Price of Abstention in Participatory Budgeting

Sagi Sameah
Department of Industrial Engineering and Management
Ben-Gurion University of the Negev
sagisa@post.bgu.ac.il

Abstract

This paper investigates the relationship between voter turnout and satisfaction in participatory budgeting using a probabilistic framework to model voter behavior. By employing modular simulation workflows, exploring aggregation algorithms, and implementing robust statistical techniques such as strata-based sampling and bootstrapping, the study examines turnout dynamics and satisfaction. Contrary to traditional expectations that higher turnout inherently increases satisfaction due to enhanced representativeness, our findings reveal a more complex dynamic: higher turnout often introduces diversity that reduces overall satisfaction. We introduce Shannon entropy as a proxy for diversity and investigate how it interacts with turnout levels under the Greedy rule. Our results suggest that while higher turnout improves satisfaction in homogeneous settings, this relationship weakens—or even reverses—when preference diversity is high. Regression analyses reveal weak predictive power of entropy alone, underscoring the importance of multi-factorial explanations. We discuss implications for aggregation rule design and highlight entropy-based diagnostics for identifying when proportional or deliberative methods may outperform majority-based aggregation. Using aggregation algorithms under varying turnout scenarios, the study highlights trade-offs between inclusivity and voter satisfaction. These insights contribute to optimizing participatory budgeting systems, offering actionable strategies for balancing representativeness and alignment with voter preferences.

1 Introduction

The relationship between turnout and satisfaction in participatory budgeting presents a critical and unresolved question: does greater participation enhance or hinder satisfaction? Participatory budgeting (PB), a democratic process where citizens influence the allocation of public resources, is heralded for its potential to increase inclusivity and community engagement. However, empirical and theoretical research suggests that higher turnout can introduce a broader spectrum of preferences, complicating consensus-building and potentially leading to diluted satisfaction in decision-making processes, particularly when heterogeneity increases [24]. Such dynamics have been explored in broader democratic participation studies [12].

This study investigates the turnout-satisfaction dynamic, employing probabilistic models, simulation-based methodologies, and robust statistical techniques such as strata-based sampling and bootstrapping. By analyzing voter behavior under varying turnout levels, this research seeks to uncover the trade-offs between inclusivity and satisfaction, contributing to the broader discourse on democratic innovation and participatory processes [16].

We extend our analysis of the 'Price of Abstention' by explicitly modeling population heterogeneity. In addition to the original turnout–satisfaction simulations using the Greedy approval rule, we introduce a diversity metric: the Shannon entropy of the distribution of approval ballots. This entropy quantifies preference heterogeneity (low entropy: homogeneous opinions, high entropy: polarized or diverse opinions). We simulate elections under varying turnout levels and extremes of population diversity (low/high entropy), evaluating how voter satisfaction behaves under Greedy aggregation. While preliminary work suggests that proportional rules like the Method of Equal Shares (MES) may perform better in heterogeneous settings [20, 3], this paper empirically evaluates only the Greedy rule, though the framework is extensible to MES and other rules. Under Greedy, average voter satisfaction improves with turnout in low-entropy settings but can plateau or even decline for high entropy (reflecting consensus difficulty in diverse groups [24]). In contrast, proportional rules such as MES are suggested by prior work to produce more stable satisfaction in all turnout (consistent with Fairstein et al.'s findings that Greedy outcomes are sensitive to participation while Equal Shares outcomes are robust [10]). We discuss these findings in terms of representativeness and fairness: simple majority rules tend to prioritize majority-supported projects, potentially at the expense of minority-preferred ones, whereas equity-focused rules (variance minimization or randomized fair lotteries) can safeguard minority views (cf. [3, 20]). Finally, we explore the use of entropy or Gini coefficients as 'consensus hardness' metrics to predict when deliberation or alternative aggregation is needed [24, 25].

Through simulations over 588 real-world PB cases, we model voter preferences using approval ballots and quantify heterogeneity via Shannon entropy.

We hypothesize: (1) Turnout increases satisfaction in homogeneous populations; (2) In diverse populations, this effect weakens or reverses; (3) Entropy moderates this relationship.

