CS 4510: Automata and Complexity

Spring 2015

Extra Home work // Due: Friday, March 27, 2015

Note: This (extra) homework will count for extra 8% towards your total grade.

1. (10 points) Problem 1.48, page 90 of the text.

   Show that the following language is regular:

   $$L = \{w \in \{0,1\}^* \mid w \text{ contains an equal number of occurrences of the substrings 01 and 10}\}.$$

   Thus, $101 \in L$ because 101 contains a single 01 and a single 10. The string $1010 \notin L$ because 1010 contains two 10s and one 01.

2. (10 points) Problem 2.28 (b), page 157 of the text.

   Give an unambiguous grammar for the following language:

   $$\{w \in \{0,1\}^* \mid \text{number of } 0's = \text{ and number of } 1's\}.$$

3. (15 points) Problem 2.37, page 158 of the text.

   Prove the following stronger form of the pumping lemma, wherein *both* pieces v and y must be nonempty when the string s is broken up.

   If A is a context-free language, then there is a number k where, if s is any string in A of length at least k, then s may be divided into five pieces, $s = uvxyz$, satisfying the consitions:

   (a) for each $i \geq 0$, $uv^ixy^iz \in A$,

   (b) $v \neq \epsilon$ and $y \neq \epsilon$, and

   (c) $|vxy| \leq k$.

   (Assume that A is generated by a CFG $G = (V, \Sigma, R, S)$ in CNF.)

4. (15 points) Given a CFG G and a DFA D, describe an algorithm to decide if $L(G) \subseteq L(D)$.

   Hint: We saw in class/text algorithms for the following problems:

   - Constructing a DFA that recognizes the complement of the language of a given DFA.
   - Constructing a PDA from a given CFG.
   - Constructing a CFG from a given PDA.
   - Constructing a PDA that recognizes the intersection of the languages of a given PDA and a given DFA.
   - Deciding if the language of a given CFG is empty.

5. Define a *Constrained Two-Headed* (CTH) machine as follows.

   An CTH machine is a DTM with one tape that is unbounded on one side and that has two heads FH and BH.

   - The head FH is a read-only head and the head BH is a write-only head.
   - Both heads can only move right.
   - If a symbol on the tape under the read head FH is read, the head FH moves right by one position.
   - If a symbol is written on the tape under the write head BH, the head BH moves right by one position.
   - In each step, the machine M being in a state and either reading the symbol on the tape under the head FH or not reading the symbol can: (a) change state, and (b) either write a symbol on the tape or not write a symbol.
   - The string on the tape in positions i through j is said to be *current* if FH is on position i and BH is on position $j + 1$.

Let $M = (Q, \Sigma, \Gamma, \delta_M, q_s, q_a, q_r)$ be a 1-tape deterministic Turing machine. Our goal is to simulate M by a RTH machine.

Let N be the simulating RTH machine. For each symbol $a \in \Gamma$ of M, there are two symbols a, a° in the tape alphabet of N. The symbol a° denotes that the tape head of M is over the symbol a.

Let $w = w_1 \cdots w_n$ be a bit string that is an input to M. A configuration of M on input w is represented using the current state of N and the current region of N's tape in the form: $u_1 u_2 \cdots {u_i}^\circ \cdots u_k\$$.

(a) (5 points) How is the input $w_1 w_2 \cdots w_n$ represented in N?

(b) (10 points) Let $\delta_M(p, a) = (q, b, R)$ where $a, b \in \{0, 1\}$. Show how to simulate this move with a RTH machine.

(c) (10 points) Let $\delta_M(p, a) = (q, b, L)$ where $a, b \in \{0, 1\}$. Show how to simulate this move with a RTH machine.