Department of Electronics and Telecommunication Engineering University of Moratuwa

EN 2040 – Random Signals and Processes

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BME

CONTENT

Question 3

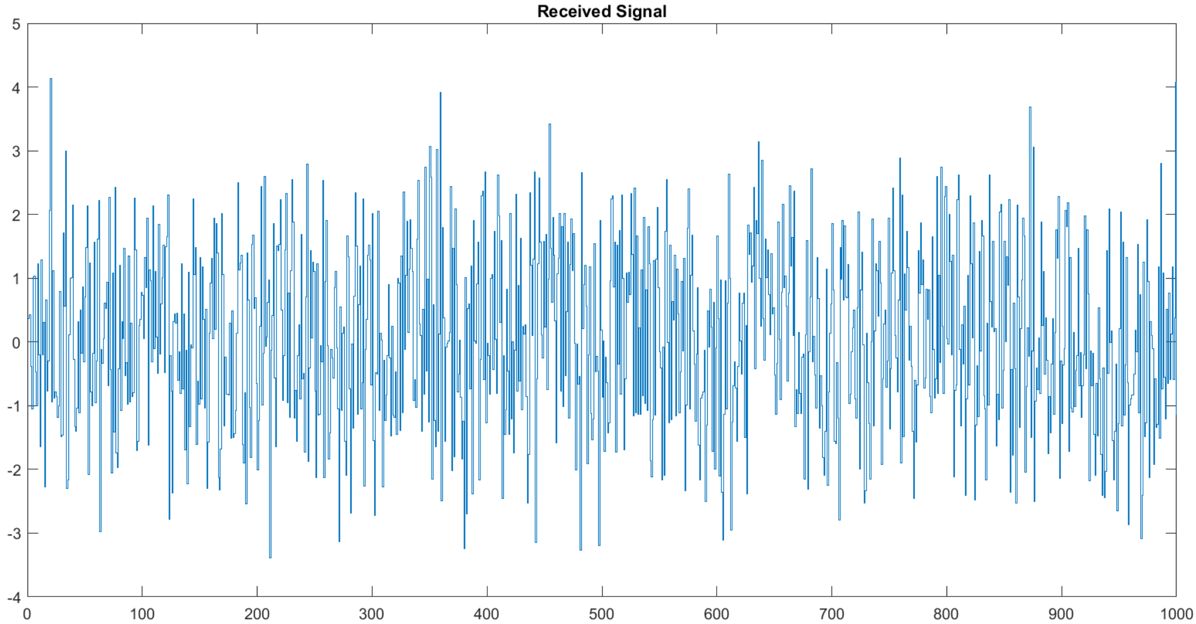

Figure 3.1: Received signal for $\sigma^2 = 1$ (L=1000)

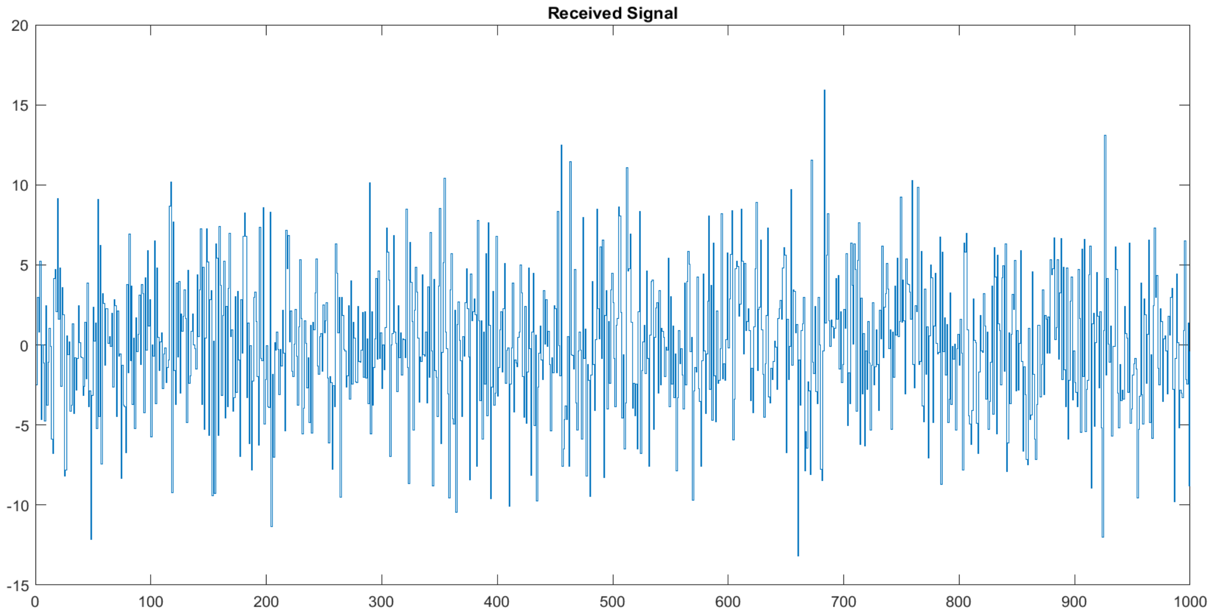

Figure 3.2: Received signal for $\sigma^2 = 4$ (L=1000)

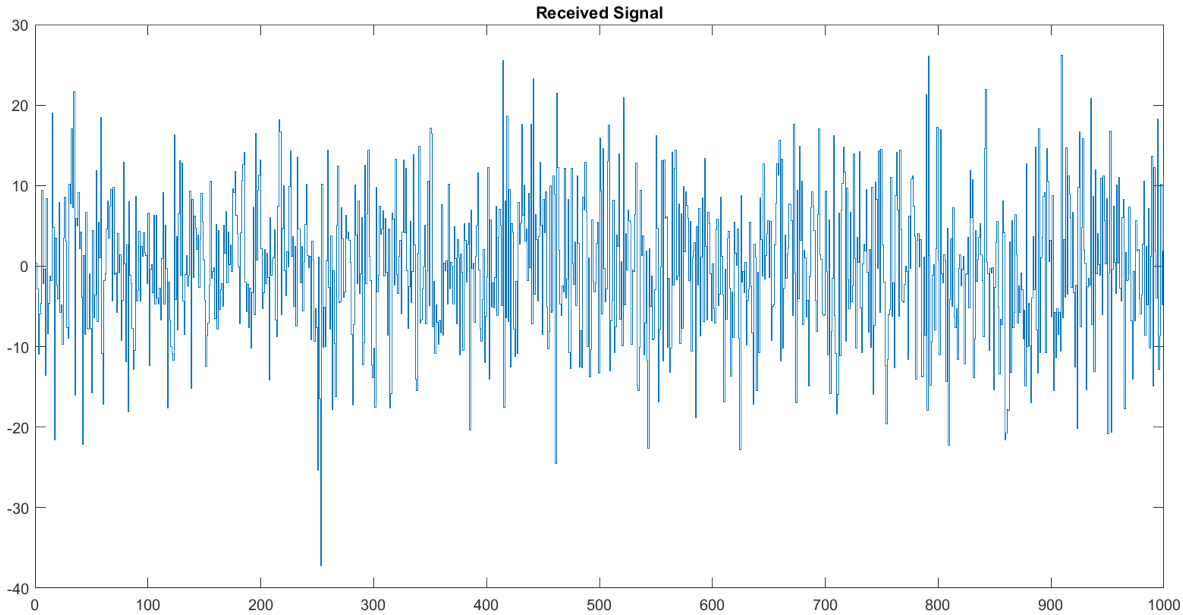

Figure 3.3: Received signal for $\sigma^2 = 9$ (L=1000)

According to above figures when increasing the variance of the noise signal is increasing, the received signal amplitudes are increasing.

