Constructing the Eiffel Tower using Geomags

By: Shubham Aggarwal

① The Eiffel tower consists of 18,038 pieces, but I was unable to find the dimensions of each of these pieces.

② For my first attempt, I tried to stack up the geomags on top of each other by creating a cube shape using the geomags. However, this process didn't work well because I couldn't account for the 54° angle.

③ My second approach provides a much more accurate description as to how to build the Eiffel tower using geomags while simultaneously accounting for the 54° angle.

- One problem with this approach is that we are ignoring the overlap of geomags breadth-wise

Assumptions: I am just constructing raw structure and not taking into account the steps, the platforms, lights, railings for the platforms, and the antenna.

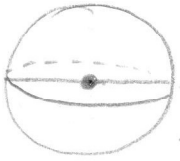
Geomag Dimensions

Geomag Rod

length = 27.00mm
= 0.027 m

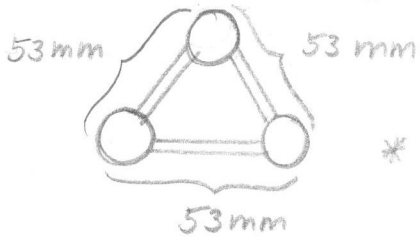
Geomag Sphere

0.013 m



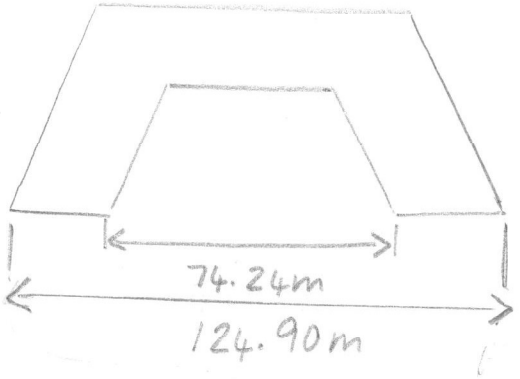
diameter = 13 mm
radius = 6.5 mm

Geomag Triangle



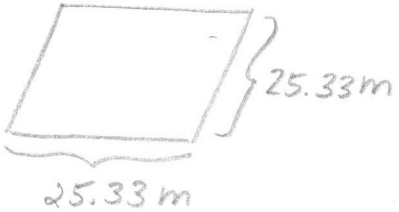
* 1 rod + 2 spheres = length

length = 27.00mm + 2(13mm)
= 27 mm + 26 mm
= 53 mm
= 0.053 m

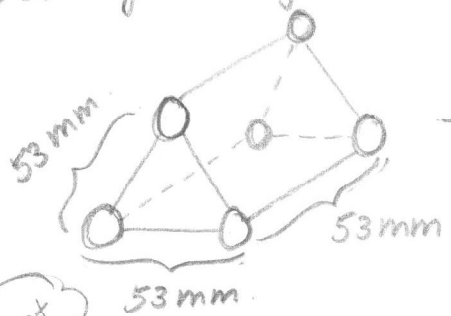


length of base: 124.90m − 74.24m
= 50.66m
2
= 25.33 m

Base of Eiffel Tower (leg)

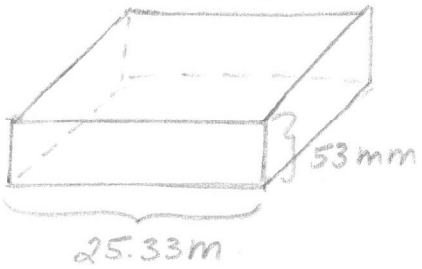


Geomag Triangular Prism



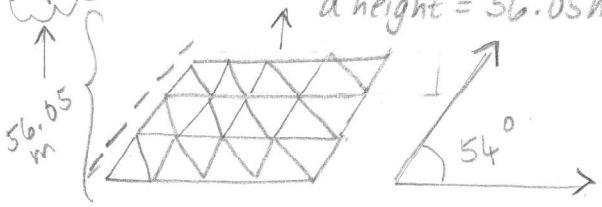
Use this shape to fill in the cube →

Box Dimensions



↳ Base of leg on eiffel tower.

