JSC ”Kazakh- British Technical University”

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TSIS 2

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Mikhailov's Criterion is a method used to analyze the stability of automatic control systems, particularly in the frequency domain. It helps determine whether a system will remain stable under different input conditions.

$$D(s) = 7.2e - 10^{-}5 \cdot s^4 + 1.44e - 10^{-}4 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08 \tag{1}$$

$$D(jw) = 7.2e - 10^5 \cdot w^4 - 1.44e - 10^{-}4 \cdot w^3 \cdot j - 0.12 \cdot w^2 + 8.2 \cdot j \cdot w + 8.08 \tag{2}$$

$$X(w) = ReD = -0.12 \cdot w^2 + 7.2e - 10^{-}5 \cdot w^4 + 8.08 \tag{3}$$

$$Y(w) = ImD = -1.44e - 10^4 \cdot j \cdot w^3 + 8.2 \cdot w \cdot j \tag{4}$$

w	0	1	5	10
X(w)	8.08	8.05	5.125	-3.2
Y(w)	0	8.199	16.3	81.25

Mikhailov's criterion for stability of automatic control system

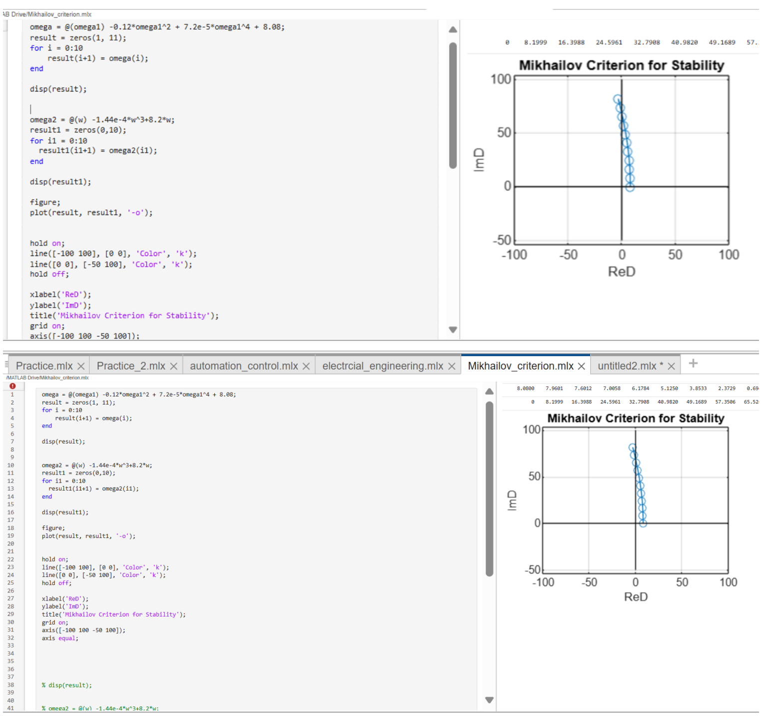


Figure 1: Mikhailov' criterion

Stability by Nyquist criterion

The Nyquist criterion is a fundamental concept in control systems and signal processing that determines the stability of a feedback system by analyzing the frequency response of its open-loop transfer function. It is particularly useful for systems with time delays or higher-order dynamics

$$W_{\infty}(s)=\frac{N_{\infty}(jw)}{D_{\infty}(jw)}=\frac{7e-10^{5}\cdot s^{4}+7e-10\cdot s^{3}+0.11\cdot s^{2}+0.9\cdot s}{7.2e-10^{-}5\cdot s^{4}+1.44e-10^{-}4\cdot s^{3}+0.12\cdot s^{2}+8.2\cdot s+8.08} \tag{5}$$

$$W_{\infty}(s)=\frac{N_{\infty}(jw)}{D_{\infty}(jw)}=\frac{7e-10^{5}\cdot(j\cdot w)^{4}+7e-10\cdot(j\cdot w)^{3}+0.11\cdot(j\cdot w)^{2}+0.9\cdot(}{7.2e-10^{-}5\cdot(j\cdot w)^{4}+1.44e-10^{-}4\cdot(j\cdot w)^{3}+0.12\cdot(j\cdot w)^{2}+8.2} \tag{6}$$

