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AST5220-Milestone II: The Universes Opacity

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Abstract

The codes for this paper can be found at:
https://github.com/SagittariusA-Star/AST5220-Milestones

1. INTRODUCTION

After having simulated the large scale background evolution of the universe in Stutzer (2020), we now want to compute the optical depth and the visibility function of the universe throughout its evolution. The computations made will expand upon the code previously made (Stutzer (2020)). The optical depth and the visibility function are important quantities as, they essentially quantify how far a photon can travel in the universe before getting absorbed (when looking back in time). We make several assumptions, which amongst other things are that extinction through Thomson scattering provides the only extinction process and that the only baryons is hydrogen atoms. Also the epoch of reionization will be neglected, as a simplification. Computing the optical depth and visibility function is in principle not that hard, however, only if one is provided the density of free electrons. Thus, most of the work is finding the electron density evolution of the universe and from it the optical depth and visibility function.

2. METHOD

The formulas and equations presented here are provided by Winther (2020), unless otherwise stated.

2.1. *The Optical Depth*

As mentioned in the introduction computing the optical depth of the universe is in theory governed by fairly simple relations. When light travels through a medium, a given amount of photons are removed and added to the beam. The amount of photons that are added depend on the emissivity of the medium. We assume here that the gas the light travels through in the universe does have a negligible emissivity. Further more the amount of attenuation is given by the extinction coefficient

$$\alpha = n_e\sigma_T, \tag{1}$$

where we assume Thomson scattering on electrons to be the only mechanism to attenuate photons. Here $\sigma_T = \frac{8\pi}{3}\frac{\alpha^2\hbar^2}{m_e^2c^2}$ and n_e are the Thomson cross section and the free electron density respectively. The optical depth is given by

$$\tau(\eta) = \int_\eta^{\eta_0} n_e\sigma_T a d\eta', \tag{2}$$

where a and η are the scale factor and the conformal time of the universe. We however will for practical reasons choose to solve for τ by solving the ordinary differential equation (ODE)

$$\frac{\mathrm{d}\tau}{\mathrm{d}x} = -\frac{n_e\sigma_T}{H} = -\frac{n_e\sigma_T}{H}c, \tag{3}$$

where $x = log(a)$ and $H = \frac{\dot{a}}{a}$ are the log-scale factor and the Hubble parameter respectively. The Hubble parameter is found by the Friedmann equation through the previously developed code in Stutzer (2020). We choose this ansatz instead of the integral, so we can use the `ODEsolver` routine provided by Winther (2020). We went from natural units to SI units in the last step. The equations provided by Winther (2020) are given in natural units, however we want them to be in SI units, so as to enable solving in C++ using the constants in `Utils.h` provided by Winther (2020). We will show the dimensional analysis to go from natural units to SI on the example above, and than just provide the equations in SI units with all the constants from now on. As one can see in eq. (3) the l.h.s. of the equation if dimensionless because τ and x both are. Thus the r.h.s. must be dimensionless too. We however see that the r.h.s. has units

$$1 = \frac{[n_e][\sigma_T]}{[H]}[c]^\alpha = \frac{\mathrm{m}^{-3}[\mathrm{m}^2]}{\mathrm{s}^{-1}}[c]^\alpha = \frac{\mathrm{s}}{\mathrm{m}}[c]^\alpha \tag{4}$$

The only physical constant that is reasonable to use here is thus the light speed c, with units m/s, thus $\alpha = 1$ in this case. Similarly, when having units for instance Js or K "too much" on one side in natural units, one can correct with the reduced Planck constant or Boltzmann constant to go to SI units.

Now, as can be seen from eq. (3), there is one problem however; we need to know the electron density. We will address this problem in the next subsection.

2.2. *The Free Electron Fraction and Density*

In order to compute the optical depth we need to know the free electron density n_e. We can compute this through the free electron fraction X_e and use the baryon, i.e. hydrogen density as a translational factor. We thus get that the electron density as a function of the electron fraction and the hydrogen (baryon) density is

$$n_e = X_e n_H, \tag{5}$$

where

$$n_H = n_b \approx \frac{\rho_b}{m_H} = \frac{\Omega_{b0}\rho_{c0}}{m_H a^3}, \tag{6}$$

where we have assumed all baryons to be in hydrogen. This should be a reasonable approximation since the hydrogen atoms comprise almost all of the baryons in the universe. Here the baryon density parameter and the critcal density today are given as their regular expressions found in and computed in Stutzer (2020). The time dependence of the baryon density is solaly in the scale factor $a^{-3} = e^{-3x}$ factor.

