FUZZY LOGIC

Presented By:

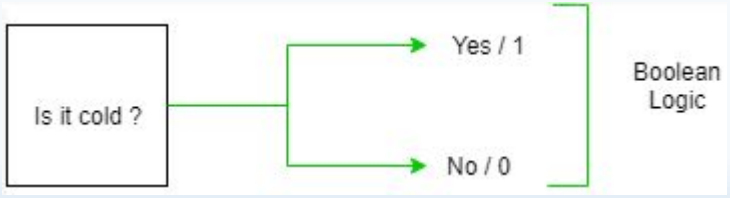
Sahaj Shakya (192919)

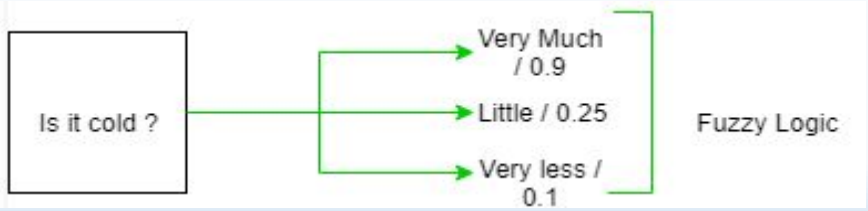
WHAT IS LOGIC?

- Logic, "the reasoning conducted or assessed according to strict principles and validity".
- Artificial Intelligence as an agent (system) to think and act humanly, and for doing so, it should be capable of taking any decision based on the current situation.
- If we talk about normal human behaviour, then a decision is made by choosing an option from the various available options.
- There are reasons behind selecting or rejecting an option. So, our artificial agent should also work in this manner.
- While taking any decision, the agent must provide specific reasons based on which the decision was taken.
- And this reasoning can be done by the agent only if the agent has the capability of understanding the logic.

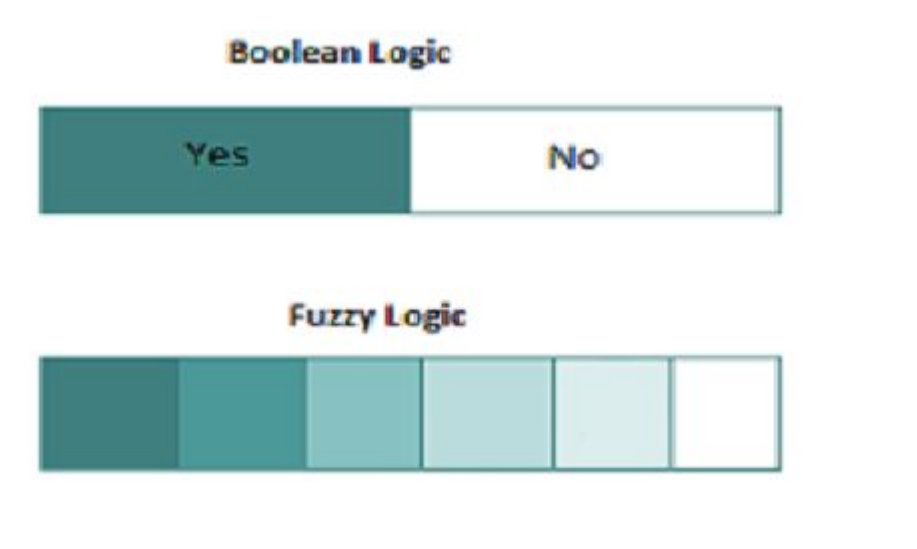
WHAT IS FUZZY LOGIC?

- The term fuzzy mean things which are not very clear or vague.
- In real life, we may come across a situation where we can't decide whether the statement is true or false.
- At that time, fuzzy logic offers very valuable flexibility for reasoning. We can also consider the uncertainties of any situation.
- Fuzzy logic algorithm helps to solve a problem after considering all available data. Then it takes the best possible decision for the given the input.
- The FL method imitates the way of decision making in a human which consider all the possibilities between digital values T and F.
- Fuzzy logic variables may have a truth value that ranges in degree.





Boolean Logic VS Fuzzy logic

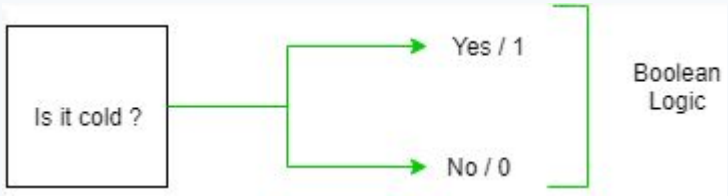


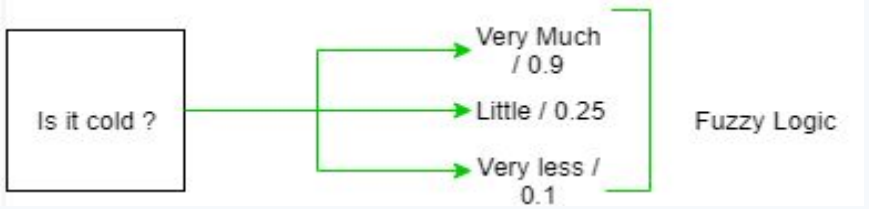
Binary set :

$$\text{High} = \begin{cases} 1 & T > 40° \\ 0 & T \leq 40° \end{cases}$$

Fuzzy set:

$$\text{High} = \begin{cases} 1 & T > 40° \\ T-30/10 & 30° < T \leq 40° \\ 0 & T \leq 30° \end{cases}$$





Boolean Logic VS Fuzzy logic

Fuzzy Set

Slow

Fast

Speed = 0

Speed = 1

```
bool speed;
get the speed
if ( speed == 0) {
        //  speed is slow
}
else {
        //  speed is fast
}
```

Slowest

[0.0 – 0.25]

Slow

[0.25 – 0.50]

Fast

[0.50 – 0.75]

Fastest

[0.75 – 1.00]

```
float speed;
get the speed
if ((speed >= 0.0)&&(speed < 0.25)) {
        //  speed is slowest
}
else if ((speed >= 0.25)&&(speed < 0.5))
{
        //  speed is slow
}
else if ((speed >= 0.5)&&(speed < 0.75))
{
        //  speed is fast
}
else // speed >= 0.75 && speed < 1.0
{
        //  speed is fastest
```

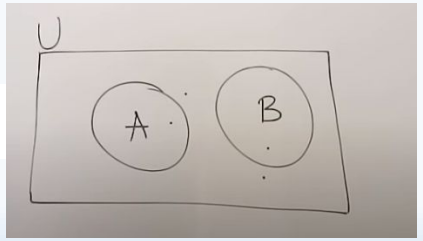
WHY TO USE FUZZY LOGIC?

- Flexible and easy to implement machine learning technique.
- Helps you to mimic the logic of human thought.
- Logic may have two values which represent two possible solutions.
- Highly suitable method for uncertain or approximate reasoning.
- Fuzzy logic views inference as a process of propagating elastic constraints.
- Fuzzy logic allows you to build nonlinear functions of arbitrary complexity.
- Fuzzy logic should be built with the complete guidance of experts.

FUZZY DATA – CRISP DATA

- Fuzzy logic is able to process incomplete data and provide approximate solutions to problems.
- A Crisp data contains either the member of set or not and is for digitally designed i.e. contains two values.
- A Fuzzy data are allowed to be partially included in the set and used for fuzzy controllers i.e. infinite value.
- The crisp set is a collection of objects (say U) having identical properties such as count ability and finiteness. A crisp set 'B' can be defined as a group of elements over the universal set U, where a random element can be a part of B or not. Which means there are only two possible ways, first is the element could belong to set B or it does not belong to set B. The notation to define the crisp set B containing a group of some elements in U having the same property P, is given below.

$$B = \{x : x \in U \text{ and } x \text{ has same property } P\}$$



- Either B will be True or False

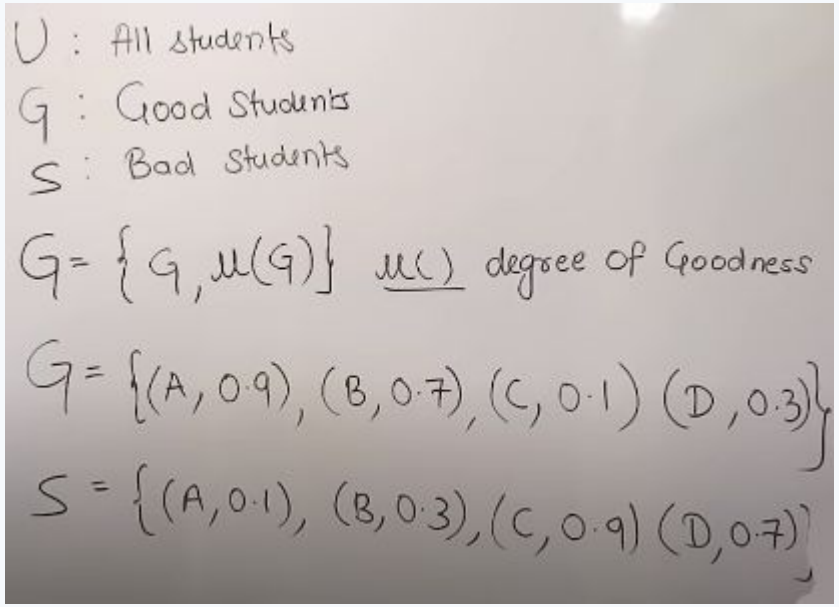
FUZZY DATA – CRISP DATA

- A fuzzy set is denoted by a text having tilde under strike. Now, a fuzzy set X would contain all the possible outcome from interval 0 to 1.
- Suppose a is an element in the universe is a member of fuzzy set X, the function gives the mapping by X(a) = [0,1]. The notion convention used for fuzzy sets when the universe of discourse U (set of input values for the fuzzy set X) is discrete and finite, for fuzzy set X is given by:

$$\mu \underset{\sim}{X}(a) = \in [0,1]$$

$$\underset{\sim}{X} = \{ \frac{\mu_A(a_1)}{a_1} + \frac{\mu_A(a_1)}{a_1} + \ldots\ldots \}$$

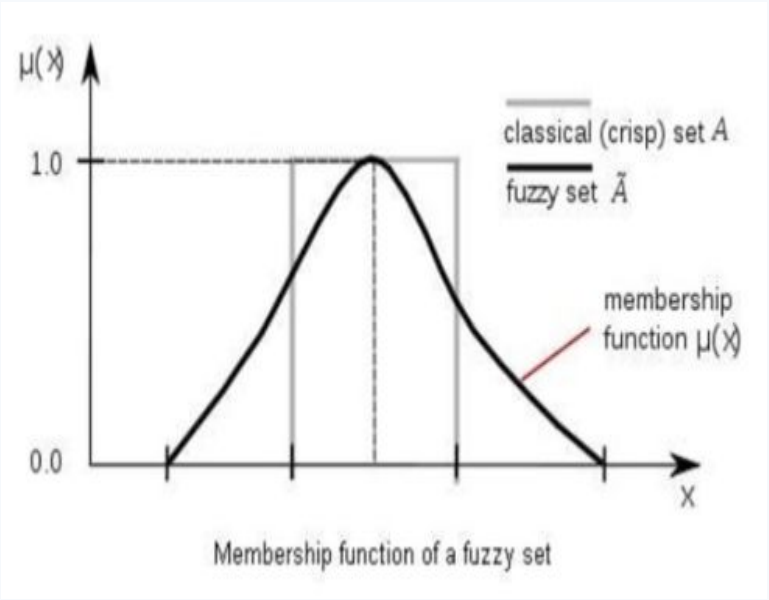
$$= \{ \sum_i \frac{\mu_X(a_i)}{a_i}$$



- Fuzzy logic corresponds to “degree of truth”, while probabilistic logic corresponds to “probability likehoods”, as these differ, fuzzy logic and probabilistic logic yield different models of the same real world situations

