Small Business Valuation with Use of Cash Flow Stochastic Modeling

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Abstract— **Enterprise can be described with vector of financial characteristics e.g. revenue, net profit, net working capital, depreciation, debt etc. Vector evolution can be modeled with use of system of recurrent equations. These equations can be combined in three groups: equations of the income statement, equations of sources and uses of funds and balance equations. System parameters can be obtained using financial performance analysis. Cash flow can be calculated using vector components. Discounted cash flow method is used for business valuation. In real systems there is an uncertainty in all parameters. This uncertainty can be modeled utilizing stochastic approach. Monte Carlo simulation can be adopted to forecast cash flow distribution and to predict the risks caused by uncertainty. We show that once simulation model is set up, it is a simple matter to analyze the principal sources of uncertainty in the cash flows and to see how much this uncertainty could be reduced by improving the forecasts of sales or costs. Practical realization of this approach is discussed in the paper. Finally, we demonstrate how changes in model parameters influence cash flows.**

Keywords—business valuation; cash flow; discounted cash flow method; stochastic modeling; parameter uncertainty; risk prediction; Monte Carlo simulation; cash flow forecast; enterprise evolution

I. FORMULATION OF THE PROBLEM. TOP-DOWN ESTIMATE

The modern concept of company management implies a constant process of major strategic and operational decision making. Before making such a decision it is necessary to estimate its impact on the company value. [1, 2, 9, 11]. Value based management is usually grounded on value maximization. Value maximization in this case is the common goal of all company`s aspirations, analytical methods and management techniques. Decision making is based on key value drivers and is usually based on decision making using key value drivers.

To conduct VBM concept one needs tools providing easy and fast value estimation, key value drivers detection and support in making decisions leading to company value gain. In this paper we discuss the practical approach to express-methods of business valuation based on imitational modeling utilization. The general model and practical approach to its realization is discussed.

As is known [3, 4, 5, 6, 7], the company value is determined by its discount future cash flow. New value appears only when companies produce returns of invested capital that exceeds that of the cost of capital. Therefore, central problems in cost estimate are cash flow forecasting [8] and risk analysis. Usually, cash flow forecasting is carried out with use of cash flow component analysis.

Brief overview of primary approaches to cash flow forecasting

Business valuation employs several methods for cash flow forecasting. These methods can be separated into three main categories:

A. Heuristic methods

B. Methods based on formalized appraisal of past data and trend detection (regression models, autoregressive models and learning models)

C. Methods based on creation of imitational models, describing company parameters interconnection

A. Heuristic methods

Heuristic methods are usually placed on specialist`s subjective opinion. Specialists usually use their experience and research that has been run in similar business spheres in order to predict future developments in the company. In budgeted years, a table of expected production volumes, sales volumes, costs, requirements for the acquisition of equipment etc., is formed. Cash flow is calculated using this table. Usually a few different scenarios are considered. Sometimes only few specific characteristics are forecasted and expected cash flow is calculated. For instance, when the growth rate is given revenue in future years can be forecasted without additional information. Forecasted revenue, profits and costs are guaranteed by the person, who makes this forecast, his experience, intuition and imagination. An expert here can be a manager or CEO of a company. A core defect of this method is the level of subjectivity, which hinders leadership to create an objective forecast of future development.

B. Regression and autoregressive methods

Regression methods belong to the class of economics and mathematics methods [14, 15]. In this methods it is assumed that the future dynamics of revenue, costs, cash flow etc. has been changing in the past and will be changing in future according to particular regression model (trend), of which the parameters could be distinguished with use of past dynamics. The data that can`t be taken into account is inserted into the model as a random deviation. The visual analysis of past data or the research of economical essence in prospective trends can form a regression model type. In [4] the three-stage model is discussed. The idea that company goes through three stages of its development: growth stage, conversion period and maturing period has been taken into consideration; the older a company becomes the lower the growth rate it has. In the maturing period, the growth rate of revenue, profits and return on equity stabilizes and is sustained at a constant level. It can be represented as algebraic dependence with unknown parameters.

