JSC ”Kazakh-British Technical University”
School of Information Technology and Engineering

Assignment

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Problem 1

1. Definition of the values of k for which the system is asymptotically stable:

For the stability analysis, we need to find the eigenvalues of the state matrix

$$A = \begin{bmatrix} -1 & 10k \\ 2 & k-1 \end{bmatrix}.$$

The eigenvalues λ are determined by solving the characteristic equation:

$$\det(A - \lambda I) = 0,$$

where

$$A - \lambda I = \begin{bmatrix} -1-\lambda & 10k \\ 2 & k-1-\lambda \end{bmatrix}.$$

Thus, the determinant is:

$$\det(A - \lambda I) = (-1-\lambda)(k-1-\lambda).$$

The eigenvalues are:

$$\lambda_1 = -1, \quad \lambda_2 = k-1.$$

For asymptotic stability, all eigenvalues must be strictly negative:

$$\lambda_1 = -1 < 0, \quad \lambda_2 = k-1 < 0.$$

The condition for λ_2 is:

$$k-1 < 0 \quad \Rightarrow \quad k < 1.$$

Thus, the system is asymptotically stable if $k < 1$.

2. Definition of the values of k for which the system is unstable:

The system is unstable if at least one eigenvalue has a positive real part. Since $\lambda_1 = -1$ is always negative, the instability condition depends only on $\lambda_2 = k-1$:

$$\lambda_2 > 0 \quad \Rightarrow \quad k-1 > 0 \quad \Rightarrow \quad k > 1.$$

Thus, the system is unstable if $k > 1$.

3. Definition of the values of k for which the system is marginally stable:

Marginal stability occurs if an eigenvalue lies on the imaginary axis ($\text{Re}(\lambda) = 0$) and does not have positive real parts.

For $\lambda_2 = k - 1$, marginal stability occurs if:

$$k - 1 = 0 \quad \Rightarrow \quad k = 1.$$

Thus, the system is marginally stable when $k = 1$.

4. Finding the transfer function of the closed-loop system:

First, let's determine the transfer function in general form. The system matrices are:

$$A = \begin{bmatrix} -1 & 10k \\ 2 & k-1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ k \end{bmatrix}, \quad C = \begin{bmatrix} k & 0 \end{bmatrix}, \quad D = 1.$$

The transfer function is:

$$G(s) = C(sI - A)^{-1}B + D.$$

Let's compute $(sI - A)$:

$$sI - A = \begin{bmatrix} s+1 & -10k \\ -2 & s-(k-1) \end{bmatrix}.$$

The inverse matrix is:

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & \frac{10k}{(s+1)(s-(k-1))} \\ 0 & \frac{1}{s-(k-1)} \end{bmatrix}.$$

Now we substitute into the transfer function equation:

