HW4

ASEN 5044 Statistical Estimation for Dynamical Systems

Fall 2020

Homework 4

Out: Thursday 02/06/2020 (posted on Canvas)

Due: Thursday 02/13/2020 (Canvas - **no credit for illegible submissions**)

Show all your work and explain your reasoning.

1. Consider the 2-mass/3-spring system presented in Lecture 4, where the CT state definition and inputs are the same, but the observed sensor outputs are now given by

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u(t).$$

(a) Find the discrete time (DT) LTI representation for this system using a step size of $\Delta T = 0.05$ sec (Note: do not use matlab c2d or similar functions, use methods shown in class) . How does this sampling rate compare to the system's Nyquist limit?

(b) Show that the DT system is observable.

(c) Suppose the system starts from some unknown initial condition $x(0)$ at $k = 0$ and is stimulated by an external set of ZOH inputs u at the $\Delta T = 0.05$ sec sampling rate from $t = 0$ to $t = 5$ secs, where $u(t) = [\sin(t),\ 0.1 \cdot \cos(t)]^T$, and the resulting output $y(k)$ at each sampling instant from $t = 0.05$ sec ($k = 1$) to $t = 5$ sec is recorded in the posted data log `hw4problem1data.mat` (the input sequence for $u(k)$ starting from $k = 0$ is also included). Derive a linear system of equations in matrix-vector form that would allow you to estimate the unknown initial condition $x(k = 0)$ using all the available logged y and u data.

(d) Estimate $x(k = 0)$ (report the vector value) and plot all the remaining states $x(k)$ for $k \geq 1$ vs. time (in secs) and separately plot their corresponding 'predicted' outputs $y(k)$ vs. time, for all $k \geq 1$ in the recorded output time series. Validate your estimate by also separately plotting the differences between the 'predicted' and recorded $y(k)$ values vs. time.

(e) How many vector measurements $y(k)$ are actually needed to estimate $x(0)$, i.e. do you need to use all available measurements, or some smaller number? Is this consistent with an analysis of the observability matrix $\mathbb{O}$ and Gramian $\mathbb{O}^T\mathbb{O}$? Explain how and why the required number of vector measurements would theoretically change if the $y(k)$ data were instead given by three different position sensors for the first mass, where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

(f) What happens to the observability of the system if only the first row of the output $y(k)$ is used for all $k \geq 1$? What if only the second row of the output vector $y(k)$ is used instead? Provide a physical explanation for the results in each case. (**Hint:**

is used instead? Provide a physical explanation for the results in each case. (**Hint:** consider how the modified outputs relate to the system's natural modal behaviors – to see the natural modes, try visualizing the state response with zero inputs for initial conditions corresponding to $x_0 = c(v_1 + v_2)$ to excite the first mode and $x_0 = d(v_3 + v_4)$ to excite the second mode, where $v_{1,2}$ and $v_{3,4}$ are the complex conjugate pairs of eigenvectors corresponding to the system's eigenvalues, and c and d are any non-zero scalar constants. Note that a diagonalizable system's state response to any initial condition is a superposition of its modal state responses, since any initial condition can be represented by a linear combination of eigenvectors. What does this mean for observability in the case of the modified outputs? A certain basis transformation can help show the effect.)

a.) Using preexisting A+B matrices from Lecture 4 3-spring 2-mass problem with new C+D matrices,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

=> Using MATLABs expm function on $A_{aug} = \text{expm}\left(\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}\right)$

$$\Rightarrow F = \begin{bmatrix} 0.9975 & 0.0500 & 0.0012 & 0 \\ -0.0999 & 0.9975 & 0.0499 & 0.0012 \\ 0.0012 & 0 & 0.9975 & 0.0500 \\ 0.0499 & 0.0012 & -0.0999 & 0.9975 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.0012 & 0 \\ -0.0499 & 0 \\ 0.0012 & 0.0012 \\ 0.0499 & 0.05 \end{bmatrix}$$

$x_k = [q_1(k), \dot{q}_1(k), q_2(k), \dot{q}_2(k)]$

$u_k = [u_1(k), u_2(k)]^T$

$y_k = [q_1(k), q_2(k)]^T$

$$\Rightarrow \boxed{x_{k+1} = F \cdot x_k + G u_k, \quad y_k = H x_k + M u_k}$$

for system, $\left(\frac{\pi}{\Delta t}\right) > 2\left|\lambda_4^{max}\right| \Rightarrow \left(\frac{\pi}{.05}\right) > 2\left|1.2321\right|$

$\Rightarrow$ sampling rate of system is good

b.) show that DT system is observable

$\Rightarrow \mathcal{O} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix}$ is rank n if observable

$\Rightarrow$ Using Matlab, it was found that the

rank of $\mathcal{O} \triangleq \underline{Y} = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^{n-1} \end{bmatrix} = 4$. Since rank$(\mathcal{O}) = n = 4$

$\Rightarrow$ system is observable

c.) unknown $x(0)$, ZOH inputs $u = [\sin(t), 0.1\cos(t)]^T$

$\Rightarrow$ Derive linear system of eqns. in matrix-vector form

that would allow $x(0)$ to be calculated based on logged y data

Using the expression for $x(0) = (\mathcal{O}^T\mathcal{O})^{-1}\mathcal{O}^T \underline{Y}$

$\Rightarrow$ need to find observation matrix $\mathcal{O}$ & $\underline{Y}$

$\Rightarrow x(k+1) = Fx(k) + Gu(k), \; y(k) = Hx(k) + Mu(k)$

$\Rightarrow x(1) = Fx(0) + Gu(0)$

$$X(2) = FX(1) + G u(1)$$

$$\Rightarrow X(2) = F(FX(0) + G u(0)) + G u(1)$$

$$X(3) = FX(2) + G u(2)$$

$$\Rightarrow X(3) = F(F(FX(0) + G u(0)) + G u(1)) + G u(2)$$

$$= F^3 X(0) + F^2 G u(0) + F G u(1) + G u(2)$$

$$\vdots$$

$$X(n) = F^n X(0) + \sum_{i=1}^{n} F^{n-i} G \cdot u(i-1)$$

In general, $y(1) = HX(1)$

$$y(2) = HX(2)$$

$$\vdots$$

$$y(n) = HX(n)$$

$$\Rightarrow y(n) = HX(n) \Rightarrow y(n) = HF^n X(0) + H\left(\sum_{i=1}^{n} F^{n-i} G u(i-1)\right)$$

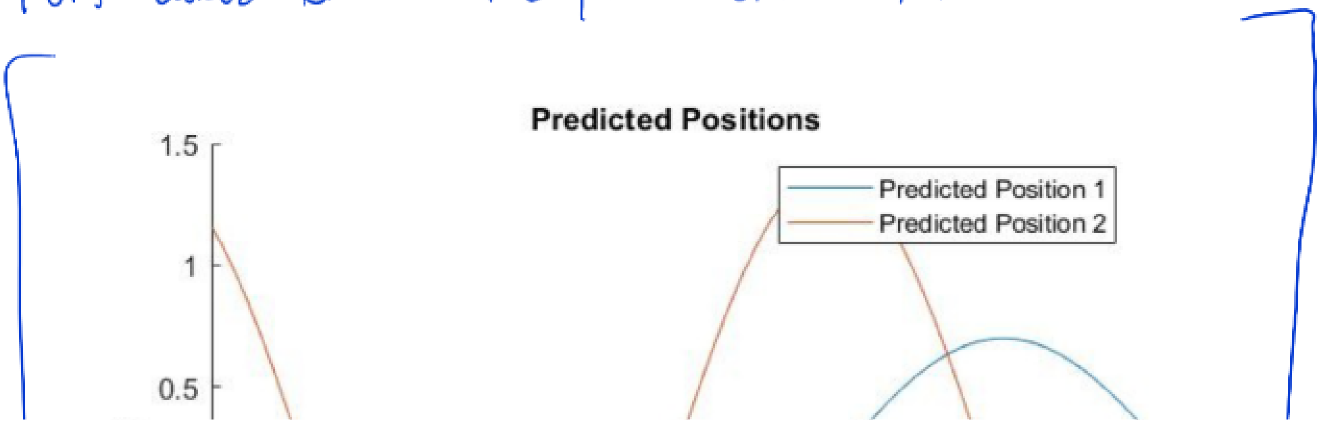
$$\Rightarrow Y(n) = \Theta X(0) + \underbrace{H \sum_{i=1}^{n} F^{n-i} G u(i-1)}_{\theta}$$

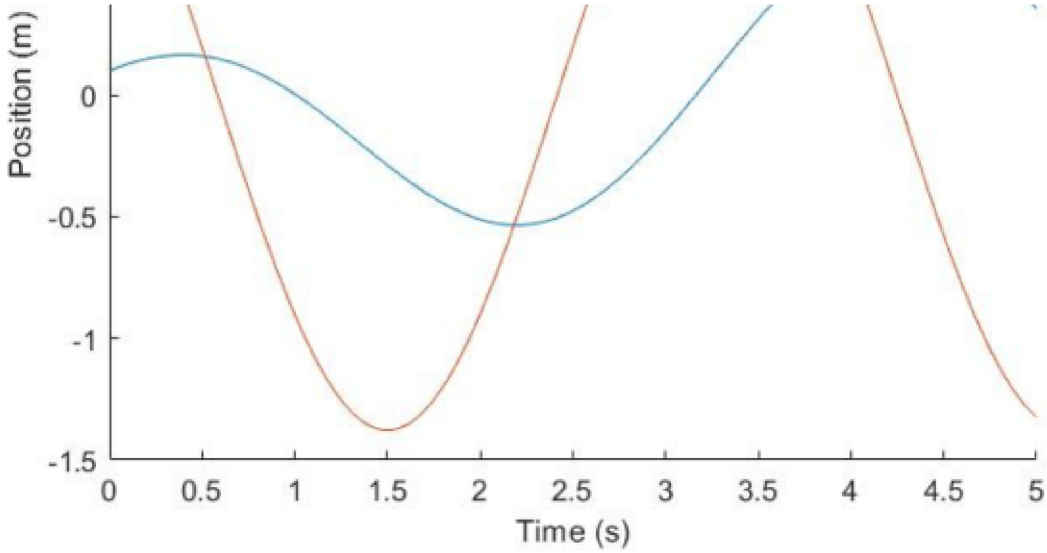
$$\Rightarrow Y(n) - \theta = \Theta X(0)$$