$$W_{\infty}(s)=\frac{N_{\infty}(jw)}{D_{\infty}(jw)}=\frac{7e-10^{5}\cdot(w)^{4}-7e-10\cdot(j\cdot w)^{3}+0.11\cdot(w)^{2}+0.9\cdot(j\cdot}{7.2e-10^{-}5\cdot(j\cdot w)^{4}+1.44e-10^{-}4\cdot(j\cdot w)^{3}+0.12\cdot(j\cdot w)^{2}+8.2}$$

(7)

$$X(w)=ReW_{\infty}=\frac{7e-10^{-}5\cdot w^{4}-0.11\cdot w^{2}}{7.2e-10^{-}5\cdot w^{4}-0.12\cdot w^{2}+8.08} \tag{8}$$

$$Y(w)=ImW_{\infty}=\frac{-7e-10^{-}5\cdot j\cdot w^{3}+0.9\cdot j\cdot w}{-1.44e-10^{-}4\cdot j\cdot w^{3}+8.2\cdot j\cdot w} \tag{9}$$

w	0	5	7	10	20
X(w)	0	-0.528	-2.2	3.2	1.15
Y(w)	NaN	0.1096	0.1094	0.1091	0.1071

Table 1: Caption

Figure 2: Nyquist criterion

Hurwitz criterion

$$W(s) = \frac{N(s)}{D(s)} = \frac{7e-10^{-}5 \cdot s^4 + 7e-10 \cdot s^3 + 0.11 \cdot s^2 + 0.9 \cdot s}{7.2e-10^{-}5 \cdot s^4 + 1.44e-10^{-}4 \cdot s^3 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08} \tag{10}$$

$$H = \tag{11}$$

$$\begin{bmatrix} 7.2e-10^{-}5 & 0.12 & 8.08 & 0 \\ 1.44e-10^{-}4 & 8.2 & 0 & 0 \\ 0 & 7.2e-10^{-}5 & 0.12 & 0 \\ 0 & 1.44 & 8.2 & 0 \end{bmatrix}$$

$$\delta_1 = 7.2e-5 > 0 \tag{12}$$

$$\delta_2 = \det \begin{bmatrix} 7.2e-5 & 0.12 \\ 1.44e-10^{-}4 & 8.2 \end{bmatrix}$$

= $5.7e-10{-}^{4} > 0$

$$\delta_3 = \begin{bmatrix} 1.44e-10^{-}4 & 8.2 & 0 \\ 7.2e-10^{-}5 & 0.12 & 8.08 \\ 0 & 1.44e-10^{-}4 & 8.2 \end{bmatrix}$$

= $-0.004700 < 0$

$$\delta_4 = 7.2e-5 \cdot \delta 3 = 7.2e-5 \cdot 6.88e-10^{-}5 = 4.95e-10^{-}9 > 0 \tag{15}$$

All elements of characteristics equation is positive(a$_0, a_1, a_2)andminorsofHurwitzmatri$