Question 4

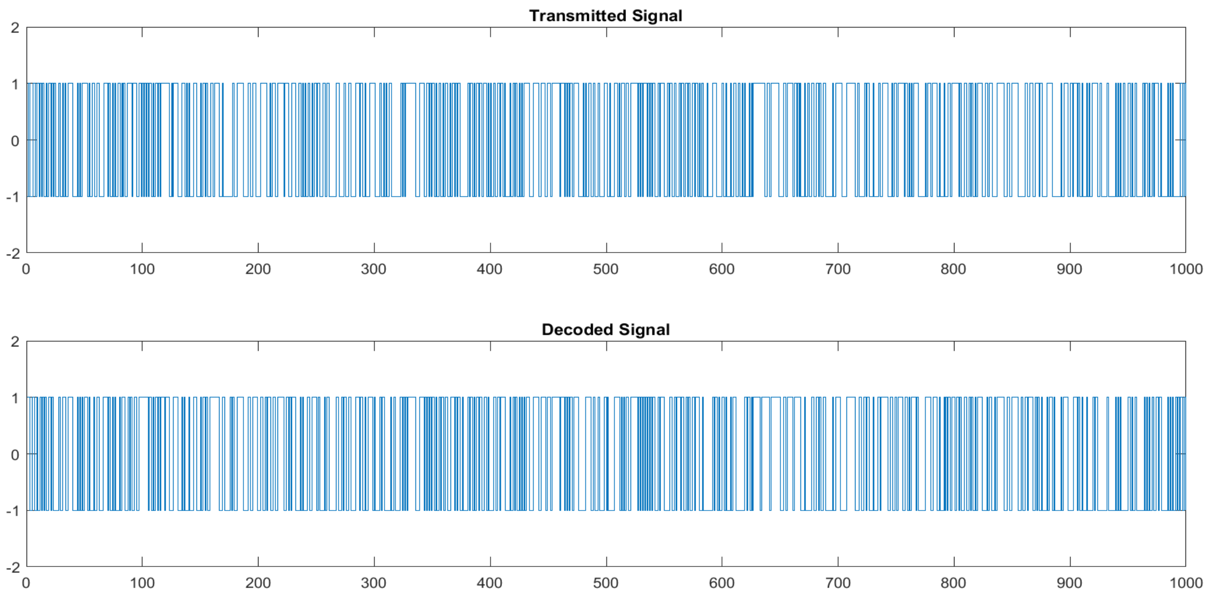

Figure 4.1: Transmitted Signal and Decoded Signal

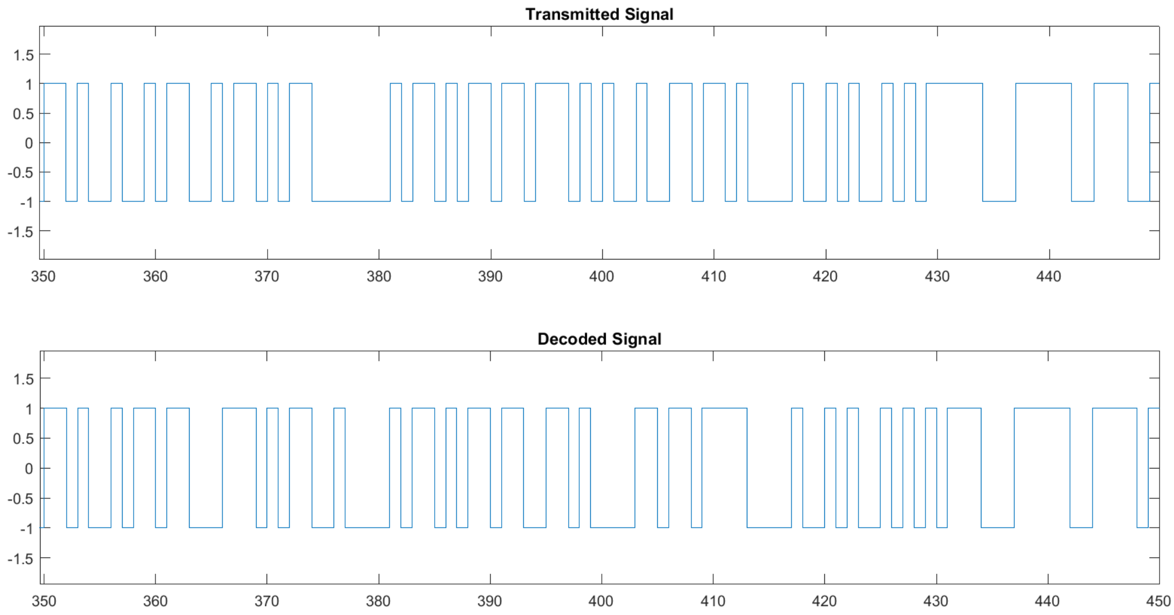

Figure 4.2: Transmitted Signal and Decoded Signal (zoomed)

Transmitted Signal and Decoded Signal is almost similar, but there are some mismatched bits are included in the decoded signal.

Question 5

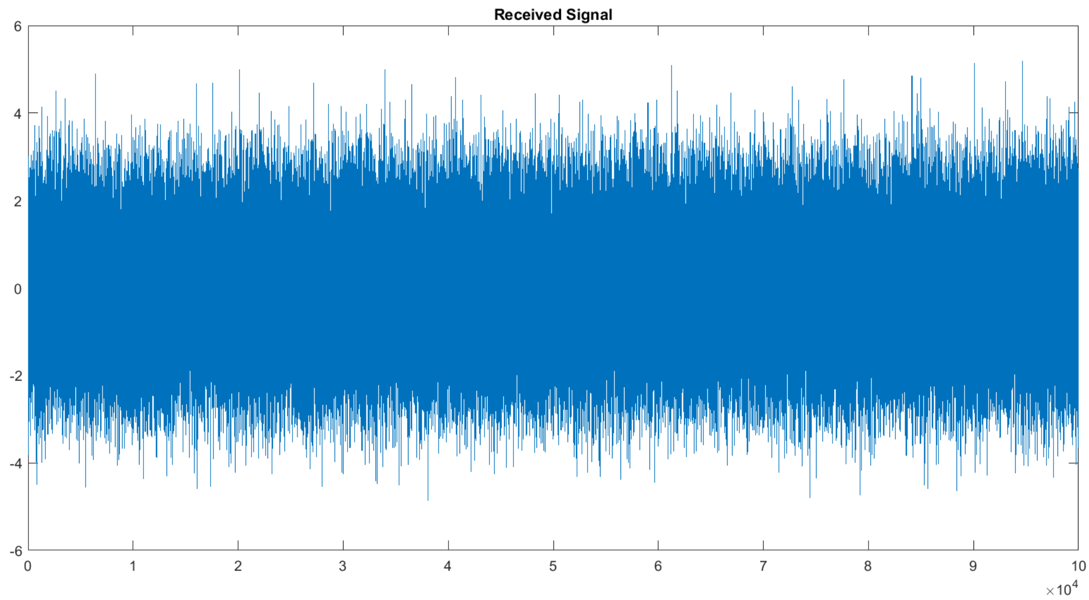

Figure 5.1.1: Received signal for $\sigma^2 = 1$ (L=100000)

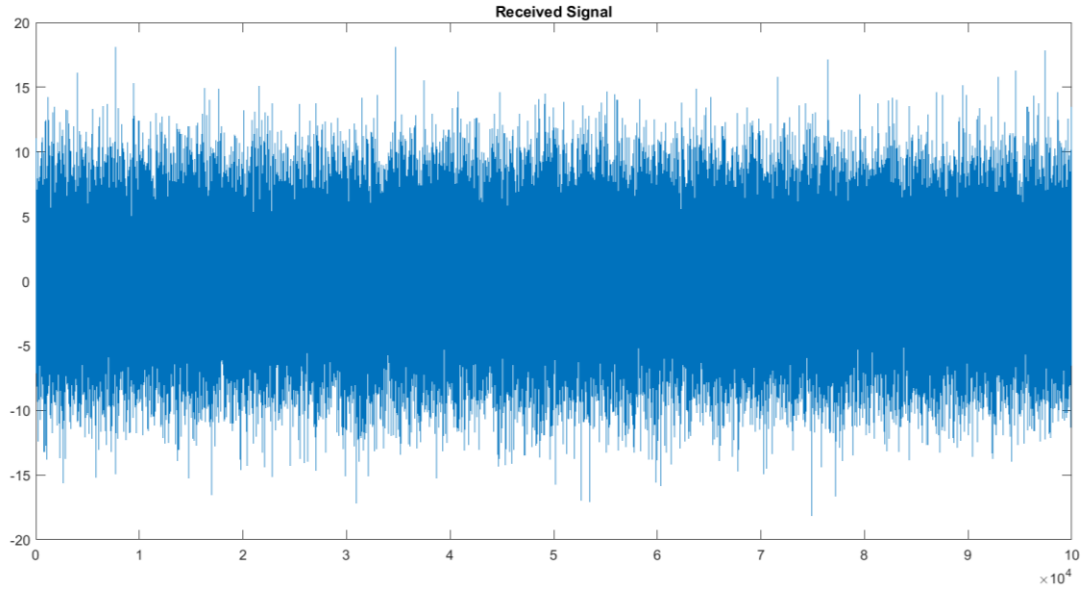

Figure 5.1.2: Received signal for $\sigma^2 = 4$ (L=100000)