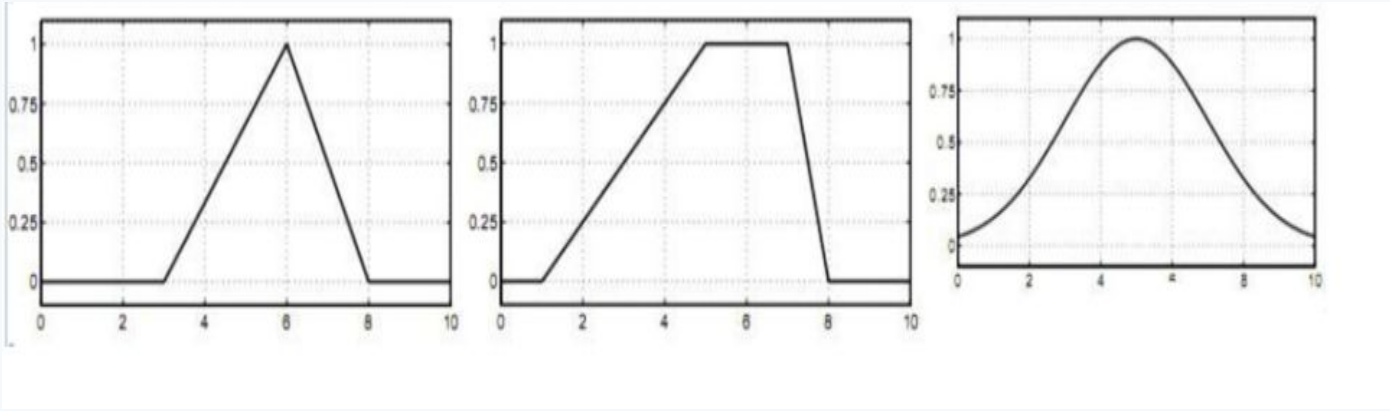
MEMBERSHIP FUNCTION

- A graph that defines how each point in the input space is mapped to membership value between 0 and 1. Input space is often referred as the universe of discourse or universal set (u), which contain all the possible elements of concern in each particular application.
- It is denoted by "μ".



Membership function of a fuzzy set

Types of Membership Function



Triangular Trapezoidal Bellshaped

LINGUISTIC VARIABLE

- It is a variable whose value are in words or in a natural language.
- Mathematical meaning of a linguistic value X is characterized by a compatibility function,

  c: U → [0,1],

  which associates with each u in U its compatibility with X
- In a standard fuzzy partition, each fuzzy set corresponds to a linguistic concept, for instance Very low, Low, Average, High, Very High.
- During reasoning the variables are referred to by the linguistic terms so defined, and the fuzzy sets determine the correspondence with the numerical values.
- Example Height(Small, Medium, Tall)

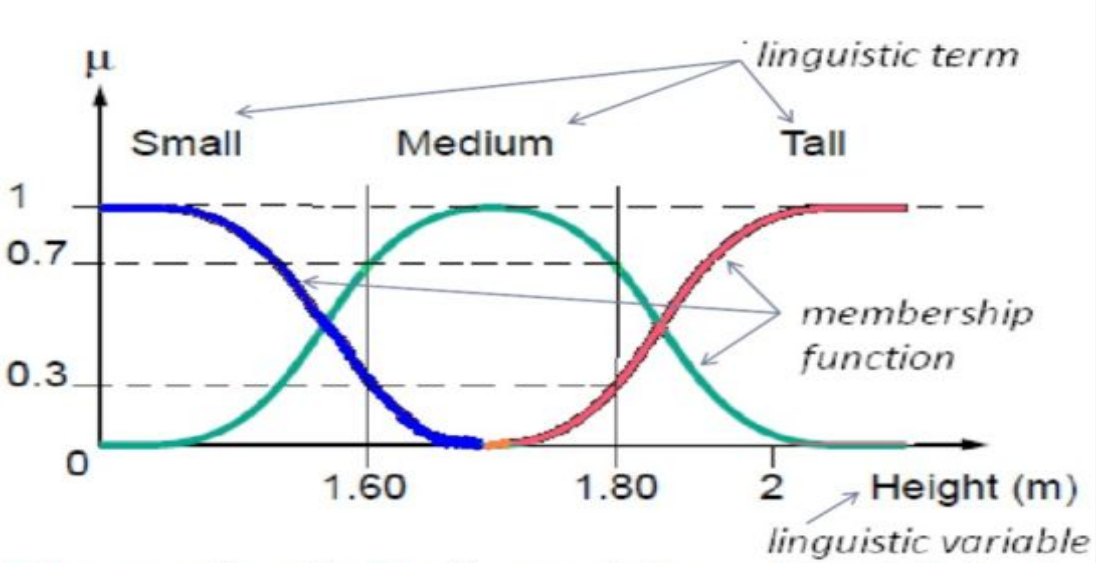
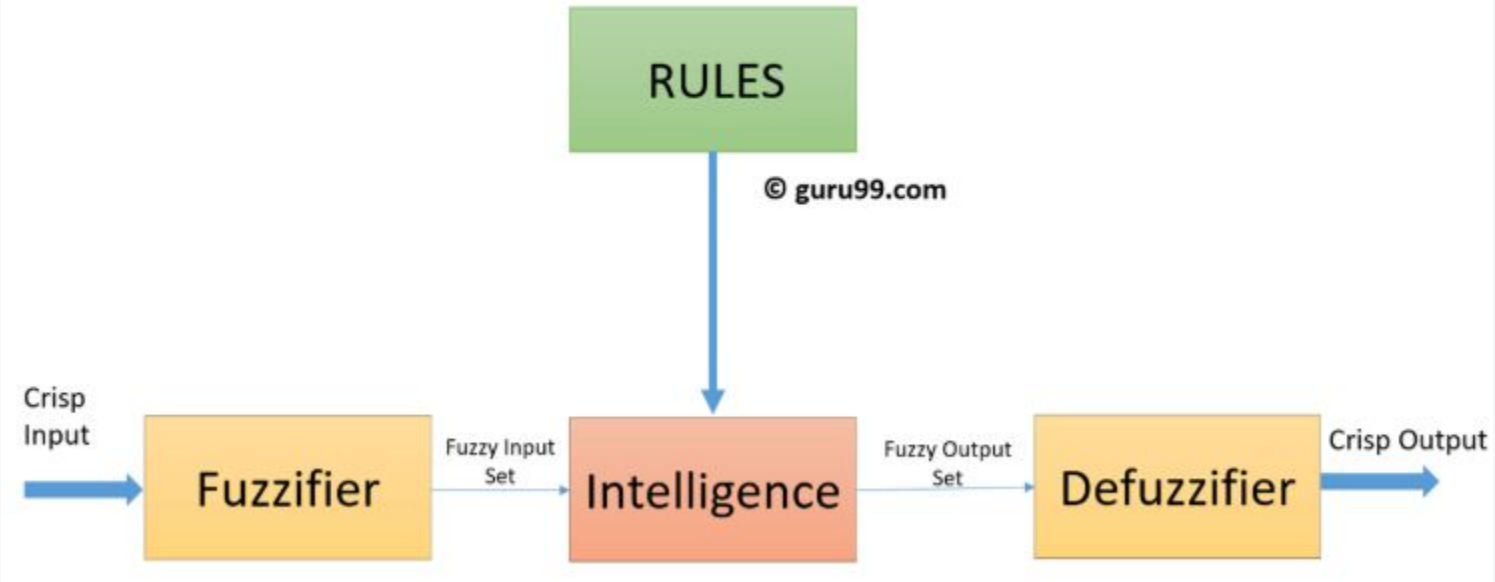


Fig. : membership function, variable and linguistic term.

FUZZY LOGIC ARCHITECTURE



- Fuzzy Logic architecture has four main parts as shown in the diagram:
  - Rule Base
  - Fuzzification
  - Interference Engine
  - Defuzzification:

RULE BASE

- It contains all the rules and the if-then conditions offered by the experts to control the decision-making system.
- The recent update in fuzzy theory provides various methods for the design and tuning of fuzzy controllers.
- This updates significantly reduce the number of the fuzzy set of rules.
- Syntax of Fuzzy Set is given by:

$A = \{x, \mu_A(x) \mid x \; X\}$

Where,
A – Fuzzy Set
x – Elements of X
X – Universe of Discourse
$\mu_A(x)$ – Membership Function of x in A

RULE BASE

- A fuzzy rule is written as **If** situation **Then** conclusion. The situation, called rule premise or antecedent, is defined as a combination of relations such as x is A for each component of the input vector.
- The conclusion part is called consequence or conclusion.
- Operators:
  - IS : the relation x is A is quantified by the membership degree of x to the fuzzy set A.
  - AND : conjunction operator, denoted ^, the most common operators are minimum and product.
  - OR : disjunction operator, the most common are maximum and sum.
- For Example :
  - If x is A then y is B
  - A and B are Linguistic values
  - x is element of Fuzzy set X and y is the element of Fuzzy set Y
- From above example,
  - Antecedent (premise): if part of rule (i.e. x is A)
  - Consequent (Conclusion): the part of rule (i.e. y is B)
  - Antecedent is interpretation and consequent is assignment.

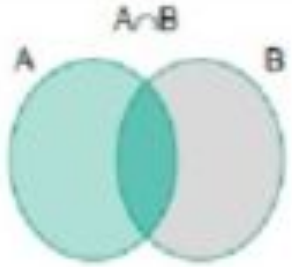
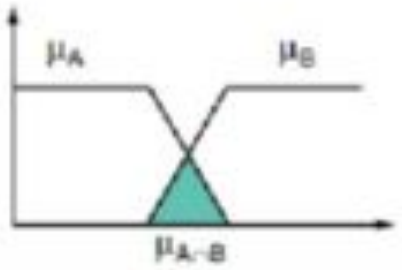
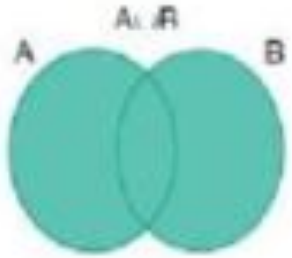
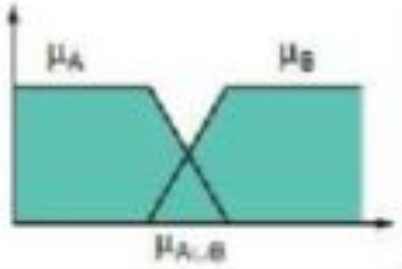
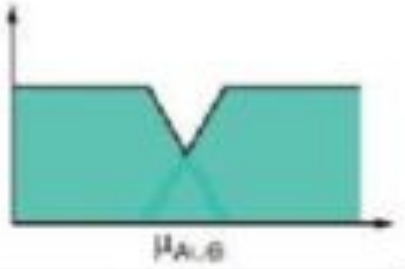
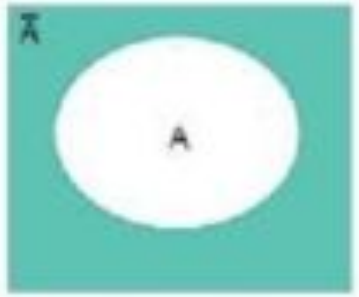
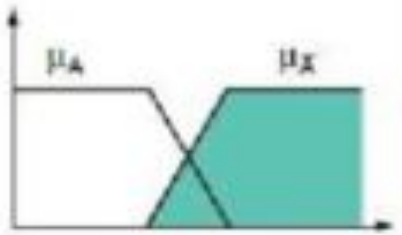
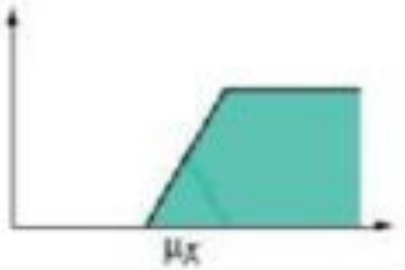
RULE BASE

- Antecedent is combination of proposals by AND, OR, NOT operators
- Consequent is combination of proposals linked by AND operators.
- Example:
  - If it is early, then John can study.
    - Universe: U = {4, 8, 12, 16, 20, 24}; time of the day
    - Input Fuzzy set: early = {(4,0), (8,1), (12,0.9), (16, 0.7), (20, 0.5), (24,0.2)}
    - Output Fuzzy set: can study = μ(study) = 1
    - i.e. At 20 (8 pm), μ(early) = 0.5

Interpreting if-then rule is a three part process.

