

# Protest tactics and organizational structure

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*Work in progress. For the latest draft, [click here](#).*

## Abstract

How do protest movements collectively choose between peaceful or violent tactics over time? I study a dynamic regime change game in which individuals decide whether to participate in a protest, given the tactic chosen by previous protesters. I assume that the power of peaceful tactics is more reliant on turnout than violent tactics. I demonstrate that tactical choice involves both prospective and retrospective considerations. On the one hand, tactical choice responds to information learned from past protests, reflecting a process of learning and experimentation. On the other hand, tactical choice must manage public optimism to preserve future opportunities for contention, reflecting the fact that tactics depend to varying extents on turnout for power. Failed protests can severely undercut public optimism, reducing the efficacy of future tactics and curtailing the possibilities for further contention. The repeated failure of protests, moreover, presents movements with a tactical dilemma. While violence may seem attractive since it is less dependent on turnout, these failures may indicate that violence is futile, leading movements to gamble on peace in hopes of securing high turnout by chance. The game also offers a solution to the problem of equilibrium multiplicity in repeated global games. Individuals' per-period participation decisions are cut off by idiosyncratic participation costs, while beliefs about the regime are commonly held across periods.

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# 1 Introduction

How and why do protests movements choose and change tactics over their lifetimes? Existing research has documented, among other questions, how methods of nonviolent and violent resistance work to weaken and overcome adversaries; the structural factors that render protests and tactics more or less likely to succeed; and how protests respond to repression. (Ackerman and DuVall 2000; Chenoweth and Stephan 2011; Cunningham 2013; Dornschneider-Elkink and Henderson 2024; Schock 2005; Sharp 1973; Sutton, Butcher and Svensson 2014). The growing empirical literature often seeks to identify each of these relationships by isolating them from the complications engendered by the ongoing dynamic interplay between tactical choice, protest events, and public opinion. This interplay, which occurs within and across episodes of resistance, causes endogeneity and path dependence to be inevitable concerns in any study of this topic: Any change in contemporaneous beliefs about the regime, observed tactical choice, and success or failure inevitably threatens a shift in each of the other quantities.

This paper seeks to systematically understand this interdependence in order to unravel the *dynamic* and *organizational* incentives at work in protests' choices of tactics. I begin with the premise that public opinion and mobilization influence and are influenced by tactical decisions over time. From here, I seek to understand how these motivate tactical decisions made by protests who have one eye on the past, and the other towards the future. To do so, I study a dynamic model of tactical choice in collective action where past protest episodes, contemporaneous beliefs about the regime, and tactical beliefs are tightly linked. I specifically focus on the choice between peaceful tactics, which are most powerful when there is massive turnout, and violent or unconventional tactics, which are less dependent on mass participation for power and visibility.

Two sets of incentives emerge from the model. The first set is *organizational*, and reflects the desire of existing protesters to retain control of the movement and use the tactics which are aligned with their personal and ideological preferences. The second set is *dynamic and success-oriented*, and reflects protesters' incentives to pick tactics which most facilitate not only chances of immediate success, but success in the future. I demonstrate that these success-oriented involves both backwards- and forwards-looking considerations. On one hand, protesters are able to learn from their past attempts and failures, and can pick tactical choices on the basis of their hard-won knowledge about the regime. On the other hand, participation is costly for individuals, and repeated failures eat away at early optimism, making individuals less likely to turn out. Thus, tactics are also chosen to *manage public optimism* – sometimes, in fact, sacrificing a more powerful protest today so as to not undercut the beliefs which make possible future protests. In other words, protesters prefer avoid situations where a failure today is so informative and discouraging that prospects of future contestation are erased.

The dynamics produced by the model mirror those which we observe in real-world social movements and their tactical choices. Importantly, they reflect the fact that protests frequently experiment with and change tactics over the course of their lifetimes. The African National Congress (ANC) initially contested apartheid using peaceful and passive tactics, but turned to violence when their nonviolent efforts failed to achieve change and overcome regime repression. The next three decades of contention were dominated by violent attacks, vandalism, and bombings, until the South African government ultimately agreed to entering into negotiations which ultimately ended apartheid. Instances where organizations considered, and ultimately resisted a transition to violence are also interesting, especially as they help shed light on how organizational incentives may conflict with success-oriented incentives: Internecine conflict over the primacy of nonviolence and organizational structure of the movement ultimately resulted in the dissolution of the Student Nonviolent Coordinating Committee (SNCC), a major organization in the American civil rights movement. These kinds of examples suggest that the ability of organizations to shift and experiment with tactics is constrained by incumbents' desire to succeed on their own terms – and to retain control of the movement in the future. However, violence is not always a terminal or permanent state, and sometimes is superseded by civil resistance, as was the case with several episodes of Latin American resistance in the 1980s (Carter 2009).

The model suggests a logic which underlies these episodes which builds upon the intuitions which have been developed and tested in existing empirical work. Importantly, it suggests that when faced with a choice between tactical repertoires which are more mobilization-dependent and less mobilization-dependent, *order matters*. Since peaceful, “people power” tactics are more sensitive to turnout for power, they are *also* more sensitive to public beliefs about the regime. When beliefs are optimistic – that is, potential participants believe that the protest has a good chance of overcoming the regime – a peaceful protest is expected to be at its most powerful. As failures accumulate and beliefs about the regime become more pessimistic, expected mobilization, and thus the expected power of peaceful protests, diminishes. Using violence early on therefore disproportionately curbs the power of peace, much more so than vice versa. Therefore, even if violence is expected to be very powerful, protesters may still eschew violent tactics, at least for a time, until public beliefs, and corresponding chances of mass mobilization, have been eroded by repeated failures. This observation provides an alternative explanation for the empirical observation that nonviolent tactics seem to more frequently result in successful protest outcomes (Ackerman, Karatnycky et al. 2005; Celestino and Gleditsch 2013; Chenoweth and Stephan 2011). If, as the model suggests, violence tends to be chosen when beliefs are already quite optimistic, then it will be used situations where a successful outcome is already very unlikely.

Yet the model shows that violence is not the inevitable choice after failures accumulate and

pessimism builds. The reason relates once again to the fact that citizens learn from what has previously been tried. The very information that the regime is strong may indicate that violent, insurgent, or unconventional tactics are simply not strong enough to present any real challenge for the regime. Therefore, it is only a massive peaceful protest which has any change of successfully triggering regime change. Movements may thus choose peaceful tactics in hopes that a stroke of luck – a politically charged national holiday, international intervention, or otherwise – will give them the massive turnout that they need. In other words, the outcome of learning and experimentation may be a return to nonviolent tactics. The conclusions of the model echo scholarship that asserts that violence, rather than being the product of a “terrorist personality,” religious fanaticism, or heightened emotional states, is a strategic choice employed by actors motivated by largely the same concerns as actors who choose nonviolence resistance (Dornschneider 2016).

On a technical level, this paper contributes to a growing literature which uses global games in order to model protests and collective action. The key difference between this paper and the canonical global games framework is that citizens do not receive a noisy signal of regime strength; rather, their per-period cost of participation is idiosyncratically drawn. As a result, individual participation decisions are cutoff in their period-specific participation cost, while beliefs about regime strength are commonly held. Hence, given a realization of cost shocks over the course of the game, there is a unique equilibrium. This structure is appealing because it resolves the multiplicity of equilibria in repeated global games, a feature which has limited the usefulness of the framework in making clear predictions in repeated or dynamics applications.

The draft proceeds as follows: Section 2 discusses related literature. Section 3 presents the model. Section 4 discusses equilibrium individual participation decisions. Section 5 presents a toy example which clarifies prospective and retrospective success-maximizing considerations. In Section 6, I offer preliminary comments on full model results, extensions, and conclusions.

## 2 Related literature

This paper intersects with two major bodies of substantive literature. These bodies ask, broadly: how do contentious politics affect citizens’ knowledge and beliefs about the regime, and impact future instances of collective action? How do tactical choices in contentious politics translate into success or failure? Findings are mixed regarding how protests reveal information, and whether this is beneficial or not for future contestation. Lohmann (1994) is amongst the earliest to document how repeated protests impacted beliefs over the regime in the context of East Germany. Lohmann argues that demonstrations created an information cascade, with contention persisting and achieving success while it revealed theretofore hidden information about the regime, and subsiding when the informational role diminished. My framework presents a slightly different

view of learning, as failed protests are only ever indicative of “bad news.” Hence, protests cannot gather momentum while they fail to result in success, although they can subside when they fail to continue revealing useful information that informs tactical choice and turnout. Recent empirical studies have also considered the impact that protests have on public opinion and beliefs about the regime. Tertychnaya and Lankina (2020) find that over the course of unsuccessful electoral protests in Russia, media coverage and regime-led repression decreased public support for the protest movement and its demands. Pop-Eleches, Robertson and Rosenfeld (2022), who study the successful Euromaidan revolution in Ukraine, finds that participation in Euromaidan protests is positively correlated with belief and agreement with protest frames. The fact that one movement ended in success and the other in failure is relevant: in the model, a successful outcome leads all members to share the same optimistic beliefs about the weakness of the regime, while failure leads all citizens to share the same pessimistic beliefs that the regime is strong.