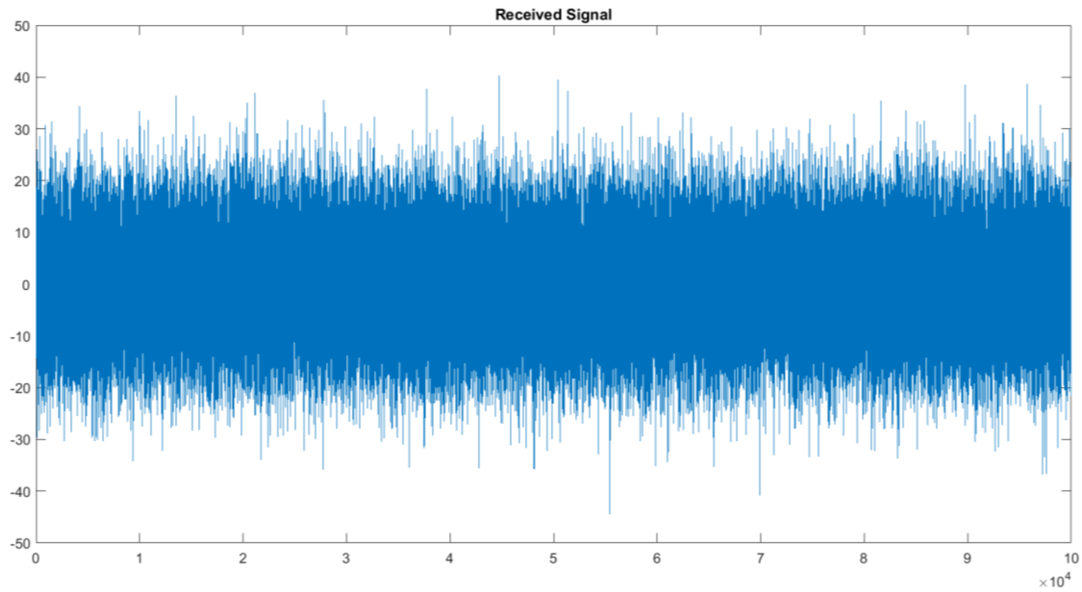

Figure 5.1.3: Received signal for $\sigma^2 = 9$ (L=100000)

According to above figures when increasing the variance of the noise signal is increasing, the received signal amplitudes are increasing. When L=1000 also same characteristics can be identified.

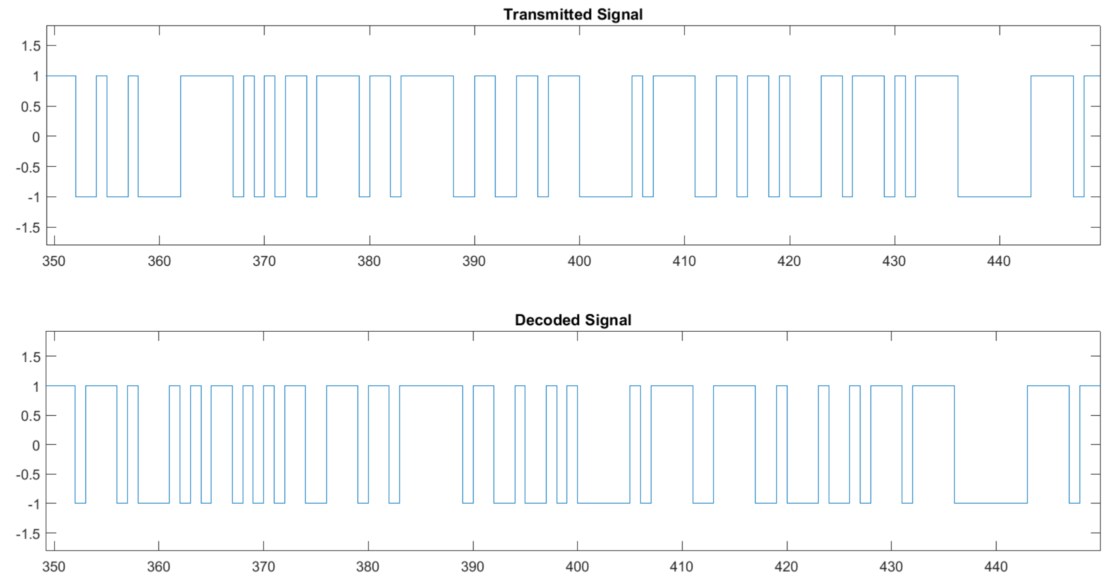

Figure 5.1.4: Transmitted and Decoded signal for $\sigma^2 = 9$ (L=100000)

There are some certain errors in decoded signal when comparing transmitted signal but the decoded signal is nearly similar to transmitted signal

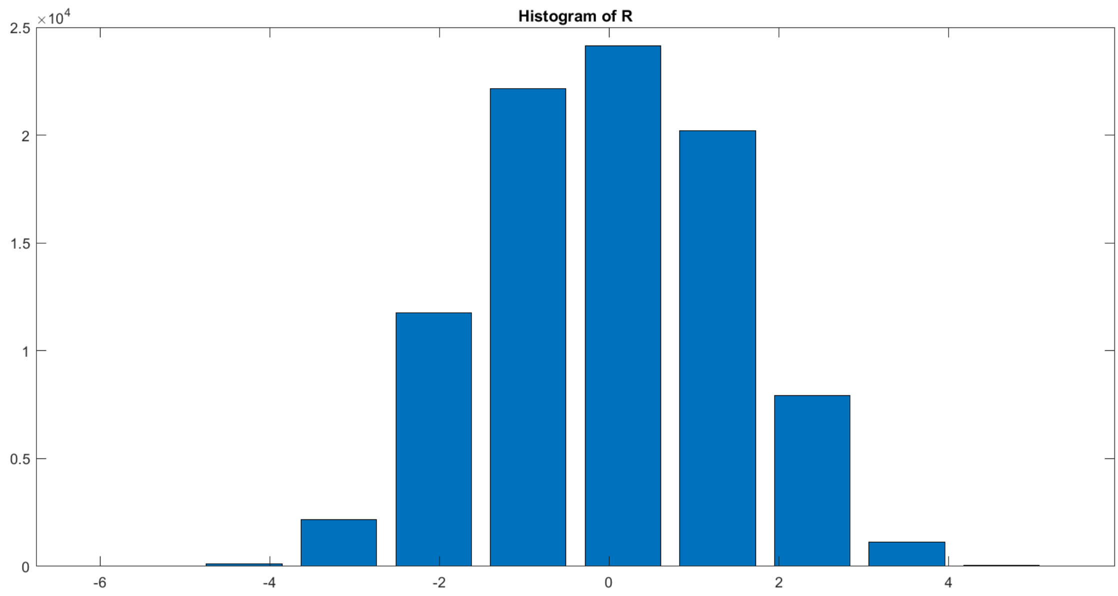

Figure 5.1.5: Histogram Plot for $\sigma^2 = 1$ (L=100000, bin=10) without hist function

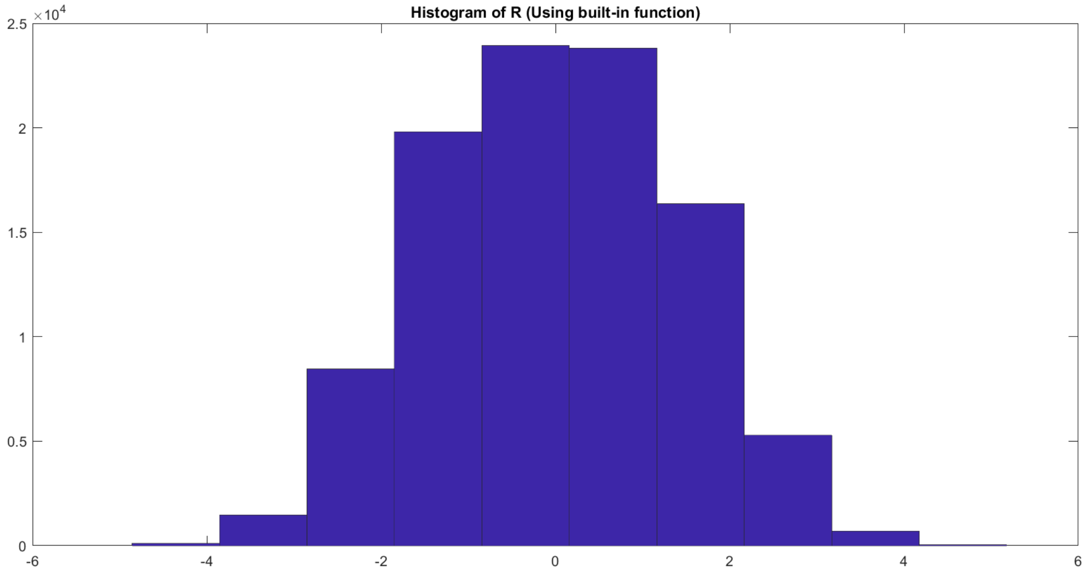

Figure 5.1.6: Histogram Plot for $\sigma^2 = 1$ (L=100000, bin=10) with hist function

Histogram figures of without hist function of MATLAB and with the hist function are almost same but there are small differences in frequency values. The reason for that incident may be the difference in the range for each bin.

