Stock Portfolio Optimization and Simulation

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1 Introduction

Stock pricing has always been an interesting topic since the earliest stock market was established in 1602, Amsterdam. At that time, it was extremely risky for sea voyages to bring back precious goods from the East. To lessen the risk of losing ships, ship owners sought out investors to financially support dangerous voyages. Investors would receive a percentage of return goods if the voyage was successful. Stock markets are modern examples of the problem of balancing risk and return on investment. Stock investment strategy continues to be added by the Internet, computational simulation, and mathematical market modeling to improve return on investment.

In this project, we simulate future stock prices after selecting our stocks and collecting historical stock prices for the past six months, while balancing between the risk and expected return. We then construct an efficient portfolio by determining the point of diminishing return for increases in risk. We collected data of 65 stocks from September 27, 2018 to March 27, 2019. Table 3 and 4 in the Appendix list all of the companies, their company symbol, and their assigned number. No issues with the historical price data such as stock splitting were found.

2 Model

Our model for stock pricing and choosing a stock portfolio was based on the last 6 months of data for a selection of 65 hand-picked stocks. In order to predict the future earnings of a chosen stock portfolio, the daily rates of return must be calculated to help choose stocks based on an risk factor set by the investor. From the daily rates of returns for each stock, we calculated an average rate of return and a covariance matrix that will be used to determine weights of each stock in our chosen portfolio returned from the quadratic optimization. Using the embedded quadratic optimization function in Matlab, the function minimizes

$$(1-\alpha)w^T C w - \alpha r^T w \tag{1}$$

where w is the vector that gives the weight fraction of our portfolio to invest in each stock, C is the positive definite covariance matrix, alpha is the specified risk parameter, and r is the vector of average daily rates of return for each stock. Within the quadratic optimization, a bound was set on the upper limit so that no stock could have more than a 20% weight in the portfolio. This forces diversification: for higher values of alpha, the optimization would only allow for a single stock to be purchased.

Once the quadratic optimization was setup, it was run for a series of alpha values between 0 and 1, where the lower bound is low risk tolerance and the upper bound is increased risk. Knowing the portfolio weights at each alpha value, the risk can be calculated as follows,

$$\text{Risk} = w^T C w \tag{2}$$

as well as the value for expected return; which is calculated using

$$\text{Expected Return} = r^T w. \tag{3}$$

This plot of risk versus expected return will later be used to find the point of diminishing return, or the point at which the slope is equal to one. The point of diminishing returns represents increase in the risk factor is greater than the resulting increase in expected return.

Once the stock portfolio has been created, it is necessary to look into the stochastic model used to determine the price of stock as a function of time. The price of stock is modeled using the following equation,

$$dP = \mu P dt + \sigma P dz \tag{4}$$

where P is the price of the stock, μ is the drift parameter, and σ is the volatility parameter.

This differential equation is a stochastic differential equation, or one that relies on a random process. In our case, z, is a Weiner process, or a time dependent random variable. Using Itô's lemma, we are able to solve the stochastic differential equation in the exponential form as follows.

$$P(t) = P(0)e^{(\mu - \sigma^2/2)t + \sigma\sqrt{t}\phi} \tag{5}$$

However, we use

$$P(t + \Delta t) \approx P(t) + \mu P(t)\Delta t + \sigma P(t)\sqrt{\Delta t}\phi \tag{6}$$

approximate the price of a stock any time step Δt into the future. This equation is a discrete solution to Equation (4) that gives a decent estimate of a stocks price change over a period of time if Δt remains small. We utilized a time step of 1 day in the model. One issue with this estimate is that it relies on using Matlab's built-in random number generator, which is not perfectly random, in order to generate ϕ.

Using this equation and historical data in order to regress other parameters, we are able to create a forward simulation of a stocks price over a given period of time. This simulation calculates the price of each of our 65 stocks by forward simulating the next month, then using the data from that month to replace the oldest month of historical data. Once that data is replaced, regression of parameters is once again computed and the next month of stock prices is calculated. This process is repeated for a total of 6 months. Using the weights of each stock returned from quadratic optimization, the change in price of our portfolio can be tracked over time in a forward simulation.

