JSC "Kazakh- British Technical University"

School of Information Technology and Engineering

TSIS 1

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Problems of TSIS1

Problem 1

A tracking system with combined control

Схема №12. СЛЕДЯЩАЯ СИСТЕМА С КОМБИНИРОВАННЫМ УПРАВЛЕНИЕМ

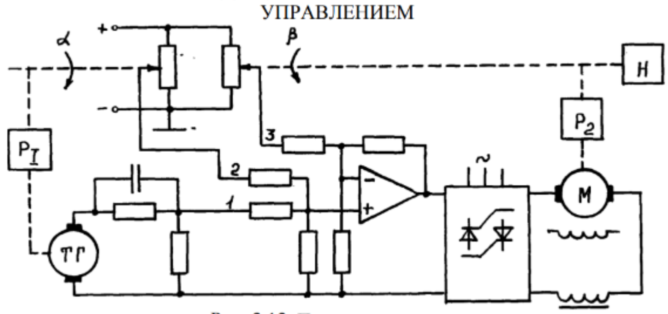


Рис. 3.12. Принципиальная схема

Таблица 12

Параметры		Значение параметров САР по вариантам									
		1	2	3	4	5	6	7	8	9	0
Кду	В/рад	16	18	20	28	21	15	24	20	30	25,2
Кду	В/рад	16	18	20	28	21	15	24	20	30	25,2
Ку1		0,216	0,202	0,192	0,03	0,154	0,04	0,3	0,12	0, 2	0,2
Ку2		0,65	0,88	0,78	2,85	1,34	3,42	0,68	1,22	0,93	1,09
Ку3		0,65	0,88	0,78	2,85	1,34	3,42	0,68	1,22	0,93	1,09
Ктп		12,8	11,2	18,6	14,8	12,8	20	13,6	16,2	10	15,2
Ттп		0,02	0,03	0,01	0,04	0,02	0,008	0,012	0,015	0,01	0,03
Кд1	рад/В*с	14,1	8,2	6,2	2,2	8,7	2,6	15,6	5,6	12,3	4,8
Кд2	рад/н*м*с	2,6	1,5	8,7	7,8	2,4	10	12	7,2	4,5	6,5
Тэ	с	0,03	0,02	0,03	0,01	0,01	0,015	0,008	0,02	0,018	0,01
Тм	с	0,15	0,12	0,09	0,2	0,24	0,18	0,14	0,21	0,12	0,15
Кр1		20	10	12	14	10	15	13,55	11,8	17,6	20,4
Кр2		0,008	0,011	0,01	0,006	0,008	0,012	0,01	0,009	0,007	0,01
Ктг	В*с/рад	0,5	0,1	0,2	0,08	0,11	0,2	0,18	0,15	0,11	0,1
δ		0,15	0,2	0,4	0,172	0,19	0,19	0,345	0,26	0,45	0,56

Brief description of the system and its components

Description of Components and Their Functions

- **PT — Setpoint Element (Input Signal)**

  This component sets the required value of the controlled parameter (such as speed, position, or voltage). It serves as the source of signals for the entire system and defines the goal that the system should strive to reach.

- **TT — Tachogenerator (Sensor)**

  The tachogenerator measures the rotational speed of the motor or output shaft and converts it into an electrical signal. This signal is used for feedback to compare the actual value with the setpoint.

- **Amplifier Blocks**

  Amplifier blocks are used to amplify the signal coming from the sensor or setpoint element. The diagram includes several amplifiers:

  - The first amplifier boosts the error signal (the difference between the setpoint and current values).
  - The second amplifier may regulate the output signal before sending it to the motor.

- **Summing Point (Located Before the Amplifier)**

  The summing point collects and compares signals from the setpoint element PT and the tachogenerator TT. This comparison generates an error signal — the difference between the setpoint and the actual value measured by the tachogenerator. The error signal is then sent to the amplifier, where it is adjusted before being sent to the motor.

- **Motor Control Block (Includes Rectifier and Switch)**

  The rectifier and switch provide control over the motor. This block converts the signal from the amplifier into the desired form, such as direct current to control an electric motor, and then sends it to M.

