CS 4510: Automata and Complexity

Spring 2015

Home work 5 // Due: Friday, April 3, 2015

1. (a) (8 points) Show that $L(G)$, where G is a context-free grammar, is infinite iff G generates a string whose length is at least p, where p is the pumping length identified in the proof of the pumping lemma.

   (b) (7 points) Let $INFINITE_{\text{PDA}} = \{M \mid M \text{ is a PDA and } L(M) \text{ is an infinite language}\}$. Show that $INFINITE_{\text{PDA}}$ is decidable.

   Solution:

   - Convert M to a context-free grammar G.
   - Compute the pumping length p as in the proof of the pumping lemma.
   - Construct a DFA D that accepts all strings of length at least p. (The DFA D, for instance, can have $p+1$ states.)
   - Construct a PDA M' that accepts $L(M) \cap L(D)$.
   - Convert M' to a CFG G'.
   - Run the decider for $EMPTY_CFG$ on G'.
   - Accept iff this decider rejects.

2. (15 points) A *Turing machine with stay put instead of left* is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{R, S\}.$$

   At each point the machine can move its head right or stay in the same position. Show that this Turing machine variation is *not* equivalent to the usual version. What class of languages do these machines recognize? Justify your answer.

   Solution: These restricted machines recognize exactly the class of regular languages.

   In one direction, a DFA can be simulated by such a machine that uses only the right move and that writes the same symbol that it reads.

   In the other direction, simulate such a machine M by a nondeterministic finite state automaton (NFA) N. Note that an NFA cannot write any symbol on the tape. Let p be the current state and a be the current input symbol.

   If M moves right going to state p' then N also moves right. Note that the symbol that is written by M need not be remembered since M will never access that position of the tape again.

   If M stays put going to state p'' writing a symbol b on the tape then N moves to state $q_{p'',b}$. If from state p'' reading the symbol b, the machine M makes a move, then N simulates it using an ϵ-move from state $q_{p'',b}$. The result of this move, as described above, depends on whether M moves right or stays put.

3. (15 points) Show that the complement $\bar{E_{\text{TM}}}$ of E_{TM} is Turing-recognizable. Explain why from this we can conclude that E_{TM} is not Turing-recognizable.

   Solution: Let $\langle M \rangle$ be the input TM. Proceed in stages. At the start of the i-th stage, the tape has the first $i-1$ strings in lexicographic order. Also, it holds that M has not accepted any of the first $i-1$ strings in $i-1$ steps. During the i-th stage, generate the next (that is the i-th) string in order and run M on all the i strings for i steps. If M accepts any of the strings then accept. Otherwise proceed to the $i+1$-th stage.

   E_{TM} is not recognizable: otherwise, E_{TM} would be decidable which is not true.

4. (15 points) On input M, w (where M is the code of a Turing machine and w is an input to M), a reduction machine constructs the Turing machine M' described below:

   On input x:

   (a) Simulate M on w for $|x|$ steps (that is, it erases one symbol of x for each step of M on w that it simulates).

   (b) Accept if M has *not* halted within that time, reject otherwise.

What is $L(M')$? Why?

Solution: If M does not halt on w, M' accepts all strings and so, $L(M') = \Sigma^*$.

If M halts on w, it does so after some m steps. M' accepts all strings x whose length is less than m. Since m is fixed for a given M, w, we conclude that $L(M')$ is finite.

5. Let $FINITE3_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a } 1-\text{tape DTM and } L(M) \text{ has } \textit{exactly} \text{ three strings}\}$.

   (15 points) Show that $A_{\text{TM}} \leq_m FINITE3_{\text{TM}}$.

   Solution: On input $\langle M, w \rangle$ construct a TM M' as follows:

   On input x,

   - If $x \neq$ 0 or 00 or 000 then reject.
   - If $x =$ 0 or 00 then accept.
   - If $x =$ 000 then accept if and only if M accepts w.

   Then, the language of M' is $\{0, 00, 000\}$ if M accepts w and is $\{0, 00\}$ if M does not accept w.