Let's See Your Digits: Anomalous-State Detection using Benford's Law

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ABSTRACT

Benford's Law explains a curious phenomenon in which the leading digits of "naturally-occurring" numerical data are distributed in a precise fashion. In this paper we begin by showing that system metrics generated by many modern information systems like Twitter, Wikipedia, YouTube and GitHub obey this law. We then propose a novel unsupervised approach called BenFound that exploits this property to detect anomalous system events. BenFound tracks the "Benfordness" of key system metrics, like the follower counts of tweeting Twitter users or the change deltas in Wikipedia page edits. It then applies a novel Benford-conformity test in real-time to identify "non-Benford events". We investigate a variety of such events, showing that they correspond to unnatural and often undesirable system interactions like spamming, hashtag-hijacking and denial-of-service attacks. The result is a technically-uncomplicated and effective "red flagging" technique that can be used to complement existing anomaly-detection approaches. Although not without its limitations, it is highly efficient and requires neither obscure parameters, nor text streams, nor natural-language processing.

CCS CONCEPTS

•**Information systems** →**Data stream mining;** *Online analytical processing;* •**Computing methodologies** →Modeling methodologies;

KEYWORDS

Anomaly detection; data streams; time series data; Benford's Law; nonparametric statistical tests

1 INTRODUCTION

In various domains it is useful to know if the interactions with a running system deviate from expected patterns. A canonical example is financial fraud, where organizations have an interest in exposing any persons attempting to deceive their funding and accounting processes. Other examples include online collaborative ecosystems like Twitter and Wikipedia, which strive to promptly detect and suppress behavior like spamming and attacks which can degrade service quality [20].

On the surface, such manipulative behavior may appear normal and be difficult to detect. A tax cheat may fabricate figures that, although dishonest, lie within expected ranges. Analogously, a spammer may use a team of fake Twitter accounts, each of which has sensible numbers of followers and friends. The manner in which the *absolute values* of such numbers are distributed is often not unusual.

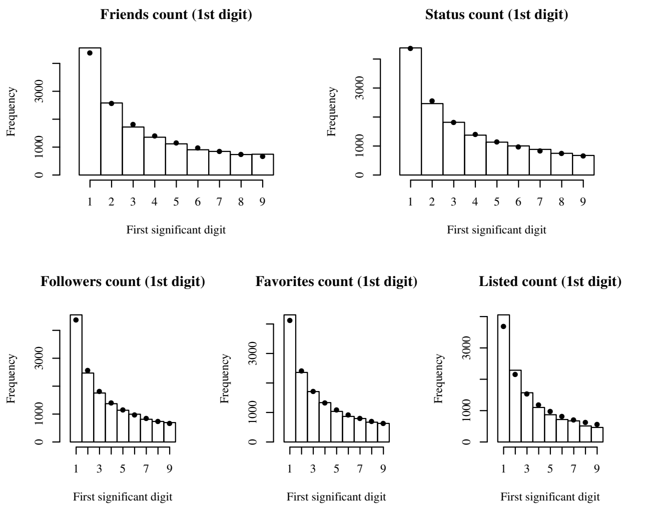

Figure 1: First-digit distributions for the five count-based metrics of 15,000 randomly-chosen Twitter users (friend-, status-, follower-, favorite- and listed-count). The bars represent the measured frequencies, and the filled bullets those predicted by Benford's Law.

The key to the effectiveness of Benford's Law (BL) in such situations lies with its observation that the *leading digits* of many sets of numbers are, by nature, distributed in a surprisingly *non-uniform* way. In contrast, if such numbers are *fabricated*, the leading digits rarely follow that same distribution. For example, the success of BL in financial auditing has shown that the naïve fraudster, in an attempt to contrive believable figures that are varied and "well spread", often fabricates figures with a more or less uniform distribution of leading digits [15]. When doing a Benford analysis, one focuses *only* on the leading digits and *purposefully discards the magnitude information.* In this work we propose that monitoring this kind of information is a novel, robust and elegant mechanism for tracking the integrity of complex systems.

To grasp the basic concept of BL, we share measurements collected from 15,000 randomly-chosen Twitter users. If we consider the number of followers that each user has, we may see that one user has 293,845 followers while another has only 320. As an exercise in curiosity, how would we expect the distribution to appear

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when tallying just the first significant digit (marked in **bold**) of each value? That is, how likely is it that one of our 15,000 users has a follower-count that begins with the digit 9 versus the digit 1? When asked, most people typically answer with "equally likely".

As Figure 1 shows, however, the distribution is far from uniform. Indeed, it defies intuition: it tells us that finding a follower-count starting with a 1 is *over six times more likely* than finding one starting with a 9. This observation is not unique to this sample. In the same figure we see the distributions for the four other Twitter count-based metrics for each user (friend, listed, status, and favorited counts). All have the same basic form, and we can visually see a strong agreement with the BL predictions in each case. Considering that data from services such as Twitter are considered noisy [2, 9] and fluctuating tails make them difficult to model with power-law distributions [7], this strong BL-conformity is a useful property.

In this work we hence consider the problem of distinguishing between "expected"/ "natural" and "unexpected"/"unnatural" states in real-time systems. To this end we exploit BL and present an efficient "red-flagging" approach based on novel conformance testing. Just as BL can expose anomalous behavior invisible to other tools, we show that the events detected by our framework are not found by state-of-the-art anomaly-detection techniques.

Contributions:
1) We show the Benfordness of various online metrics tracked by Twitter, Wikipedia, YouTube and GitHub (Sections 1, 3 and 4).
2) We propose a test for Benford's Law conformity exploiting the law's logarithmic basis and the formal Kolgomorov-Smirnov test (Section 5).
3) We present BenFound to detect "unnatural" events from streams of *numerical data*. BenFound does not require a text stream, nor any parameters other than the width w of the sliding window (Section 6). It can be deployed in real-time and has linear run-time complexity in w.
4) We present experiments on synthetic data, showing that BenFound finds events not visible to state-of-the-art change-point and event-detection techniques that feed on numerical data (Section 9).
5) We present a number of real-world case-studies showing how BenFound detects "unnatural" events not found by other techniques (Sections 4, 7, 8).