Part a

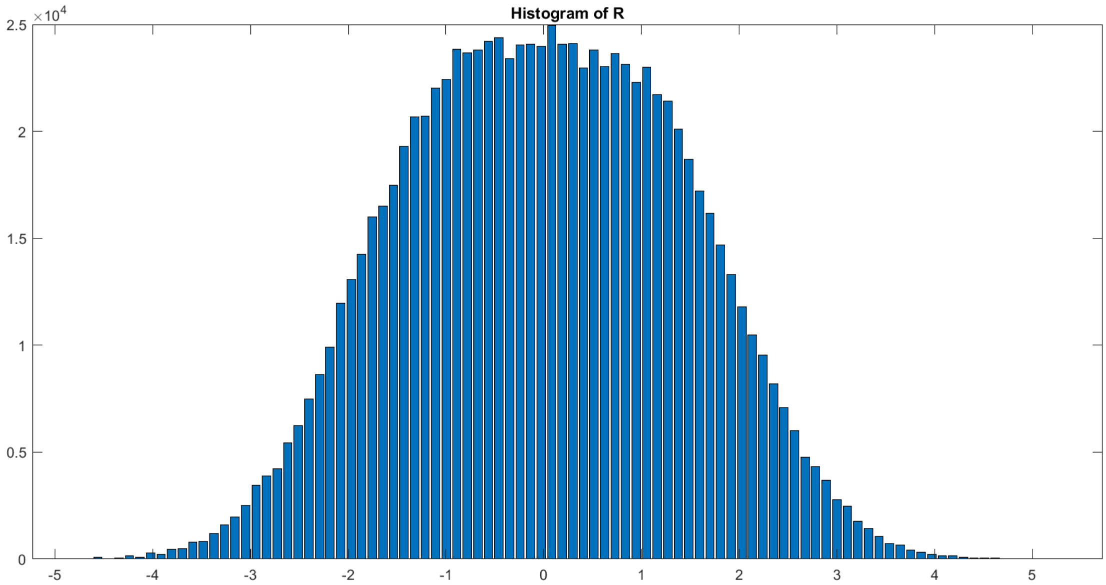

Figure 5.2.1: Histogram Plot for $\sigma^2 = 1$ (L=100000, **bin=100**)

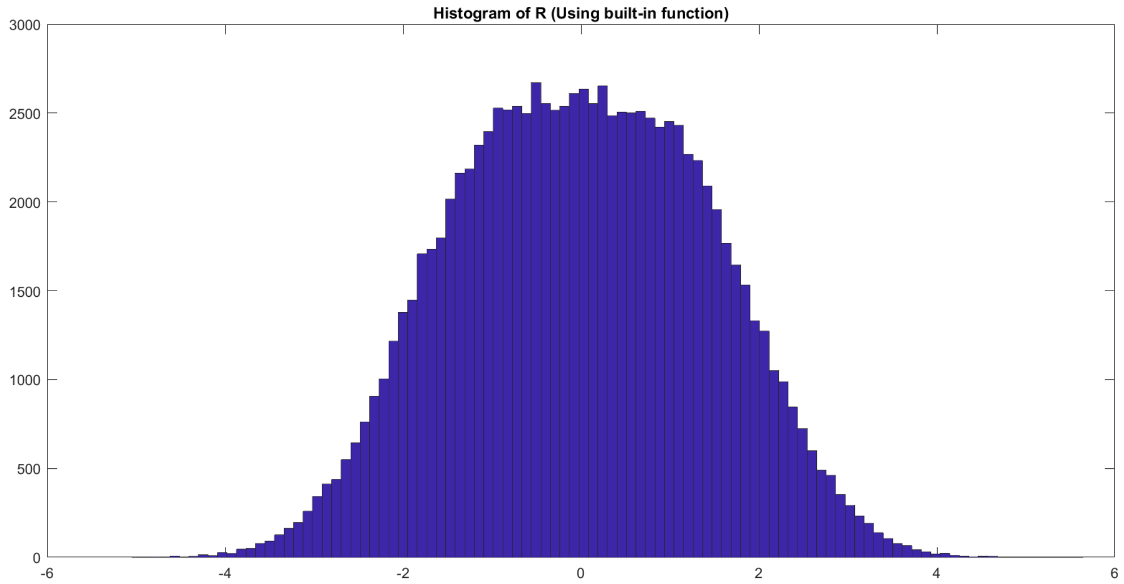

Figure 5.2.2: Histogram Plot for $\sigma^2 = 1$ (L=100000, **bin=100**) with hist function

When increasing the number of bin size two of the histograms are nearly similar shape but the values of the y axis are different. First histograms y values are nearly 10 times of second histogram y axis values. The reason for difference is the MATLAB built-in function gives a normalized frequency value.

<u>Part b</u>

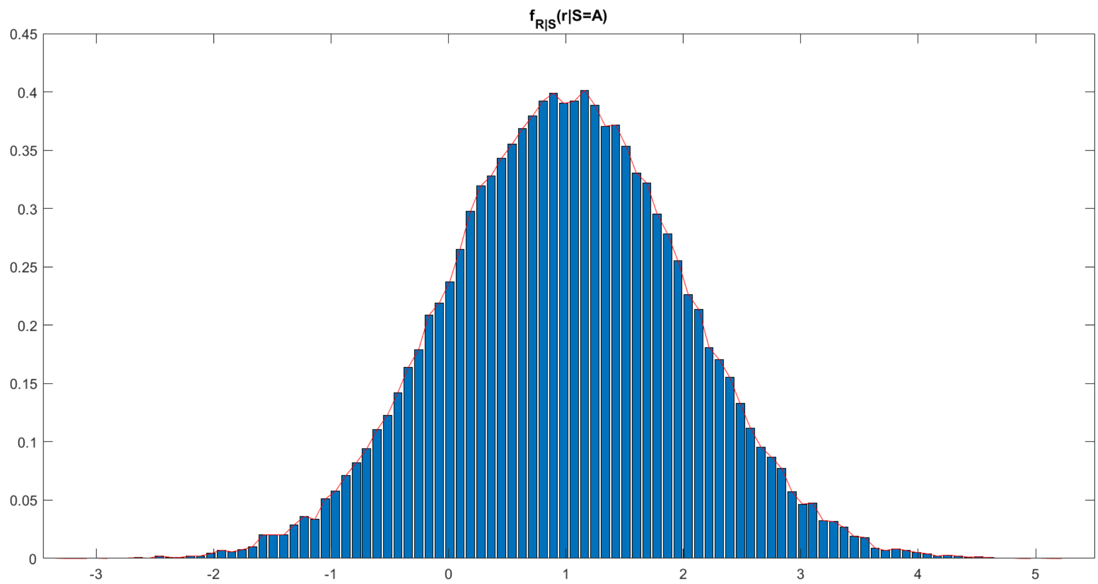

Figure 5.3.1: $f_{R|S}(r|S = A)$ histogram (**A=1**)

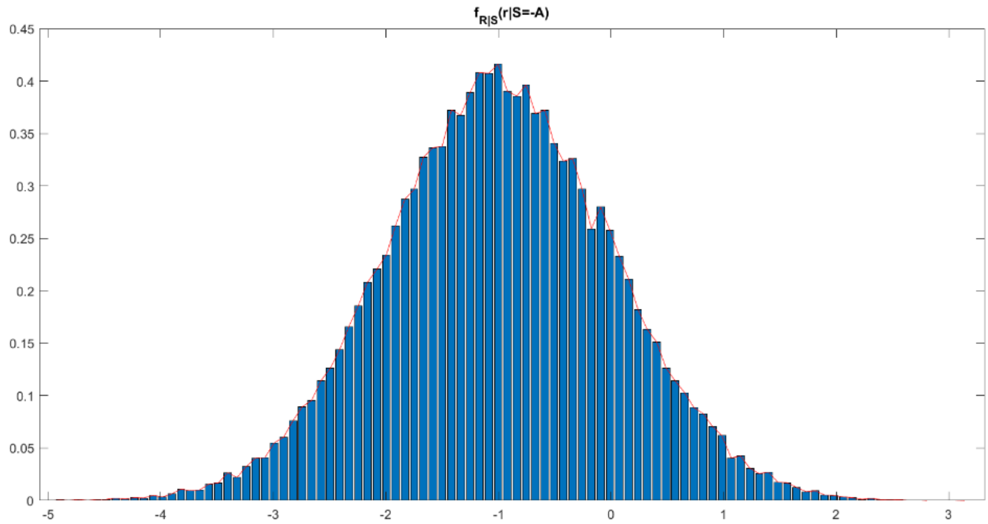

Figure 5.3.1: $f_{R|S}(r|S=-A)$ histogram (**A=1**)

