180237G

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BME

MATLAB EXERCISE 4

We can use MATLAB to calculate characteristics of a second order system, such as damping ratio, ζ; natural frequency, ω_n; percent overshoot, %OS (pos); settling time, T_s ; and peak time, T_p. Observe the MATLAB code below for the calculation of above parameters for the following pole-zero plot.

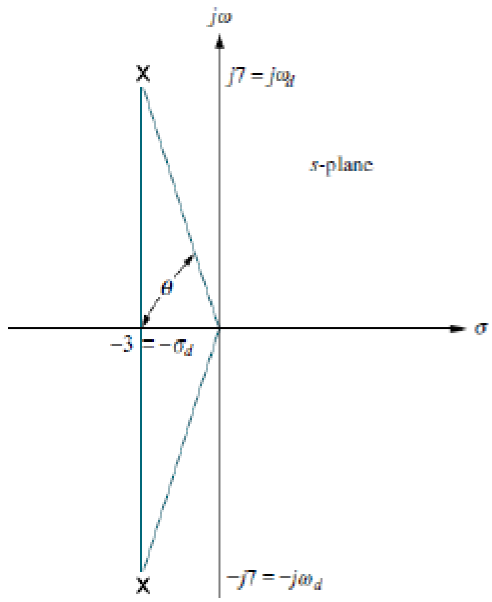


Figure M1: Pole-zero plot for M1

```
p1=[1 3+7*1j]; % Define polynomial containing first pole.
p2=[1 3-7*1j]; % Define polynomial containing second pole.
den=conv(p1 ,p2); % Multiply the two polynomials to find the 2nd order % polynomial, as^2+bs+c
omegan=sqrt (den (3) /den (1)) ;% Calculate the natural frequency, sqrt(c/a).
zeta=(den(2)/den(1))/(2*omegan); % Calculate damping ratio, ((b/a)/2*wn).
Ts=4/(zeta*omegan); % Calculate settling time, (4/z*wn).
Tp=pi/(omegan*sqrt(1-zeta^2)); % Calculate peak time, pi/wn*sqrt(l -z^2).
pos=100*exp( -zeta*pi/sqrt(1-zeta^2)); % Calculate percent overshoot (100*e^(-z*pi/sqrt(l-% z^

fprintf('Damping ratio(ζ)= %f.\n',zeta);
```

```
Damping ratio(ζ)= 0.393919.
```

```
fprintf('Natural frequence= %f.\n',omegan);
```

```
Natural frequence= 7.615773.
```

```
fprintf('Precentage overshoot(%OS)= %f.\n',pos);
```

```
Precentage overshoot(
```

```
fprintf('Settling Time(Ts)= %f.\n',Ts);
```

```
Settling Time(Ts)= 1.333333.
```

```
fprintf('Peak Time(Ts)= %f.\n',Tp);
```

```
Peak Time(Ts)= 0.448799.
```

Use the above procedure to find ζ, ω_n, %OS, T_s and T_p for the following second order system.

$$G(S) = \frac{10}{3S^2 + 8S + 24}$$

$$G(S) = \frac{\left(\frac{10}{3}\right)}{S^2 + \left(\frac{8}{3}\right)S + 8}$$

$$\text{poles} \quad S = -1.33 \pm 2.494$$

```
p1=[1 1.33+2.494*1j]; % Define polynomial containing first pole.
p2=[1 1.33-2.494*1j]; % Define polynomial containing second pole.
den=conv(p1 ,p2); % Multiply the two polynomials to find the 2nd order % polynomial, as^2+bs+c
omegan=sqrt (den (3) /den (1)); % Calculate the natural frequency, sqrt(c/a).
zeta=(den(2)/den(1))/(2*omegan); % Calculate damping ratio, ((b/a)/2*wn).
Ts=4/(zeta*omegan) % Calculate settling time, (4/z*wn).
```

```
Ts = 3.0075
```

```
Tp=pi/(omegan*sqrt(1-zeta^2)); % Calculate peak time, pi/wn*sqrt(l -z^2).
pos=100*exp(-zeta*pi/sqrt(1-zeta^2)); % Calculate percent overshoot (100*e^(-z*pi/sqrt(l-% z^2

fprintf('Damping ratio(ζ)= %f.\n',zeta);
```

```
Damping ratio(ζ)= 0.470552.
```

```
fprintf('Natural frequence= %f.\n',omegan);
```

```
Natural frequence= 2.826471.
```

```
fprintf('Precentage overshoot(%%OS)= %f.\n',pos);
```

```
Precentage overshoot(%OS)= 18.724298.
```

```
fprintf('Settling Time(Ts)= %f.\n',Ts);
```