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Lastly, in economical modeling which is connected to time series forecasting autoregressive models are commonly used [13]. Unlike traditional regression models autoregressive models consider correlations between different processes in the company. These models match real processes more precisely. The main difficulty in the realization of these methods is connected with the definition of the model parameters. These parameters are usually determined with the use of the previous data. Thus, the methods mentioned above can be attributed to learning models. Once a model type and all the parameters are defined, forecasting no longer remains a complex problem. The core problem of this method is that it doesn`t allow the comparison of different particular management decisions. Trends are preset by regression analysis of past periods of the company's life. Also, here it is hard to estimate the impact of the disparate key value drivers and their interconnection. An approach can be best applied in the case if it is presumed that the future development will sustain stability and the trends won`t change in future. This block of practices can be supplemented by neural network models of forecasting, which are also grounded on the previous data analysis. This kind of approach has appeared to be effective for technical analysis of the stock market.

Regression, autoregressive and neural network models can`t be applied to our specific problem of decision making due to fact that the decisions usually changes the dynamics.

C. Imitational models

Many authors noted expediency and the principal possibility of imitational models application to the research of enterprise economic problems. The authors [1 – 4, 10, 12] were closest to considered problem of imitational modeling of the innovational business process.

The use of imitational systems for financial process dynamics description haven`t met any principled criticism. However, practical realization of such technologies in an acting enterprise valuation encounters a number of technical difficulties. We don`t know of any evaluation reports that have implemented the imitational modeling results as a part of the valuation process. We managed to overcome the said difficulties to some extent and provide effective implementation of imitational modeling in business valuation practice. Our experience in the use of imitational modeling in numerous evaluation reports gives us ground to assume that the developed technologies and corresponding software can be widely introduced in the modern business valuation practice.

In this paper we briefly expound the main provisions of used technology and give a simplified example of imitational modeling use in one particular enterprise valuation.

II. IMITATIONAL MODELING APPLICATION TECHNICS ON FINITE-DIFFERENCE EQUATION BASIS

According to the chosen approach, a company is described as a vector, in which the components are financial parameters. Vector dynamics is described by employing finite-difference equations. These equations show how to find state at the moment "t" if the state in the moment "t - 1" is given.

$$A_t = F(A_{t-1}) \quad (1)$$

A_t – state vector, which components are net profit, fixed assets, net working capital etc. A discrete step here is one year. The state vector value in year "t" determines the state vector value in year "t + 1", value at year "t + 1" determines value at year "t + 2". Thus, having the state vector value at the moment "0", its value in future years can be calculated.

There are no principal difficulties with realization of this approach, but the practical solution faces a number of issues:

A. Construction of a model

B. Determination of parameters

C. Analysis of forecast uncertainty

A. Construction of a model

We use the set of finite-difference equations as a model of a company. Equations are based on business process analysis. Approach can be used only for small companies, which have a single process of producing products or services of the same type. It is assumed that there are no other sources of income in the company, not are there any branches or affiliated companies. Besides, all profit is divided between dividends and future investments. Thereby, the approach and model itself can be applied only for small business.

B. Initial conditions and model parameters

State vector characterizing company's financial condition future dynamics can be forecasted by solving a set of equations with preset initial conditions. Values of all the financial parameters at the moment have to be determined. These values can be obtained from the company's financial statements.

Model parameters are the financial characteristics of a company. All parameters can be divided into three groups: 1. parameters representing economic conditions that emerged in a company, which are constant during the future period, 2. parameters representing market prices and thus market situation, which are uncertain, and 3. managerial parameters, which determine the policy of the company and are defined by the manager`s solution. Examples of the economic condition parameters can be the amount of net working capital and fixed assets needed to produce a fixed number of products or services. These types of constants can be obtained from an economic activity analysis. The situation becomes more complicated when dealing with external factors such as the market situation. The market price depends on the supply and demand ratio in the market segment. These types of factors are inserted as parameters in a set of equations and can`t be determined precisely. These elements contain uncertainty and special methods are usually used to determine its mean value and dispersion. Finally, the third group is the group of managerial parameters. Managerial parameters determine the company`s policy, for instance if it is a dividend policy, which determines the share of the net profit that will be distributed among shareholders. The ratio between future cash flows behavior and the values of managerial coefficients can be researched by inserting different coefficients.