- **Fuzzify Input**: Resolve all fuzzy statements in the antecedent to a degree of membership between 0 and 1.
- **Apply Fuzzy Operator**: To multiply part antecedents. If multiple part apply fuzzy logical operator and resolve antecedent to a single number between 0 and 1.
- **Apply implication method**: Output fuzzy sets for each rules are aggregated into a single output fuzzy set. Then the resulting output fuzzy set is defuzzified or resolve to a single number.

RULE BASE

		ZADEH operator	Logic operation		
Intersection		$\mu_{A\cap B} = MIN(\mu_A, \mu_B)$	AND		
Union		$\mu_{A\cup B} = MAX(\mu_A, \mu_B)$	OR		
Negation		$\mu_{\bar{A}} = 1 - \mu_A$	NOT		

RULE BASE

- Fuzzy logical Operators are used to write logic combination between fuzzy notations.
- It performs computations on degree of membership.

Zadeh operators

1. Intersections:

   Corresponds to Intersection of sets is AND

$$\mu_{(A \text{ AND } B)} = \text{MIN}(\mu_{(A)}, \mu_{(B)})$$

2. Union:

   Logical operator corresponds to union of set is OR.

$$\mu_{(A \text{ OR } B)} = \text{MAX}(\mu_{(A)}, \mu_{(B)})$$

3. Negation:

   Logical operator corresponds to compliment of set is NOT.

$$\mu_{(\text{NOT } A)} = 1 - \mu_{(A)}$$

RULE BASE

- Let's create a fuzzy control system which models how you might choose to tip at a restaurant. When tipping, you consider the service and food quality, rated between 0 and 10. You use this to leave a tip of between 0 and 25%.
  - Input Fuzzy set (Antecedents) :

    1. Service
    - Universe(crisp value): How good the service was on the scale of 0 to 10
    - Fuzzy set: poor, acceptable, amazing

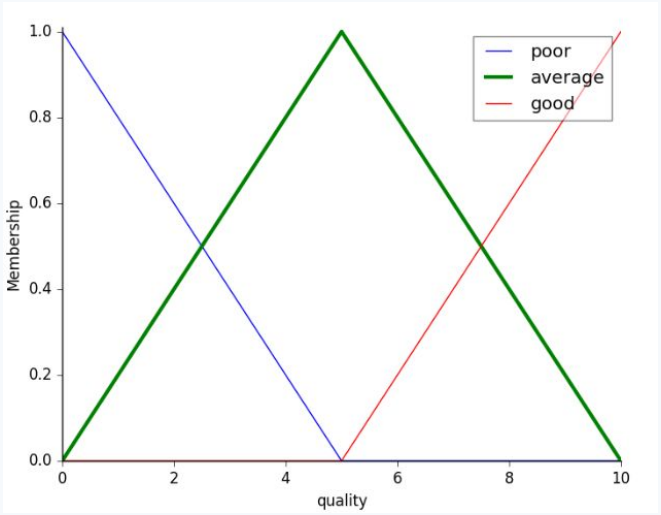
    2. Food Quality
    - Universe(crisp value): How good was food on the scale of 0 to 10
    - Fuzzy set: bad, decent, great

  - Output Fuzzy set (Consequents) : Tip
    - Universe: How much should we tip, on a scale of 0% to 25%
    - Fuzzy set: low, medium, high
  - Rules
    - IF the service was good or the food quality was good, THEN the tip will be high.
    - IF the service was average, THEN the tip will be medium.
    - IF the service was poor and the food quality was poor THEN the tip will be low.

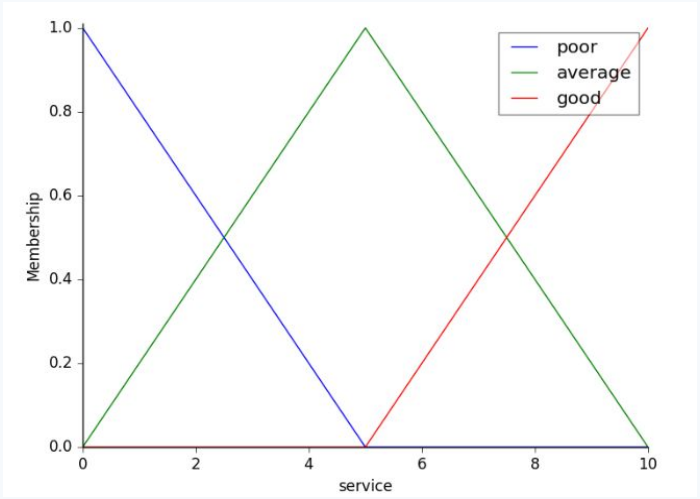
INFERENCE ENGINE

- It helps you to determines the degree of match between fuzzy input and the rules.
- Based on the percentage match, it determines which rules need implement according to the given input field.
- After this, the applied rules are combined to develop the control actions.
- It is the process of formulating the mapping from a given input to an output using fuzzy logic.
- FIS having Multidisciplinary nature are called fuzzy rule based system
- Type of FIS
  - Mamdani type: Expects the output member functions to be fuzzy sets
  - Sugeno type: Outputs member functions are either linear or constant

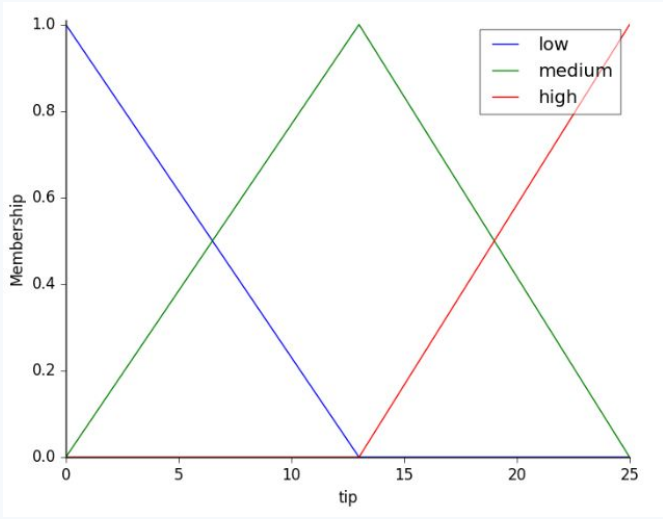
INFERENCE ENGINE



Input: quality

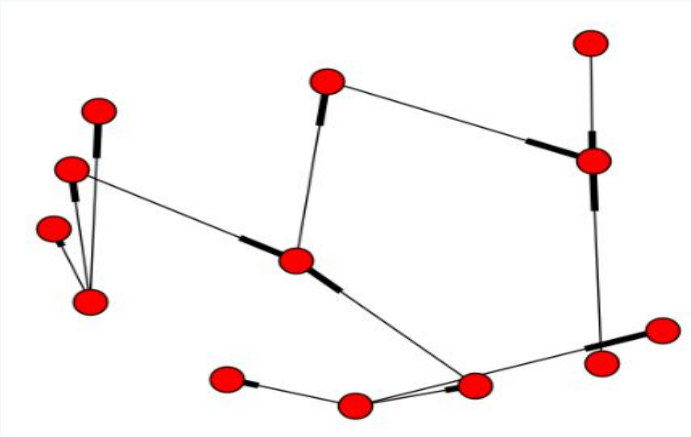


Input: service



Output: Tip

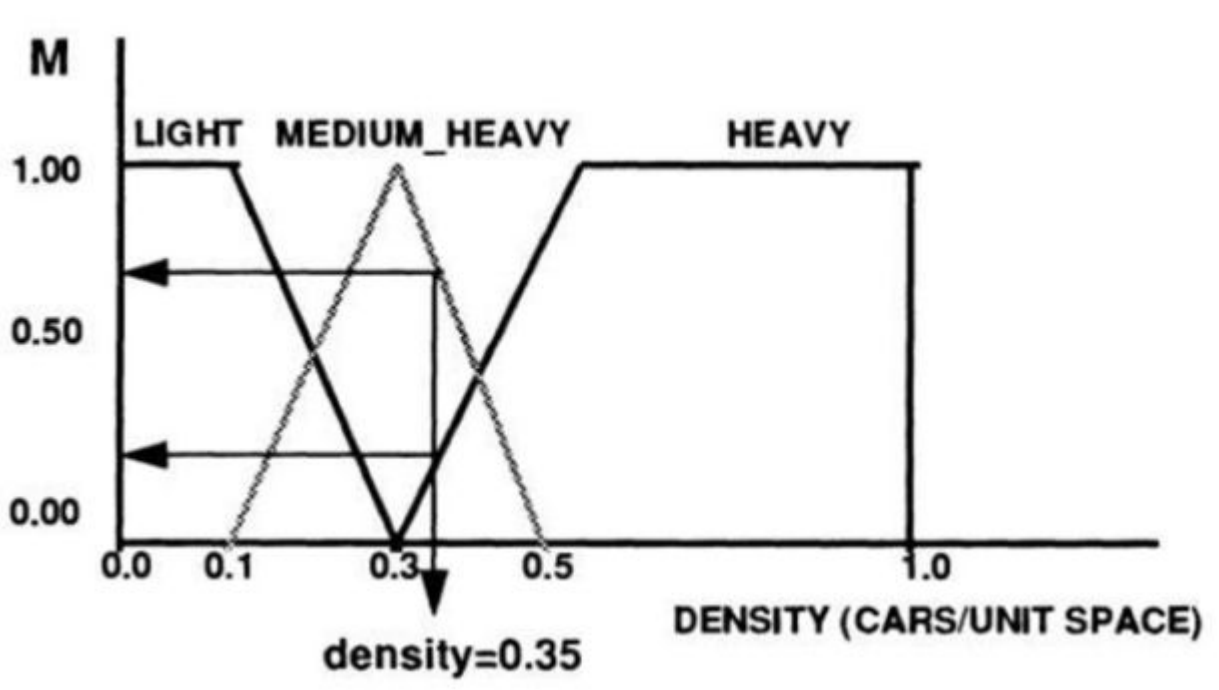
- We define the fuzzy relationship between input and output variables
- Mapping the imprecise rules into a defined, actionable tip is a challenge.
- This is the kind of task at which fuzzy logic excels.



Rule View

FUZZIFICATION

- Fuzzification step helps to convert inputs.
- It allows you to convert, crisp numbers into fuzzy sets.
- Crisp inputs measured by sensors and passed into the control system for further processing. Like services,
- Input values are translated to linguistic concepts, which are represented by fuzzy sets.