3 Results

The first stock quadratic optimization used a risk factor of 0.2 and a 20% capped maximum weight for any given stock. 6 of the 65 stocks were chosen based on the historical data as shown in Table 1.

Company Number	1	5	7	17	19	57	62
Company Symbol	BLL	DEO	KDP	INTC	LLY	SBUX	VZ
Daily Rate of Return ($\mu_{daily} \times 10^{-3}$)	2.360	1.214	1.668	1.425	1.657	2.055	1.120
Daily Volatility ($\sigma_{daily} \times 10^{-2}$)	1.499	1.072	1.616	2.141	1.597	1.566	1.292
Capped Weight	0.199	0.121	0.193	0.011	0.177	0.200	0.085
Uncapped Weight	0.606	0	0.125	0	0.006	0.258	0.001

Table 1: Weights of companies invested in with a risk factor of 0.2 with and without a cap of 20% investment.

Using the weights given by the 20% maximum weight and risk factor of 0.2 quadratic optimization problem, the next 6 months of stock prices were simulated and the returns were calculated. In this forward simulation, all of the stock choices were fixed and no reinvestment occurred for the 6 month period. Figures 1 and 2 show a singular and multiple forward simulation of the chosen stocks respectively.

For the forward simulation in Figure 1, an initial investment of \$1 turned into a final value of \$1.1928 after 6 months of forward simulation which is a 19.28% increase in a 6 month period. The simulation was run 4 more times for a total of 5 separate simulations. The average increase over the 6 month period was 21.98%.

A known risk management strategy is choosing a risk factor where the derivative of the expected return with respect to the risk factor is 1. This represents a "point of diminishing returns" where an increase in risk is greater than the resulting increase in expected return. The risk factor was varied and the expected return was plotted with and without a cap of 20% investment in a single stock as shown in Figure 3.

Using a central finite difference method, the point of diminishing returns on increasing risk occurred at a risk factor of 0.5184. The point of diminishing returns often occurs near a risk factor of 0.5 so the result seems reasonable.

Using the optimized risk factor of 0.5184, experimentation with forward simulation of stocks was conducted. We modified the forward simulation so that after each month of forward simulation, the oldest month of price data was removed and the most recent month of data was added to the historic price data. We calculated a new daily rate of return and covariance matrix, and the quadratic optimization recomputed weights for the pool of stocks. All stocks that were given weights larger than 0.01 during any of the 6 months were plotted in Figure 4. Figure 5 shows which stock numbers were invested in during each month. The value of the original investment at the end of each month is plotted in Figure 6.

The run of the simulation resulted in a 51.22% increase in value over the 6 months ignoring commission fees. The companies of Ball Corp (1:BLL), Paypal (55:PYPL), and Starbucks (57:SBUX) had weights larger than 0.01 for all 6 months. When this model was run an additional 4 times, the value of the portfolio averaged 1.334 (33.4% growth) after 6