C. Analysis of forecast uncertainty

Every model is only the approximate delineation of the real business process. The real process can deviate from the forecasted company development scenario. Deviations can be observed as a difference between the forecasted cash flow and the actual cash flows. Hence, the financial results that are expected will also vary. Inaccuracy of the model itself is strengthened by inaccuracy of the parameters affecting the development process of a company. Usually several scenarios of development are considered to estimate the risks.

However, the Monte-Carlo method is considered to be most effective. In order to employ this method, a uniformly distributed random quantity is inserted into the set of equations (1). The solution is then found many times for different realizations of uncertain parameters. The value is chosen from the range set as an uncertainty interval. As a result the histogram of forecasted financial magnitudes is obtained.

III. Thoughtful description of business processes in the company. Assumptions and presuppositions.

The example of the model of the company as a set of linear finite-difference equations of the first order with initial conditions, is based on the business process analysis of the assessed company can be found below.

It is assumed that in the beginning of the period that there is equity from the company`s own resources and liability from borrowed funds. The funds provide fixed assets and the net working capital for the first year. Revenue is attributed to the end of the period, therefore revenue for the next year is determined by fixed assets and net working capital referred to the previous year. Fixed costs are neglected and cost price is determined as a percent of revenue. Depreciation is also neglected. Earnings before interest and taxes (EBIT) equals the difference between the revenue and costs. The net profit is a measure of the profitability of a venture after accounting for all the costs which would be equal to EBIT after the deduction of taxes and payments. Tax can be determined as a fixed percentage of EBIT. It is assumed that in the beginning of the period that a bank load was obtained and the scheme of annuity payments was chosen. Thus, payments can be calculated according to the known formula. According to the dividend policy net profit is split into two parts: 1. first there are dividends and 2. the second one is the increase of fixed assets and the net working capital. In order to get the maximum efficiency from invested capital, fixed assets and the net working capital should be increased proportionally. Equity is obtained by subtracting total liabilities from the total assets of the shareholders. These assets and liabilities can be determined as: equity (beginning of year) + net income (net money one gained) − dividends how much money one gained or lost so far = equity (end of the year).

Therefore, the state of the company can be described using these parameters:

$\mathrm{REV}(t)$ – revenues from sales,

$\mathrm{NWC}(t)$– net working capital,

$\mathrm{FA}(t)$ – fixed assets,

$\mathrm{PAY}(t)$ – annual payment,

$\mathrm{CGS}(t)$ – cost of sales,

$\mathrm{TAX}(t)$ – taxes,

$\mathrm{NET}(t)$ – net profit,

$\mathrm{DIV}(t)$ – dividends.

Business-process described above, can be represented with set of equations:

$$\mathrm{REV}(t) = \frac{\mathrm{NWC}(t-1)}{a_6} = \frac{\mathrm{FA}(t-1)}{a_7}$$

$$\mathrm{PAY}(t) = D(1) * \left(a_2 + \frac{a_2}{(1+a_2)^n - 1}\right)$$

$$\mathrm{CGS}(t) = a_1 \mathrm{REV}(t)$$

$$\mathrm{TAX}(t) = a_3(\mathrm{REV}(t) - \mathrm{CGS}(t) - \mathrm{PAY}(t))$$

$$\mathrm{NET}(t) = \mathrm{REV}(t) - \mathrm{CGS}(t) - \mathrm{PAY}(t) - \mathrm{TAX}(t)$$

$$\mathrm{DIV}(t) = a_5 * \mathrm{NET}(t)$$

$$\mathrm{delta}(t) = \mathrm{NET}(t) - \mathrm{DIV}(t)$$

$$\mathrm{deltaFA}(t) = \mathrm{delta}(t) * \frac{a_7}{a_6 + a_7}$$

$$\mathrm{deltaNWC}(t) = \mathrm{delta}(t) * \frac{a_6}{a_6 + a_7}$$

$$\mathrm{FA}(t) = \mathrm{FA}(t-1) + \mathrm{deltaFA}(t)$$

$$\mathrm{NWC}(t) = \mathrm{NWC}(t-1) + \mathrm{deltaNWC}(t)$$

$$E(t) = \mathrm{FA}(t) + \mathrm{NWC}(t-1) - D(t-1)$$

Where

a_1 – cost (percentage of revenue),

a_2 – interest rate,

a_3 – tax rate,

a_5 – share of dividends,

a_6 and a_7 – coefficients that raise NWC and FA to fit balance equations,

$\mathrm{deltaFA}(t)$ – change of fixed assets,

$\mathrm{deltaNWC}(t)$ - change of net working capital.