FUZZIFICATION

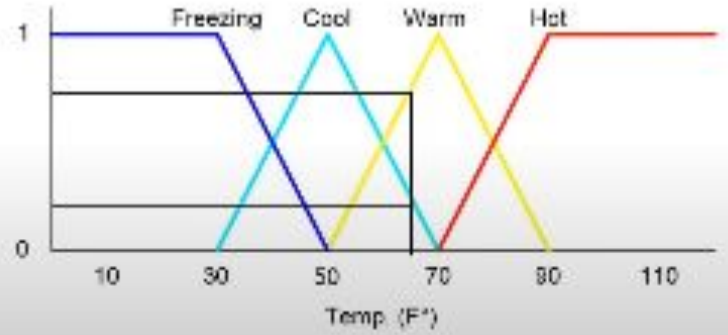
Steps to take in minds

- A fuzzy control system links fuzzy variables using a set of rules.
- These rules are simply mappings that describe how one or more fuzzy variables relates to another.
- These are expressed in terms of an IF-THEN statement; the IF part is called the antecedent and the THEN part is the consequent.
- Example
- If it's sunny and warm, drive Fast
  - Sunny(Cover)^Warm(Temp) => Fast(Speed)
- If it's Cloudy and Cool, drive Slow
  - Cloudy(Cover)^Cool(Temp) => Slow(Speed)
- How fast should I go for
  - 65 F
  - 25 % Cloud Coverage

FUZZIFICATION

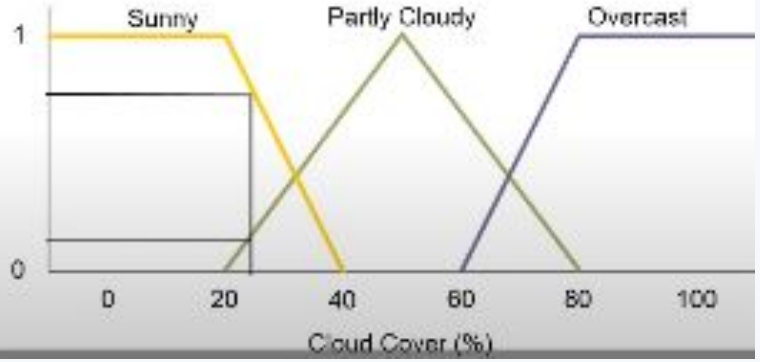
- 65 F°

⇒ Cool = 0.3, Warm= 0.7



- 25% Cover ⇒

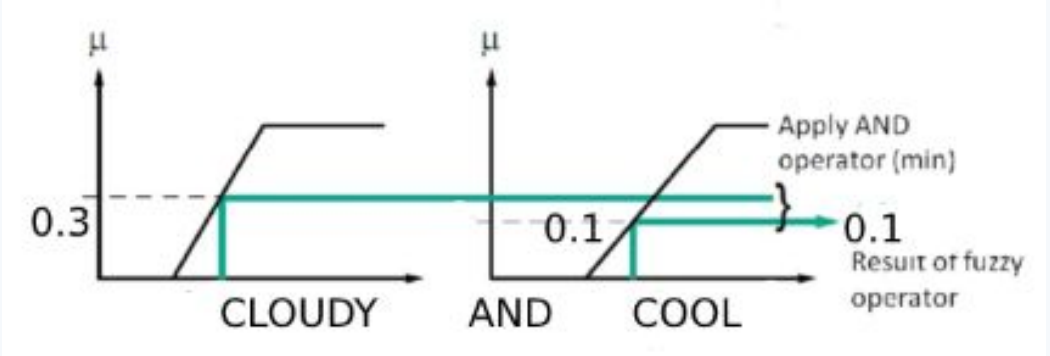
Sunny = 0.7, Cloudy = 0.1



FUZZIFICATION

Steps to take in minds (Fuzzy Operator)

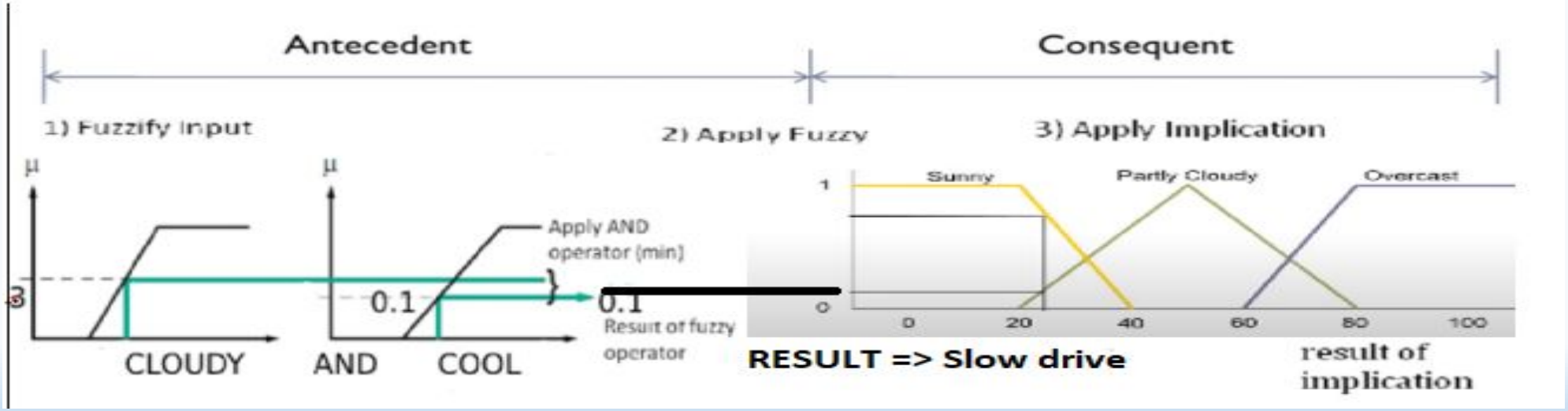
- If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule.
- The input to the fuzzy operator is two or more membership values from fuzzified input variables.
- The output is a single truth table
  - If it's Cloudy and Cool, drive Slow
  - Cloudy(Cover)^Cool(Temp) => Slow(Speed



FUZZIFICATION

Steps to take in minds (Implication Method)

- Operation in which the result of fuzzy operator is used to determine the conclusion of the rule is called as implication,
- First determine the rule's weight.
- Input for the implication is given by antecedent
- Output of the implication is a fuzzy set.
- Apply implication for each rule.

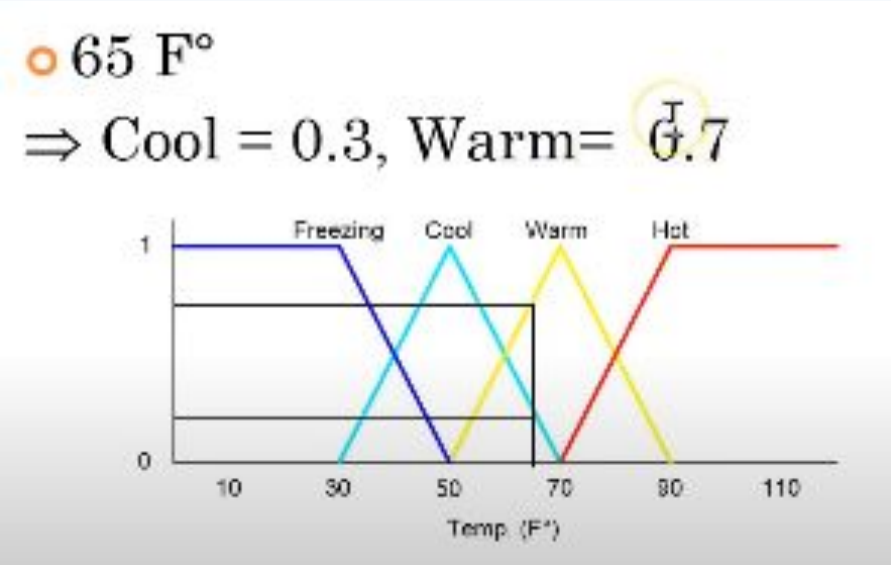


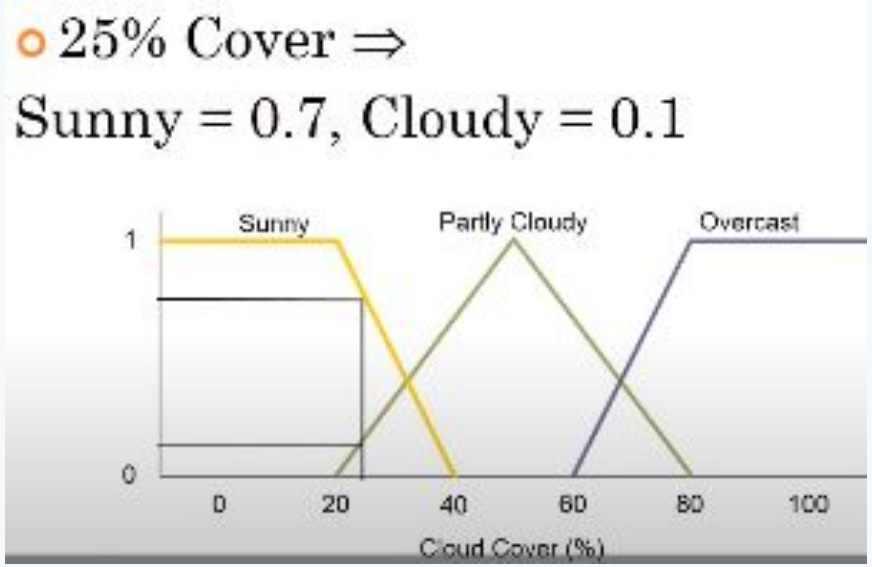
FUZZIFICATION

Steps to take in minds (Aggregate All Outputs)

- Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into single fuzzy set.
- Aggregation only occurs once for each output variable.
- The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule.
- The output of the aggregation process is one fuzzy set for each output variable.

FUZZIFICATION





- If it's sunny and warm, drive Fast
  - Sunny(Cover)^Warm(Temp) => Fast(Speed)
  - 0.7 ^ 0.7 = min(0,7m0.7) = 0.7 □ Fast = 0.7 (70 % Faster)
- If it's Cloudy and Cool, drive Slow
  - Cloudy(Cover)^Cool(Temp) => Slow(Speed)
  - 0.1 ^ 0.3 = min(0.1, 0.3) => 0.1 □ Slow = 0.1 (10 % Slower)

FUZZIFICATION

Aggregate all Outputs

- With the activity of each output membership function known, all output membership functions must be combined. This is typically done using a maximum operator.
- In the tipping example, rule might be

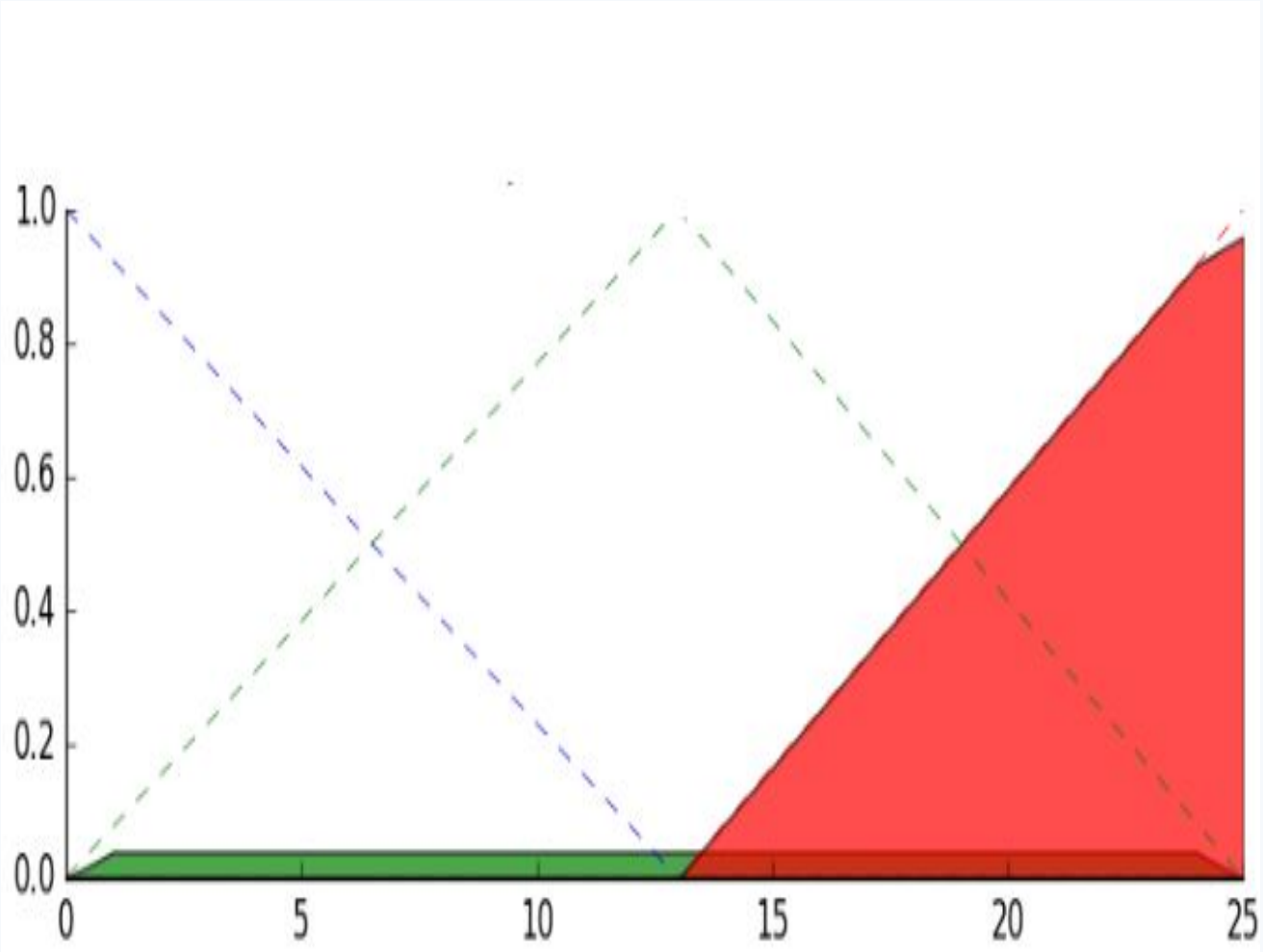
"IF the service was good or the food quality was good, THEN the tip will be high."

