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Simulation of inverted pendulum

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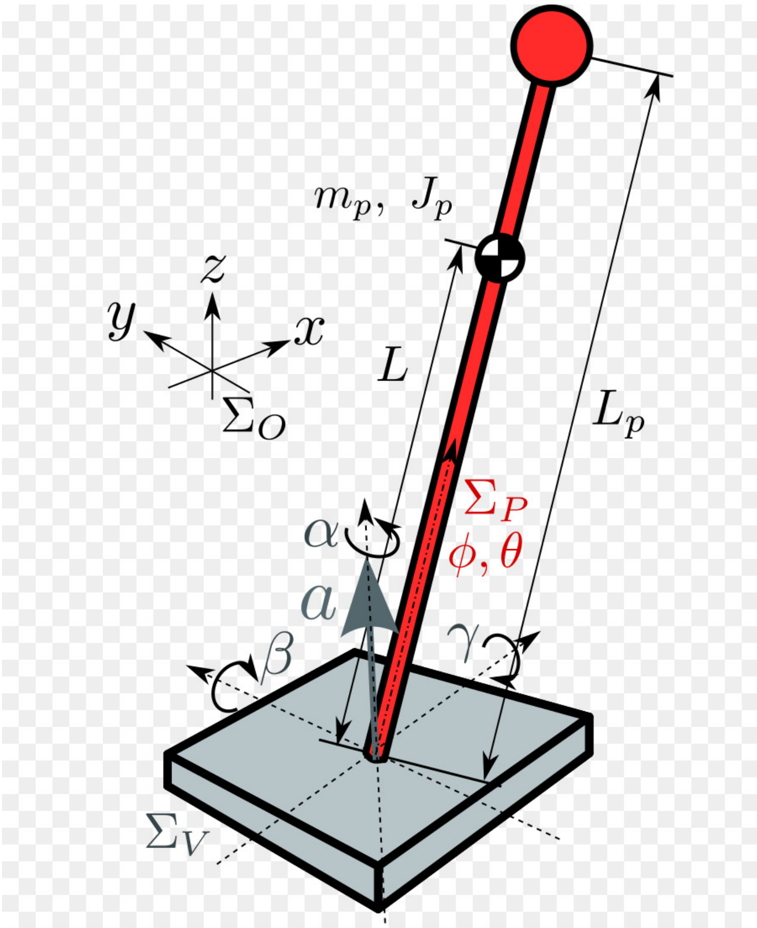


Figure 1: Inverted pendulum

Inverted pendulum An inverted pendulum is a dynamic system where a pendulum has its mass above its pivot point, rather than below. This makes it inherently unstable, meaning it requires continuous control to keep it balanced. It is a classic problem in control theory and robotics, often used to illustrate techniques for system stabilization and feedback control.

In simple terms: The pendulum wants to fall, but the LQR controller makes the platform move in such a way that it keeps the pendulum upright. The controller determines how much force is needed to move the platform to prevent the pendulum from falling while also minimizing control costs (how much and how often we move the platform).

1 Parameters

m = 0.2;
M = 1.5;
L = 0.8;
g = 9.81;
d = 0.05;

