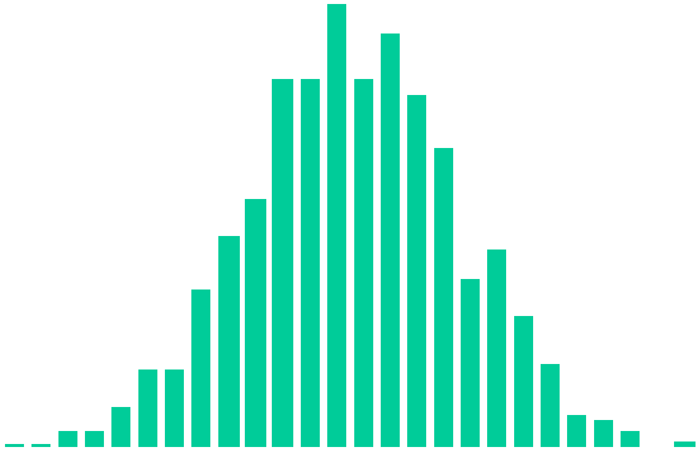
Chapter 7: Risk & Return



A Simple Bet?

Option A

- Heads you lose $10 million
- Tails you win $12 million
- Expected payoff is .5 x $12 - .5 x $10 = $1 million

Option B

- Guaranteed $1 million

A Simple Bet?

Scenario 2

- Tails you win $12 million
- Heads you lose $?? million

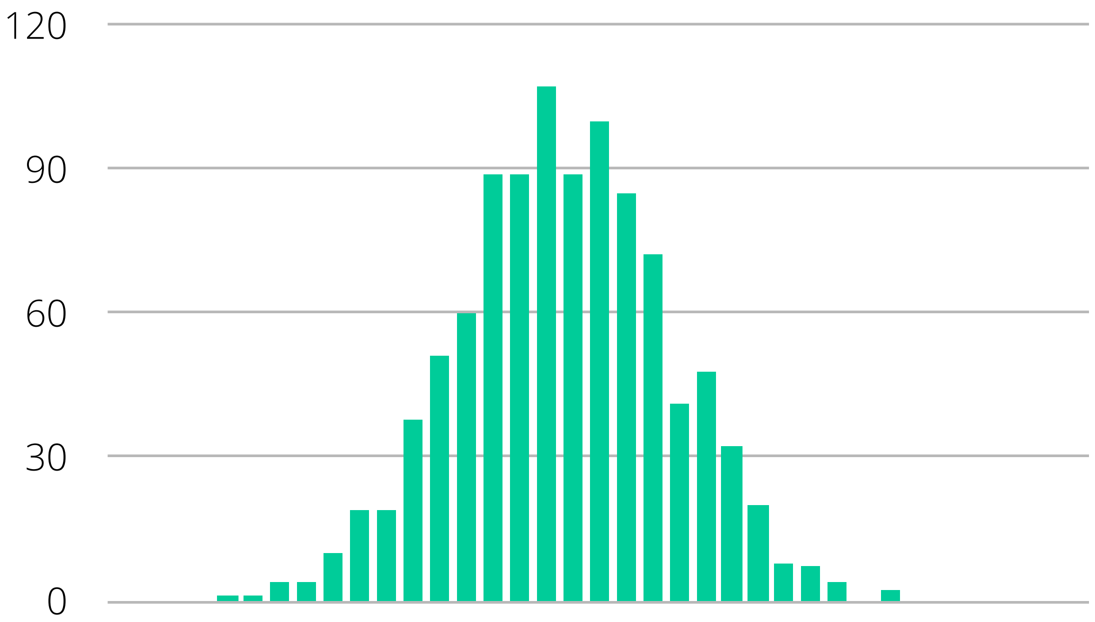
Scenario 3

- Tails you win $12
- Heads you lose $10

What is Risk?

- *Risk* is the probability of the unexpected.
  - How likely is the outcome?
  - How unexpected is the outcome?
- Individuals have different **risk preferences**.
  - **Risk averse**: Higher risk ➟ Higher return
  - Risk neutral: Higher risk ➟ No change
  - Risk seeking: Higher risk ➟ Lower return
- Preferences imply a quantitative trade-off between risk and return.