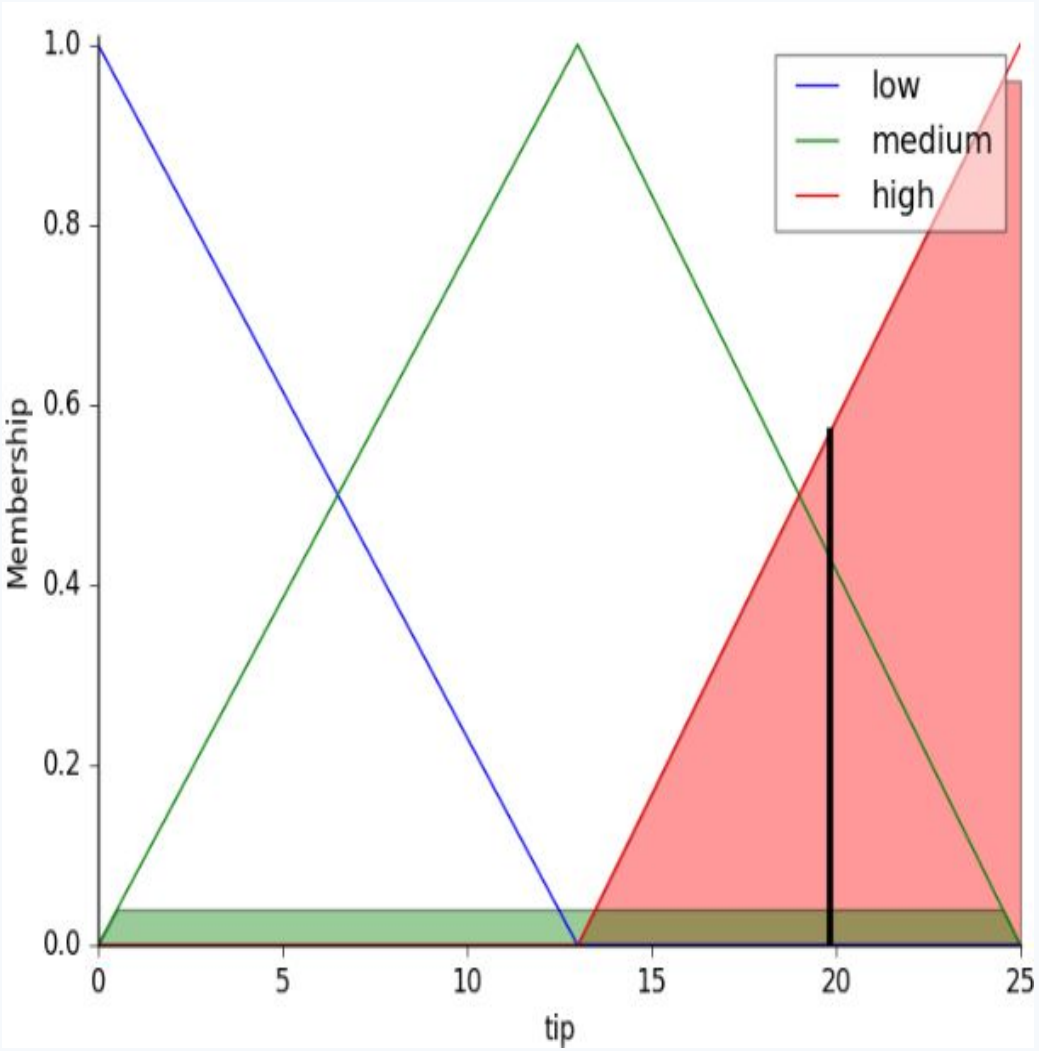
μ(service)= good or μ(food) = good => max(service, food) => good tip

"IF the service was poor and the food quality was poor THEN the tip will be low."

μ(service)= poor ^ μ(food) = poor=> min(service, food) => poor tip

FUZZIFICATION



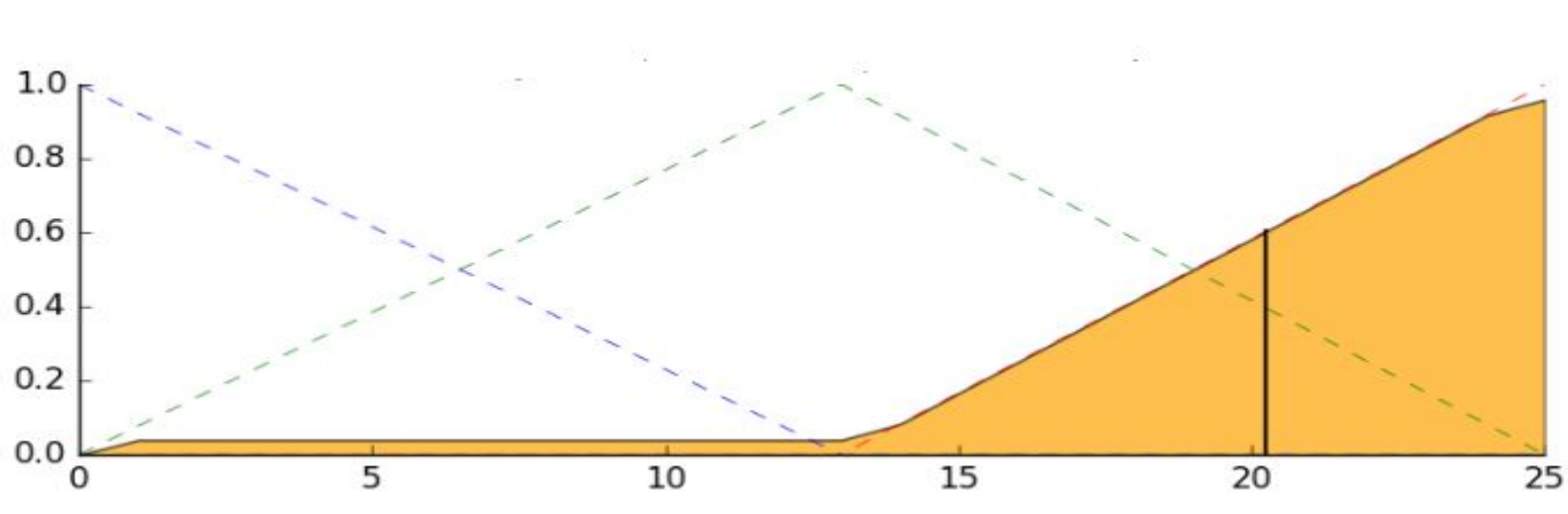


DEFUZZIFICATION

- At last the Defuzzification process is performed to convert the fuzzy sets into a crisp value
- There are many types of techniques available, so you need to select it which is best suited when it is used with an expert system.
- Move from the “fuzzy world” to the “real world” is known as Defuzzification .
- The input for the Defuzzification process is a fuzzy set.
- The output is a single number.
- Methods of Defuzzification methods are
  - Centroid Calculation
  - Bisector
  - Middle of maximum

DEFUZZIFICATION

- Finally, to get a real world answer, we return to crisp logic from the world of fuzzy membership functions.
- For the purposes of this example the centroid method will be used.
- The result is a tip of 20.2%.

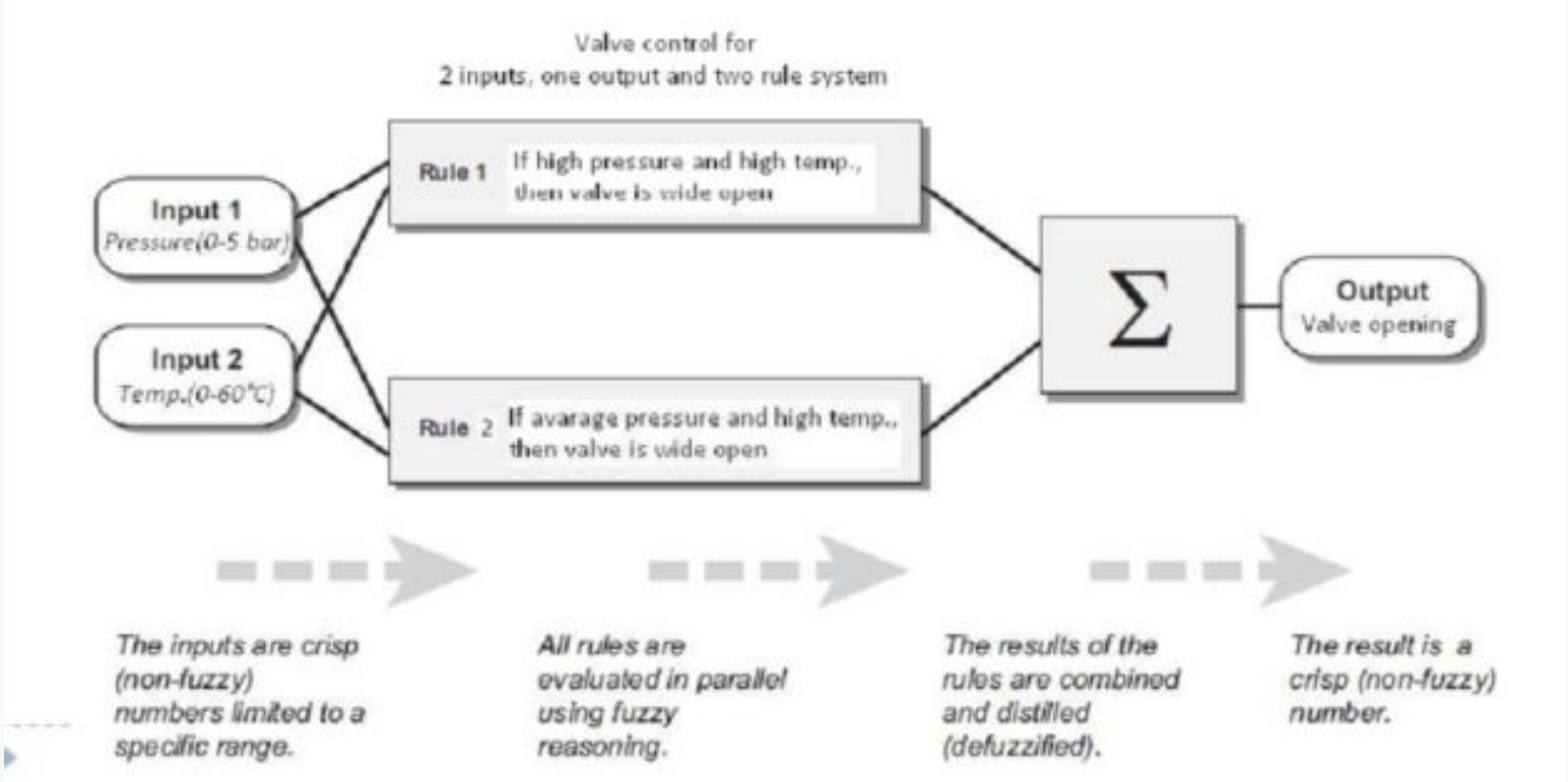


WHEN NOT TO USE FUZZY LOGIC?