2 Related Work

Participatory budgeting has been widely studied across political science, economics, and computational fields. Research on voter abstention reveals the challenges of declining participation, emphasizing its impact on democratic legitimacy [16]. Studies on PB, such as [14], highlight the trade-offs between voter engagement and representativeness. While prior work focuses on increasing participation, limited attention has been given to the implications of turnout on satisfaction. This study bridges this gap by integrating insights from democratic theory, statistical modeling (e.g., strata-based sampling [19], and bootstrapping [9]), and computational social choice to explore how turnout influences satisfaction in PB. Our study bridges these areas by focusing on the interplay between voter preferences, aggregation methods, and satisfaction metrics. By modeling turnout probabilistically and incorporating robust sampling techniques, we extend existing frameworks to address the "price of abstention."

Turnout and Voter Participation. The relationship between turnout and representativeness has been widely studied. Classical political theory often treats increased participation as inherently beneficial for democracy [8]. However, empirical studies suggest that higher turnout can introduce more heterogeneity, which may strain consensus in collective decision-making [11]. In participatory budgeting (PB), this issue might be magnified as more citizens participate; the variance in preferences over public goods tends to increase, raising questions about how aggregation rules handle such diversity.

Aggregation in Participatory Budgeting. PB presents unique aggregation challenges. The literature has developed a range of rules to translate approval votes into budgets [5, 2]. The Greedy rule, used in real-world platforms such as Stanford's PB tool [1], selects projects based on approval scores per cost unit. While it performs well in practice, it may struggle under preference fragmentation. Alternative methods, such as Maximin Support [21], seek to ensure proportionality by maximizing support for funded items. These rules differ in how they respond to increased preference diversity—a central concern of this paper.

Simulations and Satisfaction Metrics. Simulations have long been used to model decision processes in voting and budgeting [4]. Recent PB studies have focused on satisfaction metrics, particularly average satisfaction under budget constraints [5]. Our work builds on this literature by simulating elections with varying turnout levels and computing satisfaction across entropy levels. We adopt the satisfaction framework of [5] and test its interaction with voter heterogeneity, generating new hypotheses about the robustness of aggregation rules under different social configurations.

Modeling Preference Diversity. A growing body of work seeks to formalize and quantify diversity in preferences. In participatory settings, heterogeneity affects not only outcomes but also perceptions of fairness [6]. While prior research has examined how diversity shapes deliberative outcomes and committee selection [17], few have proposed generalizable measures of diversity across electoral settings. This paper introduces entropy—a standard metric from information theory [22]—as a proxy for preference diversity in PB, allowing us to link diversity to outcome satisfaction under different turnout levels.

3 Formal Model

We define a participatory budgeting system with the following components:

- n: Number of voters.
- m: Number of projects.
- B: Total budget available.
- C_j: Cost of project j, a positive real number.

Voter preferences are represented as a binary matrix $V \in \{0,1\}^{n\times m}$, where $V_{ij} = 1$ indicates that voter i supports project j. Let $S \subseteq \{1,\ldots,m\}$ denote the set of selected projects constrained by B.

The voting rule selects S that respects the total budget available, based on the input V and C. Voter satisfaction S_i is defined as:

$$S_i = |\{j \in S : V_{ij} = 1\}|,$$

where $|A|$ represents the cardinality of set A. The average satisfaction S_{avg} across the total population is computed as:

$$S_{\text{avg}} = \frac{1}{n}\sum_{i=1}^{n} S_i.$$

Aggregation Algorithm: Greedy Approval The greedy approval voting rule is applied to select projects. The algorithm ranks projects by their approval scores:

$$F_j = \sum_{i=1}^{n} V_{ij},$$

and iteratively selects projects in descending order of F_j, provided the cumulative cost does not exceed B.

While computationally efficient, this method prioritizes popularity over fairness, making it a benchmark for evaluating alternative methods [13].

Modeling Population Heterogeneity. To study how turnout interacts with preference diversity, we simulate electorates based on real-world PB data. Each simulation begins with a base population of N voters, where each voter approves a subset of m available projects. The approval ballots are modeled as binary vectors in $\{0,1\}^m$.

We assume a fixed total population and define "turnout" as the number of voters drawn without replacement from this population to form the active electorate. By varying turnout levels from low to full participation, we can examine how diversity in preferences correlates with satisfaction under different aggregation conditions.

To model preference diversity, we introduce a sampling parameter $\beta \in [0, 1]$ that governs the degree of homogeneity in the generated electorate. A value of $\beta = 0$ corresponds to maximal homogeneity, where all voters share the same preferences; $\beta = 1$ corresponds to maximal heterogeneity, where each ballot is sampled independently from a uniform distribution over $\{0, 1\}^m$. Intermediate values of β interpolate between these extremes by sampling from multiple clusters of voters with distinct modal ballots.

