BSc Computer Science CS1541 Computer Graphics

Module II

2D Transformations

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SAC

Overview

- Changing size, shape or orientation of an object on display
- Classifications
    - 2D Transformations
    - 3D Transformations
- **Geometric Transformation**: The object itself is transformed relative to the coordinate system or background. The mathematical statement of this viewpoint is defined by geometric transformations applied to each point of the object.
- Eg : Moving the automobile while keeping the background fixed

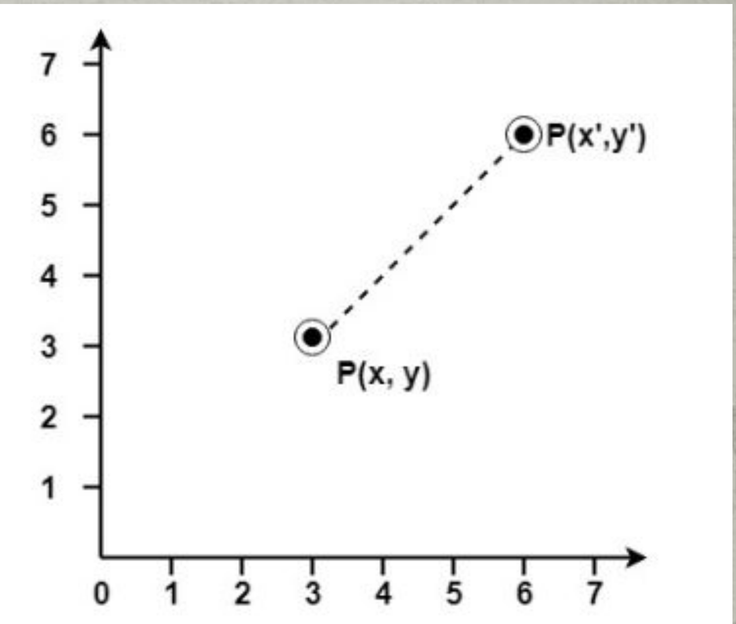
- **Coordinate Transformation**: The object is held stationary while the coordinate system is transformed relative to the object. This effect is attained through the application of coordinate transformations.
- Eg: Keep the car fixed while moving the background scenery

Types of Transformations

- Basic Transformations
  - Translation
  - Scaling
  - Rotation
- Additional Types
  - Reflection
  - Shearing

Translation

- It is the straight line movement of an object from one position to another.
- The object is positioned from one coordinate location to another.
- **Translation of point:**
- To translate a point from coordinate position (x, y) to another (x_1 y_1), add T_x and T_y algebraically the translation distances T_x and T_y to original coordinate.



$$X' = X + T_x$$
$$Y' = Y + T_y$$

($T_{x,}$ T_y) is the shift vector/Translation vector

Matrix Representation

- We can represent it into single matrix equation in column vector as;

$$P' = P + T$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

- We can also represent it in row vector form as:

$$P' = P + T$$

$$[x' \quad y'] = [x \quad y] + [t_x \quad t_y]$$

- Since column vector representation is standard mathematical notation and since many graphics package like **GKS** and **PHIGS** uses column vector we will also follow column vector representation.

Example

Example: - Translate the triangle [A (10, 10), B (15, 15), C (20, 10)] 2 unit in x direction and 1 unit in y direction.

For point (10, 10)

$$A' = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 12 \\ 11 \end{bmatrix}$$

For point (15, 15)

$$B' = \begin{bmatrix} 15 \\ 15 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 17 \\ 16 \end{bmatrix}$$

For point (10, 10)

$$C' = \begin{bmatrix} 20 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C' = \begin{bmatrix} 22 \\ 11 \end{bmatrix}$$

- Final coordinates after translation are [A′ (12, 11), B′ (17, 16), C′ (22, 11)].

Scaling

- It is a transformation that used to alter the size of an object.
- This operation is carried out by multiplying coordinate value (x, y) with scaling factor (s_x, s_y) respectively.
- So equation for scaling is given by:

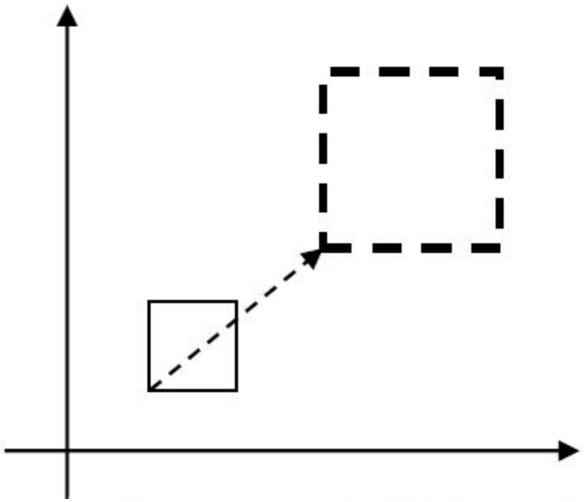
  $x' = x \cdot s_x$

  $y' = y \cdot s_y$

- These equation can be represented in column vector matrix

  $P' = S \cdot P$

  $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$



- Any positive value can be assigned to (s_x, s_y).
- Values less than 1 reduce the size while values greater than 1 enlarge the size of object, and object remains unchanged when values of both factor is 1.
- Same values of s_x and s_y will produce **Uniform Scaling**. And different values of s_x and s_y will produce **Differential Scaling**.
- Objects transformed with above equation are both scale and repositioned.
- Scaling factor with value less than 1 will move object closer to origin, while scaling factor with value greater than 1 will move object away from origin.
- We can control the position of object after scaling by keeping one position fixed called **Fix point** (x_f, y_f) that point will remain unchanged after the scaling transformation.

Example

- **Example**: - Consider square with left-bottom corner at (2, 2) and right-top corner at (6, 6) apply the transformation which makes its size half.

  As we want size half so value of scale factor are $s_x = 0.5, s_y = 0.5$ and Coordinates of square are [A (2, 2), B (6, 2), C (6, 6), D (2, 6)].

  $P' = S \cdot P$

$$P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 6 & 6 & 2 \\ 2 & 2 & 6 & 6 \end{bmatrix}$$

$$P' = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 1 & 1 & 3 & 3 \end{bmatrix}$$

- Final coordinate after scaling are [A′ (1, 1), B′ (3, 1), C′ (3, 3), D′ (1, 3)].

Rotation

- It is a transformation that used to reposition the object along the circular path in the XY - plane.
- To generate a rotation we specify a rotation angle $\boldsymbol{\theta}$ and the position of the **Rotation Point (Pivot Point)** $(x_{r,}y_r)$ about which the object is to be rotated.
- Positive value of rotation angle defines counter clockwise rotation and negative value of rotation angle defines clockwise rotation.
- We first find the equation of rotation when pivot point is at coordinate origin$(\mathbf{0},\mathbf{0})$.

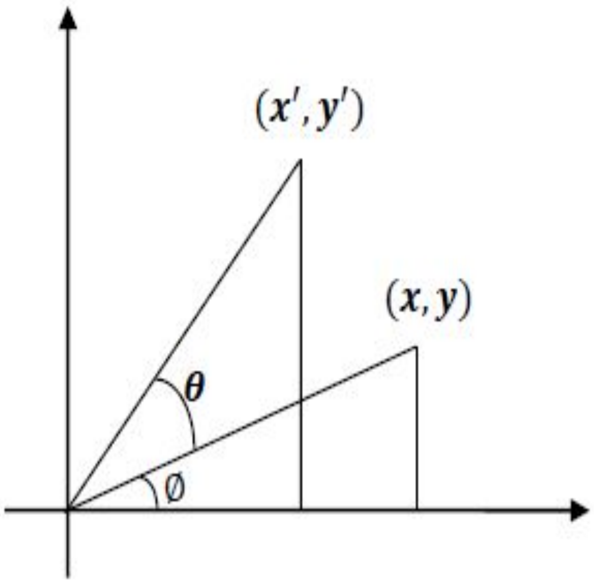


Fig. 3.2: - Rotation.

Rotation through Origin

$x = r\cos\emptyset$

$y = r\sin\emptyset$

and

$x' = r\cos(\theta + \emptyset) = r\cos\emptyset\cos\theta - r\sin\emptyset\sin\theta$

$y' = r\sin(\emptyset + \theta) = r\cos\emptyset\sin\theta + r\sin\emptyset\cos\theta$

- Now replace $r\cos\emptyset$ with x and $r\sin\emptyset$ with y in above equation.

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

- We can write it in the form of column vector matrix equation as;

$P' = R \cdot P$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Example

- **Example:** - Locate the new position of the triangle [A (5, 4), B (8, 3), C (8, 8)] after its rotation by 90º clockwise about the origin.

  As rotation is clockwise we will take $\theta = -90^{\circ}$.

  $$P' = R \cdot P$$

  $$P' = \begin{bmatrix} \cos(-90) & -\sin(-90) \\ \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

  $$P' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

  $$P' = \begin{bmatrix} 4 & 3 & 8 \\ -5 & -8 & -8 \end{bmatrix}$$

- Final coordinates after rotation are [A′ (4, -5), B′ (3, -8), C′ (8, -8)].

Homogeneous Coordinates

- To combine these three transformations into a single transformation, homogeneous coordinates are used.
- In homogeneous coordinate system, two-dimensional coordinate positions (x, y) are represented by triple-coordinates.
- Homogeneous coordinates are generally used in design and construction applications. Here we perform translations, rotations, scaling to fit the picture into proper position.
- For two-dimensional geometric transformation, we can choose homogeneous parameter h to any non-zero value. For our convenience take it as one.
- Each two-dimensional position is then represented with homogeneous coordinates (x, y, 1).