Note that this manuscript was prepared using *Knitr*, a tool for creating transparent, reproducible research. The document source includes our data, algorithms and the embedded R code necessary to reproduce all results, tables and figures. It is publicly available[1].

2 PRELIMINARIES

Benford's Law asserts that the probabilities for finding each of the nine possible digits $1, 2, \dots, 9$ as the first significant digit are *not equal*. It states that the digit 1, for example, occurs more than 30% of the time and the digit 9 less than 5% of the time. More precisely, given a vector $\vec{x} \in \mathbb{R}^n$ of numerical values, BL states that the probability of a randomly-selected element from $\vec{x}$ having the *first* significant digit $d \in \{1, \dots, 9\}$ is

$$P(D_1 = d) = \log_{10}(d + 1) - \log_{10}(d). \quad (1)$$

The bullets (•) in Figure 1 were computed using these probabilities. The general form of BL further specifies the joint probability distribution of *all* the significant digits [6][2]. The form of the probability mass function in this more general case is identical to (1), however d is then understood to be the string of digits in question (for example, the probability of finding a first digit 3 and second digit 8 is given by substituting $d = 38$ into (1)).

Although it is clear that not *all* sets of numbers obey the law (a sample of human heights in cm, for example, does not), Benford's original manuscript [4] includes evidence supporting its wide application. Data from sources as diverse as baseball statistics, death rates, lists of physical constants and randomly-selected numbers from newspapers were shown to conform. We recommend that the justifiably-skeptical reader perform a simple experiment in this light, such as randomly sampling numerical data from independent web pages.

It is worth highlighting that Equation (1) fits such data without having to estimate any distributional parameters. This is a useful property, and contrasts with modeling based on alternatives like power-law distributions where the estimation of such parameters is often difficult [7].

On a theoretical level, many familiar mathematical sequences follow BL. These include Fibonacci, the powers of (almost[3]) any integer, and the factorials $(1!, \dots, n!)$. Such sequences offer an informal starting point for *explaining* BL. It turns out that BL is often (but not always) observed to hold for sets of numbers spanning several orders of magnitude and associated with exponential or multiplicative growth processes. In this light, consider the sequence of numbers generated by the powers of 1.2 (that is, $1.2^0, 1.2^1, 1.2^2, \dots$) plotted on a logarithmic axis:

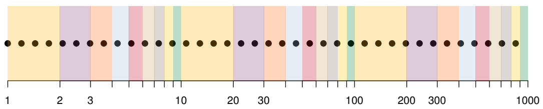


Figure 2: The geometric progression $1.2^0, 1.2^1, 1.2^2, \dots$ **plotted on a logarithmic axis.**

It is clear that the values are equally spread out when viewed on a logarithmic scale. Looking at the shaded areas, it is then obvious that a randomly-selected value will more often fall in an interval where the first significant digit is one (e.g. $[1, 2)$, $[10, 20)$, $[100, 200)$) rather than nine. We can now understand the BL formula in Equation 1 graphically: the probability for any given digit is simply the width of its interval on the logarithmic scale.

With this in mind, let us consider the physiological and psychological reaction to external stimuli that we find in the real world.

[1] github.com/samhelmholtz/benfound

[2] Note that the distributions for the second and higher digits quickly approach the uniform.

[3] For base ten it is clear that powers of any integer that is itself a power of ten (e.g. 1000) will not obey the law.

The growth of the sensation of brightness (Fechner's Law), the sense of loudness, the sense of weight, the response of the body to medicine or radiation, and the killing curves under toxins and radiation are all often logarithmic. In economics we often hear about percentage rates of growth. In social media, content can go "viral" with an exponentially growing number of views or downloads over time. As Benford himself elegantly noted: "the analogy is complete, and one is tempted to think that the $1, 2, 3, \ldots$ scale is not the natural scale; but that, invoking the base e of the natural logarithms, *Nature* counts $e^0, e^x, e^{2x}, e^{3x}, \ldots$ and builds and functions accordingly" [4].

Given that BL arises naturally, it should come as no surprise that it is base- and unit-invariant. This means that numbers obeying BL consistently obey BL after we express them in a different base (e.g. octal) or in different units (e.g. feet instead of meters). The mathematical properties of the law are intriguing, and we refer the curious reader to [6] for a formal treatment. Although many facets of BL now rest on solid ground, there remains *no unified approach* that simultaneously explains its appearance in dynamical systems, number theory, statistics and real-world data [5].

As this work is concerned with *applications* of the law, we simplify matters by focusing on decimal numbers with no scaling applied. Arguably the most classical application for BL is in fraud-detection. For example, an employee who serially alters the leading digits from 1 to 7 on reimbursement invoices (such that a hotel stay becomes $738.45 instead of $138.45, for example) introduces an artificial and detectable bias to the leading-digits distribution [15].

Our work is based on the idea that different kinds of unnatural events take place in different domains. These events may or may not be malicious in nature. In Twitter, for example, we will see that the lead-up to Father's Day is accompanied by artificial use of the **#FathersDay** hashtag for pushing product sales (spamming and advertising). In such cases we can raise a "red-flag" and investigate the items that violate the digit distribution.

3 FURTHER EVIDENCE OF BENFORD'S LAW ONLINE

Figure 3 shows that, like the Twitter data in our Introduction, the count metrics from YouTube videos and GitHub repositories are close to Benford. For this experiment we collected data from the YouTube[4] and GitHub[5] APIs.

Interestingly, we see that the level of conformity correlates with the range of the metric in question. That is, the YouTube view counts range up to the large value of 1.6×10^9 and are seen to be a close to perfect fit to Benford's prediction. YouTube likes, dislikes and comment counts, however, span a narrower range. Their maximum values are 4.9×10^6, 1.31×10^6 and 1.25×10^6 respectively. GitHub stars and forks have even narrower ranges (up to 1.567×10^5, 1.563×10^5 and 8321 respectively). This result empirically helps to support the conjecture that BL is most pronounced for data spanning many orders of magnitude.

