FE 630
Portfolio Theory and Practice
Final Examination

Due: 11:59pm, Tuesday, August 22, 2016

Total points scaled to 100%

Instructions

- **Read these instructions and follow them precisely.** Failure to follow these instructions will seriously affect your grade.
- **Independence:** All students must work independently. Checks for collaboration will be performed electronically. Improper collaboration will be reported to the Stevens Office of Graduate Academics. Please make yourself familiar with the Graduate Student Code of Academic Integrity.
- **Submission:** Submit your examination solutions via Canvas. The submission must contain the following two files:
  1. **Answer Document:** You must submit a single .pdf file that contains all of your answers including code printouts and graphs. The answer document must be typed and be in machine-readable form (i.e., it must not simply be a scanned image of handwritten answers). Do not submit your answer document in any format other than pdf. Any answer document that does not comprise a single machine-readable .pdf file will receive a grade of **zero**. Your answer document must include a *cover sheet* that states the course name, the date, and your name.
  2. **Source Code:** Your submission should also include a separate .zip file that contains all of the source code that you ran to obtain your solutions. I may run your source code to ensure that it provides the results that you claim.

  Hint: To satisfy these requirements, it may be to your advantage to use knitr in R, or to view this page for MATLAB.
- **Permissible Computer Languages:** You can use any matrix-oriented computer programming language (preferably R or MATLAB, but you could also use julia, or Python with Pandas, for example). Do not use *any* spreadsheets. Numerical problems solved manually or with spreadsheets will receive no credit.
- **Legibility and Logical Presentation:** Answer documents that are not easily legible, or not logically presented, or have a non-professional appearance, will not be graded.
- **Late Submission Policy:** No submissions will be accepted after the due date and time. Canvas will automatically reject any attempts at late submission.

  Only in extreme circumstances will exceptions be made to this late submission policy. Such extreme circumstances must be proved by verifiable documentation. Technical, network, or computer problems are not considered extreme circumstances. Therefore, you are advised to submit the homework assignment well before the due date and time.

Theory

For this part of the exam, for each answer, provide closed-form mathematical solutions and verbal commentary. You should not use numerical or computer generated results, except for Question 1, which will also require some simple arithmetic calculations.

Question 1. Portfolio Returns with Cash Flows.

Question 1.1

(Standard Time-Weighted Returns—Adjustment to Numerator). An investor gives \$1.0 million to a portfolio manager to manage for two years. Close to the end of the first year, the investor gives the manager a further \$0.1 million to manage. At the end of the first year, the portfolio value has risen to \$1.2 million. At the end of the second year, the portfolio value is \$1.44 million. Each year, the portfolio is valued *after* the external cash flow for that year.

a. Using the usual time-weighted method,

$$r = \prod_{t=1}^{T} \frac{W_t - C_t}{W_{t-1}} - 1,$$

compute the net rate of return $r = R - 1$ over the two years. (4 points)

b. Why, do you think, might this method give an estimate of return that could be too high? (1 point)

Question 1.2

(Alternative time-weighted method—Adjustment to Denominator). The basic conditions as in Question Question 1.1 apply. That is, an investor gives \$1.0 million to a portfolio manager to manage for two years. At the end of the first year, the portfolio value has risen to \$1.1 million and, at that time, the investor gives the manager a further \$0.1 million to manage. At the end of the second year, the portfolio value is \$1.44 million. Using the alternative adjustment method outlined below, compute the manager's rate of return for the two years. Show all of your working.

a. Let

- W_0 be the starting wealth,
- W_1^- be the wealth at the end of the first period *before* the extra investment is made,
- W_1^+ be the wealth at the end of the first period *after* the extra investment has been made, and
- W_2 be the wealth at the end of the second period.

Write an expression for the gross return $R = W_2/W_0$ in terms of a product of the gross returns over the two periods using *only* the quantities defined above, in which you adjust only denominators for cash flows. (3 points)

b. Substitute the numerical values into your expression, and find the net rate of return $r = R-1$ over the two years. Comment on any difference between the return computed this way, and the return you computed in Question 1.1. (1 point)

Question 1.3

(Money-Weighted Returns, IRR). An investor gives \$1.0 million to a portfolio manager to manage for two years. At the end of the first year, the investor gives the manager a further \$0.1 million to manage. At the end of the second year, the portfolio value is \$1.32 million. Using the method outlined below, compute the manager's annual Internal Rate of Return. Show all of your working.

a. Let

   - W_0 be the starting wealth,
   - W_2 be the wealth at the end of the second period, and
   - I be the extra money invested.