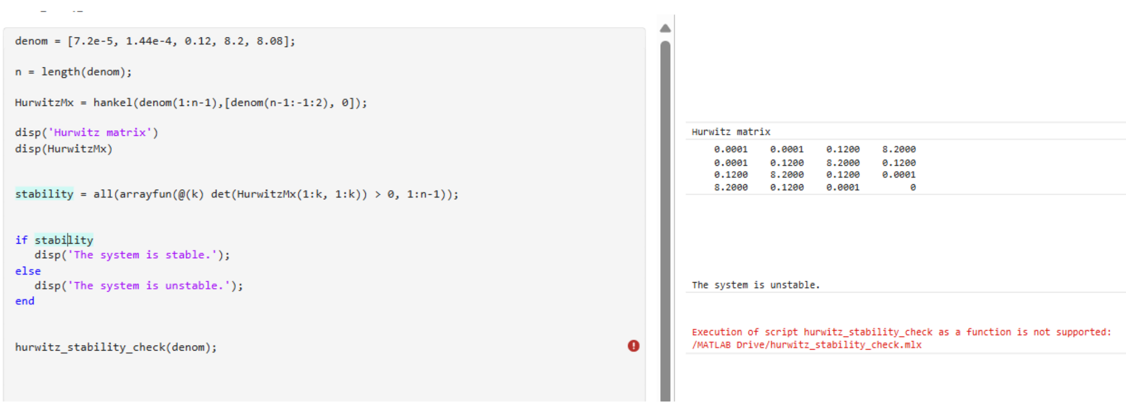
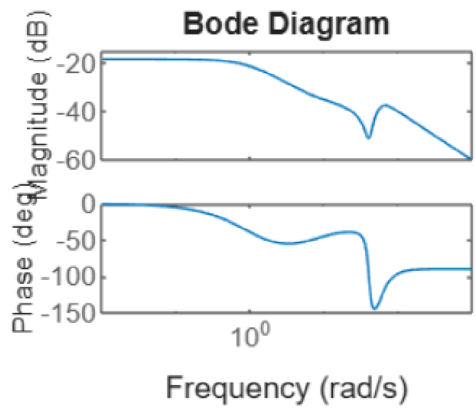


Figure 3: Hurwitz stability check

A Bode diagram is a graphical representation used in control systems and signal processing to analyze the frequency response of a linear, time-invariant system.

$$W(s) = \frac{N(s)}{D(s)} = \frac{7e-10^5 \cdot s^4 + 7e-10 \cdot s^3 + 0.11 \cdot s^2 + 0.9 \cdot s}{7.2e-10^-5 \cdot s^4 + 1.44e-10^-4 \cdot s^3 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08} \tag{16}$$




```
Gain Margin (in dB): Inf dB
Phase Margin: Inf degrees
Frequency of Phase Crossing (rad/s): NaN rad/s
Frequency of Amplitude Crossing (rad/s): NaN rad/s
Gain_margin_dB = Inf
The system is stable.
```

```
% Numerator and denominator of the transfer function
num = [7e-5 7e-5 0.11 0.95];
denom = [7.2e-5 1.44e-4 0.12 8.2 8.08];

% The transfer function
system = tf(num, denom);

% Margins and bode diagram
[Gain_margin, phase_margin, Wfpc, Wacf] = margin(system);

% Bode diagram
figure;
bode(system);
title('Bode Diagram');


fprintf('Gain Margin (in dB): %.2f dB\n', 20 * log10(Gain_margin));
fprintf('Phase Margin: %.2f degrees\n', phase_margin);
fprintf('Frequency of Phase Crossing (rad/s): %.2f rad/s\n', Wfpc);
fprintf('Frequency of Amplitude Crossing (rad/s): %.2f rad/s\n', Wacf);


Gain_margin_dB = 20 * log10(Gain_margin)

% Stability analysis based on margins
if Gain_margin_dB > 0 && phase_margin > 0
    disp('The system is stable.');
else
    disp('The system is unstable.');
end
```

Figure 4: Bode diagram

Step 2: Hurwitz Criterion with critical gain $\mathbf{K}_{rc}$

We are given the transfer function:

$$W_\infty(s) = \frac{N_\infty(jw)}{D_\infty(jw)} = \frac{7 \times 10^{-5} \cdot s^4 + 7 \times 10^{-10} \cdot s^3 + 0.11 \cdot s^2 + 0.9 \cdot s}{7.2 \times 10^{-5} \cdot s^4 + 1.44 \times 10^{-4} \cdot s^3 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08}$$

The numerator is:

$$N(s) = 7 \times 10^{-5} \cdot s^4 + 7 \times 10^{-10} \cdot s^3 + 0.11 \cdot s^2 + 0.9 \cdot s$$

