CS 4510

Spring 2015

Test 1 Practice problems

Note:

- Please write neat and legible answers.
- You can use any of the theorems/facts/lemmas that we covered in *class* without re-proving them unless explicitly stated otherwise. You can also cite homework problems from this course.
- Please state any assumptions you make.

1. Let A and B be two languages over a finite alphabet Σ. If $A \cup B$ is regular, does it follow that A and B are both regular? If so, give a proof. If not, give a counter-example.

   Solution: False. Let A be a non-regular language and B be Σ^*. Their union is Σ^* which is regular.

2. Let N be a nondeterministic finite automaton with p states. Then, is it true that any deterministic finite automaton equivalent to N must have *more* than p states? Justify your answer.

   Solution: FALSE. Any DFA is an NFA.

3. Show by giving an example that, if N is a nondeterministic finite automaton that recognizes language L, swapping the accept and nonaccept states in N does not necessarily yield a new nondeterministic finite automaton that recognizes the complement of L.

4. Show that the language $L = \{xy \mid x, y \in \{0,1\}^*, \#0(x) = \#1(y)\}$ is regular. (Here $\#0(x)$ denotes the number of $0's$ in the string x, and $\#1(y)$ denotes the number of $1's$ in the string y.)

   Solution: L is the same as $\{0,1\}^*$.

   Why?

   In one direction, $L \subseteq \{0,1\}^*$.

   For the other direction, let $w \in \{0,1\}^*$ be an arbitrary string. Let $w = w_1 w_2 \cdots w_n$, where $w_i \in \{0,1\}$ for all $1 \leq i \leq n$.

   Let $k = \#1(w)$. Consider $x = w_1 w_2 \cdots w_k$ and $y = w_{k+1} w_{k+2} \cdots w_n$.

   Then, $\#1(y) + \#1(x) = \#1(w) = k$.

   This implies that $\#1(y) = k - \#1(x) = |x| - \#1(x) = \#0(x)$.

   Therefore, $w \in L$.

5. Let $M_1 = (Q_1, \{0,1\}, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \{0,1\}, \delta_2, q_2, F_2)$ be two deterministic finite automata. Let $M = (Q, \{0,1\}, \delta, q_0, F)$ be a nondeterministic finite automata defined as follows:

   - $Q = Q_1 \times Q_2$
   - For all $p \in Q_1$, $q \in Q_2$:
     - $\delta(\langle p, q\rangle, 0) = \{\langle \delta_1(p,0), \delta_2(q,0)\rangle, \langle \delta_1(p,1), \delta_2(q,1)\rangle\}$

- $\delta(\langle p, q\rangle, 1) = \{\langle \delta_1(p, 0), \delta_2(q, 1)\rangle, \langle \delta_1(p, 1), \delta_2(q, 0)\rangle\}$

- $q_0 = \langle q_1, q_2\rangle$
- $F = F_1 \times F_2$

What is the language recognized by M in terms of $L(M_1)$ and $L(M_2)$?

Solution: $L(M) = \{x \oplus y \mid x \in L(M_1), y \in L(M_2), |x| = |y|\}$. Here $x \vee y$ is the EXCLUSIVE-OR of the two bit strings x and y. If the input symbol is a 0 then the two bits whose EXCLUSIVE-OR is 0 can be either $(0, 0)$ or $(1, 1)$. If the input symbol is a 1 then the two bits whose EXCLUSIVE-OR is 1 can be either $(0, 1)$ or $(1, 0)$.

6. Show that the following language is not regular:

$$L = \{x \mid x \in \{0, 1\}^* \text{ and if } x \text{ has odd length its middle symbol is } 0\}.$$

Solution: Intersect L with the set of all odd length strings over the alphabet. The resulting set is:

$$L' = \{x \mid x \in \{0, 1\}^* \text{ has odd length and its middle symbol is } 0\}.$$

Show that L' is not regular. Let p be the pumping lemma constant. Choose $s = 1^p01^p$. Choose $i = 0$.

7. Let A be a regular language. Show that the following language is also regular:

$$\{xy \mid \ x1y \in A\}.$$

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA for A. An NFA $M' = (Q', \Sigma, \delta', {q_0}', F')$ on input w guesses a partition of w as xy and : (a) simulates M on input x, (b) simulates M on input 1, and (c) simulates M on y.

- $Q' = Q \times \{0, 1\}$
- ${q_0}' = \langle q_0, 0\rangle$
- Transitions:
  - For all $q \in Q, a \in \Sigma$, $\delta'(\langle q, 0\rangle, a) = \langle \delta(q, a), 0\rangle$.
  - For all $q \in Q$, $\delta'(\langle q, 0\rangle, \epsilon) = \langle \delta(q, 1), 1\rangle$.
  - For all $q \in Q, a \in \Sigma$, $\delta'(\langle q, 1\rangle, a) = \langle \delta(q, a), 1\rangle$.
- $F' = \{\langle q, 1\rangle \mid q \in F\}$.

8. Show that the following languages are not regular using the pumping lemma for regular languages:

   (a) $\{a^ib^jc^{2j} \mid i, j \geq 0\}$.
   Solution: Choose the string b^pc^{2p} in L (where p is the pumping lemma constant). Choose $i = 0$.

   (b) $\{a^{2^n} \mid n \geq 0\}$. (Here, a^{2^n} means a string of 2^n a's.)
   Solution: Choose the string a^{2^p} where p is the pumping lemma constant. Choose $i = 2$.