JSC ”Kazakh- British Technical University”

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Industrial and Systems Engineering

1 SIS

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Almaty 2024

1 First problem

Equations of Motion

For the mass-spring-damper system, we apply Newton's second law to derive the equation of motion. The forces acting on the mass M are:

- Spring force: $F_k = -Ky(t)$,
- Damping force: $F_b = -B\dot{y}(t)$,
- External input force: $F(t)$.

Thus, Newton's second law gives:

$$M\ddot{y}(t) = F(t) - B\dot{y}(t) - Ky(t)$$

Rearranging the terms:

$$M\ddot{y}(t) + B\dot{y}(t) + Ky(t) = F(t)$$

This is the differential equation that governs the motion of the system.

Block Diagram Representation

The block diagram for the mass-spring-damper system consists of:

- A block representing the mass M, which converts acceleration into velocity,
- A block for the damper with damping coefficient B, which provides resistance proportional to velocity,
- A block for the spring with constant K, which generates a restoring force proportional to displacement $y(t)$.

The system is in a feedback loop, where the displacement $y(t)$ feeds back to affect both the spring and damping forces.

State-Space Representation

Define the state variables as:

$$x_1 = y(t) \quad \text{(position)}$$

$$x_2 = \dot{y}(t) \quad \text{(velocity)}$$

The state-space representation is:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{M}\left(F(t) - Bx_2 - Kx_1\right)$$

In matrix form, we have:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ \frac{-K}{M} & \frac{-B}{M} \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} F(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t)$$

Transfer Function

To derive the transfer function, we take the Laplace transform of the differential equation, assuming zero initial conditions:

$$Ms^2Y(s) + BsY(s) + KY(s) = F(s)$$

The transfer function $H(s)$ is given by:

$$H(s) = \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

For the given values $K = 1$, $M = 10$, and $B = 1$, the transfer function becomes:

$$H(s) = \frac{1}{10s^2 + s + 1}$$

Step Response of the System

The step response of the system is its response to a unit step input $F(t) = 1$. This can be obtained by applying the step function in MATLAB.

Impulse Response of the System

The impulse response is the system's response to a Dirac delta input. It can be computed using the impulse function in MATLAB.

MATLAB Script

Below is the MATLAB script to simulate the system using both the step response and ODE45 methods for numerical integration.

```
% Given parameters
K = 1;
M = 10;
B = 1;

% Define system matrices
A = [0 1; -K/M -B/M];
B_matrix = [0; 1/M];
C = [1 0];
D = [0];

% State-space system
sys_ss = ss(A, B_matrix, C, D);

% Transfer function of the system
sys_tf = tf([1], [M B K]);

% Time vector for simulation
t = 0:0.01:10;

% Step response
figure;
step(sys_tf, t);
title('Step Response');

% Impulse response
figure;
impulse(sys_tf, t);
title('Impulse Response');

% ODE45 simulation
f = @(t, x) [x(2); (1/M)*(1 - B*x(2) - K*x(1))];
x0 = [0; 0];  % Initial conditions

[t_ode, x_ode] = ode45(f, t, x0);

% Plot ODE45 results
figure;
plot(t_ode, x_ode(:, 1));
xlabel('Time (s)');
ylabel('Position y(t)');
title('ODE45 Simulation');
```

