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%Created: 4/11/12	
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Section 5.5, problem 2a

```
clear all;clc
%begin by obtaining initial values from 4th order RK method for 2-step
%AB method
clear all;clc
t = [0:.1:1];
h = .1;
y(1) = 1;
order = 2i
%define DE
f = @(t,y) (2 - 2*t*y)/(t^2 + 1);
%run RK
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/2)*(3*f(t(j),y(j)) - f(t(j-1),y(j-1)));
end
%compare approx., exact, and error
exact = (2.*t + 1)./(t.^2 + 1)
error = abs(y - exact)
%repeat process for 3-stop AB method
clear y;clc
y(1) = 1;
order = 3;
for i = 1:(order - 1)
   k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
```

```
k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/12)*(23*f(t(j),y(j)) - 16*f(t(j-1),y(j-1)) +
 5*f(t(j-2),y(j-2)));
%compare approx., exact, and error
error = abs(y - exact)
%4-step order AB method
clear y;clc
y(1) = 1;
order = 4;
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
   k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/24)*(55*f(t(j),y(j)) - 59*f(t(j-1),y(j-1)) +
 37*f(t(j-2),y(j-2)) - 9*f(t(j-3),y(j-3)));
end
%compare approx., exact, and error
error = abs(y - exact)
%5-step order AB method
clear y;clc
y(1) = 1;
order = 5i
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/720)*(1901*f(t(j),y(j)) - 2774*f(t(j-1),y(j-1))
 + 2616*f(t(j-2),y(j-2)) - 1274*f(t(j-3),y(j-3)) +
 251*f(t(j-4),y(j-4)));
%compare approx., exact, and error
error
exact =
  Columns 1 through 3
   1.0000000000000000
                       1.188118811881188
                                           1.346153846153846
```

```
Columns 4 through 6
  1.467889908256881 1.551724137931034 1.600000000000000
 Columns 7 through 9
  1.617647058823530 1.610738255033557 1.585365853658536
 Columns 10 through 11
  1.546961325966851 1.500000000000000
y =
 Columns 1 through 3
  1.00000000000000 1.188118764675268 1.349857811269072
 Columns 4 through 6
  1.473196588677409 1.556590819080956
                                    1.602988448114635
 Columns 7 through 9
  1.618098483274705 1.608646163118815 1.581123788571847
 Columns 10 through 11
  1.541126535638776 1.493132968263878
error =
 Columns 1 through 3
                   0.000000047205920 0.003703965115226
 Columns 4 through 6
  0.005306680420528 0.004866681149921
                                    0.002988448114635
 Columns 7 through 9
  Columns 10 through 11
```

y =

Columns 1 through 3		
1.000000000000000	1.188118764675268	1.346153608557930
Columns 4 through 6		
1.466184072493249	1.548505437114575	1.595794459693531
Columns 7 through 9		
1.613103413962713	1.606388445380196	1.581537429185894
Columns 10 through 11		
1.543788714803801	1.497482320601062	
error =		
Columns 1 through 3		
0	0.000000047205920	0.000000237595916
Columns 4 through 6		
0.001705835763632	0.003218700816460	0.004205540306470
Columns 7 through 9		
0.004543644860817	0.004349809653361	0.003828424472643
Columns 10 through 1	1	
0.003172611163049	0.002517679398938	
y =		
Columns 1 through 3		
1.000000000000000	1.188118764675268	1.346153608557930
	1.188118/040/3208	1.346153608557930
Columns 4 through 6	1 55154000500500	1 (00206812001080
1.467889340629606	1.551742825287962	1.600396713821270
Columns 7 through 9		
1.618495896231417		1.586784646459038
Columns 10 through 1	1	
1.548411983332391	1.501365685577498	

error = Columns 1 through 3 0 0.000000047205920 0.000000237595916 Columns 4 through 6 0.000000567627275 0.000018687356928 0.000396713821270 Columns 7 through 9 0.000848837407888 0.001222983383127 0.001418792800502 Columns 10 through 11 0.001450657365540 0.001365685577498 y =Columns 1 through 3 1.00000000000000 1.188118764675268 1.346153608557930 Columns 4 through 6 1.467889340629606 1.551723173322634 1.600339369926289 Columns 7 through 9 1.618045412727204 1.611098226129188 1.585534486294508 Columns 10 through 11 1.546992422198530 1.499907131186876 error = Columns 1 through 3 0 0.000000047205920 0.000000237595916 Columns 4 through 6 0.000000567627275 0.000018687356928 0.000396713821270 Columns 7 through 9