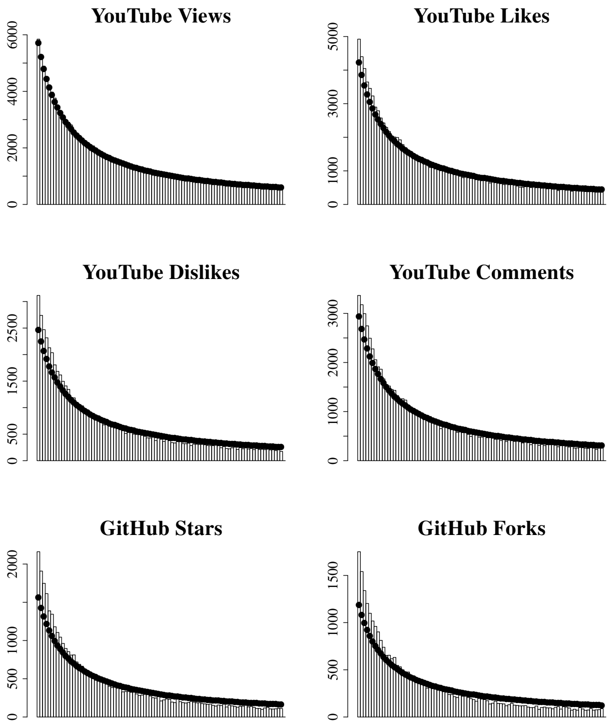


Figure 3: Leading two-digit histograms for YouTube Views, Likes, Dislikes, Comments, and GitHub Stars and Forks. The 90 histogram bins correspond to the leading two-digit combinations (10-99).

4 BOT-DETECTION USING BENFORD'S LAW

In this section we consider the application of Benford's Law in the detection of anomalous behavior in online services in the form of **bots** (where automated computer programs, rather than humans, interact with a system). As a case study we present and share data collected from Wikipedia's Recent Changes stream[6].

Between July 6 and July 11 2016 we collected $n > 2$ million page-edit events classified by Wikipedia as *non-bot* edits. We recorded the vector $\vec{\Delta} = (\delta_1, \delta_2, \ldots, \delta_n)$, where each δ_i represents the magnitude in bytes of the corresponding change. Changes ranged from a few bytes to over two megabytes.

Consider in Figure 4 the distribution of the *leading two digits* of this data. From the basic form of the leading-digits distribution it is evident that the data in question fits the general shape predicted by Benford's Law (again shown by bullets ●). However, we see some clear aberrations. For example, noticeable spikes exist for the digit pairs 11, 22, 38 and 40. This data is therefore a candidate for "red flagging" and investigation.

Filtering the data to include only the editor comments for the changes with a delta beginning with the digits 40, for example, we quickly notice a significant amount of automation happening (despite the edits being flagged by Wikipedia as *non-bot*). Table 1

[4] developers.google.com/youtube/v3/

[5] developer.github.com/v3/

[6] mediawiki.org/wiki/API:Recent_changes_stream

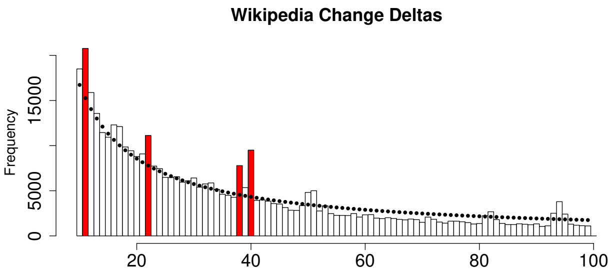


Figure 4: Leading two-digit histogram for Wikipedia page-edit deltas. Red bars are abberations from BL.

contains a sample of the offending comments. We see edits being made by “Addbots” as well as tools like HotCat and Gadget-Merge (automated tools for mass-editing Wikipedia content). The byte counts often *begin* with the digits 40 (despite their full values spanning several orders of magnitude), which manifests as the spike in Figure 4. At the very least, Benford’s Law has thus uncovered what appears to be a data-integrity issue in the Wikipedia: many automated transactions are not being suitably labeled.

Change comment	**Bytes**
`[[SpiderMum]] [[User:Addbot|Addbot]]`	**40**
`Removed using [[Gadget-HotCat|HotCat]]`	**40**
`Wbremoveclaims-remove [[P1012,Q902513]]`	**407**
`[[MediaWiki:Gadget-Merge]] 0||Q75421`	**4062**
`[[OneClickArchiver]]`	**40971**

Table 1: Examples of automated Wikipedia edits

5 TESTING CONFORMITY TO BENFORD’S LAW

To develop a real-time system, we require a mechanism to objectively assess a given numerical sample’s conformity to BL. The “state-of-the-art” for such tests was recently surveyed by Nigrini [15]. Broadly speaking, one can partition BL tests into three categories: *single-digit* tests, *all-digits-at-once* tests and tests exploiting the *logarithmic basis* of BL. To initially evaluate these tests, we consider two geometric progressions (i.e. $a^1, a^2, \ldots, a^n$) with common ratios $a = 1.2$ and 1.772. From Section 2 we know that each sequence obeys BL exactly in the limit $n \to \infty$.

Table 2 displays the conclusions (“Benford” or “Non-Benford”) that each test defined and detailed in [15] makes for our two sequences based on a significance level of 0.05 and sequence length $n = 100$.

The tests based on the *discrete digit distribution* (“Single” digit, or “All” at once) fail to label the second sequence as Benford. These techniques are the Z-Statistic, Chi-Square, Euclidean Distance, Hotelling T-square, Joenssen’s JP-square and Mean Absolute Deviation (MAD) tests. It is this latter test (MAD) which is recommended in [15]. This test yields a MAD statistic D_{MAD}, defined as the average of the magnitudes of the differences between the observed digit proportions (p_i^o) and those predicted by BL (p_i^t) across each of the $i = 1, \ldots, k$ digit bins:

Test	Type	Seq. 1 is	Seq. 2 is	Remarks
Z-Stat.	Single	**Benf.**	Non-Benf.	8 digits deviate significantly for Seq. 2
Chi-Sq.	All	**Benf.**	Non-Benf.	Seq. 2 $\chi^2 = 22.8$ (9 − 1 = 8 d.o.f.)
Euc. Dist.	All	**Benf.**	Non-Benf.	
Hot. T^2	All	**Benf.**	Non-Benf.	
JP Sq.	All	**Benf.**	Non-Benf.	
MAD	All	**Benf.**	Non-Benf.	Critical values from [15]
Mant. Arc	Log	**Benf.**	**Benf.**	
K-S Mant.	Log	**Benf.**	**Benf.**	*(Our technique)*

Table 2: Comparison of BL-conformity tests on two geometric progressions

$$D_{\text{MAD}} = \frac{\sum_{i=1}^{k} \left| p_i^o - p_i^t \right|}{k}.$$

In Figure 1, for example, the statistic is simply calculated by summing the distances between the each bullet/bar pair, and dividing by nine (the number of possible first digits). Smaller values of the MAD correspond to higher Benfordness.