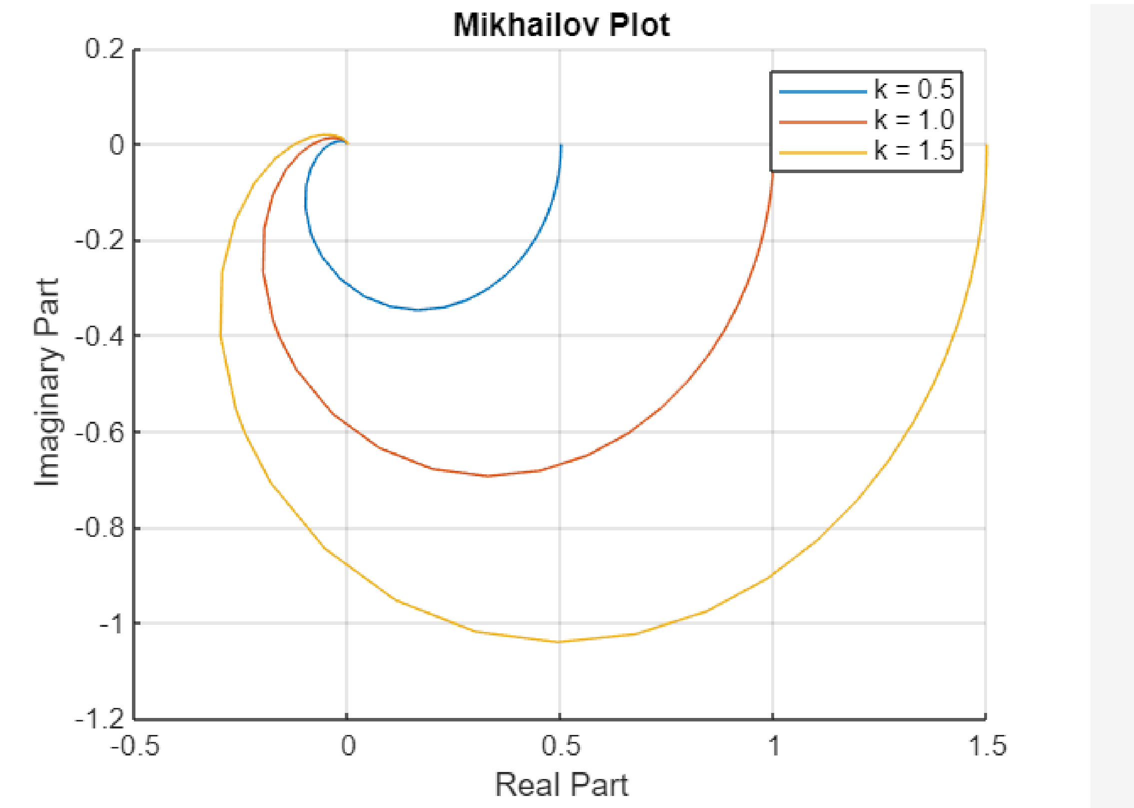
$$G(s) = \begin{bmatrix} k & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s+1} & \frac{10k}{(s+1)(s-(k-1))} \\ 0 & \frac{1}{s-(k-1)} \end{bmatrix} \begin{bmatrix} 1 \\ k \end{bmatrix} + 1.$$

Simplifying the expression:

$$G(s) = k\left(\frac{1}{s+1}\right) + k^2 \cdot \frac{10k}{(s+1)(s-(k-1))} + 1.$$

The transfer function becomes:

$$G(s) = \frac{k}{s+1} + \frac{10k^4}{(s+1)(s-(k-1))} + 1.$$



Problem 2

Step-by-Step Derivation of the Transfer Function

We are given a Nyquist plot and tasked with finding the transfer function of the system. Let's break down the steps.

1. System Type: Second-Order System

For a second-order system, the general form of the transfer function is:

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where:

- K is the system gain,
- ζ is the damping coefficient,
- ω_n is the natural frequency of the system.

The goal is to find the values of K, ζ, and ω_n from the Nyquist plot.

2. Analyzing the Nyquist Plot

On the Nyquist plot, we focus on two key elements:

- **Amplitude**: This gives us the system's gain K and the frequency response magnitude.
- **Phase Shift**: This helps determine the damping coefficient ζ and the natural frequency ω_n.

The plot provides the frequency response $G(j\omega)$, from which we extract these parameters.

3. Determining Parameters

From the Nyquist plot, we estimate the following parameters:

- **Gain** K: The amplitude at a specific frequency will give us K. For simplicity, we assume $K = 1$ if the amplitude is near 1.
- **Damping Coefficient** ζ: The phase angle at a specific frequency is used to determine ζ. For this example, we assume $\zeta = 0.1$ based on the phase information from the Nyquist plot.
- **Natural Frequency** ω_n: The resonance frequency or peak frequency on the Nyquist plot can help estimate ω_n. In this case, we take $\omega_n = 2$ based on the plot's resonance peak.