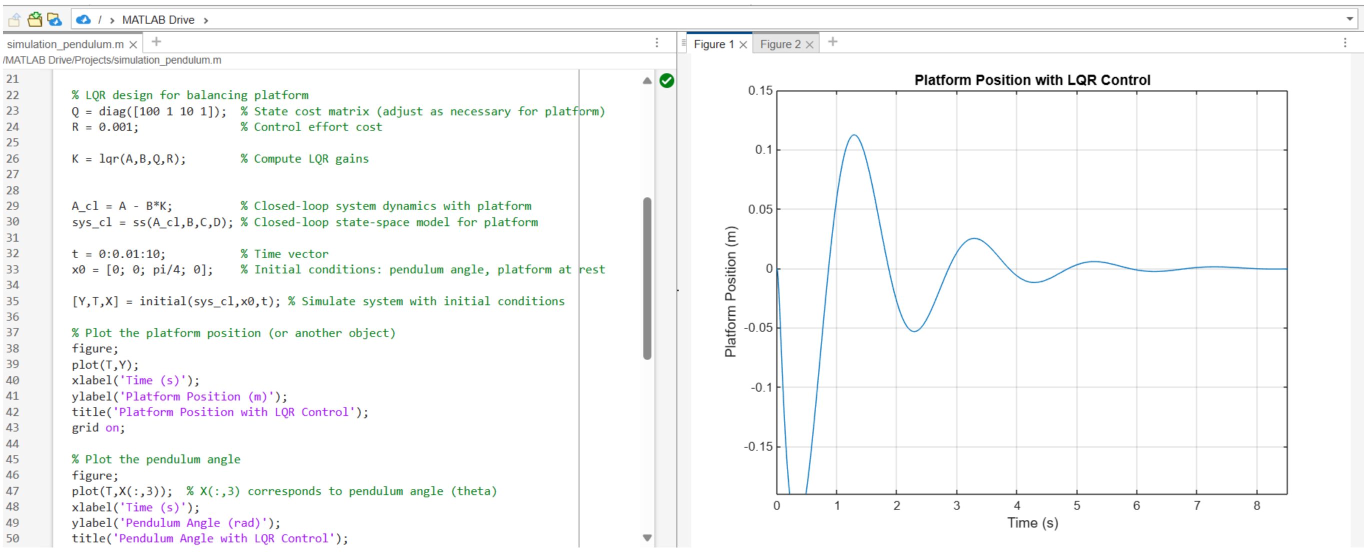


Figure 2: Caption

2 State space representation for balancing platform with pendulum

A matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-d}{M} & \frac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-d}{ML} & \frac{-(m+M)g}{ML} & 0 \end{bmatrix}$$

B matrix

$$B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{ML} \end{bmatrix}$$

C matrix

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

D = 0;

Result of matrices

$$A = [0100;$$
$$0 - d/M - m * g/M0;$$
$$0001;$$
$$0 - d/(M * L) - (m + M) * g/(M * L)0];$$
$$C = [1000];$$
$$D = 0;$$

Summary:

- A represents the system's internal dynamics (position, velocity, angle, angular velocity).

- B tells how the external force (input) influences the system's states.
- C tells what part of the state we're measuring (in this case, the cart's position).
- D indicates there is no direct effect of input on output.

Create the state-space model
sys = ss(A,B,C,D);

LQR Design for Balancing Platform

We are designing an LQR (Linear Quadratic Regulator) to stabilize a balancing platform (inverted pendulum system). The following MATLAB code represents the design and simulation:

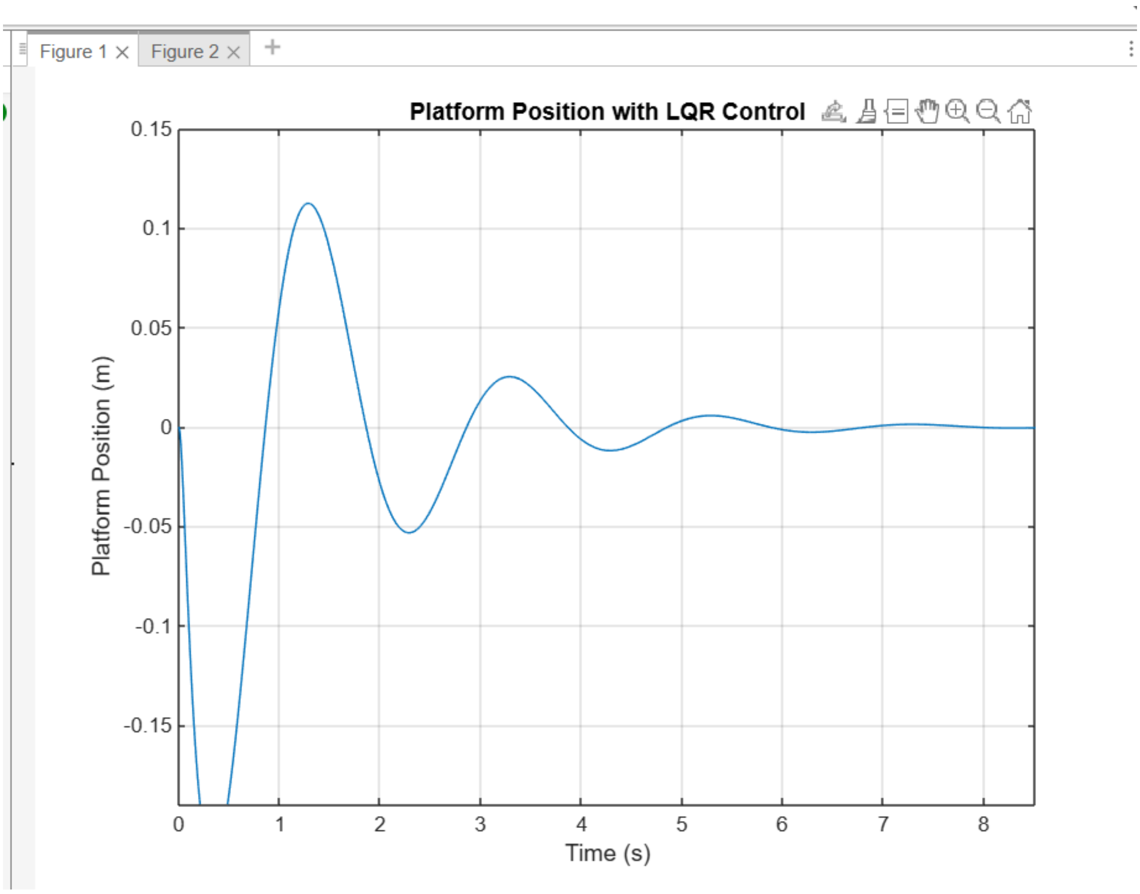
$$Q = diag([1001101]); R = 0.001; \tag{1}$$