let $Y(n) - \theta = \Phi$

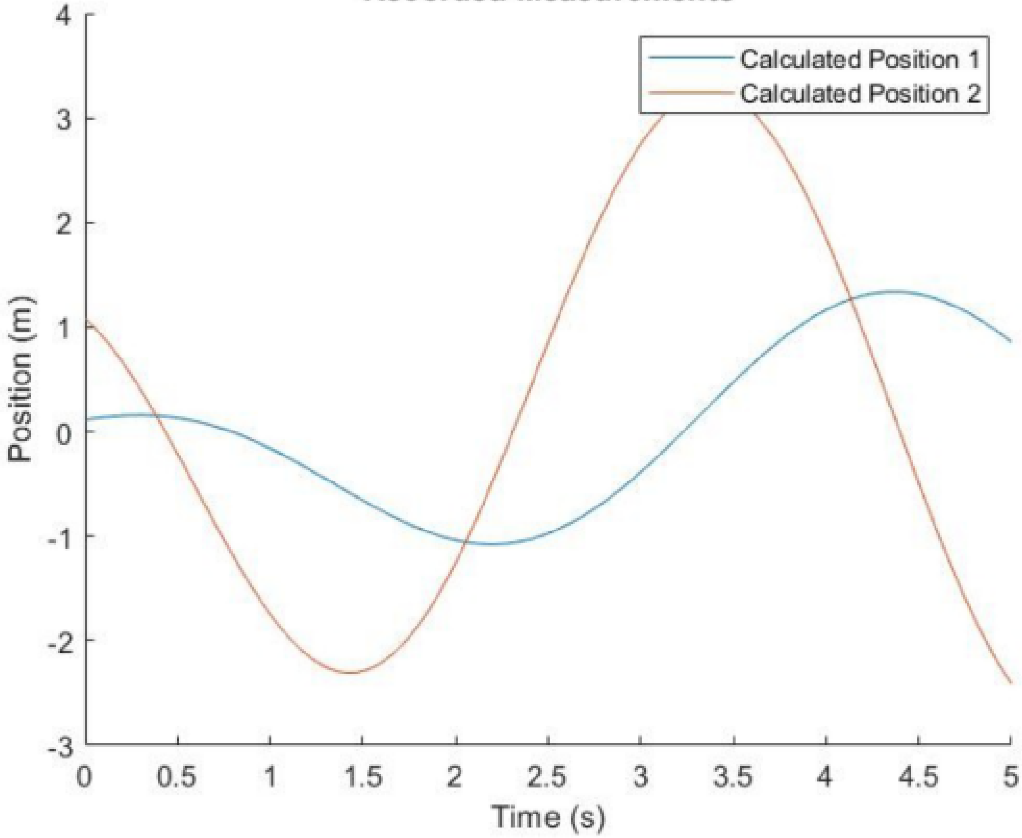
$$\Rightarrow \boxed{\Phi = \Theta X(0) \Rightarrow X(0) = (\Theta^T \Theta)^{-1} \Theta^T \Phi}$$

d.) After carrying out the operations above, the $X(k=0)$ values were determined to be $x_0 = [0.1 \;\; 0.3 \;\; -0.33 \;\; -0.86]^T$, & plots asked for in the problem statement:

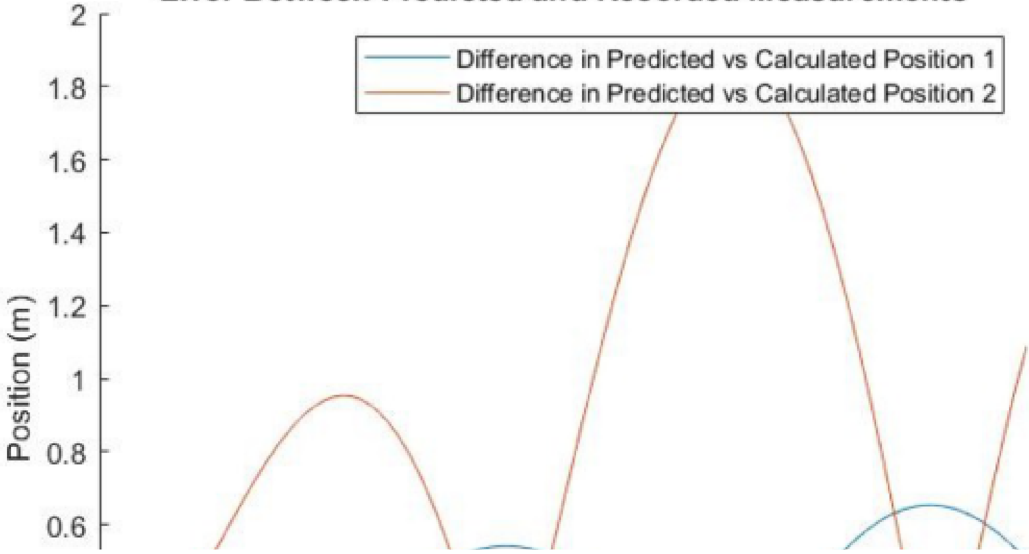


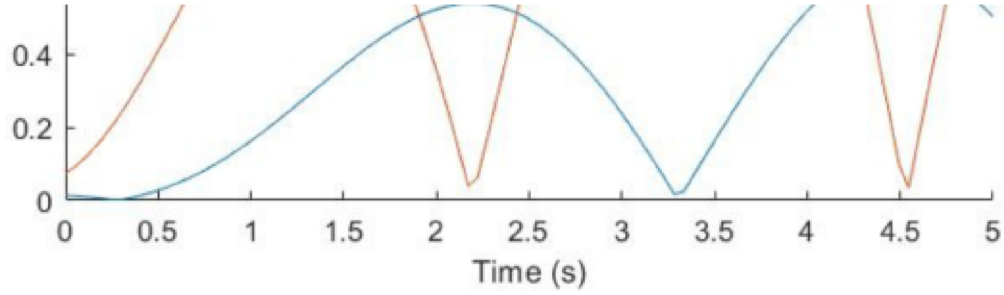


Recorded Measurements



Error Between Predicted and Recorded Measurements





e.) You could use n vector measurements (in this case 4) to estimate $x(0)$. This is consistent with the rank of the observability matrix, which is 4.

If $y(k)$ data was instead given by 3 different position vectors, i.e. $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$,

then Θ would be of dimension 12×4, so the required number of vector measurements shouldn't change

f.) Using only the first row of the output $y(k)$ for all $k \geq 1$ should not effect the observability of the system since these are all position measurements. Using only the second row of the output $y(k)$ will effect the observability of the system since these would only be velocity measurements, making the system unobservable

End problem 1

2. (Luenberger, 1979) discusses a simple model for the national income dynamics. The national income y_k in year k in terms of consumer expenditure c_k, private investment i_k and government expenditure g_k is assumed to be given by $y_k = c_k + i_k + g_k$, where the interrelations between these quantities are specified by $c_{k+1} = \alpha y_k$ and $i_{k+1} = \beta(c_{k+1} - c_k)$. The constant α is called the *marginal propensity to consume*, while β is a growth coefficient. Typically, $0 < \alpha < 1$ and $\beta > 0$.

(a) Show that these relations can be re-arranged into the following discrete time state space model with (F, G, H, M) matrix parameters:

$$x_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha \\ \beta(\alpha - 1) & \beta\alpha \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta\alpha \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + u_k$$

where $x_{1,k} \equiv c_k$, $x_{2,k} \equiv i_k$ and $u_k \equiv g_k$.

(b) Let the parameters (α, β) take on the following pairs of values: (0.6,1.1), (0.6,1.75), and (1.4,1). For each case, determine the eigenvalues of the F matrix, and also plot the states for $0 \leq k \leq 30$ when u_k is a unit step input (i.e. $u_k = 1$ for all $k > 0$) and $x_0 = [0, 0]^T$. Comment on the stability and observability of the system in each case. Can you find a pair (α, β) that will make the system unobservable?

(c) For each of the parameter cases in (b), plot the states for $k \geq 0$ when $u_k = 0$ for all time k and $x_0 = [3, 1]^T$. Comment on your results.