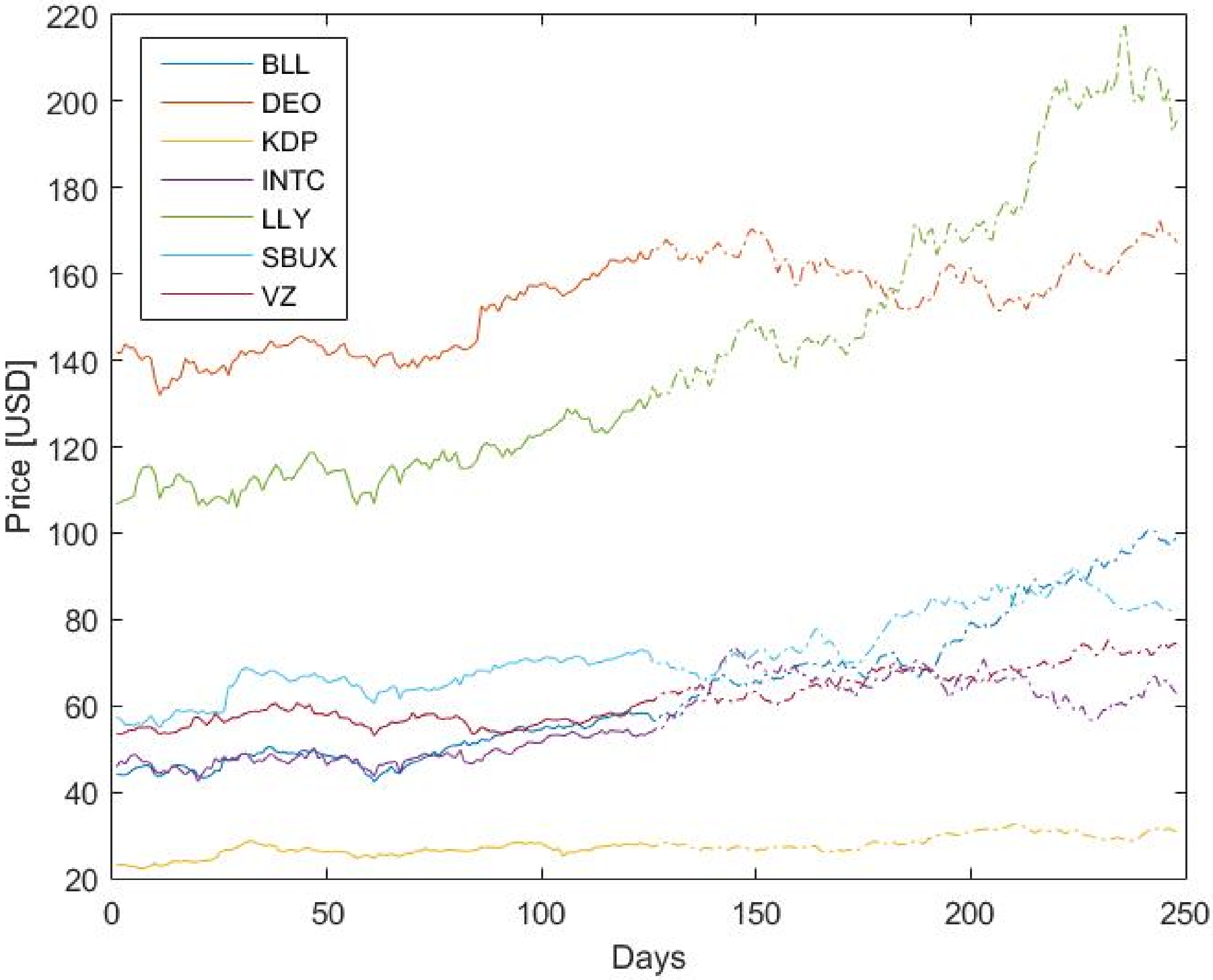

Figure 1: Single 6 month simulations using a fixed portfolio (dashed lines are simulated values).

months. The final value of the portfolio for all 5 runs is included in Table 2. This monthly reinvestment model produced desirable results with all 5 runs producing more than 10% growth over 6 months.

Trial	Average	1	2	3	4	5
Value at 6 mo	1.334	1.512	1.279	1.469	1.286	1.123

Table 2: Value of the portfolio at the end of 6 months over multiple simulation trials.