$$K = lqr(A, B, Q, R); \tag{2}$$

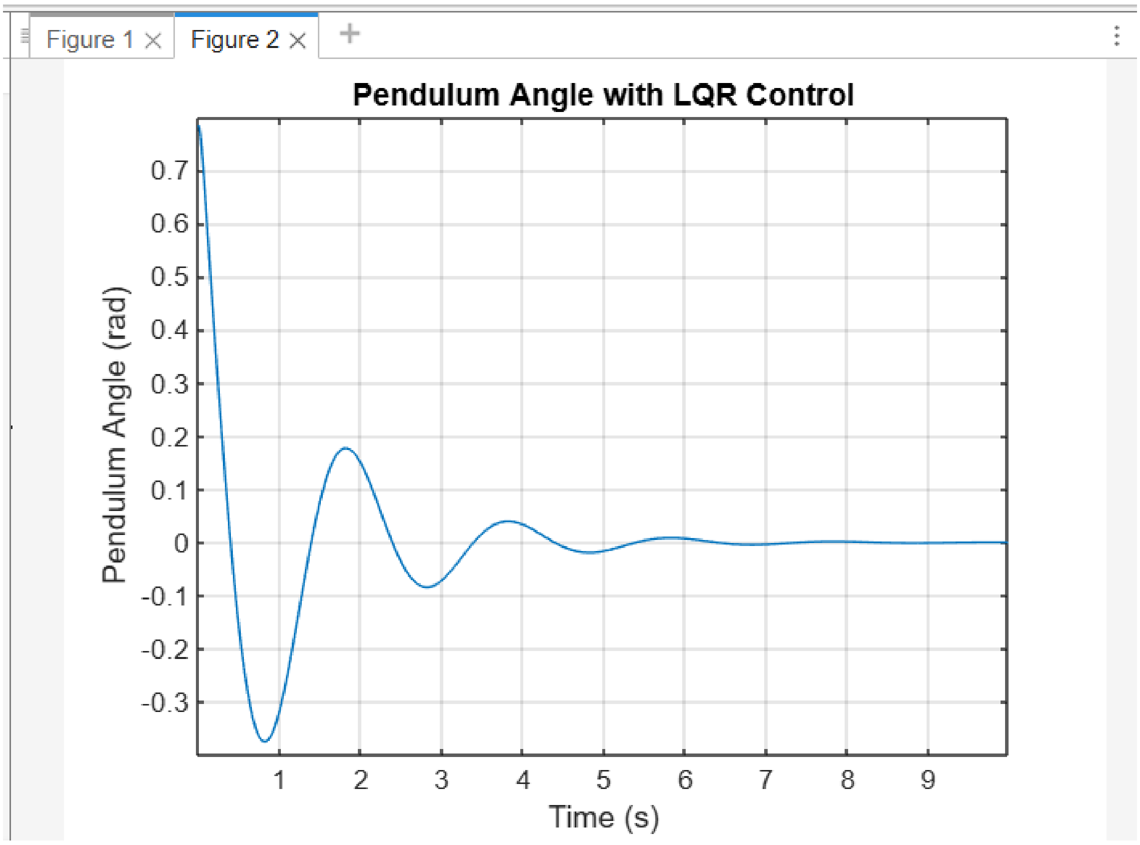
$$A_cl = A - B * K; sys_cl = ss(A_cl, B, C, D); \tag{3}$$

$$t = 0 : 0.01 : 10; x0 = [0; 0; pi/4; 0];$$

$$[Y, T, X] = initial(sys_cl, x0, t); \tag{5}$$



3 **The first graph shows how the platform's position changes over time. The second graph illustrates how the pendulum's angle changes — the controller attempts to return it to a vertical position.**



4 Conclusion

The simulation shows that the LQR controller works well for stabilizing the balancing platform. The platform stays in a steady position with only small movements, and the pendulum remains close to the upright position with minimal swinging. This means the LQR controller is effectively keeping both the platform and pendulum stable, showing that the design is successful.