5

Columns 10 through 11

Problem 2b, reusing above script and changing IVP parameters

```
clear all;clc
t = [1:.1:2];
h = .1;
y(1) = -\log(2)^{-1};
order = 2;
%define DE
f = @(t,y) y^2/(t + 1);
%run RK
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/2)*(3*f(t(j),y(j)) - f(t(j-1),y(j-1)));
end
%compare approx., exact, and error
exact = -1./(log(t + 1))
У
error = abs(y - exact)
%repeat process for 3-stop AB method
clear y;clc
y(1) = -\log(2)^{-1};
order = 3;
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/12)*(23*f(t(j),y(j)) - 16*f(t(j-1),y(j-1)) +
 5*f(t(j-2),y(j-2)));
%compare approx., exact, and error
У
error = abs(y - exact)
%4-step order AB method
clear y;clc
y(1) = -\log(2)^{-1};
order = 4;
for i = 1:(order - 1)
```

```
k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/24)*(55*f(t(j),y(j)) - 59*f(t(j-1),y(j-1)) +
 37*f(t(j-2),y(j-2)) - 9*f(t(j-3),y(j-3)));
end
%compare approx., exact, and error
error = abs(y - exact)
%5-step order AB method
clear y;clc
y(1) = -\log(2)^{-1};
order = 5i
for i = 1:(order - 1)
    k1 = h*f(t(i),y(i));
    k2 = h*f((t(i)+h/2),(y(i)+.5*k1));
    k3 = h*f((t(i)+h/2),(y(i)+.5*k2));
    k4 = h*f(t(i+1),(y(i) + k3));
    y(i+1) = y(i) + (1/6)*(k1 + 2*k2 + 2*k3 + k4);
end
for j = order:length(t) - 1
    y(j+1) = y(j) + (h/720)*(1901*f(t(j),y(j)) - 2774*f(t(j-1),y(j-1))
 + 2616*f(t(j-2),y(j-2)) - 1274*f(t(j-3),y(j-3)) +
 251*f(t(j-4),y(j-4)));
end
%compare approx., exact, and error
error
exact =
  Columns 1 through 3
  -1.442695040888963 -1.347822706464185 -1.268299403709030
  Columns 4 through 6
  -1.200611174093137 -1.142245242271581 -1.091356667937291
  Columns 7 through 9
  -1.046559939395897 -1.006794074949662 -0.971232654817011
 Columns 10 through 11
  -0.939222236853531 -0.910239226626838
y =
```

```
Columns 1 through 3
 -1.442695040888963 -1.347822674884314 -1.270097902057147
 Columns 4 through 6
 -1.203363491119358 -1.145585720370877 -1.095042885456653
 Columns 7 through 9
 -1.050436721920583 -1.010760409565795 -0.975222383960341
 Columns 10 through 11
 -0.943192007242693 -0.914160827806065
error =
 Columns 1 through 3
                 0 0.000000031579870 0.001798498348117
 Columns 4 through 6
  0.002752317026221 0.003340478099296 0.003686217519361
 Columns 7 through 9
  Columns 10 through 11
  0.003969770389162 0.003921601179227
y =
 Columns 1 through 3
 -1.442695040888963 -1.347822674884314 -1.268299355137524
 Columns 4 through 6
 -1.200137109438933 -1.141555414249868 -1.090516252148282
 Columns 7 through 9
 -1.045646517508684 -1.005846294444148 -0.970276352998182
 Columns 10 through 11
 -0.938272557686463 -0.909305377447475
```

```
error =
  1.0e-03 *
 Columns 1 through 3
              0 0.000031579870274 0.000048571506150
 Columns 4 through 6
  Columns 7 through 9
  Columns 10 through 11
  0.949679167068451 0.933849179362900
y =
 Columns 1 through 3
 -1.442695040888963 -1.347822674884314 -1.268299355137524
 Columns 4 through 6
 -1.200611116801790 -1.142395928967874 -1.091566924443562
 Columns 7 through 9
 -1.046822254707184 -1.007069701167645 -0.971516319366565
 Columns 10 through 11
 -0.939503773854617 -0.910516392292800
error =
  1.0e-03 *
 Columns 1 through 3
              0 0.000031579870274 0.000048571506150
 Columns 4 through 6
```

