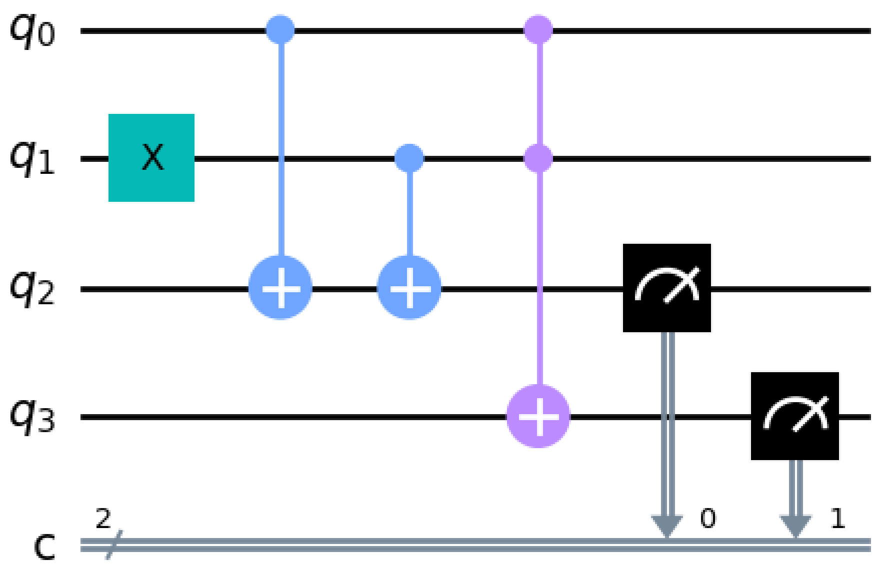
Quantum Logic Gates and Quantum Circuits



SIT111 Task 1.6HD Rishant Sharma 222458666

What are Quantum Logic Gates?

- Quantum Logic Gates are essentially Linear Maps or Matrix Transformations
- These Gates Preserve the Total Probability of a State
- Quantum Logic Gates are reversible

Quantum Logic Gates VS Classical Logic Gates

- All Quantum Gates are reversible
- Classical Logic Gates are not always reversible
- Output of a Quantum Logic Gate is *Measured*
- Output of a Classical Logic Gate is *Mapped*
- Quantum Logic Gates take advantage of Quantum Phenomenon
- Classical Logic Gates can only make use of Boolean Logic

1-Qubit Logic Gates

- 1-Qubit Logic Gates apply on single qubits only

- All 1-Qubit Logic Gates are rotations on the Bloch Sphere

Identity Gate

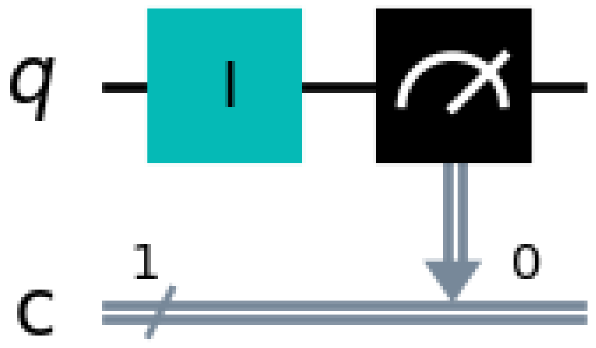
The identity gate turns $|0\rangle$ into $|0\rangle$ and $|1\rangle$ into $|1\rangle$, hence doing nothing:

$$I|0\rangle = |0\rangle$$
$$I|1\rangle = |1\rangle$$

This is a classical reversible gate (the identity gate), so it keeps states normalized and is a valid quantum gate.