- **M — Electric Motor (Control Object)**

  The motor is the main actuator that performs the work. Depending on the input signal, it changes its speed or position. The motor output is connected to the load H, which can be a mechanism moved by the motor.

- **P- Gearbox** A gearbox is a mechanical device designed to change the speed and torque transmitted from the motor to the working mechanism.

- **Motor load** Motor load- refers to the force or resistance exerted on a motor during its operation

- **A comparator** A comparator is an electronic device that compares two input voltages and outputs a signal indicating which of the input voltages is greater, lesser, or equal to the other. Comparators are commonly used in various electronic circuits, including alarm systems, analog-to-digital converters, and control systems.
- **Feedback** P_2

  The output signal P_2 is feedback sent back to the system through the tachogenerator TT. This allows the system to ”know” what is happening at the output and use this information to adjust the control signal if the actual value deviates from the setpoint.

System Operation Principle

- **Setting the Goal:** The setpoint element PT sets the required value that the system should aim to achieve.
- **Comparing Values:** The signal from PT goes to the summing point, where it is compared with the feedback signal from the tachogenerator TT. The tachogenerator measures the current motor speed and returns it as feedback.
- **Generating the Error Signal:** As a result of the comparison, an error signal is generated at the summing point, representing the difference between the setpoint and actual values.
- **Amplifying the Error Signal:** The error signal is sent to an amplifier, which boosts it to the required level.
- **Motor Control:** The amplified signal is fed to the motor control block, where it undergoes conversion and controls the motor through the rectifier and switch.
- **Executing the Task:** The motor M changes its speed or position, moving the load H, which is connected to the motor output.
- **Feedback:** The tachogenerator TT measures the motor’s performance (its speed) and sends this signal back into the system through feedback.

Role of Each Component

- **Setpoint Element** PT defines the target value.
- **Tachogenerator** TT provides information on the current state (actual value).
- **Summing Point** generates the error signal.
- **Amplifier** adjusts the error signal by amplifying it to the required level for motor control.

- **Motor** M directly performs the mechanical work by changing its speed or position.
- **Feedback** allows the system to constantly monitor the output state and adapt control.

Transfer functions of all of each components

1. PT — Setpoint Element (Input Signal)

The setpoint element defines the desired value of the controlled parameter. Its transfer function is typically represented as:

$$G_{PT}(s) = 1$$

This indicates that the output (setpoint) directly follows the input, as it's simply a constant or step signal provided to the system.

2. TT — Tachogenerator (Sensor)

The tachogenerator converts rotational speed to an electrical signal. Its transfer function can be represented as:

$$W_{(t)=\frac{K}{DU(s)}}(1)$$

where K_{TT} is a constant representing the gain (sensitivity) of the tachogenerator, relating the output voltage to the rotational speed.

3. Amplifier Blocks

- **First Amplifier (Error Signal Amplifier):** The first amplifier amplifies the error signal. Its transfer function can be given as:

  $$G_{A1}(s) = K_{A1}$$

  where K_{A1} is the gain of the amplifier.

- **Second Amplifier (Output Signal Regulator):** This amplifier regulates the output signal before sending it to the motor. Its transfer function can be represented as:

  $$G_{A2}(s) = K_{A2}$$

  where K_{A2} is the gain of the second amplifier.

4. Summing Point (Located Before the Amplifier), Adder subtractor

The summing point compares the setpoint and the feedback signal from the tachogenerator to produce the error signal. The transfer function for the summing point is not typically represented as a transfer function since it simply outputs the difference between two signals:

$$E(s) = R(s) - Y(s)$$

$$delta_{Ux}(t) = K_{u1} * delta_{U1} + K_{u2} * delta_{u2}(t) - K_{u3} * delta_{u3}(t) \qquad (2)$$

where $R(s)$ is the setpoint input and $Y(s)$ is the output from the tachogenerator.

5. Motor Control Block (Includes Rectifier and Switch)

The motor control block processes the output from the amplifiers and converts it to control the motor. The transfer function for this block can be represented as:

$$W_{tp}(s) = \frac{K_{tp}}{T_{tp} * s + 1}$$

where K_{MC} is the gain and T_{MC} is the time constant representing the dynamics of the rectifier and switch.

