

II Coin Change Problem

Find min. no. of coins to make change of given amount using given coins. (infinite supply of coins).

Here, greedy algo does not provide optimal solution or sometimes solution doesn't exist. \Rightarrow not possible

Coins = $\{1, 5, 6, 9\}$

$W = 10$.

$i \searrow j \rightarrow$ coins	0	1	2	3	4	5	6	7	8	9	10
1	0	1	2	3	4	5	6	7	8	9	10
5	0	1	2	3	4	$\min(5, 1+0) = 1$	$\min(6, 1+1) = 2$	$\min(7, 2) = 2$	$\min(8, 3) = 3$	$\min(9, 4) = 4$	$\min(10, 5) = 2$
6	0	1	2	3	4	1	$\min(2) = 1$	$\min(3, 1) = 2$	$\min(4, 2) = 2$	$\min(5, 3) = 2$	$\min(6, 4) = 2$
9	0	1	2	3	4	1	1	2	3	$\min(1, 1) = 1$	$\min(2, 2) = 2$

\rightarrow Min. no. of coins
 $(5, 5) \rightarrow$ denominations.

if $\text{coin}[i] > w$ copy value from above.
 else $\min(\text{above}, \text{exclude} + 1)$

Implementation

$a[i][0] = 0$.

for ($i \geq 0; i < \text{coins.length}; i++$) {

for ($j \geq 0; j < w; j++$) {

if ($\text{coins}[i] > j$) {

$a[i][j] = a[i-1][j]$ }

else $a[i][j] = \min(a[i-1][j], a[i][j - \text{coins}[i]] + 1)$

}

Time complexity $\rightarrow O(n * w)$, amount.
 \downarrow
 no. of coins