A related body of work examines the relationship between dissent and repression, asking how regimes choose to repress dissension and how repression either suppresses or inflames dissent (Fariss and Schnakenberg 2014; Finkel 2015; Tarrow 1998). Ritter and Conrad (2016) in particular argue that both repression and dissent are chosen strategically: governments and dissidents act not only in response to one another’s realized actions, but in anticipation of what one other *may* do. Several other studies anticipate dynamics which emerge from my model. For instance, Chiang (2021) finds that repression, particularly physical repression, deters participation in nonviolent movements, but does not find a clear effect of repression on participation in violent movements – results consistent with a model where participation in peaceful protests is more sensitive to beliefs about the strength of the regime. Butcher and Pinckney (2022) points out, as my model shows, that participation is endogenous to participants’ beliefs about the regime, so that “large protest sizes are almost certainly indicative of the widely-shared expectation of government concessions.” When controlling for this endogenous effect, they find that the positive relationship between protest size and government concessions goes away.

Nonetheless, this literature has largely abstracted away from examining how protest organizers change tactics in response to the endogeneity of beliefs and turnout. While scholarship has long acknowledged that tactical experimentation and innovation are key to the success of social movements to achieving their goals (Chenoweth, Pinckney and Lewis 2018; Tilly 1978), fewer scholars have taken up the question of how tactical change is employed *strategically* in response to evolving beliefs about the regime. A version of this question has been studied in the conflict relations in international relations. Several authors have theorized the choice of conventional or unconventional tactics by rebel groups (Bueno de Mesquita 2013; Qiu 2022). As these authors observe, rebels’ wartime strategies are responsive to their current strength – but are also consequential for the duration of fighting and the likelihood of reaching a peace settlement. These

models typically treat governments and rebels, or rebel leaders, as unitary actors. Furthermore, they are not models of collective action, and thus do not capture the incentives that arise directly from the centrality of collective action in my model (in particular, organizational incentives and prospective “public opinion management” incentives).

This paper is closely related to the literature that models mass protests using global games (De Mesquita 2010; De Mesquita and Shadmehr 2023; Gieczewski and Koçak 2024; Morris and Shin 1998, 2003; Morris and Shadmehr 2024). A limitation of this technology, however, is that the uniqueness properties that guarantee uniqueness in the stage game break down in repeated or dynamic extensions (Angeletos, Hellwig and Pavan 2007; Little 2017). In the canonical one-shot global game, agents receive heterogeneous signals about the strength of the regime. As Morris and Shin (1998) show, in the unique equilibrium of this game, each agent’s optimal action is a best response to the proportion of agents which they believe have a lower signal than they do. In a repeated global game, however, beliefs about the regime formed on the basis of private signals collide with beliefs formed from observations of past protests. This interaction produces a multiplicity of equilibria. This contrasts with my model, in which agents have common beliefs about the strength of the regime in each period, and global games properties are instead generated by heterogeneity in realized participation costs.

Another difference is that I allow both private and public benefits of participating in protest. The presence of possible public benefits, which is not canonical in global games, is intended to create incentives for incumbents to cede control of the movement if they feel that current tactics are unlikely to succeed. Without public benefits, individuals can only benefit from success if they personally participate in a protest. This adds a measure of verisimilitude – individuals benefit from regime change even if they do not personally participate in the protest – and facilitates dynamic linkages between periods. An individual can still derive utility if they, for instance, step away from a movement, allowing a new set of protesters with different tactical inclinations to enter and successfully achieve regime change.

Finally, modeling a protest movement as an organization where incumbent members vote on tactics (thereby influencing the composition of future membership) connects this paper to the literature on experimentation in organizations with endogenous membership (Bai and Lagunoff 2011; Gieczewski 2021; Gieczewski and Kosterina 2024). This literature analyzes the long-term consequences of policy choices made by organizations where membership endows voting rights over future policies. A common theme of this literature is that sub-optimal policies can become entrenched when the organization is “captured” by relatively extreme membership. This paper is the first, to my knowledge, to use a dynamic model of an organization with endogenous membership to study protests and collective action.

### 3 Model

*Overview.* A protest movement exists in discrete time  $t = 0, 1, \dots, T \leq \infty$ . In each period  $t \geq 1$  that a protest did not succeed, incumbent protesters choose one tactic to be implemented in  $[t, t + 1)$ . The choice is observed publicly. A participation stage then takes place where all agents can decide whether or not to join the protest in  $[t, t + 1)$  (there is no priority given to previous incumbents). If their protest is successful, the game ends. If it is unsuccessful, the game proceeds to the next period, where the set of agents who choose to be members at time  $t$  constitutes the new set of incumbent protesters who can choose tactics for the next period. In the beginning of the game,  $t = 0$ , the game starts with a participation stage where the movement's initial tactic is exogenously specified.

*Types.* There exist two types of agents, *peaceful* and *violent*. Let  $\tau$  denote the type of an individual agent (peaceful or violent). I assume there is a unit mass of each type. An agent's cost of taking their *aligned* action (e.g. a violent agent participating in a protest endowed with the violent tactic) is  $c_{it}^\tau = c_t^\tau + \epsilon_{it}^\tau$ . The two components of cost, which are both drawn i.i.d. each period, are a population shock  $c_t^\tau \sim Unif[\underline{c}, \bar{c}]$  which affects all agents of a given type equally, and an individual shock  $\epsilon_{it}^\tau \sim Unif[-\epsilon, \epsilon]$ , where  $\epsilon > 0$ . An agent's cost of taking their unaligned action is infinite. As such, a peaceful agent will only ever join when the peaceful tactic has been chosen, and likewise for a violent agent.

*Tactics and success.* Each tactic has a protest production function which transforms turnout,  $\ell$ , into protest power. Turnout  $\ell \in [0, 1]$  is measured as the fraction of all aligned agents who show up. I denote the production functions by  $A_V(\ell)$  and  $A_P(\ell)$ , and assume that  $A_V(0) > A_P(0)$ ,  $A_V(1) < A_P(1)$ ,  $A'_V(\ell) < A'_P(\ell)$ . The strength of the regime is given by a state variable  $\theta$ . Before the game begins,  $\theta \sim Unif[m, M]$ , which also constitutes the common prior held by agents. If  $A_P(\ell) > \theta$ , the protest succeeds with probability  $\rho_P$ , and if  $A_V(\ell) > \theta$  it succeeds with probability  $\rho_V$ . Let  $\mathcal{W}_t$  denote whether a  $t$ -period protest succeeds. An individual's flow payoff is

$$u_{it}^\tau = \underbrace{(-c_{it}^\tau)\mathbb{1}_{\{a_{it}=1\}}}_{\text{Participation cost}} + \underbrace{\mathbb{1}_{\{a_{it}=1 \cap \mathcal{W}_t=1\}}}_{\text{Private benefits}} + \underbrace{\nu\mathbb{1}_{\{\mathcal{W}_t=1\}}}_{\text{Public benefits}}$$

and their total payoff is the discounted sum of their flow payoffs  $u_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_{it}$ .

The timing of each period proceeds as follows:

1. The protest's tactic is inherited from the last period's choice.
2. Costs are drawn. Individual agents choose whether or not to join the protest.
3. Protest occurs. Agents observe turnout and the outcome. Flow payoffs are realized.

4. If the protest succeeds, the game ends. If not, current organization members vote on next period's tactics. Repeat from step 1.

The solution concept is Perfect Bayesian Equilibrium.

## 4 Equilibrium participation decisions

I first demonstrate foundational results about per-period individual participation decisions. Taking tactical choices and beliefs as given, I will demonstrate that individuals' participation decisions follow a turnout rule in each period: that is, an agent chooses to protest if their cost  $c_{it}^\tau$  is less than some threshold  $\hat{c}_t^\tau$ . I then show that  $\hat{c}_t^\tau$  is weakly decreasing over time, the consequence of increasing pessimism about  $\theta$  brought on by repeated failures. Although each tactic's threshold is decreasing, however, aggregate turnout may not. Because participation costs are re-drawn each period and tactical choices may change, either one or both of these factors could drive an increase in turnout from one period to the next even if the threshold decreases.