Quantifying Risk & Return



Measuring Risk

- **Mean** is the "expected" outcome—the average.

$$E[r] = \mu = p_1 r_1 + \ldots + p_N r_N$$

- **Variance** is related to the expected deviation from the mean.

$$var[r] = \sigma^2 = [p_1(r_1 - E[r]) + \ldots + p_N(r_N - E[r])]^2$$

- **Standard deviation** is the square-root of variance.

$$stdev[r] = \sigma$$

Estimating Risk

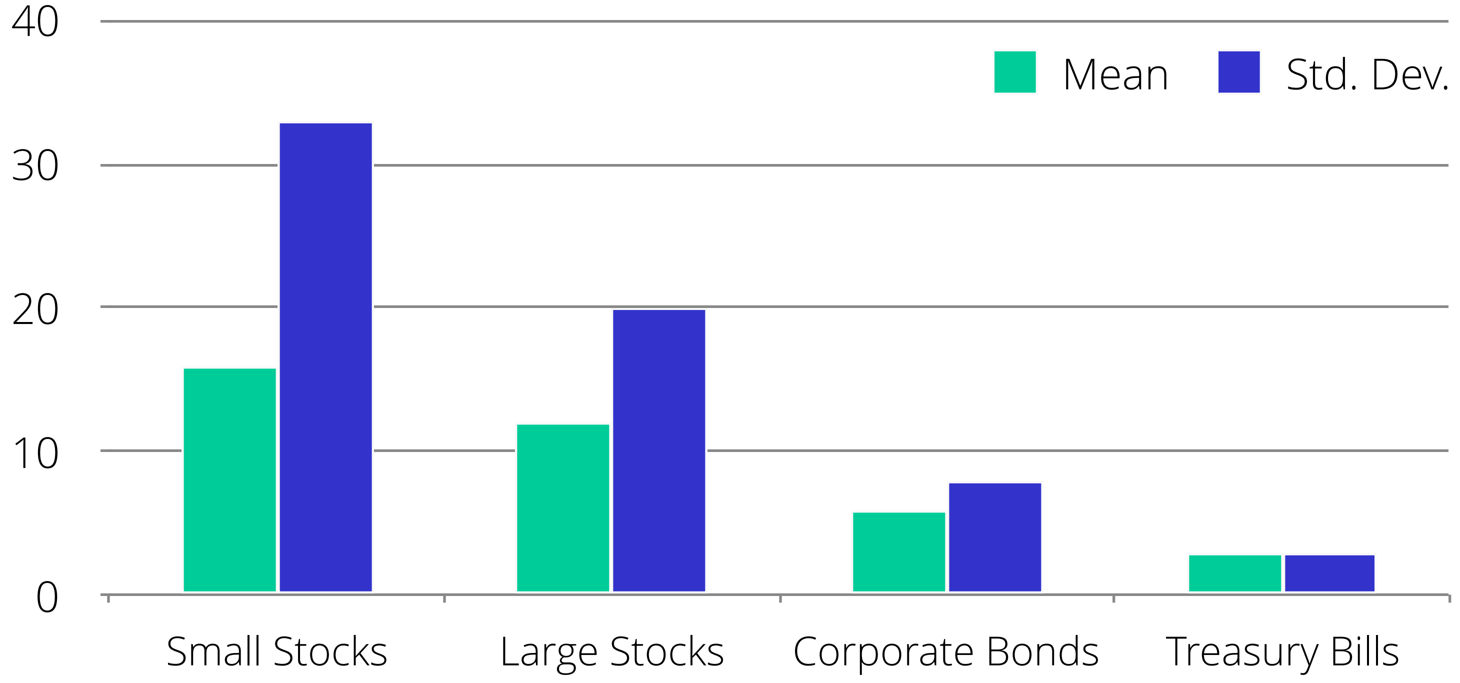
- We use historical data to estimate the mean and standard deviation.
- Assume each observation is a equally likely ($P_i = 1/N$).
- The mean is estimated by

