Chapter 4: Time Value of Money

Fundamental Rule

- Over time, invested money grows.
- $1 today is more valuable than $1 in the future because you can invest $1 and get $2.
- **We cannot directly compare dollar amounts across time.**
- How can we compare instead?

Why study this?

- We use time value of money to:
  - Calculate prices of financial assets.
  - Plan retirement savings.
  - Plan college savings.
  - Make capital budgeting decisions.
- Generally useful when considering whether an investment is a good idea.

Two Types of Calculations

- N = # of **compounding** periods
- I/YR = interest per period
- PV = sum at beginning
- PMT = **0**
- FV = sum at end

- N = # of **payment** periods
- I/YR = interest per period
- PV = sum at beginning
- PMT = **size of each payment**
- FV = sum at end

Time Value of Money Hints

- **Draw a time line!**
- Learn your calculator:
  - Read the manual.
  - Check payments per year (usually need one).
  - Negative signs on cash **outflows**.
  - Clear calculator memory after each problem.
  - Recall chart on the previous slide.

Future Value of a Lump Sum

- I deposit \$100 in a bank account that pays 10%. How much is it worth after one year? Easy—\$110.
- How to formulate using math?

Future Value of a Lump Sum

- Drawing the time line:

- Using the BA-II Plus:

Future Value: Multiple Periods

- What is \$100 worth after two years? \$121—not \$120!
- Why?

- **Compounding**: earning interest on your interest.
- After 30 years?
  - Without compounding:
  - With compounding:

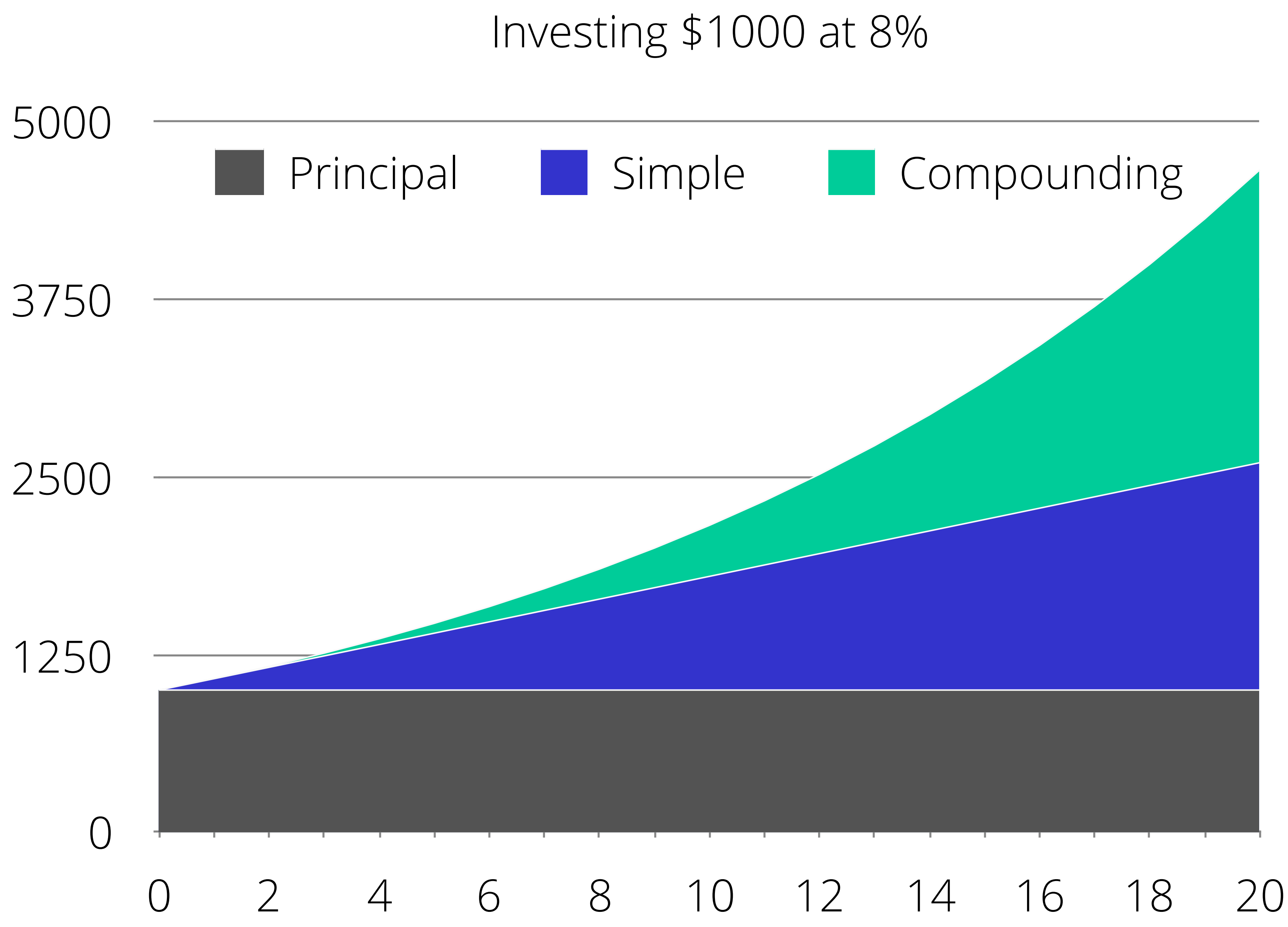
The Basic Future Value Formula

- **$FV = PV \times (1 + r)^N$**
- FV = future value
- PV = present value
- r = interest rate
- N = number of compounding periods
- All other time value of money formulas are based extensions of this.