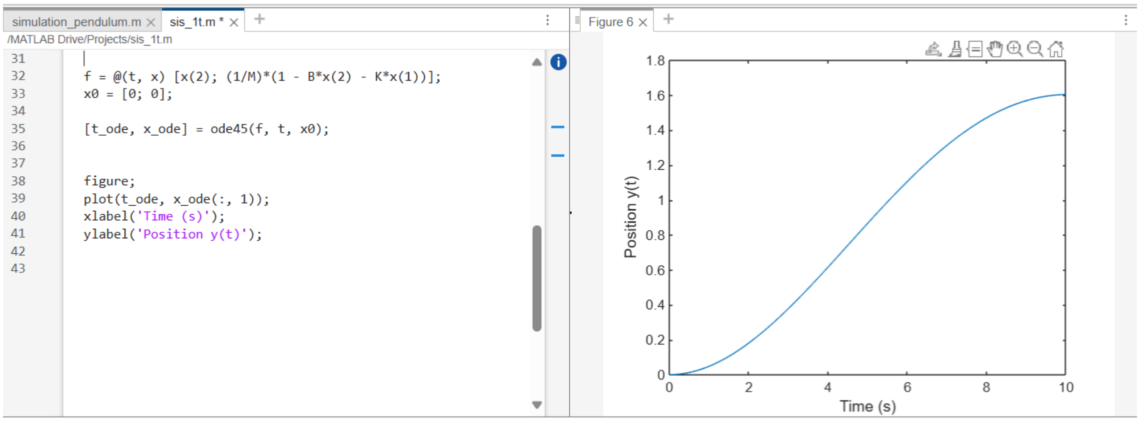


Figure 1: First Figure Description

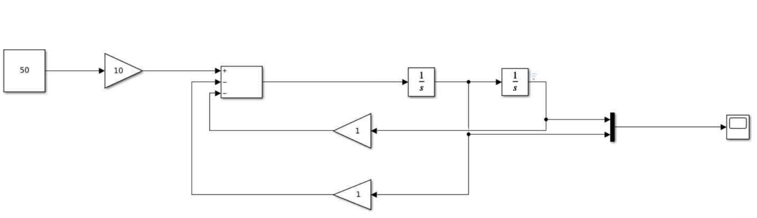


Figure 2: Simulink block diagram

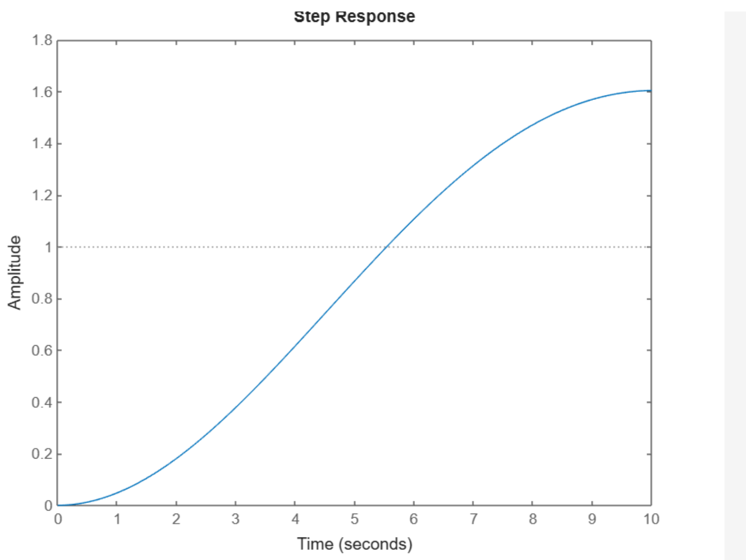


Figure 3: Step response

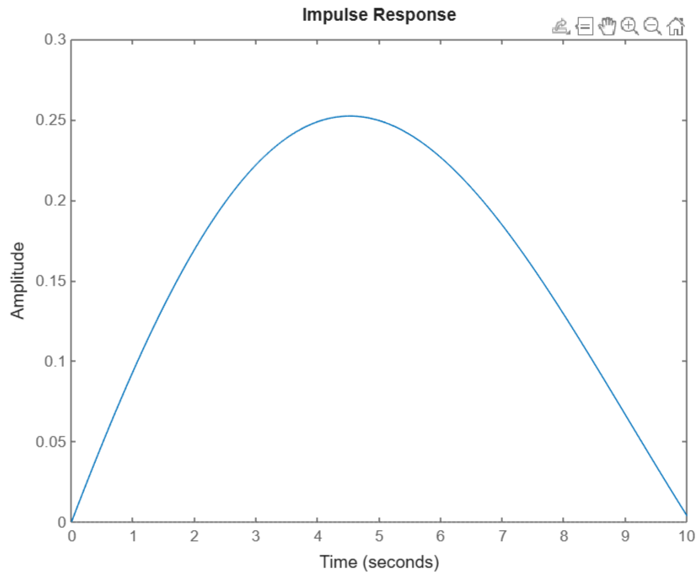


Figure 4: Impulse response

Second Problem

Equations of Motion

The equations of motion for mass M_1 and M_2 are as follows:

For mass M_1:

$$M_1\ddot{y}_1 = -B_1\dot{y}_1 + K(y_2 - y_1) - B_3(\dot{y}_1 - \dot{y}_2) + f(t) \tag{1}$$

For mass M_2:

$$M_2\ddot{y}_2 = -B_2\dot{y}_2 + K(y_1 - y_2) + B_3(\dot{y}_1 - \dot{y}_2) \tag{2}$$

State-Space Representation

We define the state variables as:

$$x_1 = y_1, \quad x_2 = \dot{y}_1, \quad x_3 = y_2, \quad x_4 = \dot{y}_2$$

Thus, the state-space form is:

$$\dot{x} = Ax + Bu$$

where the system matrices are defined as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K}{M_1} & \frac{-B_1-B_3}{M_1} & \frac{K}{M_1} & \frac{B_3}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & \frac{B_3}{M_2} & \frac{-K}{M_2} & \frac{-B_2-B_3}{M_2} \end{bmatrix}$$

and the input, output, and direct transmission matrices are:

$$B = \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

Transfer Function

The transfer function can be obtained from the state-space representation. The general formula is:

$$G(s) = C(sI - A)^{-1}B + D$$

This transfer function relates the input force $f(t)$ to the output displacement $y_1(t)$.

