SurvMeth/Surv 625: Applied Sampling

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1/8/25

Section 1

Course introduction

SurvMeth/Surv 625: Applied sampling

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- The course syllabus:
  Syllabus_SurvMeth625_Winter2025_YajuanSi.pdf

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  - Quasi-experimental: Observational
  - Survey samples: Observational

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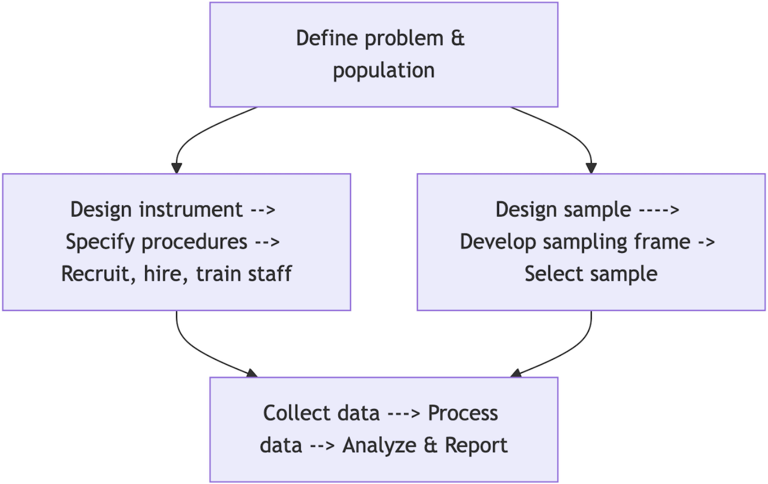
- Design characteristics
  - Realism
  - Randomization
  - Representation
- Example studies with good or bad designs

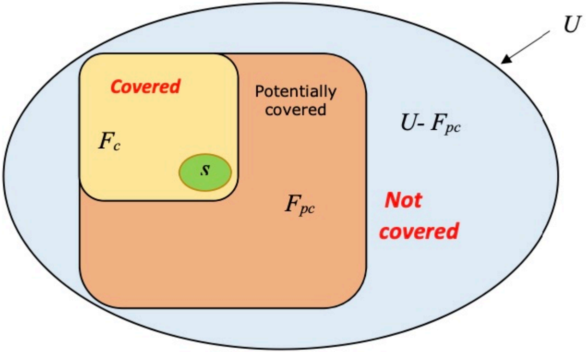
Design vs. analysis

- Sampling techniques (simple random sample, stratified, cluster sampling. etc.)
- Complex design: Multistage, area sampling
- Sampling error: sampling variance estimation
- Nonsampling errors
- Sampling **implementation, inference, and projection**

Sample surveys



Finite population: Valliant (2020)



- U = target population
- F_{pc} = potentially covered; F_c = actually covered
- $U - F_{pc}$ = not covered at all
- s = sample

Vacabulary

- Observation unit/element
- Target population
- Census
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- Response: nonresponse
- Inclusion: selected and responded
- Non-probability sampling
- Sampling error: resulting from taking one random sample instead of examining the whole population
    - Margin of error
- Nonsampling error: any errors that cannot be attributed to the sample-to-sample variability
    - Selection bias
    - Measurement error

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- Model-based sampling inference: the population is a set of random variables following some probability distribution, and the actual sample values are realizations of these random variables. *The sample data are fixed, and the population distribution is unknown.*.

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7. The coefficient of variation is defined as $CV(Y) = \frac{\sqrt{V(Y)}}{E(Y)}$, for $E(Y) \neq 0$.

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13. Total variability $V(Y) = V[E(Y \mid X)] + E[V(Y \mid X)] =$ between.ave + ave.within

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- These are the calculated/observed summary statistics based on the one sample dataset

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- Standard error: $SE(\hat{t}) = \sqrt{V(\hat{t})}$, i.e., the standard deviation of the sampling distribution of $\hat{t}$
- Confidence interval: $CI(s) = [low_s, up_s]$, if we repeatedly take samples from the population, construct a 95% CI for each possible sample, we expect 95% of the resulting intervals to include the true value, i.e., a 95% chance that the sample containing the true value

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- Statistical inference validity assessment calculates bias, variance, MSE and CI.

Lohr Example 2.6

```
library(survey)
library(sampling)
library(SDAResources)
library(tidyverse)
data(agpop)  ## Load the data set agpop
N <- nrow(agpop)
N  ## 3078 observations
```

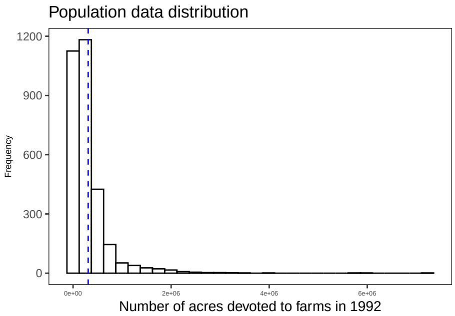
```
[1] 3078
```

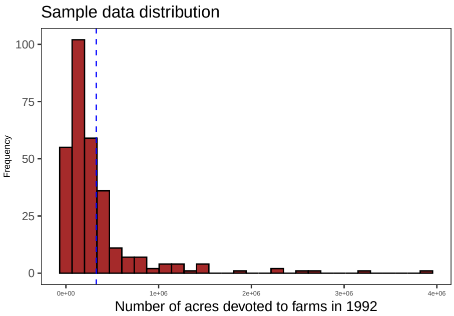
```
## Select an SRS of size n=300 from agpop
set.seed(8126834)
index <- srswor(300,N)
## each unit k is associated with index 1 or 0, with 1 indicating selectio
index[1:10]
```

```
 [1] 0 0 0 1 0 0 0 0 0 0
```

```
## agsrs is an SRS with size 300 selected from agpop
## extract the sampled units from the data frame containing the population
agsrs<- getdata(agpop,index)
```

Population distribution





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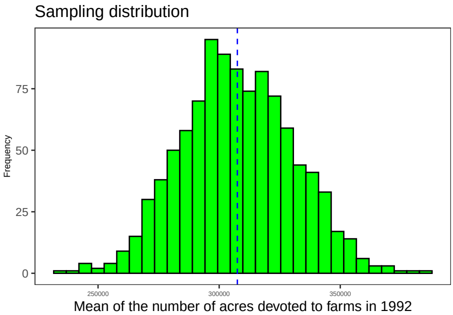

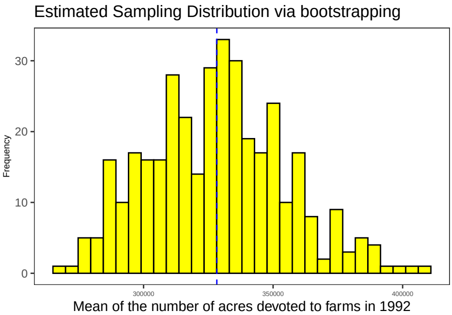

Central limit theorem!!!

References

Valliant, Richard. 2020. "Comparing Alternatives for Estimation from Nonprobability Samples." *Journal of Survey Statistics and Methodology* 8 (2): 231–63.