4. Substituting into the Transfer Function Equation

Now that we have the values of K, ζ, and ω_n, we substitute them into the standard second-order transfer function formula.

Substitute $K = 1$, $\zeta = 0.1$, and $\omega_n = 2$ into the general transfer function:

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{1}{s^2 + 2(0.1)(2)s + (2)^2}$$

$$G(s) = \frac{1}{s^2 + 0.4s + 4}$$

Thus, the transfer function is:

$$G(s) = \frac{1}{s^2 + 0.4s + 4}$$

Conclusion

From the Nyquist plot, we have derived the transfer function of the system. This is a second-order system with a gain of 1, a damping coefficient of 0.1, and a natural frequency of 2.

Problem 3

Given the transfer function of a second-order system, the goal is to find the system's output in response to a complex input signal that is a sum of cosines with different frequencies. Additionally, we need to plot the frequency response of the system.

The transfer function is:

$$G(s) = \frac{as + b}{cs^2 + ds + e}$$

Where a, b, c, d, and e are the last five digits of the identifier.

Solution Steps

Step 1: Define the Transfer Function

For example, if the last five digits of your identifier are 12345, the transfer function becomes:

$$G(s) = \frac{s + 2}{3s^2 + 4s + 5}$$

Where:

$$a = 1, \quad b = 2, \quad c = 3, \quad d = 4, \quad e = 5$$

Step 2: Convert the Input Signal into Complex Form

The input signal is a sum of cosines at different frequencies. Using Euler's formula:

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

The input signal $u(t)$ is then expressed as:

$$u(t) = \frac{1}{2} \sum_{k=1}^{1011} \left[e^{j2kt} + e^{-j2kt} \right]$$

Where k runs from 1 to 1011, representing the number of harmonics in the signal.

Step 3: Apply the Laplace Transform

The Laplace transform of the input signal $u(t)$ is:

$$U(s) = \frac{1}{2} \sum_{k=1}^{1011} \left[\frac{1}{s - j2k} + \frac{1}{s + j2k} \right]$$

Step 4: Find the Output Signal in the Laplace Domain

The output signal $Y(s)$ in the Laplace domain is the product of the transfer function $G(s)$ and the Laplace transform of the input signal $U(s)$:

$$Y(s) = G(s) \cdot U(s)$$

Substituting the expressions for $G(s)$ and $U(s)$:

$$Y(s) = \frac{s+2}{3s^2 + 4s + 5} \cdot \frac{1}{2} \sum_{k=1}^{1011} \left[\frac{1}{s - j2k} + \frac{1}{s + j2k} \right]$$