Importantly, it is the *conflicting* conclusions of the digit-based tests that is the most troublesome, because both sequences are the same size, non-random and follow ideal exponential growth. The reason for the conflict is that we selected the second common ratio to maximize the *information loss during the discretization*, and hence highlight the weakness of these tests. For example, we have a large information-loss when discretizing the sequence value 9.8595 to the “first digit 9” histogram bin. For moderate sample sizes, these discretization errors are enough to change the leading-digit distribution to the extent that these digits-based tests believe that the sequence is non-Benford.

To overcome this kind of weakness, we argue that a BL conformity test should avoid such an unnecessary discretization. In this light, we can design tests around the *logarithmic basis* of BL, an alternative way of thinking about Benford’s Law which states that the mantissae of a Benford set are uniformly distributed over $[0, 1)$. To see this, we first note that the mantissa $X(a)$ of a number a is given by the fractional part of its common logarithm:

$$X(a) := \log_{10}(a) - \lfloor \log_{10}(a) \rfloor.$$

With reference to Figure 2, we recognize that the mantissa approach exploits the fact that each “order of magnitude” interval (e.g. $[10, 100)$, $[100, 1000)$) has an equal length on the logarithmic scale. The mantissa mapping thus effectively removes this common and irrelevant magnitude information by mapping each point to the standard interval $[0, 1)$. On this interval, the test for BL conformity reduces to a test of *uniformity*. The leading-digit distributions defined in (1) are a direct consequence of such a uniform mantissa distribution, however the same cannot be said for the reverse case. Specifically, we can construct samples which, although conforming exactly to (1), do not have a uniform mantissa distribution. Consider the set of values for arbitrarily large n such that, for a small value $\epsilon > 0$, $\epsilon \ll 1$, a fraction $P(D_1 = 1)$ of the values are equal to $2 - \epsilon$, a fraction $P(D_1 = 2)$ of the values are equal to 2, a fraction $P(D_1 = 3)$ of the values are equal to $4 - \epsilon$, a fraction $P(D_1 = 4)$ of the values are equal to 4, and so on until the final condition that a fraction $P(D_1 = 9)$ of the values are equal to $10 - \epsilon$. By definition, this set satisfies (1) and hence the single-digit variant of Benford's Law. If we look at the empirical cumulative distribution function (ECDF) of this sample's mantissae, however, we see below that it is far from uniform (here choosing $n = 10^6$ and $\epsilon = 10^{-6}$). Note also that analogous samples can easily be constructed for the two-digit and higher cases. Given such examples, we argue that conformance tests based on the logarithmic representation have the potential to be statistically more robust.

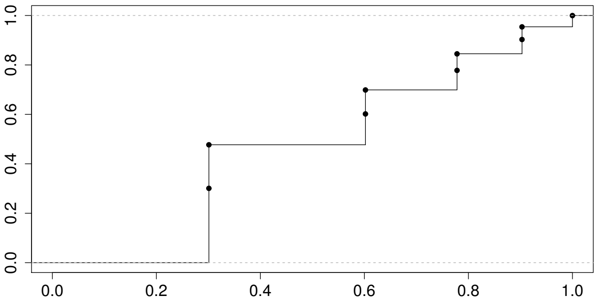


Figure 5: Mantissa ECDF of the described synthetically-constructed set of numbers that conform perfectly to the single-digit variant of BL.

Nigrini [15] discusses BL tests that exploit the logarithmic basis. The first involves naïvely testing for two necessary conditions of mantissae with a uniform distribution – namely that the mean is 0.5 and the variance is $\frac{1}{12}$. Nigrini correctly comments that these conditions are insufficient (numerous non-uniform distributions satisfy them) and dismisses this approach. The second approach involves ordering the mantissae, performing a linear regression and testing for expected values of the intercept and slope. This technique, however, "should only be used after more research" [15].

A more interesting logarithmic-basis test discussed by Nigrini is the recent Mantissa-Arc (Mant. Arc) test [1]. It involves projecting the mantissae around the circumference of the unit circle and testing if the center-of-gravity differs significantly from zero. We see in Table 2 that the Mantissa-Arc test is able to correctly diagnose both sequences as Benford. At first glance, this test appears promising, however we soon realize that the center-of-gravity condition is insufficient. Consider, for example, the arbitrary-length sample of $10^{1/4}, 10^{3/4}, 10^{1/4}, 10^{3/4}, \ldots, 10^{1/4}, 10^{3/4}$ having only two unique values. These two values are mapped to opposite sides of the unit circle, giving a zero center-of-gravity and incorrectly signaling *perfect* Benford conformity. We therefore suggest avoiding this test.

Why not apply a test which directly compares the ECDF of the mantissae with the cumulative distribution function of the uniform? To the best of our knowledge, this approach has not been suggested. Specifically, we propose applying the formal Kolmogorov-Smirnov (K-S) one-sample test [12] for this purpose. The first requirement for the test is the empirical cumulative distribution function $F_n(x)$ of the n mantissa values $X_1, \ldots, X_n$, which is given by (with indicator function I):

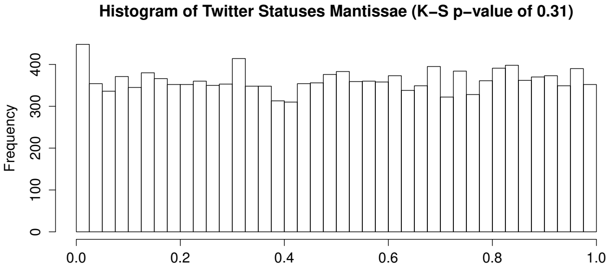
$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{[-\infty, x]}(X_i).$$

The second requirement is the cumulative distribution function $F(x)$ of the reference uniform. The K-S test statistic D_n is then defined as the supremum of the difference between the curves:

$$D_n = \sup_x |F_n(x) - F(x)| .$$

Using the corresponding null (Kolmogorov) distribution, the probability p of observing such a D_n under the null hypothesis is evaluated. If the probability is less than a significance threshold α, the null hypothesis is rejected. Adopting the common significance threshold of $\alpha = 0.05$, we hence interpret a $p < \alpha$ as an indicator for non-Benfordness. We note that the asymptotic statistical power of the K-S test is 1.