MATLAB Code

Below is the MATLAB code used to simulate the step and impulse responses of the system, as well as to numerically simulate the system using `ode45`.

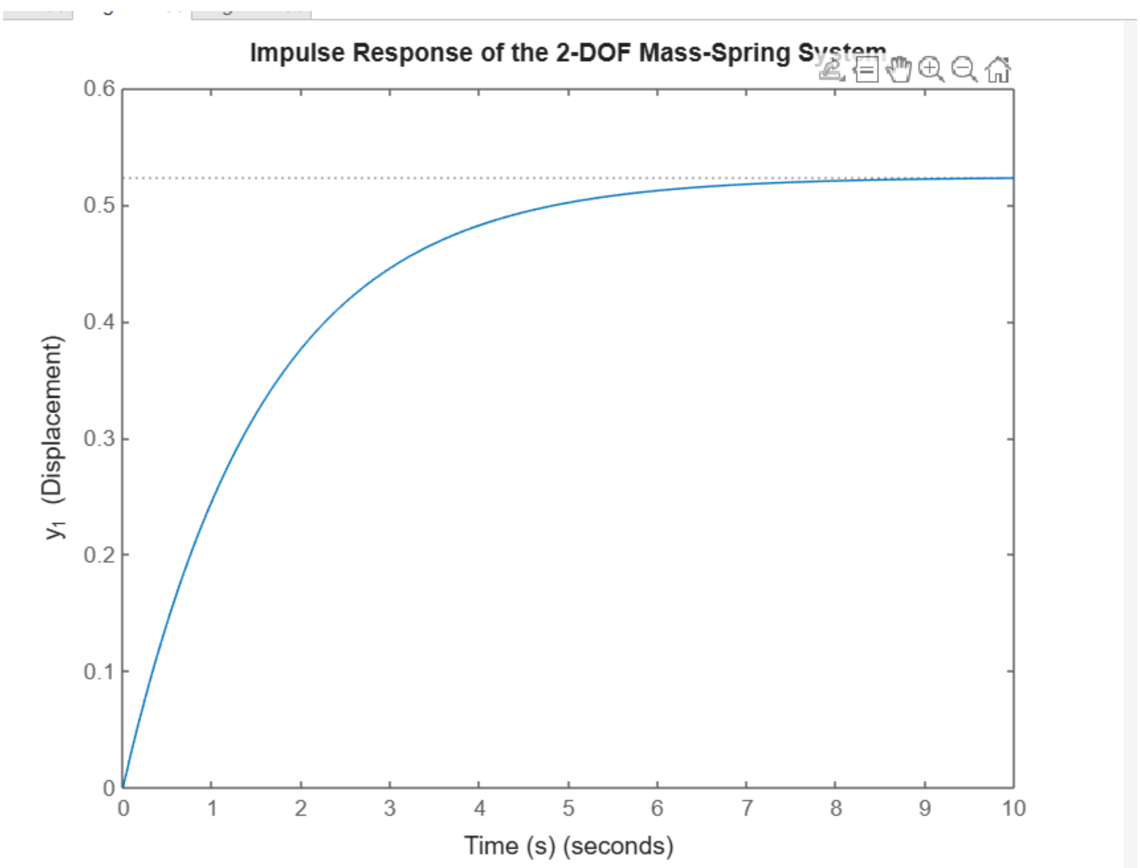


Figure 5: Impulse Response of the 2-DOF Mass-Spring System

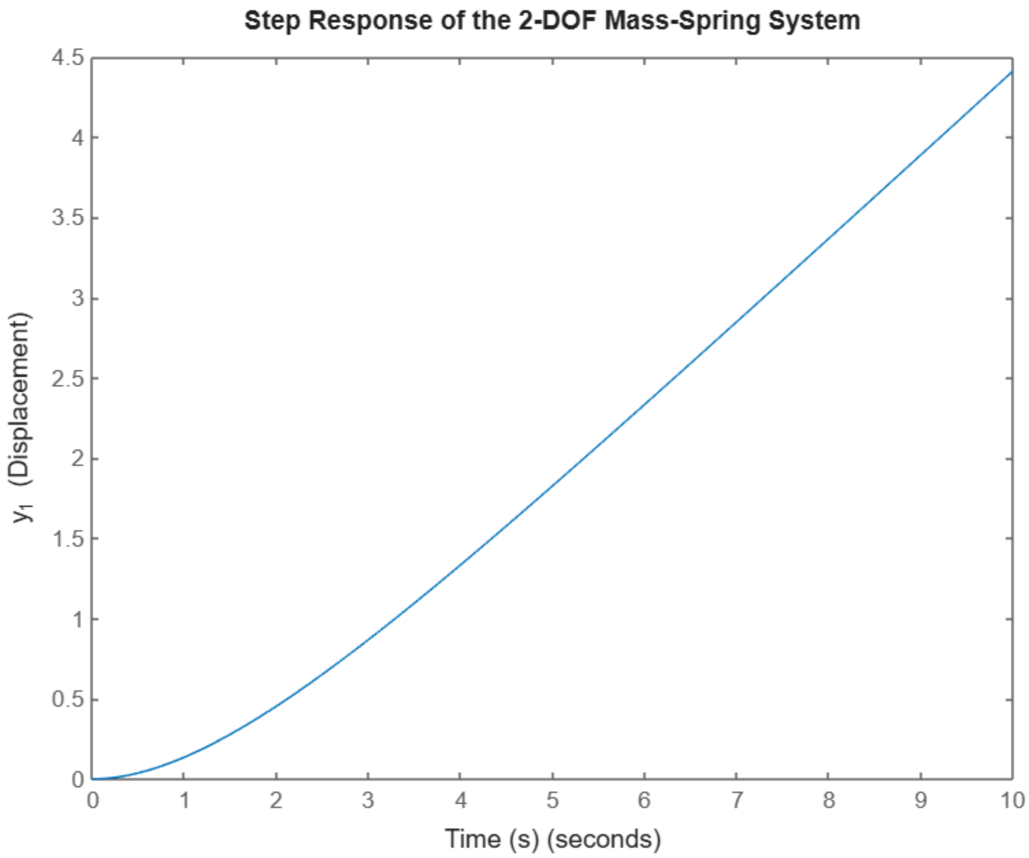


Figure 6: Step Response of the 2-DOF Mass-Spring System

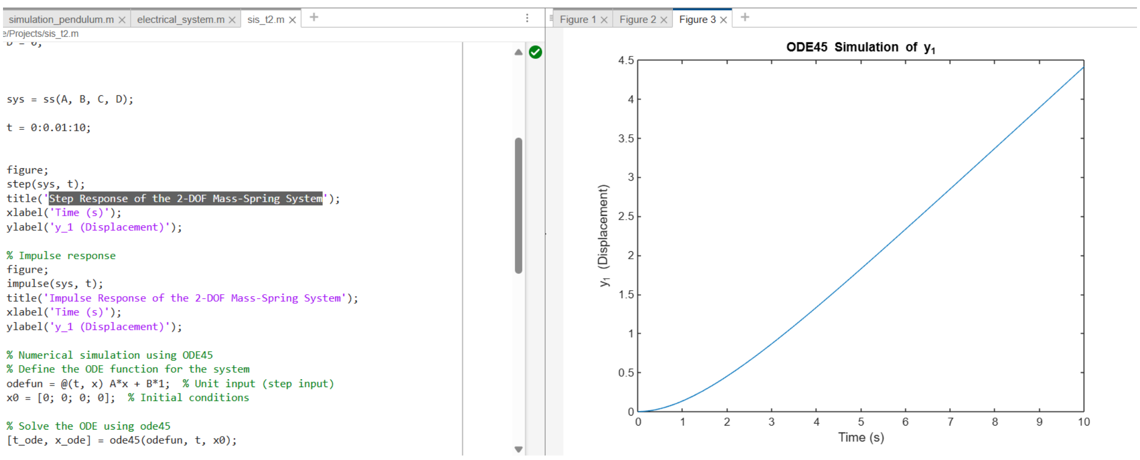


Figure 7: Simulation with ODE45

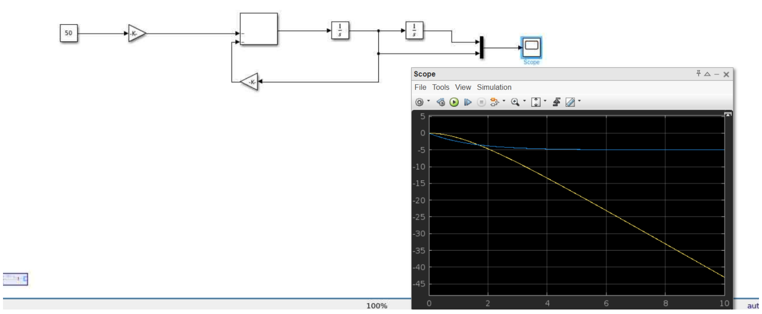


Figure 8: Block diagram of Mechanical system

Problem 3

Input-Output Relationship

Applying Kirchhoff's voltage law (KVL) around the circuit, the total voltage across all components equals the input voltage $V_{in}(t)$:

$$V_{in}(t) = i(t)r + L_1\frac{di(t)}{dt} + \frac{1}{C_1}\int i(t)dt + L_2\frac{di(t)}{dt} + \frac{1}{C_2}\int i(t)dt + Ri(t)$$

Using the relationship $i(t) = C_2\frac{dV_{out}(t)}{dt}$, we can write this in terms of $V_{out}(t)$.

$$V_{in}(t) = ri(t) + L_1\frac{di(t)}{dt} + \frac{1}{C_1}\int i(t)dt + L_2\frac{di(t)}{dt} + \frac{1}{C_2}\int i(t)dt + Ri(t)$$

After simplifying the equation, the input-output relationship becomes a differential equation relating $V_{in}(t)$ and $V_{out}(t)$.

