CS 4510: Automata and Complexity

Spring 2015

Home work 3 // Due: Friday, February 20, 2015

1. Consider the DFA $M = (Q, \Sigma, \delta, p_0, F)$ where

$$
\begin{aligned}
Q &= \{p_0, p_1, p_2, p_3, p_4, p_5\} \\
\Sigma &= \{0, 1\} \\
F &= \{p_0, p_1\})
\end{aligned}
$$

δ is defined below:

$$
\begin{aligned}
\delta(p_0, 0) &= p_1 \\
\delta(p_0, 1) &= p_2 \\
\delta(p_1, 0) &= p_1 \\
\delta(p_1, 1) &= p_3 \\
\delta(p_2, 0) &= p_3 \\
\delta(p_2, 1) &= p_5 \\
\delta(p_3, 0) &= p_2 \\
\delta(p_3, 1) &= p_4 \\
\delta(p_4, 0) &= p_4 \\
\delta(p_4, 1) &= p_1 \\
\delta(p_5, 0) &= p_5 \\
\delta(p_5, 1) &= p_0
\end{aligned}
$$

Minimize this DFA using the table-construction algorithm described in class (this algorithm is in the class notes). Let T be a table with 6 rows and 6 columns indexed by the states of the DFA.

(a) (3 points) Initially, what are the entries of the table T?

(b) (7 points) Until no more entries are updated, show the entries with value k that are updated to $k + 1$, for each $k \geq 0$.

(c) (3 points) What is the resulting DFA?

(d) (2 points) What is the language accepted by this DFA?

2. (5 points) What is the language generated by the CFG below. Give a brief justification for your claim.

$$
\begin{aligned}
S &\rightarrow 0S1 \mid 1A \mid A0 \\
A &\rightarrow 1A \mid 0A \mid \epsilon
\end{aligned}
$$

Solution: This grammar generates all strings that are not of the form 0^k1^k for $k \geq 0$. All the A-rules together generates all strings over $\{0,1\}^*$. Using the second rule for S the grammar generates all strings that start with a 1. Similarly, using the third rule for S, the grammar generates all string that end with a 0. The first rule ensures that any string x that starts with a 0 and ends with a 1 can be writted in the form $x = 0y1$ where y is not of the form 0^k1^k inductively.

3. (10 points) Write a context-free grammar for the following language:

$$\{a^m b^n c^p d^q \mid m + n = p + q\}.$$

Briefly justify your constrcution.

Solution:

$$\begin{array}{rcl} S & \rightarrow & aSd|\ A \mid B \\ A & \rightarrow & aAc \mid C \\ B & \rightarrow & bBd \mid C \\ C & \rightarrow & bCc \mid \epsilon \end{array}$$

Generate some number of a's with the same number of d's using the first S-rule. Then, guess if $m \geq q$ or $m \leq q$ using the S-rules 2 and 3 respectively.

4. Let w^r denote the reversal of a string w. Thus, if $w = 011$ its reversal $w^r = 110$.

   Consider the language:
   $$l = \{ww^R x \mid w \in \{0,1\}^+, x \in \{0,1\}^*\}.$$

   (a) (10 points) Prove that L is not regular using Myhill-Nerode theorem.

   Solution: Argue that for $m < n$ the string $(01)^m$ is not equivalent to the string $(01)^n$ under $\equiv_L$. (The string $(10)^m$ concatenated to $(01)^m$ is in L and concatenated to the string $(10)^n$ is not in L.) That is, the equivalence relation $\equiv_L$ has infinite index. Thus, by Myhill-Nerode theorem L is not regular.

   (b) (10 points) Give a contex-free grammar that generates L. Briefly justify why your grammar generates L.

   Solution:

   $$\begin{array}{rcl} S & \rightarrow & AB \mid A \\ A & \rightarrow & 0A0 \mid 1A1 \mid 00 \mid 11 \\ B & \rightarrow & 0B \mid 1B \mid 0 \mid 1 \end{array}$$

   The variable A generates all even length palindromes and the variable B generates all strings over $\{0,1\}^*$.

5. (10 points) Construct a PDA for the following language:

   $$\{w \in \{0,1\}^* \mid \#_0(w) \leq \#_1(w) \leq 2\#_0(w)\}.$$

6. (10 points) Show that, if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

   Solution: Let $G = (V, \Sigma, R, S)$.

   Let $w \in L(G), w \neq \epsilon$. with $|w| = n$.

   $n = 1$: There has to be a rule $S \rightarrow w$.

   $n \geq 2$: Any derivation of w will have a first step as $S \rightarrow AB$, for some $A, B \in V$.

   Then there will be exactly $n-2$ steps of the form $X \rightarrow YZ$, for some $X, Y, Z \in V - \{S\}$.

   And exactly n steps of the form $W \rightarrow a$, for some $W \in V$ and $a \in \Sigma$.

   This is because of the fact that G is written in Chmosky normal form.