Transfer functions of each signals

$$W_{y1}(S) = K_{y1}W_{y2}(S) = K_{y2}W_{y3}(S) = K_{y3} \qquad (3)$$

Divider

$$W_d(s) = K_p \qquad (4)$$

6. M — Electric Motor (Control Object)

The electric motor's dynamics can be modeled as a first-order system:

$$G_M(s) = \frac{K_M 1}{T_{*m*s^2+T_m*s+1} * s(5)}$$

where K_M is the gain of the motor and T_M is the time constant representing the motor's response.

7. P — Gearbox

The gearbox can also be modeled as a first-order system with gain:

$$G_P(s) = \frac{K_P}{T_P s + 1}$$

where K_P is the gearbox gain and T_P is its time constant.

8. Motor Load

The motor load is usually considered as an additional dynamic element affecting the motor's performance, which can be modeled similarly to the motor itself. It might not have a distinct transfer function but can be represented as a load torque $T_L(s)$ affecting the motor.

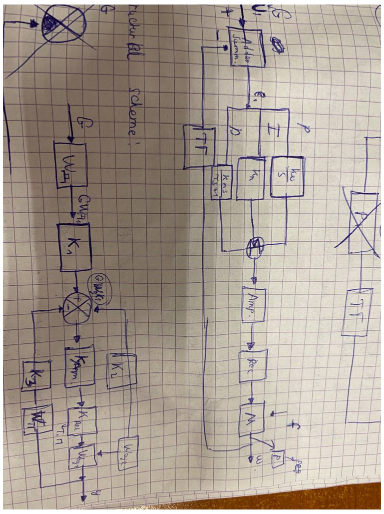
9. Comparator

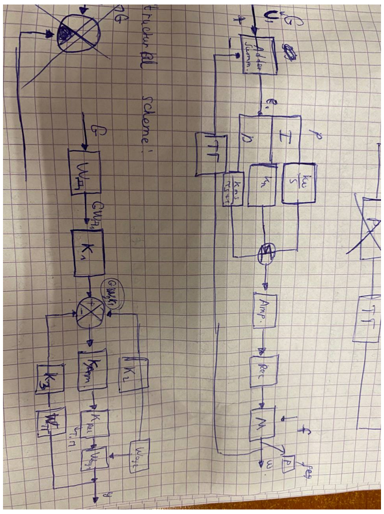
The comparator's function is typically to provide a binary output based on the input conditions. In the context of transfer functions, it is often represented as:

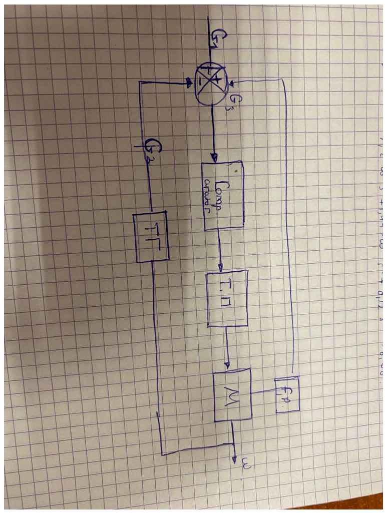
$$G_C(s) = \begin{cases} 1, & \text{if } V_{in} > V_{ref} \\ 0, & \text{if } V_{in} \leq V_{ref} \end{cases}$$

where V_{in} is the input voltage and V_{ref} is the reference voltage.

Structural and Functional scheme







Open and closed system transfer function

To check the stability of the system based on the roots of the characteristic equation

To check the stability of the system based on the roots of the characteristic equation, we need to consider the denominator of the transfer function $Y(s)$ and find its roots.

