Nondeterminism

GUESS and VERIFY

Example:
$\{w \mid \text{the second bit from last is } 1\}$

On input $w = w_1 w_2 \cdots w_n$, an NFA does:

Repeat until end of input:

Current bit is 1: GUESS if it is the second last bit.

guess YES: read the next bit and accept.

guess NO:

Non-deterministic Finite Automata

$N = (Q, \Sigma, \delta, q_0, F)$:

Q is a finite set (states)

Σ is a finite set (alphabet)

$\delta : Q \times (\Sigma \cup \epsilon) \rightarrow \mathcal{P}(Q)$ (transition function)
(That is, $\delta(q, a)$ is a subset of Q.)

q_0: distinguished state (start state)

$F \subseteq Q$ (accepting/final states)

DFA is a special case of an NFA

A DFA is an NFA with:

- no ϵ transitions, and
- for all $a \in \Sigma$, $q \in Q$, $|\delta(q, a)| = 1$.

Language of a non-deterministic finite automaton

A string $w = w_1 w_2 \cdots w_n$, where each $w_i \in \Sigma$, is accepted by N if w can be written as $w = y_1 y_2 \cdots y_m$, where each $y_i \in (\Sigma \cup \{\epsilon\})$, and there exists a sequence of states $r_0, r_1, \cdots, r_m$ such that:

- (start right) $r_0 = q_0$,
- (move right) for all $0 \leq i \leq m - 1$, $r_{i+1} \in \delta(r_i, y_{i+1})$, and
- (finish right) $r_m \in F$.

The NFA N *recognizes* language A if $A = \{w \mid N \text{ accepts } w\}$. We denote the language of N as $L(N)$.

Computation tree of a NFA on an input

Computation tree of an NFA M on input w:

- nodes labeled with states
- children of a node labeled with a state q are labeled with states that follow from q on the next input symbol
- root labeled with initial state
- leaves labeled with states with no next possible state
- accept leaves and reject leaves

The computation tree of a DFA on an input is a path.

M accepts input w if and only if there exists a computation tree with at least one accepting path.