The initial condition for the set of equations is E(1) and D(1).

IV. Business valuation

Cash flow (CF) dynamics can be evaluated by solving the set of equations above for forecast horizon (T).

$$\mathrm{CF}(t) = \mathrm{REV}(t) - \mathrm{CGS}(t) - \mathrm{PAY}(t) - \mathrm{TAX}(t) - \mathrm{deltaNWC}(t),$$

$$t = 1..T$$

Present value (PV) depends on cash flow (CF), future value (FV), the cost of equity (r) and can be determined with the use of this formula

$$\mathrm{PV} = \sum_{t=1}^{T} \frac{\mathrm{CF}(t)}{(1+r)^n} + \frac{\mathrm{FV}}{(1+r)^T}$$

FV is calculated in terms of the model of free cash flow with the constant growth rate (g)

$$\mathrm{FV} = \frac{\mathrm{CF}(T)}{r - g}$$

V. Software implementation

There is a wide variety of applied programs that could be employed to solve the problem. In this paper the MATLAB system was used for implementation of the stated methods and algorithms. MATLAB is appropriate for the solving of problems appearing in mathematical modeling and analysis of financial applications including cash flow forecasting, risk analysis and business valuating. We represent example in which application written in MATLAB with interfaces using MS Excel files is used to implement methods brought up above.

There are two modes in which the program can work: in the first one, a precise realization of development is modeled, in the second one, many realizations can be researched. Let`s take a closer look on these modes.

A. First mode

The result of the work of the program in the first mode is a consolidated reporting table in which you can find: FA, NWC, REV, NET, deltaFA, deltaNWC, DIV, TAX, D, PAY, E, a, PV, where a – coefficients, PV – present value.

Input data needed for program work is taken from MS Excel file and is represented on Table 1.

Forecast period, years	5
Initial capital, units	250
Credit period, years	10
Discount rate, percent	10
Market growth rate, percent	2
Number of parameters	8
Cost, share of revenue	0,8
Debt interest rate, share	0,1
Tax rate, share	0,2
Depriciation rate, share	0
Dividends, share	0,2
NWC needed to get REV, share	0,5
FA needed to get REV, share	1,5
Mortgage constant, share of equity	0,8

Table 1- Input data mode 1

A typical graph of forecasted values is represented on the Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, and Figure 6.

