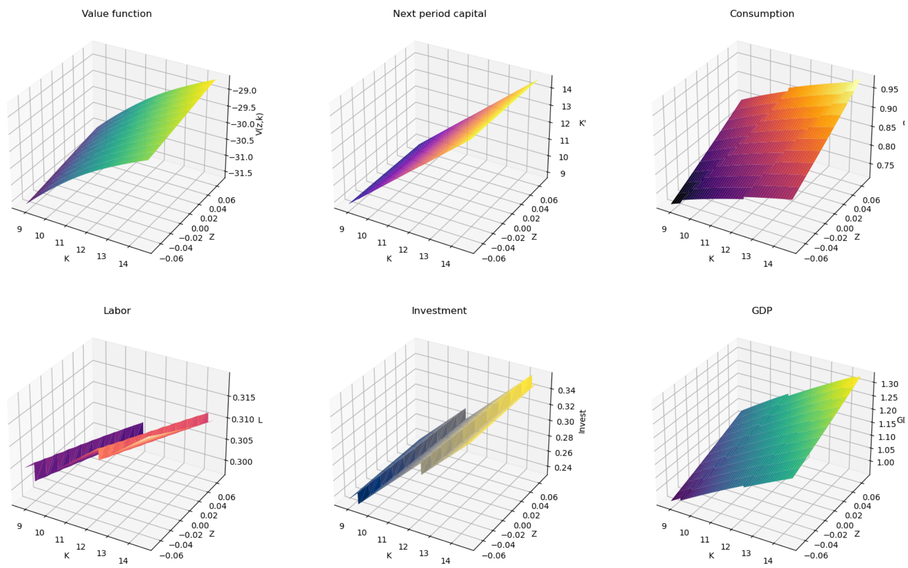

Figure 3: Value and Policy Functions: $\phi = 25$, nz = 11, nk = 400

current capital is also higher - which is represented by the upward-sloping investment graph.

One last comment about this set of graphs is for the GDP. Its natural that when the current capital increases or there is a larger positive productivity shock, the output increases. This is the reason behind upward sloping GDP with current capital and the productivity shock.

2 Distribution of Capital Stock:

Figures 4-6 represent the distribution of capital stock for $\phi = 0$, 0.025, and 25 respectively. We can see that when there is no or very little adjustment cost, the distribution of capital appears normal (there are many spikes/multiple modes though). This is expected as the productivity shock is log-normally distributed. However, when the adjustment cost becomes higher, we can see two peaks in the distribution of capital - one around 11 and the other around 12.5. This means that when the adjustment cost is higher, at the stationary distribution, the capital tends to be concentrated around some capital rather than spreading out. Just out of curiosity, I tried looking for the distribution when $\phi = 25$, nk = 1000, and nz = 25, and found that the distribution is concentrated around 11.75 (only one peak - the figure is not shown). So, when the adjustment cost is very high, the stationary distribution is very sensitive towards the number of discrete states (nz and nk).