These results provide the close link between beliefs and participation behavior. In brief summary, the realization of cost shocks combined with prior beliefs determines turnout. Turnout, produced with the chosen tactic, produces protest power, which provides the information upon which beliefs are updated and induces path dependence in the model. Since this entire process hinges upon the realization of cost shocks, the path of the population-level costs generates path dependence in the model.

Beliefs in any given period about  $\theta$  are simply the posterior after viewing any number of rounds of failed protests. The variation which produces "Laplacian" beliefs is over private *costs*: players' choice to protest is a best response to the proportion of agents who they expect to have a lower participation *cost* than they do. To see this result, note that an agent aligned with the current tactic possessing cost  $c_{it}^\tau$  protests if

$$\begin{aligned} \mathbb{P}(\mathcal{W}_t = 1) (1 + \nu) - c_{it}^\tau &> \mathbb{P}(\mathcal{W}_t = 1) (\nu) \\ \iff \mathbb{P}(\mathcal{W}_t = 1) &> c_{it}^\tau \end{aligned}$$

Let  $F_t$  denote the CDF of the (public) posterior belief about  $\theta$  at time  $t$ . At the point of indifference between participating and not participating,

$$\begin{aligned} \mathbb{P}(\mathcal{W}_t = 1) &= c_{it}^\tau \\ \rho_\tau \int_{\tilde{c}} F_t(A_\tau(\ell(\tilde{c}))) g(\tilde{c} | c_{it}^\tau) d\tilde{c} &= c_{it}^\tau \end{aligned}$$

Here,  $g(\tilde{c} | c_{it}^\tau)$  denotes a player's belief about  $\tilde{c}$ , (the population-level cost) conditional on their own observed  $c_{it}^\tau$ .  $\ell(\tilde{c})$  is the proportion of the population that would mobilize if the population cost was



indeed  $\tilde{c}$ . Recall that  $A_\tau(\cdot)$  indicates the tactic-specific protest production function. Therefore, the condition is the average expected *success* of all possible realizations of protest power induced by possible cost shocks. We can simplify this expression substantially using standard global games properties. We know that the marginal agent who is indifferent between participation and non-participation believes that the proportion of agents with cost less than hers is distributed  $Unif[0, 1]$ . Therefore, we can abstract away from explicit beliefs about costs and shift instead to beliefs directly about turnout,  $\ell$ :

$$\rho_\tau \int_0^1 F_t(A_\tau(\ell)) d\ell = c_{it}^\tau \equiv \hat{c}_t^\tau \quad (1)$$

This condition yields a per-period cutoff participation rule: an agent protests in time  $t$  iff his cost  $c_{it}^\tau < \hat{c}_t^\tau$ . The key element is  $F_t$ , agents' commonly held posterior beliefs about  $\theta$ , which is what is used to compute the expected success of a protest produced with tactic  $\tau$  and with mobilization  $\ell$ . Changes in the cost threshold over time therefore directly reflect updates in posterior beliefs. Beliefs are only ever updated with bad news. Since a success ends the game, beliefs are only ever updated in the case of failure. As a result,  $F_t$  at each period first-order must (weakly) stochastically dominates  $F_{t-1}$ , placing more and more (relative) weight on higher values of  $\theta$ . The consequence is that the average expected success over all possible realization of protests must fall – that is,  $\hat{c}_t^\tau$  must fall. These results are summarized in the following statement:

**Lemma 1.** *An individual with cost  $c_{it}^\tau$  protests in period  $t$  if*

$$c_{it}^\tau < \rho \int_0^1 F_t(A_\tau(\ell)) d\ell \equiv \hat{c}_t^\tau$$

*Furthermore,  $\hat{c}_t^\tau \leq \hat{c}_{t-1}^\tau \forall t$ .*

Note that the statement  $\hat{c}_t^\tau \leq \hat{c}_{t-1}^\tau \forall t$  applies *within* a tactic – it does not make a statement about the relationship *between* the different tactical thresholds, i.e. for a given  $t$ , whether  $c_{it}^p > c_{it}^v$  or vice versa.

## 5 Success-maximizing tactical choice

I now demonstrate how the structure of equilibrium participation maps onto tactical decisions. The aim of this section is to clarify the intertemporal incentives to make *success-maximizing* tactical decisions. The game as it is written contains many other possible incentives: for instance, individual incentives to acquire private participation benefits, or conversely to free-ride off the costly participation of others. This sections abstracts away from those incentives to isolate how tactics can be chosen to maximize the movement's chances at success. To this end, I focus on the decision of a social planner who is solely concerned with maximizing success across the total (two-period) lifetime of the protest. The social planner does not discount, and does not care

about the payoffs incurred by individual agents. The social planner can choose tactics in both periods, but cannot control turnout.

I show that there are, broadly, two forces that affect the social planner’s choice: retrospective and prospective incentives. The retrospective incentives are strongly reminiscent of the experimentation literature: essentially, the more accurate the information that the social planner has about the regime, the better able he is to make an “accurate” tactical choice. The prospective incentive originates from the coordination dynamic. It reflects the fact that tactics’ (expected) power in the future depends on beliefs that will be informed by what happens today. Since the social planner is not myopic, he therefore has incentive to “manage public opinion” so as to not sabotage his chances of succeeding in the future if there is a failure today. Note that while an experimentation incentive would be present even without coordination concerns, the latter category of incentives – to preserve future chances at contesting the regime – is purely driven by coordination concerns. The social planner knows that citizens will learn from what happens today, and that that learning will affect their turnout decisions, changing the potential of both tactics. In short, coordination means that the tactics available to use tomorrow are not the same tactics available to use today.

One temporary simplification will make it possible to cleanly isolate these two incentives: assume that the output of violence is *independent of turnout*. The reason this works is because retrospective (“experimental”) and prospective (“coordination”) concerns map onto uncertainty over  $\theta$  and uncertainty over turnout, respectively. The tactical assumption made up until now is that the peaceful tactic is *more* sensitive to turnout than the violent tactic. The simplification is simply to make this difference stark: to assume that turnout uncertainty does not apply at all to the violent tactic. As a result, we can simply consider the choice between the violent tactic whose success is subject *only* to regime strength uncertainty, and the peaceful tactic whose success depends on both regime strength uncertainty and turnout uncertainty (peace). The result is starkly illustrated in the following lemma:

**Lemma 2.** *Suppose that  $A_v(\ell) = \bar{v} \forall \ell$ ,  $A_p(\ell)$  is linear, and that there is no backfiring ( $\rho_P = \rho_V = 1$ ). Consider the choice of a success-maximizing social planner who does not discount. The following facts are true:*

- (i) *The social planner will never use violence in the first period.*
- (ii) *The range of failed first-period protests for which violence is used in the second period is increasing in  $\bar{v}$ .*

These two facts isolate how prospective and retrospective concerns map onto success-maximizing tactical choice. The first fact – that violence is never used in the first period – comes directly

from *prospective* concerns. The reason for this is that turnout in the second period is affected by the realization of the first-period protest, and therefore the order in which tactics are used is not neutral. A failed first-period violent protest deterministically reduces the expected power of peace in the second period. However, a failed first-period peaceful protest has no effect on the power of violence in the second period. This is true for *any* level of  $\bar{v}$ , even if  $\bar{v}$  is very high. Hence, forward-looking concerns can lead the social planner to make choices which are not myopically optimal: even if violence has much higher expected success than peace in the first period, using it in the first period takes away the chance that a large peaceful turnout will be able to overcome a regime too strong to be defeated even by the strong violent technology.

This emphasizes the importance of choosing tactics to “manage public optimism.” Since the power of the peaceful tactic is sensitive to turnout, it is also vulnerable to being impacted by public posteriors about the regime formed after the first protest. Using violence first essentially *guarantees* that, in the case of failure, the “peaceful tactic” available to tomorrow will be weaker in expectation than the peaceful tactic available today. By contrast, since the powerful violence is not sensitive to turnout in this simplified example, it offers, essentially, a risk-free guarantee of protest of power  $\bar{v}$ . Therefore, forward-looking concerns are directly connected to the *collective action* dynamic established in Section 4. Because tactics depend (to varying extents) on collective action to generate power, the versions of tactics available at time  $t + 1$  are *not* the same versions of tactics available at time  $t$  – and that this impacts optimal tactical choice at time  $t$ .

