Computational Methods in Finance

- Please note: for the problems using data we are not interested in all the minor steps performed when analyzing the data. However, you should explain your broad reasoning and hand in a final report.
- Do not include any failed attempts at modeling just to give volume to the final report.
- You need to argument with graphs or selected numerical results all the conclusions you draw. Put important plots and tables within the report and relegate any non essential ones to an appendix at the end of the document.
- Please add supporting arguments (e.g., output) in the appendix. The whole report should be constructed as a regular journal article.
- The final write-up (excluding the appendix) should not be more than 10-15 pages.
- Communication with other students either physical or virtual is strictly forbidden.

For instructor's use only

Problem	Points	Score
A	100	
B	100	
Bonus 1	20	
Bonus 2	20	
Total	200	

Problem A: Asian Option Pricing using Monte Carlo Control Variate. The payoff of an arithmetic Asian call option is:

$$\left(\frac{1}{N+1}\sum_{i=0}^{N} S_{t_i} - K\right)_+ .$$

Its value may be computed using straight Monte Carlo simulations. However, in order to obtain a small standard error, the number of simulations must be very high. To solve this computationally extensive problem, we will use the payoff of a geometric Asian call option as the control variate:

$$\left(\left(\prod_{i=0}^{N} S_{t_i}\right)^{\frac{1}{N+1}} - K\right)_+$$

The idea is to use the known analytic price of the geometric Asian and the distance between MC simulations to obtain an approximate for the analytical formula for the arithmetic Asian price.

In this problem we consider $r = 3\%,\ \sigma = 0.3,\ S_0 = 100$, and assume the goal is to price an arithmetic Asian call option with strike $K = 100$ and maturity $T = 5$.

We also assume the asset follows the standard log-normal/geometric Brownian motion model, the stock price dynamic may be written as:

$$S(\Delta t) = S(0)e^{((\mu - \frac{\sigma^2}{2})\Delta t + (\sigma\sqrt{\Delta t})\epsilon)}$$

(a) The price of a geometric Asian option in the Black-Scholes model is given by:

$$P_g = e^{-rT}\left(S_0 e^{\rho T} N(d_1) - K N(d_2)\right)$$

where:

$$\rho = \frac{1}{2}\left(r - \frac{1}{2}\sigma^2 + \hat{\sigma}^2\right)$$

$$\hat{\sigma} := \sigma\sqrt{\frac{2N+1}{6(N+1)}}$$

such that $\hat{\sigma}$ is adjusted sigma and N is the total number of trading days ($T * 252$).

$$d_1 := \frac{1}{\sqrt{T}\hat{\sigma}}\left(\ln\left(\frac{S_0}{K}\right) + \left(\rho + \frac{1}{2}\hat{\sigma}^2\right)T\right)$$

$$d_2 := \frac{1}{\sqrt{T}\hat{\sigma}}\left(\ln\left(\frac{S_0}{K}\right) + \left(\rho - \frac{1}{2}\hat{\sigma}^2\right)T\right)$$

Use the above formula to price this geometric Asian call option.

(b) Implement a Monte Carlo scheme to price an arithmetic Asian call option (P_a^{sim}). Use $M = 1,000,000$ simulations. Record the answer, a confidence interval and the time it takes to obtain the result.

(c) Implement a Monte Carlo scheme to price a geometric Asian Call option (P_g^{sim}).

(d) Using $M = 10,000$ simulations and the same exact random variables create the following:

- numbers X_i which are M replications for the arithmetic Asian Option price
- numbers Y_i which are M replication for the geometric Asian Option price

Finally, calculate b^* such that:

$$b^* = \frac{\sum_{i=1}^{M}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{M}(X_i - \overline{X})^2}$$

Note that b^* is actually the slope of the regression line $Y = a + bX + \varepsilon$. Please also record the price of the arithmetic P_a^{sim} and the geometric P_g^{sim}.

(e) Calculate the error of pricing for the geometric Asian: $E_g = P_g - P_g^{sim}$

(f) Calculate the modified arithmetic option price (P_a^*) as:

$$P_a^* = P_a^{sim} - b^* E_g$$

Compare with the results in (b). Comment. Vary the value of M in part (d). What do you observe.