9

```
Columns 7 through 9
  Columns 10 through 11
  y =
 Columns 1 through 3
 -1.442695040888963 -1.347822674884314 -1.268299355137524
 Columns 4 through 6
 -1.200611116801790 -1.142245181015467 -1.091301753330997
 Columns 7 through 9
 -1.046486049761775 -1.006698439099802 -0.971138529060901
 Columns 10 through 11
 -0.939123426102761 -0.910144223217906
error =
  1.0e-03 *
 Columns 1 through 3
                 0.000031579870274 0.000048571506150
 Columns 4 through 6
  0.210256506270490
 Columns 7 through 9
  0.262315311286621 0.275626217982827 0.283664549553975
 Columns 10 through 11
```

5.9, question 2a

clear y;clc
ul(1) = -1;

```
u2(1) = 0;
h = 0.1;
t = [0:0.1:1];
f1 = @(t,u1,u2) u1 - u2 + 2;
f2 = @(t,u1,u2) -u1 +u2 +4*t;
for i = 1:length(t) - 1
    for j = 1:2
       k(1,j) = h*funcgrab1(j,t(i),u1(i),u2(i));
   end
   for j = 1:2
       k(2,j) =
 h*funcgrab1(j,t(i)+h/2,u1(i)+(k(1,1)*.5),u2(i)+(k(1,2)*.5));
    for j = 1:2
       k(3,j) =
 h*funcgrab1(j,t(i)+h/2,u1(i)+(.5*k(2,1)),u2(i)+(.5*k(2,2)));
    for j = 1:2
       k(4,j) = h \cdot funcgrab1(j,t(i)+h,u1(i)+k(3,1),u2(i)+k(3,2));
    end
  u1(i+1) = u1(i) + (k(1,1) + 2*k(2,1) + 2*k(3,1) + k(4,1))/6;
  u2(i+1) = u2(i) + (k(1,2) + 2*k(2,2) + 2*k(3,2) + k(4,2))/6;
end
u1
u2
exact u1 = -.5*exp(2.*t) + t.^2 + 2.*t - .5
exact_u2 = .5*exp(2.*t) + t.^2 - .5
%compare approx. and actual soln. with error
error_u1 = abs(exact_u1 - u1)
error_u2 = abs(exact_u2 - u2)
u1 =
  Columns 1 through 3
  Columns 4 through 6
  -0.721053228172000 -0.652760412889281 -0.609125568302967
  Columns 7 through 9
  -0.600035969125244 -0.637567932689573 -0.736471472987045
  Columns 10 through 11
  -0.914762257106377 -1.194444620829729
```

u2 =Columns 1 through 3 0 0.12070000000000 0.285908980000000 Columns 4 through 6 0.501053228172000 0.772760412889281 1.109125568302967 Columns 7 through 9 1.520035969125245 2.017567932689574 2.616471472987045 Columns 10 through 11 3.334762257106377 4.194444620829729 exact_u1 = Columns 1 through 3 -1.00000000000000 -0.900701379080085 -0.805912348820635 Columns 4 through 6 $-0.721059400195254 \quad -0.652770464246234 \quad -0.609140914229523$ Columns 7 through 9 -0.600058461368274 -0.637599983422337 -0.736516212197557Columns 10 through 11 -0.914823732206473 -1.194528049465325 exact_u2 = Columns 1 through 3 0 0.120701379080085 0.285912348820635 Columns 4 through 6 0.501059400195255 0.772770464246234 1.109140914229523Columns 7 through 9 1.520058461368273 2.017599983422337 2.616516212197558

Columns 10 through 11

```
3.334823732206473 4.194528049465325
error_u1 =
 1.0e-04 *
 Columns 1 through 3
           0 0.013790800849067 0.033688206352300
 Columns 4 through 6
 Columns 7 through 9
 0.224922430294150 0.320507327639108 0.447392105121791
 Columns 10 through 11
 error u2 =
 1.0e-04 *
 Columns 1 through 3
             0.013790800849484 0.033688206352300
 Columns 4 through 6
 Columns 7 through 9
 Columns 10 through 11
```