not slant height



- fill in box with triangular prisms
- Stack up the cuboid on top of each other and account for the 54° angle
- height until the first platform = 56.05 m
- stack cuboids until it reaches a height of 56.05 m

Volume of Box:

Volume = $l \times w \times h$

$= 25.33\,m\,(25.33\,m)(53\,mm)$

$= 25.33\,m\,(25.33\,m)\,(0.053\,m)$

$= 641.6089\,m\,(0.053\,m)$

$= \boxed{34.005\,m^3}$

* Assumption: I have used the triangular prism as my base structure because while I was doing research, I found that this structure is rigid and doesn't break easily

Volume of Triangular Prism:

Volume = (base x height x length)/2

base = 53 mm or 0.053 m

height = 0.053 m

length = 0.053 m

Volume = $[(0.053\,m)(0.053\,m)(0.053\,m)]/2$

$= \dfrac{1.488 \times 10^{-4}\,m^3}{2}$

$= \boxed{7.4438 \times 10^{-5}\,m^3}$

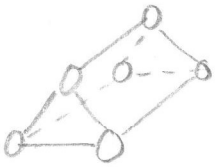
How many prisms fit into the box (cuboid)?

$$\frac{\text{Volume of Box}}{\text{volume of Triangular prism}} = \frac{34.005\,m^3}{(7.4438 \times 10^{-5}\,m^3)}$$

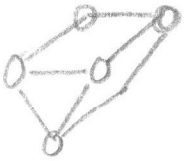
$= \boxed{456,823 \text{ prisms}}$

Number of Geomags needed to fill one cuboid (box):

First layer:



+ 5 rods & 2 balls ⟹

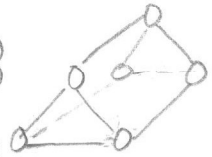


* accounts for overlap of rods and balls going left to right

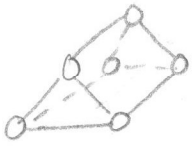
- This approach will work when we stack the cuboid boxes on top of each other to form multiple layers.

* This approach does not account for overlap of geomags front to back

1st prism has 6 rods and 3 balls so we take away 1 prism and calculate for remaining prisms



+ 6 rods & 3 balls ⟹



FRONT TO BACK!

* There is overlap of geomags *.

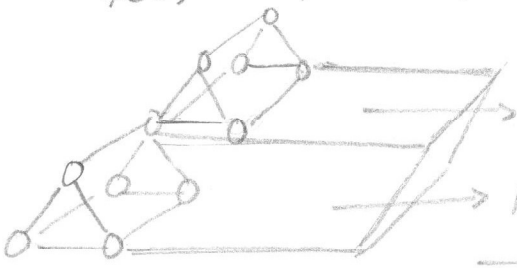
456,822 prisms (5 rods) + 9 = 2284119 rods

456,822 prisms (2 balls) + 6 = 913650 balls

From second layer onwards: 456,822 prisms (5 rods) + 5 = 2284115 rods

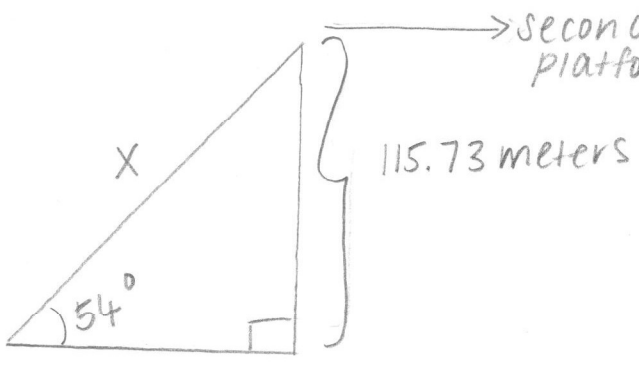
456,822 prisms (2 balls) + 2 = 913646 balls

Diagram:



Calculating Slant height:

- height until second platform = 115.73 meters
- create a right triangle and use SOH CAH TOA to determine slant height



