Exam 5

CS 3510A, Spring 2014

These solutions are being provided **for your personal use only**. They are not to be shared with, or used by, anyone outside this class (Spring 2014 section of Georgia Tech CS 3510A). Deviating from this policy will be considered a violation of the GT Honor Code. **Instructions:**

- **Do not open this exam packet until you are instructed to.**
- **Write your name** on the line below.
- You will have **30 minutes** to earn **30 points**. Pace yourself!
- You may use one double-sided 8.5-by-11" study sheet. **You may not use any other resources, including electronic devices.**
- Do your best to fit your final answers into the space provided below each question. You may use the backs of the exam pages, and the final page of the exam, as scrap paper.
- **You may refer to any facts from lectures or the textbook without re-deriving them.**
- Your work will be graded on correctness and clarity. Please write legibly!

Question	Points	Score
1	7	
2	23	
Total:	30	

Your name: ____________________________________

1. (7 points) Suppose you discover a problem A in NP that you cannot manage to prove is NP-complete. However, you are able to show that A does *not* have a polynomial-time algorithm. Would you have resolved the P versus NP question, or not? Explain your answer.

> **Solution:** You *would* have resolved the P versus NP question! Since $A \in NP$ but $A \notin P$ by the first and third facts given above, this means $P \neq NP$. In this setting it does not matter whether A is NP-complete or not. However, notice that since $A \leq$ CIRCUIT-SAT (along with every other NP-complete problem), and A is not in P, CIRCUIT-SAT is also not in P.

2. The EVEN-CLIQUE decision problem is defined very similarly to the CLIQUE problem, except that it is restricted to ask about cliques of *even* size. Formally, the EVEN-CLIQUE problem is: given an undirected graph G and an *even* integer $k \geq 0$, is there a clique of k vertices in G?

   (a) (7 points) Prove that EVEN-CLIQUE is in NP. Be sure to clearly describe the form of a witness and the efficient verification algorithm for the problem.

   > **Solution:** A witness for a "yes" EVEN-CLIQUE instance (G, k) is a set of k vertices in G, which should form a clique. The verifier algorithm is given an instance (G, k) and a witness, i.e., a set of vertices v_i in G. The verifier checks that k is even, that the v_i are k distinct vertices, and that there is an edge $(v_i, v_j) \in E$ between every pair of these vertices. If so it outputs "yes," otherwise it outputs "no." This procedure is clearly polynomial time.
   >
   > This is a valid verifier because by definition of the EVEN-CLIQUE problem, for a "yes" instance (G, k) there exists a set of vertices (a k-clique) that satisfies the above verifier, whereas for a "no" instance (including one where k is odd, regardless of whether there is a clique of size k), no witness can cause the verifier to accept.

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(b) (8 points) Describe an efficient way of modifying a given undirected graph G into an undirected graph G', such that G has a clique of ℓ vertices *if and only if* G' has a clique of $\ell + 1$ vertices (where ℓ is arbitrary). Prove that your method has this property.

Solution: We construct G' by copying G, adding one new vertex v, and adding an edge between v and each one of the original vertices u in G. (Clearly this procedure is polynomial time.)

If G has a clique $C \subseteq V$ of size ℓ, then G' has a clique $C \cup \{v\}$ of size $\ell + 1$. It is a clique because all vertices in C have edges between them, and there also are edges between v and every vertex in C. In the other direction, if G' has a clique C' of size $\ell + 1$, then either $v \in C'$ and so $C' \setminus \{v\}$ is a clique in G of size ℓ, or $v \notin C'$ and any choice of ℓ vertices in C' is also a clique in G.

(c) (8 points) Using the previous part, prove that EVEN-CLIQUE is NP-complete. (To save time, you can assume without proof that EVEN-CLIQUE is in NP.)

Solution: We give a reduction algorithm *from* the NP-complete CLIQUE problem *to* EVEN-CLIQUE, which shows that CLIQUE $\leq$ EVEN-CLIQUE. This (along with EVEN-CLIQUE $\in NP$) will prove that EVEN-CLIQUE is NP-complete.

Our reduction takes as input an instance (G, k) of CLIQUE, and outputs an instance (G', k') of EVEN-CLIQUE (so k' must be even) as follows. If k is even, the reduction just outputs $(G' = G, k' = k)$. If k is odd, the reduction modifies G as in the previous part to get G', and outputs $(G', k' = k + 1)$. This procedure clearly runs in polynomial time, and k' is always even, as required.

To show that this is a correct reduction, we need to prove that the input graph G has a clique of size k *if and only if* the output graph G' has a clique of size k'. When k is even this is trivial (because $G' = G$ and $k' = k$). When k is odd, we showed in the previous part that this property holds. So this completes the proof.

Scrap paper — no exam questions here.