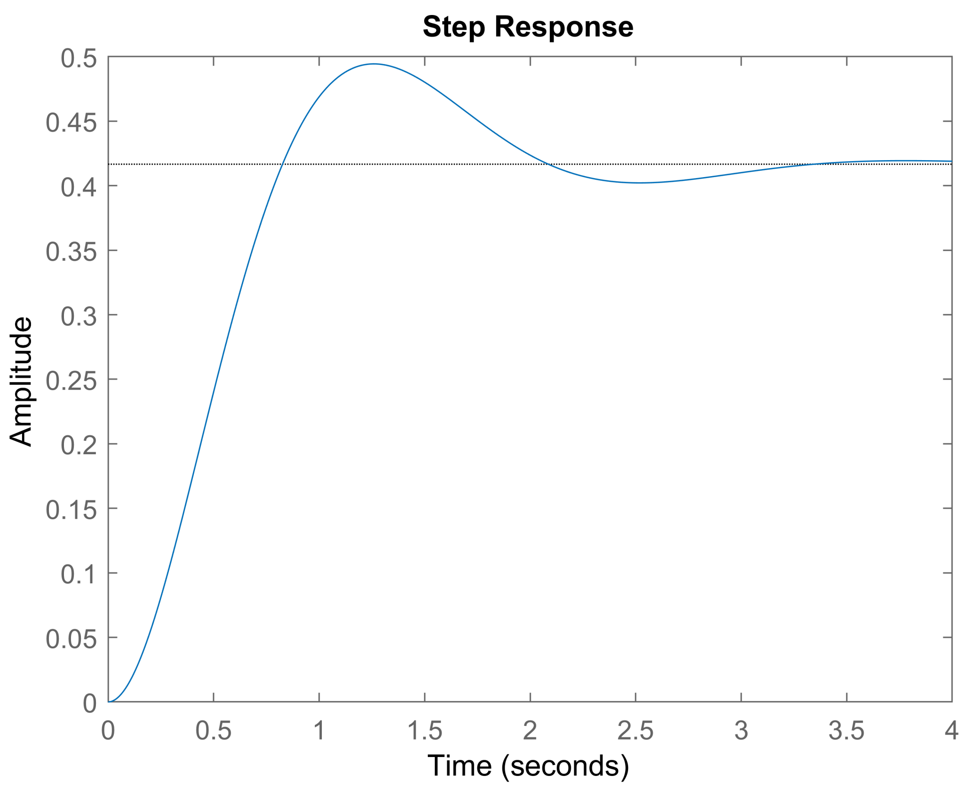
```
Settling Time(Ts)= 3.007519.
```

```
fprintf('Peak Time(Ts)= %f.\n',Tp);
```

```
Peak Time(Ts)= 1.259660.
```

Use tf and stepplot or step functions to plot the step response, and compare with the results obtained before. Note: You may left click the mouse on the curve to get more information about a particular coordinate. Moreover, by right clicking away from the curve brings up a menu, from which you can obtain the characteristics of the step response curve by mouse pointing at the appearing dots.

```
num=10;
deno=[3 8 24];
H=tf(num,deno);
stepplot(H,0:0.001:4);
```