   Write an equation that is quadratic in R linking W_2, W_0 and I.

b. Solve the quadratic for R in terms of the quantities defined above.

c. Using the numerical values, find the IRR.

(5 points)

Question 2. Performance Attribution.

Brinson, Hood, and Beebower suggested a method of separating returns from asset allocation, security selection, and the benchmark. Let

- h_j be the weight that the asset allocator assigns to asset class j,
- h_j^{B} be the asset allocator's benchmark allocation for asset class j,
- r_j be the return that the security selector obtains by investing in securities from asset class j, and
- r_j^{B} be the benchmark return for asset class j.

Show ***mathematically*** (i.e., not graphically or geometrically) how a portfolio's return can be separated into a sum of four terms representing asset allocation return r_{AA}, security selection return r_{SS}, benchmark return r_{B}, and an interaction effect r_{INT}. In the geometric representation, give expressions for r_{AA}, r_{SS}, r_{B}, and r_{INT}.

(6 points)

Question 3. Quadratic Programming.

Question 3.1

Suppose you are using a quadratic program solver called `qps` with syntax

```
x = qps(Q, mu, A, b)
```

that performs the following operation: Find $\boldsymbol{x}$ so as to

$$\begin{array}{ll} \text{minimize} & \boldsymbol{x}^\top \boldsymbol{Q} \boldsymbol{x} - \boldsymbol{\mu}^\top \boldsymbol{x} \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} \geq \boldsymbol{b}. \end{array}$$

However, for your particular portfolio, you want to impose the constraints $\boldsymbol{B}\boldsymbol{x} \geq \boldsymbol{c}$, $\boldsymbol{D}\boldsymbol{x} < \boldsymbol{e}$, and $\boldsymbol{F}\boldsymbol{x} = \boldsymbol{g}$.

Write the matrix $\boldsymbol{A}$ in terms of $\boldsymbol{B}$, $\boldsymbol{D}$, and $\boldsymbol{F}$, and write the vector $\boldsymbol{b}$ in terms of $\boldsymbol{c}$, $\boldsymbol{e}$, and $\boldsymbol{g}$, so that you can use `qps` to solve your problem. [*Hint:* An equality constraint can be written as two inequality constraints. For example, $ux = v$ can be written as $ux \geq v$ and $ux \leq v$.] (3 points)

Question 3.2

In the context of quadratic programming, explain what is meant by the terms

a. free variable,

b. slack variable, and

c. surplus variable.

(3 points)

Question 4. Arbitrage Pricing.

Question 4.1

Assume that every stock's return can be expressed in the single-factor form $r_i = a_i + x_i\phi$, where a_i is a constant for stock i, ϕ is a random variable, and x_i is the exposure of stock i to the random variable. Assume that you construct a portfolio with two different stocks, i and j. You use weights w_i and w_j whose sum is 1. Find an expression for the portfolio's return in terms of a risk-less part and a risky part that involves the random variable ϕ. (4 points)

Question 4.2

Create a risk-free portfolio by setting the coefficient of ϕ equal to zero. Find expressions for w_i and w_j in terms of x_i and x_j. (2 points)

Practice

For this part of the exam, for each answer, be sure to include a **printout of your code** in your answer document, and also to provide a separate .zip archive containing your source code.

Question 5. Portfolio Construction.

You have been given a mandate to form a portfolio using the following ten companies:

No.	Name	α_i	β_i	ω_i	b_i
1	ABC	10%	1.2	30%	10%
2	DEF	9%	1.1	25%	12%
3	GHI	11%	1.3	27%	12%
4	JKL	5%	0.8	20%	6%
5	MNO	8%	0.9	22%	21%
6	PQR	7%	0.7	20%	8%
7	STU	5%	0.6	20%	6%
8	VWX	4%	0.6	29%	8%
9	YZA	8%	0.8	24%	6%
10	BCD	3%	0.5	20%	11%

where α_i, β_i, and ω_i have their usual meanings, and b_i is your benchmark's weights for security i. The standard deviation of the market return is 20%, the expected return of the market is 7%, and the risk-free rate is 3%.