(d) For each of the parameter cases in (c), simulate a sequence of observations $y(0), ..., y(9)$ and use all the data to estimate the initial condition x_0 as if it were unknown (assume u_k is the same as in case c). Show a plot of your observation sequence vs. time, and report your resulting estimates in each case (be sure to explain the approach used to get the estimates, and validate your estimates by plotting the resulting predicted $y(k)$ outputs vs. time against the 'real' $y(k)$ data you generated in each case).

a.) given $y_k = c_k + i_k + g_k$, where $c_{k+1} = \alpha y_k + i_{k+1} = \beta(c_{k+1} - c_k)$,
typically $0 < \alpha < 1 + \beta > 0$

let $x_{1,k} \equiv c_k$, $x_{2,k} \equiv i_k$, $u_k \equiv g_k$

$\Rightarrow c_{k+1} = \alpha y_k \Rightarrow x_{1,k+1} = \alpha y_k = \alpha(c_k + i_k + g_k)$

$\Rightarrow \alpha(c_k + i_k) + \alpha g_k \Rightarrow \alpha x_{1,k} + \alpha x_{2,k} + \alpha u_k$

$i_{k+1} = \beta(c_{k+1} - c_k) \Rightarrow \beta[(\alpha(c_k + i_k + g_k)) - c_k] = i_{k+1}$

$$\Rightarrow \beta\alpha c_k + \beta\alpha i_k + \beta\alpha g_k - \beta c_k = i_{k+1}$$

$$\Rightarrow \beta\alpha x_{1,k} + \beta\alpha x_{2,k} + \beta\alpha u_k - \beta x_{1,k} = i_{k+1}$$

$$\Rightarrow \beta x_{1,k}(\alpha - 1) + \beta\alpha x_{2,k} + \beta\alpha u_k$$

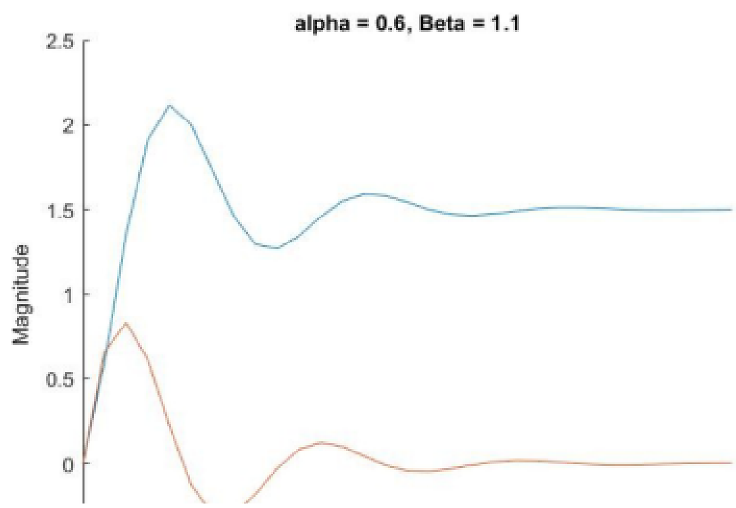
$$\Rightarrow \beta(\alpha-1)x_{1,k} + \beta\alpha x_{2,k} + \beta\alpha u_k$$

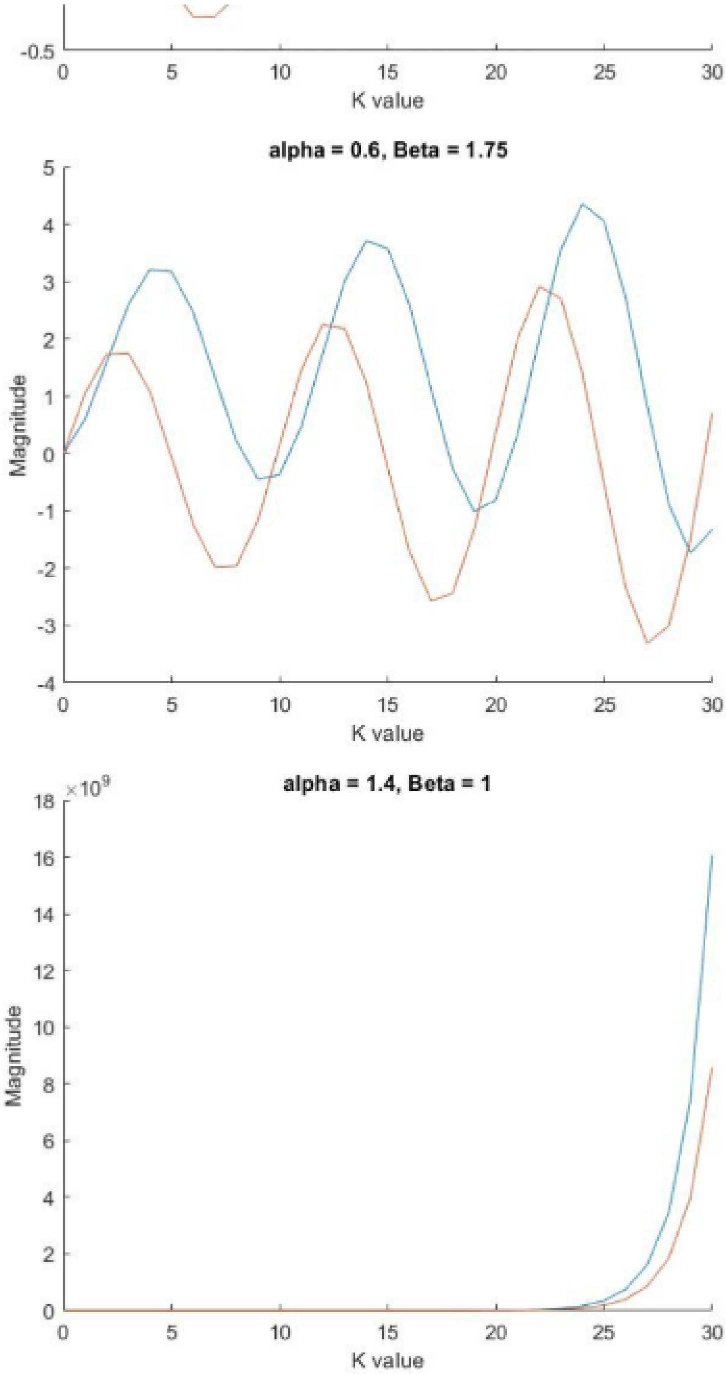
putting all this in matrix form

$$\Rightarrow \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} \alpha & \alpha \\ \beta(\alpha-1) & \beta\alpha \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} \alpha \\ \beta \end{bmatrix} u_k$$

$$y_k = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + u_k$$

b.) let α, β take on pairs of values: $(0.6, 1.1)$, $(0.6, 1.25)$, & $(1.4, 1)$. Determine evals. of F & plot states for $0 \leq k \leq 30$ for each case, when $u_k = 1$ for all $k \geq 0$ & $x_0 = [0, 0]^T$ comment on stability & observability in each case & try to find (α, β) pair that makes system unobservable





Given in code appendix

according to the eigenvalues of $\tilde{F}$ for each of the 3 cases, the first 2 scenarios result in a stable system, while the 3rd $\alpha + \beta$ combo, (1.4, 1), result in an unstable system.

constructing the O matrix for each system &

knowing $n=2$ resulting in all three systems being observable since they were all found to be of rank 2 (using MATLAB)

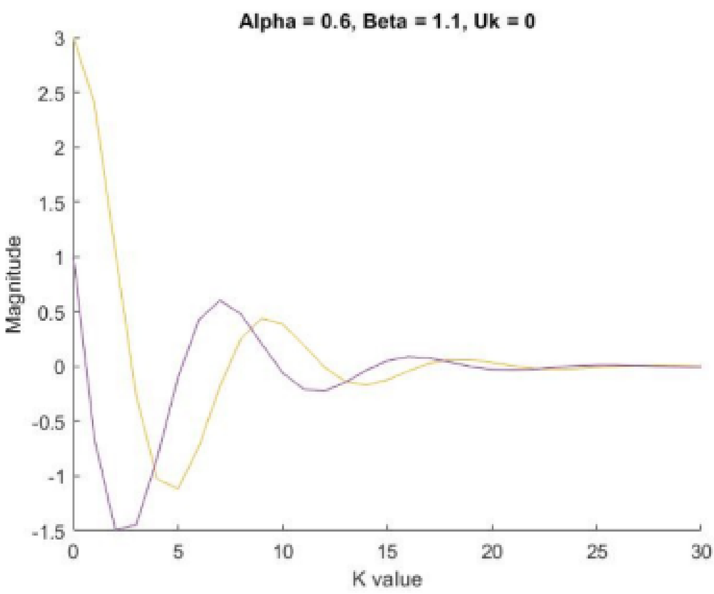
given ⓐ $\begin{bmatrix} H \\ HF \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ \alpha+\beta(\alpha-1) & \alpha+\beta\alpha \end{bmatrix}$

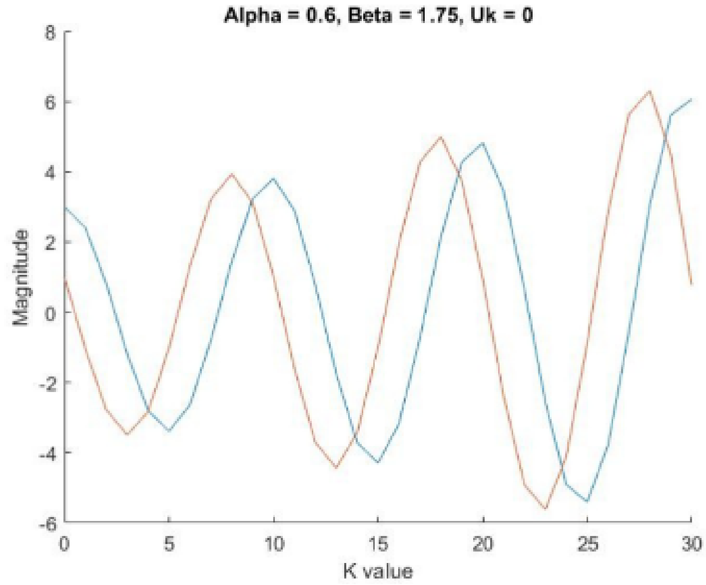
to not be observable, $\lambda \cdot 1 = (\alpha+\beta(\alpha-1))$ & $\lambda \cdot 1 = (\alpha+\beta\alpha)$

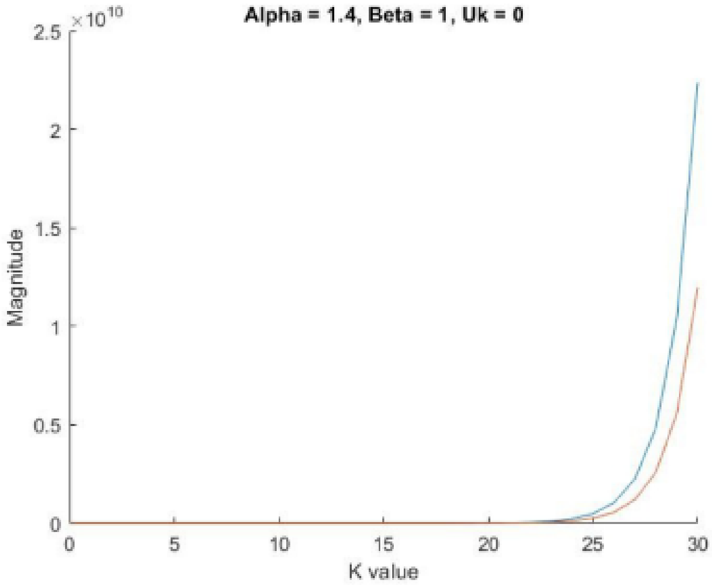
$\Rightarrow \alpha+\beta(\alpha-1) = \alpha+\beta\alpha \Rightarrow \beta(\alpha-1) = \beta\alpha = \alpha-1 \neq \alpha$