Its Matrix Form is:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



Pauli X-Gate

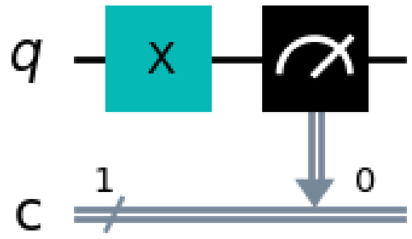
The Pauli X gate, or NOT gate, turns $|0\rangle$ into $|1\rangle$, and $|1\rangle$ into $|0\rangle$:

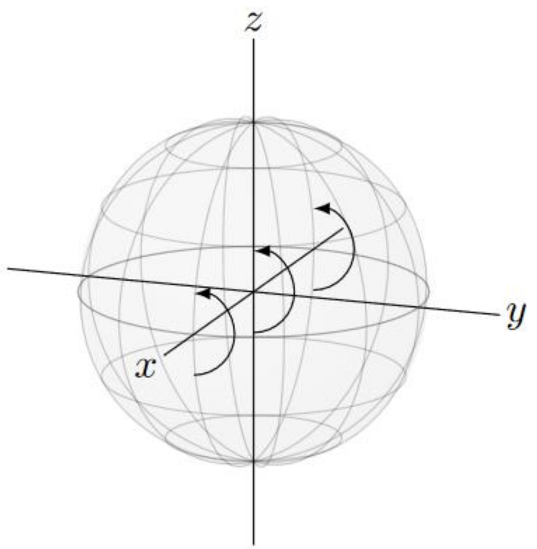
$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

This is a classical reversible gate (the NOT gate), so it keeps states normalized and is a valid quantum gate.

Its Matrix Form is:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$





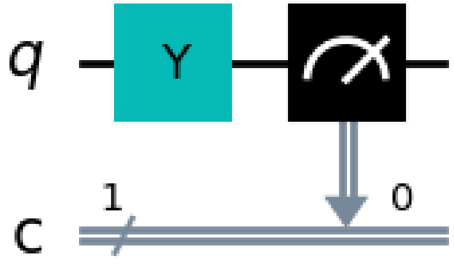
Pauli Y-Gate

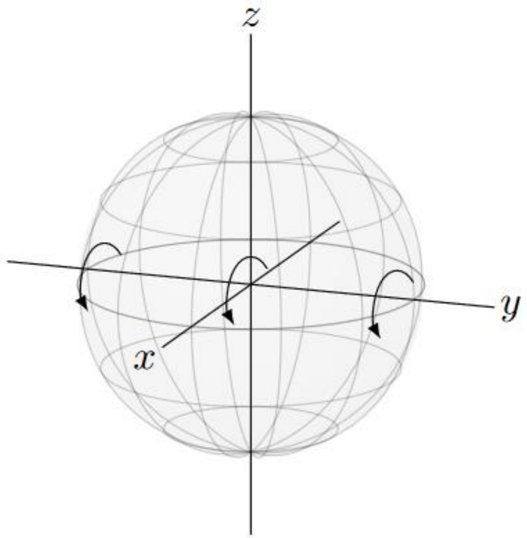
The Pauli Y gate turns |0⟩ into i|1⟩, and |1⟩ into -

$$Y\,|0\rangle = i|1\rangle$$
$$Y\,|1\rangle = -i|0\rangle$$

Its Matrix Form is:

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$





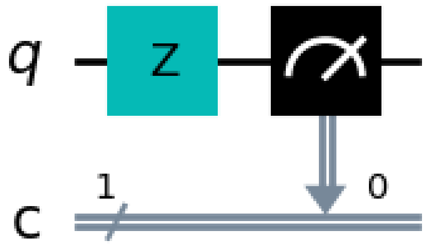
Pauli Z-Gate

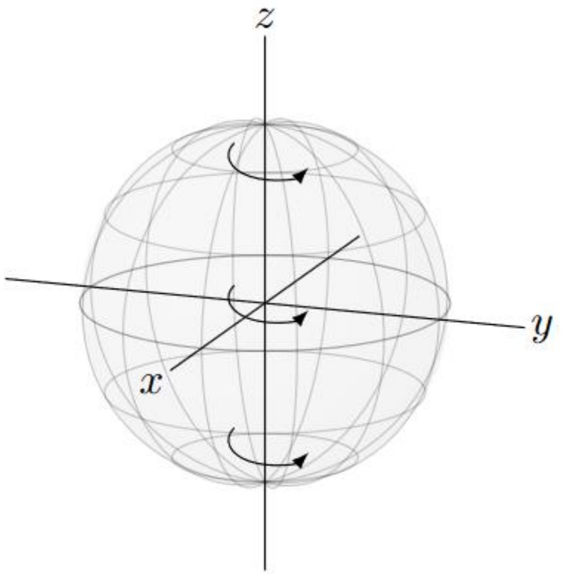
he Pauli Z gate keeps |0⟩ as |0⟩ and turns |1⟩ into −|1⟩:

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

Its Matrix Form is:

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$





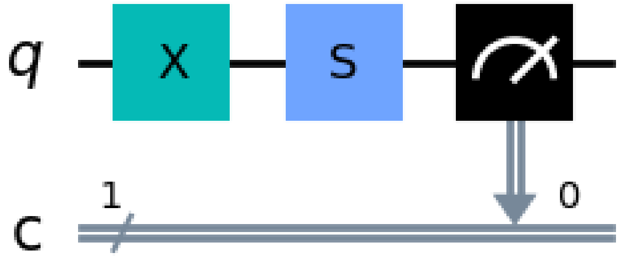
Phase Gate

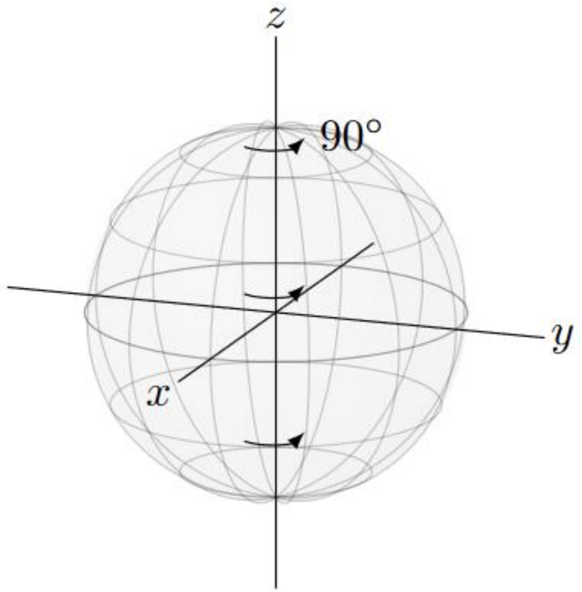
Phase gate is the square root of the Z gate (i.e., $S^2 = Z$):

$$S|0\rangle = |0\rangle$$
$$S|1\rangle = i|1\rangle$$

Its Matrix Form is:

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$





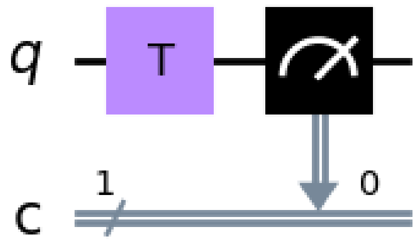
T Gate

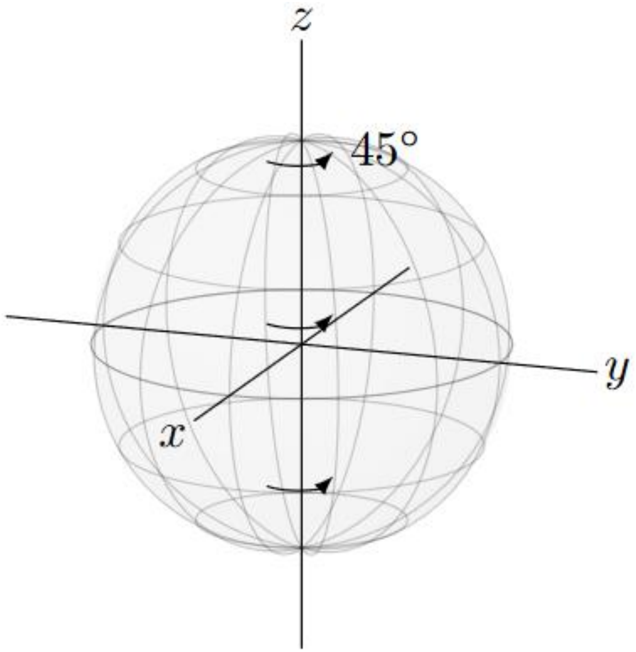
T gate (also called π/8 gate) is the square root of the S gate (i.e., $T^2 = S$) or fourth root of the Z gate:

$$T\,|0\rangle = |0\rangle$$
$$T\,|1\rangle = e^{i\pi/4}|1\rangle$$

Its Matrix Form is:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$





Hadamard Gate

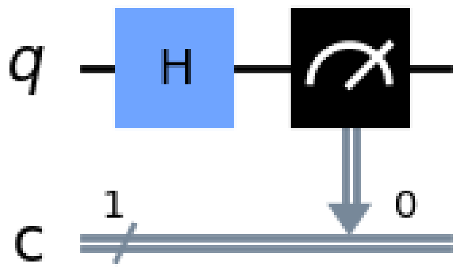
The Hadamard gate turns $|0\rangle$ into $|+\rangle$, and $|1\rangle$ into $|-\rangle$:

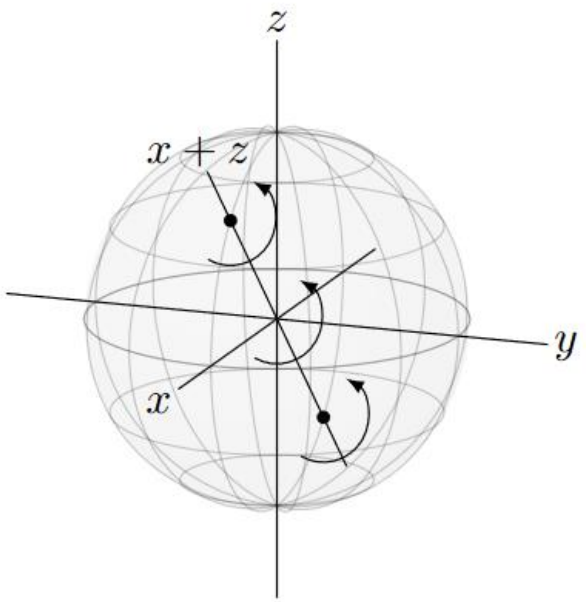
$H|0\rangle = 1/\sqrt{2}\,(|0\rangle + |1\rangle) = |+\rangle$,

$H|1\rangle = 1/\sqrt{2}\,(|0\rangle - |1\rangle) = |-\rangle$

Its Matrix Form is:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$





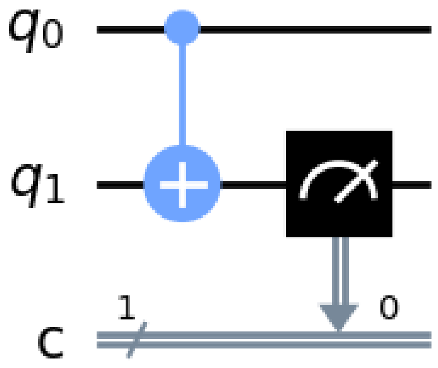
Multi-Qubit Gates

- These Logic Gates operate on Multiple Qubits
- These often involve controlled operations
- They can create and manipulate entanglement in Qubits

CNOT Gate

The CNOT gate or controlled-NOT gate inverts the right qubit if the left qubit is 1:

$$\mathrm{CNOT}|00\rangle = |00\rangle$$
$$\mathrm{CNOT}|01\rangle = |01\rangle$$
$$\mathrm{CNOT}|10\rangle = |11\rangle$$
$$\mathrm{CNOT}|11\rangle = |10\rangle$$



Thus, CNOT is a quantum XOR gate. Also, since the X gate is the NOT gate, the CNOT gate is also called the CX gate or controlled-X gate.