$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin 54° = \frac{115.73\,m}{x}$

solve for x: $\frac{\sin 54°\,(x)}{\sin 54°} = \frac{115.73\,m}{\sin 54°}$

$\boxed{x = 207.1085 \text{ meters}}$

Determine number of layers (until second platform):

height until 2nd platform = 115.73 meters

each cube is 0.053 meters tall

$\therefore$ number of layers $= \frac{115.73 \text{ meters}}{0.053 \text{ meters}}$

$= 2183.58$ layers

$\approx$ (2184 layers)

Total number of geomags per layer:

Total # of layers = 2184

1st layer $\Rightarrow$ 2284119 rods and 913650 balls

2nd layer onwards: 2183 layers (2284115 rods)

= (4986223045 rods)

— AND —

2183 layers (913646 balls)

= (1994489218 balls)

Total # of Geomags $\Rightarrow$ 2284119 rods + 4986223045 rods

= 4988507164 rods

— AND —

913650 balls + 1994489218 balls

= 1995402868 balls

* NOTE THAT THIS IS ONLY FOR ONE LAYER *

- The Eiffel tower has 4 legs so multiply number of balls and rods by 4 legs

TOTAL $\Rightarrow$ 4988507164 rods (4) = (19954028660 rods)

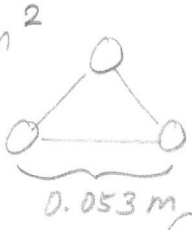
1995402868 balls (4) = (7981611472 balls)

Calculating number of geomags needed for each platform:

* I am assuming that the platforms are flat

Area of 1st platform = $4,010\ m^2$

Area of geogmag triangle



$$\text{Area} = \frac{1}{2}bh$$
$$= \frac{1}{2}(0.053\,m)(0.053\,m)$$
$$= 1.4 \times 10^{-3}\ \text{meters}^2$$

* We will be constructing the platform by using triangle shaped geomags to fill up the area of the platform (I am accounting for overlap of geomags)

Total # of triangles for platform ①:

$4010\ m^2 \div 1.4 \times 10^{-3} = 2855108.58$ triangles

≈ 2855109 triangles

does not include 1st triangle

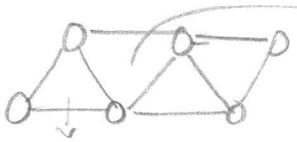
Total # of geomags: 2855108 triangles (2 rods) + 3 = 5710219 rods

accounts for the 3 rods in the first triangle and overlap of geomags from left to right

2855108 triangles (1 ball) + 3 = 2855111 balls

same logic but for sphere balls

Diagram:



2nd triangle needs 2 rods & 1 ball

1st triangle has 3 rods & 3 balls

Total number of geomags needed for platform ②:

* similar calculation to platform ①

Area of 2nd platform = $1360\ m^2$

Total # of triangles = $\frac{1360\ m^2}{1.4 \times 10^{-3}\ m^2}$

$= 971428.5714$ triangles ≈ 971429 triangles

Total # of Geomags: 971428 triangles (2 rods) + 3 = 1942859 rods

971428 triangles (1 ball) + 3 = 971431 balls

Total number of geomags needed for platform ③:

Area of 3rd platform = $264.31\ m^2$

Total # of triangles = $\frac{264.31\ m^2}{1.4 \times 10^{-3}\ m^2}$ = 188792.8571 triangles

≈ 188793 triangles

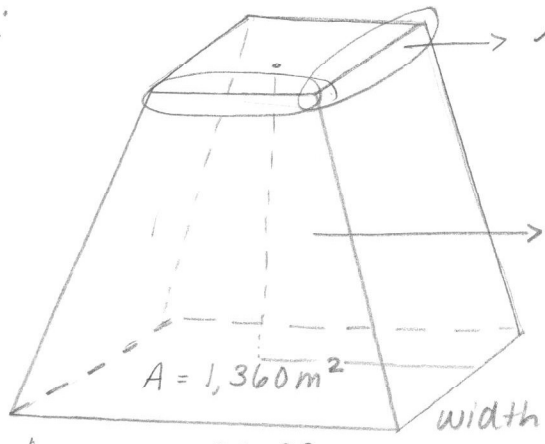
Total # of Geomags: 188792 triangles (2 rods) + 3 = (377587 rods)

188792 triangles (1 ball) + 3 = (188795 balls)

Truncated Square Pyramid

- the truncated square pyramid extends above the 2nd platform

Diagram:



length & width = $\sqrt{264.31}$ = 16.26 m (upper base)

height = 160.4 m

width = 36.89 m (lower base)

length = 36.89 m

$\sqrt{1360}$ = 36.89 m.