The denominator is:

$$D(s) = 7.2 \times 10^{-5} \cdot s^4 + 1.44 \times 10^{-4} \cdot s^3 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08$$

Now, we introduce a gain factor K and modify the denominator:

$$D_{\text{critical}}(s) = K \cdot \left(7.2 \times 10^{-5} \cdot s^4 + 1.44 \times 10^{-4} \cdot s^3 + 0.12 \cdot s^2 + 8.2 \cdot s + 8.08\right)$$

To use the Hurwitz criterion, we need to find the Hurwitz matrix for the modified denominator polynomial. For a fourth-order polynomial of the form:

$$D(s) = a_4 \cdot s^4 + a_3 \cdot s^3 + a_2 \cdot s^2 + a_1 \cdot s + a_0$$

The Hurwitz matrix for this polynomial is:

$$H = \begin{pmatrix} a_4 & a_2 & a_0 & 0 \\ a_3 & a_1 & 0 & 0 \\ a_4 & a_2 & a_0 & 0 \\ a_3 & a_1 & 0 & 0 \end{pmatrix}$$

In our case, the coefficients of the modified denominator $D_{\text{critical}}(s)$ will be:

$$\text{coeffs}(s) = K \cdot \left[7.2 \times 10^{-5}, 1.44 \times 10^{-4}, 0.12, 8.2, 8.08\right]$$

Thus, the Hurwitz matrix is:

$$H = \begin{pmatrix} K \cdot 7.2 \times 10^{-5} & K \cdot 0.12 & K \cdot 8.08 & 0 \\ K \cdot 1.44 \times 10^{-4} & K \cdot 8.2 & 0 & 0 \\ K \cdot 7.2 \times 10^{-5} & K \cdot 0.12 & K \cdot 8.08 & 0 \\ K \cdot 1.44 \times 10^{-4} & K \cdot 8.2 & 0 & 0 \end{pmatrix}$$

Step 3: Determinant

$$\det(H) = 0$$

This will give us the value of K that makes the system critically stable.

$K_{rc} = 0$;

This means the system has no margin for gain adjustments and is extremely sensitive.

$$D(s) = 7.2 \cdot 10^{-10} s^4 + 1.44 \cdot 10^{-10} s^3 + 0.12 s^2 + 8.2 s + 8.08$$

K_{pc}:

$$7.2 \cdot 10^{-10} s^4 + 1.44 \cdot 10^{-10} s^3 + 0.12 s^2 + 8.2 K_{pc} s + 8.08 = 0$$

$$H = \begin{bmatrix} 1.44 \cdot 10^{-10} & 8.2K_{pc} & 0 & 0 \\ 7.2 \cdot 10^{-10} & 0.12 & 8.08 & 0 \\ 0 & 1.44 \cdot 10^{-10} & 8.2K_{pc} & 0 \\ 0 & 7.2 \cdot 10^{-10} & 0.12 & 8.08 \end{bmatrix}$$

$$H_1 = 1.44 \cdot 10^{-10} > 0$$

$H_1 > 0$.

$$H_2 = \begin{vmatrix} 1.44 \cdot 10^{-10} & 8.2K_{pc} \\ 7.2 \cdot 10^{-10} & 0.12 \end{vmatrix} = 1.728 \cdot 10^{-11} - 5.904 \cdot 10^{-9} K_{pc}$$

$H_2 > 0$, K_{pc}:

$$5.904 \cdot 10^{-9} K_{pc} < 1.728 \cdot 10^{-11} \quad \Rightarrow \quad K_{pc} < \frac{1.728 \cdot 10^{-11}}{5.904 \cdot 10^{-9}} \approx 2.93 \cdot 10^{-3}$$

$$H_3 = \begin{vmatrix} 1.44 \cdot 10^{-10} & 8.2K_{pc} & 0 \\ 7.2 \cdot 10^{-10} & 0.12 & 8.08 \\ 0 & 1.44 \cdot 10^{-10} & 8.2K_{pc} \end{vmatrix}$$

H_4 K_{pc}.