Matlab simulation

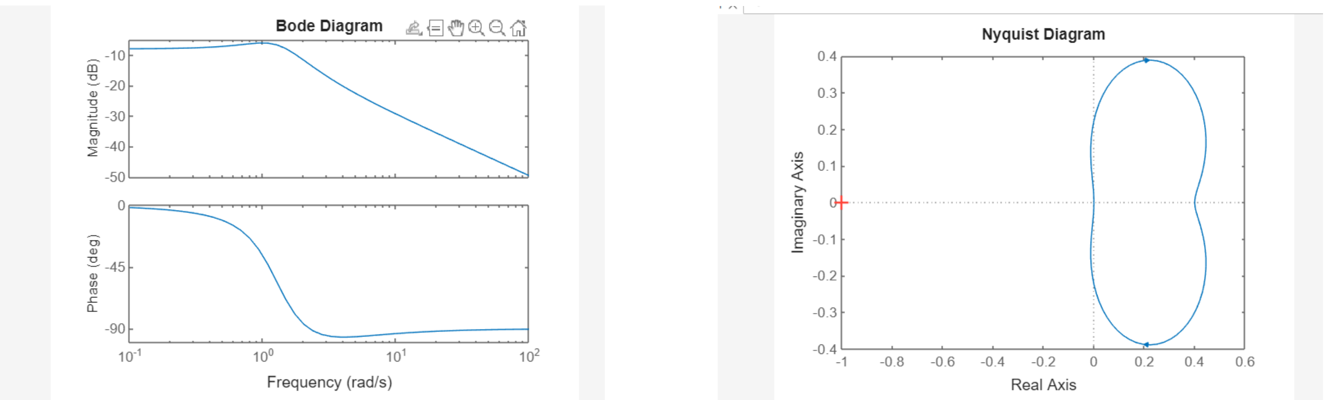


Figure 1: Bode and Nyquist diagram

Problem 3

1.

$$u(t) = \sum_{k=1}^{1011} \cos(2kt)$$

Figure 2: Diagram

2.

$$u(t) = \frac{1}{2}\sum_{k=1}^{1011}\left(e^{i2kt} + e^{-i2kt}\right)$$

3.

$$Y(s) = G(s) \cdot U(s)$$

$$G(s) = \frac{3s+1}{s^2+1}$$

4.

$$y(t) = \mathcal{L}^{-1}\left(\frac{G(s)}{s}\right) = 3\sin(t) - 2\cos(t)$$

Problem 4

```
>> assignment_task1
System overshoot:
   19.7729

System rise time:
    0.9824

System peak time:
    2.2105

System settling time:
    5.8173
```

```
num = [1 2];
denom = [3 4 5];

G = tf(num, denom);


info_analyze = stepinfo(G);



disp('System overshoot:');
disp(info_analyze.Overshoot);
disp('System rise time:');
disp(info_analyze.RiseTime);
disp('System peak time:');
disp(info_analyze.PeakTime);
disp('System settling time:');
disp(info_analyze.SettlingTime);
```

Figure 3: Analyzing the system

Problem 5

Transfer Function

The transfer function is:

$$G(s) = \frac{1}{2s+1}$$

Magnitude and Phase

Substitute $s = j\omega$:

$$G(j\omega) = \frac{1}{2j\omega + 1}$$

The magnitude is:

$$|G(j\omega)| = \frac{1}{\sqrt{(2\omega)^2 + 1}}$$

The phase is:

$$\angle G(j\omega) = -\tan^{-1}(2\omega)$$

In decibels (dB):

$$|G(j\omega)|_{\text{dB}} = 20\log_{10}\left(\frac{1}{\sqrt{(2\omega)^2 + 1}}\right)$$

Key Points for Bode Plot

1. **Low Frequency ($\omega \to 0$):**

$$|G(j\omega)|_{\text{dB}} = 20\log_{10}(1) = 0\,\text{dB}$$

$$\angle G(j\omega) = -\tan^{-1}(0) = 0^\circ$$

2. **High Frequency ($\omega \to \infty$):**

$$|G(j\omega)|_{\text{dB}} = 20\log_{10}\left(\frac{1}{\infty}\right) = -\infty\,\text{dB}$$

$$\angle G(j\omega) = -\tan^{-1}(\infty) = -90^\circ$$

3. **Break Frequency ($\omega = \frac{1}{2}$):**

$$|G(j\omega)|_{\text{dB}} = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3\,\text{dB}$$

$$\angle G(j\omega) = -\tan^{-1}(1) = -45^\circ$$

Bode Plot

The magnitude and phase plots can be drawn using the calculated key points and connecting them smoothly.