For each simulated population, we compute a measure of entropy (defined formally in the next section) to quantify its preference diversity. We then analyze how average satisfaction under the Greedy rule varies as a function of turnout and entropy.

To measure preference diversity, we introduce an entropy-based metric. Consider the empirical distribution of unique ballots in the population. Let $B_1, \ldots, B_k$ denote the distinct approval types, and let p_j be the fraction of voters with ballot B_j. The (Shannon) entropy of the population is defined as:

$$H(\mathcal{A}) = -\sum_{j=1}^{k} p_j \log_2 p_j.$$

This entropy captures the effective diversity of preferences: it is minimized (zero) when all voters submit identical ballots and maximized when each ballot is unique and equally likely.

Together, S_{avg} and $H(\mathcal{A})$ allow us to study how satisfaction varies with turnout under different levels of preference heterogeneity.

4 Simulation Design

The simulation is divided into the following modules:

1. **Data Generation:** Generate n voter preference vectors V and m project costs C. The binary matrix V is created using a probability p for $V_{ij} = 1$, reflecting voter preferences. Costs C_j are sampled uniformly from a predefined range.

2. **Turnout Sampling:** A subset of voters is sampled probabilistically based on turnout level t, ensuring proportional representation of the population.

3. **Bootstrapping and Replications:** Bootstrapping is a common replication method for variance estimation, which uses repeated resampling with replacement from the original full sample. Each replicate is created by sampling the primary sampling units (PSUs) within each stratum with replacement in order to enhance accuracy and minimize sampling error [18, 15]. Each replication computes satisfaction metrics independently, allowing for more robust population-level results.

4. **Aggregation:** Apply the greedy approval algorithm to the sampled subset, selecting projects S while adhering to budget B.

5. **Evaluation:** Compute average satisfaction S_{avg} across the total population.

We simulate PB elections using real-world data from the Jagiellonian and Stanford universities' Participatory Budgeting Platforms [23, 1]. Each dataset contains a list of proposed projects, their costs, and voter-submitted approval votes.

We run simulations on 588 PB instances, varying both the size and structure of the electorate. For each instance, we generate synthetic electorates of fixed size $N = 1000$ voters by resampling from the original voting data using bootstrapping. This retains the empirical distribution of approvals while allowing repeated trials under controlled conditions.

To explore the interaction between turnout and heterogeneity, we fix a sampling rate β to control population diversity (see Section 3), then vary turnout precentage $t \in \{10, 20, ..., 100\}$ to represent different levels of voter participation. At each turnout level, we sample t voters without replacement from the simulated population and apply the Greedy rule to compute the winning set.

We record:

- The average satisfaction across voters.
- The entropy of the sampled electorate.
- The rate of vote uniqueness (number of distinct votes / total votes).
- The budget utilization and number of funded projects.

Each configuration (PB instance, β, turnout level) is repeated 50 times to ensure statistical robustness. We aggregate results across trials and analyze how satisfaction trends vary as a function of entropy and turnout.

5 Simulation Framework

Although we reference three aggregation rules in our discussion—Greedy approval, Method of Equal Shares (MES), and variance-minimization—our current simulation framework only implements and evaluates the Greedy rule. Comparative evaluation of alternative aggregation methods is left for future work.

The simulation proceeds as follows:

1. Generate voter preferences V and project costs C.

2. Sample turnout t and generate a subset of voters.

3. Apply the greedy approval algorithm to the sampled subset.

4. Evaluate S_{avg} for the total population.

5. Repeat for multiple replications and average results to compute bootstrapped satisfaction values.

6 Results

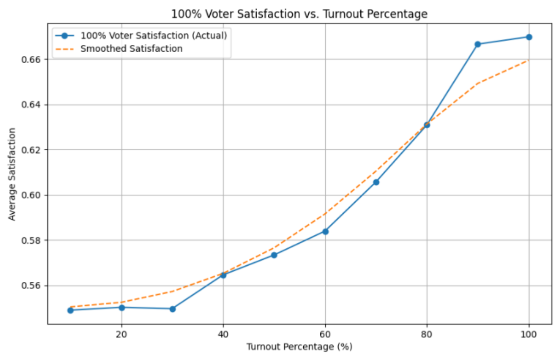

Figure 1: Average satisfaction across varying turnout levels. The orange dashed line represents a smoothed curve generated using Gaussian filtering, which reduces noise to reveal overall trends.

