Simulating an NTM by a DTM

On input w, a DTM traverses the computation tree of the NTM on w.

Breadth-first search - level by level traversal.

N be a 1-tape NTM.

 δ_N : its transition function.

Assume: two or zero transitions from each configuration.

Order the two transitions as the first and the second.

On input w, a 3-tape DTM D simulates N as described below.

Tape 1 of D: the input w.

Tape 2 of D: contents of N's tape .

Tape 3 of D: a bit string that describes a path of N's computation tree.

- 1. Tape $3 \leftarrow \epsilon$;
- 2. Set ToNextLevel to FALSE;
- 3. Tape 2 \leftarrow Tape 1; Tape 3 \leftarrow next string; Let the length of the string on Tape 3 be k.
- 4. Simulate N on w for k steps guided by the bit string on Tape 3:

(If current Tape 3 bit is 0 (1), use the first (second) immediate successor of the current transition.)

Current move results in an accepting state: HALT and ACCEPT;

No move possible or current move results in a rejecting state: go to step 5;

Current move results in a non-halting state: ToNextLevel ← TRUE; go to step 5;

5. IF (this is the last path of length k) Tape 3 contains all 1's THEN

IF ToNextLevel is TRUE THEN go to step 2;

ELSE HALT and REJECT;

6. ELSE go to step 3;

Nondeterministic Turing Machines

Theorem: A language L is recognizable iff there is an NTM M such that L = L(M).

A decider NTM halts on all branches of computations on all inputs.

Theorem: A language L is decidable iff there is a decider NTM M such that L = L(M).

Simulating an NTM by a Guess-Verify NTM

N be a 1-tape NTM.

 δ_N : its transition function.

Assume:

Two or zero transitions from each configuration.

Every computation path has same length t.

Order the two transitions as the first and the second.

On input w, an NTM V simulates N as described below.

- 1. GUESS a bit vector of length t and write on Tape 2.
- 2. Simulate N on w for t steps guided by the bit string on Tape 2:

(If current Tape 2 bit is 0 (1), use the first (second) immediate successor of the current transition.)

Current move results in an accepting state: HALT and ACCEPT;

No move possible or current move results in a non-accepting state: HALT and REJECT.