Calculate volume:

$V = \frac{1}{3}(a^2 + ab + b^2)(h)$

a = length of lower base

b = length of upper base

h = height

* I will be using triangular prisms to construct the truncated square pyramid
* volume of truncated pyramid divided by volume of prisms

Volume = $\frac{1}{3}\left[(36.89)^2 + (36.89)(16.26) + (16.26)^2\right](160.4)$

= $\frac{1}{3}(1360.8721 + 599.8314 + 264.3876)(160.4)$

= $\frac{1}{3}(2225.091)(160.4)$

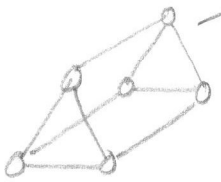
= $(118968.1988\ m^3)$

Total number of triangular prisms needed to construct truncated square pyramid:

$$\frac{118968.1988\ m^3}{7.4438 \times 10^{-5}\ m^3}$$ → volume of truncated square pyramid
→ volume of triangular prism

= **1598218636 prisms.**

First layer of truncated pyramid ⇒



→ 6 balls and 9 rods to start

+ 5 rods and 2 balls for prisms added left to right

From second layer onwards ⇒ adding 5 rods and 2 balls

* 1st prism on 2nd layer will only need 5 rods and 2 balls

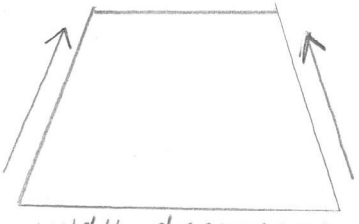
Total # of Geomags needed for truncated square pyramid:

1598218635 prisms (5 rods) + 9 = **7991093184 rods**

1598218635 prisms (2 balls) + 6 = **3196437276 balls**

- This method will have a lot of overlap because the truncated square pyramid doesn't have a constant breadth (width decreases as you go up → hence why we cannot use stack approach!

Diagram:

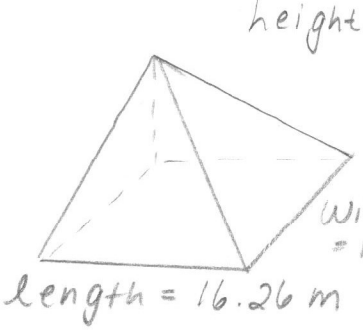


Tip of Eiffel Tower

- I am assuming it is a square pyramid

Volume = $\frac{1}{3}Bh$, B = base area

Base area = $l \times w$
= 16.26 m (16.26 m)
= 264.3876 m^2



Volume = $\frac{1}{3}(264.38)(24)$

V = 2115.1008 m^3

Number of triangular prisms enclosed in square pyramid:

$$\frac{\text{Volume of square pyramid}}{\text{Volume of triangular prism}} = \frac{2115.1008\ m^3}{7.4438 \times 10^{-5}\ m^3}$$

$$= 28414261.53 \text{ prisms}$$

$$\approx \boxed{28414262 \text{ prisms.}}$$

Total # of Geogmags in square pyramid:

28414261 prisms (5 rods) + 9 = 142071314 rods

28414261 prisms (2 balls) + 6 = 56828528 balls

* Not accounting for overlap of prisms

WE ARE NOT CALCULATING THE # OF GEOMAGS NEEDED FOR ANTENNA!