The results, depicted in Figure 1, show a continuous increase in average voter satisfaction as turnout rises. The data does not exhibit sharp plateaus or diminishing returns but shows variability at low turnout levels and a slower rate of growth at higher turnout. Overall, satisfaction improves gradually and stabilizes at higher turnout levels.

Key observations include:

- At **low turnout levels** (e.g., 0–30%), there are noticeable fluctuations in the actual satisfaction data, which may result from sampling variability. These fluctuations diminish in significance when smoothed and replicated.
- At **mid-range turnout levels** (e.g., 30–80%), the average satisfaction increases more consistently as the sampled voters better reflect the population's preferences.
- At **high turnout levels** (e.g., 80–100%), satisfaction continues to grow, though the rate of increase slows slightly, indicating minimal additional improvements despite increased participation.

The smoothed satisfaction curve (orange dashed line) highlights the overall trend, reducing noise present in the raw data. The smoothing is achieved using **Gaussian filtering** [7], which applies a weighted average to each data point:

$$G[i] = \sum_{j=-k}^{k} x[i+j] \cdot w[j] \tag{1}$$

where $x[i]$ is the original data point, and $w[j]$ is the Gaussian weight:

$$w[j] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{j^2}{2\sigma^2}} \tag{2}$$

with σ controlling the smoothness. The weights are normalized so their sum equals 1.

6.1 City-Level Results

To assess whether the turnout–satisfaction dynamics observed in our simulations hold across real-world participatory budgets, we examined individual PB files from major Polish cities. Each dataset contains voter preferences, project costs, and demographic metadata.

The figure below compares satisfaction curves under Greedy aggregation rule in Warszawa and Lodz. Notably, while prior studies suggest that MES tends to maintain stable satisfaction even with high entropy in settings like Warszawa [10], our simulations with the Greedy rule show substantial decline in satisfaction through turnout in Lodz. These patterns, observed under Greedy aggregation, support the broader understanding that heterogeneous electorates challenge winner-take-all aggregation.

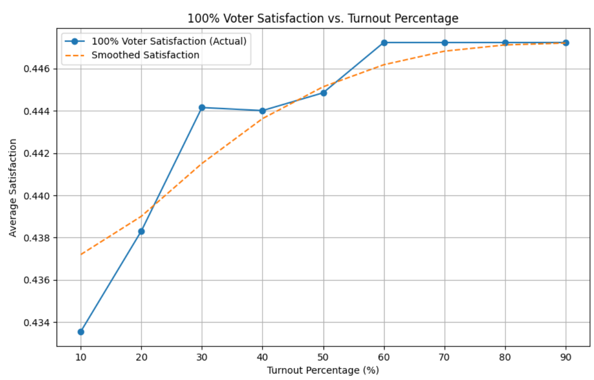

Figure 2: Voter satisfaction vs. turnout in Warszawa 2023. The orange dashed line represents smoothed satisfaction using Gaussian filtering.

Figure 2 demonstrates a logarithmic-like increase in satisfaction as turnout increases. This result exemplifies a case where voter preferences are relatively aligned, allowing Greedy aggregation to maintain high satisfaction at near-universal turnout.

City-level contrast: Lodz vs. Warszawa

In contrast to Warszawa, Figure 3 illustrates the trend in Lodz 2024, where increasing turnout appears to slightly decrease satisfaction — a shape resembling a reciprocal function ($\sim 1/x$), possibly indicating fragmentation of voter preferences at higher participation levels.

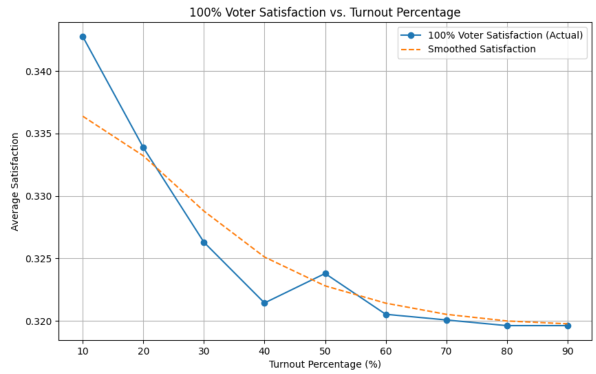

Figure 3: Satisfaction vs. turnout in Lodz 2024. Unlike Warszawa, the curve exhibits decreasing satisfaction with more voters.