$T_p = 1.263s$
$Y_{\text{peak}} = 0.4944$
$Y_{\text{final}} = 0.4169$

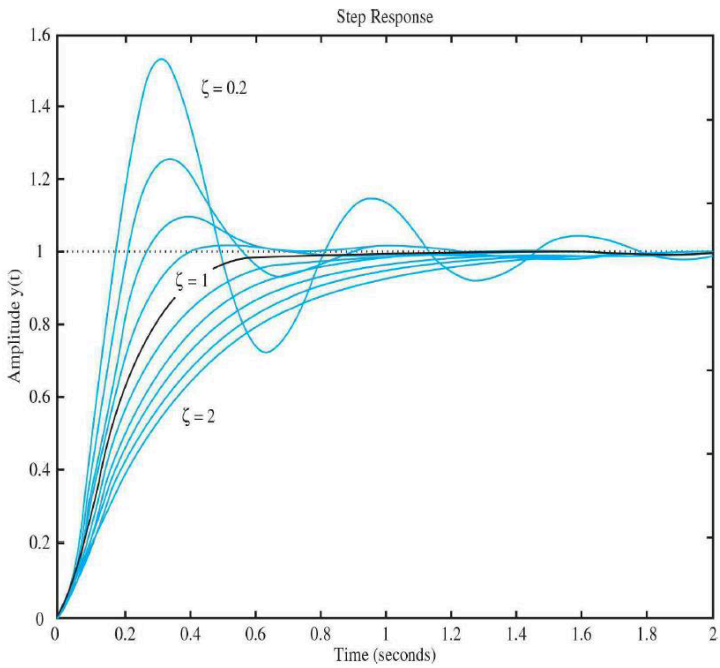
$t_1 = 0.172 \ < - - - - - - - - \ Y = 0.1 \times 0.4169 = 0.04169$
$t_1 = 0.732 \ < - - - - - - - - \ Y = 0.9 \times 0.4169 = 0.37521$
$t_2 - t_2 = 0.732 - 0.172 = 0.56s$
$Therefore \ \ T_r = 0.56$

T_s =2.931s $< - - - - - - - - \; Y = 0.98 \times 0.4169 = 0.4085$

$$\%\mathrm{OS} = \frac{C_{\max} - C_{\text{final}}}{C_{\text{final}}} \times 100\% = \frac{(0.4944 - 0.4169)}{0.4169} \times 100\% = 18.58\%$$

Therfore calculated results are nearly equals to results from graphs.

2. For a 2nd order system, obtain the following response curves by changing ζ from 0.2 to 2.



$T_p = 0.3s$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} - - - - - > \omega_n = 10.688$$