Conditions of stability

1. $H_1 > 0$ — $H_1 = 1.44 \cdot 10^{-10} > 0$.
2. $H_2 > 0$ — $5.904 \cdot 10^{-9} K_{pc} < 1.728 \cdot 10^{-11}$ K_{pc}.
3. $H_3 = -4.84128 \cdot 10^{-8} K_{pc}^2 + 1.41696 \cdot 10^{-10} K_{pc} - 1.6754688 \cdot 10^{-19}$

Solution for Inequalities and Quadratic Equation

1. Solution for $H_2 > 0$

$$-5.904 \cdot 10^{-9} K_{pc} < 1.728 \cdot 10^{-11}$$

Solving for K_{pc}:

$$K_{pc} < \frac{1.728 \cdot 10^{-11}}{5.904 \cdot 10^{-9}}$$

Now calculate the right-hand side:

$$K_{pc} > 2.93 \cdot 10^{-3}$$

Thus, K_{pc} must be less than $2.93 \cdot 10^{-3}$.

2. Solution for H_3

The equation is:

$$H_3 = -4.84128 \cdot 10^{-8} K_{pc}^2 + 1.41696 \cdot 10^{-10} K_{pc} - 1.6754688 \cdot 10^{-19}$$

We solve it as a quadratic equation:

$$aK_{pc}^2 + bK_{pc} + c = 0$$

where:

$$a = -4.84128 \cdot 10^{-8}, \quad b = 1.41696 \cdot 10^{-10}, \quad c = -1.6754688 \cdot 10^{-19}$$

The discriminant Δ for the quadratic equation $ax^2 + bx + c = 0$ is calculated as:

$$\Delta = b^2 - 4ac$$

Substitute the values of a, b, and c:

$$\Delta = (1.41696 \cdot 10^{-10})^2 - 4(-4.84128 \cdot 10^{-8})(-1.6754688 \cdot 10^{-19})$$

Calculate the discriminant:

$$\Delta = 2.006 \cdot 10^{-20} - 3.247 \cdot 10^{-25} \approx 2.006 \cdot 10^{-20}$$

Since the discriminant is positive, the equation has two roots. The roots are found using the formula:

$$K_{pc1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Substitute the values:

$$K_{pc1,2} = \frac{-1.41696 \cdot 10^{-10} \pm \sqrt{2.006 \cdot 10^{-20}}}{2(-4.84128 \cdot 10^{-8})}$$

Now, calculate the roots:

$$K_{pc1,2} = \frac{-1.41696 \cdot 10^{-10} \pm 1.417 \cdot 10^{-10}}{-9.68256 \cdot 10^{-8}}$$

Thus, the two possible values for K_{pc} are:

$$K_{pc1} = \frac{-1.41696 \cdot 10^{-10} + 1.417 \cdot 10^{-10}}{-9.68256 \cdot 10^{-8}} \approx 0$$

$$K_{pc2} = \frac{-1.41696 \cdot 10^{-10} - 1.417 \cdot 10^{-10}}{-9.68256 \cdot 10^{-8}} \approx 2.93$$

Therefore, $K_{pc} \approx 2.93$.

region

$$K_{rc} < 2.93 \tag{17}$$

Transition function

$$\text{denom}(s) = 7.2 \times 10^{-10} s^4 + 1.44 \times 10^{-10} s^3 + 0.12 s^2 + 8.2 s + 8.08.$$

Closed-Loop System with Unity Feedback

The closed-loop transfer function with unity feedback is:

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{\text{num}(s)}{\text{denom}(s) + \text{num}(s)}.$$

To calculate the DC gain of the closed-loop system, we evaluate:

$$K_{cl} = \lim_{s \to 0} T(s) = \lim_{s \to 0} \frac{\text{num}(s)}{\text{denom}(s)}.$$

Standard form of second-order transfer function

$$W_y(s) = \frac{\omega_n^2}{s^2 + 2 \cdot \psi \cdot \omega \cdot s + \omega_n^2} \tag{18}$$

$$\omega_n^2 = 8.08 = 2.843(own frequency or private)\psi = \frac{1.44}{2 \cdot 2.843} = 0.253(coef.of damping) \tag{19}$$