Key findings:

- For **low entropy** (homogeneous populations), under the Greedy rule, satisfaction consistently rises with turnout. Prior studies suggest that in such settings, proportional rules may also perform similarly [10].
- For **high entropy** (heterogeneous populations), Greedy often plateaus or declines in satisfaction. Prior studies suggest that MES may maintain stable satisfaction even in high entropy settings, while our simulations under Greedy aggregation reveal substantial declines in such cases (see Figures 3 and 2).
- **Consensus hardness** is theorized to increase with H, making high-diversity electorates more challenging to satisfy, as suggested by studies on group dynamics [24].

To further examine the role of diversity metrics such as entropy or KL divergence in mediating the relationship between turnout and satisfaction, we conducted regression analyses across all simulated elections (n=5880 across 588 budgets and 10 turnout levels each). For each budget file, we computed the slope of satisfaction as a function of turnout, and regressed this slope against entropy and KL divergence (measured at full turnout).

Surprisingly, both linear and non-linear regression models (Random Forest) showed extremely weak explanatory power: the entropy–slope correlation was $\rho = 0.096$ ($p = 0.0196$), and the linear model's R^2 was below 0.003. Feature importance in the Random Forest model was nearly split between entropy and

KL divergence, yet predictive performance was poor ($R^2 < 0$ on test set). A one-way ANOVA of slope values grouped by low, medium, and high entropy categories likewise yielded non-significant differences ($p = 0.8397$).

These null results are important: while theoretical expectations and prior studies [24, 10] highlight the impact of diversity, our simulations indicate that such effects may be contingent or masked by other factors (e.g., budget tightness, project overlap, or voter clustering). This implies that entropy alone may be insufficient as a predictor and should be combined with other contextual variables such as budget tightness, project overlap, or demographic segmentation. These results may suggest the need for latent clustering structures or composite heterogeneity indicators beyond entropy alone. The absence of strong predictive patterns underscores the need for further multi-factorial modeling.

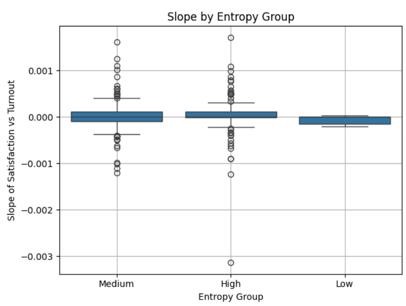

Figure 4: Slope by Entropy Group - Box Plot (a)

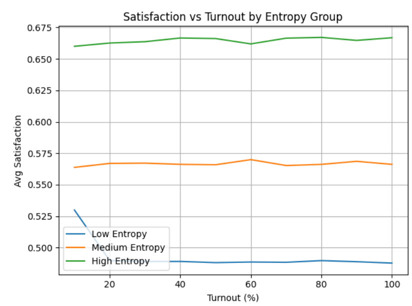

Figure 5: Satisfaction vs Turnout by Entropy Group - Line Graph (b)

Comparison of the slope of satisfaction vs. turnout across entropy groups, sampled from 588 PB instances withdrawn from PB library [23]. Both Figure 4 and Figure 5 show the same figure for demonstration.

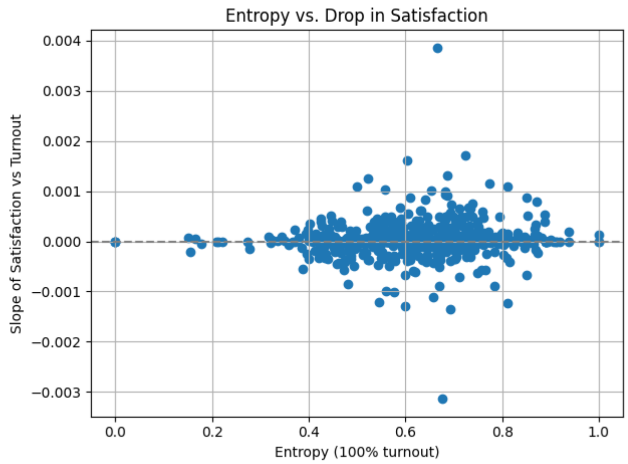

Figure 6: Entropy vs. slope of satisfaction (Spearman correlation) in 588 PB simulations. Weak but significant positive correlation observed ($\rho = 0.096$, $p < 0.02$).