$\Rightarrow$ only α, β combo to make system unobservable would be trivial $\beta=0$ solution.

c.) Changing the initial conditions to $X_0 = [3, 1]^T$ & $U_k = 0$ for all K, neither observability nor stability changes, only shifts in the plots appear, as shown below.







d.) Starting with generic eqn. forms :

$$x(k+1) = Fx(k) + \overset{0}{\cancel{Gu(k)}} \quad \& \quad y(k) = Hx(k) + \overset{0}{\cancel{Mu(k)}}$$

$$\Rightarrow y(0) = Hx(0)$$

$$y(1) = Hx(1)$$
$$= HFx(0)$$

$$y(2) = Hx(2)$$
$$= HF^2x(0)$$

$$\vdots$$

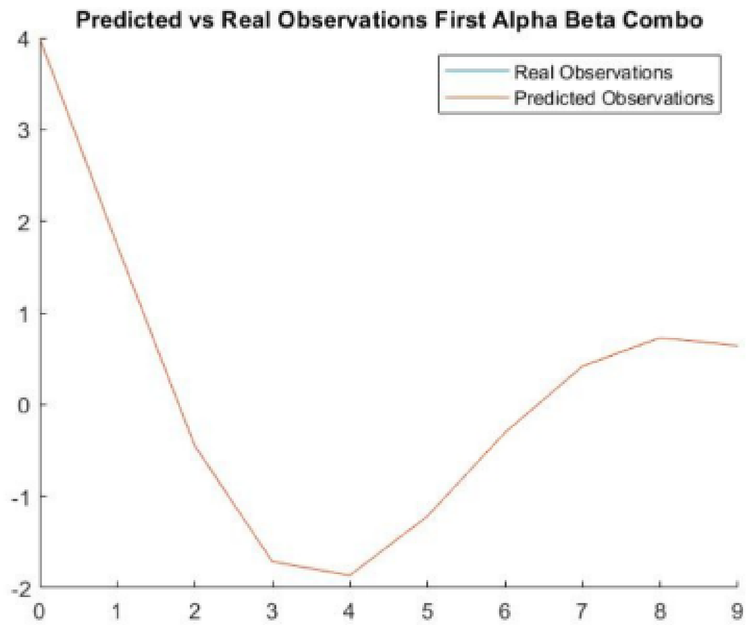
$$y(9) = HF^9x(0)$$

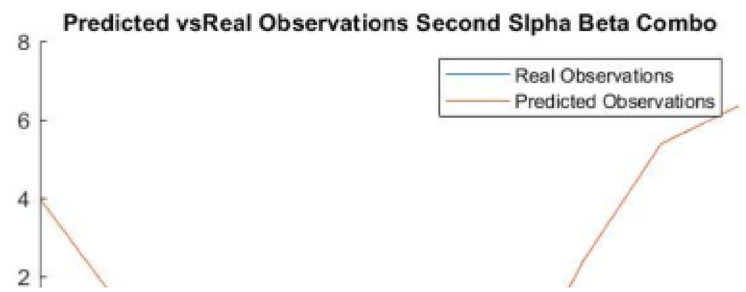
using form previously used in Q2 to obtain 'real' y observations, $x(k+1)=Fx(k)+Gu(k)$, with $u(k)=0$,
$\Rightarrow y(k)=x(k+1)$

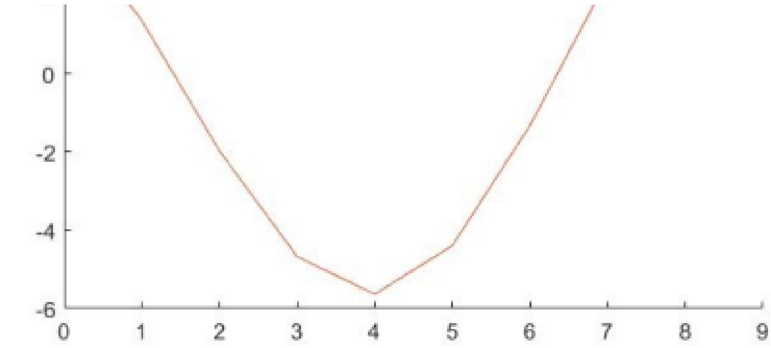
x_0 values are calculated using Grammian

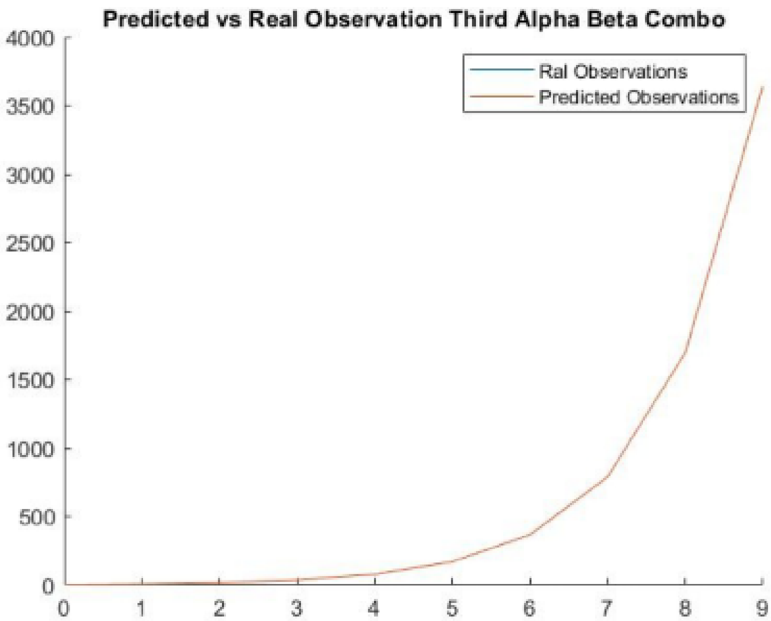
$\Rightarrow x_0 = (\Theta^T\Theta)^{-1}\Theta^T \cdot y$

to get second set of predicted y values, previously determined x_0 values are used in $y(n)=H\cdot F^{n-1}x(0)$ to determine 'predicted' y values, & the plots comparing the 2 sets of y values for the 3 different $\alpha\ \beta$ combos are shown below.









3.)

3. You are waiting around Gary Slick's dealership while your new Ferrari is being serviced, when you notice local folk hero Pladimir Vutin walk in and start haggling with Slick over a vintage GAZ-13 Chaika that is prominently on display. You can't help but overhear their

intense discussion, and record the following discrete time series for Slick's offers (z_1, in tens of thousands of dollars) and Vutin's offers (z_2, in tens of thousands of dollars), stacked into the vector $z(k) = [z_1(k), z_2(k)]^T$,

$$z(0) = \begin{bmatrix} 100.0000 \\ 20.0000 \end{bmatrix}, z(1) = \begin{bmatrix} 44.4160 \\ 40.5440 \end{bmatrix}, z(2) = \begin{bmatrix} 41.7257 \\ 41.5383 \end{bmatrix},$$
$$z(3) = \begin{bmatrix} 41.5955 \\ 41.5865 \end{bmatrix}, z(4) = \begin{bmatrix} 41.5892 \\ 41.5888 \end{bmatrix}, z(5) = \begin{bmatrix} 41.5889 \\ 41.5889 \end{bmatrix}.$$

Having conducted your own negotiation with Slick recently, you assume the dynamics of $z(k)$ could be reasonably modeled as

$$z_1(k+1) = z_1(k) + \lambda[z_1(k) - z_2(k)]$$

$$z_2(k+1) = z_2(k) + \mu[z_1(k) - z_2(k)],$$

where the constants λ and μ describe Slick's and Vutin's update parameters, respectively. You don't know what values of λ and μ apply for this particular negotiation – but, you'd sure like to figure out how Vutin galloped away with Slick's shirt on this deal...

(a) Suppose the 'unknown state' to be estimated in this case is $x(k) = [\lambda, \mu]^T$, and (based on what you recorded) the 'observed output' for $k = 0, 1, 2, \ldots$ is taken to be $y(k+1) = [g(k+1), p(k+1)]^T$, where $g(k+1) = z_1(k+1) - z_1(k)$ and $p(k+1) = z_2(k+1) - z_2(k)$. Find a suitable DT linear state space system model for this problem, and discuss any interesting features of the (F, G, H, M) matrix parameters for the model (e.g. which are time invariant or time varying?).

(b) Under what conditions can the λ and μ parameters be identified? Give a careful precise mathematical justification and interpretation.