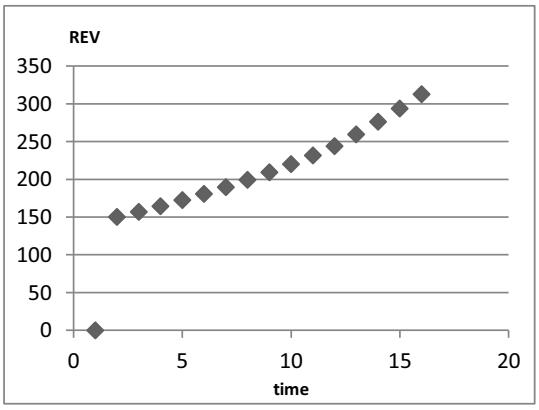

Figure 1

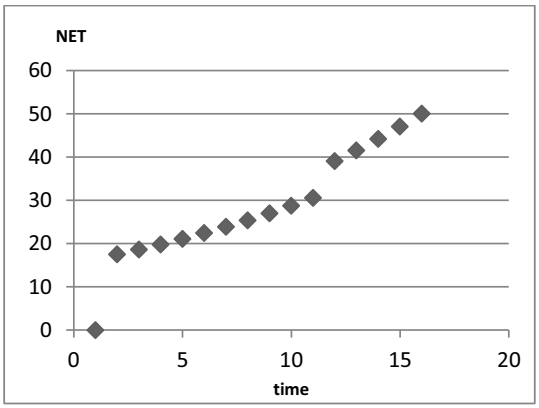

Figure 2

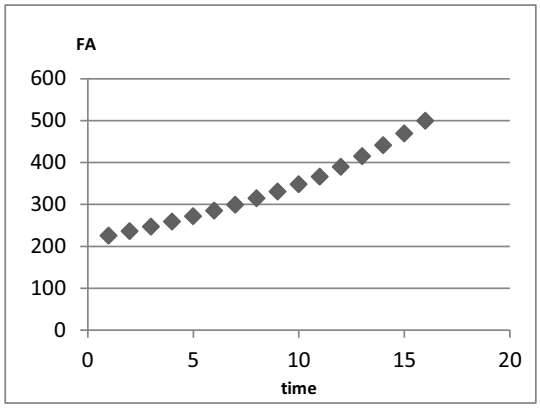

Figure 3

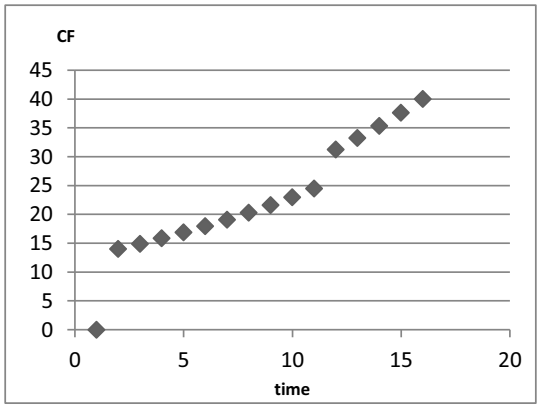

Figure 4

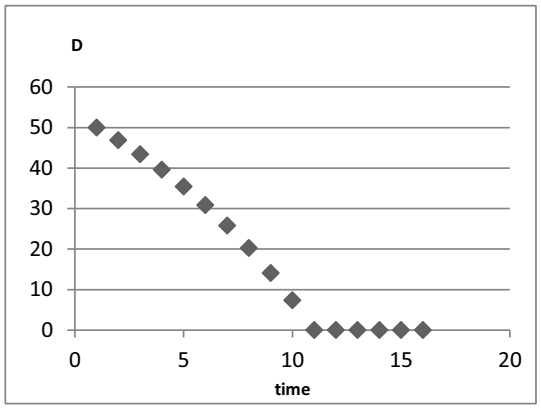

Figure 5

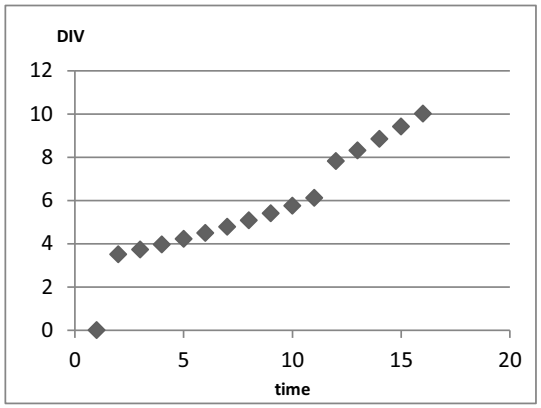

Figure 6

Exponential growth of revenue, the net profit and cash flow can be seen. Besides, the decrease in debt has a typical form for annuitant payments. The rupture in the graphs is visible at the moment when all credit payments are completed.

The present value of the company is estimated based on data obtained from modeling. In this example, the company`s market value is 234,2 units.

Thus, the model stated above allows us to forecast the future financial parameters of the company. It is important to note that the manager can model different scenarios of company development, thus changing the inserted managerial parameters, for instance is the share of dividends.

B. Second mode

Second mode is designed for risk analysis. The Monte-Carlo method is used here for random quantity formation. The table below represents the configuration of data given to the program. Here three types of parameters are represented: domestic and external parameters which are uncertain and are set as intervals and managerial factors that are preset. Input data needed for program work is taken from MS Excel file and is represented on Table 2.