As an example, consider our Twitter status-count mantissae:



The p-value corresponding to the application of the K-S test on this sample is 0.31. We hence accept our null hypothesis that the sample was taken from a uniform distribution, and hence that the sample conforms to BL. This same approach correctly diagnoses both our test sequences as Benford (final row in Table 2). We note that the application of the K-S test on a sample involves no obscure parameters – we only need a significance threshold α in order to reach a conclusion for a sample.

6 FROM A TIME SERIES TO A BENFORDNESS SIGNAL

Equipped with a tool to measure the Benfordness of a univariate sample, we now turn to the real-time detection of "non-Benford" events. That is, we wish to be alerted when a significant deviation from the "natural" state of a running system occurs.

To achieve this, we track key metrics over time (e.g. follower counts of tweeting users) and apply the K-S Test on a sliding time window. The time window contains a fixed number w of the newest numeric values. For the sample of size w corresponding to each window, we compute the mantissae and apply the K-S test as discussed in Section 5. The resulting p-value is the quantity that we then follow over time. We interpret a drop of the p-value below a significance threshold α as an indicator for the beginning of "non-Benford" behavior in the system (something unnatural that should be investigated).

Window Size w and Threshold α: The value for w is user-specified. The K-S test we use for BL-conformity has an asymptotic power of 1. For $w \rightarrow \infty$ this implies that there is zero probability of rejecting a true null hypothesis. In practice, however, a large value of w means that we miss out on potentially important dynamics of the system under investigation. At the other extreme, if w is too small, we lack statistical power and thus have a higher false negative rate. As always, w should be chosen considering this trade-off and the application. For our upcoming Twitter case studies (Sections 7 and 8), we will see that a common window size ($w = 2000$) was sufficient.

The value for α can likewise be user-specified, however we will see in real-world case studies (Figure 7) and synthetic experiments (Figure 9) that similar results are obtained for the commonly-chosen threshold levels ($\alpha = 0.01, 0.05$). We use $\alpha = 0.05$ in all our work.

6.1 Performance

Assuming a fixed significance threshold α and window size w, the K-S critical value can be computed in advance using existing tables for the K-S statistic distribution (for $\alpha = 0.05$ it is $\frac{1.358}{\sqrt{w}}$, for example). Our optimized implementation of BenFound hence works directly with the K-S statistic, avoiding the need to compute p-values.

To calculate the K-S statistic for the current window we would normally need to sort its contents. Using a comparison-based sorting procedure like Heapsort implies a $O(w \cdot \log w)$ run-time in the worst case for this step. In practice, this sorting only need be performed once for the first window. Afterwards, we use standard indexing structures to update our sorted window in $O(1)$ operations. For each new stream value the calculation of the K-S statistic is hence reduced to $O(w)$, which is the time to determine the maximum difference between the window's ECDF and the reference uniform distribution.

7 CASE STUDY: HASHTAG HIJACKING

In the context of social media, Hashtag Hijacking occurs when a hashtag is (ab)used for purposes other than intended. We now demonstrate BenFound's ability to detect this phenomenon in real-time.

Consider the hashtag **#FathersDay**. The intended purpose of this hashtag is to allow Twitter users to celebrate their father. Father's Day occurred in most countries on 19 June 2016. During the time leading up to Father's Day, we listened for and logged all tweets referencing this hashtag. The raw follower counts of the corresponding tweeters are traced in Figure 6. The same Figure shows events, anomalies and change-points found by BenFound and three state-of-the-art approaches ([11, 13, 21], discussed further in Section 9 and 10). These three approaches consider the *absolute values* of the streamed values (tweeter follower counts).

The BenFound signal shows that the behavior begins Benford, turning non-Benford before Father's Day, and finally regressing