7 Discussion

The findings confirm that voter satisfaction increases with turnout in a monotonic but non-linear manner, with some exceptions. In our metric of evaluating homogeneous populations like Warszawa, Greedy aggregation aligns well with voter preferences even at full turnout. However, in heterogeneous populations (e.g., Lodz), increasing turnout may introduce preference fragmentation, which Greedy fails to reconcile effectively.

These observations support the hypothesis that *preference entropy moderates the turnout–satisfaction relationship*. At low entropy, turnout improves satisfaction due to alignment; at high entropy, turnout may surface latent conflicts and reduce average satisfaction. Figures 4, 5 and 6 illustrate this statistically across a large dataset of 588 real-world PB files.

Future work should investigate whether aggregation rules prioritizing proportionality or fairness are more resilient to entropy-driven volatility. Local variations in the satisfaction curve suggest avenues for further study, including the effects of subgroup diversity and project overlap.

Preference heterogeneity moderates the turnout-satisfaction relationship. Our simulations under the Greedy rule show that increased turnout in fragmented electorates can reduce overall satisfaction. This contrasts with findings from prior studies where MES and fairness-oriented rules demonstrate greater robustness to heterogeneity [10, 3]. These observations, combined with theoretical expectations [24], suggest that measures such as entropy and satisfaction variance could serve as useful diagnostics for fairness risks and can guide the use of proportional methods or deliberation to improve outcomes.

While prior studies suggest that diversity hampers consensus, our null findings point to a more nuanced picture: heterogeneity alone may not be a sufficient predictor of satisfaction outcomes. Future work should consider interaction effects and multi-dimensional explanations.

7.1 Policy Implications and Practical Relevance

These findings suggest that cities employing Greedy aggregation rules should carefully monitor indicators of preference diversity, such as entropy, within their electorates. In contexts characterized by high diversity or polarization, considering a transition to proportional methods like MES could lead to more stable and equitably perceived outcomes. Furthermore, the observed non-linear relationship between turnout and satisfaction highlights the importance of targeted engagement strategies rather than simply maximizing participation, focusing on ensuring representative turnout that genuinely reflects community preferences.

8 Conclusion

This study demonstrates that voter satisfaction in participatory budgeting increases with turnout, but not always linearly or uniformly. In some cases, satisfaction may even decrease due to increased heterogeneity. These findings have implications for aggregation rule design and turnout incentives. Based on a novel simulation framework incorporating entropy and turnout analysis across 588 real-world participatory budgeting datasets, our results highlight the importance of balancing inclusivity with achievable satisfaction in PB processes. The main conclusions are:

1. In homogeneous (low-entropy) contexts, turnout reliably improves satisfaction.
2. In heterogeneous settings, turnout may lead to conflicting preferences and reduced satisfaction under simple aggregation rules.
3. Low turnout levels risk volatility of satisfaction due to limited representativeness.
4. Moderate turnout levels strike the best balance between representativeness and voter satisfaction.
5. High turnout levels provide limited additional satisfaction gains or losses (depends on Population composition).
6. Policy-makers should consider preference diversity when designing participatory systems.

Future research should focus on exploring alternative aggregation methods, analyzing different population distributions, and applying the framework to real-world participatory budgeting scenarios.

9 Future Work

Future research should investigate alternative voting rules such as Cumulative Single Transferable Vote (CSTV) and Equal Shares, and incorporate real-world complexities including dynamic budgets and evolving project costs. Exploring diverse satisfaction metrics and simulating heterogeneous population environments will improve the framework's generalizability. We plan to integrate MES and other proportional rules into the simulation framework to empirically validate the theoretical claims and facilitate comparative evaluation.

Our simulations under Greedy aggregation underscore the moderating role of preference heterogeneity on turnout–satisfaction dynamics, showing that increased turnout can degrade outcomes in polarized electorates. This contrasts with findings from recent literature where MES and fairness-oriented rules demonstrate greater resilience [10, 3]. These observations align with theoretical expectations regarding diversity's impact [24]. Utilizing metrics like entropy and satisfaction variance may serve as diagnostic tools to identify fairness risks and guide the adoption of proportional aggregation or deliberative interventions.