$$E[r] \approx (1/N)r_1 + ... + (1/N)r_N$$

- Variance is estimated by

$$var[r] \approx [(1/N)(r_1 - E[r]) + ... + (1/N)(r_N - E[r])]^2$$

Historical Risk & Return

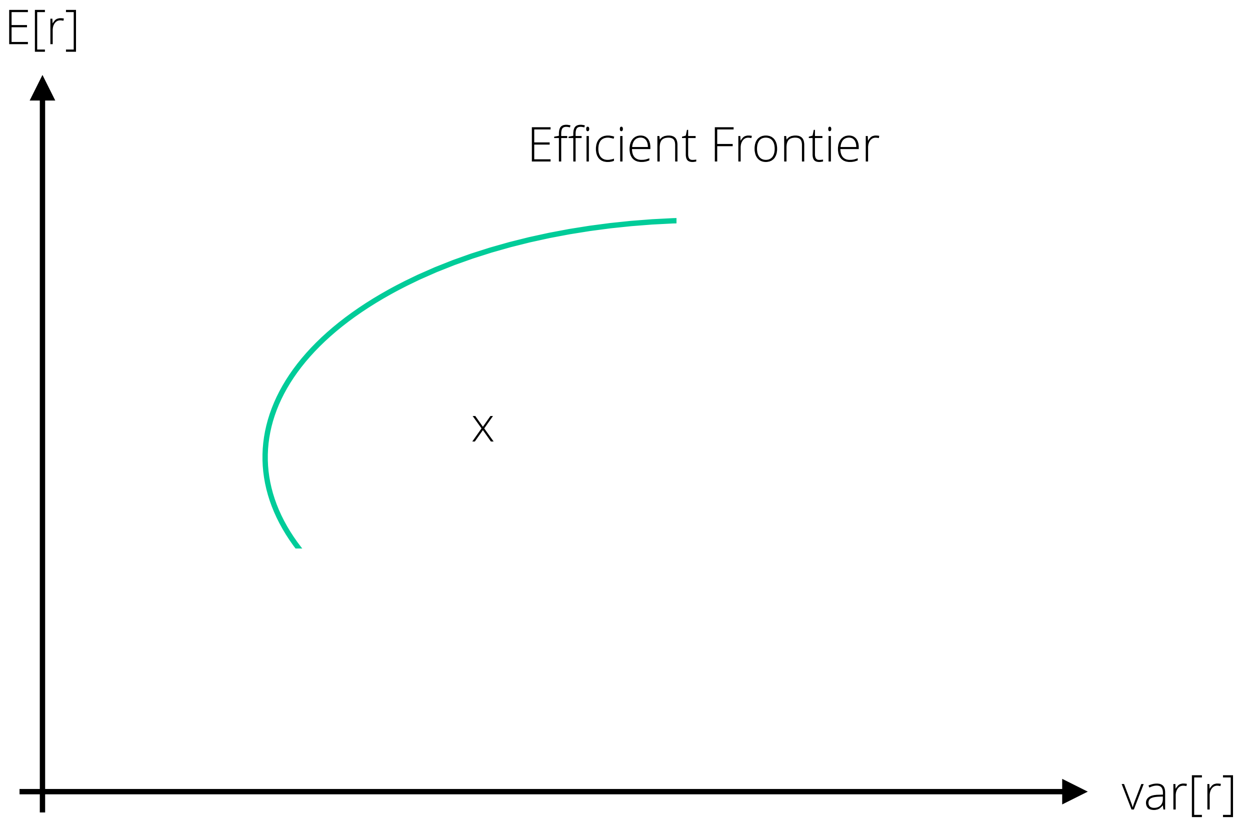


Decision Rules

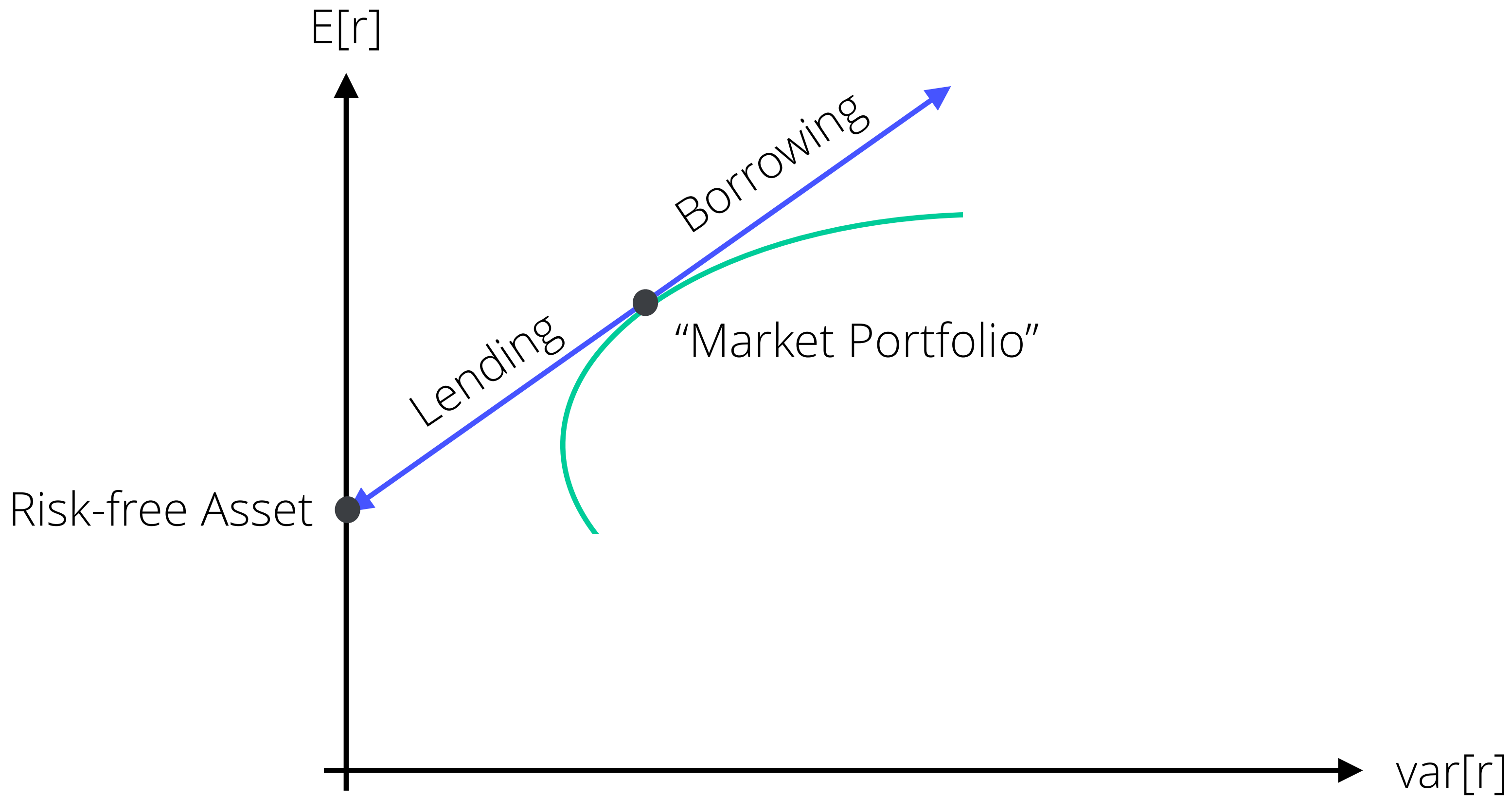
1. **Return maximization**: For a given level or risk (standard deviation), choose the highest return (mean).
2. **Risk minimization**: For a given level or return (mean), choose the lowest risk (standard deviation).