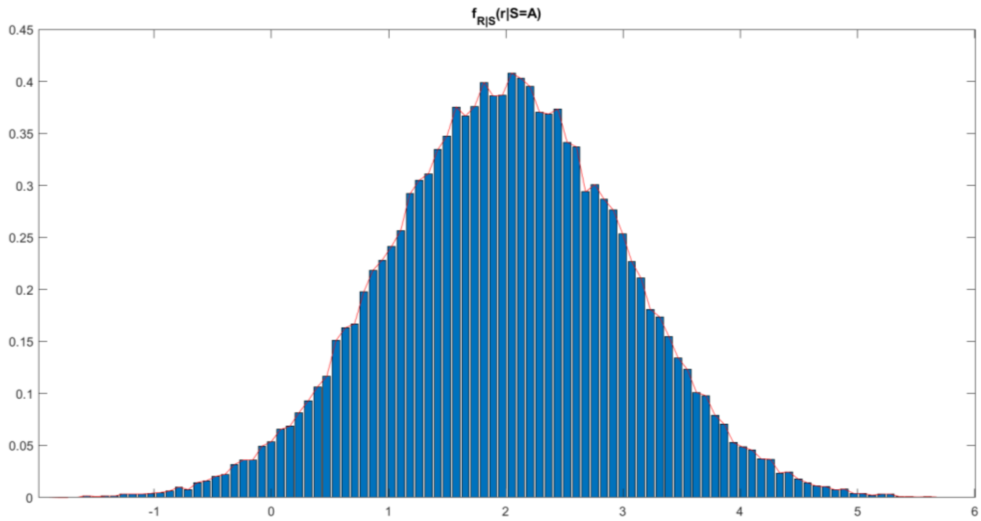

Figure 5.3.1: $f_{R|S}(r|S=A)$ histogram (**A=2**)

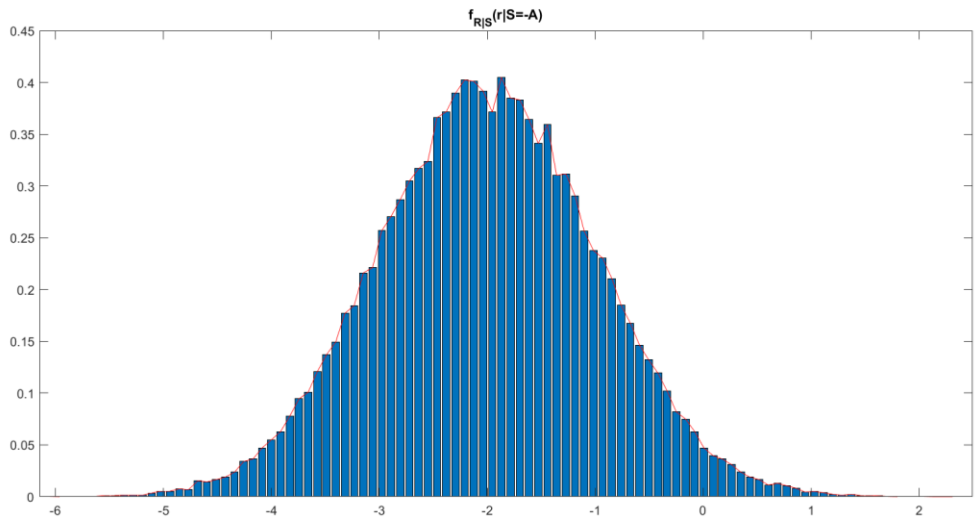

Figure 5.3.1: $f_{R|S}(r|S=-A)$ histogram (**A=2**)

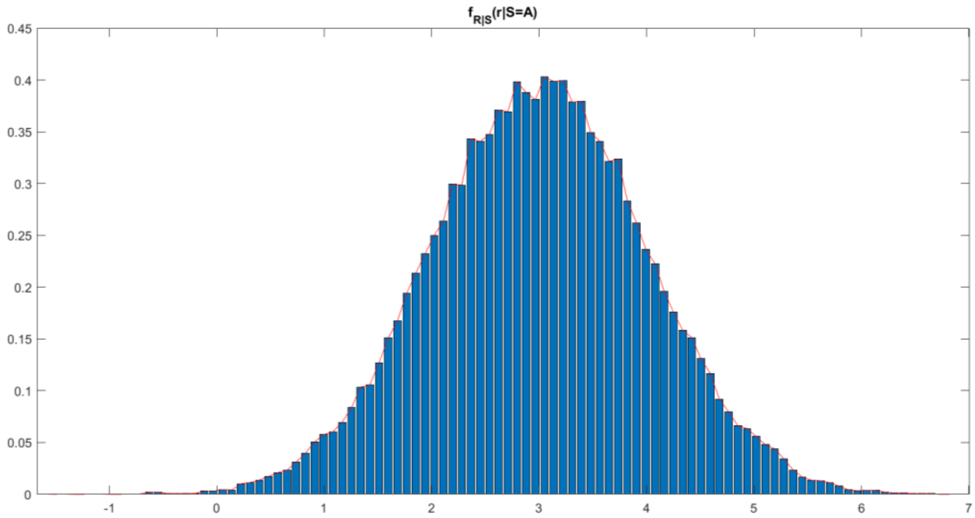

Figure 5.3.1: $f_{R|S}(r|S = A)$ histogram (**A=3**)

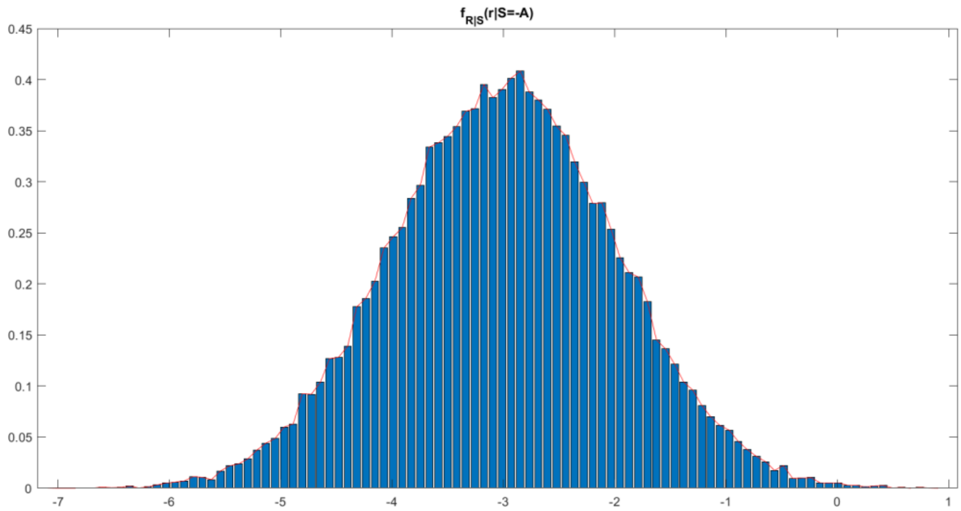

Figure 5.3.1: $f_{R|S}(r|S = -A)$ histogram (**A=3**)

When increasing the amplitude, the histogram shifting from the positive side or the negative side. The histogram shifts to the positive side if the function is $f_{R|S}(r|S = A)$ and shifts to the negative side if the function is $f_{R|S}(r|S = -A)$. Mean also is shifting regarding to histogram. Magnitude of the mean is equal to the Amplitude.

Part c

Expected value calculation.

$$E(X) = \sum_{i=1}^{N} x_i f_{xi}(x_i)$$

A	E[R|S=A]	E[R|S=-A]	E[A]
1	0.995471	-0.999311	-0.001865
2	1.999273	-2.004848	-0.003017
3	2.996237	-3.004463	-0.004182

Table 5.4.1: Expected values for Amplitude changes

Part d

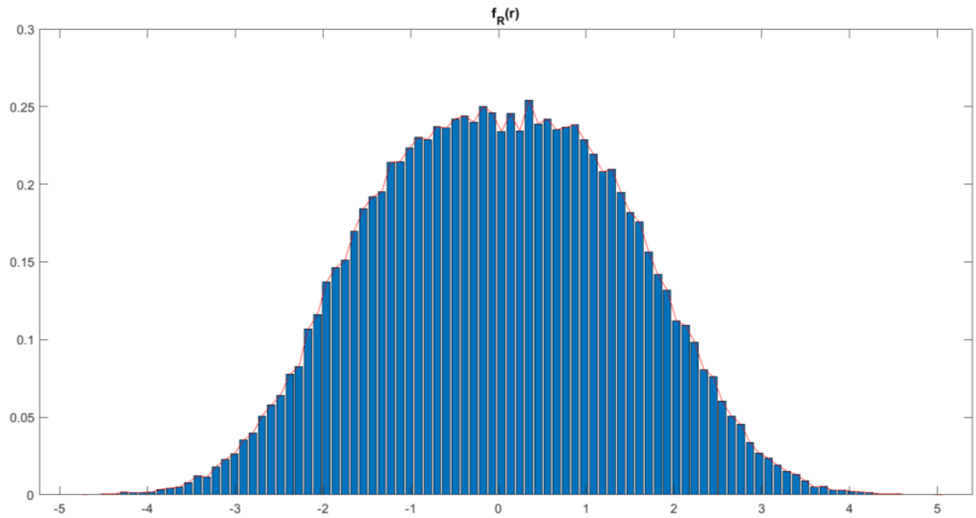

Figure 5.4.1: $f_R(r)$ histogram (**A=1**)