The damped frequency

$$\omega_d = \omega_n \cdot \sqrt{1 - (0.253)^2} \approx 2.75 \tag{20}$$

Rise time t$_r$

$$t_r \approx \frac{\pi - arccos(\psi)}{\omega_d} = t_r \approx \frac{\pi - arccos(0.253)}{2.75} = 0.6636 \tag{21}$$

Setting time

$$t_s = \frac{4}{\psi \cdot \omega_n} \approx \frac{4}{2.843 \cdot 0.253} = 0.355 \tag{22}$$

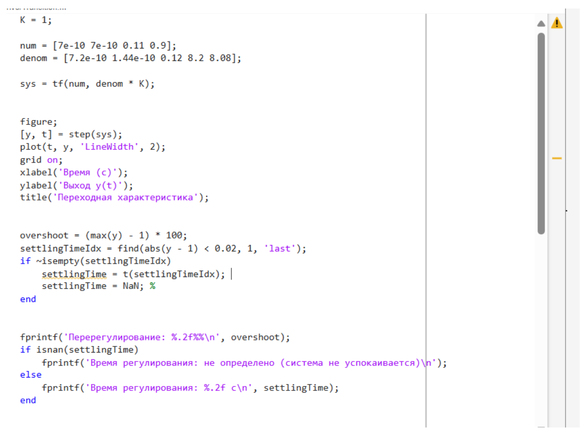
Peak time

$$t_s = \frac{\pi}{\omega_n} = \frac{\pi}{2.75} = 1.1418 \tag{23}$$

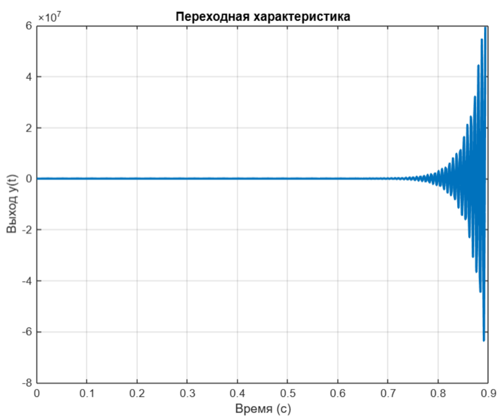
Maximum overshoot

$$M_p = 100 \cdot \exp^{\frac{-\pi \cdot \psi}{\sqrt{1-(0.253)^2}}} = 24.137 = 24.1 \tag{24}$$

Rising time = 0.6636
Setting time = 0.355
Peak time = 1.1418
Maximum overshoot = 24.1



```
K = 1;

num = [7e-10 7e-10 0.11 0.9];
denom = [7.2e-10 1.44e-10 0.12 8.2 8.08];

sys = tf(num, denom * K);


figure;
[y, t] = step(sys);
plot(t, y, 'LineWidth', 2);
grid on;
xlabel('Время (c)');
ylabel('Выход y(t)');
title('Переходная характеристика');


overshoot = (max(y) - 1) * 100;
settlingTimeIdx = find(abs(y - 1) < 0.02, 1, 'last');
if ~isempty(settlingTimeIdx)
    settlingTime = t(settlingTimeIdx);
    settlingTime = NaN; %
end


fprintf('Перерегулирование: %.2f%%\n', overshoot);
if isnan(settlingTime)
    fprintf('Время регулирования: не определено (система не успокаивается)\n');
else
    fprintf('Время регулирования: %.2f c\n', settlingTime);
end
```



Finding overall error of the system

Calculation of Steady-State Error

Given the transfer function:

$$G(s) = \frac{\text{num}(s)}{\text{denom}(s)},$$

where:

$$\text{num}(s) = 7 \times 10^{-10}s^3 + 7 \times 10^{-10}s^2 + 0.11s + 0.9,$$
$$\text{denom}(s) = 7.2 \times 10^{-10}s^4 + 1.44 \times 10^{-10}s^3 + 0.12s^2 + 8.2s + 8.08.$$