The remaining job to find n_e is to find the electron fraction X_e. This turns out to be the hardest part of the total work. We can find X_e through solving the Boltzmann equation for a recombination/ionization reaction of the type $e^- + p^+ \rightleftharpoons H + \gamma$. At early times the energies of the constituent particles were high enough to keep this reaction in equilibrium. As long at the reaction is close to equilibrium Saha's approximation (Dodelson 2003, p. 70) ensures that

$$\frac{n_e n_p}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}}, \tag{7}$$

where the $n_i^{(0)}$ denote the equilibrium distribution species i. As the universe's charge is net zero, we can safely assume their to be as many free electrons as protons, i.e. $n_e = n_p$. Using this assumption we get the Saha equation for recombination given as

$$\frac{X_e^2}{1 - X_e} = \frac{1}{n_b}\left(\frac{m_e T_b k_B}{2\pi\hbar^2}\right)^{3/2} e^{-\epsilon_0/T_b k_B}. \tag{8}$$

Here m_e and ϵ_0 are the electron mass and the ionization energy of hydrogen respectively (Winther 2020). The temperature T_b is the baryon temperature, which in principle should be found separately, however we can approximate it to the radiation temperature T_r (Winther 2020). It is is thus given as $T_b = T_r = T_{\rm CMB}/a = 2.7255\text{K}/a$

3. RESULTS/DISCUSSION

4. CONCLUSION

REFERENCES

Dodelson, S. 2003, Modern Cosmology, Elsevier Science

Stutzer, N.-O. 2020, AST5220-Milestone I: The Background Cosmology, , , https://github.com/SagittariusA-Star/AST5220-Milestones/blob/master/doc/Milestone_I_Report_Nils_Stutzer.pdf, Last visited: 10.03.2020

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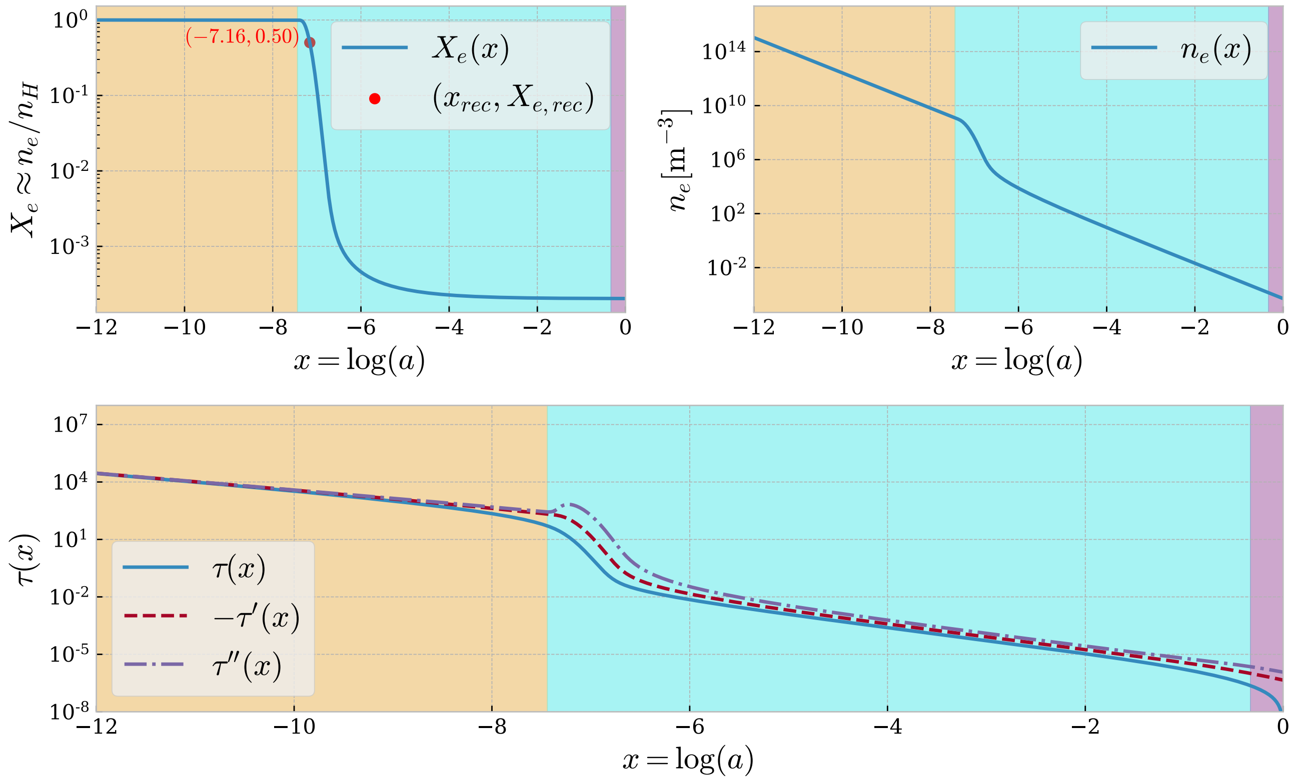