Beyond the null results discussed above, future work should pursue three directions:

1. **Interactive Models:** Investigate whether entropy interacts with other parameters (e.g., budget tightness, voter clustering) to influence satisfaction trajectories. This aligns with findings by Zhang et al. [25] and Zhang et al. [24], who showed that heterogeneity effects depend on network or spatial structures.

2. **Expanded Predictors:** Extend regression and ML models to include additional variables such as number of projects, average approval set size, project cost distribution, and demographic diversity. These could reveal latent confounders that moderate or suppress the effects of preference entropy.

3. **Clustering and Subgroup Analysis:** Apply unsupervised learning (e.g., KMeans, DBSCAN) to identify subtypes of electorates with distinct turnout–satisfaction profiles. Entropy may be informative within specific subgroups (e.g., urban vs. rural budgets or age-stratified electorates).

10 Appendix A: General Example

This section demonstrates the operation of the greedy algorithm with a specific example.

10.1 Parameters

- $n = 4$, $m = 5$, $B = 5000$.
- Costs $C = [1000, 2000, 1500, 3000, 2500]$.

- Preferences matrix V:

$$V = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

- Approval scores $F = [2, 3, 3, 1, 4]$.

10.2 Selection Order

- Select Project 5 ($F_5 = 4$, cost = \$2500). Remaining budget: \$2500.
- Select Project 2 ($F_2 = 3$, cost = \$2000). Remaining budget: \$500.

Selected projects: $\{5, 2\}$, satisfaction vector $S = [0.5, 0.667, 0.667, 0.334]$, and $S_{\text{avg}} = 0.2167$.

11 Appendix B: Example for Satisfaction decrease with Turnout

This section demonstrates the effect of turnout on satisfaction using a small example.

11.1 Parameters

- $n = 10$, $m = 4$, $B = 6,000$.
- Project costs $C = [3000, 3000, 3000, 3000]$, for projects A, B, C, D respectively.
- Preferences matrix V:

$$V = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

- Approval scores $F = [4, 5, 6, 4]$.

11.2 Turnout Scenario

- 20% turnout: Voters 4–5 participate.
- 30% turnout (in addition): Voters 1–3 participate.

11.3 Greedy Selection (based on approval score)

Approval scores among 20% turnout participants:

Project A: 0 approvals
Project B: 0 approvals
Project C: 2 approvals (Voters 4–5)
Project D: 2 approvals (Voters 4–5)

- Select Project C (score = 2, cost = 3). Remaining budget: 3.
- Select Project D (score = 2, cost = 3). Remaining budget: 0.
- Satisfaction vector (number of approved selected projects per voter):

$$S = [0.5, 0.5, 0.5, 1, 1, 0.5, 0.5, 0.5, 0.5, 0]$$

- Average satisfaction: $S_{\text{avg}} = \frac{5.5}{10} = 0.55$

Approval scores among 30% turnout participants:

Project A: 3(Voters 1–3) approvals
Project B: 3(Voters 1–3) approvals
Project C: 0 approvals
Project D: 0 approvals

- Select Project A (score = 3, cost = 3). Remaining budget: 3.
- Select Project B (score = 3, cost = 3). Remaining budget: 0.
- Satisfaction vector (number of approved selected projects per voter):

$$S = [1, 1, 1, 0.5, 0.5, 0, 0, 0, 0, 1]$$

- Average satisfaction: $S_{\text{avg}} = \frac{5}{10} = 0.5$

11.4 Outcome

- Selected projects for 20%: $\{C, D\}$
- Average satisfaction: $S_{\text{avg}} = \frac{5.5}{10} = 0.55$
- Selected projects for 30%: $\{A, B\}$
- Average satisfaction: $S_{\text{avg}} = \frac{5}{10} = 0.5$

Average satisfaction reduced from $S_{\text{avg}} = 0.55$ to $S_{\text{avg}} = 0.5$ while turnout increased by 10%

11.5 Illustration of Heterogeneity Effects

In this example, the preference matrix V represents a relatively homogeneous electorate, where voters share many project approvals. Under such conditions, the greedy algorithm selects projects with broad support, yielding relatively high average satisfaction.

However, in highly heterogeneous populations—where voter approvals are more polarized or diverse—the greedy approach may disproportionately favor certain groups, potentially reducing overall satisfaction and fairness. This motivates the need for alternative aggregation methods, such as proportional or variance-minimizing rules, which better accommodate preference diversity as explored in the main text.

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