Question 5.1

What is the minimum variance portfolio, and what is the standard deviation of that portfolio's return? (4 points)

Question 5.2

Use the Elton-Gruber-Padberg algorithm with Sharpe's Single Index Model to choose a long-only portfolio. What is that portfolio? (4 points)

Question 5.3

Extend Question 5.2 above to produce a long-short portfolio using the Lintnerian definition of shorts. What is your optimal portfolio now? What are the expected value and standard deviation of that portfolio's return (2 points)

Question 5.4

Using the data provided in the table above, construct the covariance matrix $\boldsymbol{Q}$, and use quadratic programming to find the optimal portfolio that satisfies a long-only constraint (i.e., no short positions), the budget constraint, and the constraint $\beta_{\mathrm{P}} = 1$ for a range of risk tolerances from $\tau = 0.001$ to $\tau = 1$. Plot the resulting efficient frontier.

(6 points)

Question 5.5

Repeat Question 5.4 above, but now include the constraint that the portfolio should have no weight should be greater that 20%. Superimpose this efficient frontier on the efficient frontier from Question 5.4 and explain any differences. (6 points)

Question 6. Active Portfolio Construction.

Use the same data from Question 5.

Question 6.1

Find the active portfolio that maximizes your expected alpha, α_{A}, subject to the constraints that the sum of the active weights is zero, and that the tracking error is 3%. (4 points)

Question 6.2

Assuming that the returns of the stocks are exactly equal to the expectations given in the table, what is the information ratio? (2 points)

Question 7. Performance Analysis with Cash Flows.

Your fund's initial value at the beginning of January 2014 is $500 million. You do your accounting at the end of each month. Over the year, at the end of each month, you have the fund values given in the table below. The month-end values include the cash inflows and cash outflows as shown below. Finally, the last column in the table shows the market returns. All values and inflows are all measured in millions of dollars.

Date	Value	Inflow	Outflow	r_{M}
January 2015	520	50	0	10%
February 2015	560	100	0	8%
March 2015	600	80	20	5%
April 2015	540	0	0	12%
May 2015	480	0	10	-3%
June 2015	400	0	20	-1%
July 2015	700	250	50	10%
August 2015	710	0	10	4%
September 2015	705	0	0	5%
October 2015	715	10	0	-5%
November 2015	640	20	60	-8%
December 2015	670	15	15	7%

Question 7.1

What are the annual returns using the time weighted method, the Simple Deitz method, and the Modified Deitz method? (6 points)

Question 7.2

Compute the IRR (Write your own program. Do not use any built-in function designed specifically to find IRR. *Hint:* try using the `roots` function in MATLAB, or its equivalent in your language of choice). (6 points)

Question 7.3

Using your monthly returns adjusted for cash flows, what are your alpha, beta, R^2, Sharpe Ratio, Treynor Ratio, IR, and M^2 measures? (6 points)

Question 8. Bond Portfolio Construction.

A company has the following expected liabilities (in millions of dollars).

Time	Liability
6 Months	10
12 Months	15
18 Months	20
24 Months	25
30 Months	20
36 Months	15

To meet these obligations, it is allowed to buy from the following bonds (each with a face value of $1,000):

Bond	Maturity	Coupon	Price
A	12 Months	0.0%	$997
B	12 Months	1.0%	$1,000
C	12 Months	1.2%	$1,050
D	24 Months	0.0%	$992
E	24 Months	1.3%	$1,100
F	24 Months	1.4%	$1,150
G	36 Months	0.0%	$990
H	36 Months	1.3%	$1,200
I	36 Months	1.4%	$1,280

Question 8.1

Use linear programming to find the least-cost portfolio of bonds that will meet the company's obligations. Show your code. State the portfolio as the number of each type of bond to be bought. (6 points)

Question 8.2

What is the cost of the portfolio? (2 points)

Question 8.3

What is the duration of each of the bonds? (State any assumptions that you need to make).

(4 points)

Question 8.4

What is the duration of the portfolio? (4 points)