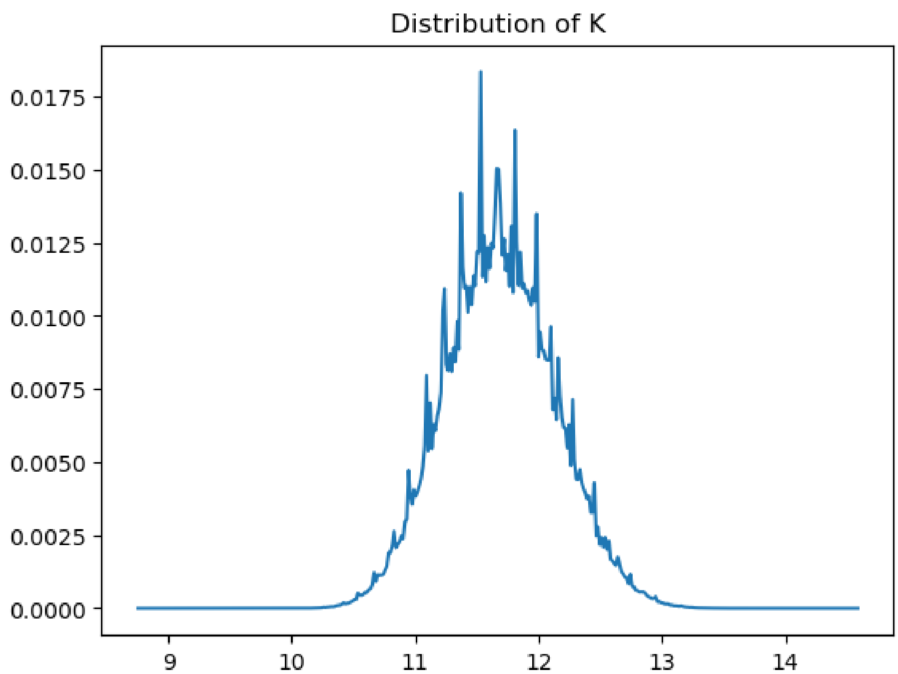

Figure 4: Distribution of Capital stock: $\phi = 0$, nz = 11, nk = 400

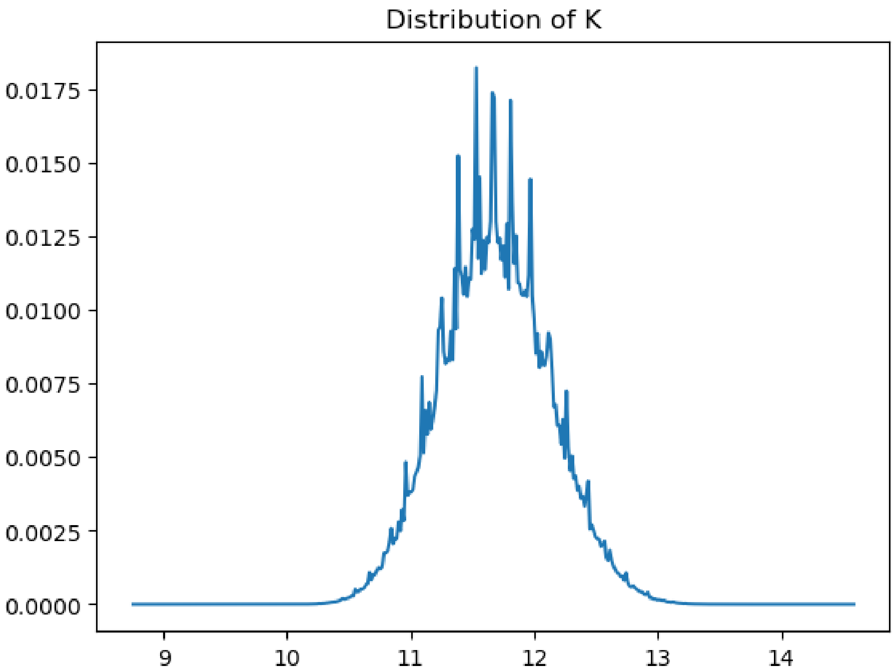

Figure 5: Distribution of Capital stock: $\phi = 0.025$, nz = 11, nk = 400

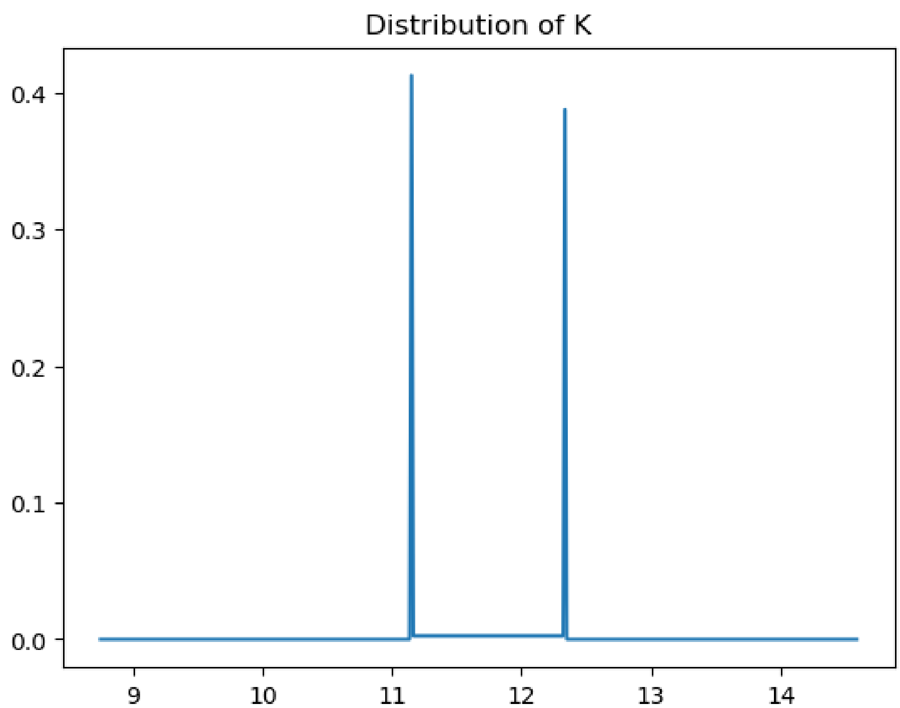

Figure 6: Distribution of Capital stock: ϕ = 25, nz = 11, nk = 400

3 Moments:

Table 1 - 2 shows the moments (correlation, autocorrelations, and relative standard deviation) of GDP, Consumption, Labor, and Investment for $\phi = 0$, $\phi = 0.025$, and $\phi = 25$.

We can see from Table 1 that the autocorrelation of GDP increases when the adjustment cost increases. The correlation between GDP and Consumption increases with the increase in adjustment cost, but the correlation between GDP and labor, and GDP and investment decreases with the increase in adjustment cost.

When the GDP increases, two effects come into play which affect the labor - the substitution effect and the income effect. Substitution effect: if the income (GDP) increases, you want to work more (increase labor); and income effect: if income (GDP) increases, you want to work less (decrease labor). When there is a very high adjustment cost, the income effect dominates i.e., a decrease in labor due to the income effect is higher than an increase in labor due to the substitution effect, compared to the case when there is no adjustment cost involved. So, when the adjustment cost is very high, the labor and GDP have a lower correlation compared to low/no adjustment cost.

Similarly, the correlation between GDP and investment decreases with the increase in adjustment cost. This is because when there is a very high adjustment cost involved, people want to maintain a consistent investment pattern (they don't want to invest to avoid higher adjustment costs). So, for the same level of output, the investment decreases when adjustment cost increases - thereby decreasing the correlation.