(c) If the conditions hold for identifying λ and μ, set up a linear system of equations that could be solved to determine the parameters. What is your estimate, based on the observed sequence of bids recorded above? If the required conditions do not hold, explain why they do not hold in this case.

a.) Find DT linear SS system model & discuss any interesting features of (F, G, H, M) matrices

=> assuming general DT system form

=> $x(k+1) = Fx(k) + Gu_k$, $y(k) = Hx(k) + Mu_k$

=> $z_1(k+1) = z_1(k) + \lambda[z_1(k) - z_2(k)]$

$z_2(k+1) = z_2(k) + \mu[z_1(k) - z_2(k)]$

$$\Rightarrow \begin{bmatrix} \lambda(k+1) \\ \mu(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{F} \begin{bmatrix} \lambda \\ \mu \end{bmatrix}$$

$y(k) = H x(k)$

$$\Rightarrow y(k) = \begin{bmatrix} g(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} z_1(k+1) - z_1(k) \\ z_2(k+1) - z_2(k) \end{bmatrix} = \begin{bmatrix} \lambda[z_1(k) - z_2(k)] \\ \mu[z_1(k - z_2(k)] \end{bmatrix}$$

$$\Rightarrow y(k) = \underbrace{\begin{bmatrix} z_1 - z_2 & 0 \\ 0 & z_1 - z_2 \end{bmatrix}}_{H} \begin{bmatrix} \lambda \\ \mu \end{bmatrix}$$

Interesting that F is just an identity matrix & F is time invariant while H is time variant

b.) conditions that λ & μ can be identified $\Rightarrow$ observability

$$\Rightarrow \mathcal{O} = \begin{bmatrix} H \\ HF \end{bmatrix} \Rightarrow \begin{bmatrix} H \\ H \end{bmatrix} \begin{bmatrix} z_1 - z_2 & 0 \\ 0 & z_1 - z_2 \\ z_1 - z_2 & 0 \\ 0 & z_1 - z_2 \end{bmatrix}$$

$\Rightarrow \lambda$ & μ are observable as long as $z_1 \neq z_2$

Since $z_1 = z_2$ implies that Pladimir & Slick have reached the same prize

c.) Set up linear system of eqns. that could be solved to determine λ & μ & give estimate based on bids recorded.

given that $y(k+1) = \begin{bmatrix} g(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} z_1(k+1) - z_1(k) \\ z_2(k+1) - z_2(k) \end{bmatrix}$

$\Rightarrow$ using $k=0$ & $k=1$ data points

$$\Rightarrow y = \begin{bmatrix} 44.4160 - 100.00 \\ 40.5440 - 20.000 \end{bmatrix}$$

using $y = H \cdot \begin{bmatrix} \lambda \\ \mu \end{bmatrix}$

$$\Rightarrow 44.4160 - 100.00 = \lambda(100 - 20 \quad)$$
$$40.5440 - 20.000 = \mu(100 - 20 \quad)$$

$$\Rightarrow \boxed{\begin{aligned} \lambda &= -0.6948 \\ \mu &= 0.2568 \end{aligned}}$$

```
clear all;close all;clc

A = [0 1 0 0;-2 0 1 0;0 0 0 1;1 0 -2 0];
B = [0 0; -1 0; 0 0; 1 1];

A_aug = [A B];
pad = zeros(2,6);
A_aug = [A_aug;pad];

A_hat = expm(A_aug*.05);

F = A_hat([1:4],[1:4]);
G = A_hat([1:4],[5:6]);

[V,D] = eig(A);
```

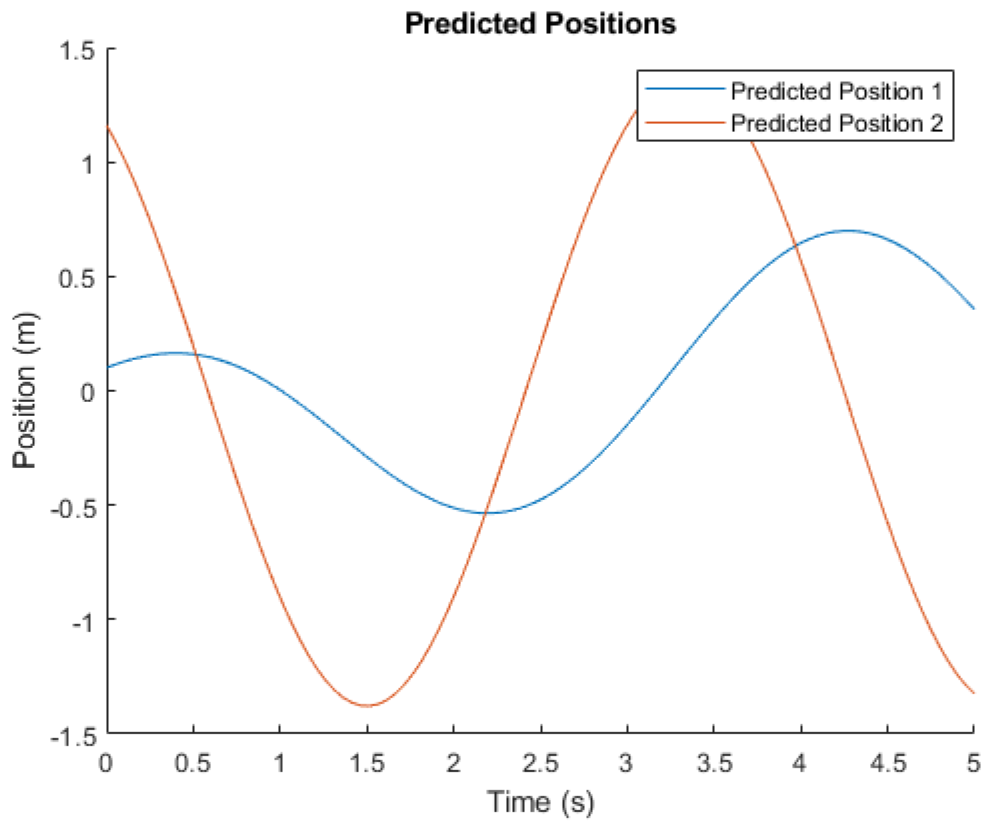
Part b

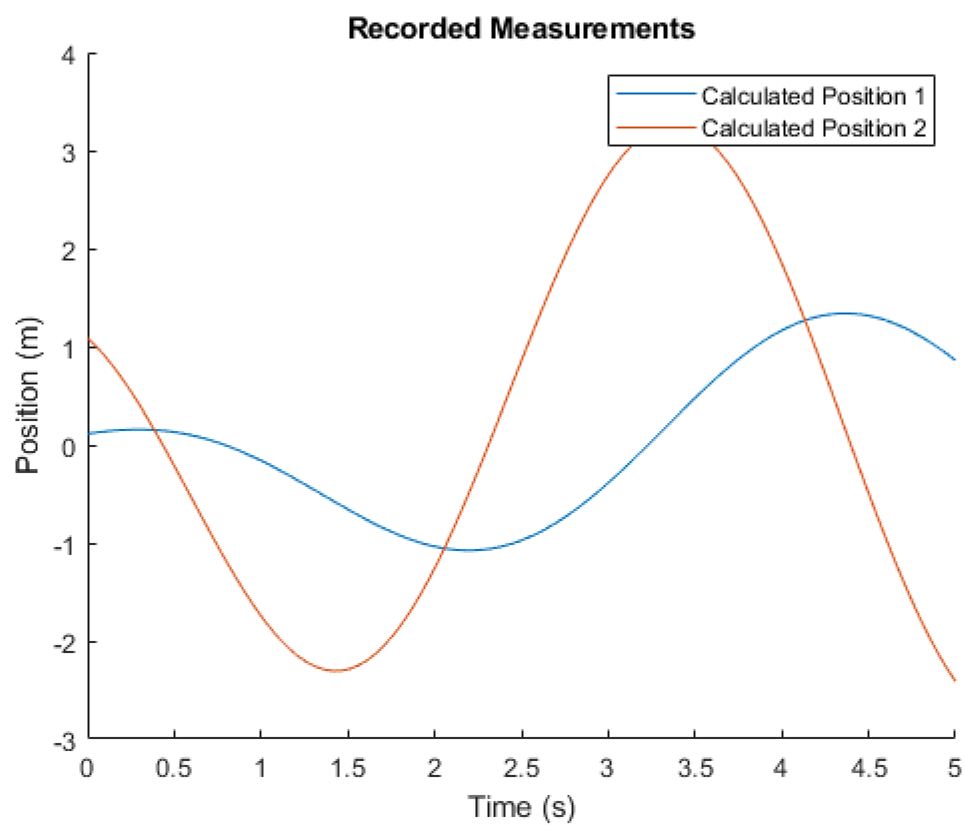
```
H = [1 0 0 0;0 1 0 -1];
Y = [H;H*F;H*F^2;H*F^3];

rank(Y)


ans =

     4
```

Part c/d

```
load('hw4problem1data.mat');
Udata(end,:) = [];
O = [];
% theta = 0;
for i = 1:size(Ydata,1)
    theta = 0;
    temp = H*F^i;
    O = [O;temp];
    for j = 1:i
        theta = theta + F^(i - j)*(G*Udata(j,:)');
    end
    theta = H*theta;
    phi(i,:) = Ydata(i,:) - theta';
end
phi = reshape(phi',[200,1]);
x0 = inv(O'*O)*O'*phi;

for i = 1:100
    y(:,i) = H*F^(i-1)*x0;
end
t = linspace(0,5,size(Ydata,1));
```

```
figure
hold on
plot(t,y(1,:))
plot(t,y(2,:))
title('Predicted Positions')
xlabel('Time (s)')
ylabel('Position (m)')
hold off
legend('Predicted Position 1','Predicted Position 2')
figure
hold on
plot(t,Ydata(:,1))
plot(t,Ydata(:,2))
title('Recorded Measurements')
xlabel('Time (s)')
ylabel('Position (m)')
hold off
legend('Calculated Position 1','Calculated Position 2')
figure
hold on
plot(t,abs(y(1,:)-Ydata(:,1)'))
plot(t,abs(y(2,:)-Ydata(:,2)'))
title('Error Between Predicted and Recorded Measurements')
xlabel('Time (s)')
ylabel('Position (m)')
legend('Difference in Predicted vs Calculated Position 1','Difference
 in Predicted vs Calculated Position 2')
```


Predicted Positions
Predicted Position 1
Predicted Position 2
Position (m)
Time (s)
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Recorded Measurements
Calculated Position 1
Calculated Position 2
Position (m)
Time (s)
4
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