Total # of Geomags needed to construct Eiffel Tower:

# of rods for 4 legs : 19954028660 rods

rods of platform ① : 5710219 rods

platform ② : 1942859 rods

platform ③ : 377587 rods +

rods in truncated square pyramid : 7991093184 rods

rods in square pyramid : 142071314 rods

$$\boxed{28095223820 \text{ rods}}$$

# of Balls — 7981611472 balls → balls for 4 legs

2855111 balls → balls of platform ①

971431 balls → platform ②

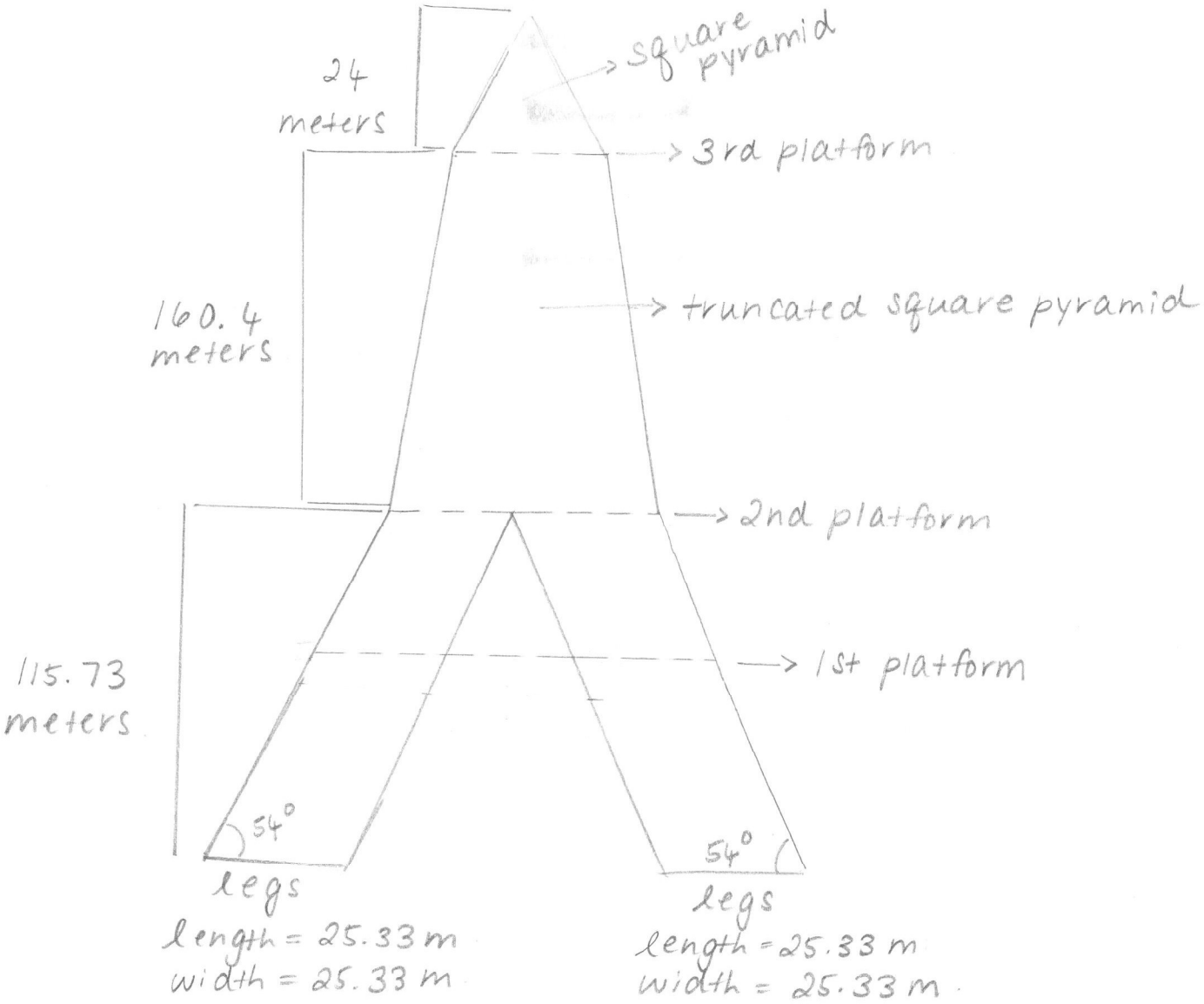
188795 balls + → platform ③

3196437276 balls → truncated square pyramid

56828528 balls → square pyramid

$$\boxed{11238892610 \text{ balls}}$$

Diagram of Eiffel Tower based on what we calculated and assume it will look like using Geomags



NOTE: I know this doesn't really resemble the Eiffel tower, but I personally think it's better... Am I right!? ☺