Let's denote the numerator and denominator of the function $Y(s)$:

$$Y(s) = \frac{N(s)}{D(s)}$$

$$W_{3c}^{g}(S) = \frac{W_{\Pi P}^{g}}{1+W_{pc}} = \frac{\frac{K_{T\Pi}}{T_{T\Pi}\cdot S+1}\cdot\frac{U_{out}}{U_{in}}\cdot K_3\cdot\frac{K_{TT}}{DU_{TT}(S)}\cdot K_q}{1+\frac{8{,}08}{0{,}72\cdot 10^{-4}(S+1)\cdot S^2+0{,}12(S+1)S}} =$$

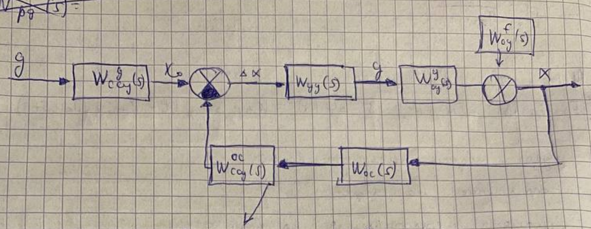
$$= \frac{\frac{11{,}2\cdot 0{,}88\cdot 0{,}1}{T_{T\Pi}\cdot S+1}}{1+\frac{8{,}08}{0{,}72\cdot 10^{-4}(S^3+S^2)+0{,}12S^2+0{,}12S}} = \frac{0{,}9856\cdot(0{,}72\cdot 10^{-4}(S^3+S^2))+0{,}12S+1S}{(0{,}72\cdot 10^{-4}(S^3+S^2)+0{,}12S^2+0{,}12S+8{,}08)\,0{,}03S+1}$$

Переход к дифференциальному уровнению

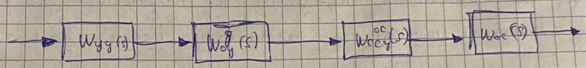
$$\frac{\varphi(S)}{V(S)} = \frac{7{,}09\cdot 10^{-5}S^4+7{,}09\cdot 10^{-5}S^3+0{,}115S^2+0{,}9}{7{,}2\cdot 10^{-5}S^4+1{,}44\cdot 10^{-4}S^3+0{,}12S^2+8{,}08}$$

Transfer function of system (разомкнутой и замкнутой)

~~$W_{pg}(S)=$~~



Open, разомкнутой системы

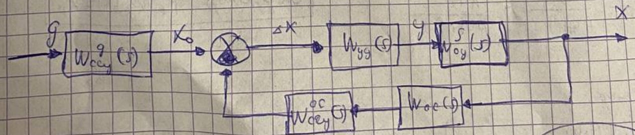


$$W_{pc}(S) = \frac{K_{T\Pi}}{T_{T\Pi}(S+1)}\cdot\cancel{K_\Pi}\cdot\frac{K_{Д}}{T_Э\cdot T_M\cdot S^2+T_M\cdot S+1\cdot S}\cdot\frac{K_{TT}}{DU_{TT}(S)}\cdot K_3$$

$$W_{pc}(S) = \frac{K_{T\Pi}\cdot K_{Д1}\cdot K_{TT}\cdot K_3}{T_{T\Pi}(S+1)\cdot T_Э\cdot T_M\cdot S^2+T_M\cdot S+1\cdot S} = \frac{11{,}2\cdot 8{,}2\cdot 0{,}1\cdot 0{,}88}{0{,}03\cdot(S+1)\cdot 0{,}02\cdot 0{,}12\cdot S^2}$$

$$= \frac{11{,}2\cdot 8{,}2\cdot 0{,}1\cdot 0{,}88}{0{,}03(S+1)\cdot 0{,}02\cdot 0{,}12\cdot S^2+0{,}12(S+1)S} = \frac{8{,}08192}{0{,}72\cdot 10^{-4}(S+1)\cdot S^2+0{,}12(S+1)S}$$

$W_{3g}(S) =$ ______



$$W_{3c}^{g}(S) = \frac{W_{yy}(S)\cdot \overset{\text{о. усилитель}}{W_{oy}}\cdot W_{ccy}(S)}{1+W_{yy}(S)\cdot W_{oy}(S)\cdot W_{oc}(S)\cdot W_{ccy}^{oc}(S)} = \frac{W_{\Pi P}^{g}(S)}{1+W_{pc}}$$

$$= \cancel{\frac{K_{T\Pi}}{T_\Pi S+1}\cdot W_{oy}\cdot K_T(}$$

where:

$$N(s) = 7.2 \times 10^{-5}s^4 + 1.44 \times 10^{-4}s^3 + 0.120072s^2 + 8.2s + 8.08$$

$$D(s) = 7.09632 \times 10^{-5}s^4 + 7.09632 \times 10^{-5}s^3 + 0.118272s^2 + 0.9856s$$

Step 1: Find the Roots of the Characteristic Equation

The characteristic equation of the system can be expressed as $D(s) = 0$. We need to find the roots of this equation.