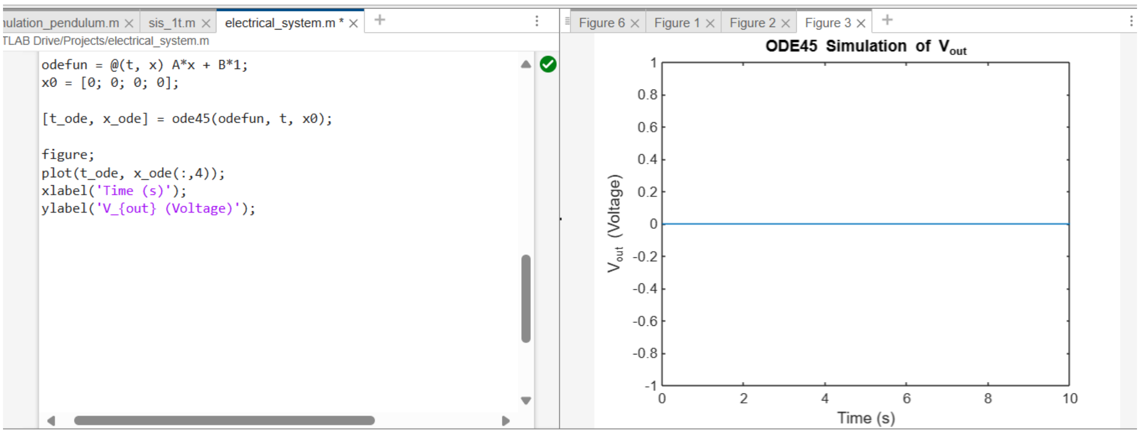
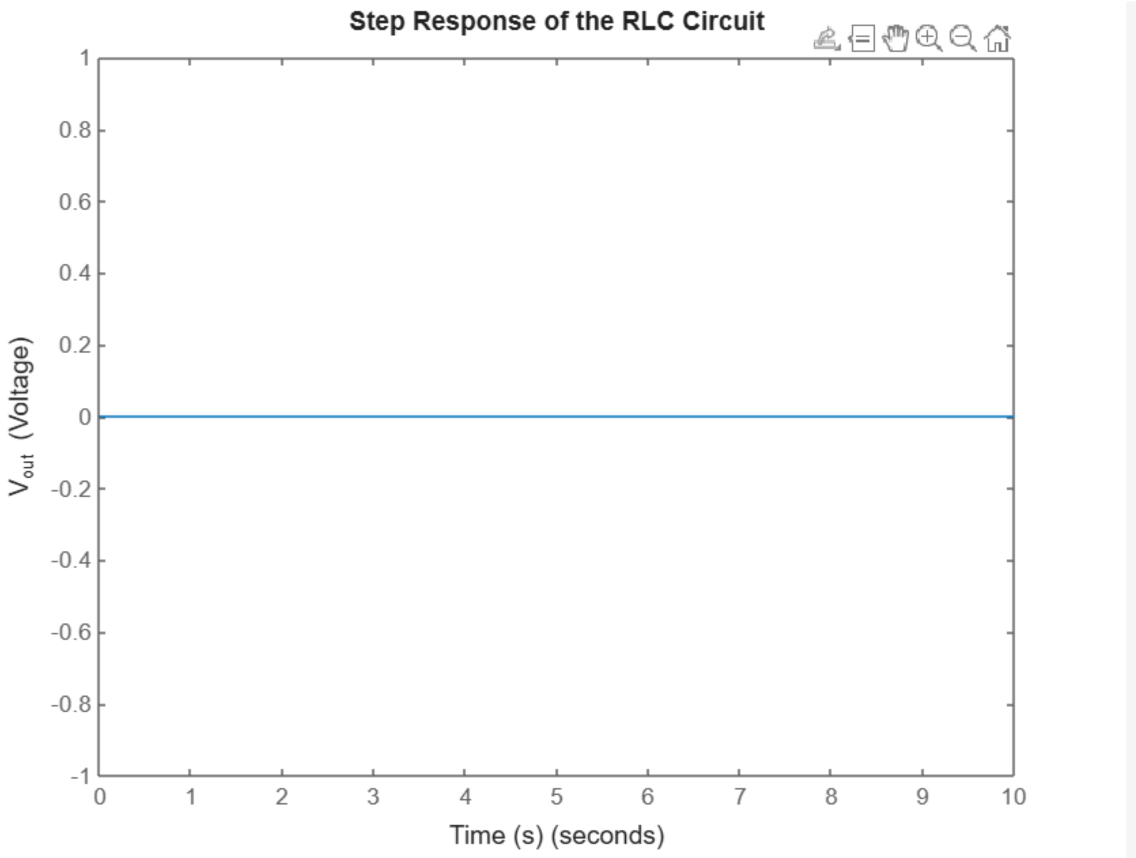