Compound Interest

- Choose between: (a) $10,000 each day for one month, and (b) Starting with $0.01, double it each day for one month.

Compound Interest



Present Value of a Lump Sum

- Let's reverse the question: How much should I pay today to receive $110 in one year if I expect a 10% return?
- Another way to say this: how much do I need to start with so that I will have $110 in one year if I invest at 10%?
- Do the math:

Present Value of a Lump Sum

- Drawing the time line:

- Using the BA-II Plus:

The Basic Present Value Formula

- **$FV = PV \times (1 + r)^N$**
- FV = future value
- PV = present value
- r = interest rate
- N = number of compounding periods
- This is a rearrangement of the basic future value formula.

Example

- How long does it take to double your money if you invest at 8% interest?
- There are four variables in this formula. We know three, so we can find the fourth:

Example

- What interest rate will double your money in 9 years?

Perpetuities & Annuities

Perpetuities

- Make payments **perpetually**—forever.

- How to compute the present value of all the cash flows?

Example: Preferred Stock

- Firms preferred stock promises a $2 annual dividend —one a year, every year. Estimate its price if investors expect a 9% return.

- If the actual price of the stock is $25.24, what is the implied required rate of return?

Annuities

- Pays multiple payments, evenly spaced out.

- Compute the present value using the same strategy as before (discount each payment individually).

Annuity Example: Finding PMT

- A bank lends you $200,000 to buy a home and charges 5% interest. You pay back the loan and interest in equal annual payments (this is called **amortization**). The mortgage term is 30 years. What is the size of the payment?

Annuity Example: Finding PMT

- Step 1: Make a time line.

- Step 2: Plug in values.

Annuity Example: Finding FV

- How much will be in your bank account if you deposit \$100 at the end of every year for the next four years and your account earns 10% per year?

Math Tricks

- Suppose you the number of periods (N), the interest rate (r), and the present value (PV) of an annuity.
- How can you find its future value? Do you need to know the payments (PMT)?

Math Tricks

- Let’s check the math:

Another Example

- How much do you need to put into your retirement account each year if you plan to retire in 30 years and you need \$2 million at the start of retirement, assuming you earn a 12% return?

Another Example

- Step 1: Make a time line.

- Step 2: Plug in values.

Annuity Due

- "Due" means payments arrive at the beginning (t=0).
- Example: Assuming a 10% discount rate, how much would you pay for a five year annuity that begins making payments today?

Annuity Due

- The payments are a regular annuity at $t = -1$.
- Each cash flow is discounted one too many times. We can fix this by multiplying each by $(1 + r)$.

Annuity Due

- Alternatively, use the math trick from before and "push" the PV forward one period.

- (Or use the BA-II Plus in "begin" mode).

Interest Rates

Types of Interest Rates

- Many rates are stated as Annual Interest Rates (APR).
  - Credit cards (e.g. 12% monthly compounding)
  - CD rates (e.g. 1% daily compounding)
  - Coupon rates (e.g. 10% semiannual compounding)
- Stated rates are “in name only” or **nominal** rates—they cannot be used for present or future value calculations.

Translating Rates: Nominal to Periodic

- An APR of 12% with monthly compounding means you pay 12% / 12 = 1% per month for twelve months.
- We call 1% the **periodic rate**: it is the amount of interest earned per compounding period.
- In general, $\mathbf{r_{period} = r_{nominal} / N}$.

Translating Rates: Periodic to Effective

- How much interest do we earn per year if we earn1% per month for 12 months (i.e. 1% periodic rate with monthly compounding)?

- We call this the **effective (annual) rate**: it is the amount of interest that we actually earn in a year.
- In general, $\mathbf{r_{effective} = (1 + r_{period})^N - 1}$.

Translating Rates: Effective to Nominal

- Now we complete the circle: from nominal to effective is $\mathbf{r_{effective} = (1 + r_{nominal}/N)^N - 1}$.
- To get from effective back to nominal, we just solve for the nominal rate above.

Interest Rates: Summary

- The effective rate is what you actually earn during the year (or whatever the duration in the nominal rate is—usually we use APR).
- Without knowing the compounding period, nominal rates are meaningless; with the compounding period, we can calculate effective rates and do TVM work.
- The BAII-Plus will also convert rates for you (see examples to follow).

Interest Rates: Summary

- Here is a nice way to keep track of all this:

Examples

- Is effective rate higher or lower than the APR?

Compounding	N	Formula	Effective Rate
Annual			
Semiannual			
Monthly			
Daily			
Continuous			

Converting Rates: BA-II Plus

- There is a P/Y function on the BA-II Plus: don't use it.
- Calculate the effective rate using ICONV:
  - [2nd][ICONV]
  - 10 [ENTER]
  - [↓] (twice)
  - 2 [ENTER]
  - [↑][CPT] 10.25 (safe to use this rate)

Example: Effective to Nominal

- What interest rate does an investment pay per month if the effective annual rate is 10% with monthly compounding?
- [2nd][ICONV], [↓] (once)
- 10 [ENTER], [↓] (once)
- [↑] (twice)
- [CPT] 9.76
- 9.76% is the annual rate—0.81% is the monthly rate.

Compounding Examples

Home Mortgage

- You need \$100,000 to buy a house. You're offered a loan at 7% APR for 30 years with monthly installments. You also have \$100,000 in a bank account that earns 7.15% per year. Which payment option is cheaper?
- (Basic idea) The present value of the loan is \$100,000 by definition. What is the present value of the interest you would earn on your account?
- We need to know the discount rate implied by the loan to do this.

Home Mortgage (continued)

- Step 1) Compute the effective interest rate on the loan.

- Step 2) Compute the present value of the account’s interest.