5.9, question 2b

```
clear y;clc
u1(1) = -3;
u2(1) = 5;
h = 0.2;
t = [0:0.2:2];
```

```
f1 = @(t,u1,u2) u1 - u2 + 2;
f2 = @(t,u1,u2) -u1 +u2 +4*t;
for i = 1:length(t) - 1
   for j = 1:2
       k(1,j) = h*funcgrab2(j,t(i),u1(i),u2(i));
   end
   for j = 1:2
       k(2,j) =
h*funcgrab2(j,t(i)+h/2,u1(i)+(k(1,1)*.5),u2(i)+(k(1,2)*.5));
   end
   for j = 1:2
       k(3,j) =
h*funcgrab2(j,t(i)+h/2,u1(i)+(.5*k(2,1)),u2(i)+(.5*k(2,2)));
   end
   for j = 1:2
       k(4,j) = h*funcgrab2(j,t(i)+h,u1(i)+k(3,1),u2(i)+k(3,2));
  u1(i+1) = u1(i) + (k(1,1) + 2*k(2,1) + 2*k(3,1) + k(4,1))/6;
  u2(i+1) = u2(i) + (k(1,2) + 2*k(2,2) + 2*k(3,2) + k(4,2))/6;
end
u1
u2
exact_u1 = -3*exp(t) + t.^2
exact_u^2 = 4*exp(t) - 3.*t + 1
%compare approx. and actual soln. with error
error u1 = abs(exact u1 - u1)
error_u2 = abs(exact_u2 - u2)
u1 =
 Columns 1 through 3
  Columns 4 through 6
  -5.106319397087947 -6.036562515167024 -7.154753457644198
 Columns 7 through 9
  -8.520215872797515 -10.205407664632787 -12.298828917101721
 Columns 10 through 11
 -14.908573632741899 -18.166667826251675
u2 =
```

```
Columns 1 through 3
  5.00000000000000 5.2856000000000 5.767271839999999
 Columns 4 through 6
  6.488425825375999 7.502083303114246 8.873004546423740
 Columns 7 through 9
 10.680287753001956 13.020543461516588 16.011771783896361
 Columns 10 through 11
 19.798098056851018 24.555556966637834
exact_u1 =
 Columns 1 through 3
 Columns 4 through 6
 -5.106356401171527 -6.036622785477403 -7.154845485377137
 Columns 7 through 9
 -8.520350768209642 -10.205599900534024 -12.299097273185344
 Columns 10 through 11
-14.908942393238840 -18.167168296791949
exact u2 =
 Columns 1 through 3
  5.00000000000000 5.285611032640679 5.767298790565081
 Columns 4 through 6
  6.488475201562036 7.502163713969871 8.873127313836182
 Columns 7 through 9
 10.680467690946189 13.020799867378699 16.012129697580459
 Columns 10 through 11
 19.798589857651784 24.556224395722602
```

```
error u1 =
 1.0e-03 *
 Columns 1 through 3
            0
               0.008265335562196 0.020194428420162
 Columns 4 through 6
 0.037004083579539 0.060270310379806
                            0.092027732939037
 Columns 7 through 9
 Columns 10 through 11
 0.368760496941789 0.500470540274023
error_u2 =
 1.0e-03 *
 Columns 1 through 3
            0
               Columns 4 through 6
 Columns 7 through 9
 0.179937944233544 0.256405862110398 0.357913684098321
 Columns 10 through 11
```