The correlation between GDP and Consumption increases with the increase in adjustment cost because, with higher adjustment costs, instead of investing more and incurring higher costs, they want to consume more.

Table 1: Correlations and Autocorreltaion

	$\phi = 0$	$\phi = 0.025$	$\phi = 25$
Autocorrelation (GDP)	0.917	0.918	0.948
Cor(GDP, Consumption)	0.797	0.812	0.927
Cor(GDP, labor)	0.81	0.81	0.095
Cor(GDP, Investment)	0.918	0.92	0.702

Table 2: Relative Standard Deviations (COV)

	$\phi = 0$	$\phi = 0.025$	$\phi = 25$
% std GDP	4.277	4.218	2.978
% std Consumption	2.62	2.603	3.093
% std Labor	1.871	1.799	0.806
% std Investment	11.569	11.224	4.695

Table 2 shows the relative standard deviations of GDP, Consumption, Labor, and Investment. When there is no adjustment cost, we can see that investment is about 3 times more volatile than GDP. This is expected because investment is the most volatile part of output and is forward-looking and depends heavily on the current economic situation, the expectations about the future, etc. Investment is more volatile than consumption too because individuals want to smooth out their consumption over time and if there is some change in economic conditions, investment reacts to these changes more rapidly than the current consumption.

However, if there is a higher adjustment cost, individuals do not want to change their investment as much and thus the investment volatility decreases. The volatility of investment comes from the capital distribution, and as we saw in the figures that the distribution of capital stock is more concentrated when there is a higher adjustment cost, it justifies less volatility in investment.