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### 3.17

```
clear all;close all;clc
phi = 45;
e = [1 0 0]';

etil = [0 -e(3) e(2);e(3) 0 -e(1);-e(2) e(1) 0];

C = expm(-deg2rad(phi)*etil);

compC = eye(3)*cosd(phi) - sind(phi)*etil + (1 -
    cosd(phi))*e*transpose(e);
```

### 3.22

```
clear all;close all;clc
syms e1 e2 e3 phi b0 b1 b2 b3
c = cos(phi);
s = sin(phi);
sig = 1 - c;

C = [e1^2*sig+c e1*e2*sig+e3*s e1*e3*sig-e2*s;e2*e1*sig-e3*s e2^2+c
    e2*e3*sig+e1*s;e3*e1*sig+e2*s e3*e2*sig-e1*s e3^2+c];

c = 2*cos(phi/2)^2 - 1;
s = 2*sin(phi/2)*cos(phi/2);
sig = 1 - c;

C = [e1^2*sig+c e1*e2*sig+e3*s e1*e3*sig-e2*s;e2*e1*sig-e3*s e2^2*sig
+c e2*e3*sig+e1*s;e3*e1*sig+e2*s e3*e2*sig-e1*s e3^2*sig+c];

% C =
%
% [
%           2*cos(phi/2)^2 - e1^2*(2*cos(phi/2)^2 - 2) - 1,
%       2*e3*cos(phi/2)*sin(phi/2) - e1*e2*(2*cos(phi/2)^2 - 2), -
%       2*e2*cos(phi/2)*sin(phi/2) - e1*e3*(2*cos(phi/2)^2 - 2)]
% [ - 2*e3*cos(phi/2)*sin(phi/2) - e1*e2*(2*cos(phi/2)^2 - 2),
%           2*cos(phi/2)^2 + e2^2 - 1,
%       2*e1*cos(phi/2)*sin(phi/2) - e2*e3*(2*cos(phi/2)^2 - 2)]
% [   2*e2*cos(phi/2)*sin(phi/2) - e1*e3*(2*cos(phi/2)^2 - 2), -
%       2*e1*cos(phi/2)*sin(phi/2) - e2*e3*(2*cos(phi/2)^2 - 2),
%           2*cos(phi/2)^2 + e3^2 - 1]

C = subs(C,e1*sin(phi/2),b1);
```

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```

C = subs(C,e2*sin(phi/2),b2);
C = subs(C,e3*sin(phi/2),b3);
% substitue (2*cos(phi/2)^2 - 2) expressions in C
% using trig identity cos(phi/2)^2 = 1 - sin(phi/2)^2
%=>
% C =
% [ 2*cos(phi/2)^2 + 2*b1^2 - 1,    2*b3*cos(phi/2) - e1*e2*(2*(1 -
    sin(phi/2)^2) - 2), - 2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) -
    2)]
% [ - 2*b3*cos(phi/2) - e1*e2*(2*(1 - sin(phi/2)^2) - 2),
    2*cos(phi/2)^2 - e2^2*(2*(1 - sin(phi/2)^2) - 2) - 1,
    2*b1*cos(phi/2) - e2*e3*(2*(1 - sin(phi/2)^2) - 2)]
% [    2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) - 2), -
    2*b1*cos(phi/2) - e2*e3*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2 -
    e3^2*(2*(1 - sin(phi/2)^2) - 2) - 1]

C = ...
[ 2*cos(phi/2)^2 + 2*b1^2 - 1,    2*b3*cos(phi/2) - e1*e2*(2*(1 -
    sin(phi/2)^2) - 2), - 2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) -
    2)
    - 2*b3*cos(phi/2) - e1*e2*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2
    - e2^2*(2*(1 - sin(phi/2)^2) - 2) - 1,    2*b1*cos(phi/2) -
    e2*e3*(2*(1 - sin(phi/2)^2) - 2)
    2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) - 2), - 2*b1*cos(phi/2)
    - e2*e3*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2 - e3^2*(2*(1 -
    sin(phi/2)^2) - 2) - 1];

%Now make substitution from equation 3.97, b0^1+b1^2+b2^2+b3^2 = 1
%=>
% C =
% [ 2*cos(phi/2)^2 + 2*b1^2 - (b0^1+b1^2+b2^2+b3^2),    2*b3*cos(phi/2)
    - e1*e2*(2*(1 - sin(phi/2)^2) - 2), - 2*b2*cos(phi/2) - e1*e3*(2*(1 -
    sin(phi/2)^2) - 2)]
% [ - 2*b3*cos(phi/2) - e1*e2*(2*(1 - sin(phi/2)^2) -
    2), 2*cos(phi/2)^2 - e2^2*(2*(1 - sin(phi/2)^2) - 2) -
    (b0^1+b1^2+b2^2+b3^2),    2*b1*cos(phi/2) - e2*e3*(2*(1 -
    sin(phi/2)^2) - 2)]
% [    2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) - 2), -
    2*b1*cos(phi/2) - e2*e3*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2 -
    e3^2*(2*(1 - sin(phi/2)^2) - 2) - (b0^1+b1^2+b2^2+b3^2)]

