MATLAB Assignment 3

Continuous and Discrete Wavelet Transforms

- **This is an individual assignment.**
- **Use data files (.mat) that are attached as instructed throughout the assignment.**
- **Submission guidelines**

Submission document	**Submission method**	**Notes**
Report	Upload the softcopy to Moodle	Should include observations and discussions with relevant plots to support your answers.
MATLAB scripts	Upload a single ZIP file including all the .m files to Moodle	Name each script according to the question number. Also, include necessary comments on the scripts for better read-ability.

Introduction

This assignment allows,

- Basic implementation of continuous and discrete wavelet transforms
- Use of built-in MATLAB functions for wavelet transforms
- Denoising and compression using wavelet techniques

I. Continuous Wavelet Transform

1.1. Introduction

Continuous wavelet transform is defined by the following equation.

$$W(s,\tau) = \int x(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

$$where\ s = scaling\ factor, \qquad \tau = translation\ \ and\ \ \psi\ = wavelet\ function$$

There are many wavelet families defined such as Haar, Shannon, Mexican hat, Morlet, Daubechies, etc. depending on the application. Three of such wavelets are shown in Figure 1.

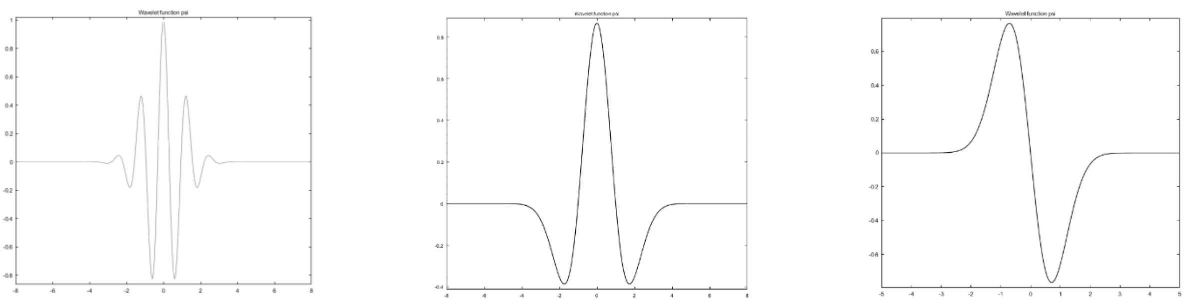

Figure 1.a. Morlet Wavelet

Figure 1.b. Mexican Hat Wavelet

Figure 1.c. 1st derivative of Gaussian Wavelet

Figure 1: Wavelets

In this section, you will construct the Mexican hat mother wavelet to check wavelet properties and then implement CWT on a non-stationary signal.

and scaled versions and apply CWT to generate wavelet coefficients. Mexican hat wavelet are given by the following equations.

1.2.Wavelet properties

i. Given the Gaussian function $g(t)$, derive the Mexican hat function $m(t)$

$$m(t) = -\frac{d^2}{dt^2} g(t)$$

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

Where $\mu = 0$ and $\sigma = 1$.

ii. Calculate the normalizing factor of $m(t)$ such that the energy E:

$$E = \int_{-\infty}^{\infty} m^2(t)\, dt = 1$$

iii. Hence, write the normalized Mexican hat mother wavelet $\psi(t)$. Include the scaling factor (s) in the generic wavelet function.

iv. Using the provided script *wavelet_construction.m*, generate the Mexican hat daughter wavelet for scaling factors of 0.01:0.1:2. Report the time-domain waveforms.

v. Verify the wavelet properties of zero mean, unity energy and compact support (observe) for each of the above daughter wavelets.

vi. Using the same script, plot and comment the spectra of daughter wavelets.

1.3. Continuous Wavelet Decomposition

i. Create a waveform on MATLAB as defined below with the following parameters.

$$x[n] = \begin{cases} sin(0.5\pi n), & 1 \le n < \frac{3N}{2} \\ sin(1.5\pi n), & \frac{3N}{2} \le n < 3N \end{cases}$$

Sampling Frequency = 250 Hz

ii. Apply the scaled Mexican hat wavelets to $x(n)$ with the following specifications. To achieve translations, for each wavelet scale, convolve the signal with the constructed wavelet. Note: increase the scale resolution to 0.01:0.01:2.

iii. To visualized the spectrogram, plot the derived coefficients using the `pcolor()` command. The spectrogram should look similar to Figure 2.

iv. Comment on the plot regarding the continuous wavelet coefficients representing the frequency content of $x(n)$.