The second part of the lemma deals with retrospective considerations, and isolates the effect of period 1 learning on optimal period 2 choices. This is essentially an *experimentation* result: the more information that is revealed about the state variable,  $\theta$ , the better-informed that the organizer’s choice is. Just because the organizer should eschew violence in the first period does not mean that he necessarily will, or will not use violence in the second period – this depends on the information revealed about the state by the power, and outcome of the period 1 protest. Without backfiring, learning about the state is especially stark: the failure of a protest of size  $A_1$  confirms that  $\theta > A_1$ .

Figure 1 plots the expected second-period success of each tactic as a function of the power of a (failed) first-period protest. The figure makes several dynamics clear: Holding  $\bar{v}$  constant, the larger the (peaceful) first-period protest, the more likely it is that peace will be chosen in the second period. Part of this is mechanical: if the first period protest exceeds  $\bar{v}$ , then violence has zero chance of success, while peace has a positive chances of success – as such, peace will be chosen in the second period. However, it is not entirely mechanical: even when  $A_1$ , the size of the first period protest, is smaller than  $\bar{v}$ , the social planner has enough doubt that  $\bar{v} > \theta$  to choose peace. By contrast, when the first period protest is small and the social planner learns

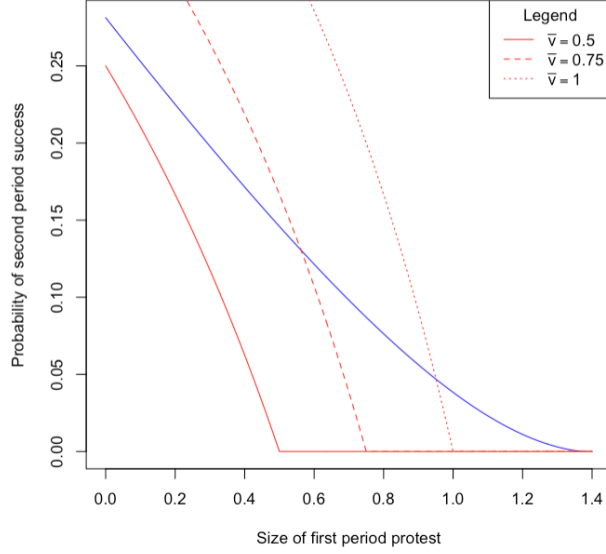


Figure 1: Expected success of peaceful (in blue) and violent (in red) tactics as a function of first period protest power.  $A_p(\ell) = 1.5\ell$ , different levels of violence displayed in legend. Other starting parameter values are:  $\theta \sim Unif[0, 2]$ ,  $\epsilon = 0.1$ ,  $c_t^p \sim Unif[0, 1]$ .

relatively little, violence is a more appealing “safe” option. The more powerful that  $\bar{v}$  is, however, the wider the range of failed first-period protests for which violence is chosen the second period. In the figure, when violence is relatively weak ( $\bar{v} = 0.5$ ), there is no failed first-period protest for which violence is chosen. As we progressively increase the power of violence to 0.75, then to 1, so too we increase the range of histories for which violence is chosen in the second period.

The existence of a collective action dynamic, however, sets this model apart from existing models of experimentation. Unlike models of experimentation where the technologies or tactics available to experimenter are fixed, a key component of this model is that the potential participants are *also* learning from past protests – not only the experimenter, and adjusting their own participation thresholds in response. Hence, the tactics available for an experimenter to “use” in the future are not identical to the tactics available in the past. The retrospective calculus that may lead organizers to choose peace in the second period is fundamentally different from the prospective calculus that drives organizers to choose peace in the first period.

A dynamic that obvious in this example is the idea that *failures do not necessarily lead towards violence*. In particular, the failure of a peaceful protest that is more powerful than  $\bar{v}$  teaches protesters that a violent protest is futile. The cleanliness with this is illustrated here is owed mostly to the starkness of learning: without backfiring, or other processes which would render learning about the regime more fuzzy, it is not only unlikely but impossible that a protest

of power  $\bar{v}$  would succeed. In a fuller version of the game, where learning is less stark and both tactics are responsive to turnout uncertainty, this dynamic is significantly more complex.

This example also shows how path dependence in the model can take on non-trivial forms. In particular, the likelihood of success over the lifetime of the movement can vary non-monotonically with the realization of first-period costs. To illustrate this with a toy example, I continue with the parameterization displayed in Figure 1, with  $\bar{v} = 0.5$  (so that violence is never chosen in the second period). In this case, the first-period participation threshold is  $\int_0^1 \frac{1.5\ell-0}{2-0} d\ell = 0.375$ . Suppose that the first-period aggregate shock is low enough that all citizens turn out. Since this is the maximally powerful protest, its failure indicates with certainty that it is *impossible* to succeed. Nonetheless, since all plausibly defeatable values of  $\theta$  would have fallen, there is also no “money left on the table.”

Conversely suppose that the first-period aggregate shock is high enough that no citizens turn out. In this case, nothing is learned from the protest, and as a result, expectations of success in the second period are identical to those in the first period: in particular, the second-period attendance participation cost is once again 0.375. Finally, suppose that the cost draw is intermediate. Consider what happens if  $c_1^p = \hat{c}_1^p$ , so exactly half the population turns out. If this protests fails, then the second period cost threshold is 0 – and in the very best scenario, then only half the population turns out again. The first period experience shows that this is futile, but also that violence with  $\bar{v} = 0.5$  is also futile. Indeed, it is this intermediate cost draw that is most detrimental for *total* chances of success over the lifetime of the protest.

## 6 Preliminary conclusions

*Organizational incentives.* The natural next step is to extend the example provided in the previous section to consider the decision that would actually be made if peaceful incumbents were installed period 1 and had control over period 2 decisions. Under what conditions would they be willing to cede control of the movement, and under what conditions would they be willing to sacrifice chances of success in order to succeed on their own terms? In the two-period example, the condition under which an incumbent would be willing to change tactics in the second period is:

$$F_{t+1}\left(\mathbb{E}[A_v(l_{t+1})]\right)(\nu) > F_{t+1}\left(\mathbb{E}[A_p(l_{t+1})]\right)\left[\mathbb{P}\left(c_{i,t+1}^p < \hat{c}_{t+1}^p\right) + \nu\right] \\ + \mathbb{P}\left(c_{i,t+1}^p < \hat{c}_{t+1}^p\right)\left(-\mathbb{E}_{t+1}[c_{it}^p | c_{it}^p < \hat{c}_{t+1}^p]\right)$$

For reference, the condition for success-maximizing planner to change tactics to violence was simply  $F_{t+1}\left(\mathbb{E}[A_v(l_{t+1})]\right) > F_{t+1}\left(\mathbb{E}[A_p(l_{t+1})]\right)$ . This comparison provides us with some clarity. Peaceful incumbents’ unwillingness to change tactics originates from the likelihood that they will

participate in a successful protest and acquire public benefits – the first term of the right-hand side, but is curbed by cost that necessarily accompanies participation. Since participants never show up if the cost is higher than the expected benefits, it is clear that this calculus tilts in the favor of recalcitrance – failure to change tactics even in some instances where it would be success-maximizing. Hence, it is a reasonable conjecture that *incumbents respond less to retrospective incentives than a success-maximizing social planner*. The desire to acquire private benefits dilutes incumbents’ ability to capitalize off learning and make decisions maximally conducive to a social outcome. Of course, a decrease in success probability is not identical in this model to a decrease in *global welfare* – and a further task of this exercise is to characterize precisely the degree of success probability loss and a comparison in welfare. This portion of the exercise, once proven, offers preliminary and suggestive evidence regarding the difference between movements with decentralized decisionmaking structures and those with explicitly defined leadership. However, a more complete exercise, which looks at leaders with more realistic goals (e.g. those who are biased towards one tactic or the other, or care more about participants’ welfare than success) in the context of the full model, is necessary to make definitive statements.

After completing the exercise introduced in Section 5, the next task is to provide a characterization of all three sets of incentives – prospective, retrospective, and organizational – map onto equilibrium behavior in a full version of the model with all the elements described in Section 3. With this characterization complete, it will be possible to look at situations where some of these incentives reinforce one another – such as cases when both retrospective and prospective considerations both push organizers towards initial usage of peaceful tactics – and when they conflict – such as organizational incentives interfering with retrospective incentives. It will then also be possible to isolate effects on likelihood of success, as well as implications for citizens’ welfare.

Several comparative statics and extensions will then also be possible. For instance, how does tactical choice change when public benefits are large relative to private benefits, or vice versa? It is also possible to look at systematic differences between the two populations: what is the result when the distribution of possible cost shocks for peaceful and violent engagement are very different, as opposed to relatively similar? What about when one type has a larger population than the other (rather than both being a unit mass)? Finally, a number of interesting and realistic extensions to the model are worth investigating. For instance, the model provides a natural framework for understanding how multiple rebel groups might interact when collectively contesting the same cause. One can imagine, for instance, a simple extension when two groups with different fundamentals organize subsequent protests, learning from their counterparts’ choices as well as their own. This extension has the potential to address questions on, for instance, the emergence of violent flanks in protest movements.