Step 2: Analyze the Roots

A system is considered stable if all roots of the characteristic equation have negative real parts (i.e., lie in the left half of the complex plane).

Finding the Roots of the Denominator

The roots of the characteristic equation $D(s) = 0$ are as follows:

$$\begin{array}{ll} 1. & 3.5292 + 41.3653j \\ 2. & 3.5292 - 41.3653j \\ 3. & -8.0583 \\ 4. & 0 \end{array}$$

Stability Analysis

To determine the stability of the system, we look at the real parts of the roots:

- The first two roots have positive real parts (≈ 3.53), indicating instability.
- The third root has a negative real part (≈ -8.06), indicating stability.
- The fourth root is equal to zero (0), indicating marginal stability.

Conclusion

The system is **unstable** because there are roots with positive real parts. For the system to be stable, all roots must lie in the left half-plane, meaning they should have negative real parts.

Differential Equation of the System

$$\frac{Y(s)}{U(s)} = \frac{7.09632 \times 10^{-5}s^4 + 7.09632 \times 10^{-5}s^3 + 0.118272s^2 + 0.9856s}{7.2 \times 10^{-5}s^4 + 1.44 \times 10^{-4}s^3 + 0.120072s^2 + 8.2s + 8.08}$$

Step 1: Move the Denominator to the Left Side

We can express the transfer function as:

$$\left(7.2 \times 10^{-5}s^4 + 1.44 \times 10^{-4}s^3 + 0.120072s^2 + 8.2s + 8.08\right) Y(s) = \left(7.09632 \times 10^{-5}s^4 + 7\right.$$

Step 2: Rewrite in Time Domain

We will use the inverse Laplace transform for each term. We replace $s^nY(s)$ with $\frac{d^nY(t)}{dt^n}$ and $s^nU(s)$ with $\frac{d^nU(t)}{dt^n}$.

Left Side (for $Y(s)$)

$$\begin{aligned}
7.2 \times 10^{-5}s^4Y(s) &\rightarrow 7.2 \times 10^{-5}\frac{d^4Y(t)}{dt^4} \\
1.44 \times 10^{-4}s^3Y(s) &\rightarrow 1.44 \times 10^{-4}\frac{d^3Y(t)}{dt^3} \\
0.120072s^2Y(s) &\rightarrow 0.120072\frac{d^2Y(t)}{dt^2} \\
8.2sY(s) &\rightarrow 8.2\frac{dY(t)}{dt} \\
8.08Y(s) &\rightarrow 8.08Y(t)
\end{aligned}$$

Thus, the left side of the equation in the time domain will be:

$$7.2 \times 10^{-5}\frac{d^4Y(t)}{dt^4} + 1.44 \times 10^{-4}\frac{d^3Y(t)}{dt^3} + 0.120072\frac{d^2Y(t)}{dt^2} + 8.2\frac{dY(t)}{dt} + 8.08Y(t)$$

Right Side (for $U(s)$)

$$7.09632 \times 10^{-5} s^4 U(s) \to 7.09632 \times 10^{-5} \frac{d^4U(t)}{dt^4}$$
$$7.09632 \times 10^{-5} s^3 U(s) \to 7.09632 \times 10^{-5} \frac{d^3U(t)}{dt^3}$$
$$0.118272 s^2 U(s) \to 0.118272 \frac{d^2U(t)}{dt^2}$$
$$0.9856 s U(s) \to 0.9856 \frac{dU(t)}{dt}$$