Bonus 1 For this problem apply the Asian option pricing in practice. Download from the Bloomberg terminal Asian option data for an equity at your choice. Consider five different maturities and five strike prices. Construct a table and repeat the parts (a) to (f) using your data. In order to access Asian options for your equity, follow the instruction below. For example, if you want to access Asian options on IBM Us Equity:

- Enter IBM EQUITY OMON. OMON function stands for Option Monitor.
- On the OMON page, type OVME. OVME function (Option Valuation) is Bloomberg main pricing application for options on equity, indexes, funds, bonds, bond futures and short term interest rate futures.
- On the top of the screen, you will find "Product" tab. Under this tab, click on the "show all styles". You will find all types of options.
- Click on number 17 which is Asian option.
- You can click on the "Matrix" subtab to see your data as a matrix. You can modify this matrix by checking/unchecking variables at the right side of the screen. For this specific question, you can check strike prices and maturities.

Problem B. Principal Component Analysis.

1. Download daily prices for the components of DJIA for the last 5 years. Construct the corresponding matrix of standardized returns.

$$Y_{it} = \frac{R_{it} - \overline{R}_i}{\overline{\sigma}_i}$$

$$i = 1, ..., N \text{ and } t = 1, ..., T$$

with

$$\overline{R}_i = \frac{\sum_{t=1}^{T} R_{it}}{T} \text{ and } \overline{\sigma}_i = \sqrt{\frac{\sum_{t=1}^{T} (R_{it} - \overline{R}_i)^2}{T}}$$

2. Calculate the sample correlation matrix

$$C_{ij} = \frac{1}{T} \sum_{t=1}^{T} Y_{it} Y_{jt}$$

3. Calculate the eigenvalues and eigenvectors of the matrix C_{ij} and graph the eigenvalues $\lambda_1 > \lambda_2 > ....$ What percent of the trace is explained by summing the first 5 eigenvalues?

4. Consider the first eigenvector and denote it by $(V_1, V_2, ..., V_N)$. Define the factor

$$F_t = \frac{1}{\sqrt{\lambda}} \sum_{i=1}^{N} \frac{V_i}{\overline{\sigma}_i} R_{it} \text{ with } t = 1, 2, ..., T$$

Note: F_t represents the returns of a portfolio which invests $\frac{1}{\sqrt{\lambda_1}} \frac{V_i}{\overline{\sigma}_i}$ dollars in the i^{th} stock. Calculate the sample mean and sample standard deviation of the factor F.

5. Consider a series of daily returns of the DIA ETF for the same time period. Perform a linear regression of the returns of F with the standardized returns of DIA. Calculate the R-squared of the regression and discuss whether F and the capitalization-weighted market portfolio are good proxies for each other. Discuss the result and argue why F and the particular market index might be related.

6. Consider the 5 eigenportfolios (factors) corresponding to the top 5 eigenvalues in the PCA conducted previously. That is, each of the 5 factors are constructed in a similar way with the construction in part4. Let F_{kt} denote the daily return of the eigenportfolio k on date t. For each of the 30 equity in the DJIA do the following:

   - Estimate the mean μ_s and standard deviation σ_s for return of each equity R^s. Standardize the returns $r_k^s = (R_k^s - \mu_s)/\sigma_s$, where k denote the time.
   - Run a regression with the 5 factors and obtain the parameters β_{sk} in:
   $$r_k^s = \sum_{i=1}^{5} \beta_{sk} F_k + \varepsilon_k^s.$$
   Please note that since the returns are standardized the regression intercept should be zero.

   Next we will be using the following factor model for stock returns:
   $$R^s = \mu_s + \sigma_s\Big(\sum_{k=1}^{5} \beta_{sk} F_k\Big) + \sigma_s \sqrt{\Big(1 - \sum_{k=1}^{5} \beta_{sk}^2\Big)} G_s$$
   where F_k with $k = 1, 2, ..., 5$ are uncorrelated Student-t variables with 3.5 degrees of freedom. We assume that G_s is also a Student-t variable with the same degrees of freedom, which is uncorrelated with F_k and with $G_{s'}$, if s' is a different stock. We will assume that each day return follows this structure. Generate realizations for R^s for the next 10 days for all the index components. Calculate the return of a sample portfolio equally weighted in its components. Repeat the experiment 100,000 times. Calculate 99% one-week VAR.

Bonus 2. Research paper For this problem refer to the paper at: `https://www.sciencedirect.com/science/article/pii/S0378437108010479`. The paper may be accesed from the Stevens network and a preprint may be found at: `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2066888`. In this paper the authors apply a Re-scaled Range (R/S) method and a Detrended Fluctuation (DFA) method to estimate the degree of long memory effects in high frequency financial data. The study uses High frequency data during a particular day. Repeat the two studied with any day data. Use a Bloomberg terminal to download high frequency data.