Question 6

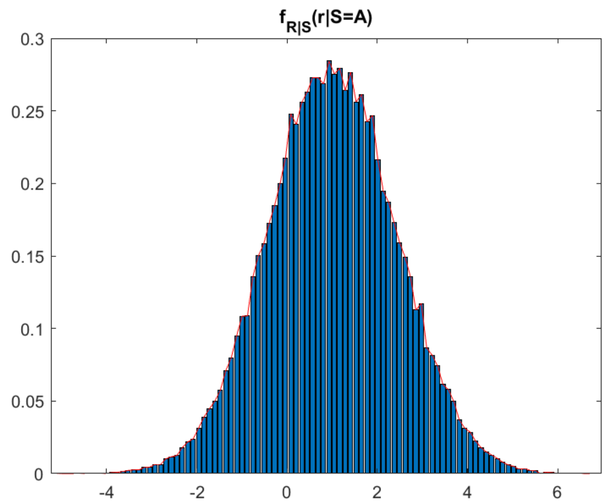

Figure 6.1.1: $f_{R|S}(r|S = A)$ histogram (**A=1**)with interference

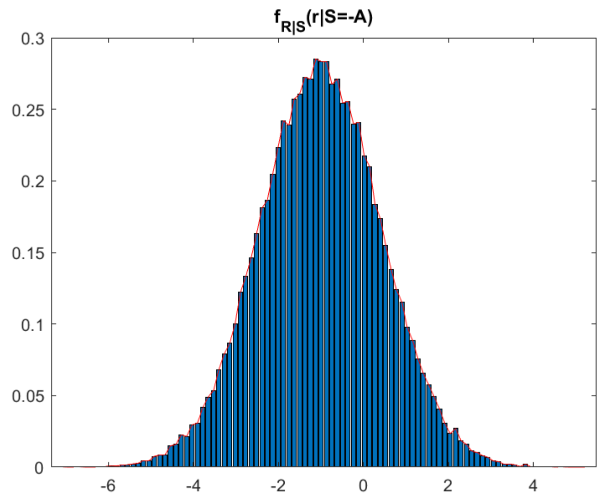

Figure 6.1.2: $f_{R|S}(r|S = -A)$ histogram (**A=1**)with interference

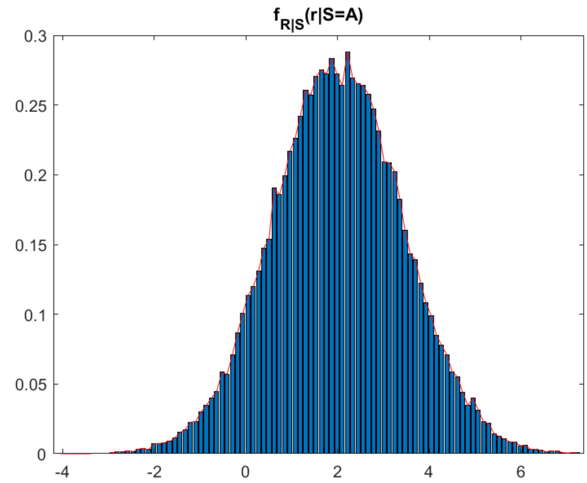

Figure 6.1.3: $f_{R|S}(r|S=A)$ histogram (**A=2**)with interference

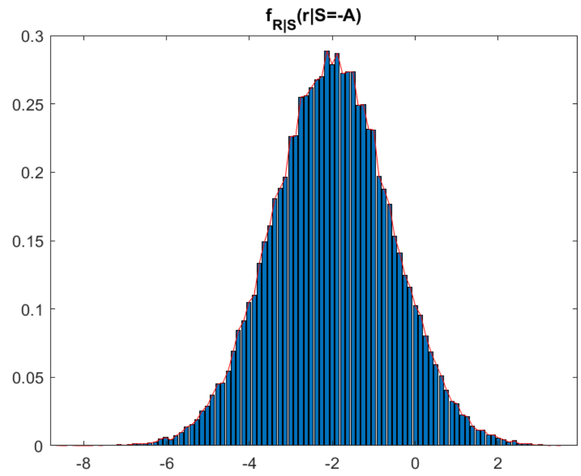

Figure 6.1.4: $f_{R|S}(r|S=-A)$ histogram (**A=2**)with interference

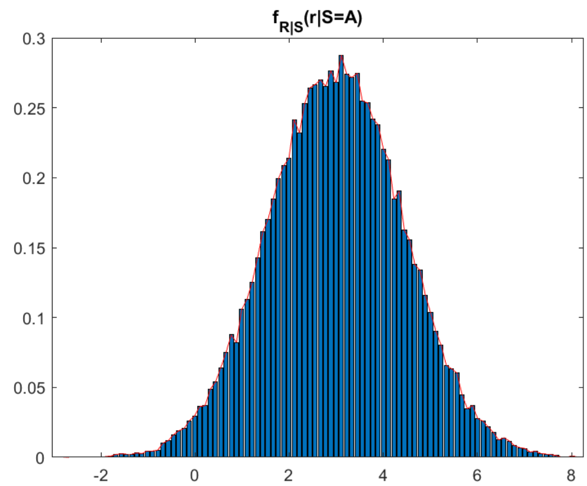

Figure 6.1.5: $f_{R|S}(r|S=A)$ histogram (**A=3**)with interference

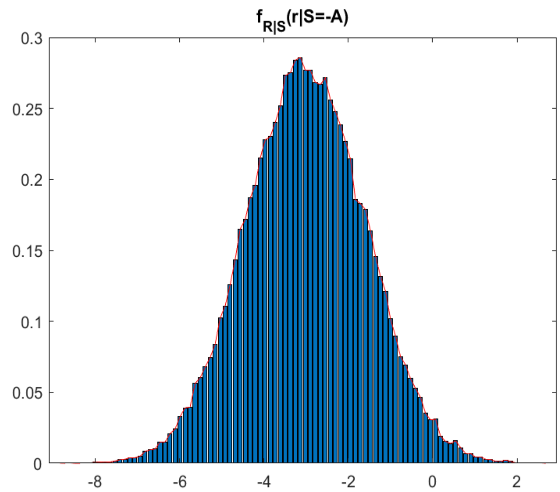

Figure 6.1.6: $f_{R|S}(r|S=-A)$ histogram (**A=3**)with interference

A	E[R|S=A]	E[R|S=-A]	E[A]
1	0.997343	-1.010493	-0.006438
2	2.010156	-2.005859	0.002084
3	2.997258	-3.008902	-0.005958

Table 6.2.1: Expected values for Amplitude changes

Question 5 characteristics are similar in the above figures. Histograms are shifting left side or Right side regarding to the PDF function and the amplitude. **E[R|S=A] value** and **E[R|S=-A]** value is nearly the value of the amplitude of the transmitting signal.

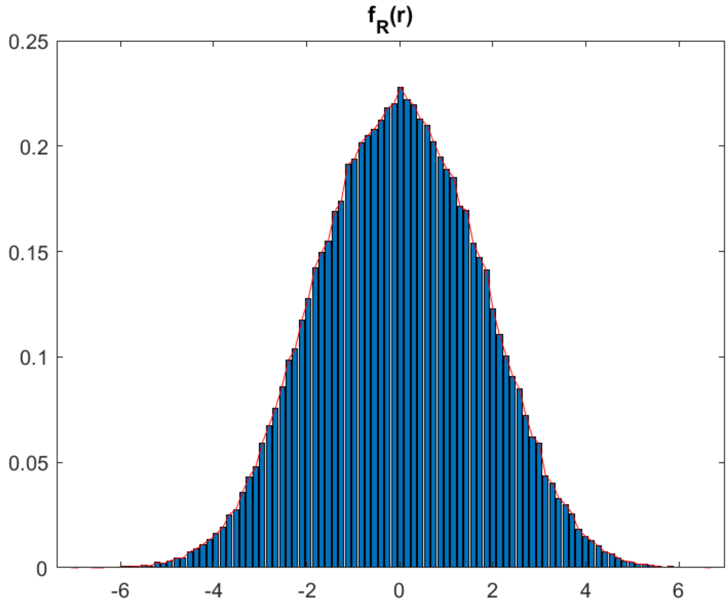

Figure 6.3.1: $f_R(r)$ histogram (**A=1**) with interference

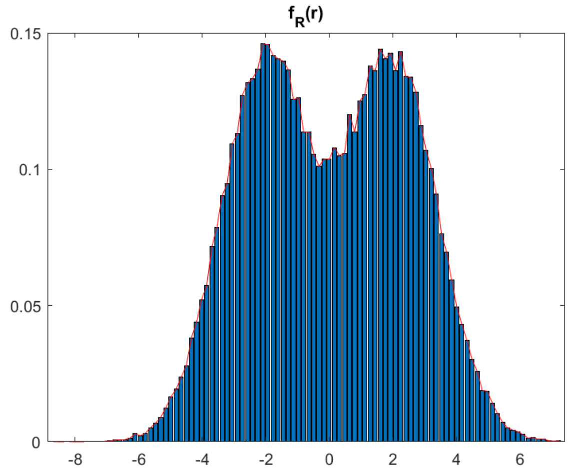

Figure 6.3.2: $f_R(r)$ histogram (**A=2**) with interference

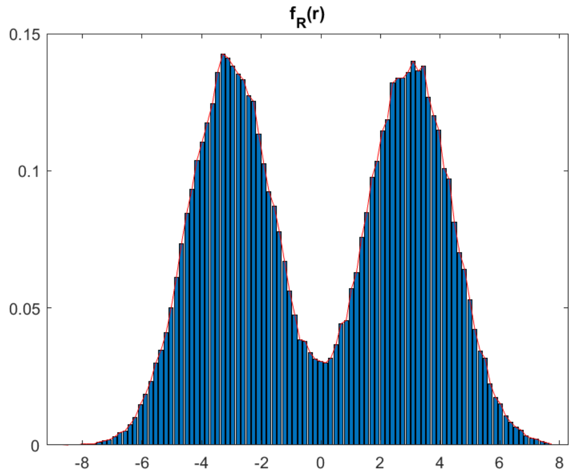

Figure 6.3.2: $f_R(r)$ histogram (**A=2**) with interference

When increasing the amplitude(A), histogram peak value is decreasing and start to split two parts which has two peaks.