Thus, the right side of the equation in the time domain will be:

$$7.09632 \times 10^{-5} \frac{d^4U(t)}{dt^4} + 7.09632 \times 10^{-5} \frac{d^3U(t)}{dt^3} + 0.118272 \frac{d^2U(t)}{dt^2} + 0.9856 \frac{dU(t)}{dt}$$

Final Differential Equation

Now we can write the final differential equation that relates $Y(t)$ and $U(t)$:

$$7.2\times10^{-5}\frac{d^4Y(t)}{dt^4}+1.44\times10^{-4}\frac{d^3Y(t)}{dt^3}+0.120072\frac{d^2Y(t)}{dt^2}+8.2\frac{dY(t)}{dt}+8.08Y(t) = 7.09632\times10$$

Checking stability of the system with methods Routh-Hurwitz methods

Stability Analysis of the Differential Equation

The given differential equation is:

$$7.2\times10^{-5}\frac{d^4Y(t)}{dt^4}+1.44\times10^{-4}\frac{d^3Y(t)}{dt^3}+0.120072\frac{d^2Y(t)}{dt^2}+8.2\frac{dY(t)}{dt}+8.08Y(t) = 7.09632\times10$$

To simplify, we rearrange it to standard form (i.e., all terms on the left):

$$7.2\times10^{-5}\frac{d^4Y(t)}{dt^4}+1.44\times10^{-4}\frac{d^3Y(t)}{dt^3}+0.120072\frac{d^2Y(t)}{dt^2}+8.2\frac{dY(t)}{dt}+8.08Y(t)-7.09632\times10^{-}$$

Step 1: Characteristic Equation

For stability analysis, we typically consider the homogeneous part of the system. Therefore, we can ignore the input $U(t)$ terms for the characteristic equation.

The characteristic polynomial can be derived from the differential equation:

$$7.2 \times 10^{-5}s^4 + 1.44 \times 10^{-4}s^3 + 0.120072s^2 + 8.2s + 8.08 = 0$$

Step 2: Applying Routh-Hurwitz Criterion

To check stability using the Routh-Hurwitz criterion, we construct the Routh array. The first step is to write down the coefficients of the characteristic polynomial.

1. Coefficients:

$$\begin{aligned} a_4 &= 7.2 \times 10^{-5}, \\ a_3 &= 1.44 \times 10^{-4}, \\ a_2 &= 0.120072, \\ a_1 &= 8.2, \\ a_0 &= 8.08. \end{aligned}$$

2. Routh Array Construction: - The first row consists of coefficients of even powers of s:

$$s^4 : 7.2 \times 10^{-5}, \quad s^2 : 0.120072$$

- The second row consists of coefficients of odd powers of s:

$$s^3 : 1.44 \times 10^{-4}, \quad s^1 : 8.2$$

Now, we calculate the elements of the Routh array:

$$\begin{array}{c|cc} s^4 & 7.2 \times 10^{-5} & 0.120072 \\ s^3 & 1.44 \times 10^{-4} & 8.2 \\ s^2 & & \\ s^1 & & \\ s^0 & 8.08 & \end{array}$$

3. For s^2:

$$Routh(s^2) = \frac{(1.44 \times 10^{-4})(0.120072) - (7.2 \times 10^{-5})(8.2)}{(1.44 \times 10^{-4}) + (7.2 \times 10^{-5})}$$

4. For s^1:

$$Routh(s^1) = \frac{(0.120072)(8.2) - (0)(8.08)}{(0.120072) + (0)}$$

Step 3: Stability Condition

The system is unstable (at least one first column element is non-positive).” This indicates that the system may exhibit unbounded behavior or oscillations in response to inputs or disturbances, leading to potential instability in practical applications.

```
>> tsis1
Routh Array:
    0.0001    0.1201    8.0800
    0.0001    8.2000         0
   -3.9799    8.0800         0
    8.2003         0         0
    8.0800         0         0

The system is unstable (at least one first column element is non-positive).
Eigenvalues of the system matrix:
   1.0e+03 *

   -1.5964
   -0.0010
   -0.0703
         0

>>
```