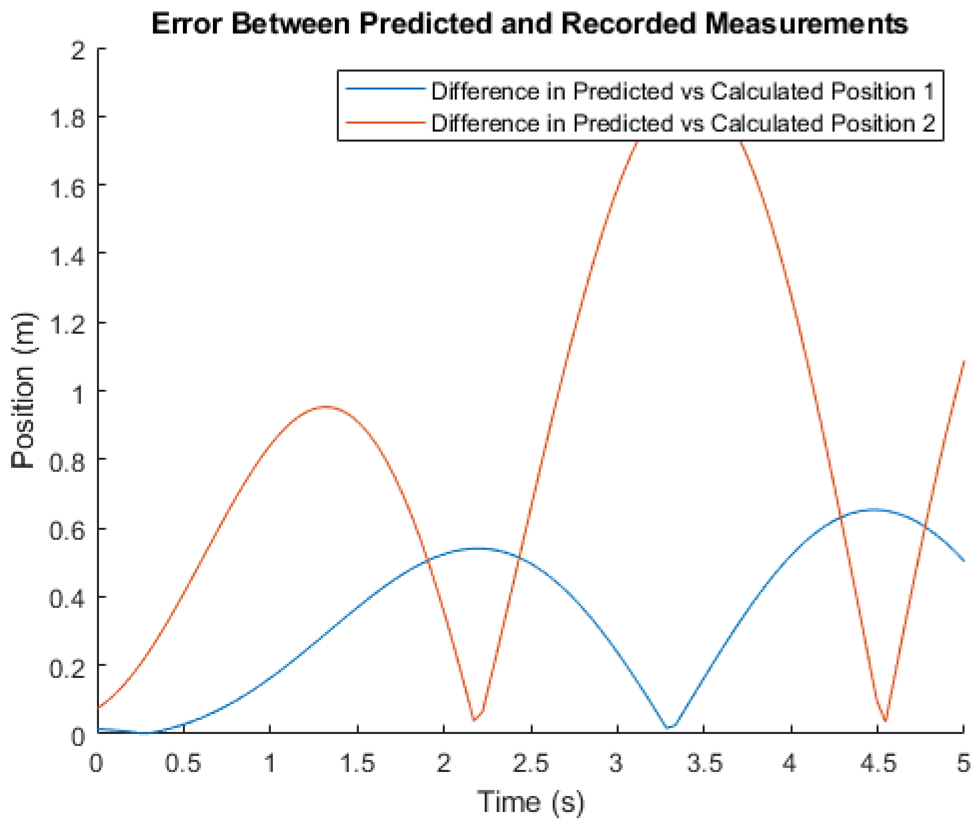


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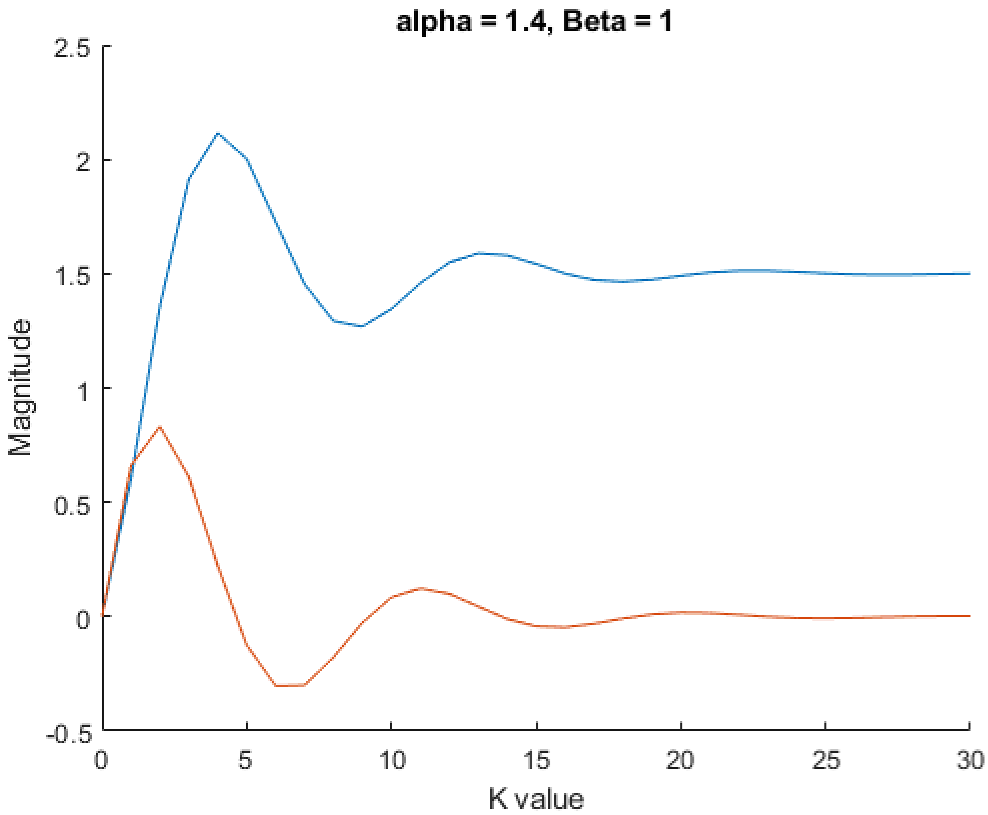
Table of Contents

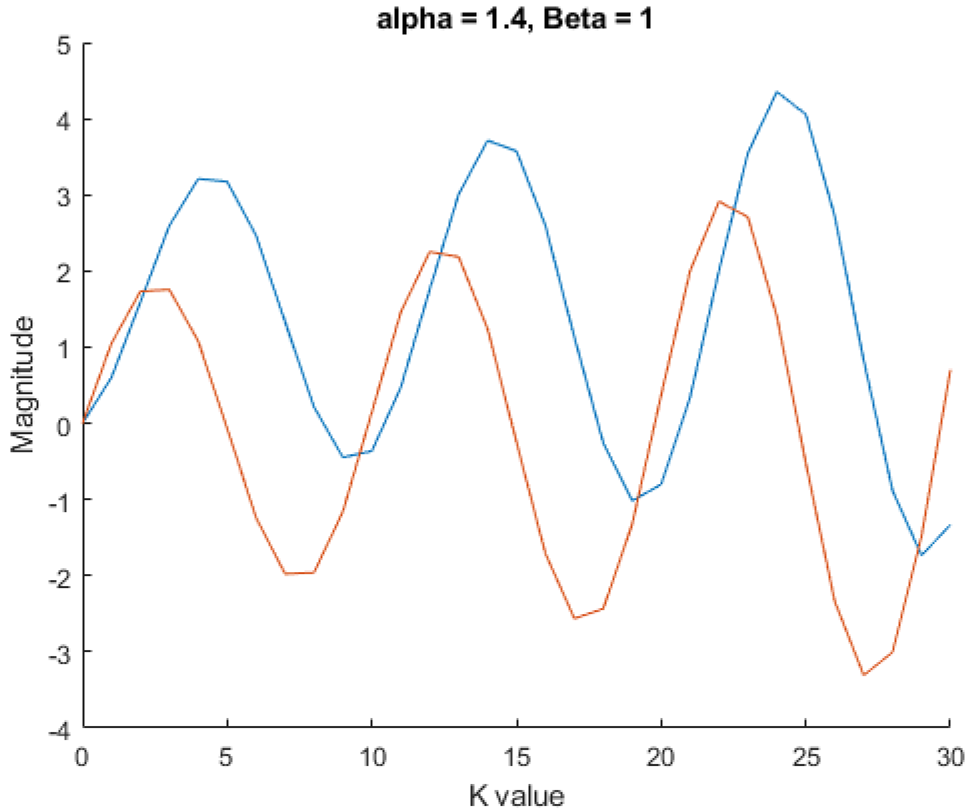
Problem 2, part b

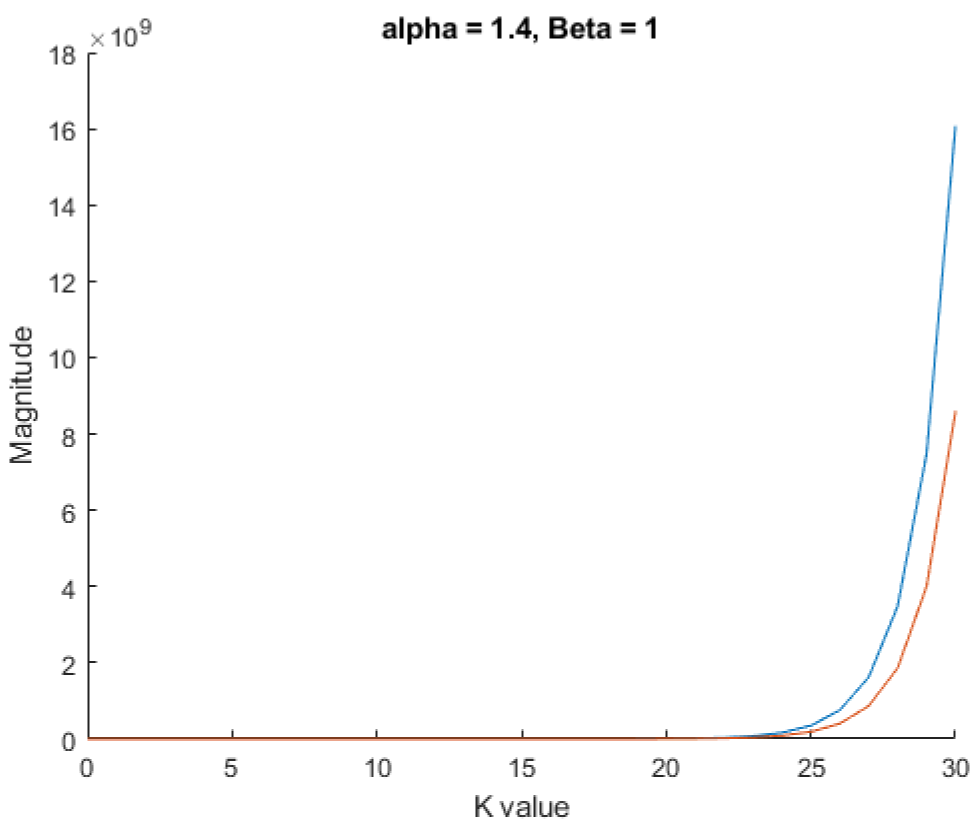
```
clear all;close all;clc
alpha = [0.6,0.6,1.4];
beta = [1.1,1.75,1];
x = [0 0]';

k = [0:30];

evals = zeros(2,3);
for j = 1:3
    for i = 1:numel(k) - 1
        F = [alpha(j) alpha(j); beta(j)*(alpha(j) - 1)
 beta(j)*alpha(j)];
        G = [alpha(j);beta(j)*alpha(j)];
        H = [1 1];
        Uk = 1;

        x(:,i+1) = F*x(:,i) + G*Uk;
    end
        figure(j)
        hold on
        plot(k,x(1,:))
        plot(k,x(2,:))
        evals(:,j) = eig(F)
        title('alpha = 1.4, Beta = 1')
        xlabel('K value');
        ylabel('Magnitude');
%         title('alpha = 0.6, Beta = 1.1 ')
        Y = [H;H*F];
        rank(Y)
end
```

```
evals =

   0.6300 + 0.5129i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.6300 - 0.5129i   0.0000 + 0.0000i   0.0000 + 0.0000i


ans =

     2
```

```
evals =

   0.6300 + 0.5129i   0.8250 + 0.6078i   0.0000 + 0.0000i
   0.6300 - 0.5129i   0.8250 - 0.6078i   0.0000 + 0.0000i


ans =

     2


evals =

   0.6300 + 0.5129i   0.8250 + 0.6078i   2.1483 + 0.0000i
   0.6300 - 0.5129i   0.8250 - 0.6078i   0.6517 + 0.0000i


ans =

     2
```