Question 7

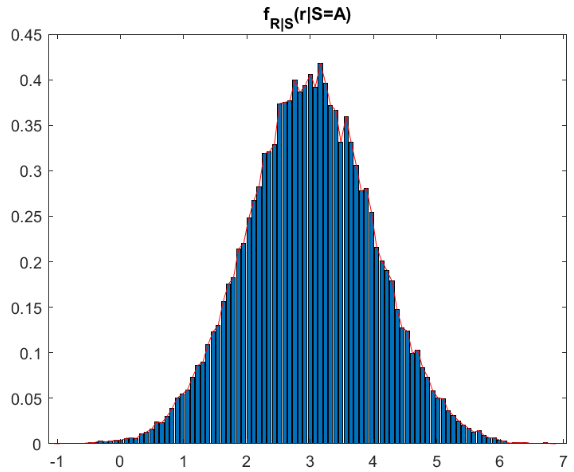

Figure 7.1.1: $f_{R|S}(r|S = A)$ histogram (**A=1, alpha=3**)with Amplification

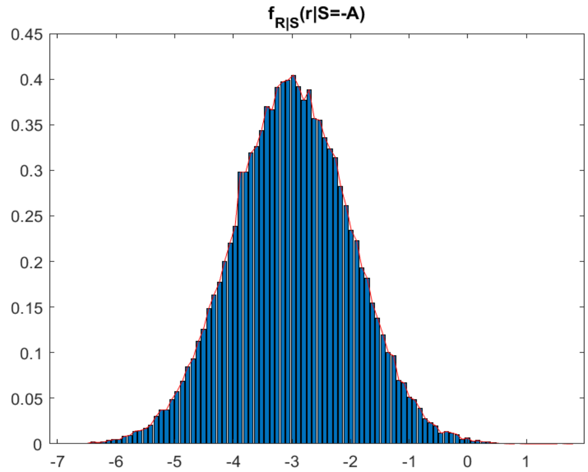

Figure 7.1.2: $f_{R|S}(r|S = -A)$ histogram (**A=1, alpha=3**)with Amplification

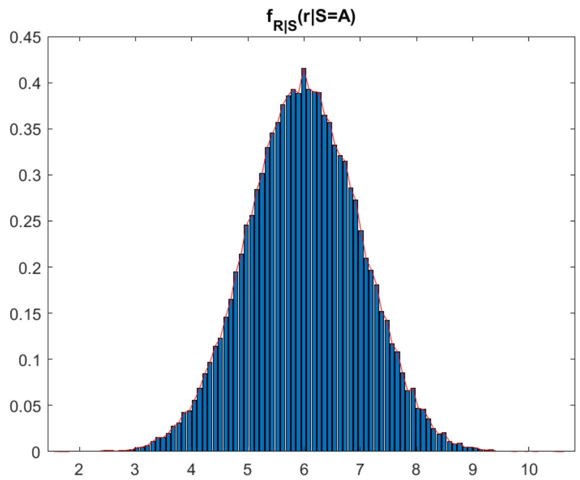

Figure 7.1.3: $f_{R|S}(r|S = A)$ histogram (**A=2, alpha=3**)with Amplification

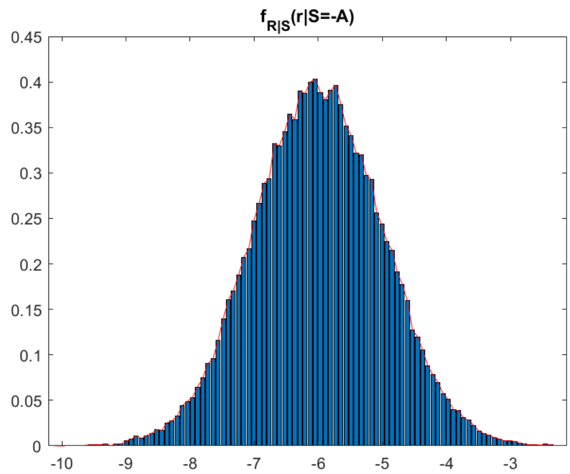

Figure 7.1.4: $f_{R|S}(r|S = -A)$ histogram (**A=2, alpha=3**)with Amplification

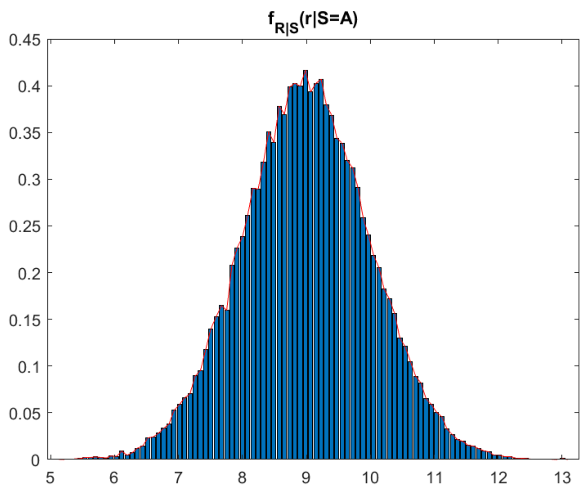

Figure 7.1.5: $f_{R|S}(r|S = A)$ histogram (**A=3, alpha=3**)with Amplification

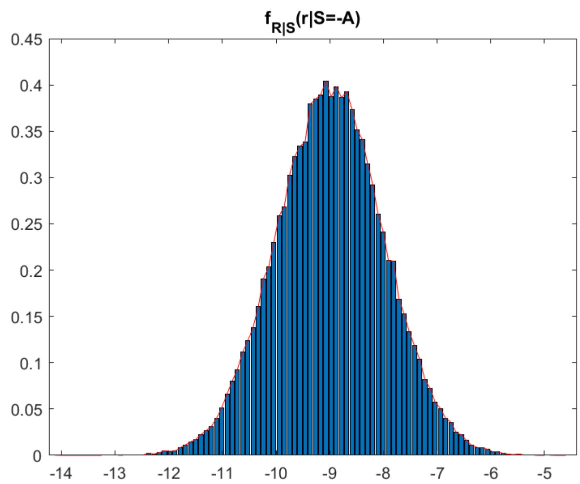

Figure 7.1.6: $f_{R|S}(r|S = -A)$ histogram (**A=2, alpha=3**)with Amplification

A	E[R|S=A]	E[R|S=-A]	E[A]
1	2.996645	-2.999221	-0.001279
2	6.003021	-6.001802	0.002084
3	8.991909	-8.991024	0.000813

Table 7.2.1: Expected values for Amplitude changes

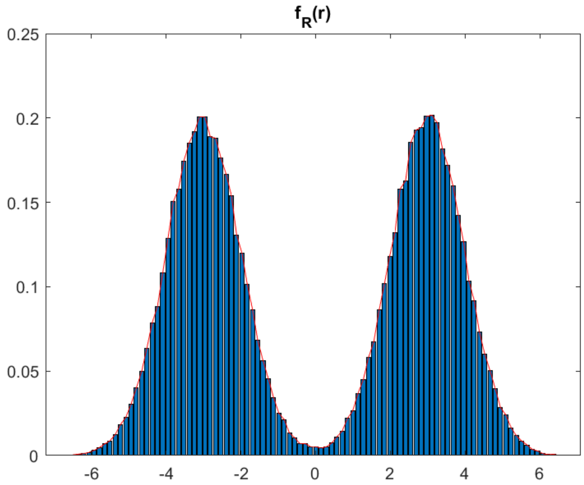

Figure 7.3.1: $f_R(r)$ histogram (**A=1**) with Amplification

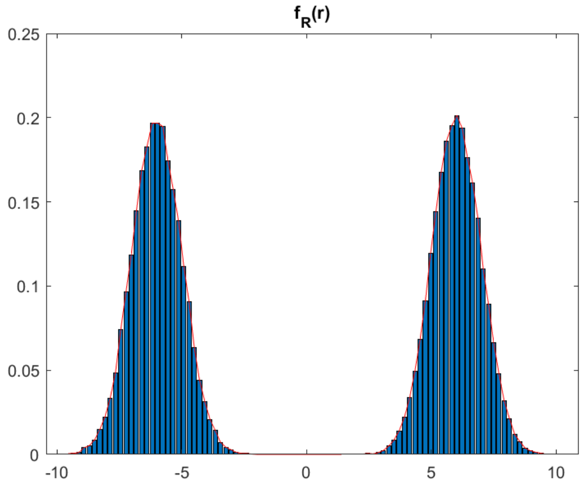

Figure 7.3.2: $f_R(r)$ histogram (**A=2**) with Amplification

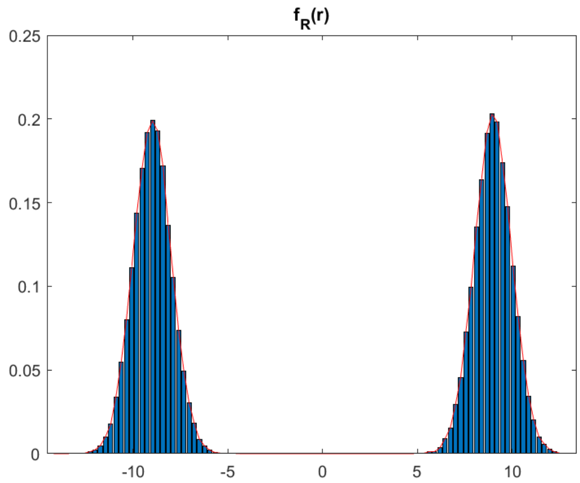

Figure 7.3.3: $f_R(r)$ histogram (**A=3**) with Amplification (**alpha =3**)

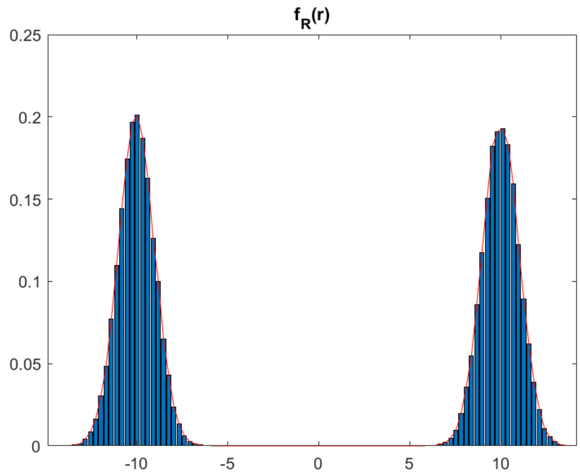

Figure 7.3.4: $f_R(r)$ histogram (**A=1**) with Amplification (**alpha =10**)

E[R|S=A] value and **E[R|S=-A]** value is nearly three times (**alpha=3**) of the value of the amplitude of the transmitting signal and when **alpha=10** those values become 10 times. Therefore, scaling factor occurs expected values. When considering above figures amplitudes of transmitting signal is increasing split the graph two parts and means are nearly equal to scaling factor times the transmitting signal amplitude. Split histograms are behaving as mirror object around the y axis. When increasing the scaling factor bandwidth is increasing but the width of a single component is decreasing.

Appendix

```
%%%%%%%%%%%%%%===Defining Variables===%%%%%%%%%%%%%%%%%%%%%%%

L = 100000; %Binary sequence of length
D = zeros(1,L); %10^3 or 10^5 zeros sequence
p = randperm(L,L/2); %Selecting the placement indexes for ones(1-1000)
D(p) = ones(1,L/2); %Replaces the zeros to ones reguarding to above placement
indexes
A = 1; %Amplitude

%%%%%%%%%%%%%%===Generating the Transmitting signals===%%%%%%%%%%%%%%%%%%%%%%%

S = zeros(1,L); %10^3 or 10^5 zeros sequence
for i = 1:L
    if D(i) == 1 %if D = 0 amplitude set => A
        S(i) = A;
    else
        S(i) = -1*A; %if D = 1 amplitude set => -A
    end
end

%%%%%%%%%%%%%%===Generating the AWGN===%%%%%%%%%%%%%%%%%%%%%%%
M = 0;
sigma = 1;
N = M + sigma*randn(1,L);

%%%%%%%============Question 6 interference part======%%%%%%%%%%
%generating intterference
MI = 0;
sigmaI = 1;
I = MI + sigmaI*randn(1,L);
%%%%%%%%=============================================%%%%%%%%%%%

alpha = 10;
%%%%%%%%%%%%%%===Generating the Recived signals===%%%%%%%%%%%%%%%%%%%%%%%
%R = S + N; %for Question 5
%R = S + N + I; %for Question 6
R = alpha*S + N; %for Question 7
%%%%%%%%%=========================================%%%%%%%%%%