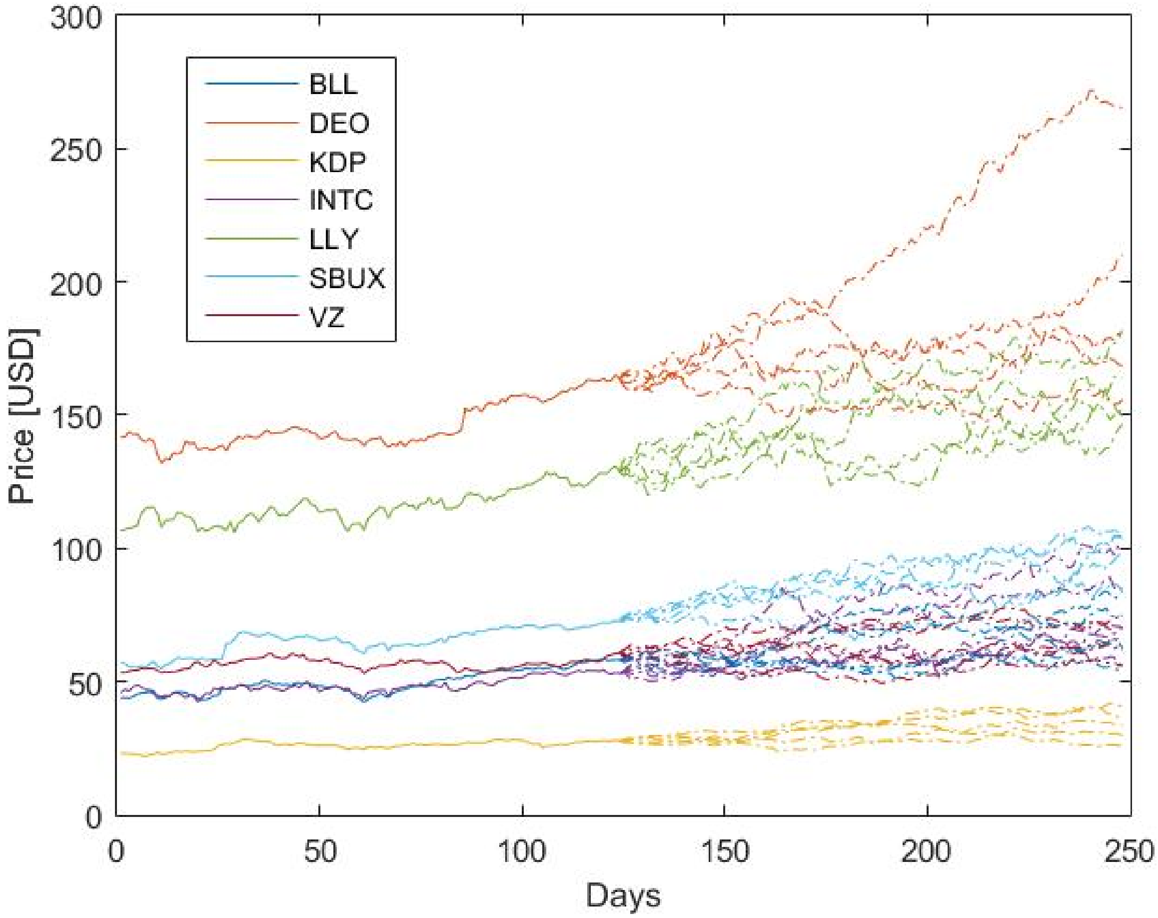

Figure 2: Multiple 6 month simulations using a fixed portfolio.

4 Conclusion

Both the fixed portfolio and monthly reinvestment portfolios produced an increase in value over 6 months on average. The fixed portfolio had an average return of 21.98% over 6 months and the monthly reinvestment portfolio had an average return of 33.4% over 6 months. This provides evidence that reinvesting in stocks after each month using the quadratic optimization method produces a better return on investment. However, we did not consider the commission costs associated with reinvesting each month which would remove some of the reinvesting benefit. Future ways the model could be investigated include varying ranges of maximum weight constraints, including commission costs in the quadratic optimization model, and utilizing the non-diagonal entries of the covariance matrix to correlate the stocks during forward simulation.