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# Appendix

## A Proofs

### A.1 Proof of Lemma 1

*Proof.* Recall that we defined

$$\hat{c}_t^\tau = \rho_\tau \int_0^1 F_t(A_\tau(l)) dl$$

where  $A_\tau$  is an increasing function of  $l$  and  $F_t$  denotes the CDF of the public belief about  $\theta$  with which agents make their participation decision in time  $t$ .

A protest of size  $l$  succeeds if  $A_\tau(l) > \theta$ . Suppose  $F$  FOSD  $F'$ . Then, for some fixed  $a$ ,  $F(a) < F'(a)$ . This must be true of any size of mobilization  $a$ . Hence, in order to show that  $\hat{c}_{t+1} < \hat{c}_t$ , it suffices to show that  $F_{t+1}$  FOSD  $F_t \forall t$ .

I proceed by induction on  $t$ .

**Base case:**  $t = 1$ . At  $t = 0$ ,  $F_0(\theta) = \frac{\theta - m}{M - m}$ . After one failed protest of size  $A_1 \in [m, M]$ :

$$F_1(\theta) = \begin{cases} \frac{(1-\rho)(\theta-m)}{(1-\rho)(A_1-m)+(M-A_1)} & \text{if } \theta \in (m, A_1) \\ \frac{\theta-A_1}{(1-\rho)(A_1-m)+(M-A_1)} + \frac{(1-\rho)(A_1-m)}{(1-\rho)(A_1-m)+(M-A_1)} & \text{if } \theta \in (A_1, M) \end{cases}$$

We will show that in both cases,  $F_0 > F_1$ . For the first case,

$$\frac{(1-\rho)(\theta-m)}{(1-\rho)(A_1-m)+(M-A_1)} < \frac{\theta-m}{M-m} \iff \rho > \rho\left(\frac{A_1-m}{M-m}\right)$$

which is true. For the second case,

$$\begin{aligned} & \frac{\theta-A_1}{(1-\rho)(A_1-m)+(M-A_1)} + \frac{(1-\rho)(A_1-m)}{(1-\rho)(A_1-m)+(M-A_1)} \\ &= \frac{(\theta-A_1) + (1-\rho)(A_1-m)}{(1-\rho)(A_1-m)+(M-A_1)} < \frac{\theta-m}{M-m} \\ &\iff \frac{M-m}{\theta-m} > 1 \end{aligned}$$

which is true for all  $\theta \in [A_1, M]$ .

**Inductive step:** Prior to the start of period  $t + 1$  there exists some past sequence of sizes of past (unsuccessful mobilizations):  $A_\tau(l_1), A_\tau(l_2), \dots, A_\tau(l_t)$ . Order these from **smallest to**

**largest** and denote them  $b_1, \dots, b_t$ . Remove any mobilizations from the sequence that less than  $m$ , since these do not affect beliefs. Thus, we have some non-decreasing sequence of mobilizations of length  $s$ , where  $t \geq s \geq 1$ . Add the cut-points of the support of the distribution  $\theta$ , so the non-decreasing sequence of intervals consists of  $m, \{b\}_{r=1}^s, M$ .

The density of the belief  $f_t$  about  $\theta$  is

$$f_t(\theta) = \begin{cases} \frac{(1-\rho)^s}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [m, b_1] \\ \frac{(1-\rho)^{s-1}}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [b_1, b_2] \\ \dots & \\ \frac{1}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [b_s, M] \end{cases} \quad (\text{A1})$$

where the generic interval  $b_r - b_{r-1}$  has density

$$\frac{(1-\rho)^{s-(r-1)}}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)}$$

The corresponding CDF is

$$F_t(\theta) = \begin{cases} \frac{(1-\rho)^s(\theta-m)}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [m, b_1] \\ \frac{(b_1-m)(1-\rho)^s + (\theta-b_1)(1-\rho)^{s-1}}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [b_1, b_2] \\ \vdots & \\ \frac{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (\theta-b_s)}{(b_1-m)(1-\rho)^s + (b_2-b_1)(1-\rho)^{s-1} + \dots + (b_s-b_{s-1})(1-\rho) + (M-b_s)} & \text{if } \theta \in [b_s, M] \end{cases} \quad (\text{A2})$$

Consider now the mobilization that is initiated at time  $t+1$ :

$$A_\tau \left( \frac{\hat{c} - (c_t^\tau - \epsilon)}{2\epsilon} \right) \equiv b'$$

If  $b' < m$  then agents learn nothing and beliefs do not change from one period to the next. If  $b' > M$ , replace it with the value of  $M$  (the upper bound on the support of  $\theta$ ). If  $b' \in (m, M)$ , then it must be that  $b'$  is contained within an existing interval. Denote this interval by  $[b_k, b_{k+1}]$ . Let  $\underline{B}$  denote the set of all intervals from  $[m, b_1] \dots [b_{k-1}, b_k]$  and  $\bar{B}$  denote the set of all intervals from  $[b_{k+1}, b_{k+2}] \dots [b_s, M]$ . Note that the new  $(t+1)$  sequence of cutpoints  $m, b_1, \dots, b_k, b', b_{k+1}, \dots, M$  has  $s+1$  terms.

I will show that the FOSD relation must hold at least weakly in the interval  $[b', b_k + 1]$  and must hold strictly everywhere else.

(1) First consider all intervals in  $\underline{B}$ . For any  $\theta \in [b_j, b_{j+1}] \in \underline{B}$ ,  $F_t$  FOSD  $F_{t+1}$  if

$$\frac{(1-\rho)^{j-1}(\theta - b_j) + \dots + (1-\rho)^s(b_1 - m)}{N_t} > \frac{(1-\rho)^j(\theta - b_j) + \dots + (1-\rho)^{s+1}(b_1 - m)}{N_{t+1}}$$

where

$$\begin{aligned}
N_t &= (b_1 - m)(1 - \rho)^s + (b_2 - b_1)(1 - \rho)^{s-1} + \cdots + (b_{k+1} - b_k)(1 - \rho)^{s-k} + \cdots + (M - b_s) \\
N_{t+1} &= (b_1 - m)(1 - \rho)^{s+1} + (b_2 - b_1)(1 - \rho)^s + \cdots \\
&\quad + (b' - b_k)(1 - \rho)^{s+1-k} + (b_{k+1} - b')(1 - \rho)^{s-k} + \cdots + (M - b_s)
\end{aligned}$$

Note that  $N_t$  has one fewer term than  $N_{t+1}$ . Rearrange the initial inequality:

$$\begin{aligned}
\iff \frac{N_{t+1}}{N_t} &> \frac{(1 - \rho)^j(\theta - b_j) + \cdots + (1 - \rho)^{s+1}(b_1 - m)}{(1 - \rho)^{j-1}(\theta - b_j) + \cdots + (1 - \rho)^s(b_1 - m)} \\
\iff \frac{N_{t+1}}{N_t} &> (1 - \rho) \frac{(1 - \rho)^{j-1}(\theta - b_j) + \cdots + (1 - \rho)^s(b_1 - m)}{(1 - \rho)^{j-1}(\theta - b_j) + \cdots + (1 - \rho)^s(b_1 - m)} \\
\iff \frac{N_{t+1}}{N_t} &> (1 - \rho)
\end{aligned}$$

To see why this is true, note that for  $N_t$  restricted to terms preceding the “split term”, i.e.  $(b_1 - m)(1 - \rho)^s + \cdots + (b_k - b_{k-1})(1 - \rho)^{s-(k-1)}$  it is true that (with some abuse of notation)  $(1 - \rho)N_t = N_{t+1}$ . For  $N_t$  restricted to terms following the split, i.e.  $(b_{k+2} - b_{k+1})(1 - \rho)^{s-(k+1)} + \cdots + (b_1 - m)$ , we have (with the same abuse of notation) a strict inequality:  $(1 - \rho)N_t < N_{t+1}$ .

