# REAL NUMBERS

## Important formulas

**Theorem 1.1 (Euclid's Division Lemma):** Given positive integers *a* and *b*, there exist unique integers *q* and *r* satisfying

$$a = bq + r, \quad 0 \le r < b.$$

**Euclid's Division Algorithm** is based on Euclid's Division Lemma and this algorithm is a technique to compute the Highest Common Factor (**HCF**) of two given positive integers.
To obtain the HCF of two positive integers, *a* and *b*, with *a* > *b*, follow the steps below:

**Step 1:** Apply Euclid's division lemma, to *a* and *b*. So, we find whole numbers, *q* and *r* such that $a = bq + r, \; 0 \le r < b$

**Step 2:** If $r = 0$, *b* is the HCF of *a* and *b*.
If $r \neq 0$, apply the division lemma to *b* and *r*.

**Step 3:** Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

**Theorem 1.2 (Fundamental Theorem of Arithmetic):** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

1. HCF = Product of the smallest power of each common prime factor in the numbers.
2. LCM = Product of the greatest power of each prime factor, involved in the numbers.
3. For any two positive integers *a* and *b*,

$$HCF(a,b) \times LCM(a,b) = a \times b$$

4. Product of three numbers is not equal to the product of their HCF and LCM.

5. For three positive integers *p*, *q* and *r*

$$LCM(p,q,r) = \frac{pqr \times HCF(p,q,r)}{HCF(p,q) \times HCF(q,r) \times HCF(p,r)}$$

$$HCF(p,q,r) = \frac{pqr \times LCM(p,q,r)}{LCM(p,q) \times LCM(q,r) \times LCM(p,r)}$$

**Theorem 1.3**: Let *p* be a prime number. If *p* divides $a^2$ then *p* divides *a*, where *a* is a positive integer.

**Theorem 1.5**: Let *x* be a rational number whose decimal expansion terminates.
Then *x* can be expressed in the form, $\frac{p}{q}$ where *p* and *q* are co-prime, and the prime factorisation of *q* is of the form $2^n 5^m$ , where *n*, *m* are non-negative integers.

**Theorem 1.6** Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of *q* is of the form $2^n 5^m$, where *n*, *m* are non-negative integers.
Then *x* has a decimal expansion which terminates.

**Theorem 1.7**: Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of *q* is not of the form $2^n 5^m$, where *n*, *m* are non-negative integers.

Then, *x* has a decimal expansion which is non-terminating repeating (recurring).

# POLYNOMIALS

## Important formulas

**1.** Polynomials:

Linear polynomial: $ax+b$
Quadratic polynomial: $ax^2+bx+c$
Cubic polynomial: $ax^3+bx^2+cx+d$

*where a, b, c and d are real numbers and* $a \neq 0$

**2.** The zero of the linear polynomial, $ax+b$ is $\frac{-b}{a} = \frac{-(\text{Constant term})}{\text{Coefficient of } x}$

**3.** If α and β are the zeroes of the quadratic polynomial

$$p(x) = ax^2+bx+c$$

Then,

Sum of the zeros $= \alpha + \beta = \frac{-b}{a} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of the zeros $= \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

**4.** If α, β, γ are the zeroes of the cubic polynomial

$$ax^3 + bx^2 + cx + d$$

Then,

Sum of the zeros $= \alpha + \beta + \gamma = \dfrac{-b}{a} = \dfrac{-\left(\text{Coefficient of } x^2\right)}{\text{Coefficient of } x^3}$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

Product of the zeros $= \alpha\beta\gamma = \dfrac{-d}{a} = \dfrac{-\left(\text{Constant term}\right)}{\text{Coefficient of } x^3}$

**5.** The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x) \times q(x) + r(x)$$

$$\textit{where} \quad r(x) = 0 \quad \textit{or} \quad \text{degree } r(x) < \text{degree } g(x)$$

# PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

## Important formulas

**1.** Linear equation in two variables is given by,

$$ax + by + c = 0$$

*where a, b and c are real numbers and* $a \neq 0, b \neq 0$

**2.** The general form for a pair of linear equations in two variables *x* and *y* is given by,

$a_1x + b_1y + c_1 = 0$ and
$a_2x + b_2y + c_2 = 0$

**3.** If the lines represented by the pair of linear equations,

$a_1x + b_1y + c_1 = 0$ and
$a_2x + b_2y + c_2 = 0$ are

(i) Intersecting, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ and we get a unique solution

(ii) Coincident, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and we get infinitely many solutions.

(iii) Parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and there is no solution.

