Nondeterminism

GUESS and **VERIFY**

Nondeterministic Turing Machines

$M = (Q, \Sigma, \Gamma, \delta, q_0, q_a, q_r)$, where

$$\delta : Q \times \Gamma \longrightarrow \mathcal{P}(Q \times \Gamma \times \{L, R, S\})$$

Every DTM is an NTM

Nondeterministic Turing Machines

Input $w = w_1 w_2 \cdots w_n$

a configuration on input w: current state, current head location, tape contents

current configuration: $uaqbv$, $a, b \in \Gamma$

Example: $\delta(q, b) = \{(p, c, L), (r, d, R), (s, e, R)\}$

three possible next configurations: $upacv$, $uadrv$, and $uaesv$

normalize: if not a halting configuration, exactly two possible next configurations

$\delta(p, a) = \{(q, b, L)\}$ changed to $\delta(p, a) = \{(q, b, L), (q, b, L\}$

$\delta(p, a) = \{(q_1, b, L), (q_2, c, L), (q_3, d, L), (q_4, e, L)\}$ changed to

$$
\begin{aligned}
\delta(p, a) &= \{(p', a, S), (q_1, b, L)\} \\
\delta(p', a) &= \{(p'', a, S), (q_2, c, L)\} \\
\delta(p'', a) &= \{(q_3, d, L), (q_4, e, L)\}
\end{aligned}
$$

Nondeterministic Turing Machines

An NTM M accepts input w iff $\exists$ a sequence of configurations $C_0, \cdots, C_t$ such that:

1. C_0 is the initial configuration of M on input w
2. For all $1 \leq i \leq t$, the machine M moves from C_{i-1} to C_i
3. C_t is an accepting configuration

Nondeterministic Turing Machines

Computation tree of an NTM M on input w:

- nodes labeled with configurations
- children of a node labeled C are labeled with configurations that follow from C
- root labeled with initial configuration
- leaves labeled with configurations with no next possible configuration
- accept leaves and reject leaves

The computation tree of a normalized NTM on an input is a full binary tree.

The computation tree of a DTM on an input is a path.

M accepts input w iff there exists a computation tree with at least one accepting path.