5 Appendix of Tables and Figures

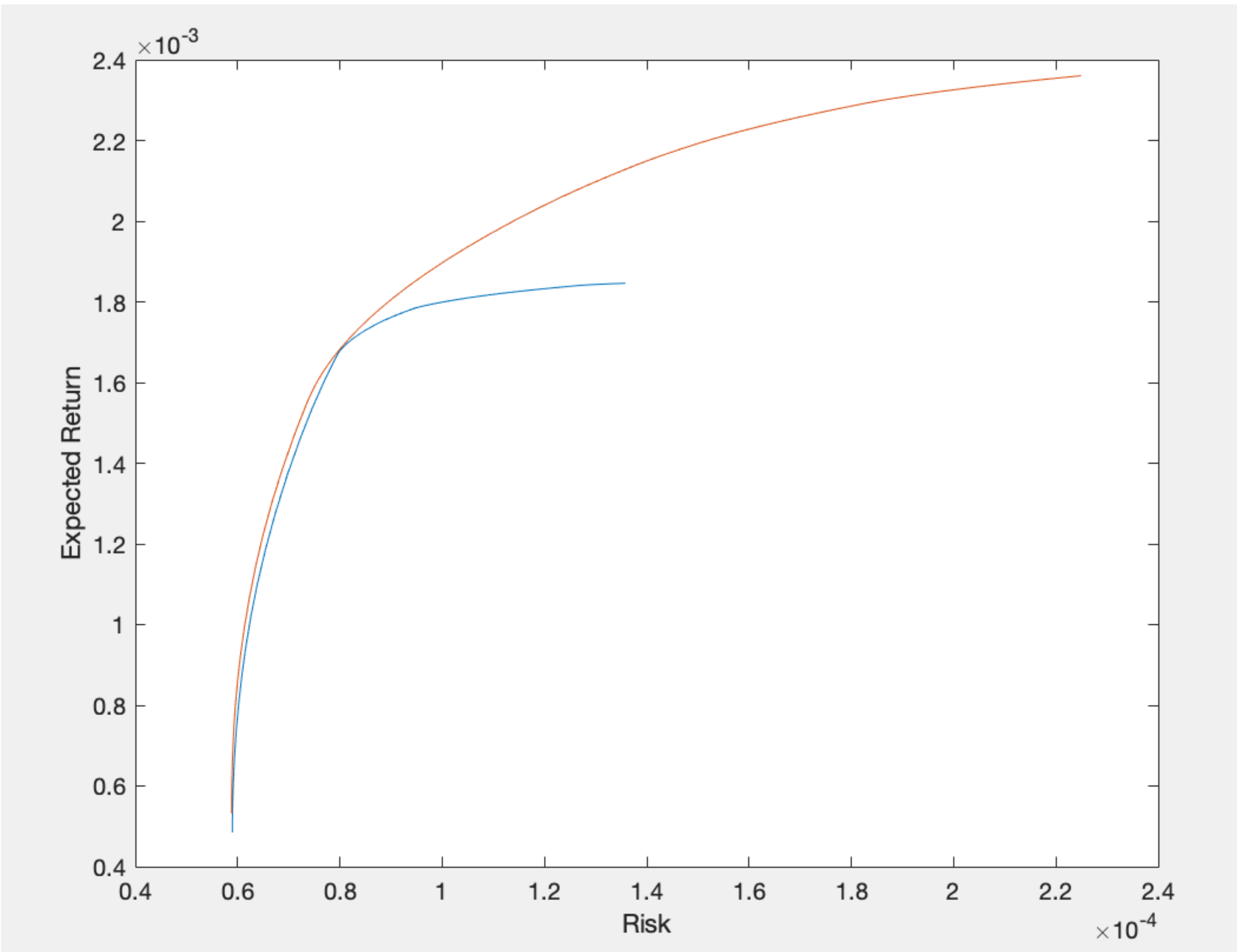

Figure 3: Expected Return with (blue) and without (red) 20% investment cap (maximum weight) as the risk factor is varied.