**4.** Solution of a pair of linear equations can be found by,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

**5.** Cross multiplication method of solving a pair of linear equations

$a_1x + b_1y + c_1 = 0$ and
$a_2x + b_2y + c_2 = 0$ is given by

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

*where* $a_1b_2 - a_2b_1 \neq 0$

# QUADRATIC EQUATIONS

## Important formulas and statements

**1.** The standard form of a quadratic equation is,

$$ax^2 + bx + c = 0$$

*where a, b and c are real numbers and* $a \neq 0$

**2.** Then the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

known as **Quadratic Formula**.

Since $b^2 - 4ac$ determines whether the quadratic equation $ax^2 + bx + c = 0$ has real roots or not, $b^2 - 4ac$ is called the **discriminant** of this quadratic equation and denoted by D. i.e., $D = b^2 - 4ac$.

**3.** A quadratic equation $ax^2 + bx + c = 0$ has

(i) two distinct real roots, if $b^2 - 4ac > 0$

(ii) two equal real roots, if $b^2 - 4ac = 0$

(iii) no real roots, if $b^2 - 4ac < 0$

# ARITHMETIC PROGRESSIONS

## Important formulas and statements

**1.** The general form of an AP is:

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots$$

*where a is the first term and d is the common difference*

**2.** $d = a_{k+1} - a_k$

*where* $a_{k+1}$ *and* $a_k$ *are the* $(k+1)th$ *and the* $kth$ *terms respectively.*

**3.** The *n*th term $a_n$ of the AP with first term *a* and common difference *d* is given by

$$a_n = a + (n-1)d$$

$a_n$ *is also called general term of an AP*

**4.** The sum of the first *n* terms of an AP is given by

$$\mathbf{S} = \frac{n}{2}\left[2a + (n-1)d\right]$$

or $\mathbf{S} = \frac{n}{2}(a + a_n)$

or $\mathbf{S} = \frac{n}{2}(a + l)$ $\quad [a_n = l]$

*where a is the first term, d is the common difference, n is the number of terms and l is the last term.*

**5.** The sum of first *n* positive integers is given by

$$S_n = \frac{n(n+1)}{2}$$

**6.** If *a*, *b* and *c* are in AP, then $b = \frac{a+c}{2}$ and *b* is called the arithmetic mean of *a* and *c*.

# TRIANGLES

## Important formulas & thesis

**1.** All congruent figures are similar, but the similar figures need not be congruent.

**2.** Two polygons of the same number of sides are similar, if
(i) their corresponding angles are equal and
(ii) their corresponding sides are in the same ratio (or proportion).

**3.** Two triangles are similar, if
(i) their corresponding angles are equal and
(ii) their corresponding sides are in the same ratio (or proportion).

**4.** The ratio of any two corresponding sides in two equiangular triangles is always the same.

**Theorem 6.1**: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

**Theorem 6.2**: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

In $\Delta ABC$ and $\Delta DEF$, if

(*i*) $\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$ and

(*ii*) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

then two triangles are similar

**Theorem 6.3**: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two
triangles are similar. (AAA similarity criterion)

**Theorem 6.4**: If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. (SSS similarity criterion)

**Theorem 6.5**: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. (SAS similarity criterion)

**Theorem 6.6**: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

**Theorem 6.7**: If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

**Theorem 6.8**: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras theorem).

**Theorem 6.9**: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

# TRIGONOMETRY

## Important formulas

**The trigonometric ratios of the $\angle A$ in right triangle ABC.**

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}}$$
$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$$
$$\tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A}$$
$$\cot A = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A}$$
$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A}$$
$$\operatorname{cosec} A = \frac{\text{hypotenuse}}{\text{side opposite to } \angle A}$$

**Note:** The ratios cosec A, sec A and cot A are respectively, the reciprocals of the ratios sin A, cos A and tan A.

$$\Rightarrow \sin A = \frac{1}{\operatorname{cosec} A} \qquad \Rightarrow \operatorname{cosec} A = \frac{1}{\sin A}$$
$$\Rightarrow \cos A = \frac{1}{\sec A} \qquad \Rightarrow \sec A = \frac{1}{\cos A}$$
$$\Rightarrow \tan A = \frac{1}{\cot A} \qquad \Rightarrow \cot A = \frac{1}{\tan A}$$

We can also observe that:

$$\Rightarrow \tan A = \frac{\sin A}{\cos A} \qquad \Rightarrow \cot A = \frac{\cos A}{\sin A}$$