5.9, question 4a

```
clear all;clc
ul(1) = 3;
u2(1) = -1;
u3(1) = 9;
h = 0.2;
t = [0:0.2:2];
% f1 = @(t,u1,u2) u1 - u2 + 2;
```

```
% f2 = @(t,u1,u2) -u1 +u2 +4*t;
for i = 1:length(t) - 1
    for j = 1:3
       k(1,j) = h*funcgrab3(j,t(i),u1(i),u2(i),u3(i));
    for j = 1:3
       k(2,j) =
 h*funcgrab3(j,t(i)+h/2,u1(i)+(k(1,1)*.5),u2(i)+(k(1,2)*.5),u3(i)+(k(1,3)*.5));
    end
    for j = 1:3
       k(3,j) =
 h*funcqrab3(j,t(i)+h/2,u1(i)+(.5*k(2,1)),u2(i)+(.5*k(2,2)),u3(i)+(.5*k(2,3)));
    end
    for j = 1:3
        k(4,j) =
 h*funcgrab3(j,t(i)+h,u1(i)+k(3,1),u2(i)+k(3,2),u3(i)+k(3,3));
    end
  u1(i+1) = u1(i) + (k(1,1) + 2*k(2,1) + 2*k(3,1) + k(4,1))/6;
  u2(i+1) = u2(i) + (k(1,2) + 2*k(2,2) + 2*k(3,2) + k(4,2))/6;
  u3(i+1) = u3(i) + (k(1,3) + 2*k(2,3) + 2*k(3,3) + k(4,3))/6;
end
111
exact = exp(-t) + exp(2.*t) + exp(-2.*t)
error = abs(exact - u1)
u1 =
  Columns 1 through 3
   3.0000000000000000
                       2.98086666666667
                                          3.345028768888889
  Columns 4 through 6
  4.169625781639407
                       5.603146665472463
                                         7.890094976913602
  Columns 7 through 9
  11.411109228288309 16.745061449784984 24.763215867629949
  Columns 10 through 11
  36.770723451050408 54.718398568739595
exact =
 Columns 1 through 3
   3.0000000000000 2.980875496754892 3.345189938645329
  Columns 4 through 6
```

```
4.170122770742776 5.604257906506992 7.8922708233338706
 Columns 7 through 9
 Columns 10 through 11
 36.790857054346866 54.751800955269587
error =
 Columns 1 through 3
                0.000008830088225 0.000161169756440
 Columns 4 through 6
  0.000496989103369
                0.001111241034529
                              0.002175846425104
 Columns 7 through 9
  0.011973051452426
 Columns 10 through 11
```

5.9, question 4b

```
clear all;clc
u1(1) = 2i
u2(1) = 8;
u3(1) = 6;
h = 0.1;
t = [1:0.1:2];
f1 = @(t,u1,u2) u1 - u2 + 2;
f2 = @(t,u1,u2) -u1 +u2 +4*t;
for i = 1:length(t) - 1
    for j = 1:3
        k(1,j) = h \cdot funcgrab \cdot 4(j,t(i),u1(i),u2(i),u3(i));
    end
    for j = 1:3
        k(2,j) =
 h*funcgrab4(j,t(i)+h/2,u1(i)+(k(1,1)*.5),u2(i)+(k(1,2)*.5),u3(i)+(k(1,3)*.5));
    end
    for j = 1:3
        k(3,j) =
 h*funcgrab4(j,t(i)+h/2,u1(i)+(.5*k(2,1)),u2(i)+(.5*k(2,2)),u3(i)+(.5*k(2,3)));
```

```
end
    for j = 1:3
       k(4,j) =
h*funcgrab4(j,t(i)+h,u1(i)+k(3,1),u2(i)+k(3,2),u3(i)+k(3,3));
  u1(i+1) = u1(i) + (k(1,1) + 2*k(2,1) + 2*k(3,1) + k(4,1))/6;
  u2(i+1) = u2(i) + (k(1,2) + 2*k(2,2) + 2*k(3,2) + k(4,2))/6;
  u3(i+1) = u3(i) + (k(1,3) + 2*k(2,3) + 2*k(3,3) + k(4,3))/6;
end
u1
exact = 2.*t - t.^{-1} + t.^{2} + t.^{3} - 1
error = abs(exact - u1)
u1 =
 Columns 1 through 3
  2.0000000000000000
                      2.831908073282223 3.734666312717679
 Columns 4 through 6
  4.717770472743966 5.789717699109209
                                          6.958339315905182
 Columns 7 through 9
  8.231008857995716 9.614776697439808 11.116459802216719
 Columns 10 through 11
  12.742703153800006 14.500022741313533
exact =
 Columns 1 through 3
  2.0000000000000000
                      2.831909090909091 3.7346666666666
 Columns 4 through 6
  4.717769230769231 5.789714285714284 6.9583333333333333
 Columns 7 through 9
  8.23100000000000 9.614764705882351 11.1164444444444445
 Columns 10 through 11
  12.742684210526313 14.500000000000000
error =
```

1.0e-04 *

Columns 1 through 3

0 0.010176268685491 0.003539489865645

Columns 4 through 6

Columns 7 through 9

0.088579957147061 0.119915574572360 0.153577722734610

Columns 10 through 11

0.189432736927841 0.227413135327481

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