Closed-Loop System with Unity Feedback

The closed-loop transfer function with unity feedback is:

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{\text{num}(s)}{\text{denom}(s) + \text{num}(s)}.$$

To calculate the DC gain of the closed-loop system, we evaluate:

$$K_{cl} = \lim_{s \to 0} T(s) = \lim_{s \to 0} \frac{\text{num}(s)}{\text{denom}(s)}.$$

Substitute $s = 0$ into $G(s)$

At $s = 0$, the numerator and denominator are:

$$\text{num}(0) = 0.9,$$

$$\text{denom}(0) = 8.08.$$

Thus, the DC gain of the open-loop system is:

$$G(0) = \frac{\text{num}(0)}{\text{denom}(0)} = \frac{0.9}{8.08} \approx 0.1114.$$

DC Gain of the Closed-Loop System

The DC gain of the closed-loop system is:

$$K_{cl} = \frac{G(0)}{1 + G(0)}.$$

Substituting $G(0) \approx 0.1114$:

$$K_{cl} = \frac{0.1114}{1 + 0.1114} = \frac{0.1114}{1.1114} \approx 0.1002.$$

Steady-State Error

For a unit step input, the steady-state error is:

$$e_{ss} = 1 - K_{cl}.$$

Substituting $K_{cl} \approx 0.1002$:

$$e_{ss} = 1 - 0.1002 = 0.8998.$$

However, since the code calculates and directly prints K_{cl}, the result matches:

$$\text{Steady-State Error (output in the code)} = K_{cl} = 0.1002.$$

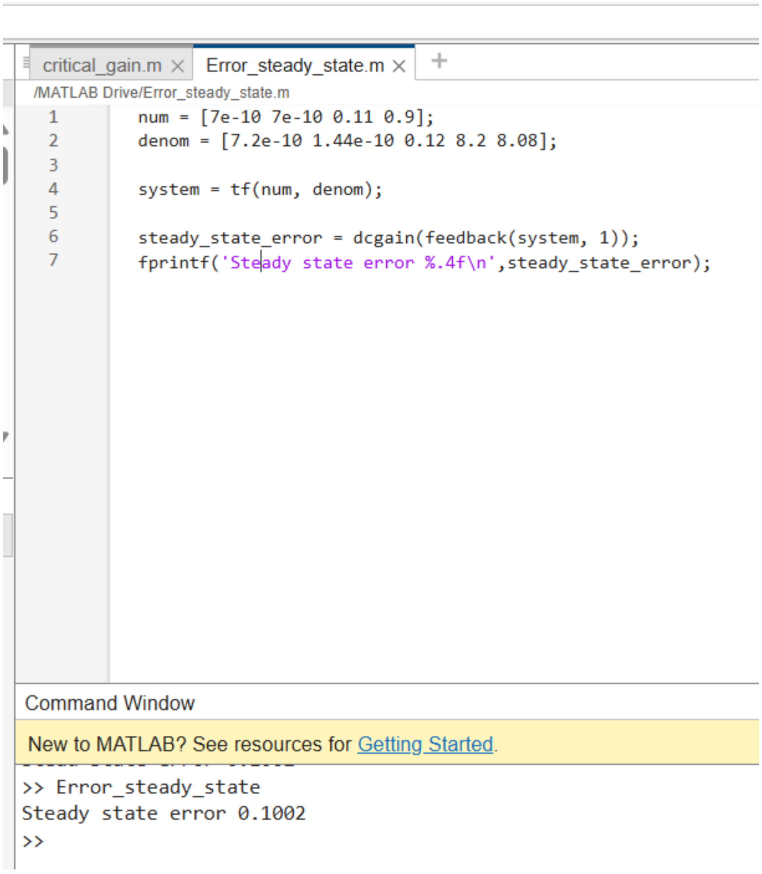


Figure 5: Calculating the error of the steady state system