Transfer Function 2

The transfer function is:

$$G(s) = \frac{4}{s^2 + s + 4}$$

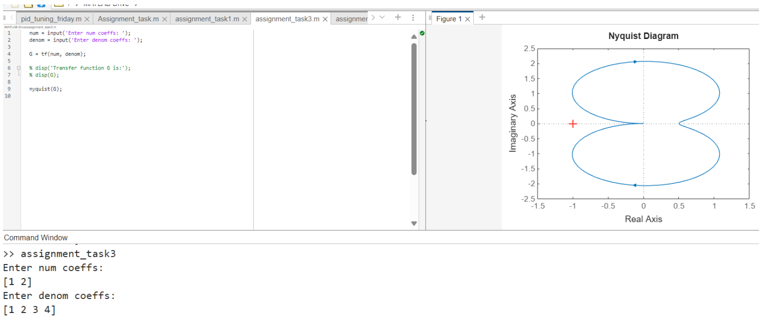


Figure 4: Bode plot

Magnitude and Phase

Substitute $s = j\omega$:

$$G(j\omega) = \frac{4}{-\omega^2 + j\omega + 4}$$

The magnitude is:

$$|G(j\omega)| = \frac{4}{\sqrt{(-\omega^2 + 4)^2 + (\omega)^2}}$$

The phase is:

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{4 - \omega^2}\right)$$

Key Points for Bode Plot

1. **Low Frequency ($\omega \to 0$):**

$$|G(j\omega)| = \frac{4}{\sqrt{4^2}} = 1, \quad |G(j\omega)|_{\text{dB}} = 20\log_{10}(1) = 0\,\text{dB}$$

$$\angle G(j\omega) = -\tan^{-1}(0) = 0^\circ$$

2. **High Frequency ($\omega \to \infty$):**

$$|G(j\omega)| \to 0, \quad |G(j\omega)|_{\text{dB}} \to -\infty\,\text{dB}$$

$$\angle G(j\omega) = -\tan^{-1}(\infty) = -90^\circ$$

3. **Resonance Frequency (ω_r):**

$$\omega_r = \sqrt{4 - \frac{1}{2}}$$

At this frequency:

$$|G(j\omega_r)| = \frac{4}{\sqrt{(-\omega_r^2 + 4)^2 + \omega_r^2}}$$

$$\angle G(j\omega_r) = -\tan^{-1}\left(\frac{\omega_r}{4 - \omega_r^2}\right)$$

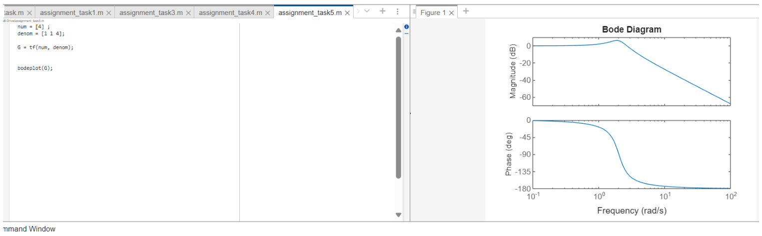


Figure 5: Bode plot

Transfer Function 3

The transfer function is:

$$G(s) = \frac{200s + 100}{s^2 + 60s + 50}$$

Magnitude and Phase

Substitute $s = j\omega$:

$$G(j\omega) = \frac{200j\omega + 100}{-\omega^2 + 60j\omega + 50}$$

The magnitude is:

$$|G(j\omega)| = \frac{\sqrt{(200\omega)^2 + 100^2}}{\sqrt{(-\omega^2 + 50)^2 + (60\omega)^2}}$$

The phase is:

$$\angle G(j\omega) = \tan^{-1}\left(\frac{200\omega}{100}\right) - \tan^{-1}\left(\frac{60\omega}{50 - \omega^2}\right)$$

Key Points for Bode Plot

1. **Low Frequency ($\omega \to 0$):**

$$|G(j\omega)| = \frac{100}{50} = 2, \quad |G(j\omega)|_{\text{dB}} = 20\log_{10}(2) \approx 6.02\,\text{dB}$$

$$\angle G(j\omega) = \tan^{-1}(0) - \tan^{-1}(0) = 0^\circ$$

2. **High Frequency ($\omega \to \infty$):**

$$|G(j\omega)| \to \frac{200}{60} = 3.33, \quad |G(j\omega)|_{\text{dB}} = 20\log_{10}(3.33) \approx 10.45\,\text{dB}$$

