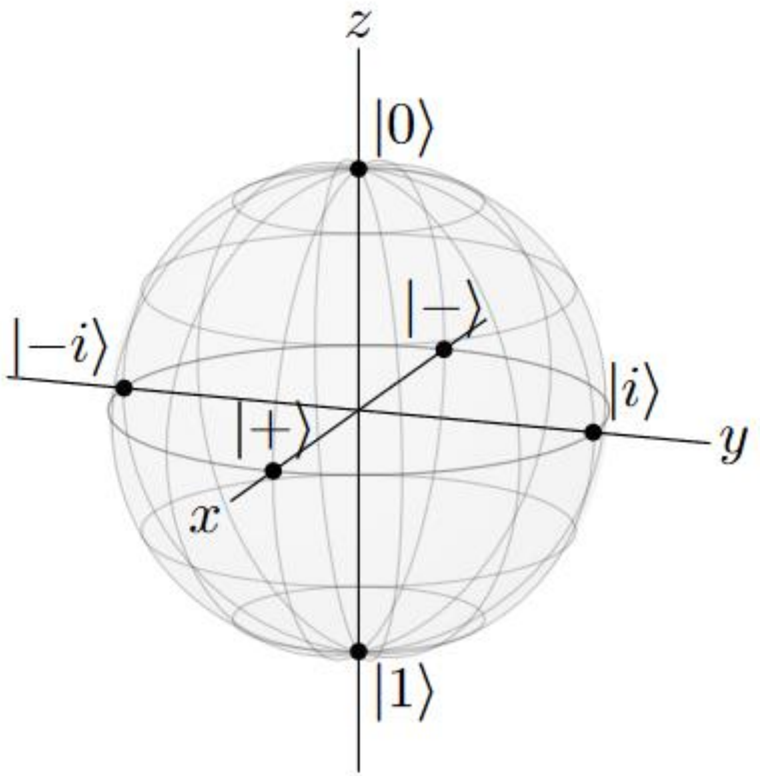
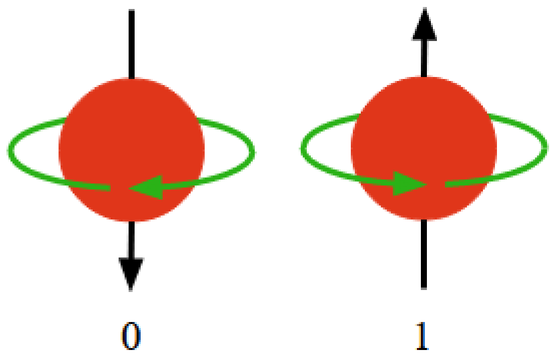
Qubits



Source: Wong, T.G. (2022) ‘One Quantum Bit’, in *Introduction to classical and quantum computing*. Omaha: Rooted Groove, p. 78.

What are Qubits?

Physically: Any Quantum System with two distinct states like spins, polarization, energy levels



Computationally: A value that is both 0 and 1 till it collapses into one of them upon measurement

Image SRC: http://davidbkemp.github.io/QuantumComputingArticle/

How do Qubits Differ from Bits?

- Bits can either be 1 <u>OR</u> 0 at a time
- Qubits can be both 1 <u>AND</u> 0 simultaneously.
- Bits have Binary Discrete States
- Qubits are a Linear Combination of States

Representation of Qubits

Qubits or Quantum Bits have the same values of 0 and 1 like classical bits, but are written in Bra-ket Notation as follows:

$$|0\rangle, |1\rangle.$$

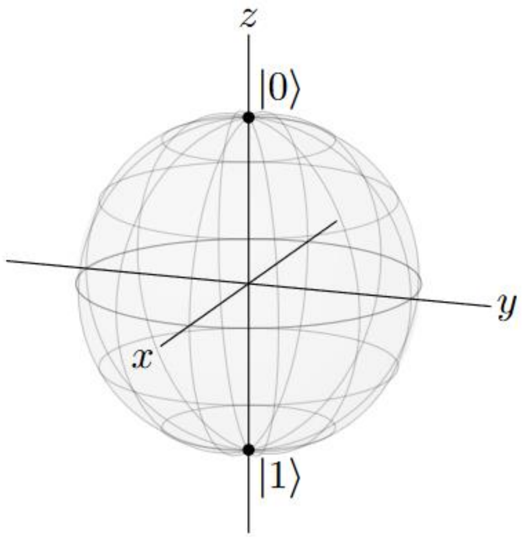
As mentioned earlier, a qubit state is a Linear Combination of both $|0\rangle$ and $|1\rangle$, a superposition if you will. Hence a qubit state is expressed as:

$$\alpha|0\rangle + \beta|1\rangle$$

Where α and β are complex coefficients that are used for measuring the Qubit.

