

1. Partial Derivatives (Partial Derivatives)
 - Partial Derivatives: Derivatives of a function with respect to one variable, treating other variables as constants.
 - Notation: $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$
 - Example: $f(x, y, z) = x^2 + y^2 + z^2$
 $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 2y$, $\frac{\partial f}{\partial z} = 2z$
 - Chain Rule: If $f(x, y, z)$ is a function of x, y, z and x, y, z are functions of t , then
 $\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$
 - Implicit Differentiation: If $f(x, y, z) = 0$, then
 $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx} + \frac{\partial f}{\partial z} \frac{dz}{dx} = 0$
 - Directional Derivatives: Derivatives of a function in a specific direction.
 - Gradient: A vector of partial derivatives, $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$
 - Example: $f(x, y, z) = x^2 + y^2 + z^2$
 $\nabla f = (2x, 2y, 2z)$
 - Directional Derivative: $\frac{\partial f}{\partial \mathbf{u}} = \nabla f \cdot \mathbf{u}$, where \mathbf{u} is a unit vector in the direction of interest.
 - Example: $\frac{\partial f}{\partial \mathbf{u}} = (2x, 2y, 2z) \cdot (1, 0, 0) = 2x$

2. Maxima and Minima (Maxima and Minima)
 - Local Extrema: Points where a function has a local maximum or minimum.
 - Necessary Conditions: If $f(x, y, z)$ has a local extremum at (x_0, y_0, z_0) , then
 $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$, $\frac{\partial f}{\partial z} = 0$
 - Sufficient Conditions: If the Hessian matrix is positive definite, then (x_0, y_0, z_0) is a local minimum. If negative definite, then it is a local maximum.
 - Hessian Matrix: $H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{pmatrix}$
 - Example: $f(x, y, z) = x^2 + y^2 + z^2$
 $H = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$
 - Global Extrema: Points where a function has a global maximum or minimum.
 - Example: $f(x, y, z) = x^2 + y^2 + z^2$
 - Global Minimum: $(0, 0, 0)$

3. Applications (Applications)
 - Optimization: Finding the maximum or minimum of a function subject to constraints.
 - Lagrange Multipliers: A method for finding the local maxima and minima of a function subject to equality constraints.
 - Example: Find the maximum of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $g(x, y, z) = x + y + z = 1$.
 - Solution: $\nabla f = \lambda \nabla g$
 $(2x, 2y, 2z) = \lambda (1, 1, 1)$
 $2x = \lambda$, $2y = \lambda$, $2z = \lambda$
 $x = y = z = \frac{\lambda}{2}$
 - Substitution: Solving for one variable in terms of others and substituting into the function.
 - Example: Find the maximum of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $g(x, y, z) = x + y + z = 1$.
 - Solution: $z = 1 - x - y$
 $f(x, y) = x^2 + y^2 + (1 - x - y)^2$
 $\frac{\partial f}{\partial x} = 2x + 2(1 - x - y)(-1) = 0$
 $\frac{\partial f}{\partial y} = 2y + 2(1 - x - y)(-1) = 0$
 $x = y = \frac{1}{2}$
 $z = 1 - \frac{1}{2} - \frac{1}{2} = 0$
 - Real-world Applications: Optimization problems in physics, engineering, economics, etc.
 - Example: Finding the maximum volume of a rectangular box with a fixed surface area.

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