- Either decision rule leads to the same “efficient frontier” of assets/portfolios.

Decision Rules



Decision Rules



Decision Rules

- Investors only need to decide how much to invest in the market portfolio (w_{mkt}) and how much to put in the risk-free asset.
- The mean of the portfolio is

$$E[r_p] = w_{mkt} \times E[r_{mkt}] + (1 - w_{mkt}) \times r_{rf}$$

- The standard deviation of the portfolio is

$$\sigma_p = w_{mkt} \times \sigma_{mkt}$$

Portfolio Risk

Stock	Investment ($)	Investment (%)	Expected Return	Variance
GM	2.7	25%	7%	0.6
CSCO	1.5	14%	12%	1.5
DE	3.0	28%	8%	0.4
WY	1.2	11%	7%	0.8
PG	2.5	23%	9%	0.6
Portfolio	**10.9**	**100%**	**8.4%**	**69.1%**

Portfolio Risk

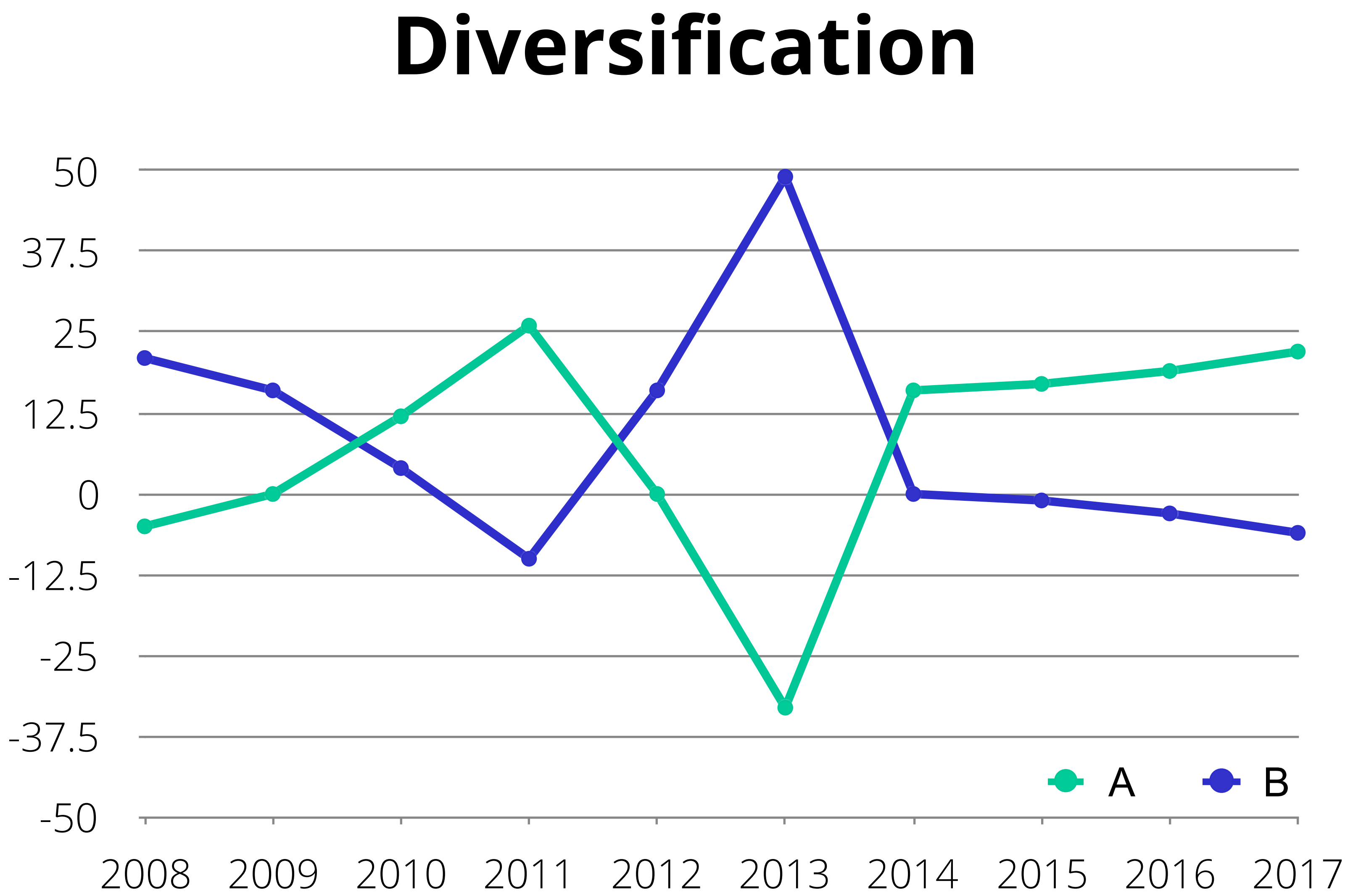
- Portfolio means are weighted averages:

$$E[r_p] = w_1\ E[r_1] + \ldots + w_N\ E[r_N]$$

- Adding a high (low) return stock to a portfolio increases (decreases) the portfolio mean return.
- Portfolio variances are **not** weighted averages!

$$\mathrm{var}[r_p] \neq w_1\ \mathrm{var}[r_1] + \ldots + w_N\ \mathrm{var}[r_N]$$

- The effect of adding a high (low) variance stock to a portfolio does not necessarily increase (decrease) the portfolio variance.


Diversification
50
37.5
25
12.5
0
-12.5
-25
-37.5
-50
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
A
B

Diversification

	Stock A	Stock B	Portfolio (A+B)
2008	-5	21	8
2009	0	16	8
2010	12	4	8
2011	26	-10	8
2012	0	16	8
2013	-33	49	8
2014	16	0	8
2015	17	-1	8
2016	19	-3	8
2017	22	-6	8
Mean	**7**	**9**	**8**
Std. Dev.	**18**	**18**	**0**

Diversification

- By themselves, the stocks are not that appealing.
  - High risk, modest return (low ➟ price).
- As a portfolio, they make an attractive investment.
  - If you bought them both, you made 8% every year!
  - Low risk, modest return (high ➟ price).
- Suggests two prices for the stocks. Which price prevails in the market?

Diversification

- **Example** Invest \$50 in Stock A and \$50 in Stock B. What are the risk and return of the portfolio?

Diversification

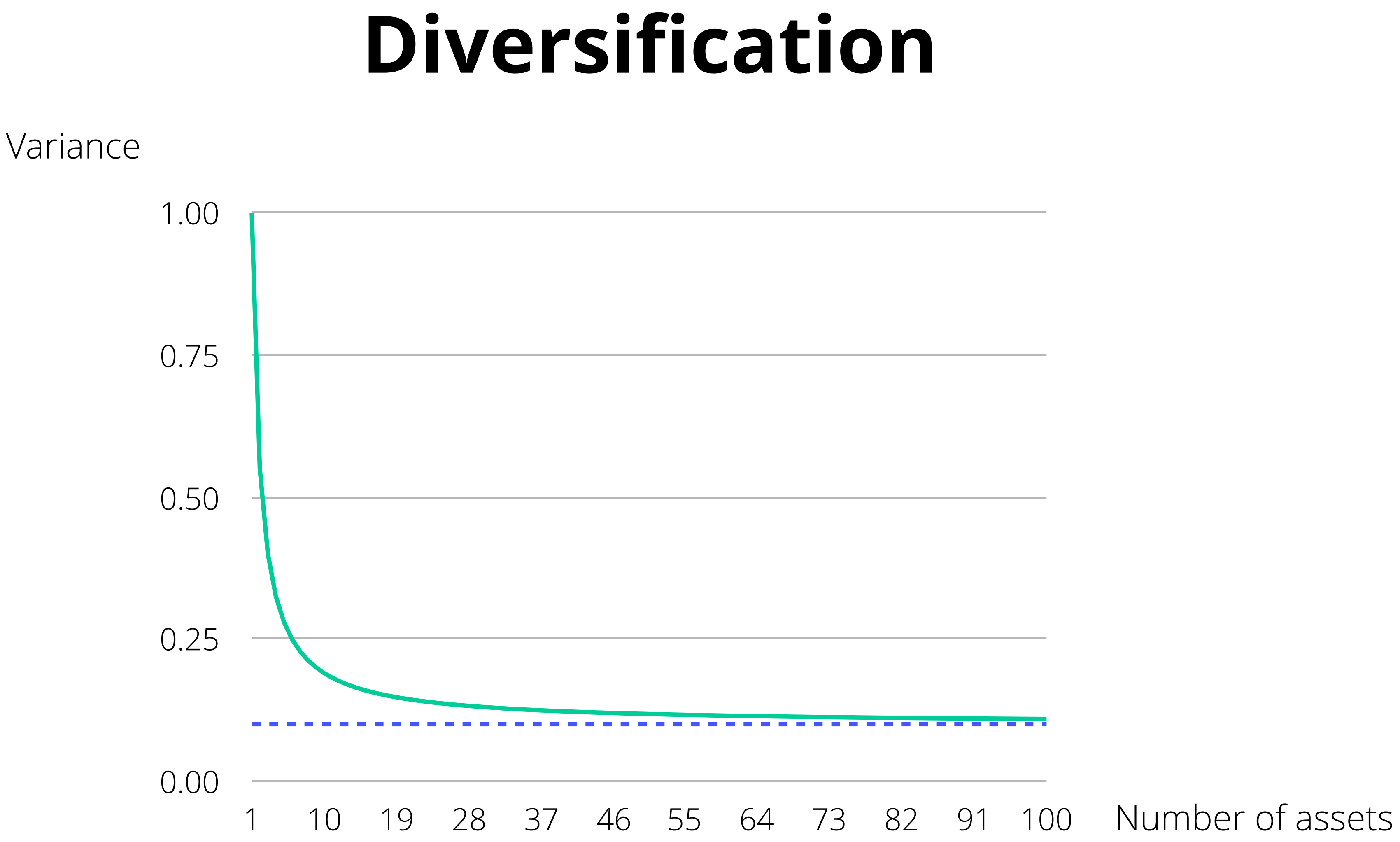
- Assume we have N assets with equal weights invested in each.
- The portfolio mean is