Now I compare the term in  $N_t$  that is split into two terms in  $N_{t+1}$ . For  $N_t$  and  $N_{t+1}$  restricted to these terms, first note that

$$\begin{aligned}
(1 - \rho)N_t &= (b_{k+1} - b_k)(1 - \rho)^{s-k}(1 - \rho) \\
&= (b_{k+1} - b_k)(1 - \rho)^{s-k+1}
\end{aligned}$$

By definition,  $N_{t+1} = (b' - b_k)(1 - \rho)^{s-k+1} + (b_{k+1} - b')(1 - \rho)^{s-k}$ . This implies that

$$\begin{aligned}
N_{t+1} &> (b' - b_k)(1 - \rho)^{s-k+1} + (b_{k+1} - b')(1 - \rho)^{s-k+1} \\
&= (1 - \rho)^{s-k+1}(b' - b_k + b_{k+1} - b') \\
&= (1 - \rho)^{s-k+1}(b_{k+1} - b_k) \\
&= (1 - \rho)N_{t+1}
\end{aligned}$$

We have therefore proved that there must be (weakly greater than zero) terms where  $N_{t+1} = (1 - \rho)N_t$  and there must be at least one term where  $N_{t+1} > (1 - \rho)N_t$ . This proves the claim that for intervals in  $\underline{B}$ ,  $F_t > F_{t+1}$ .

(2) Suppose  $\theta \in [b_k, b']$ . In this region,  $F_t$  FOSD  $F_{t+1}$  if

$$\begin{aligned}
\frac{(1-\rho)^{s-k}(\theta - b_k) + \dots + (1-\rho)^s(b_1 - m)}{N_t} &> \frac{(1-\rho)^{s-k+1}(\theta - b_k) + \dots + (1-\rho)^{s+1}(b_1 - m)}{N_{t+1}} \\
\iff \frac{N_{t+1}}{N_t} &> \frac{(1-\rho)^{s-k+1}(\theta - b_k) + \dots + (1-\rho)^{s+1}(b_1 - m)}{(1-\rho)^{s-k}(\theta - b_k) + \dots + (1-\rho)^s(b_1 - m)} \\
\iff \frac{N_{t+1}}{N_t} &> (1-\rho)
\end{aligned}$$

which was already been proved above.

(3) Suppose  $\theta \in [b', b_k + 1]$ . I will show that in this region,  $F_t \geq F_{t+1}$  if

$$\begin{aligned}
&\frac{(1-\rho)^{s-k}(\theta - b_k) + \dots + (1-\rho)^s(b_1 - m)}{N_t} \\
&\geq \frac{(1-\rho)^{s-k}(\theta - b') + (1-\rho)^{s-k+1}(b' - b_k) + \dots + (1-\rho)^{s+1}(b_1 - m)}{N_{t+1}} \\
\iff &\frac{(1-\rho)^{s-k}(\theta - b') + (1-\rho)^{s-k}(b' - b_k) + \dots + (1-\rho)^s(b_1 - m)}{N_t} \\
&\geq \frac{(1-\rho)^{s-k}(\theta - b') + (1-\rho)^{s-k+1}(b' - b_k) + \dots + (1-\rho)^{s+1}(b_1 - m)}{N_{t+1}} \\
\iff &\frac{N_{t+1}}{N_t} \geq \frac{(1-\rho)^{s-k}(\theta - b') + (1-\rho)^{s-k+1}(b' - b_k) + \dots + (1-\rho)^{s+1}(b_1 - m)}{(1-\rho)^{s-k}(\theta - b') + (1-\rho)^{s-k}(b' - b_k) + \dots + (1-\rho)^s(b_1 - m)} \\
\iff &\frac{N_{t+1}}{N_t} \geq \frac{(1-\rho)^{s-k}(\theta - b') + (1-\rho) \left[ (1-\rho)^{s-k}(b' - b_k) + \dots + (1-\rho)^s(b_1 - m) \right]}{(1-\rho)^{s-k}(\theta - b') + \left[ (1-\rho)^{s-k}(b' - b_k) + \dots + (1-\rho)^s(b_1 - m) \right]}
\end{aligned}$$

Let  $\mathcal{S} \equiv \left[ (1-\rho)^{s-k}(b' - b_k) + \dots + (1-\rho)^s(b_1 - m) \right]$ . Note that

$$\frac{N_{t+1}}{N_t} = \frac{(M - b_s) + \dots + (1-\rho)^{s-k}(b_{k+1} - b') + (1-\rho)\mathcal{S}}{(M - b_s) + \dots + (1-\rho)^{s-k}(b_{k+1} - b') + \mathcal{S}}$$

Restating the problem:

$$\frac{\left[ (M - b_s) + \dots + (1-\rho)^{s-k}(b_{k+1} - b') \right] + (1-\rho)\mathcal{S}}{\left[ (M - b_s) + \dots + (1-\rho)^{s-k}(b_{k+1} - b') \right] + \mathcal{S}} \geq \frac{\left[ (1-\rho)^{s-k}(\theta - b') \right] + (1-\rho)\mathcal{S}}{\left[ (1-\rho)^{s-k}(\theta - b') \right] + \mathcal{S}}$$

Collecting terms, we can restate the problem as:

$$\begin{aligned}
& \frac{\mathcal{M} - (1 - \rho)\mathcal{S}}{\mathcal{M} + \mathcal{S}} \geq \iff \frac{\alpha - (1 - \rho)\mathcal{S}}{\alpha + \mathcal{S}} \\
& \iff (\mathcal{M} + (1 - \rho)\mathcal{S})(\alpha + \mathcal{S}) \geq (\alpha + (1 - \rho)\mathcal{S})(\mathcal{M} + \mathcal{S}) \\
& \iff \mathcal{M}\mathcal{S} + \alpha(1 - \rho)\mathcal{S} \geq \alpha\mathcal{S} + \mathcal{M}(1 - \rho)\mathcal{S} \\
& \iff \mathcal{M} + \alpha(1 - \rho) \geq \alpha + \mathcal{M}(1 - \rho) \\
& \iff \mathcal{M}(1 - 1 + \rho) \geq \alpha(1 - 1 + \rho) \\
& \iff \mathcal{M} \geq \alpha \\
& \iff (M - b_s) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') \geq (1 - \rho)^{s-k}(\theta - b')
\end{aligned}$$

Since all terms are positive and  $\theta \in [b', b_{k+1}]$ , we must have

$$(1 - \rho)^{s-k}(b_{k+1} - b') \geq (1 - \rho)^{s-k}(\theta - b')$$

This holds with “=” if  $\theta = b_{k+1}$  and with “>” if  $\theta \in [b', b_{k+1})$ .

(4) Last, consider all intervals in  $\bar{B}$ . For any  $\theta \in [b_j, b_{j+1}] \in \bar{B}$ ,  $F_t$  FOSD  $F_{t+1}$  if

$$\begin{aligned}
& \frac{(1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b_k) + \dots + (1 - \rho)^s(b_1 - m)}{N_t} \\
& > \frac{(1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k+1}(b_{k+1} - b') + (1 - \rho)^{s-k+1}(b' - b_k) + \dots + (1 - \rho)^{s+1}(b_1 - m)}{N_{t+1}}
\end{aligned}$$

$$\iff \frac{N_{t+1}}{N_t} > \frac{(1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') + (1 - \rho)\mathcal{S}}{(1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_1 - b') + \mathcal{S}}$$

Using the same definition of  $\mathcal{S}$  that we employed previously, we can restate the problem:

$$\begin{aligned}
& \frac{\left[ (M - b_s) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') \right] + (1 - \rho)\mathcal{S}}{(M - b_s) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') + \mathcal{S}} \\
& > \frac{\left[ (1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') \right] + (1 - \rho)\mathcal{S}}{\left[ (1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_1 - b') \right] + \mathcal{S}}
\end{aligned}$$

Collecting terms, we can restate this expression as

$$\frac{\mathcal{M} - (1 - \rho)\mathcal{S}}{\mathcal{M} + \mathcal{S}} > \frac{\alpha' - (1 - \rho)\mathcal{S}}{\alpha' + \mathcal{S}}$$

Which holds whenever  $\mathcal{M} > \alpha'$ , i.e.

$$(M - b_s) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b') > (1 - \rho)^{s-j}(\theta - b_j) + \dots + (1 - \rho)^{s-k}(b_{k+1} - b')$$

Note that terms  $(M - b_s) + \dots + (1 - \rho)^{s-j+1}(b_{j+2} - b_{j+1})$  on the LHS have no analogue on the RHS, so we can cancel terms and simplify the inequality to

$$(M - b_s) + \dots + (1 - \rho)^{s-j+1}(b_{j+2} - b_{j+1}) > (1 - \rho)^{s-j}(\theta - b_j)$$

since  $\theta \in [b_j, b_{j+1}]$ , the inequality must hold strictly.