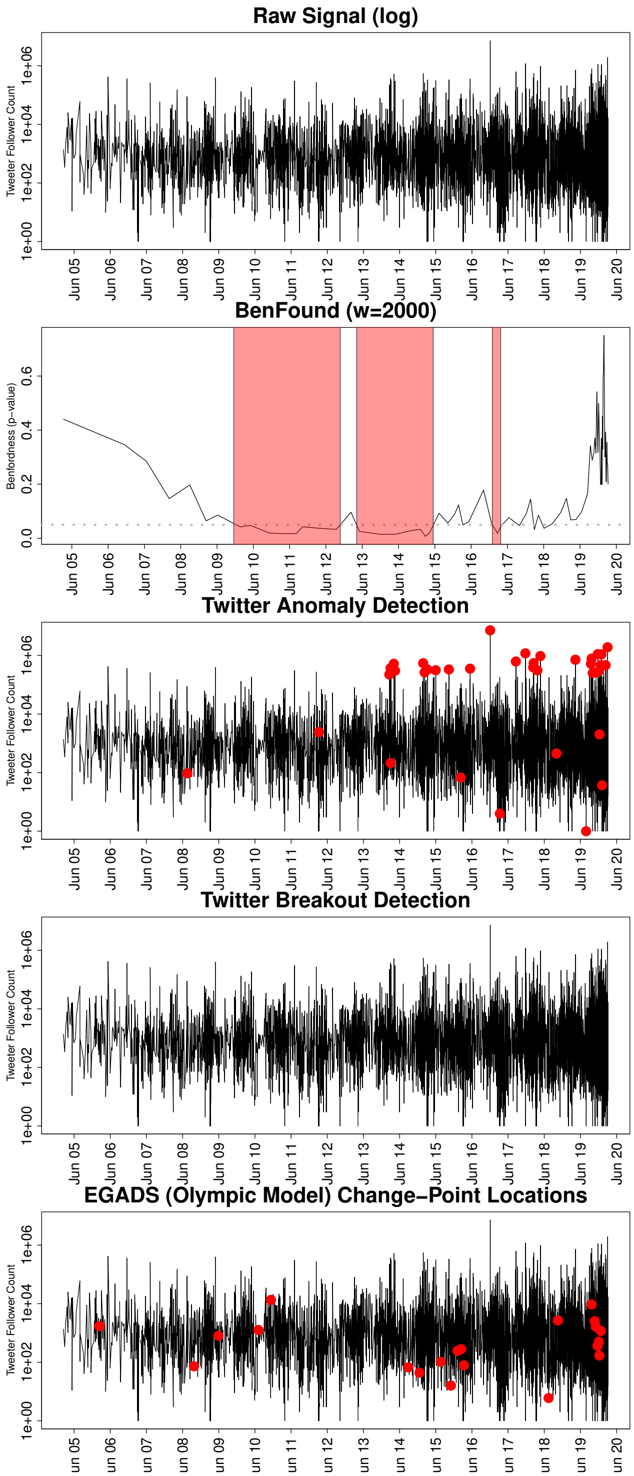


Figure 6: Raw follower counts (top) for users tweeting the hashtag FathersDay. The remaining plots show events, anomalies and changepoints detected by BenFound and other approaches.

to Benford *on* Father's Day. Looking at the data, we quickly see the behavior that the Benfordness signal describes. In the lead-up to Father's Day, a large number of spammers and advertising accounts hijacked the hashtag. The follower-counts of these accounts have a non-Benford (unnatural) digit distribution. This behavior is reflected in the low p-value of the Benfordness signal. It first drops below our threshold on 10 June 2016[7], the beginning of the weekend before Father's Day and a logical time for those spammers and advertisers unscrupulously trying to generate profit from the event to begin pushing their content. Example tweets from this time are in Table 3.

On Father's Day itself, spammers and advertisers presumably recognized that time had run out to generate profit from this hashtag. The tweet behavior shifted to the more organic kind of content expected for this hashtag. In Table 3 we find examples of the tweets from 19 June 2016 (users celebrating the efforts of fathers).

The other techniques struggle to extract meaningful information from the raw signal. Two of the techniques find an indigestible number of changepoints/anomalies; the third (Breakout Detection) finds none at all.

Father's Day lead-up:
"Oh Yea! I just entered to #Win LED GlowBowl"; "Check out Libbey Ceramic Tiki Mug Blue 7 x 3 16 Ounces"; "Enter the #giveaway to #win"; "I just entered to #win a $50 giftcard to @cuffdaddy #giveaway";
Father's Day itself:
"I love you Dad, Happy Father's Day"; "The best thing a man can do for his children is to love their mother"; "To all of the champion fathers... Happy #FathersDay!"; "Hope you had a great #FathersDay, lads!"

Table 3: Tweets *leading up to* and *on* Father's Day

8 CASE STUDY: OUTAGE DETECTION

Shortly before the time of data collection, the popular **#PokemonGO** game had been released. We tracked the tweets to this hashtag over a number of days. The Benfordness signal is shown in Figure 7.

The signal shows a sharp drop on July 15 and 16, suggesting with high certainty that the underlying distribution had changed from Benford to non-Benford. In the colloquial context of Benford's earlier quote, this implies that tweeting behavior against this hashtag had become "unnatural". An analysis of the data reveals that the Pokemon GO application suffered severe outages during this time period. The tweets during this time period were largely revolving around these problems (see Table 4) and had a non-Benford digit distribution. The official **@PokemonGoApp** account confirmed the problems on July 16 (Figure 8). As a comparison, the state-of-the-art Twitter Anomaly Detection result is shown in Figure 7, and events/topics detected using two state-of-the-art Twitter text-mining approaches are shown in Table 5.

9 EXPERIMENTS ON SYNTHETIC DATA

In this section we use synthetically-generated numerical time-series data to compare our proposed method with state-of-the-art techniques from the field of numerical event- and anomaly-detection.

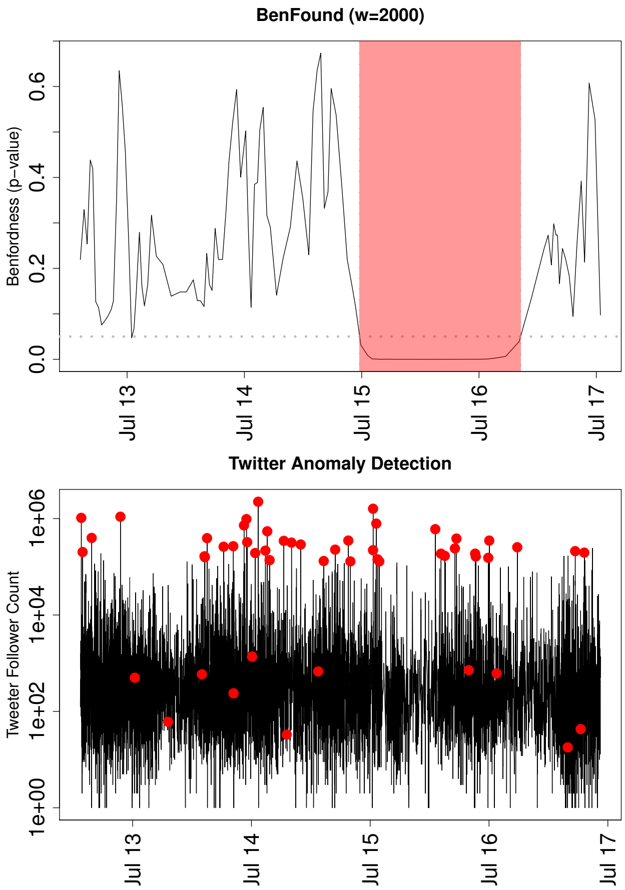


Figure 7: Tracking the Benfordness of follower-counts associated with users tweeting against the #PokemonGO hashtag.

SERVER UPDATE: According to @Independent, #PokemonGO servers have been taken down in a DDOS attack.
The #PokemonGO servers are down due to a DDOS attack. No word on when they will be back. Stay with us for the latest.
Niantic trying to fix the #PokemonGO servers is as successful as Team Rocket trying to kidnap Pikachu.
#PokemonGO is down. What do I do now with my time?