The Bloch Sphere

- A Qubit can be visualized as a point on the Bloch Sphere
- A Bloch Sphere is a sphere of radius 1 unit
- The surface of the sphere represents all possible states of the qubit



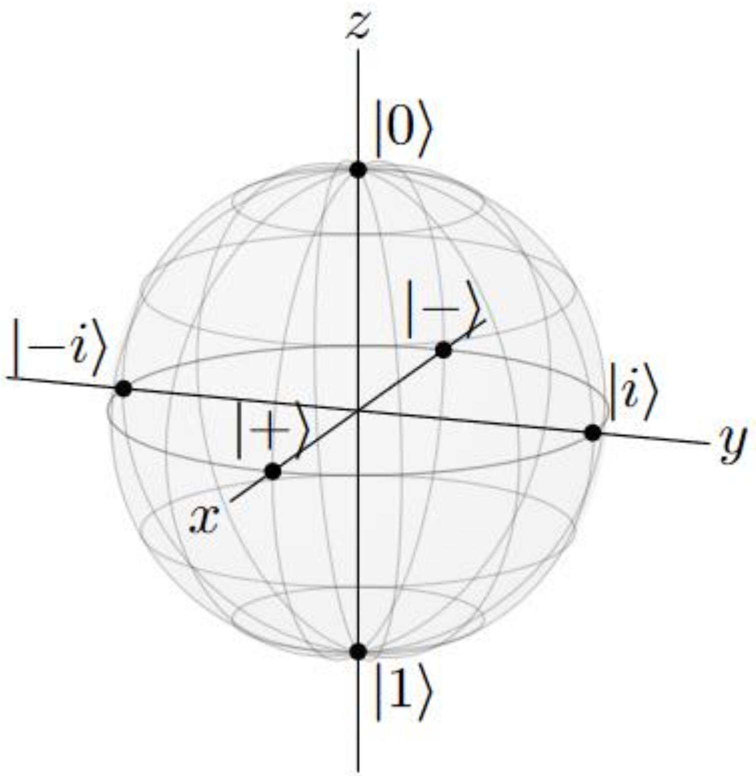
Some Common Qubit States

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),$$

$$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle),$$

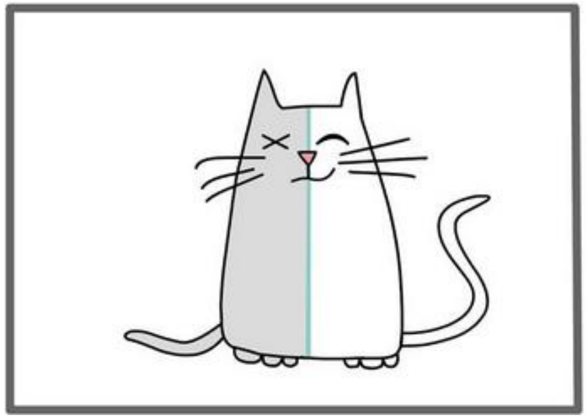
$$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle).$$



Measuring a Qubit

Measurement collapses a Qubit into a $|0\rangle$ or $|1\rangle$.

Example:



Qubits as Vectors

- Qubits can be written as column vectors:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Hence Quantum States can also be represented as column vectors:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Multi-Qubit Representation

- A Quantum Computer has multiple Qubits in it just like Classical Computers can handle multiple bits
- The states of the Multiple Qubits are represented by a Tensor Product $\otimes$, written as $|0\rangle \otimes |0\rangle$, which is often compressed to $|00\rangle$
- Multiple Qubit states follow the same rule as Classical Bits when it comes to the number of possible values:

$$Possible\ States = 2^n, where\ n\ is\ the\ number\ of\ Qubits$$

Multiple Qubits VS Multiple Bits

- Given 2 Qubits, the possible states in the z-axis are:

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

- With the Superposition Defined as:

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

- A Classical 2-bits State can only be one of 00, 01, 10 or 11
- A 2-Qubit Quantum State is all of them at the same time!
- This scales up as the No. of Qubits increases and is a very important result in Quantum Computing.

Measuring Multiple Qubits

Thanks for Watching!

See You in the Next Video