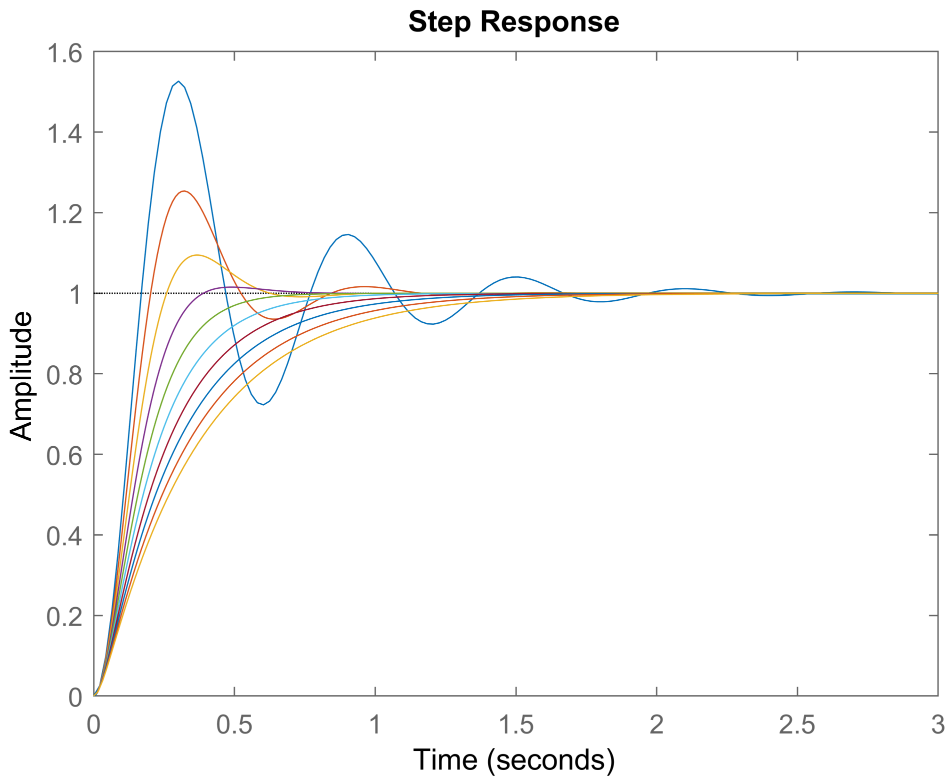
$\omega_n^2 = 114.23$

$$T(S) = \frac{\omega_n^2}{S^2 + a\omega_n S + \omega_n^2} \quad a = 2\zeta$$

```
num=114.23;
omega=10.688;
H1=tf(num,[1 2*omega*0.2 114.23]);
H2=tf(num,[1 2*omega*0.4 114.23]);
H3=tf(num,[1 2*omega*0.6 114.23]);
H4=tf(num,[1 2*omega*0.8 114.23]);
H5=tf(num,[1 2*omega*1.0 114.23]);
H6=tf(num,[1 2*omega*1.2 114.23]);
```

```
H7=tf(num,[1 2*omega*1.4 114.23]);
H8=tf(num,[1 2*omega*1.6 114.23]);
H9=tf(num,[1 2*omega*1.8 114.23]);
H10=tf(num,[1 2*omega*2.0 114.23]);

stepplot(H1,H2,H3,H4,H5,H6,H7,H8,H9,H10)
```

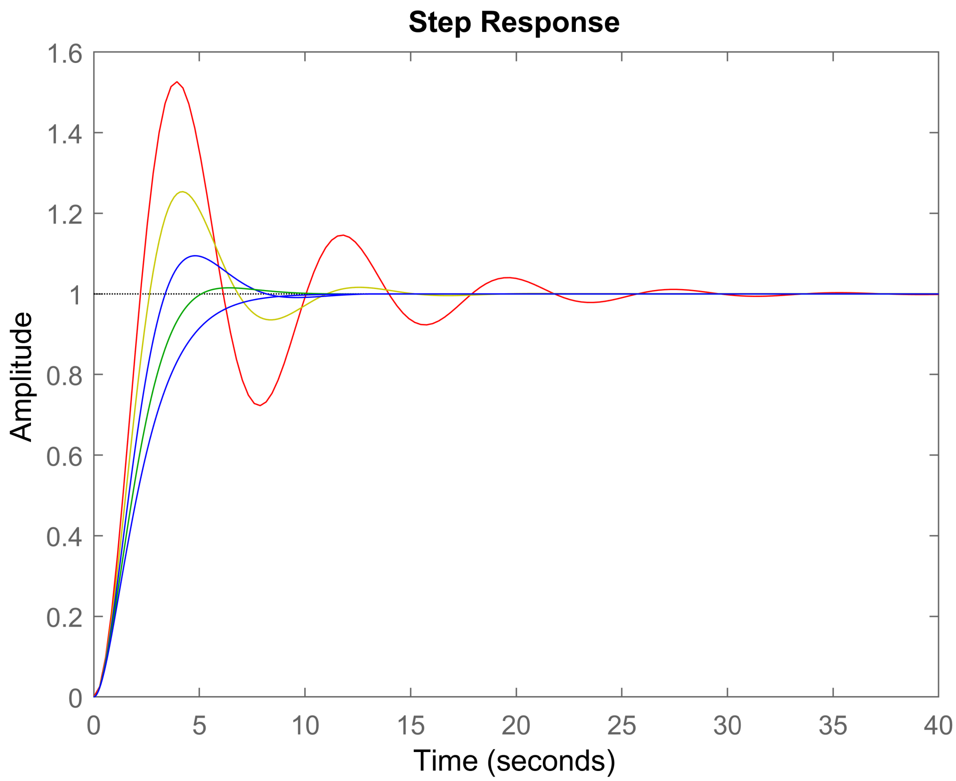


3. For a 2nd order system, figure out what happens to %OS when ζ changes from 0 to 1 (slide #42). Plot the result.

```
num=0.669;
omega=0.818;

H2=tf(num,[1 2*omega*0.2 0.669]);
H3=tf(num,[1 2*omega*0.4 0.669]);
H4=tf(num,[1 2*omega*0.6 0.669]);
H5=tf(num,[1 2*omega*0.8 0.669]);
H6=tf(num,[1 2*omega*1.0 0.669]);


stepplot(H2,'r',H3,'y',H4,'b',H5,'g',H6,'bl');
```



When ζ is increasing, $C_{\max}$ is decreasing but C_{final} is constant. Therfore %OS is decreasing.