$$E[r_p] = (1/N)\ r_1 + \ldots + (1/N)\ r_N$$

- The portfolio variance is

$$\mathrm{var}[r_p] = (1/N)\ E[\mathrm{var}[r_i]] + (N\text{-}1)/N\ E[\mathrm{cov}[r_i, r_j]]$$

- As the portfolio gets big, only the covariance matters.


Diversification
Variance
1.00
0.75
0.50
0.25
0.00
1
10
19
28
37
46
55
64
73
82
91
100
Number of assets

Diversification

- **Firm-specific (idiosyncratic) risk** is risk that can be diversified away.
  - For example, the risk that a factory catches on fire is uncorrelated with the rest of the market.
  - As the number of assets in a portfolio increases, this risk becomes irrelevant.
- **Systematic (market) risk** is risk that cannot be diversified away.
  - It depends on how the asset relates to the rest of the assets in the portfolio.

Implications of Diversification

- “Efficient” investors own a combination of the risk-free asset and the market portfolio.
- These investors should primarily care about the covariance of each security with the rest of the market portfolio.
    - The firm-specific risk is diversified away (“for free”).
- Therefore, asset returns should reflect their systematic risk.

Beta

- Risk should be measured as the covariance of the asset's return with the market portfolio.

$$\sigma_{i,Mkt} = \mathrm{cov}[r_i, r_{Mkt}] = \rho_{i,Mkt}\, \sigma_i\, \sigma_{Mkt}$$