- However, fuzzy logic is never a cure for all. Therefore, it is equally important to understand that where we should not use fuzzy logic.
- Here, are certain situations when you better not use Fuzzy Logic:
  - If you don't find it convenient to map an input space to an output space.
  - Fuzzy logic should not be used when you can use common sense.
  - Many controllers can do the fine job without the use of fuzzy logic.
- Disadvantages of Fuzzy Logic:
  - Many researchers proposed different ways to solve a given problem through fuzzy logic which lead to ambiguity.
  - There is no systematic approach to solve a given problem through fuzzy logic. Proof of its characteristics is difficult or impossible in most cases because every time we do not get mathematical description of our approach.
  - As fuzzy logic works on precise as well as imprecise data so most of the time accuracy is compromised.
  - Fuzzy systems don't have the capability of machine learning as-well-as neural network type pattern recognition.
  - Validation and Verification of a fuzzy knowledge-based system needs extensive testing with hardware.
  - Setting exact, fuzzy rules and, membership functions is a difficult task Some fuzzy time logic is confused with probability theory and the terms

IMPLEMENTATION OF FUZZY LOGIC

1. Industrial Automation for valve control based on pressure and Temperature.



IMPLEMENTATION OF FUZZY LOGIC

Step 1: Rule based

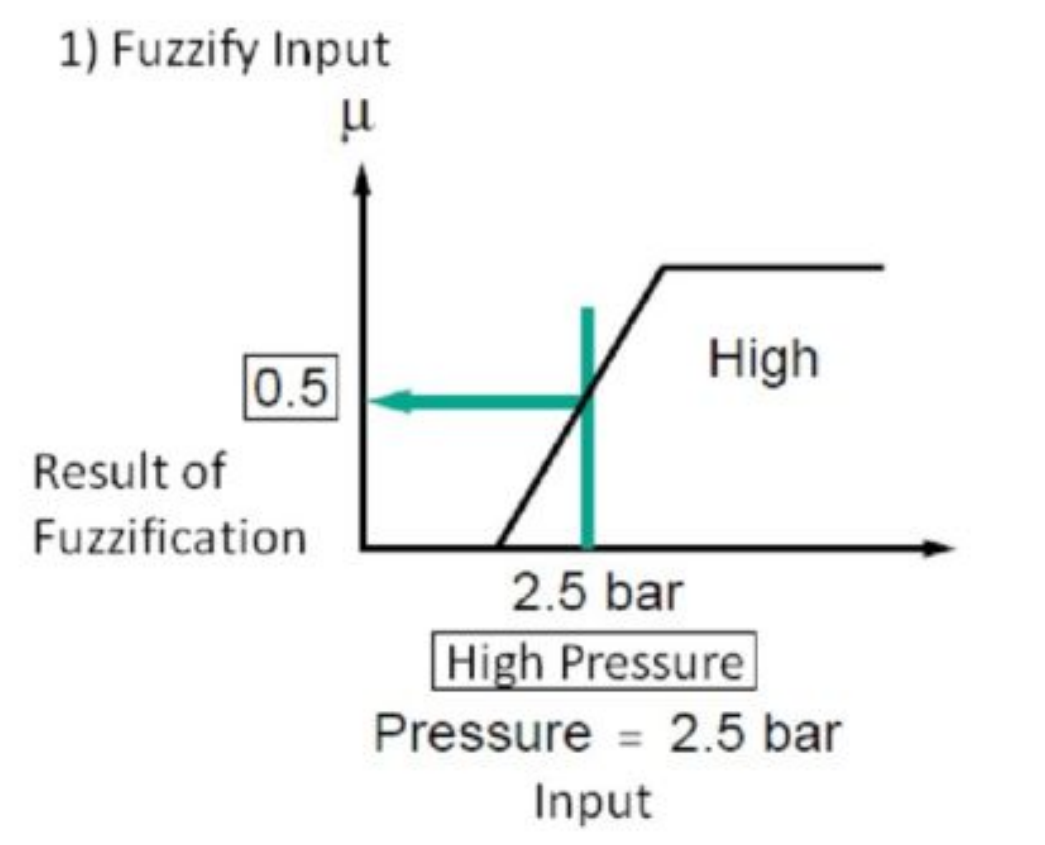
1. **Input 1: Pressure**
    - Universe(crisp value): Pressure value ranging from 0 to 5
    - Fuzzy set: High, Average
2. **Input 2: Temperature**
    - Universe(crisp value): Temperature form 0 to 60
    - Fuzzy set: High, Average

- Output Fuzzy set (Consequents) : Valve
    - Universe: Valve opening
    - Fuzzy set: Wide, Average
- Rules
    - IF high pressure and high temperature then valve is wide open
    - IF average pressure and high temperature then valve us wide open.

IMPLEMENTATION OF FUZZY LOGIC

Step 2: Fuzzify Input

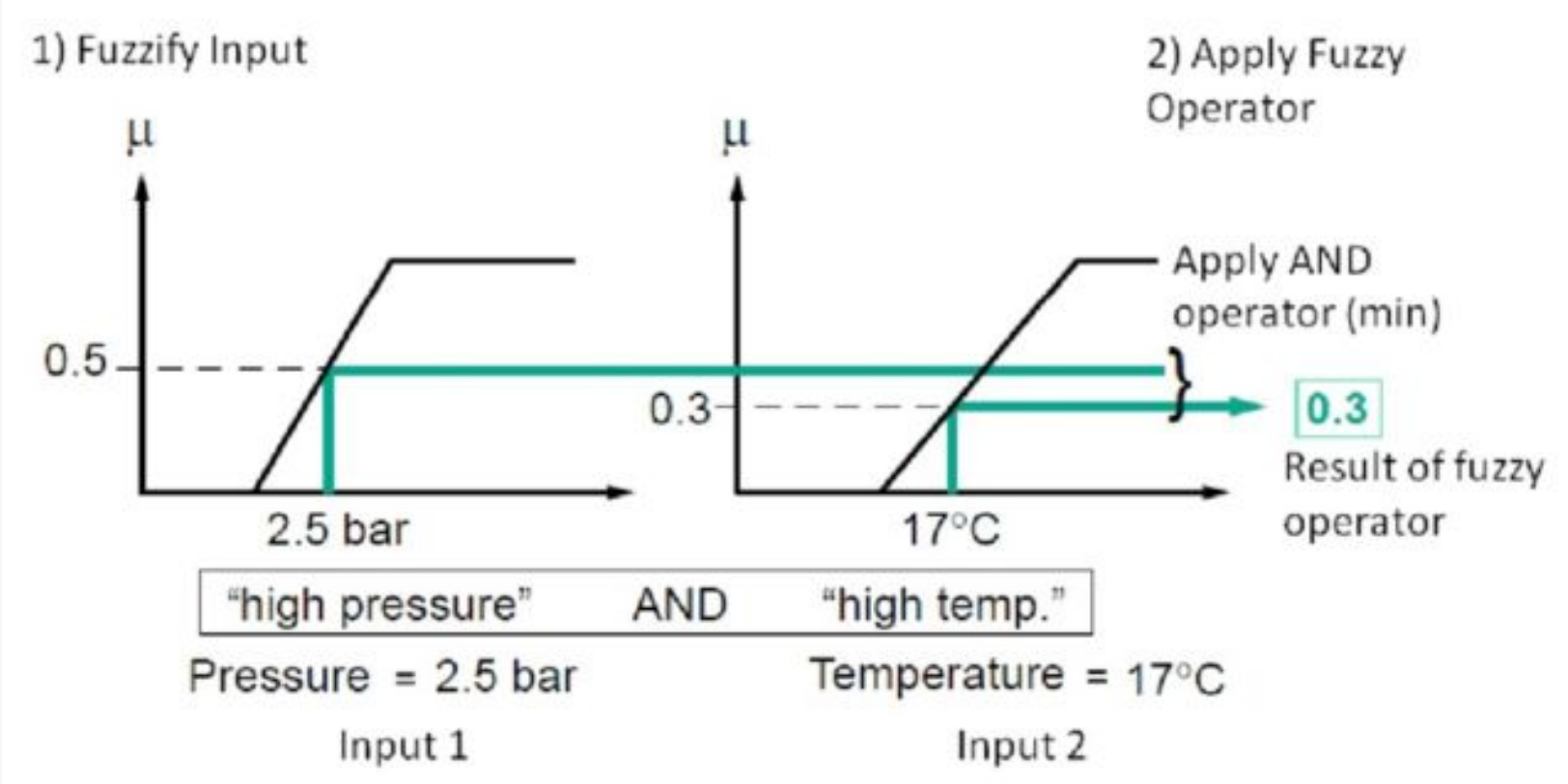
- Input Fuzzy set (Antecedents) :