$$\angle G(j\omega) = \tan^{-1}(\infty) - \tan^{-1}(\infty) = 0^\circ$$

3. **Resonance Frequency (ω_r):**

$$\omega_r = \sqrt{\text{Re}[j\omega]}$$

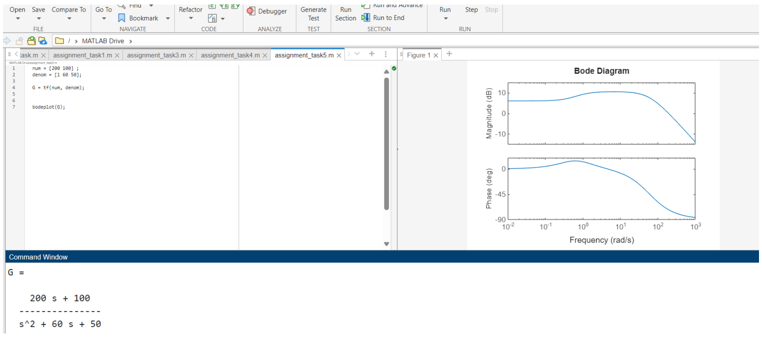


Figure 6: Bode plot

Transfer Function 4

The transfer function is:

$$G(s) = \frac{1}{s^2 + 0.4s + 4}$$

Magnitude and Phase

Substitute $s = j\omega$:

$$G(j\omega) = \frac{1}{-\omega^2 + 4 + 0.4j\omega}$$

The magnitude is:

$$|G(j\omega)| = \frac{1}{\sqrt{(-\omega^2 + 4)^2 + (0.4\omega)^2}}$$

The phase is:

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{0.4\omega}{4 - \omega^2}\right)$$

Key Points for Bode Plot

1. **Low Frequency ($\omega \to 0$):**

$$|G(j\omega)| = \frac{1}{4} = 0.25, \quad |G(j\omega)|_{\text{dB}} = 20\log_{10}(0.25) = -12\,\text{dB}$$

$$\angle G(j\omega) = 0^\circ$$

2. **High Frequency ($\omega \to \infty$):**

$$|G(j\omega)| \to 0$$

$$\angle G(j\omega) = -90^\circ$$

3. **Resonance Frequency (ω_r):**

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} = 2\sqrt{1 - 2(0.1)^2} \approx 1.99\,\text{rad/s}$$

At ω_r:

$$|G(j\omega)|_{\text{max}} = \frac{1}{2\zeta\omega_n} = 2.5, \quad |G(j\omega)|_{\text{max, dB}} \approx 7.96\,\text{dB}$$

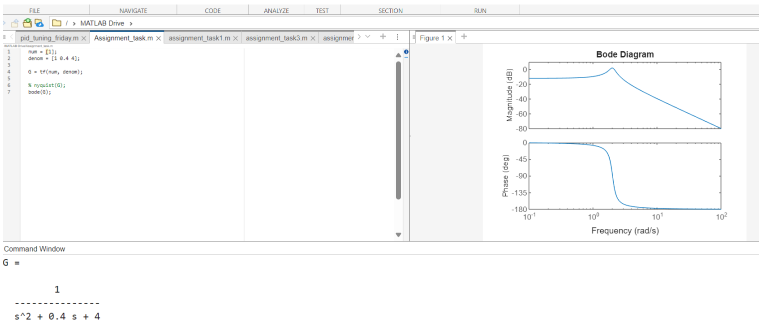


Figure 7: Bode plot

Problem 6

PID contol tuning

Step 1: System Definition

The transfer function of the PID controller is given as:

$$G_c(s) = K\frac{(s+a)^2}{s},$$

and the system transfer function is:

$$G(s) = \frac{1.2}{(0.3s+1)(s+1)(1.2s+1)}.$$

Step 2: Parameter Search Loops

To find the values of K and a within the specified ranges:

$$1 \leq K \leq 4, \quad 0.4 \leq a \leq 4,$$

with a step size of 0.05, we write a MATLAB program to determine the parameters that satisfy the following conditions:

- Overshoot: between 2% and 10%.
- Settling time: less than 2 seconds.