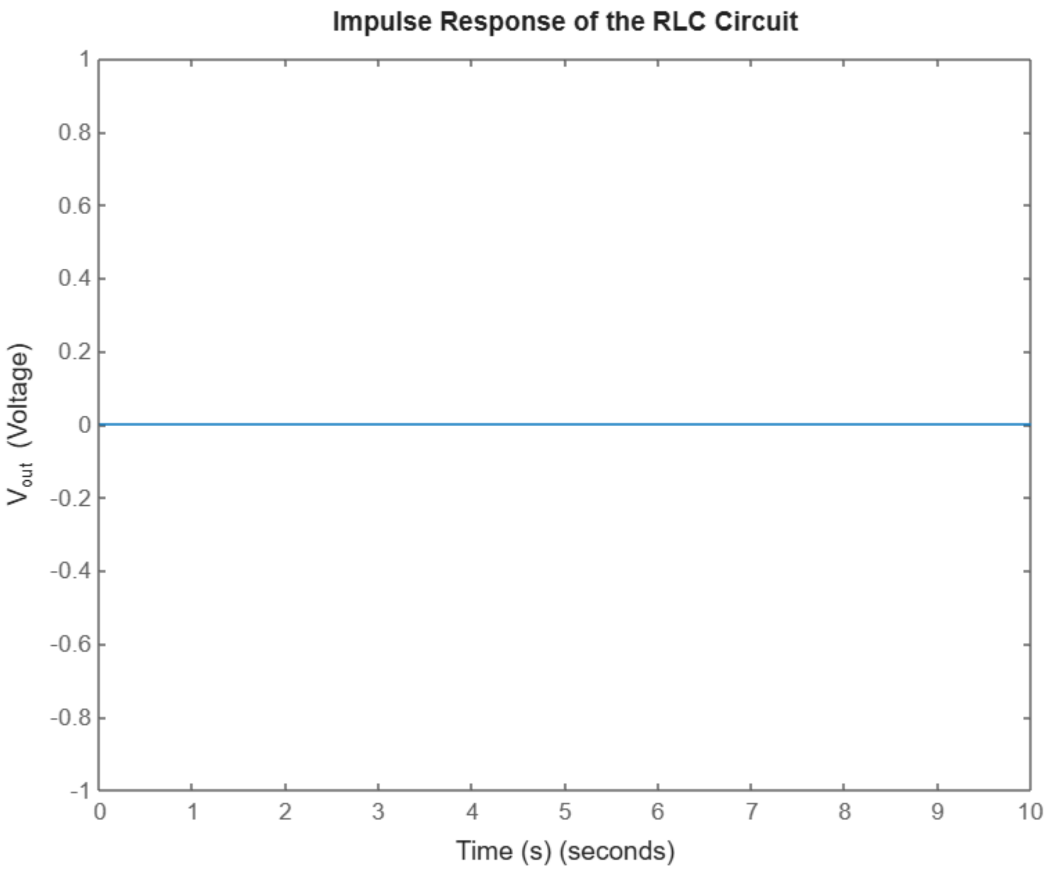