IMPLEMENTATION OF FUZZY LOGIC

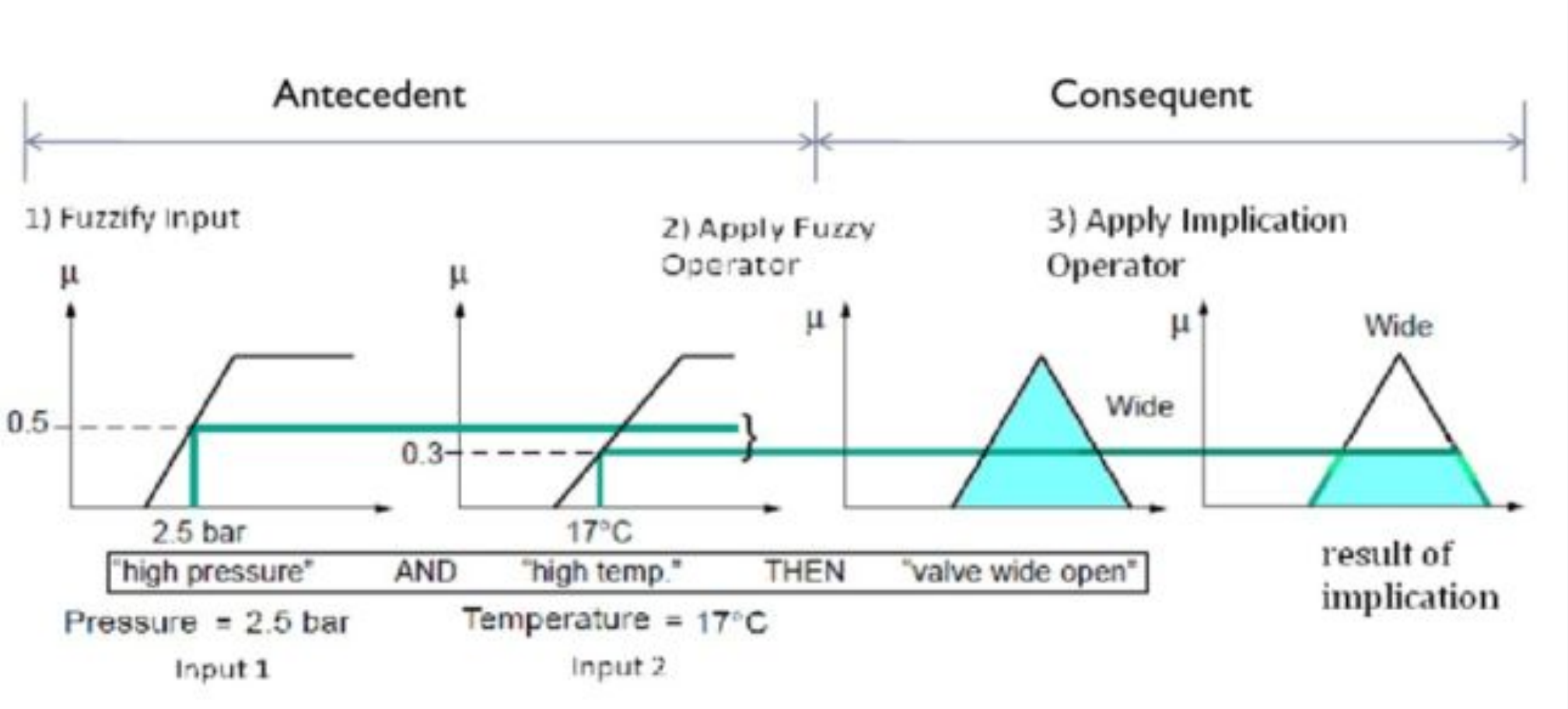
Step 3: Apply Fuzzy Operator

- Input Fuzzy Input
- Result : Fuzzy operator



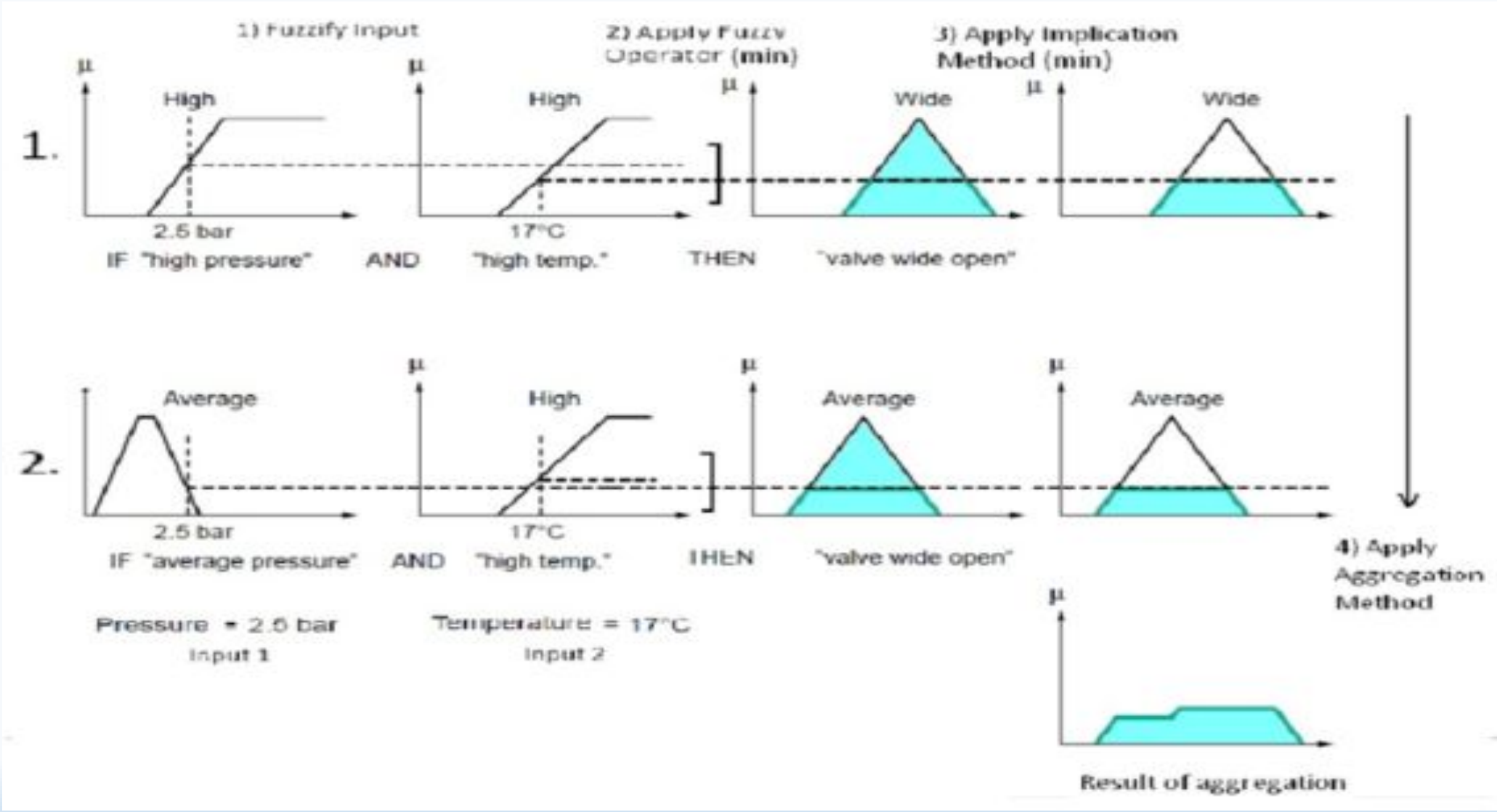
IMPLEMENTATION OF FUZZY LOGIC

Step 4: Apply Implication Method



IMPLEMENTATION OF FUZZY LOGIC

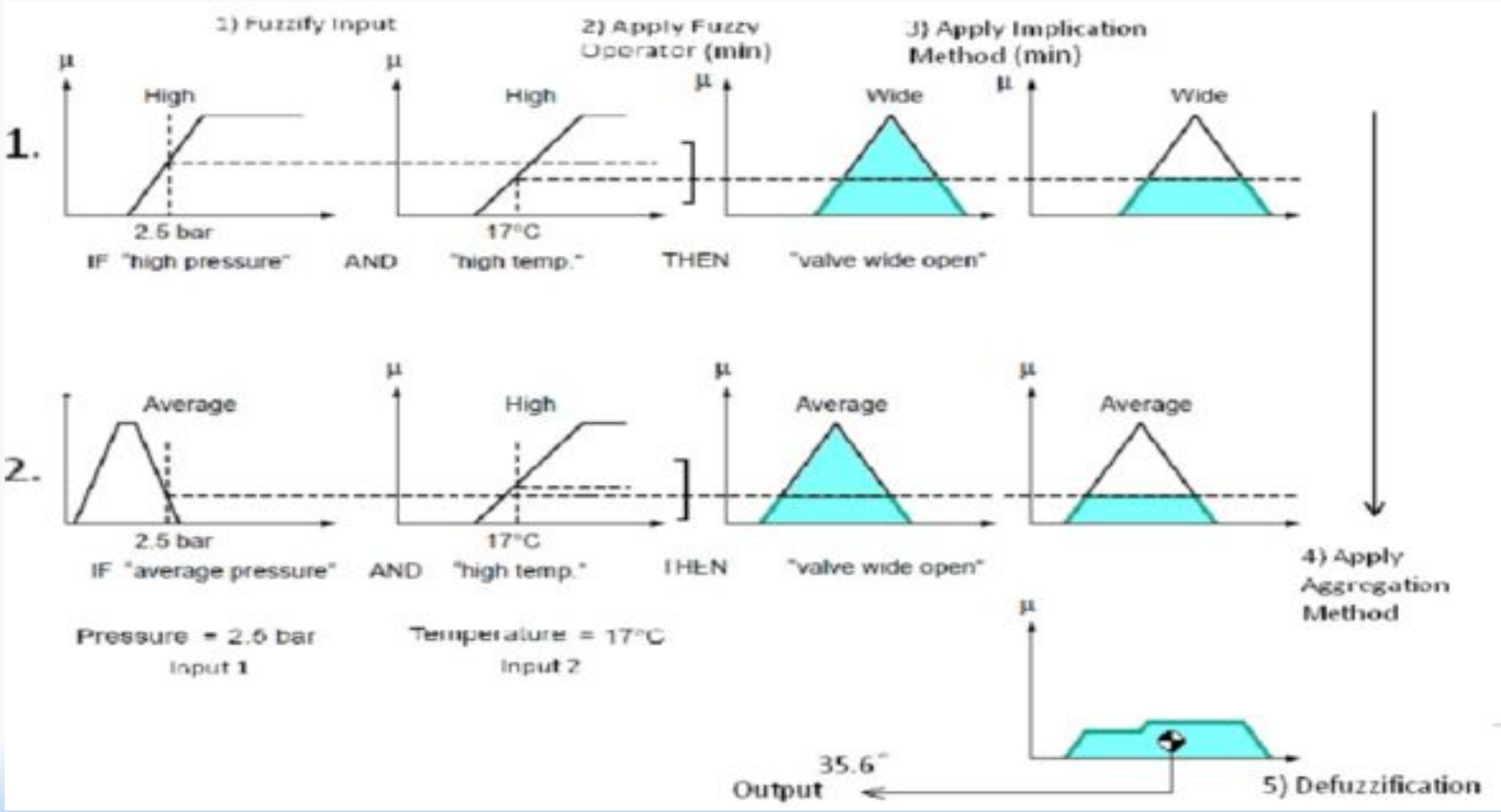
Step 5: Aggregate All Outputs



IMPLEMENTATION OF FUZZY LOGIC

Step 5: Defuzzify

- Method: Centroid Calculation



IMPLEMENTATION OF FUZZY LOGIC

2. Speed controllers

Step 1: Rule based

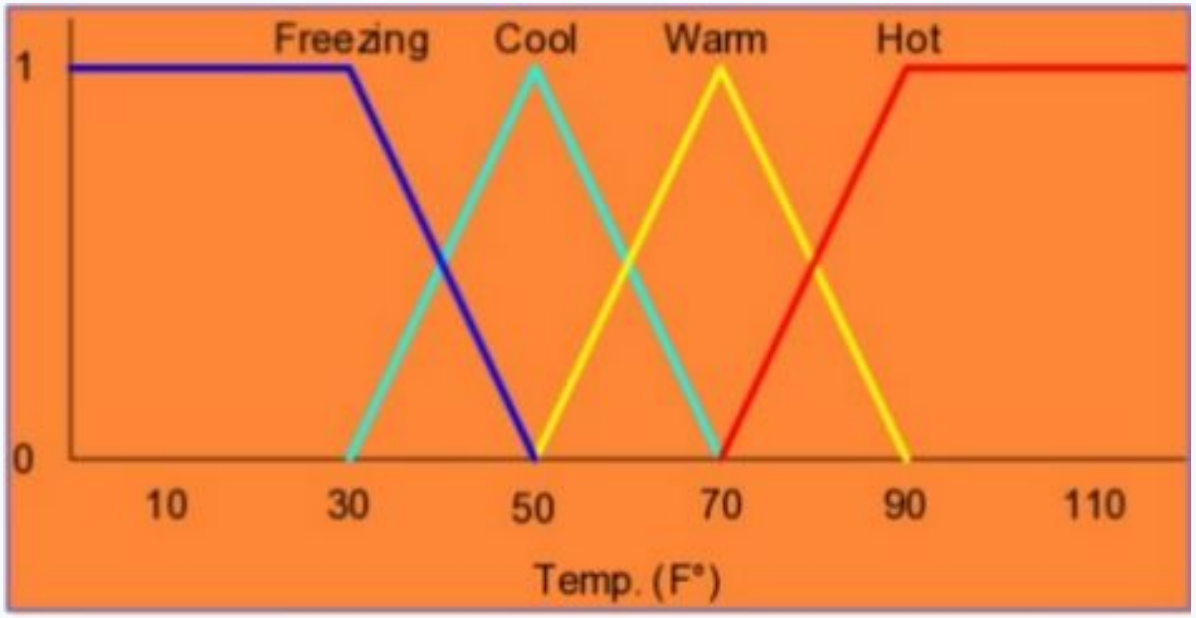
1. **Input 1: Cloud Coverage**
    - Universe(crisp value): Pressure value ranging from 0 to 100
    - Fuzzy set: Overcast, partly cloudy, sunny
2. **Input 2: Temperature**
    - Universe(crisp value): Temperature form 0 to 1
    - Fuzzy set: Freezing, cool, warm, hot