$$\beta_i = \mathrm{cov}[r_i, r_{Mkt}] / \mathrm{var}[r_{Mkt}]$$

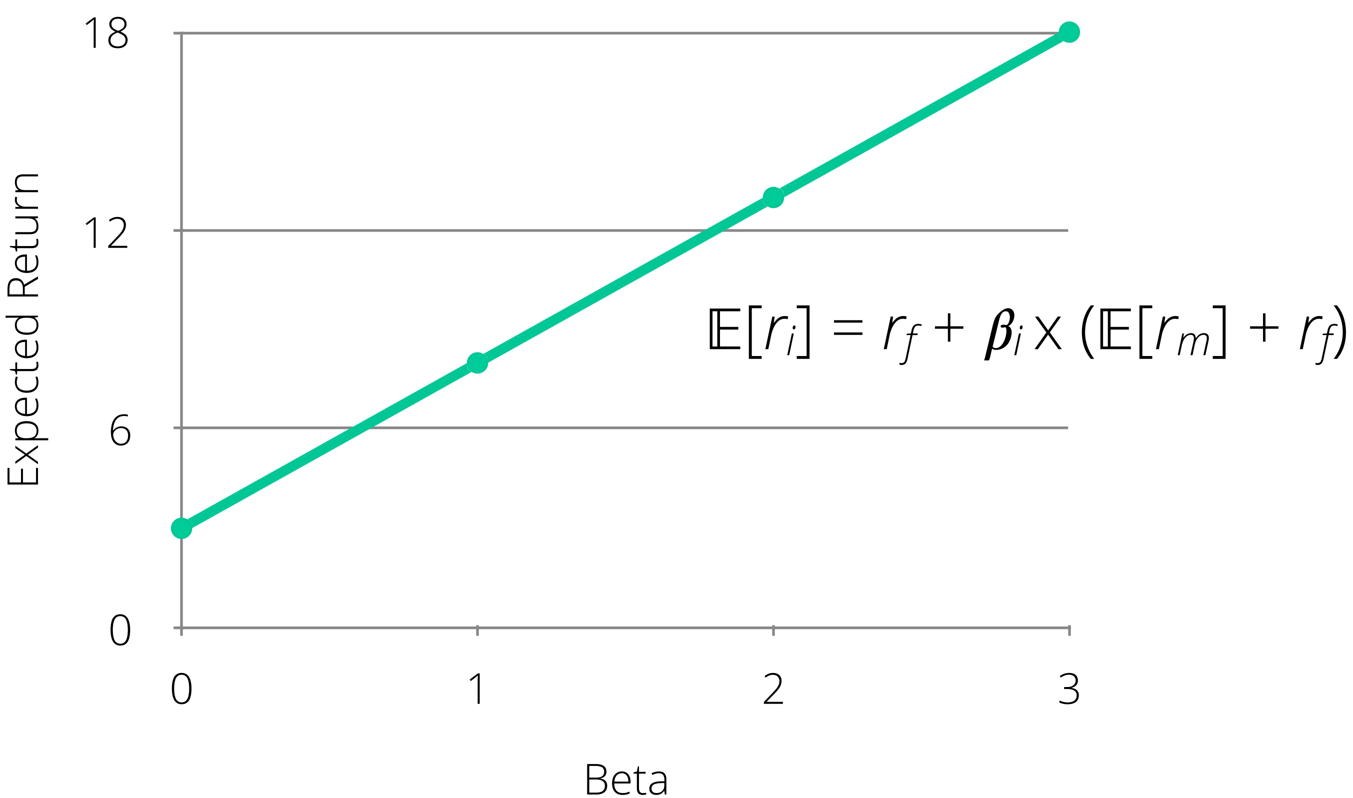
- Security betas are the same as simple regression coefficients!
- What is the market portfolio beta? The risk-free asset beta?

Capital Asset Pricing Model

- Investors require a higher return for stocks with greater systematic risk.
- Investors ignore firm specific risk because it can be diversified away).

$$E[r_i] = r_f + \text{"price per unit of risk"} \times \text{"risk units"}_i$$

CAPM



18
12
6
0
Expected Return
$\mathbb{E}[r_i] = r_f + \boldsymbol{\beta}_i \times (\mathbb{E}[r_m] + r_f)$
0
1
2
3
Beta

Security Market Line

- The CAPM is a line with slope equal to the **market risk premium**—the excess return of the market portfolio over the risk-free asset.
- The graph of the CAPM is called the **security market line**:

$$\mathbb{E}[r_i] = r_f + \boldsymbol{\beta}_i \times (\mathbb{E}[r_m] + r_f)$$

Applications

- **Example** After detailed analysis, you determine the IBM's dividend next year will be \$6, and you estimate the dividend will grow at 8% in perpetuity. If IBM's beta is 1.3, the current T-Bond rate is 7%, and the return on the market is expected to be 15%, what would you pay for a share of IBM?