Number of iterations, count	15
Number of realizations, count	5000
Initial capital, units	250
Credit period, years	10
Discount rate, percent	5 - 15
Market growth rate, percent	1,5 – 2,5
Number of parameters	8
Cost, share of revenue	0,8
Debt interest rate, share	0 – 0,2
Tax rate, share	0,1 – 0,3
Dividends, share	0,2
NWC needed to get REV, share	0,4 – 0,6
FA needed to get REV, share	1,4 – 1,6
Mortgage constant, share of equity	0,8

Table 2 Input data mode 2

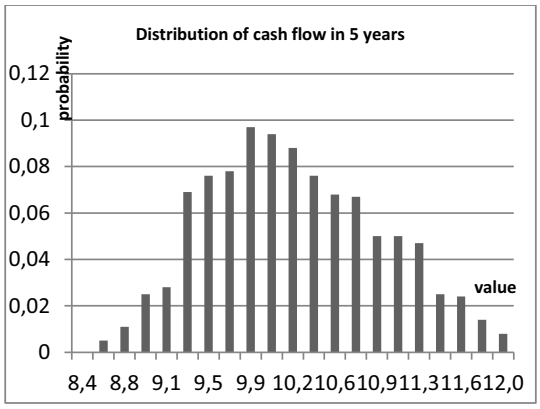

Figure 7

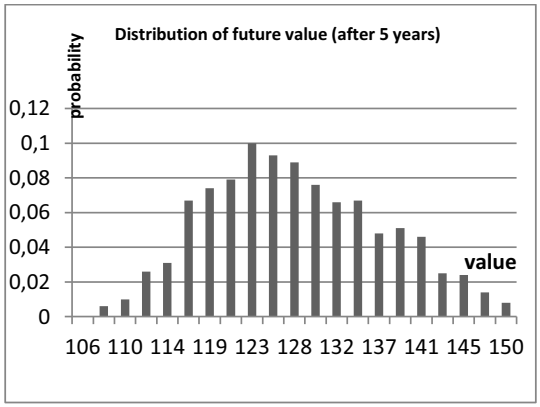

Figure 8

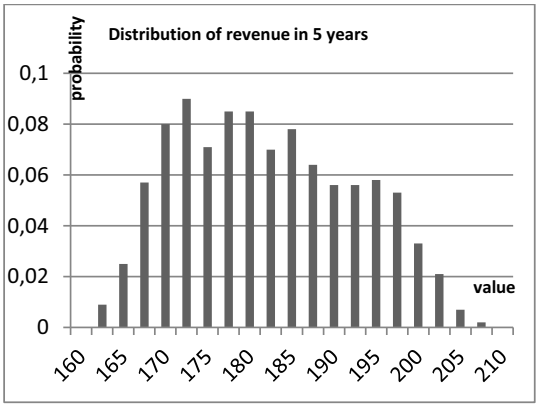

Figure 9

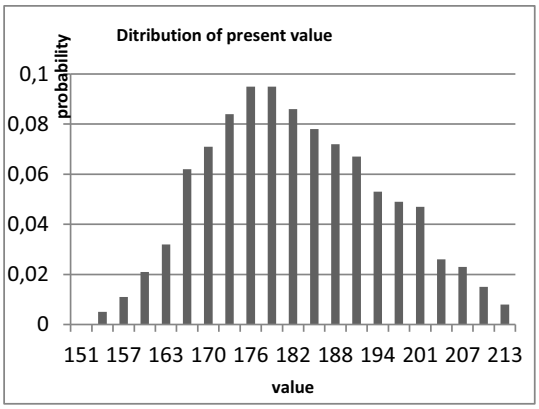

Figure 10

Histograms on Figure 7, Figure 8, Figure 9, and Figure 10 show potential financial characteristics in conditions of uncertainty on a number of parameters, describing external and internal environment.

VI. Discussion

The use of stochastic modeling for risk analysis and company development forecasting meets no principal difficulties. However, the practical implementation faces a number of technical difficulties connected with the formalization of the problem. In this paper it is suggested how to describe the company as a set of financial characteristics united in a state vector, in which dynamics can be described employing finite-difference equations. Wherein it is important to distinguish the parameters that can be changed by a manager and parameters that are independent. Thus, the best managerial solutions can be made by running mathematical modeling. The utilization of this approach allows us to actualize a popular concept of value based management. In this case, the control parameters will be key value drivers.

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