Equivalence of NFA and DFA

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA.

- Assume that there are no ϵ transitions.
  - *Construct* a DFA $M = (Q', \Sigma, \delta', {q_0}', F')$ as follows:
    * $Q' = \mathcal{P}(Q)$.
    * ${q_0}' = \{q_0\}$.
    * For $R \in Q'$ and $a \in \Sigma$, define $\delta'(R, a) = \{q \in Q \mid q \in \delta(r, a) \text{ for some } r \in R\}$.
    * $F' = \{S \in Q' \mid S \cap F \neq \emptyset\}$.
  - **Correctness Claim:** For $R, S \in Q'$ and $w \in \Sigma^*$, $\hat{\delta}'(R, w) = S$ if and only if S is the largest set such that for all $s \in S$, there exists $r \in R$ such that $s \in \hat{\delta}(r, w)$.
    * Show this for $w \in \Sigma$ and then use induction on the length of w.
    * For $w \in \Sigma$, this is just the definition of $\hat{\delta}'$.
  - **Corollary:** M accepts w if and only if N accepts w.
    * $w \in L(M)$ if and only if $\hat{\delta}'(\{q_0\}, w) = S$ for some $S \in F'$.
    * From the claim, $\hat{\delta}'(\{q_0\}, w) = S$ for some $S \in F'$ if and only if for all $s \in S$, $s \in \hat{\delta}(q_0, w)$.
      · In the claim put $R = \{q_0\}$.
    * That is, $w \in L(M)$ if and only if there exists $S \in F'$ such that for all $s \in S$, $s \in \hat{\delta}(q_0, w)$.
    * Now, S includes some $f \in F$, since $S \in F'$. Therefore, since for all $s \in S$, $s \in \hat{\delta}(q_0, w)$, we have $f \in \hat{\delta}(q_0, w)$.
    * That is, $w \in L(N)$.

- Suppose there are epsilon transitions.
  - For any $R \in Q'$, define:
    $E(R) = \{q \in Q \mid q \text{ can be reached from } R \text{ using only } \epsilon \text{ transitions}\}$.
  - ${q_0}' = E(q_0)$.
  - For $R \in Q'$ and $a \in \Sigma$, define $\delta'(R, a) = \{q \in Q \mid q \in E(\delta(r, a)) \text{ for some } r \in R\}$.
  - **Correctness Claim:** For $R, S \in Q'$ and $w \in \Sigma^*$, $\hat{\delta}'(R, w) = S$ if and only if S is the largest set such that for all $s \in S$, there exists $r \in R$ such that $s \in E(\hat{\delta}(r, w))$.