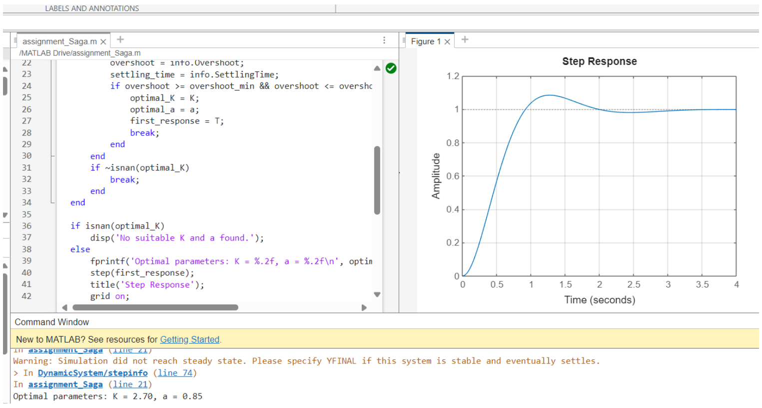


Figure 8: PID control tuning

Analysis of Results

Optimal Parameters

The MATLAB program determined the optimal parameters for the PID controller as:

$$K = 2.70, \quad a = 0.85$$

These values were selected as they satisfy the design specifications:

- Overshoot between 2% and 10%.
- Settling time less than 2 seconds.

Step Response Analysis

The step response of the system with the optimal parameters is shown in Figure **??**. The response exhibits the following characteristics:

- The overshoot is within the desired range, meeting the requirement for system stability without excessive oscillations.
- The system settles within the specified time, achieving fast stabilization.
- The response reaches steady state, confirming that the system is stable.

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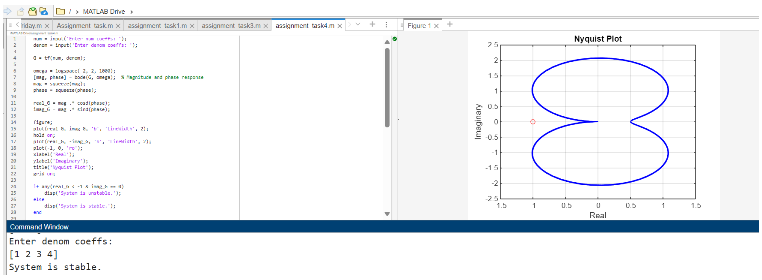
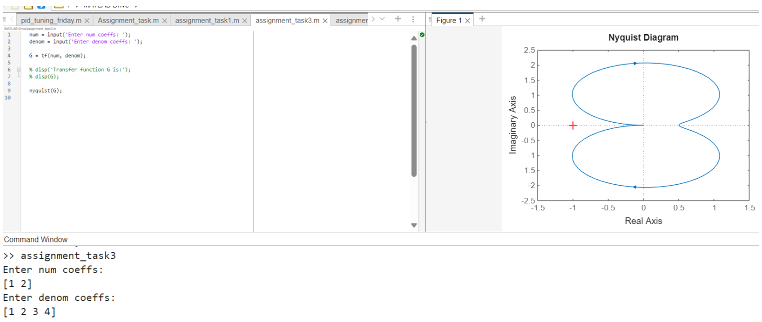
Problem 7


Nyquist Plot
Imaginary
Real
Command Window
Enter denom coeffs:
[1 2 3 4]
System is stable.


Nyquist Diagram
Imaginary Axis
Real Axis
Command Window
>> assignment_task3
Enter num coeffs:
[1 2]
Enter denom coeffs:
[1 2 3 4]