## Trigonometric Ratios of Some Specific Angles:

The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90° are given in below table:

| **∠ A** | **0°** | **30°** | **45°** | **60°** | **90°** |
|---|---|---|---|---|---|
| **sin A** | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| **cos A** | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| **tan A** | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not defined |
| **cosec A** | Not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| **sec A** | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not defined |
| **cot A** | Not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

## Trigonometric Ratios of Complementary Angles:

$$\sin\left(90^\circ - A\right) = \cos A \qquad \cos\left(90^\circ - A\right) = \sin A$$
$$\tan\left(90^\circ - A\right) = \cot A \qquad \cot\left(90^\circ - A\right) = \tan A$$
$$\sec\left(90^\circ - A\right) = \operatorname{cosec} A \qquad \operatorname{cosec}\left(90^\circ - A\right) = \sec A$$

## Trigonometric Identities:

$$\sin^2 A + \cos^2 A = 1 \qquad \text{for } 0^\circ \le A \le 90^\circ$$
$$\sec^2 A - \tan^2 A = 1 \qquad \text{for } 0^\circ \le A < 90^\circ$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A \qquad \text{for } 0^\circ < A \le 90^\circ$$

# SOME APPLICATIONS OF TRIGONOMETRY

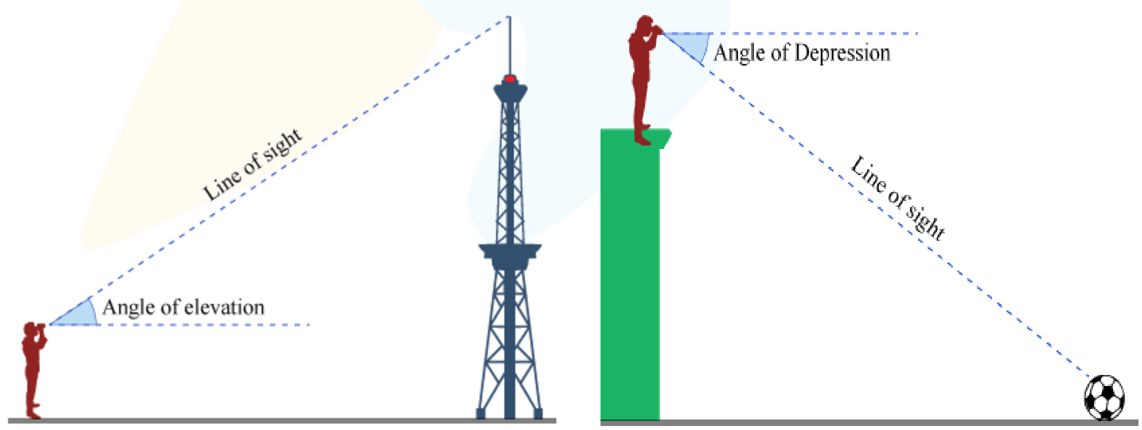
## Important formulas

**Heights and Distances:**
The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

**Line of Sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

**Angle of Elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object.

**Angle of Depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.



# COORDINATE GEOMETRY

## Important formulas and statements

**1.** The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is,

$$\mathbf{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ , known as distance formula.

**2.** The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$ known as section formula.

**3.** The mid-point of a line segment divides the line segment in the ratio 1:1. Therefore, the coordinates of the mid-point P of the join of the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**4.** ABC be any triangle whose vertices are $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$, the area of $\Delta ABC$ is the numerical value of the expression,

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

# CIRLCES

## Important formulas and statements

**Tangent to a circle:**

**Theorem 10.1:** The tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Theorem 10.2:** The lengths of tangents drawn from an external point to a circle are equal.

# AREAS RELATED TO CIRLCES

*r is the radius and* $\pi = \frac{22}{7}$ *or* $3.14$

1. Circumference of a circle,
   $C = 2\pi r$

2. Area of a circle,
   $A = \pi r^2$

3. Circumference of a semicircle,
   $C = r(2+\pi)$

4. Area of a semicircle,
   $A = \frac{1}{2}\pi r^2$

5. Area of the sector of angle θ,
   $A = \frac{\theta}{360} \times (\pi r^2)$

6. Length of an arc of a sector of angle θ,
   $l = \frac{\theta}{360} \times (2\pi r)$

7. Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle.