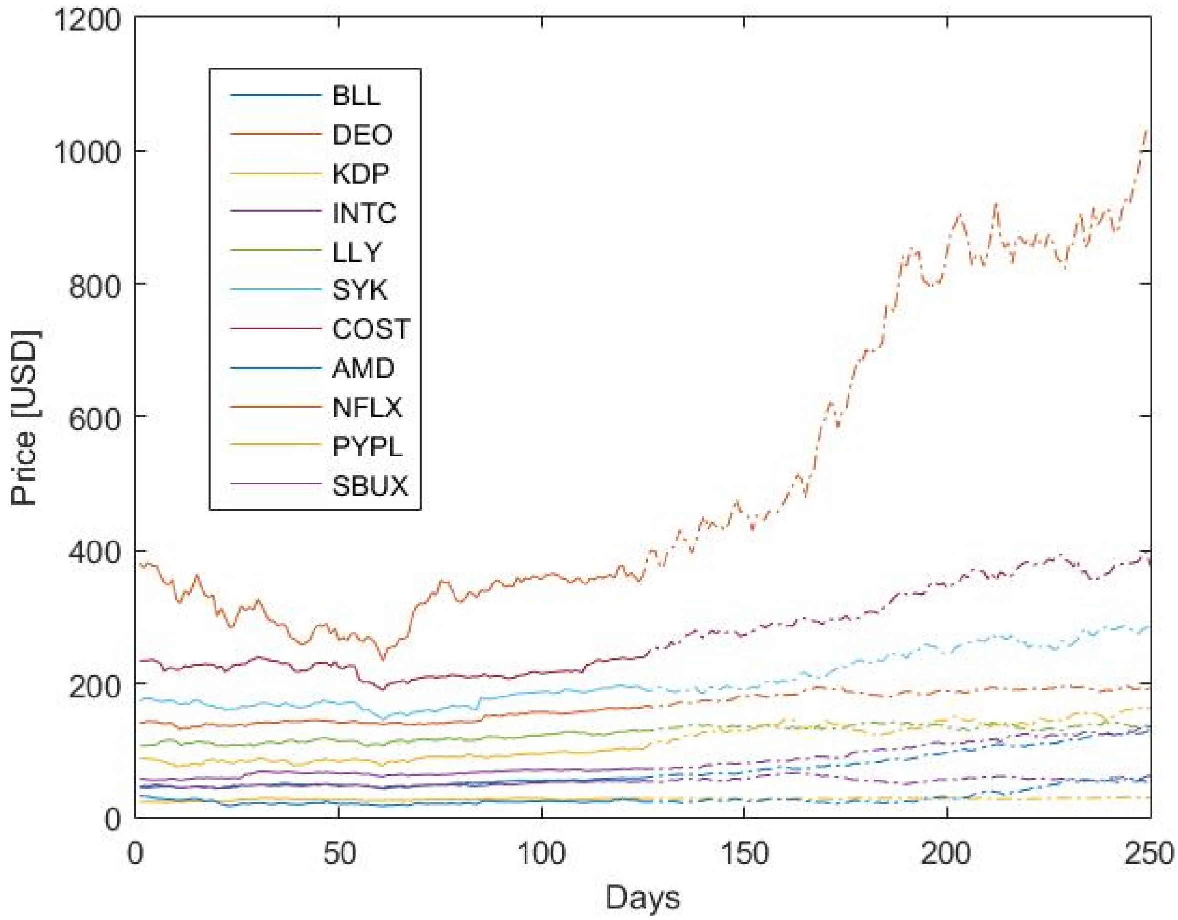

Figure 4: Six month simulation of invested stocks when using a monthly reinvest model (dashed lines are simulated values).

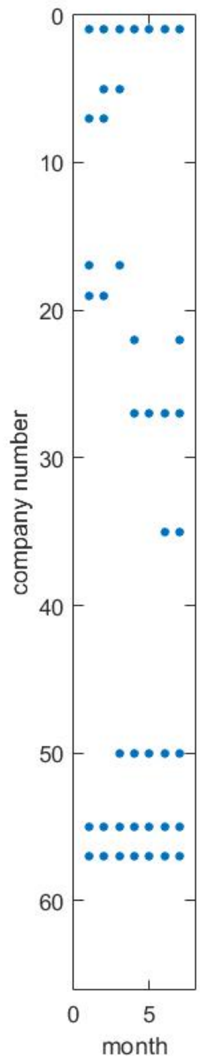

Figure 5: Stocks invested in during each month including the next future month (month 7).