Its Matrix Form is:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

SWAP Gate

The SWAP gate simply swaps the two qubits:

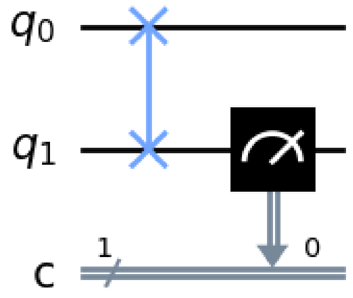
$$SWAP|00\rangle = |00\rangle$$
$$SWAP|01\rangle = |10\rangle$$
$$SWAP|10\rangle = |01\rangle$$
$$SWAP|11\rangle = |11\rangle$$

In other words,

$$SWAP|a\rangle|b\rangle = |b\rangle|a\rangle$$

Its Matrix Form is:

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



Toffoli Gate

A three-qubit gate that often appears in quantum computing is the Toffoli gate, or controlled-controlled-NOT gate. It flips the right qubit if the left and middle qubits are 1:

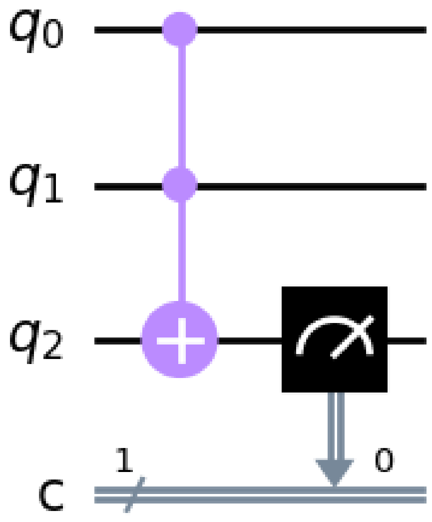
$$\text{Toffoli}|000\rangle = |000\rangle$$
$$\text{Toffoli}|001\rangle = |001\rangle$$
$$\text{Toffoli}|010\rangle = |010\rangle$$
$$\text{Toffoli}|011\rangle = |011\rangle$$
$$\text{Toffoli}|100\rangle = |100\rangle$$
$$\text{Toffoli}|101\rangle = |101\rangle$$
$$\text{Toffoli}|110\rangle = |111\rangle$$
$$\text{Toffoli}|111\rangle = |110\rangle$$

In other words,

$$\text{Toffoli}|a\rangle|b\rangle|c\rangle = |a\rangle|b\rangle|ab \oplus c\rangle$$

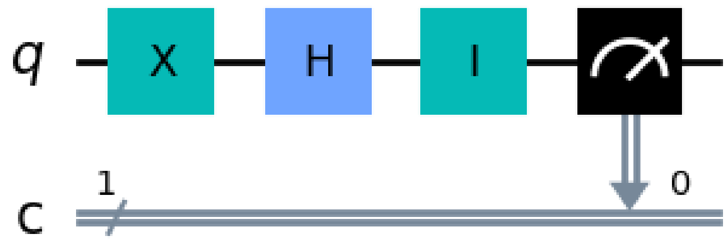
Its Matrix Form is:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



Quantum Circuits

- Quantum Circuits are essentially consecutive linear mapping performed on Qubits
- They are made by arranging the gates in a specific order
- The order and arrangement of gates determine the quantum computation performed by the circuit



Quantum Circuits and Classical Circuits

- Quantum Circuits perform *transformations* on Qubits
- Classical Circuits perform *operations* on Bits
- Quantum Circuits outputs are *probabilistic*
- Classical Circuits outputs are *deterministic*
- Quantum Circuits are better at computations that require large-scale parallelism, quantum algorithms, and certain types of optimization problems
- Classical Circuits are better at computations that require serial processing, Large-Scale Data Processing and involve classical logic and arithmetic

Half Adders: A Comparative Case Study