Figure 9: Caption


Step Response of the RLC Circuit
V_{out} (Voltage)
Time (s) (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10


Impulse Response of the RLC Circuit
V_{out} (Voltage)
Time (s) (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6
7
8
9
10

Problem 4

Step 1: KCL at Node 1

Applying Kirchhoff's Current Law (KCL) at Node 1:

$$i_1 + i_2 = i_3$$

Step 2: KVL for Loop 1 and Loop 2

Applying Kirchhoff's Voltage Law (KVL) for Loop 1:

$$v_1 - R_1 i_1 - v_{C1} = 0$$

Applying Kirchhoff's Voltage Law (KVL) for Loop 2:

$$v_{C1} - v_{C2} - L_1 \frac{di_3}{dt} = 0$$

Step 3: Expressing Currents in Terms of Voltages

Using the relationships between current and voltage for capacitors and inductors:

$$i_1 = C_1 \frac{dv_{C1}}{dt}$$

$$i_2 = C_2 \frac{dv_{C2}}{dt}$$

$$i_3 = C_2 \frac{dv_{C2}}{dt}$$

Step 4: Substituting Currents into KCL and KVL Equations

Substituting the current expressions into the KCL and KVL equations:

$$C_1 \frac{dv_{C1}}{dt} + C_2 \frac{dv_{C2}}{dt} = C_2 \frac{dv_{C2}}{dt}$$

$$v_1 - R_1 C_1 \frac{dv_{C1}}{dt} - v_{C1} = 0$$

$$v_{C1} - v_{C2} - L_1 C_2 \frac{d^2 v_{C2}}{dt^2} = 0$$

Step 5: Simplifying the Equations

Simplifying the equations:

$$C_1\frac{dv_{C1}}{dt} = 0$$

$$v_1 - R_1C_1\frac{dv_{C1}}{dt} - v_{C1} = 0$$

$$v_{C1} - v_{C2} - L_1C_2\frac{d^2v_{C2}}{dt^2} = 0$$

Obtaining State-Space Representation

Choosing the state variables as v_{C1} and v_{C2}, we can rewrite the equations in state-space form:

$$\frac{dx_1}{dt} = -\frac{1}{R_1C_1}x_1 + \frac{1}{R_1C_1}v_1$$

$$\frac{dx_2}{dt} = x_1 - \frac{1}{L_1C_2}x_2$$

$$y = x_2$$

where:

- $x_1 = v_{C1}$
- $x_2 = v_{C2}$
- $y =$ output voltage (v_{C2})

Obtaining Transfer Function

Taking the Laplace transform of the state-space equations and assuming zero initial conditions, we get:

$$sX_1(s) = -\frac{1}{R_1C_1}X_1(s) + \frac{1}{R_1C_1}V_1(s)$$

$$sX_2(s) = X_1(s) - \frac{1}{L_1C_2}X_2(s)$$

$$Y(s) = X_2(s)$$

Solving for $\frac{Y(s)}{V_1(s)}$, we obtain the transfer function:

$$G(s) = \frac{Y(s)}{V_1(s)} = \frac{1}{R_1C_1L_1C_2s^2 + L_1C_2s + 1}$$

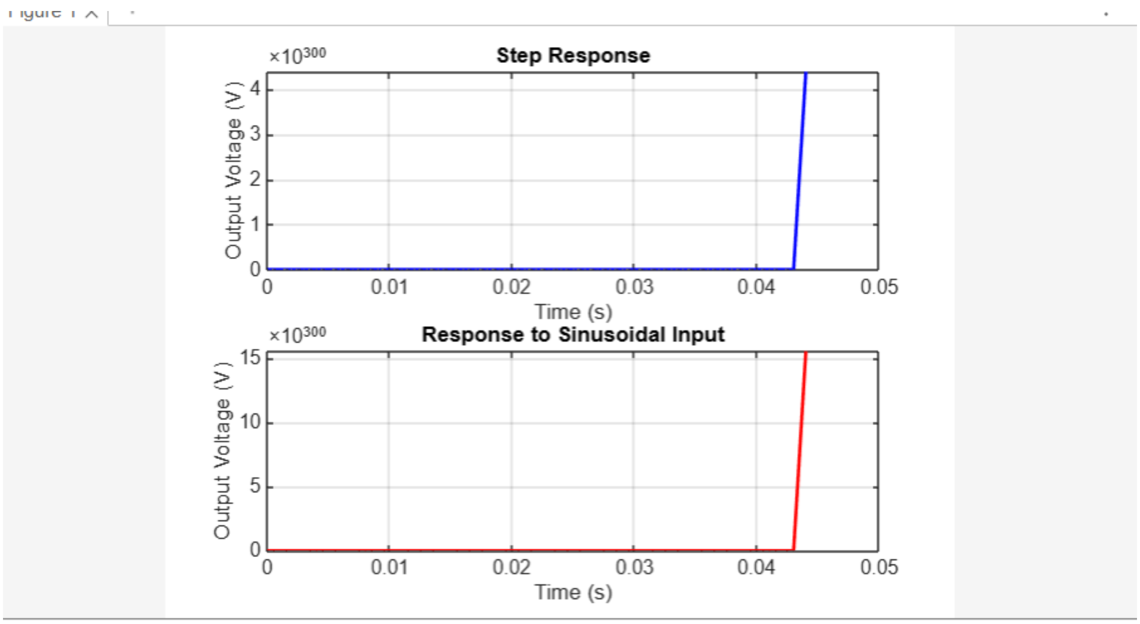


Figure 10: Caption

Finding Step Response

The step response can be found by taking the inverse Laplace transform of the transfer function with $V_1(s) = \frac{1}{s}$:

$$y(t) = \mathcal{L}^{-1}\left\{G(s) \cdot \frac{1}{s}\right\}$$

Finding Response to Sinusoidal Input

The response to a sinusoidal input can be found by substituting $V_1(s) = \frac{A}{s^2+\omega^2}$ into the transfer function and taking the inverse Laplace transform.

Finding Impulse Response

The impulse response can be found by taking the inverse Laplace transform of the transfer function with $V_1(s) = 1$.

This MATLAB script will simulate the system's response to a step input and a sinusoidal input using the state-space representation and the ode45 function.

[htbp]

Problem 5: MATLAB-based

Solve the following system of linear equations, with MATLAB:

$$6x_1 + 9x_2 + 7x_3 + 5x_4 = 250 \tag{3}$$
$$6x_1 + 4x_2 + 7x_3 + 3x_4 = 195 \tag{4}$$
$$4x_1 + 5x_2 + 3x_3 + 2x_4 = 145 \tag{5}$$
$$4x_1 + 3x_2 + 8x_3 + 2x_4 = 125 \tag{6}$$

We will solve this system of equations using the following methods:

- Solve using the `linsolve` method, a built-in function of MATLAB.
- Solve without `linsolve`, using the basic inverse matrix method.

Solution Using `linsolve` in MATLAB

The system of equations can be represented in matrix form as:

$$Ax = b$$

Where A is the coefficient matrix, x is the vector of variables, and b is the result vector. In this case:

$$A = \begin{bmatrix} 6 & 9 & 7 & 5 \\ 6 & 4 & 7 & 3 \\ 4 & 5 & 3 & 2 \\ 4 & 3 & 8 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 250 \\ 195 \\ 145 \\ 125 \end{bmatrix}$$

In MATLAB, we can use the `linsolve` function to solve for x:

MATLAB Code

Solutions

Solution using linsolve:

$$x_1 = 24.0000$$
$$x_2 = 5.0000$$
$$x_3 = -2.0000$$
$$x_4 = 15.0000$$

Solution using inverse matrix method:

$$x_1 = 24.0000$$
$$x_2 = 5.0000$$
$$x_3 = -2.0000$$
$$x_4 = 15.0000$$

Verification

Both solutions match.

MATLAB Code Example

```
% Define a simple value
x = 5;

% Display the value
disp('Value of x is:');
disp(x);
```

Problem 6

¿ Sagingaly: J = $\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$

¿ Sagingaly: $\mathrm{X}_{eq} = \begin{bmatrix} x_{1eq} \\ x_{2eq} \\ \vdots \\ x_{neq} \end{bmatrix}$

Solutions

Solution using linsolve:

$$x_1 = 24.0000$$
$$x_2 = 5.0000$$
$$x_3 = -2.0000$$
$$x_4 = 15.0000$$

Solution using inverse matrix method:

$$x_1 = 24.0000$$
$$x_2 = 5.0000$$
$$x_3 = -2.0000$$
$$x_4 = 15.0000$$

Both solutions match.