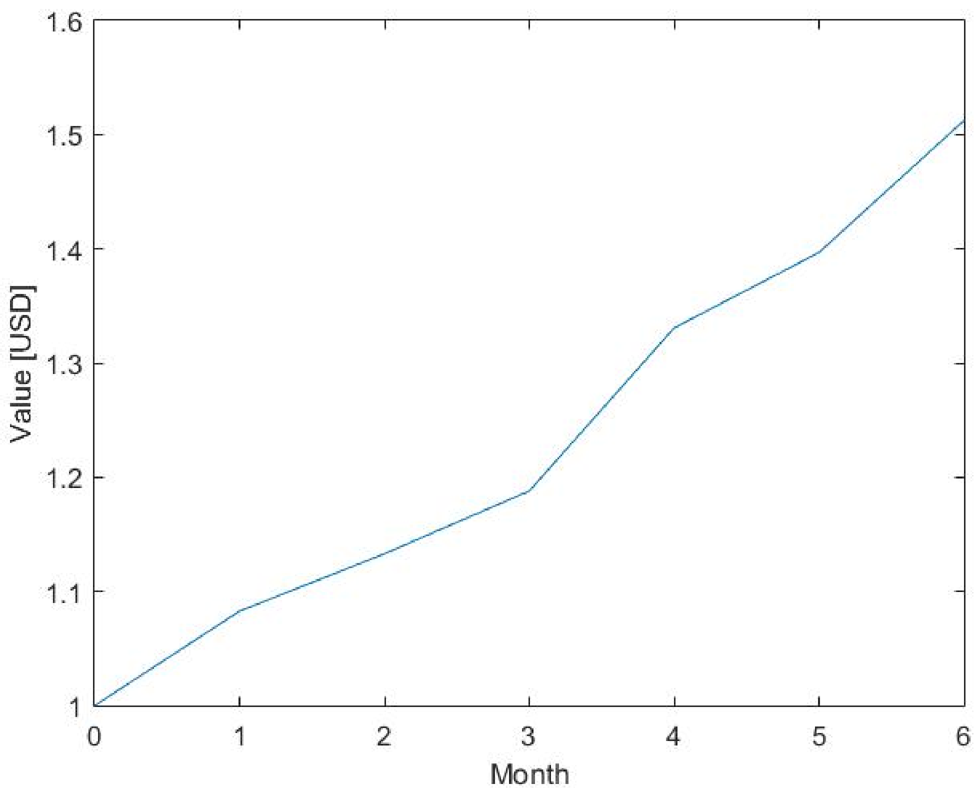

Figure 6: Value of the varying portfolio simulation at the end of each month.

Company Number	Company Symbol	Company Name
1	BLL	Ball Corporation
2	BREW	Craft Brew Alliance
3	BUD	Anheuser Busch
4	COT	Cott Corporation
5	DEO	Diageo
6	FIZZ	National Beverage Company
7	KDP	Keurig Dr Pepper
8	KO	Coca Cola
9	MNST	Monster Beverage
10	MT	ArcelorMittal
11	PE	PepsiCo, Inc.
12	PRMW	Primo Water Corp
13	SAM	Boston Beer Company
14	STZ	Constellation Brand
15	TAP	Molson Coors Brewing Company
16	HK	Techtronic Industries Company Limited
17	INTC	Intel Corporation
18	JNJ	Johnson & Johnson
19	LLY	Eli Lilly and Company
20	RYCEY	Rolls-Royce Holdings plc
21	SNE	Sony Corporation
22	SYK	Stryker Corporation
23	ZBH	Zimmer Biomet Holdings, Inc.
24	ALSN	Allison Transmission Holdings, Inc.
25	AMZN	Amazon.com, Inc.
26	BSX	Boston Scientific Corporation
27	COST	Costco Wholesale Corporation
28	DG	Dollar General Corporation
29	GOOG	Alphabet Inc.
30	DASTY	Dassault Systèmes SE
31	AAPL	Apple
32	ADBE	Adobe
33	ADDYY	Adidas
34	ADSK	AutoDesk
35	AMD	Advanced Micro Devices
36	FIZZ	National Beverage Company
37	AXP	American Express
38	BA	Boeing
39	BRK-B	Berkshire Hathaway
40	C	Citigroup

Table 3: Pool of Company Information.

Company Number	Company Symbol	Company Name
41	CRM	salesforce.com
42	CSCO	Cisco
43	DIS	Walt Disney
44	FB	Facebook
45	GS	Goldman Sachs Group
46	JPM	JPMorgan Chase
47	LMT	Lockheed Martin Corporation
48	MA	Mastercard
49	MSFT	Microsoft
50	NFLX	Netflix
51	NKE	Nike
52	NVDA	NVIDIA Corporation
53	ORCL	Oracle
54	PGRE	Paramount Group
55	PYPL	PayPal Holdings
56	QCOM	QUALCOMM
57	SBUX	Starbucks
58	T	AT&T
59	TGT	Target
60	TSLA	Tesla
61	V	Visa
62	VZ	Verizon Communications
63	WFC	Wells Fargo
64	WMT	Walmart
65	XOM	Exxon Mobil Corporation

Table 4: Continued Pool of Company Information.