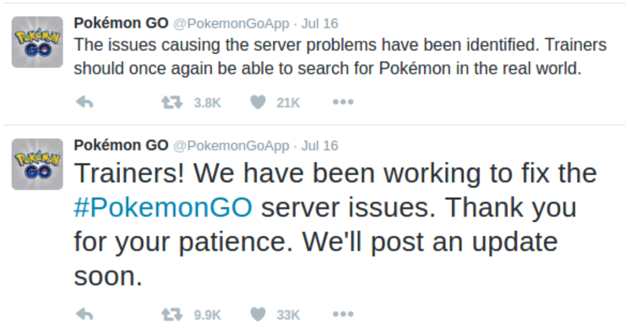
Table 4: Sample of Tweets with the hashtag #PokemonGO on July 15 and 16, 2016

Top 10 (Twitter NLP [18]): has in, strict parents, bringing out, **are down**, came over, get off, **go down**, looking for, **been down**, get out
Top 10 (Twitter Topic Detection (Streaming NMF) [9]): [argentina, coins, nintendome], [team, argentina, spark], [pokemongonews, argentina, community], [pokecoins, try, need], [argentina, giveaway, must], [try, **servers**, pokecoins], [syrian, hopes, saved], [argentina, coins, lucky], [argentina, need, try], [argentina, **unstable**, lucky]

Table 5: Top 10 events/topics found by two state-of-the-art Twitter text-mining techniques [9, 18].

Our generative model is inspired by our observations from online services such as Wikipedia and Twitter. From our introduction we

[7] We note that times were not measured in an American timezone (rather UTC), hence the reason for the slight shift in the data.



Pokémon GO @PokemonGoApp · Jul 16

The issues causing the server problems have been identified. Trainers should once again be able to search for Pokémon in the real world.

Pokémon GO @PokemonGoApp · Jul 16

Trainers! We have been working to fix the #PokemonGO server issues. Thank you for your patience. We'll post an update soon.

Figure 8: Tweets from the verified PokemonGoApp account on July 16, 2016.

know that normal usage patterns in these services lead to highly-Benford metrics. In contrast, the set of such values coming from "unnatural" usage patterns like bots and hashtag-hijacking is often non-Benford. We therefore consider the task of detecting changes from one of these states to the other using synthetically-generated data. That is, our goal is to be alerted when such non-Benford changes occur (denoted a *negative* event), as well as when things return to being Benford (denoted a *positive* event).

We make the simplifying assumption that one interaction (e.g. page edit in Wikipedia or tweet in Twitter) occurs per unit of time. The n interactions with the system fall at times $t_1, \ldots, t_n$. To evaluate the detection of both *negative* and *positive* events, we divide time into **three intervals**. The first interval spans the range $t_1, \ldots, t_{\lfloor \frac{n}{3} \rfloor}$, the second $t_{\lfloor \frac{n}{3} \rfloor + 1}, \ldots, t_{\lfloor \frac{2n}{3} \rfloor}$ and the third $t_{\lfloor \frac{2n}{3} \rfloor + 1}, \ldots, t_n$. Inside the first and third intervals we generate "natural" (Benford) data. For the second interval we generate non-Benford data. In this way we synthesize a negative event at the transition from the first interval to the second, followed by a positive event at the transition from the second interval to the third.

To generate Benford data for **intervals one and three**, we begin with a uniformly-distributed sample $\vec{u} \in [0, 1)^s$. With reference to Section 5, this sample $\vec{u}$ represents our mantissae. Next we generate the sample $\vec{m} \in \{0, 1, \ldots, 10\}^s$ of integers (uniformly sampled with replacement). This sample $\vec{m}$ represents our "magnitude" information. Our final Benford set of values then satisfies $\log_{10}(\vec{b}) = \log_{10}(\vec{u} + \vec{m})$, thus $\vec{b} = (10^{u_1 + m_1}, \ldots, 10^{u_s + m_s})$.

To generate *non-Benford* data for **interval two**, we follow [5] and *uniformly* sample in the range $[0, 10^{11}]$.

Figure 9 compares BenFound with 22 different approaches to event and change-point detection. For this data we are not able to compare with techniques specific for social media (e.g. Twitter) because they rely on the text feed and particular features (hashtags, retweets). BenFound is the only approach able to identify the correct number of changepoints (two), their locations and the fact that they form a single "event". With respect to **runtime**, BenFound's bandwidth is 8900 streamed values per second when using our R prototype with window size $w = 250$. Doubling to $w = 500$ reduces the bandwidth to 7850 streamed values per second. As a real-world reference, we note that around 6000 tweets are tweeted on Twitter every second[8].

10 RELATED WORK AND DISCUSSION

Recent work has shown BL to have practical uses beyond the classical application of forensic accounting. In [10] the authors investigate the measurement of keystroke-dynamics for the purposes of authentication and identification, observing that latency values follow BL. In [3] the authors consider the inter-arrival times of UDP packets and flows in networks, analyzing three significant digits with a digit-based sum-of-squared-errors (SSE) approach to detect anomalies (although an appropriate threshold is not discussed). The year 2015 also saw BL investigated for the first time in the context of online services [16]. The primary contribution was to show that snapshots of randomly-sampled user metrics in *social networks* (Facebook, Twitter, Pinterest) are Benford. The measurements in our introduction support this conclusion. In addition, we have presented results showing that metrics from Wikipedia, YouTube and GitHub are likewise Benford, and focused on the *real-time* detection of BL *deviations* in such systems.

A small set of dedicated BL reference volumes now exists. For theoretical and practical perspectives we respectively refer to [6] and [15]. The latter focuses heavily on BL in forensic accounting, auditing and fraud detection. It treats BL-conformity tests and also compares BL to the well-known Zipf's Law; however it does not treat event-detection. In Section 5 we re-evaluate the conformity tests and propose an alternative based on the formal Kolmogorov-Smirnov test.

The highly-cited work of Clauset et al. [7] looks at techniques for detecting and characterizing power laws in nature. The authors show applications to kinds of data also seen in classical BL demonstrations (e.g. populations of cities). The authors identify two key challenges in the task of detecting if such data follow power laws, namely 1) that large fluctuations can occur in the tail of the distributions and 2) it is difficult to determine the range over which the power law does hold. The authors discuss techniques for solving these problems, but interestingly, viewing the data through the lens of BL enables us to avoid them because the magnitude information is intentionally discarded (we need only consider the mantissae over $[0, 1)$).