Applications

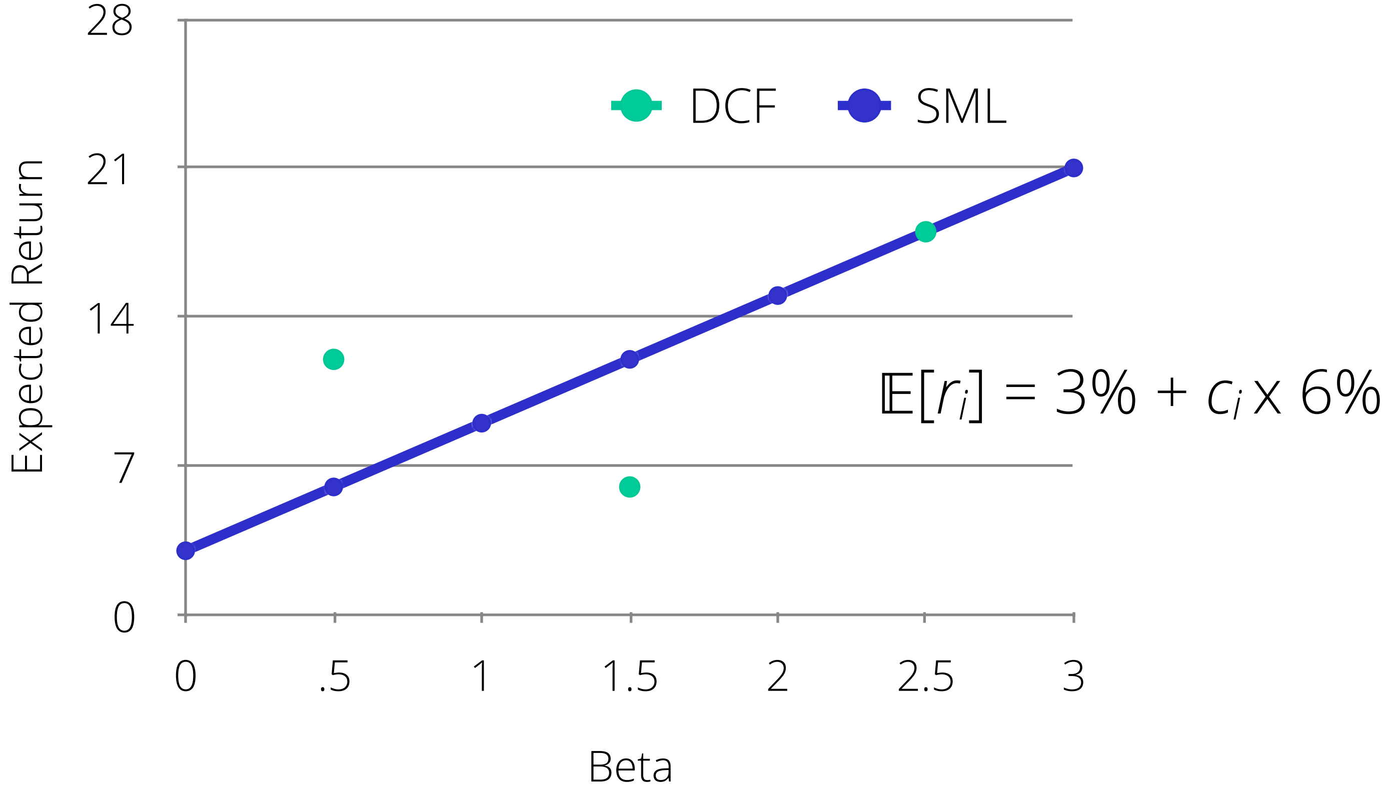
- **Example** Xerox just paid an annual dividend of $2. You know Xerox's beta is 0.75, the expected return on the market is 12%, and the T-Bond rate is 5%. What growth rate is implied by the current market price of $27?

Implications

- The market return is expected to be 9%, and the U.S. Treasury Bond yield is 3%.

Security	Estimated Return (DCF)	Estimated Beta	Required Return (CAPM)
A	6%	1.5	12%
B	12%	0.5	6%
C	18%	2.5	18%

CAPM



28
21
14
7
0
DCF
SML
Expected Return
$\mathbb{E}[r_i] = 3\% + c_i \times 6\%$
0
.5
1
1.5
2
2.5
3
Beta

Portfolio Betas

- What is the risk (beta) of a portfolio of that is 40% IBM with a beta of 2 and 60% AAPL with a beta of 1.3?

$$\mathbb{E}[r_p] = .40 \text{ x } \mathbb{E}[r_{\mathrm{IBM}}] + 0.60 \text{ x } \mathbb{E}[r_{\mathrm{AAPL}}]$$

$$\mathbb{E}[r_{\mathrm{IBM}}] = r_{\mathrm{f}} + 2 \text{ x } (\mathbb{E}[r_{\mathrm{Mkt}}] - r_{\mathrm{f}})$$

$$\mathbb{E}[r_{\mathrm{AAPL}}] = r_{\mathrm{f}} + 1.3 \text{ x } (\mathbb{E}[r_{\mathrm{Mkt}}] - r_{\mathrm{f}})$$

$$\Rightarrow \mathbb{E}[r_p] = r_{\mathrm{f}} + 1.58 \text{ x } (\mathbb{E}[r_{\mathrm{Mkt}}] - r_{\mathrm{f}}) \Rightarrow \boldsymbol{\beta}_{\mathrm{p}} = 1.58$$

Portfolio Betas

- More generally, the beta of a portfolio of N assets is the weighted-average of the individual assets:

$$\boldsymbol{\beta}_p = w_1\boldsymbol{\beta}_1 + \ldots + w_N\boldsymbol{\beta}_N$$

- If we add a stock with a greater beta than the current portfolio beta, the risk of the portfolio increases.

Portfolio Betas

	Investment ($)	Weight	Expected Return (%)	Beta
GM	2.7	24.8%	0.07	0.80
CSCO	1.5	13.8%	0.12	1.8
DE	3.0	27.5%	0.08	1.00
WY	1.2	11.0%	0.07	0.80
PG	2.5	22.9%	0.09	1.2
Portfolio	10.9	100%	8.4%	1.08

- What are r_f and r_{Mkt}?