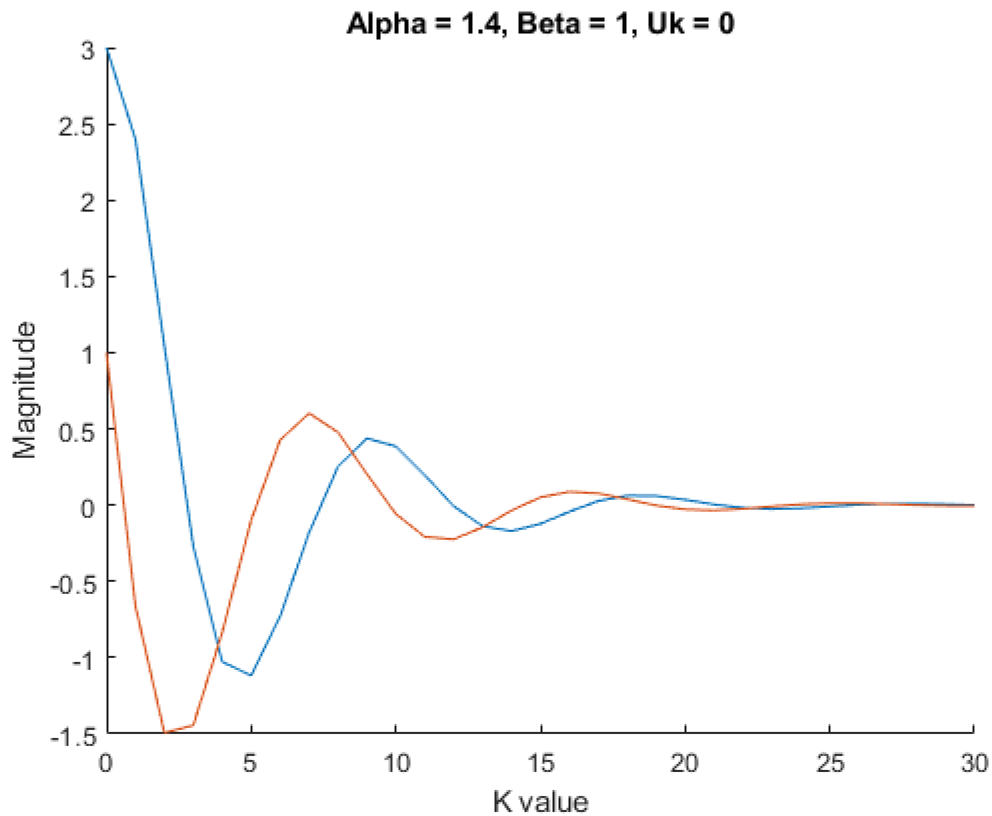

alpha = 1.4, Beta = 1
Magnitude
K value
5
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30

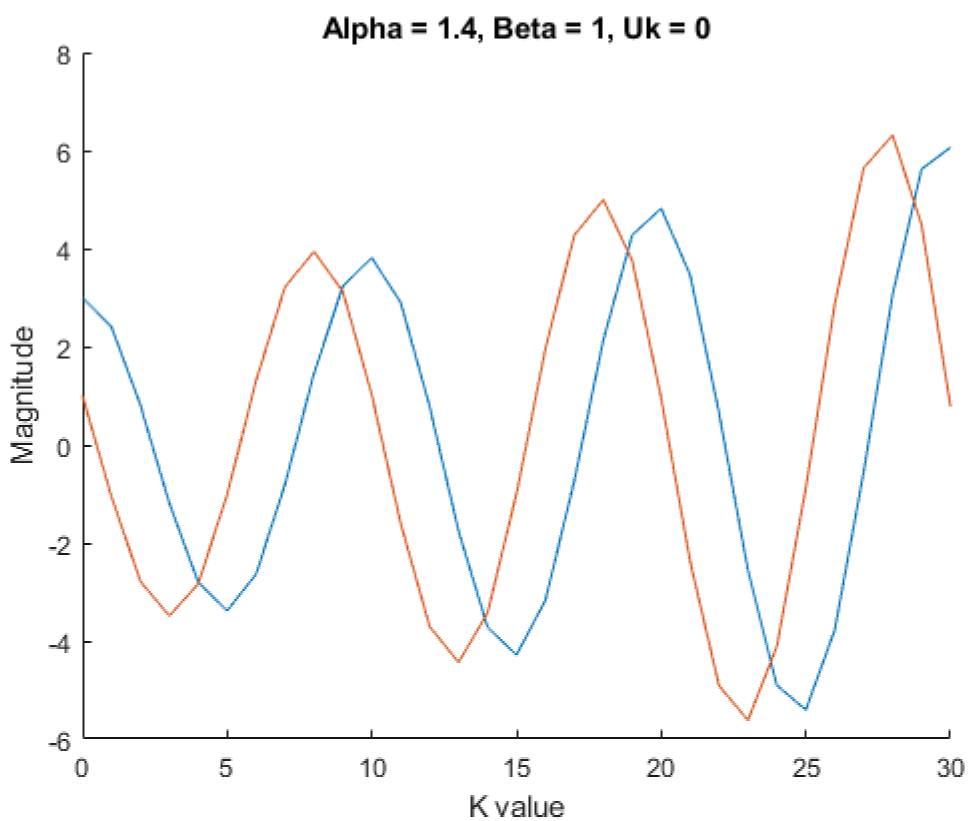

alpha = 1.4, Beta = 1
×10^9
Magnitude
K value
18
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30

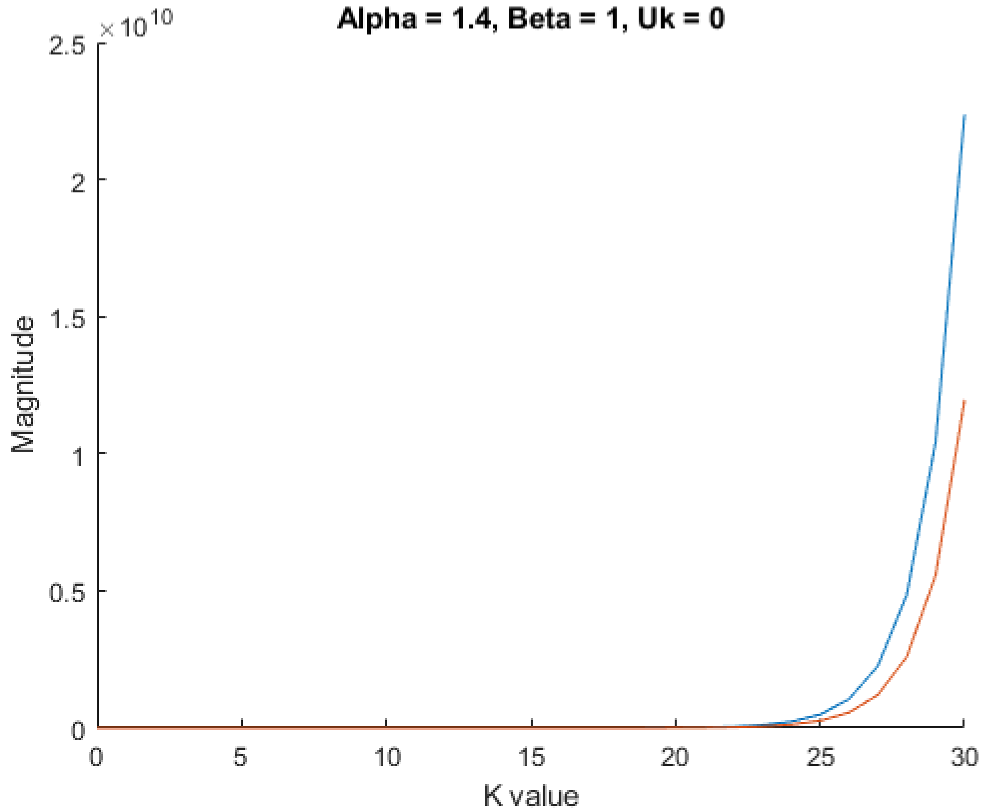
part c

```
x = [3 1]';

k = [0:30];

evals = zeros(2,3);
for j = 1:3
    for i = 1:numel(k) - 1
        F = [alpha(j) alpha(j); beta(j)*(alpha(j) - 1)
 beta(j)*alpha(j)];
        G = [alpha(j);beta(j)*alpha(j)];
        H = [1 1];
        Uk = 0;

        x(:,i+1) = F*x(:,i) + G*Uk;
    end
        figure(j+3)
        hold on
        plot(k,x(1,:))
        plot(k,x(2,:))
        evals(:,j) = eig(F);
        xlabel('K value');
        ylabel('Magnitude');
        title('Alpha = 1.4, Beta = 1, Uk = 0')
        Y = [H;H*F];
        rank(Y)
end
```

```
ans =

     2


ans =

     2


ans =

     2
```


