In [ ]:
```
import talk.config as con

# environment variable for MOSEK
con.config_mosek()
con.config_configManager()
```

Leveraged Portfolios

https://en.wikipedia.org/wiki/130%E2%80%9330_fund
(https://en.wikipedia.org/wiki/130%E2%80%9330_fund)

Thomas Schmelzer

A 130/30 Equity Portfolio

- Allocate capital $C = 1$. Sell short at most $c = 0.3$ to finance a long position of $1 + c$.
- Universe of n assets: \begin{align}\mathbf{x}^{*}=\arg\max_{\mathbf{x} \in \mathbb{R}^n}& \mu^{T}\mathbf{x}\ \text{s.t. } &\Sigma\,x_i=1\

  ```
   &\Sigma\,\lvert x_i\rvert \leq 1 + 2c\\
  &\sqrt{\mathbf{x}^T\mathbf{C}\mathbf{x}} \leq \sigma_{\max}
  ```

  \end{align}

Cholesky decomposition of the covariance matrix:

$$\sqrt{\mathbf{x}^T \mathbf{C} \mathbf{x}} = \sqrt{\mathbf{x}^T \mathbf{G} \mathbf{G}^T \mathbf{x}} = \|\mathbf{G}^T \mathbf{x}\|_2$$

Introduce the cone $[y, G^T x] \in \mathcal{Q}_{n+1}$.

The absolute value:

$$|x_i| = \sqrt{x_i^2} = \|x_i\|_2$$

The sum of absolute values (e.g. the 1-norm) is replaced by n cones of dimension 2, e.g. $[t_i, x_i] \in \mathcal{Q}_2$. Now

$$\Sigma\, t_i \leq 1 + 2c$$

implies

$$\Sigma\, |x_i| \leq 1 + 2c$$

$$
\begin{aligned}
\mathbf{x}^* = \arg \max_{\mathbf{x}\in\mathbb{R}^n} & \mu^T \mathbf{x} \\
\text{s.t. } & \Sigma\, x_i = 1 \\
& \Sigma\, t_i \le 1 + 2c \\
& [y, G^T x] \in \mathcal{Q}_{n+1} \\
& y \le \sigma_{\max} \\
& [t_i, x_i] \in \mathcal{Q}_2, \quad i \in [1, \ldots, n]
\end{aligned}
$$

Summary

- Leverage is here a constraint for the 1-norm of the weight vector.
- We replace the 1-norm of a vector of length n with n cones each of dimension 2.
- We compute the Cholesky decomposition of a $n \times n$ covariance matrix to introduce a cone of dimension $n + 1$.

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