□

## A.2 Proof of Lemma 2

- (i) Claim: *The social planner will never use violence in the first period.*

*Proof.* Recall the prior is that  $\theta \sim \text{Unif}[m, M]$ . I will show that there does not exist any  $\theta' \in [m, M]$  such that there is a strictly higher probability of defeating  $\theta'$  if violence is used in the first period and peace in the second period rather than using peace in the first period.

Suppose  $\theta' \in [m, \bar{v}]$ . If violence is used initially,  $\theta$  is defeated with certainty. If peace is used in the first period and generates power less than  $\theta$ , then the social planner uses violence and defeats  $\theta'$  with certainty. Hence there is no difference in the probability of success.

Suppose  $\theta \in [\bar{v}, M]$ . If violence is used initially, then the likelihood of defeating  $\theta$  is the probability that the peaceful protest in the second period is greater than  $\theta'$ , which is

$$\mathcal{P}_2 \equiv \frac{\hat{c}_2^p(\bar{v}) + \epsilon - (2\epsilon)A_p^{-1}(\theta') - \underline{c}}{\bar{c} - \underline{c}}$$

where  $\hat{c}_2^p(\bar{v}) = \int_0^1 \frac{A_p(l) - \bar{v}}{M - \bar{v}} dl$

If peace is used initially, then the likelihood of defeating  $\theta$  is the probability that the peaceful protest in the first period is greater than  $\theta'$ , which is

$$\mathcal{P}_1 \equiv \frac{\hat{c}_1^p + \epsilon - (2\epsilon)A_p^{-1}(\theta') - \underline{c}}{\bar{c} - \underline{c}}$$

where  $\hat{c}_1^p = \int_0^1 \frac{A_p(l) - m}{M - m} dl$

Note that  $\hat{c}_2^p(\bar{v}) < \hat{c}_1^p$ , and besides that all the other terms are identical between  $\mathcal{P}_1$  and  $\mathcal{P}_2$ . Hence,  $\mathcal{P}_1 < \mathcal{P}_2$ , so there is a strictly smaller probability of defeating  $\theta$  if peace is used first.

Now note that there is no benefit to using violence more than once. Any  $\theta$  which cannot be defeated using violence in one period cannot be defeated using violence in another period,

and any  $\theta$  which can be defeated using violence in one period can also be defeated using violence in another period. By contrast, using peace in the first period affords positive probability of defeating values of  $\theta$  greater than  $\bar{v}$ , so it is strictly preferable to use peace first and violence second than using violence twice.

Thus, it is never optimal to use violence in the first period.  $\square$

- (ii) Claim: *The range of failed first-period protests for which violence is used in the second period is increasing in  $\bar{v}$ .*

*Proof.* Let the power of the first-period protest be denoted by  $A_1$ . The game only continues if the first-period protest failed. Then, the expected success of peaceful protest is given by:

$$\int_{\underline{c}}^{\bar{c}} \frac{1}{\bar{c} - \underline{c}} \frac{\max \left\{ A_p \left( \min \left( \max \left( \frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0 \right), 1 \right) \right) - A_1, 0 \right\}}{M - A_1} dc \quad (\star)$$

where

$$\hat{c}_2^p = \int_0^1 \frac{\max(A_p(l) - A_1, 0)}{M - A_1} dl$$

Note that  $\underline{c} < \bar{c}$ ,  $0 \leq A_1 \leq A_p(1) \leq M$ . I further assume that  $A_p$  is a linear function with slope  $p$  and intercept 0.

The expected success of a violent protest is given by

$$\frac{\max\{\bar{v} - A_1, 0\}}{M - A_1}$$

The social planner chooses peace for the second period if  $(\star) > \frac{\max\{\bar{v} - A_1, 0\}}{M - A_1}$ . If  $\bar{v} < A_1$  then this is trivially true. Consider the case where  $\bar{v} \geq A_1$ :

$$\begin{aligned} & \int_{\underline{c}}^{\bar{c}} \frac{1}{\bar{c} - \underline{c}} \frac{\max \left\{ A_p \left( \min \left( \max \left( \frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0 \right), 1 \right) \right) - A_1, 0 \right\}}{M - A_1} dc > \frac{\max\{\bar{v} - A_1, 0\}}{M - A_1} \\ \iff & \frac{1}{\bar{c} - \underline{c}} \int_{\underline{c}}^{\bar{c}} \max \left\{ A_p \left( \min \left( \max \left( \frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0 \right), 1 \right) \right) - A_1, 0 \right\} dc > \bar{v} - A_1 \\ H(A_1) \equiv & \frac{1}{\bar{c} - \underline{c}} \int_{\underline{c}}^{\bar{c}} \max \left\{ A_p \left( \min \left( \max \left( \frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0 \right), 1 \right) \right) - A_1, 0 \right\} dc + A_1 > \bar{v} \end{aligned}$$

The task now reduces to proving that  $H(A_1)$  is convex in  $A_1$ . Since the RHS ( $\bar{v}$ ) is constant in  $A_1$ , this suffices to show that the set of  $A_1$  for which the inequality is true is decreasing in  $\bar{v}$  (equivalently, the set of  $A_1$  for which violence is the success-maximizing choice in the second period is increasing in  $\bar{v}$ ).



To differentiate  $A_1$ , it is helpful to adjust the bounds of integration.

$$A_p\left(\min\left(\max\left(\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0\right), 1\right)\right) < A_1$$

$$\iff c > \hat{c}_2^p + \epsilon - 2\epsilon\left(\frac{A_1}{p}\right)$$

Note that  $\min\{\max\{\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}, 0\}, 1\}$  implies that

$$\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon} > 1 \iff c < \hat{c} - \epsilon$$

$$\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon} < 0 \iff c > \hat{c} + \epsilon$$

Taken together, this yields the following constraints:

$$\hat{c} - \epsilon < c < \hat{c} + \epsilon \quad \text{and} \quad \hat{c} + \epsilon - 2\epsilon\frac{A_1}{p} < c$$

Note that the latter constraint is the stricter lower bound on  $c$ . Hence, we can simplify to:

$$\hat{c} + \epsilon - 2\epsilon\frac{A_1}{p} < c < \hat{c} + \epsilon$$

Hence, adjusting the bounds of integration yields

$$H(A_1) = \int_{\max\{\underline{c}, \hat{c} + \epsilon - 2\epsilon(A_1/p)\}}^{\min\{\bar{c}, \hat{c} + \epsilon\}} \frac{1}{\bar{c} - \underline{c}} \left[ A_p\left(\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}\right) - A_1 \right] dc + \int_{\underline{c}}^{\hat{c} + \epsilon - 2\epsilon(A_1/p)} \frac{1}{\bar{c} - \underline{c}} \left[ A_p(1) - A_1 \right] dc + A_1$$

Assuming that  $A_p$  is linear,

$$= \int_{\max\{\underline{c}, \hat{c} + \epsilon - 2\epsilon(A_1/p)\}}^{\min\{\bar{c}, \hat{c} + \epsilon\}} \frac{1}{\bar{c} - \underline{c}} \left[ p\left(\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}\right) - A_1 \right] dc + \int_{\underline{c}}^{\hat{c} + \epsilon - 2\epsilon(A_1/p)} \frac{1}{\bar{c} - \underline{c}} \left[ p - A_1 \right] dc + A_1$$

Case (1): Both of the  $\hat{c}$  constraints bind. Then, the first integral is simply a *linear* function of  $c$  which takes value ranging from 0 to  $A_p(1) - A_1$ :

$$\int_{\hat{c} + \epsilon - 2\epsilon(A_1/p)}^{\hat{c} + \epsilon} \left[ p\left(\frac{\hat{c}_2^p - c + \epsilon}{2\epsilon}\right) - A_1 \right] dc$$

Note that the integrand is a linear function of  $x$ . Therefore, this integral evaluates to:

$$\begin{aligned} & \left( \hat{c} + \epsilon - (\hat{c} + \epsilon - 2\epsilon(A_1/p)) \right) \frac{1}{2} \left[ p\left(\frac{\hat{c} - (\hat{c} + \epsilon) + \epsilon}{2\epsilon}\right) - A_1 + p\left(\frac{\hat{c} - (\hat{c} + \epsilon - 2\epsilon(A_1/p)) + \epsilon}{2\epsilon}\right) - A_1 \right] \\ &= 2\epsilon\left(\frac{A_1}{p}\right) \frac{1}{2} \left[ 0 - A_1 + p\left(\frac{A_1}{p}\right) - A_1 \right] \\ &= \epsilon\left(\frac{A_1}{p}\right) (-A_1) \\ &= -\frac{\epsilon A_1^2}{p} \end{aligned}$$