Alpha = 1.4, Beta = 1, Uk = 0
Magnitude
K value
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30

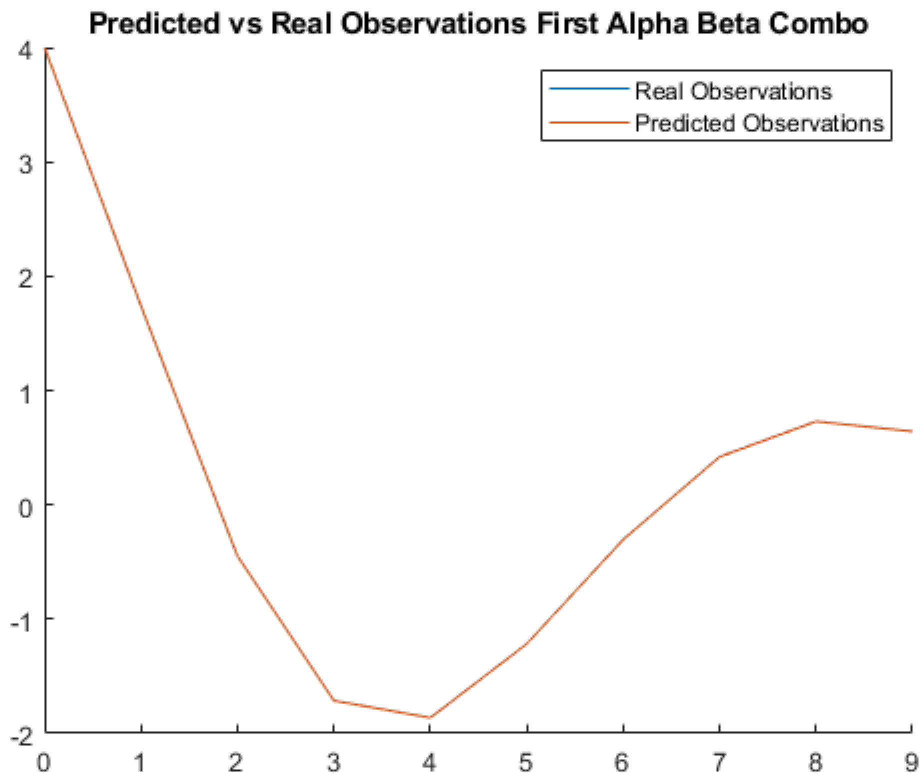

Alpha = 1.4, Beta = 1, Uk = 0
Magnitude
K value
8
6
4
2
0
-2
-4
-6
0
5
10
15
20
25
30

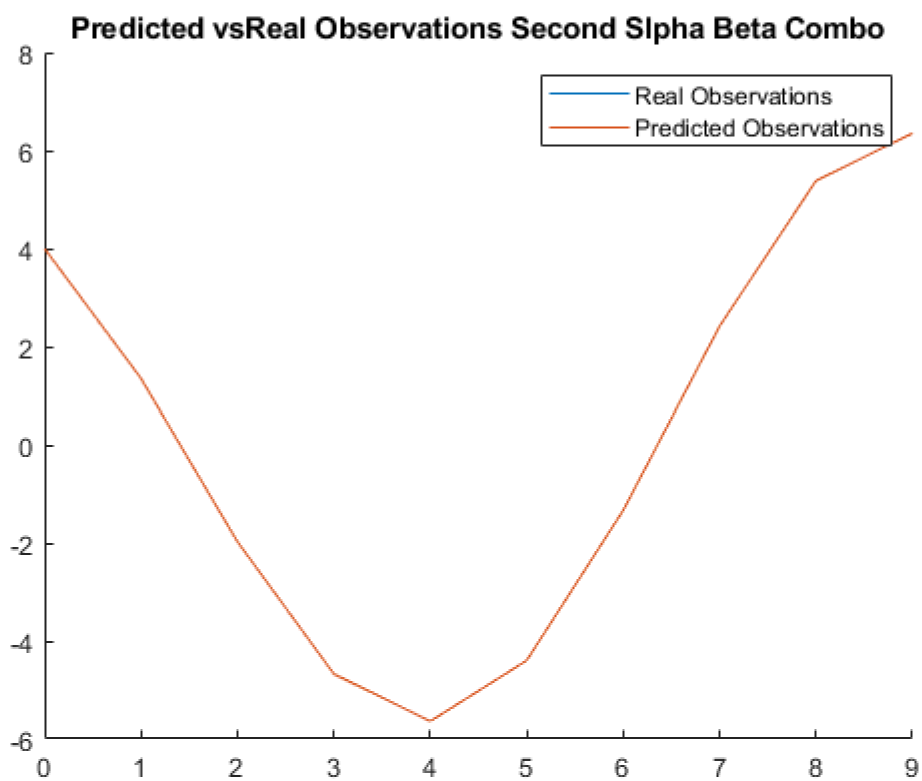


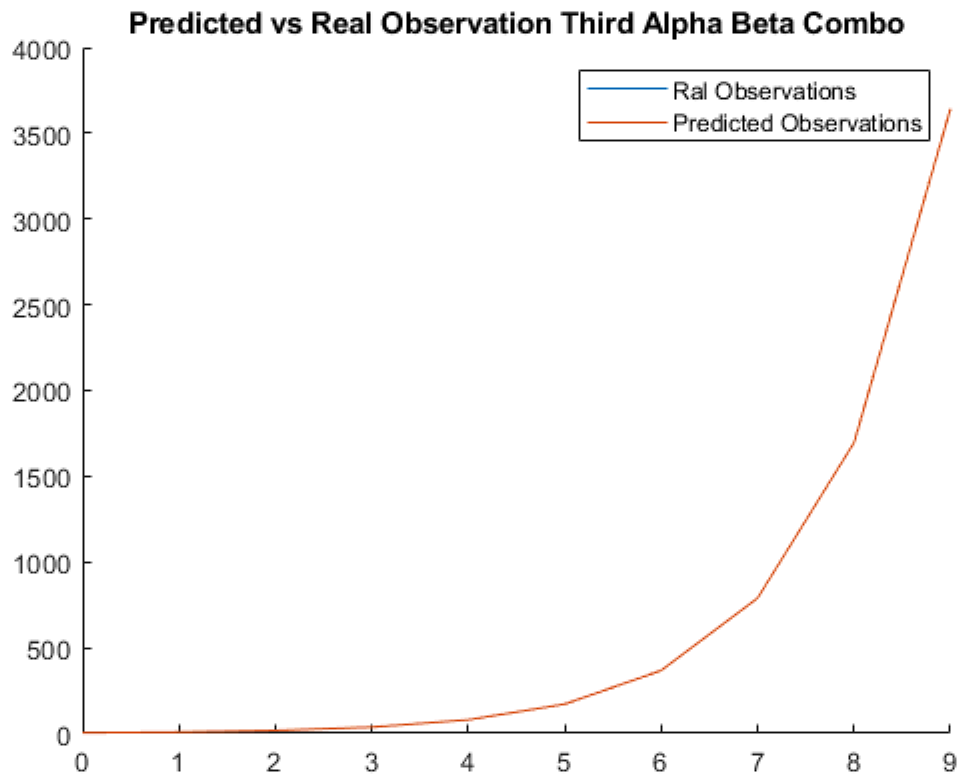
part c

```
x = [3 1]';
y = [];
X0 = [];
for j = 1:3
    O = [];
    for i = 1:10
        F = [alpha(j) alpha(j); beta(j)*(alpha(j) - 1)
 beta(j)*alpha(j)];
        G = [alpha(j);beta(j)*alpha(j)];
        H = [1 1];
        Uk = 0;

        x(:,i+1) = F*x(:,i) + G*Uk;

        O = [O;H*F^(i-1)];
    end
    y = [y;sum(x)];
    x0 = inv(O'*O)*O'*(y(j,[1:end-1]))';
    temp = x0';
    X0 = [X0;temp];
end
Y = [];
for j = 1:3
```

```
    O = [];
    F = [alpha(j) alpha(j); beta(j)*(alpha(j) - 1) beta(j)*alpha(j)];
    G = [alpha(j);beta(j)*alpha(j)];
    for i = 1:10
        O = [O;H*F^(i-1)];
    end
    Y(:,j) = O*((X0(j,:))');
end
t = [0:9];
close all
figure
hold on
plot(t,y(1,[1:end-1]))
plot(t,Y(:,1))
title('Predicted vs Real Observations First Alpha Beta Combo')
legend('Real Observations','Predicted Observations')
figure
hold on
plot(t,y(2,[1:end-1]))
plot(t,Y(:,2))
title('Predicted vsReal Observations Second Slpha Beta Combo')
legend('Real Observations','Predicted Observations')
figure
hold on
plot(t,y(3,[1:end-1]))
plot(t,Y(:,3))
title('Predicted vs Real Observation Third Alpha Beta Combo')
legend('Ral Observations','Predicted Observations')
```


Predicted vs Real Observations First Alpha Beta Combo
Real Observations
Predicted Observations
4
3
2
1
0
-1
-2
0
1
2
3
4
5
6
7
8
9


Predicted vsReal Observations Second Slpha Beta Combo
Real Observations
Predicted Observations
8
6
4
2
0
-2
-4
-6
0
1
2
3
4
5
6
7
8
9



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