BASIC 2D TRANSFORMATIONS

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

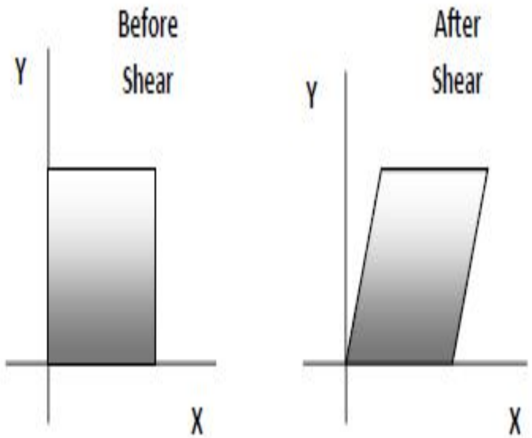
Rotate

Shear

- A transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other is called **shear**.
- Two common shearing transformations are those that shift coordinate x values and those that shift y values.

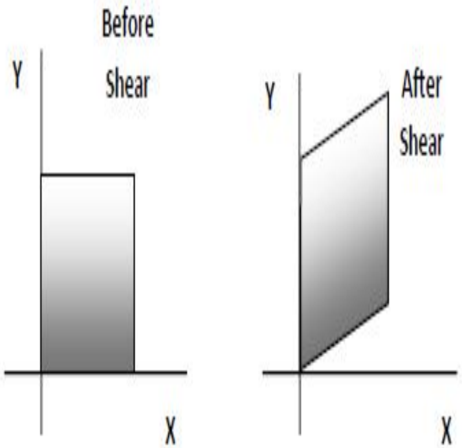
Shear in $x-direction$.

$$\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Shear in $y-direction$.

$$\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection

- Transformation which produces a mirror image of an object.
- The mirror image can be either about x-axis or y-axis.
- The object is rotated by180°.

- **Types of Reflection:**
  - Reflection about the x-axis
  - Reflection about the y-axis
  - Reflection about an axis perpendicular to xy plane and passing through the origin
  - Reflection about line y=x

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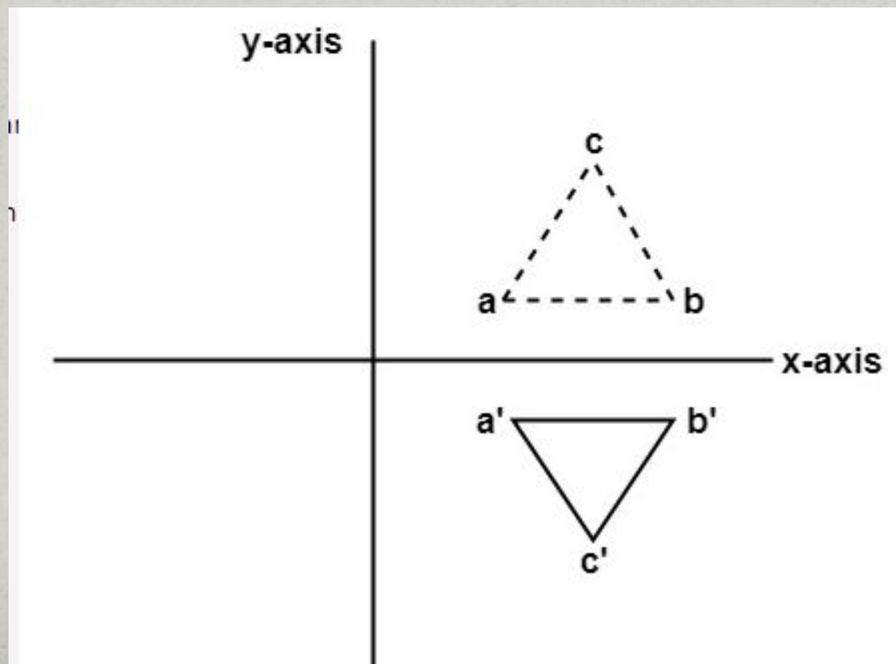
Reflection about X axis

- The object can be reflected about x-axis with the help of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Value of x will remain same whereas the value of y will become negative.