# SURFACE AREAS & VOLUMES

## Important formulas

Terms to remember: -

- CSA = Curved Surface Area
- LSA = Lateral Surface Area
- TSA = Total Surface Area

### 1. Cube

(a) Volume = $a^3$, *where a is the side of the cube*

(b) Surface Area = $6a^2$

(c) Lateral Surface Area = $4a^2$

### 2. Cuboid

(a) Volume = $l \times b \times h$, *where l, b & h are the length, breadth and height of the cuboid respectively.*

(b) Surface Area = $2(lb + bh + hl)$

(c) Lateral Surface Area = $2h(l + b)$

### 3. Cone

(a) Volume = $\frac{1}{3}(\pi r^2 h)$, *where r is the radius of circular base and h is height of the cone*

(b) Total Surface Area = $\pi r(l + r)$, *where l is the slant height of the cone*

(c) Curved Surface Area = $\pi rl$

(d) Slant height of cone, $l = \sqrt{r^2 + h^2}$

### 4. Frustum of a cone

(a) Volume = $\frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$, *where h is the height, $r_1$ and $r_2$ are the radii of the frustum of the cone & $r_1 > r_2$*

(b) Total Surface Area = $\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$, *where l is the slant height*

(c) Curved Surface Area = $\pi l(r_1 + r_2)$

(d) Slant height of frustum of a cone, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

5. Sphere

(a) Volume = $\frac{4}{3}\pi r^3$, *where r is the radius of the sphere'*

(b) Surface Area = $4\pi r^2$

6. Hemisphere

(a) Volume = $\frac{2}{3}\pi r^3$, *where r is the radius of the hemisphere*

(b) Total Surface Area = $3\pi r^2$

(c) Curved Surface Area = $2\pi r^2$

7. Cylinder

(a) Volume = $\pi r^2 h$, *where r is the radius of circular base and h is the height of the cylinder*

(b) Total Surface Area = $2\pi r\left(r+h\right)$

(c) Curved Surface Area = $2\pi rh$

**Surface Areas & Volumes of combinations of two or more solids –**

Examples:

*(i)* An ice-cream filled cone constitutes a cone and a hemisphere-shaped ice-cream.

Hence, Total surface area of the Ice-cream cone = Curved Surface Area of Hemisphere + Curved Surface Area of the Cone i.e. ($2\pi r^2 + \pi rl$)

*(ii)* The capsule-shaped container loaded on a truck is a combination of a cylinder with adjoined hemispheres on both sides.

Hence, The volume of a capsule container = Volume of hemisphere + Volume of Cylinder + Volume of Hemisphere i.e. ($\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$)

# STATISTICS

## Important formulas

**1.** Mean data is given by,

(a) Direct Method => $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$, *where the Greek letter Σ (capital sigma) means summation and* $x_i$ *and* $f_i$ *are the observations and frequencies respectively.*

(b) Assumed Mean Method => $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, *where 'a' is the assumed mean and 'd' is the deviation i.e.* $d_i = (x_i - a)$

(c) Step Deviation Method => $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$, *where* $u_i = \frac{x_i - a}{h}$ *and 'h' is the size of the class interval*

**2.** Mid-point or class mark of a class is given by,

Mid-point/Class Mark $= \dfrac{\text{Upper Class Limit} + \text{Lower Class Limit}}{2}$

**3.** The mode is a value inside the modal class (class with the maximum frequency), and is given by the formula,

Mode $= l + h\left(\frac{f_1 - f_o}{2f_1 - f_o - f_2}\right)$, *where,*

*$l$ = lower limit of the modal class*
*$h$ = size of the class interval*
*$f_1$ = frequency of modal class*
*$f_0$ = frequency of class preceding the modal class and*
*$f_2$ = frequency of class succeeding the modal class*

**4.** Median

(a) If $n$ is odd, then median is the $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation

(b) If $n$ is even, then median is the average of $\left(\frac{n}{2}\right)^{\text{th}}$ & $\left(\frac{n}{2}+1\right)^{\text{th}}$ observations.

(c) Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$, *where*

*l = lower limit of median class*
*h = size of class interval*
*cf = cumulative frequency of class preceding the median class*
*n = number of observations and*
*f = frequency of the median class*

**5.** Empirical relationship between the three measures of central tendency is as follows:

$$Mode = 3 \times Median - 2 \times Mean$$

# PROBABILITY

## Important formulas and statements

1. The probability ***P(E)*** of an event ***E*** happening, is given by

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

2. The probability of an event ***E*** is a number ***P(E)*** such that, $0 \le P(E) \le 1$

3. The probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.

4. The probability of an event which is sure or certain to occur is 1. Such an event is called a **sure** or **certain event**.

5. An event having only one outcome is called an **elementary event**. The sum of the probabilities of all the elementary events of an experiment is 1.

6. For any event ***E***, $P(E) + P(\overline{E}) = 1$, where $\overline{E}$ stands for '**not *E***'. $E$ and $\overline{E}$ are called **complementary events**.