C = ...
[ 2*cos(phi/2)^2 + 2*b1^2 - (b0^2+b1^2+b2^2+b3^2),    2*b3*cos(phi/2) -
    e1*e2*(2*(1 - sin(phi/2)^2) - 2), - 2*b2*cos(phi/2) - e1*e3*(2*(1 -
    sin(phi/2)^2) - 2)
    - 2*b3*cos(phi/2) - e1*e2*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2
    - e2^2*(2*(1 - sin(phi/2)^2) - 2) - (b0^2+b1^2+b2^2+b3^2),
    2*b1*cos(phi/2) - e2*e3*(2*(1 - sin(phi/2)^2) - 2)
    2*b2*cos(phi/2) - e1*e3*(2*(1 - sin(phi/2)^2) - 2), -
    2*b1*cos(phi/2) - e2*e3*(2*(1 - sin(phi/2)^2) - 2), 2*cos(phi/2)^2 -
    e3^2*(2*(1 - sin(phi/2)^2) - 2) - (b0^2+b1^2+b2^2+b3^2)];

%Now substitue in the given expressions for b in terms of e1,e2, and
e3 in

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```
%equation 3.96
```

```
C = subs(C,cos(phi/2),b0);
C = subs(C,e1*sin(phi/2),b1);
C = subs(C,e2*sin(phi/2),b2);
C = subs(C,e3*sin(phi/2),b3)
```

```
C =
```

```
[ b0^2 + b1^2 - b2^2 - b3^2,          2*b0*b3 + 2*b1*b2,
  2*b1*b3 - 2*b0*b2]
[          2*b1*b2 - 2*b0*b3, b0^2 - b1^2 + b2^2 - b3^2,
  2*b0*b1 + 2*b2*b3]
[          2*b0*b2 + 2*b1*b3,          2*b2*b3 - 2*b0*b1, b0^2 - b1^2 -
  b2^2 + b3^2]
```

## 3.23

```
clear all;close all;clc
%Use eqn 3.98 with eqn 3.102 and equate values
syms b0 b1 b2 b3 b4 bp0 bp1 bp2 bp3 bpp0 bpp1 bpp2 bpp3
%Create the FN matrix in eqn 3.102
FN = [b0^2+b1^2-b2^2-b3^2, 2*(b1*b2+b0*b3), 2*(b1*b3-b0*b2);
      2*(b1*b2-b0*b3), b0^2-b1^2+b2^2-b3^2, 2*(b2*b3+b0*b1);
      2*(b1*b3+b0*b2), 2*(b2*b3-b0*b1), b0^2-b1^2-b2^2+b3^2];
%Create the FB matrix in eqn 3.102
BN = [bp0^2+bp1^2-bp2^2-bp3^2, 2*(bp1*bp2+bp0*bp3), 2*(bp1*bp3-
bp0*bp2);
      2*(bp1*bp2-bp0*bp3), bp0^2-bp1^2+bp2^2-bp3^2,
      2*(bp2*bp3+bp0*bp1);
      2*(bp1*bp3+bp0*bp2), 2*(bp2*bp3-bp0*bp1), bp0^2-bp1^2-
bp2^2+bp3^2];
%Create the BN matrix in eqn 3.102
FB = [bpp0^2+bpp1^2-bpp2^2-bpp3^2,
      2*(bpp1*bpp2+bpp0*bpp3), 2*(bpp1*bpp3-bpp0*bpp2);
      2*(bpp1*bpp2-bpp0*bpp3), bpp0^2-bpp1^2+bpp2^2-bpp3^2,
      2*(bpp2*bpp3+bpp0*bpp1);
      2*(bpp1*bpp3+bpp0*bpp2), 2*(bpp2*bpp3-bpp0*bpp1), bpp0^2-
bpp1^2-bpp2^2+bpp3^2];

A = [bpp0 -bpp1 -bpp2 -bpp3;
      bpp1 bpp0 bpp3 -bpp2;
      bpp2 -bpp3 bpp0 bpp1;
      bpp3 bpp2 -bpp1 bpp0];

b = [bp0;bp1;bp2;bp3];

eqn3p103 = A*b;

C = FB*BN;
%Observe how ANS.b0(1) through ANS.b3(1) are equivalent to the four
rows
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    %of eqn3p103, verifying the composite rotation of equations 3.103 and
    %3.104 using equation 3.98 in 3.102
ANS = solve(FN == C,[b0 b1 b2 b3]);
ANS.b0(1)
ANS.b1(1)
ANS.b2(1)
ANS.b3(1)
eqn3p103

ans =

bp0*bpp0 - bp1*bpp1 - bp2*bpp2 - bp3*bpp3

ans =

bp0*bpp1 + bp1*bpp0 + bp2*bpp3 - bp3*bpp2

ans =

bp0*bpp2 + bp2*bpp0 - bp1*bpp3 + bp3*bpp1

ans =

bp0*bpp3 + bp1*bpp2 - bp2*bpp1 + bp3*bpp0

eqn3p103 =

bp0*bpp0 - bp1*bpp1 - bp2*bpp2 - bp3*bpp3
bp0*bpp1 + bp1*bpp0 + bp2*bpp3 - bp3*bpp2
bp0*bpp2 + bp2*bpp0 - bp1*bpp3 + bp3*bpp1
bp0*bpp3 + bp1*bpp2 - bp2*bpp1 + bp3*bpp0

```

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