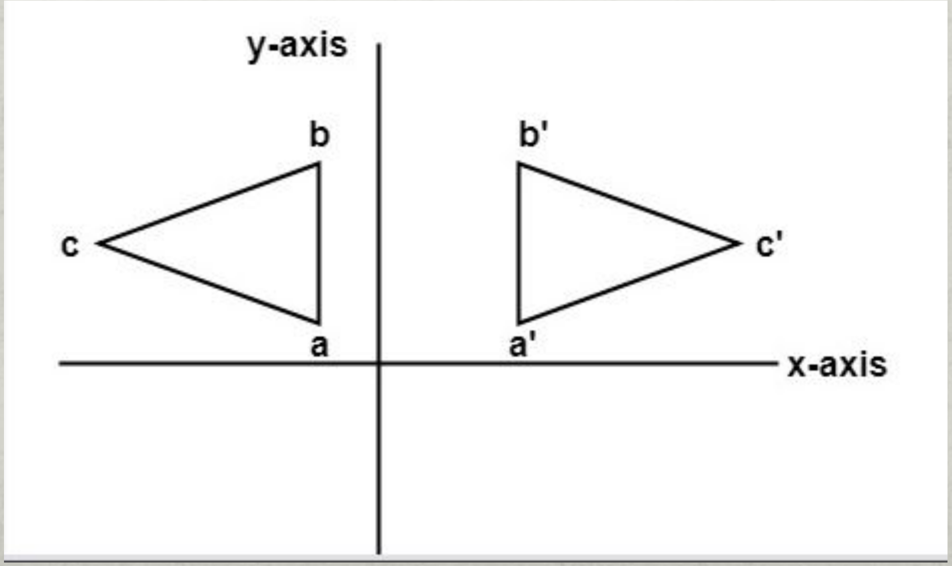


Reflection about Y axis

- The object can be reflected about y-axis with the help of following transformation matrix

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Value of Y will remain same whereas the value of X will become negative.



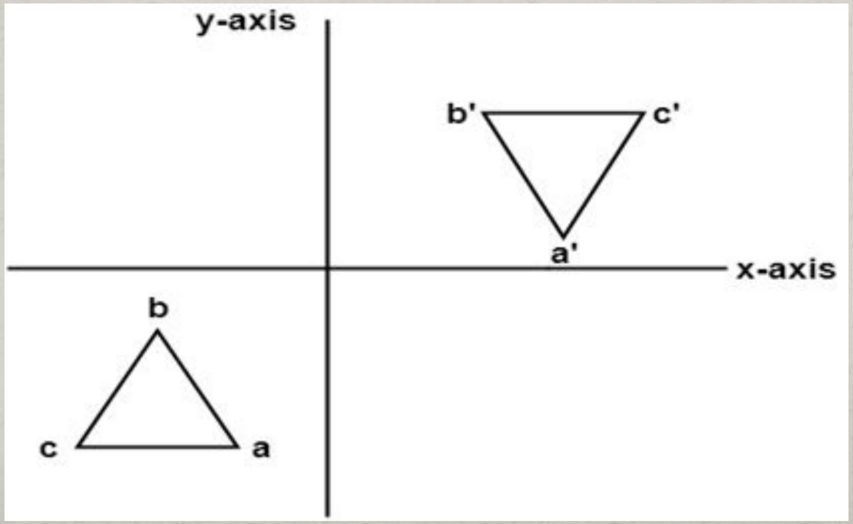
Reflection about parpandicular axis

- **Reflection about an axis perpendicular to xy plane and passing through origin**

In the matrix of this transformation is given below

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

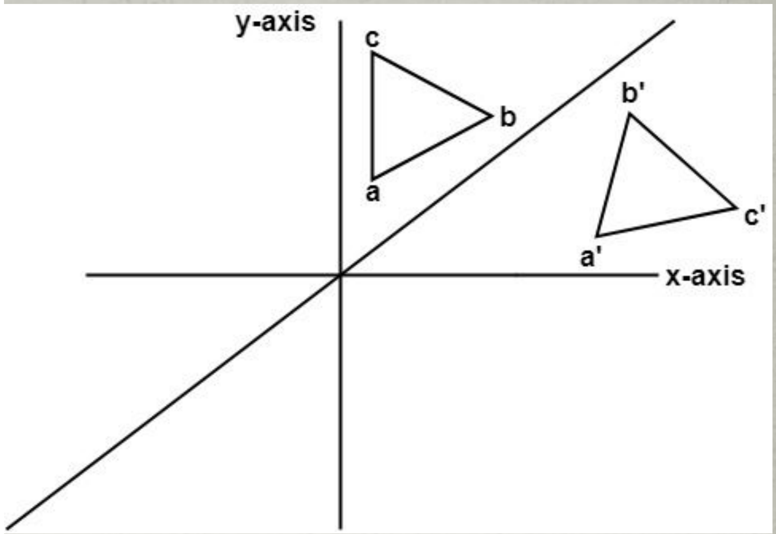
In this value of x and y both will be reversed. This is also called as half revolution about the origin.



Reflection about line y=x

The object may be reflected about line y = x with the help of following transformation matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



First of all, the object is rotated at 45°. The direction of rotation is clockwise. After it reflection is done concerning x-axis. The last step is the rotation of y=x back to its original position that is counterclockwise at 45°.

Thank You