1) Calculate the angular resolution of a telescope with a 10-meter diameter mirror observing a binary star system at a wavelength of 550nm.

Solution: By using Rayleigh criterion we have

$$\theta = 1.22 \frac{\lambda}{D}$$

where $\theta \rightarrow$ angular resolution

Given: $D = 10m$

$\lambda = 550nm$

$$\theta = 1.22 \times \frac{550 \times 10^{-9}}{10}$$

$$\theta = 6.71 \times 10^{-8} \text{ radians}$$

$$\theta = \left(\frac{360 \times 3600}{2\pi} \times 6.71 \times 10^{-8}\right)''$$

$$\theta = 0.0138''$$

2. Compare and contrast the advantages and limitations of radio telescopes versus optical telescopes. Give a few examples of where you would use each.

Radio telescopes and optical telescopes are both important tools in astronomy, yet they differ significantly in how they observe the universe and what they show. Optical telescopes detect visible light, providing high-resolution images of stars, planets, galaxies, and nebulae. Their short wavelengths allow for detailed observations, making them ideal for studying the fine structure and composition of celestial objects. However, optical telescopes face limitations such as they only work effectively at night, require clear skies, and are affected by atmospheric turbulence and light pollution. Additionally, visible light cannot penetrate dust clouds, preventing the observation of many regions in space.

On the other hand, radio telescopes detect much longer wavelengths, which allows them to observe the universe regardless of weather or daylight. Radio waves can pass through dust clouds, revealing hidden phenomena such as pulsars, quasars, and the cosmic microwave background. However, the long wavelengths mean radio telescopes generally have lower resolution, requiring very large dishes or arrays of telescopes working together through interferometry to achieve detailed images. They are also susceptible to interference from human-made radio signals, necessitating remote locations and advanced filtering.

Both types of telescopes complement each other. Optical telescopes are best for high-resolution imaging of objects emitting visible light, while radio telescopes excel at uncovering phenomena invisible to optical instruments and providing continuous observations. Together, they provide a more complete understanding of the universe, each unveiling unique and vital aspects of cosmic phenomena.

3. Explain the concept of the event horizon of a black hole. In your explanation, include: what happens to space-time at this boundary, why it's called a "point of no return”.

The event horizon of a black hole is the boundary in space-time that marks the outer edge of the black hole, beyond which nothing can escape its gravitational pull. This boundary is not a physical surface but a mathematically defined region where the escape velocity exceeds the speed of light. According to general relativity, as one approaches the event horizon, the curvature of space-time becomes so extreme that all possible paths, even those of light, are bent inward toward the center of the black hole, known as the singularity. For an outside observer, an object falling toward the event horizon appears to slow down and fade away due to gravitational time dilation and redshift; the object never seems to actually cross the boundary and eventually becomes invisible. However, for the object itself, it would cross the event horizon in a finite amount of its own time without experiencing anything unusual at that precise moment. The event horizon is often called the "point of no return" because once anything—matter, light, or information—passes this boundary, it is lost to the outside universe forever and cannot affect anything outside the black hole. This unique property is why the event horizon is considered the defining characteristic of a black hole, as described by Stephen Hawking in "The Theory of Everything": it is the ultimate limit, beyond which the known laws of physics break down and all events are hidden from the rest of the universe.