Figure 1. Upper left: Figure showing the free electron fraction X_e as a function of the log-scale factor x. **Upper right**: Figure showing the free electron density n_e as a function of the log-scale factor x. **Lower panel**: Figure showing the optical depth $\tau(x)$ of the universe due to Thomson scattering on free electrons only, as well at the derivative $\tau'(x)$ and second oder derivative $\tau''(x)$ w.r.t and as functions of the log-scale factor x. **Note**: The background color in all the plots shows the epoch of dominance for reference. Yellow markes the era of radiation dominance, blue the epoch of matter domination and purple corresponds to the epoch of dark energy. The color domain is found by checking when the corresponding density parameter is dominant, however, in reality the transitions between each epoch should be smoother than shown.

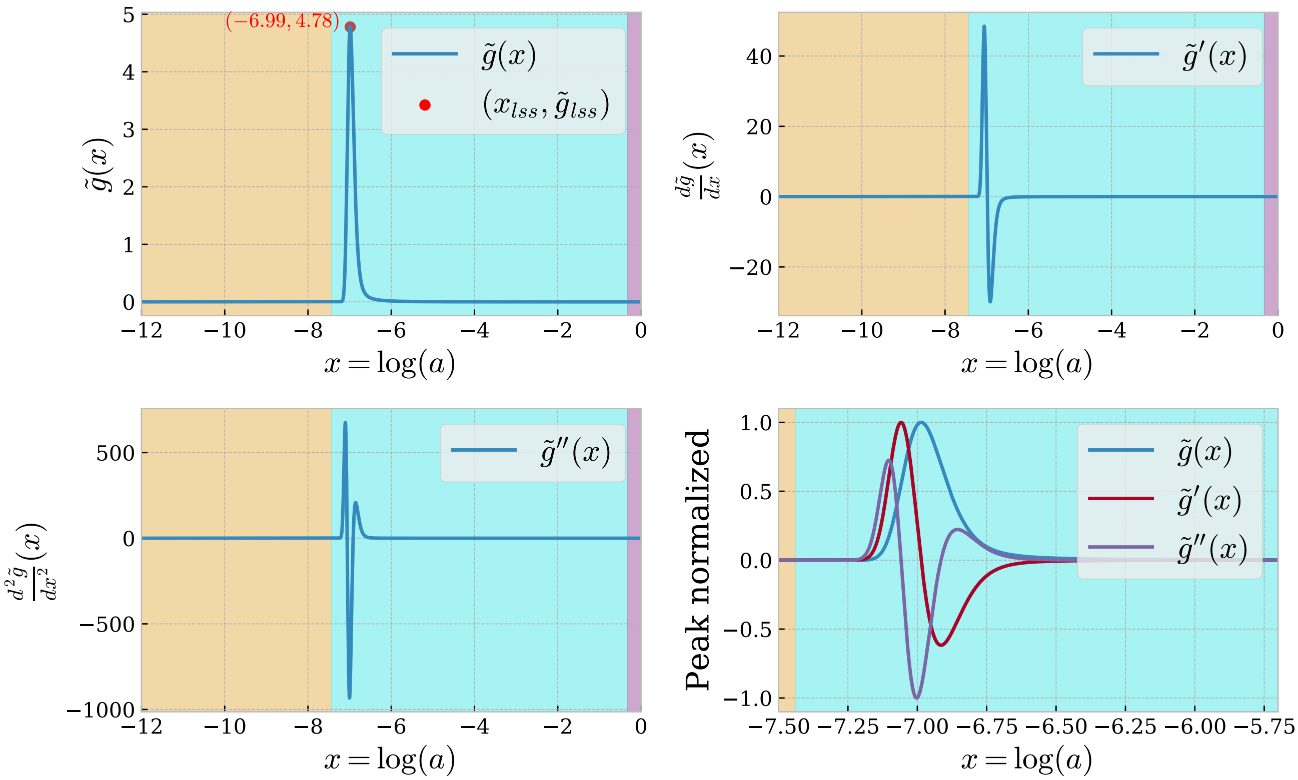

Figure 2. **Upper left**: Figure showing the un-normalized visibility function $\tilde{g}$ as a function of the log-scale factor x. The red dot markes the peak of $\tilde{g}$ and its x value corresponds to the log scale factor of the surface of last scattering. **Upper right**: Figure showing the un-normalized first order derivative of $\tilde{g}$ w.r.t. and as a function of the log-scale factor. **Lower left**: Figure showing the un-normalized second order derivative of $\tilde{g}$ w.r.t. and as a function of the log-scale factor. **Lower Right**: Figure showing the visibility function and its first two derivative together in one plot, where all functions are normalized to their respective extremum value (peak or valley). Also we have zoomed in somewhat to show more details.