The second integral is easy to evaluate (as the argument does not appear). Thus,  $H(A_1)$  is

$$H(A_1) = \frac{1}{\bar{c} - \underline{c}} \left( -\frac{\epsilon A_1^2}{p} \right) + \frac{1}{\bar{c} - \underline{c}} \left( (p - A_1)(\hat{c} + \epsilon - 2\epsilon(A_1/p) - \underline{c}) \right) + A_1$$

$$\begin{aligned} \text{Note that: } \hat{c} &= \int_{A_p^{-1}(A_1)}^1 \frac{A_p(x) - A_1}{M - A_1} dx \\ &= \int_{A_1/p}^1 \frac{px - A_1}{M - A_1} dx \text{ (with linear assumption)} \\ &= \frac{(p - A_1)^2}{2p(A_1 - M)} \end{aligned}$$

Hence,  $H(A_1)$  is

$$\frac{1}{\bar{c} - \underline{c}} \left( \frac{-\epsilon(A_1)^2}{p} + (p - A_1) \left( \frac{(p - A_1)^2}{2p(A_1 - M)} - 2\epsilon \frac{A_1}{p} + \epsilon - \underline{c} \right) \right) + A_1$$

Differentiate with respect to  $A_1$ :

$$\begin{aligned} \frac{dH(A_1)}{dA_1} &= \frac{1}{\bar{c} - \underline{c}} \left[ \frac{-2\epsilon A_1}{p} + \frac{(p - A_1)^2(p - 3M + 2A_1)}{2p(A_1 - M)^2} + \frac{2\epsilon(2A_1 - p)}{p} - \epsilon + \underline{c} \right] + 1 \\ &= \frac{1}{\bar{c} - \underline{c}} \left[ \frac{2\epsilon(A_1 - p)}{p} + \frac{(p - A_1)^2(p - 3M + 2A_1)}{2p(A_1 - M)^2} - \epsilon + \underline{c} \right] + 1 \end{aligned}$$

Differentiate once again to obtain the second derivative:

$$\begin{aligned} &\frac{1}{\bar{c} - \underline{c}} \left[ \frac{2\epsilon}{p} + \frac{(p - A_1)(3M^2 + p^2 + pA_1 + A_1^2 - 3M(p + A_1))}{p(M - A_1)^3} \right] \\ &= \frac{1}{\bar{c} - \underline{c}} \left[ \frac{2\epsilon}{p} + \frac{(p - A_1)(3M(M - p - A_1) + p^2 + pA_1 + A_1^2)}{p(M - A_1)^3} \right] \end{aligned}$$

Every term is positive (since  $M \geq p + A_1$ ). Hence, the entire expression is positive and  $H(A_1)$  is convex.

Case (2):  $\underline{c}$  and  $\bar{c}$  constraints bind. In this case, full turnout is never obtained, so we drop the second term (which describes instances where the cost draw is such that full turnout is obtained). Then,

$$\begin{aligned} H(A_1) &= \int_{\underline{c}}^{\bar{c}} \frac{1}{\bar{c} - \underline{c}} \left( \frac{p(\hat{c} - c + \epsilon)}{2\epsilon} - A_1 \right) dc + A_1 \\ &= \frac{1}{\bar{c} - \underline{c}} \frac{1}{2} \left( \frac{p(\hat{c} - \bar{c} + \epsilon)}{2\epsilon} - A_1 + \frac{p(\hat{c} - \underline{c} + \epsilon)}{2\epsilon} - A_1 \right) + A_1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{2p(\hat{c} + \epsilon)}{2\epsilon} + \frac{p(\bar{c} - \underline{c})}{2\epsilon} - 2A_1 \right) + A_1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p(\hat{c} + \epsilon)}{\epsilon} + \frac{p(\bar{c} - \underline{c})}{2\epsilon} \right) - \frac{A_1}{\bar{c} - \underline{c}} + A_1 \end{aligned}$$

Differentiate wrt  $A_1$ :

$$\begin{aligned}\frac{dH(A_1)}{dA_1} &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p}{2\epsilon} \right) \left( \frac{d\hat{c}(A_1)}{dA_1} \right) + \frac{1}{\bar{c} - \underline{c}} + 1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p}{2\epsilon} \right) \left( \frac{1}{2p} \frac{p^2 - 2Mp - A_1^2 + 2MA_1}{(M - A_1)^2} \right) - \frac{1}{\bar{c} - \underline{c}} + 1\end{aligned}$$

Differentiate again:

$$= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p}{2\epsilon} \right) \left( \frac{1}{p} \frac{(M - p)^2}{(M - A_1)^2} \right) > 0$$

Case (3):  $\underline{c}, \hat{c} + \epsilon$  constraints bind. In this case, full turnout is again never obtained, so we drop the second term. Then,

$$\begin{aligned}H(A_1) &= \int_{\underline{c}}^{\hat{c} + \epsilon} \frac{1}{\bar{c} - \underline{c}} \left( \frac{p(\hat{c} - c + \epsilon)}{2\epsilon} - A_1 \right) dc + A_1 \\ &= \frac{1}{\bar{c} - \underline{c}} \frac{1}{2} \left( \frac{p(\hat{c} - (\hat{c} + \epsilon) + \epsilon)}{2\epsilon} - A_1 + \frac{p(\hat{c} - \underline{c} + \epsilon)}{2\epsilon} - A_1 \right) + A_1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p(\hat{c} - \underline{c} + \epsilon)}{2\epsilon} - 2A_1 \right) + A_1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p(\hat{c} - \underline{c} + \epsilon)}{2\epsilon} \right) - \frac{A_1}{\bar{c} - \underline{c}} + A_1\end{aligned}$$

Differentiate wrt  $A_1$ :

$$\frac{dH(A_1)}{dA_1} = \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p}{2\epsilon} \right) \left( \frac{1}{2p} \frac{p^2 - 2Mp - A_1^2 + 2MA_1}{(M - A_1)^2} \right) - \frac{1}{\bar{c} - \underline{c}} + 1$$

Differentiate again:

$$= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p}{2\epsilon} \right) \left( \frac{1}{p} \frac{(M - p)^2}{(M - A_1)^2} \right) > 0$$

Case (4):  $\hat{c} - \epsilon + 2\epsilon(A_1/p), \bar{c}$  constraints bind. Then,

$$\begin{aligned}H(A_1) &= \int_{\hat{c} + \epsilon - (2\epsilon)(A_1/p)}^{\bar{c}} \frac{1}{\bar{c} - \underline{c}} \left( \frac{p(\hat{c} - c + \epsilon)}{2\epsilon} - A_1 \right) dc + \int_{\underline{c}}^{\hat{c} + \epsilon - 2\epsilon(A_1/p)} \frac{p - A_1}{\bar{c} - \underline{c}} dc + A_1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left( \frac{p(\hat{c} - \bar{c} - \epsilon + \epsilon(A_1/p))}{2\epsilon} - 2A_1 \right) + \frac{1}{\bar{c} - \underline{c}} \left( (p - A_1)(\hat{c} + \epsilon - 2\epsilon(A_1/p) - \underline{c}) \right) + A_1\end{aligned}$$

Differentiating wrt  $A_1$ :

$$\begin{aligned}\frac{dH(A_1)}{dA_1} &= \frac{1}{2(\bar{c} - \underline{c})} \left[ \frac{p}{2\epsilon} \left( \frac{d\hat{c}}{dA_1} + \frac{2\epsilon}{p} \right) - 2 \right] \\ &\quad + \frac{1}{\bar{c} - \underline{c}} \left[ + \frac{(p - A_1)^2(p - 3M + 2A_1)}{2p(A_1 - M)^2} + \frac{2\epsilon(2A_1 - p)}{p} - \epsilon + \underline{c} \right] + 1 \\ &= \frac{1}{2(\bar{c} - \underline{c})} \left[ \frac{p}{2\epsilon} \left( \frac{d\hat{c}}{dA_1} \right) - 1 \right] + \frac{1}{\bar{c} - \underline{c}} \left[ + \frac{(p - A_1)^2(p - 3M + 2A_1)}{2p(A_1 - M)^2} + \frac{2\epsilon(2A_1 - p)}{p} - \epsilon + \underline{c} \right] + 1\end{aligned}$$

Differentiate once again to obtain the second derivative:

$$= \frac{1}{\bar{c} - \underline{c}} \left[ \frac{p}{2\epsilon} \frac{d^2 \hat{c}}{dA_1^2} + \frac{(p - A_1)(3M(M - p - A_1) + p^2 + pA_1 + A_1^2)}{p(M - A_1)^3} \right]$$

By previous arguments we know all these terms are positive.

Hence in all four cases,  $H(A_1)$  is convex.

□