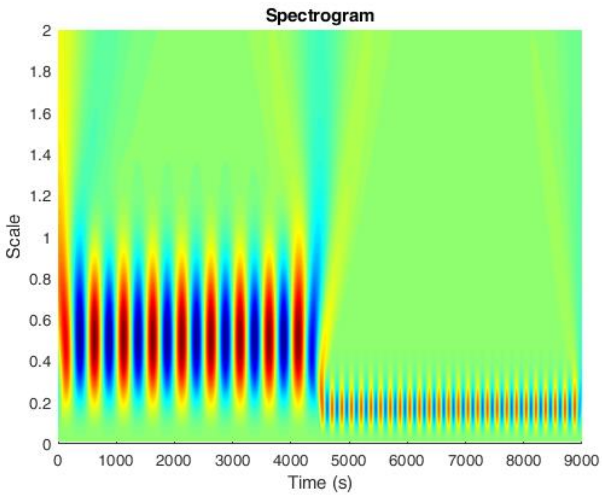

Figure 2: Spectrogram from CWT

II. Discrete Wavelet Transform

2.1. Introduction

The drawbacks of CWT include highly redundant computations which leads to the requirement of additional computational power and time consumption. Avoiding this, in discrete wavelet transform (DWT), the scaling and translation are performed in a discrete manner.

For DWT, the equation for CWT is modified as follows.

$$\psi_{m,n}(t) = \frac{1}{\sqrt{s_0^m}}\psi\left(\frac{t - n\tau_0 s_0^m}{s_0^m}\right)$$

$$s_0 = scaling\ step\ size, \qquad \tau_0 = translation\ step\ size$$

Usually $s_0 = 2$ and $\tau_0 = 1$ are used for efficient analysis. m and n are corresponding multiplier integers.

2.2.Applying DWT with the Wavelet Toolbox in MATLAB

i. Create following waveforms on MATLAB.

$$x_1[n] = \begin{cases} 2\sin(20\pi n) + \sin(80\pi n)\,, & 0 \le n < 512 \\ 0.5\sin(40\pi n) + \sin(60\pi n)\,, & 512 \le n < 1024 \end{cases}$$

$$x_2[n] = \begin{cases} 1 & 0 \le n < 64 \\ 2 & 192 \le n < 256 \\ -1 & 128 \le n < 512 \\ 3 & 512 \le n < 704 \\ 1 & 704 \le n < 960 \\ 0 & otherwise \end{cases}$$

Sampling Frequency = 512 Hz

Corrupt these signals with AWGN of 10 dB SNR, call these signal y_1, y_2. Plot corresponding $x[n]$ and $y[n]$ on the same figure. Use the command `awgn(x,snr,'measured')`.

ii. Observe the morphology of the wavelet and scaling functions of Haar and Daubechies tap 9 using `wavefun()` command and the `waveletAnalyzer` GUI.
iii. Calculate the 10-level wavelet decomposition of the signal using wavelet 'db9' and 'haar'. Use the command `wavedec()`.
iv. Use inverse DWT to reconstruct A^{10}, D^{10}, D^9, …., D^2, D^1 and verify that $y = \Sigma D^i + A$ by calculating energy between original and reconstructed signal. Explain the steps followed.

2.3. Signal Denoising with DWT

i. Plot the magnitude of wavelet coefficients (stem plot) of the above signal in the descending order.

ii. Select a threshold by observation assuming low magnitude coefficients contain noise. Reconstruct the signal with suppressed coefficients.

iii. Calculate the root mean square error (RMSE) between the original and denoised signal. Plot the two signals on the same plot and interpret the results.

iv. Repeat the same procedure with 'haar' wavelet (make sure the same signal corrupted with is used for this step since noise is random. You may fix the random generator `rng(seed)`)

v. Compare the RMSE and the reconstructed wave morphology of the two waveforms with two wavelets and comment on the suitability of the wavelets used.

2.4. Signal Compression with DWT

i. You are given the aV_R lead of an ECG sampled at 257 Hz in `ECGsig.mat`. Obtain the discrete wavelet coefficients of the signal (use 'db9' and 'haar' wavelets).

ii. Arrange the coefficients in the descending order and find the number of coefficients which are required to represent 99% of the energy of the signal.

iii. Compress the signal and find the compression ratio. Comment on the morphology of the reconstructed signal and the compression ratio.