Advanced Engineering Mathematics - Solved Paper

Section A: Short Notes (15 x 1 = 15 Marks)

i. Cayley Hamilton Theorem: Every square matrix satisfies its own characteristic equation.

ii. Eigen values: Scalars λ such that for matrix A and vector $x \neq 0$, $Ax = \lambda x$.

iii. Real and imaginary parts of Log Z: If z = re^{iθ}, Log z = ln(r) + iθ.

iv. Expand sin z by Taylor's theorem: $\sin(z) = z - z^3/3! + z^5/5! - z^7/7! + ...$

v. Real and imaginary parts of sinh(x+iy): sinh(x+iy) = sinhx cos y + i coshx sin y.

vi. Evaluate cos π: cos π = -1.

vii. Ordinary differential equation: An equation involving derivatives of a function of one variable.

viii. e^iπ = -1: This is Euler's identity.

ix. Bessel's Differential Equation: $x^2y'' + xy' + (x^2 - n^2)y = 0$.

x. Exact Differential Equation: If Mdx + Ndy = 0 is exact, then $\partial M/\partial y = \partial N/\partial x$.

xi. Residue: Coefficient of 1/(z-a) in the Laurent series expansion around singularity a.

xii. Perfect number: A number equal to the sum of its proper divisors (e.g., 6, 28).

xiii. Poles: Points where a function goes to infinity.

xiv. Singularities: Points where a function is not analytic.

xv. sin(ix) = i sinh(x).

Section B: Solve Any Five (5 x 5 = 25 Marks)

Q1. Reduce to normal form and find rank of matrix A = [[1,1,2],[1,2,3],[0,-1,-1]]

Solution: Perform row operations to reduce to row echelon form. Final rank = 2.

Q2. Prove that Log(i) = 4m+1 / 4n+1, where m and n are integers.

Solution: Log(i) = iπ/2 + 2nπi; solve for rational form by assigning integer values.

Q3. Solve the differential equation (y cos x + 1)dx + sin x dy = 0

Solution: Convert into exact form and integrate. Answer involves integration constants.

Q4. Find the solution of $(D^3 - 6D^2 + 11D - 6)y = e^x$

Solution: Solve homogeneous and particular solution using operator method.

Q5. Expand log z using Taylor's series: $\log(1+z) = z - z^2/2 + z^3/3 - \ldots$ for $|z| < 1$

Section C: Attempt Any Two (15 x 2 = 30 Marks)

Q1. Solve the matrix equation system:

$x + y + z = 9$
$2x - 3y + 4z = 13$
$3x + 4y + 5z = 40$

Solution: Use Gaussian elimination or inverse matrix method. Final answer: $x = 2$, $y = 1$, $z = 6$

Q2. Find residue of $z^3 / ((z-1)^4(z-2)(z-3))$ at its poles.

Solution: Identify poles, compute residue using limit methods or Laurent series.

Q3. Prove $\tan^{-1}(b/a) = \frac{1}{2}\pi x + y \log 2$ for complex $(1+i)/(1-i)^x$.

Solution: Use polar form and argument analysis.

Q4. Solve differential equation:

$(2x-1)^3\, d^3y/dx^3 + (2x-1)\, dy/dx - 2y = \log(2x-1)$

Solution: Use variable substitution and operator method. Solution involves complementary function and particular integral.