figure;
stairs([1:L],R);
title("Received Signal");




%%%%%%%%%%%%%%===Decoded signals===%%%%%%%%%%%%%%%%%%%%%%%

tau = 0;
Y = zeros(1,L);
for i = 1:L
    if R(i) > tau
        Y(i) = A;
    else
        Y(i) = -1*A;
    end
end

%%==plotting transmitted signal==%
figure;
subplot(2,1,1);
stairs([1:L],S);
title("Transmitted Signal");
xlim([0 L]);
ylim([-1*A-1 A+1]);
```

```
%%==plotting Decoded signal==%
subplot(2,1,2);
stairs([1:L],Y);
title("Decoded Signal");
xlim([0 L]);
ylim([-1*A-1 A+1]);


%%%%%%%%%%%%%===Bin sequence===%%%%%%%%%%%%%%%%%%%%%%%%

bin0 = 100; %Number of bins
Rmin = min(R); %minimum value of list for width calculation
Rmax = max(R); %maximum value of list for width calculation
width = (Rmax-Rmin)/(bin0-1); %width calculation
bins = [Rmin-width/2:width:Rmax]; %Histogram value list

y = zeros(1,bin0);% Bins y values
for i = 1:L
    for j = 1:bin0
        if (R(i) >= bins(j)-width/2) && (R(i) < bins(j)+width/2)
            y(j) = y(j) + 1;
        end
    end
end

new = y/width;

%histogram plot
figure;
bar(bins,new);
title("Histogram of R");

%using the MATLAB buit in function hist()
figure;
hist(R,bin0);
title("Histogram of R (Using built-in function)");


%%%%%%%%==========plotting the PDF of f_R|S(r|S=A)=========%%%%%%%

list1 = []; %creating a list for assign values when S=A
index = 1;

for i = 1:L
    if S(i) == A
        list1(index) = R(i);
        index = index + 1;
    end
end

bin = 100;
Rmax1 = max(list1);%maximum value of list for width calculation
Rmin1 = min(list1);%minimum value of list for width calculation
width1 = (Rmax1-Rmin1)/(bin-1); %width calculation
bins1 = [Rmin1-width1/2:width1:Rmax1]; %Histogram value list
[y1,x1] = hist(list1,bins1); %plotting the histogram
y1 = y1/((index-1)*width1);
figure;
bar(x1,y1);
hold on;
plot(x1,y1,'r'); %plotting the pdf
title("f_{R|S}(r|S=A)");
```

```matlab
%%%%%%%%%==========plotting the PDF of f_R|S(r|S=-A)=========%%%%%%%%

list2 = []; %creating a list for assign values when S=-A
index = 1;

for i = 1:L
    if S(i) == -1*A
        list2(index) = R(i);
        index = index + 1;
    end
end


Rmax2 = max(list2);%maximum value of list for width calculation
Rmin2 = min(list2);%minimum value of list for width calculation
width2 = (Rmax2-Rmin2)/(bin-1); %width calculation
bins2 = [Rmin2-width2/2:width2:Rmax2]; %Histogram value list
[y2,x2] = hist(list2,bins2); %plotting the histogram
y2 = y2/((index-1)*width2);
figure;
bar(x2,y2);
hold on;
plot(x2,y2,'r'); %plotting the PDF
title("f_{R|S}(r|S=-A)");



%%%%%%%%%==========plotting the PDF of f_R|S(r|S=-A)=========%%%%%%%%

%E[R|S=A] calculation
ERlSA = 0;
for i = 1:bin
    ERlSA = ERlSA + (x1(i)*y1(i)*width1); %mean function
end
fprintf("E[R|S=A] = %f\n",ERlSA);

%E[R|S=-A] calculation
ERlS_A = 0;
for i = 1:bin
    ERlS_A = ERlS_A + (x2(i)*y2(i)*width2); %mean function
end
fprintf("E[R|S=-A] = %f\n",ERlS_A);

%E[R] calculation
[y,x] = hist(R,bins);
y = y/(L*width);
ER = 0;
for i = 1:bin0
    ER = ER + (x(i)*y(i)*width); %mean function
end
fprintf("E[R] = %f\n",ER);

%plotting the PDF of f_R(r)
figure;
bar(x,y);
hold on;
plot(x,y,'r');
title("f_R(r)");
```