Figure 1: Results

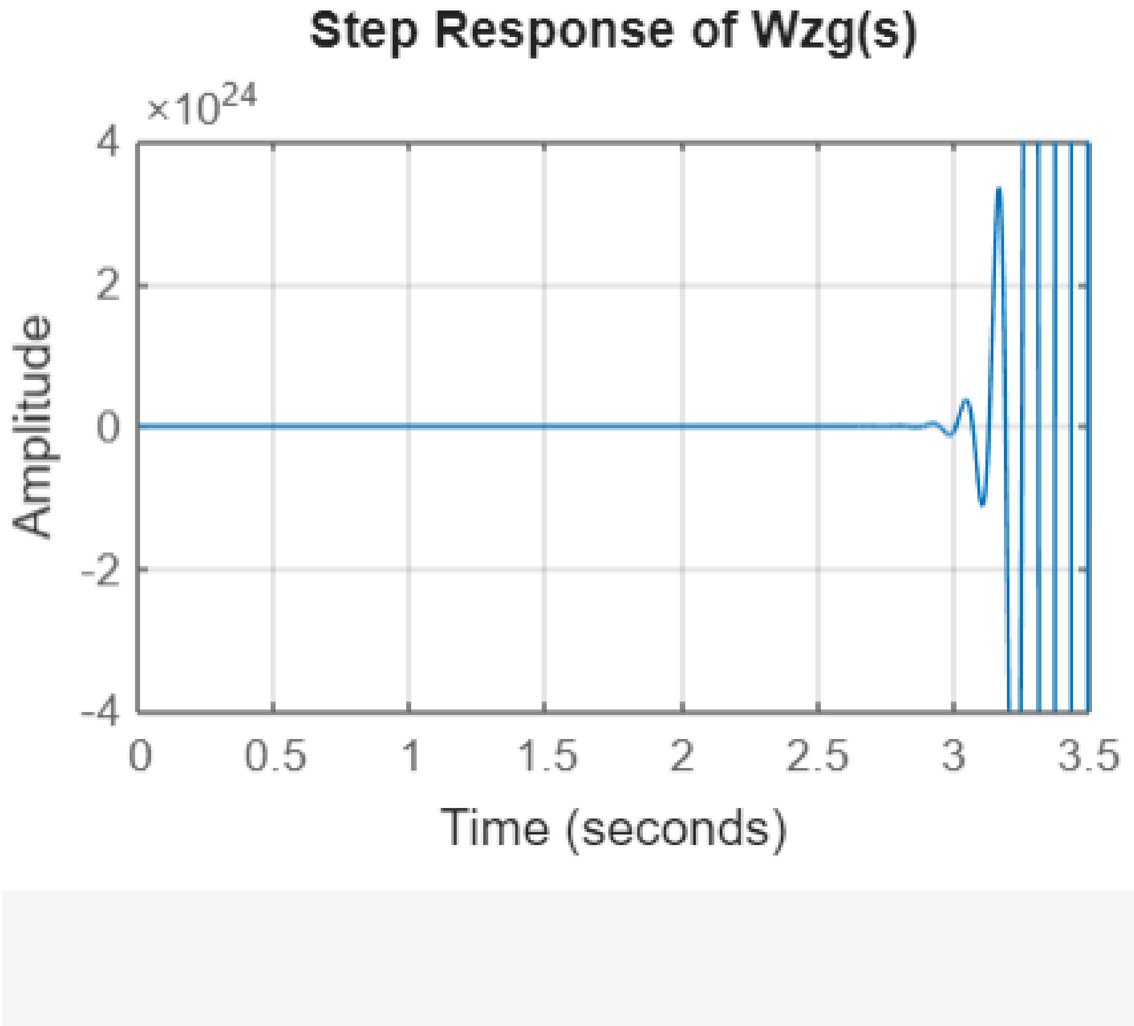


Figure 2: Results of step

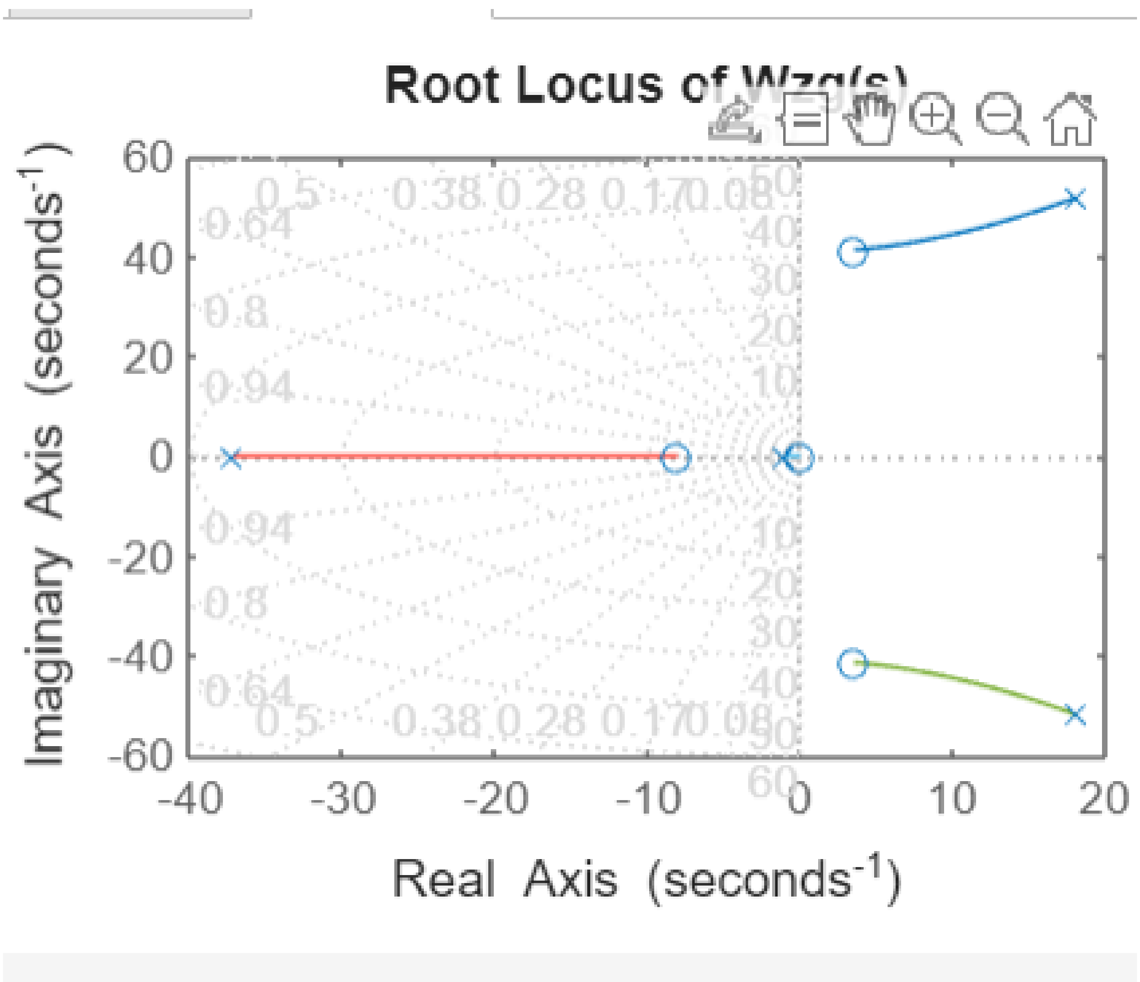


Figure 3: Results

```
numerator = [7.09632e-5, 7.09632e-5, 0.118272, 0.9856, 0];
denominator = [7.2e-5, 1.44e-4, 0.120072, 8.2, 8.08];


Wzg = tf(numerator, denominator);


disp('Transfer Function Wzg(s):');
Wzg


poles = pole(Wzg);
disp('Poles of Wzg(s):');
disp(poles);


if all(real(poles) < 0)
    disp('The system is stable.');
else
    disp('The system is unstable.');
end


figure;
step(Wzg);
title('Step Response of Wzg(s)');
grid on;


figure;
rlocus(Wzg);
title('Root Locus of Wzg(s)');
grid on;
```

Figure 4: Results