Age-structured model for Alaska herring stocks

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Abstract

1 Introduction

2 Model deconstruction

This section is intended to serve three purposes: 1) to document the model structure given the code in model.tpl, 2) to detail proposed changes to the model code to improve overall numerical stability, and 3) provide statistical approach that is amenable to maximum likelihood and Bayesian methods.

In Table 1, I start with the objective function that is being minimized, then with each component of the objective function work backwards to the parameters that are being estimated given the data. There are four components defined in (T1.1), where three of the four components are scaled by coefficients λ.

3 Methods

3.1 Input Data

3.2 Population dynamics

Estimated parameters for the population dynamics model include the initial abundance of ages 3-9+ for the intial year, abundance of age-3 recruits each year, and the natural mortality rate. In the original parameterizeation of the model, these initial recruitments and the vector of initial numbers-at-age result in creating $(N + A - 1)$ scaling parameters. To reduce the potential confounding with other global scaling parmaters, updates to the model code include estimation of two recruitment scaling parameters, and two vectors of deviates that represent deviations from the mean. This modification reduces the potential for parameter confounding among the many paraemters that affect global scaling (i.e., catchabiltiy coefficients, natural mortlaity rates).

Table 1: Model equations implied from the code in model.tpl.

Objective function		
$f = \lambda_C QC + \lambda_S QS + WQE + \lambda_R QR$		(T1.1)
$QC = \sum_i res_cc omp$	catch-at-age residuals	(T1.2)
$QS = \sum_i res_s p_c omp$	spawning catch-age residuals	(T1.3)
WQE	Egg deposition residuals	(T1.4)
$QR = \sum_i r_i - \alpha S_i \exp(-\beta S_i)$	recruitment residuals	(T1.5)
Estimated parameters		
$\vec{r}, \vec{\ddot{n}},$		(T1.6)
Initial states ($i = 1$)		
$N_{i,j} = \begin{cases} r_i & j = 1 \\ \ddot{n}_j & j \in \{2, \ldots, A\} \end{cases}$	Numbers-at-age	(T1.7)
$O_{i,j} = N_{i,j}\varphi_{i,j}$	Vulnerable numbers-at-age	(T1.8)
$o_i = \sum_j O_{i,j}$	Vulnerable population numbers	(T1.9)
$P_{i,j} = O_{i,j}/o_i$	Vulnerable proportion-at-age	(T1.10)
		(T1.11)

Table 2: Notation and equations for population dynamics model.

Model parameters	
$\theta = \{\ln(M), \ln(\bar{R}), \ln(\ddot{R}), \ln(\alpha), \ln(\beta)\}$	(T2.1)
Initial States ($t = 1$)	

Table 3: Mathematical notation, symbols and descriptions.

Symbol	Description
Index	
g	group
h	sex
i	year
j	time step (years)
k	gear or fleet
l	index for length class
m	index for maturity state
o	index for shell condition.
Leading Model Parameters	
M	Instantaneous natural mortality rate
$\bar{R}$	Average recruitment
$\ddot{R}$	Initial recruitment
α_r	Mode of size-at-recruitment
β_r	Shape parameter for size-at-recruitment
R_0	Unfished average recruitment
κ	Recruitment compensation ratio
Size schedule information	
$w_{h,l}$	Mean weight-at-length l
$m_{h,l}$	Average proportion mature-at-length l
Per recruit incidence functions	
ϕ_B	Spawning biomass per recruit
ϕ_{Q_k}	Yield per recruit for fishery k
ϕ_{Y_k}	Retained catch per recruit for fishery k
ϕ_{D_k}	Discarded catch per recruit for fishery k
Selectivity parameters	
$a_{h,k,l}$	Length at 50% selectivity in length interval l
$\sigma_{s_{h,k}}$	Standard deviation in length-at-selectivity
$r_{h,k,l}$	Length at 50% retention
$\sigma_{y_{h,k}}$	Standard deviation in length-at-retention
$\xi_{h,k}$	Discard mortality rate for gear k and sex h

References