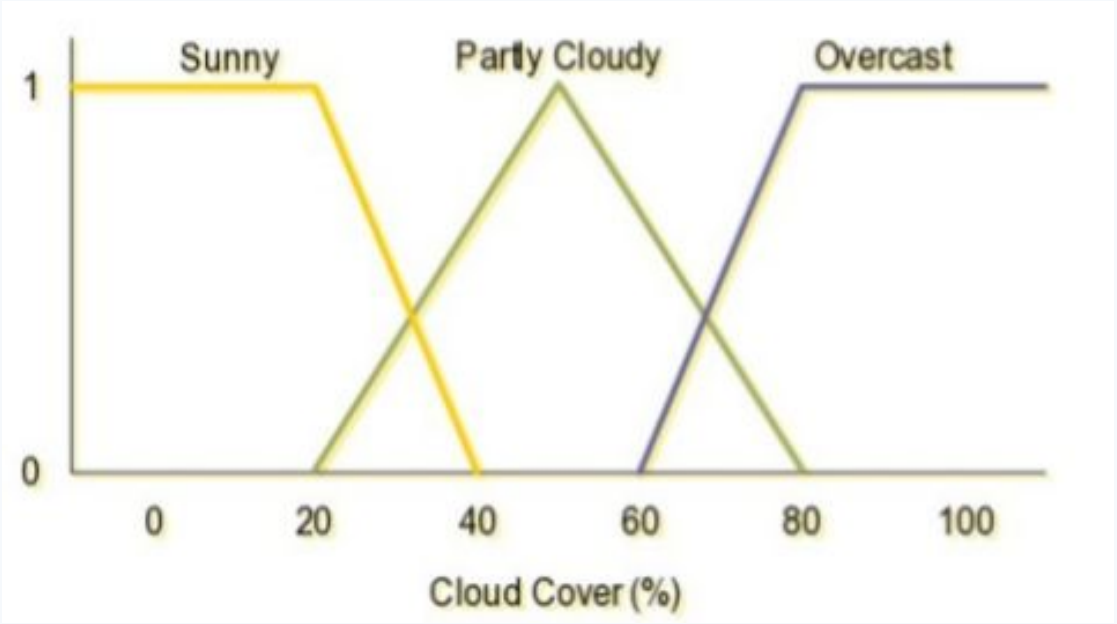
- Output Fuzzy set (Consequents) : Speed
    - Universe: Speed control
    - Fuzzy set: slow, fast
- Rules
    - IF it's sunny and warm then drive Fast.
        - Sunny(Cover)^Warm(Temp) => Fast(Speed)
    - IF it's Cloudy and cool then drive Slow.
        - Cloudy*cover)^Cool(Temp) => Slow(speed)

IMPLEMENTATION OF FUZZY LOGIC

Step 2: Fuzzify Input

- Input Fuzzy set (Antecedents) :



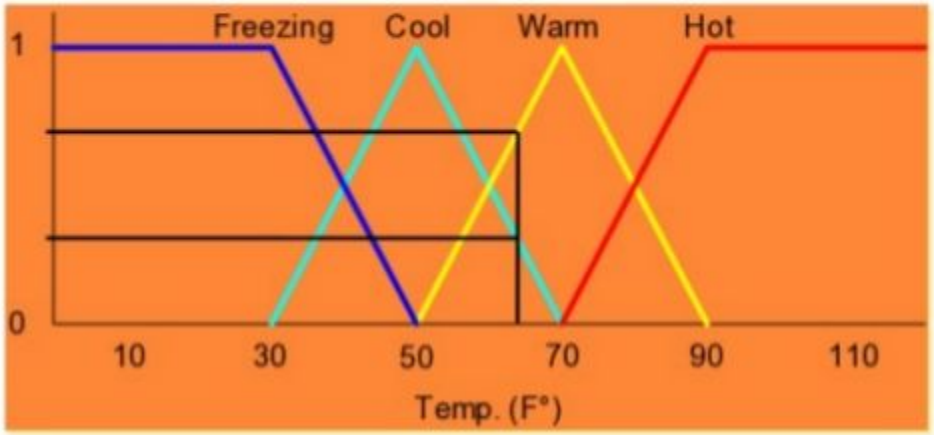


IMPLEMENTATION OF FUZZY LOGIC

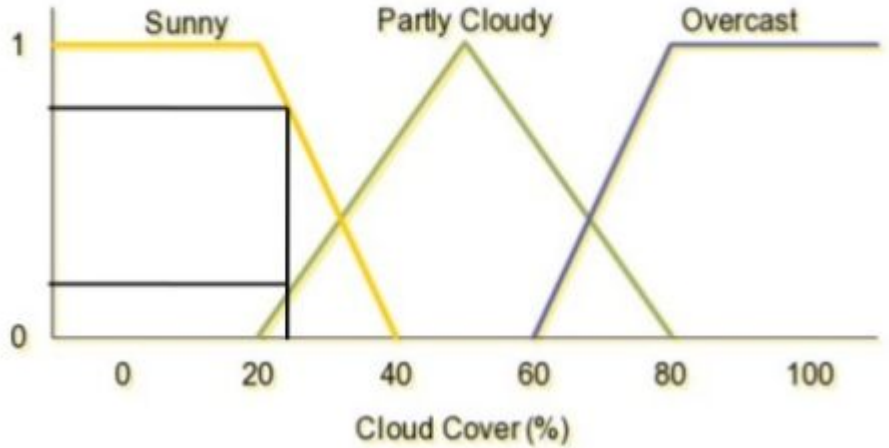
Step 3: Apply Fuzzy Operator

- Input Fuzzy Input
- Result : Fuzzy operator

65 F° ⇒ Cool = 0.4, Warm= 0.7

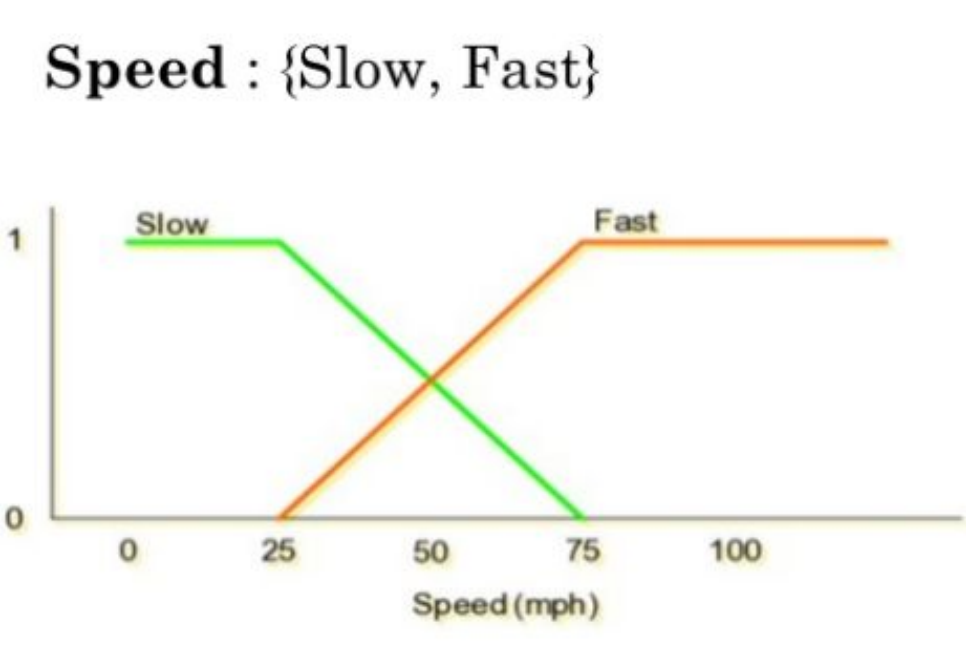


25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



IMPLEMENTATION OF FUZZY LOGIC

Step 4: Apply Implication Method



Step 5: Aggregate All Outputs

IMPLEMENTATION OF FUZZY LOGIC

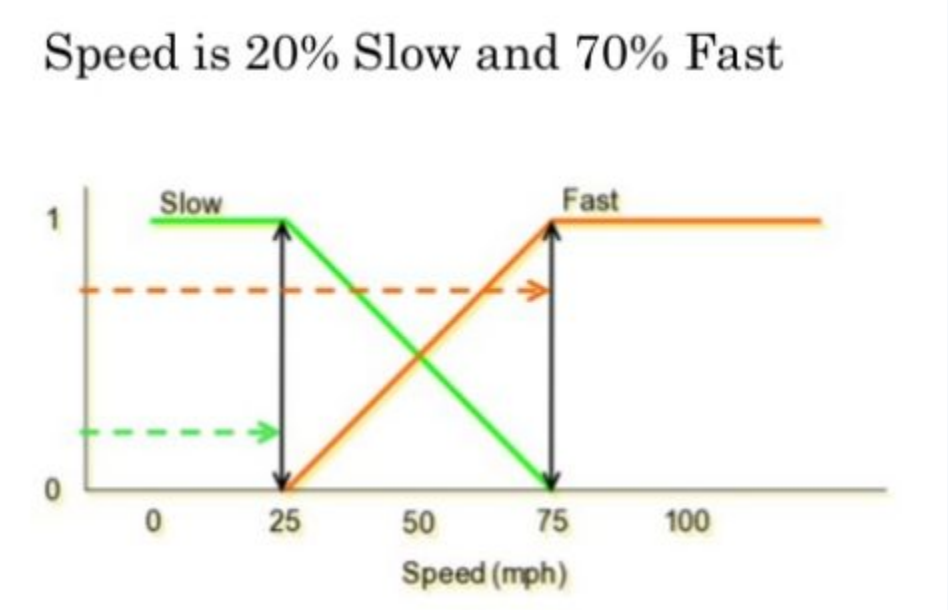
- Step 6 : Defuzzify
- Weighted Calculations:
- Speed = weighted mean

  = (20*25 + 70 *75)/(90)

  = (5750)/(90)

  = 63.889

- Speed is approximately 63.8 mph



Fuzzy Logic VS Neural Network

- Reasoning methodology that resembles the human decision making and deals with vague and imprecise information
- Helps to perform pattern recognition and classification tasks.
- Simpler Neural Network
- Creates behavioral system

- System which is inspired by biological neurons in the human brain that can perform computing tasks faster.
- Helps to perform prediction, recognition and classification tasks
- Complex than fuzzy
- Creates behavioral system

SUMMARY

- The term fuzzy mean things which are not very clear or vague.
- Fuzzy logic is a flexible and easy to implement machine learning technique.
- Fuzzy logic should not be used when you can use common sense.
- Fuzzy Logic architecture has four main parts
  - 1) Rule Base 2) Fuzzification 3) Inference Engine 4) Defuzzification
- Fuzzy logic takes truth degrees as a mathematical basis on the model of the vagueness while probability is a mathematical model of ignorance.
- Crisp set has strict boundary T or F while Fuzzy boundary with a degree of membership.
- A classical set is widely used in digital system design while fuzzy set Used only in fuzzy controllers.
- Auto transmission, Fitness management, Golf diagnostic system, Dishwasher, Copy machine are some applications areas of fuzzy logic.
- Fuzzy logic helps you to control machines and consumer products

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