The topics of event-, anomaly- and changepoint-detection have seen numerous contributions. Focusing firstly on Twitter-specific techniques, we find in [18] and [9] two state-of-the-art approaches for the extraction of event phrases and topics from Twitter. Unlike BenFound, both rely on processing the tweet text stream. Additionally, whilst BenFound only raises a "red flag" when the digit distribution violates BL, these techniques *continuously* yield a set of e.g. top-10 topics.

As BenFound is not restricted to text and graph structures, we stress that it is by no means restricted to social networks. BenFound can be applied to any numerical data stream obeying BL. In searching for comparison techniques we therefore extend our scope to the state-of-the-art in *numerical* anomaly- and changepoint-detection from data-mining and statistics.

[8] internetlivestats.com/twitter-statistics

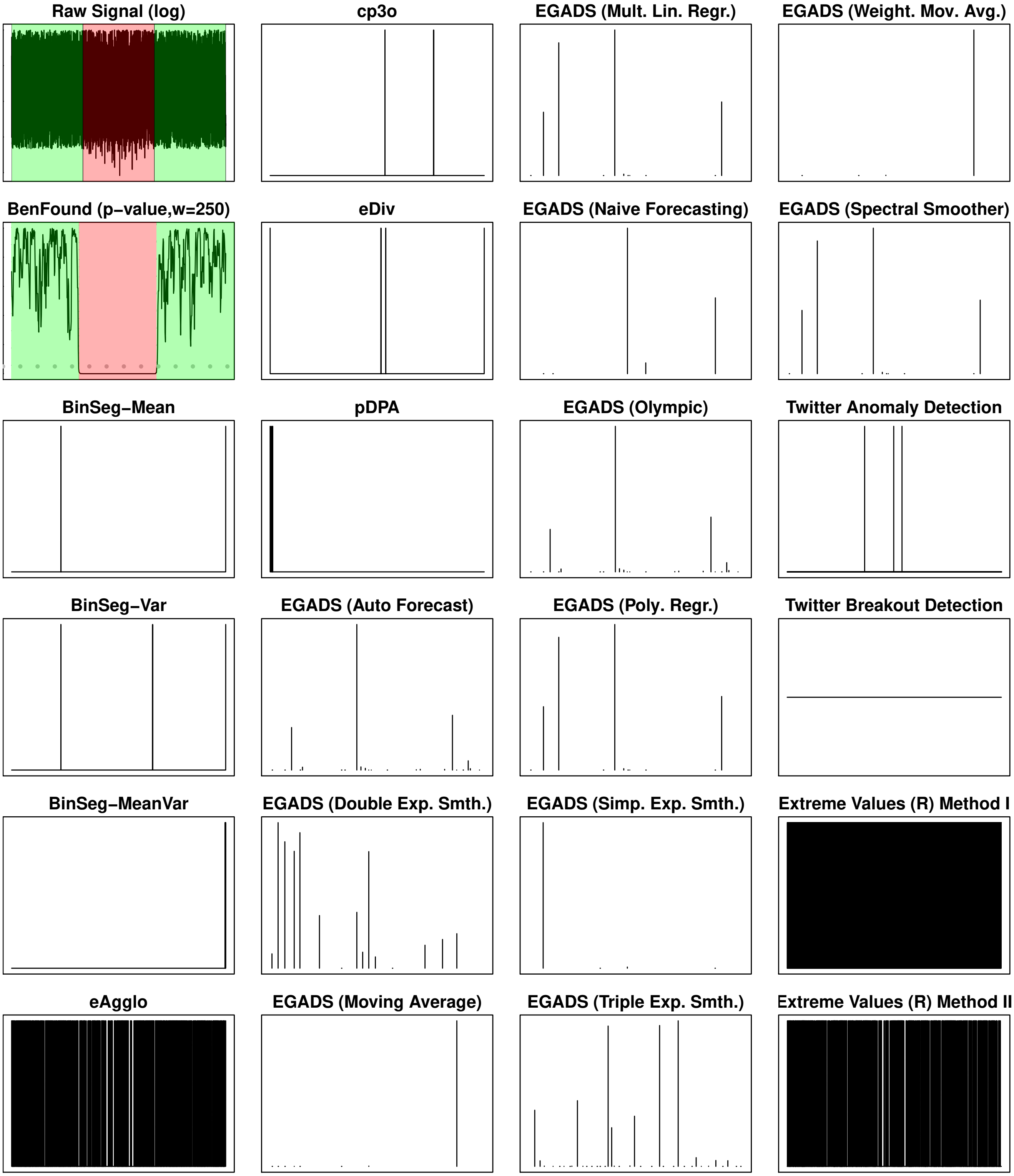


Figure 9: A synthetic time signal (top left) with Benford (green) and non-Benford (red) segments. BenFound (row 2, col 1) is the only technique able to correctly detect the state-change and its regression.

The most recent high-level contribution from the data-mining community is named the Extensible Generic Anomaly Detection System (EGADS in Figures 6 and 9) and stems from work at Yahoo [13]. EGADS is a comprehensive framework that supports anomaly-detection based on a variety of configurable models, features and metrics (synthetic experiments on 11 of which are included in Figure 9). Two further approaches come from Twitter [11, 21]. Twitter Anomaly Detection [11], for example, employs time-series decomposition using statistical metrics to detect both global and local anomalies in the presence of seasonality and an underlying trend.

From the recent statistical literature we find in [14] the non-parametric approach eDiv. Unlike previous methods which typically focus on detecting a change in mean, variance or kurtosis, eDiv is able to detect any distributional change. From the same work we find eAgglo, the bottom-up variant of eDiv, and another probabilistic pruning method cp3o. These three methods are compared to BenFound in Figure 9. Each is designed for an offline setting and has a quadratic time complexity in the series length *n*.

The pruned dynamic programming algorithm [17] (pDPA in Figure 9) is an offline approach which uses a functional cost representation to infer both the number and positions of the change-points from the data. It requires a user-specified parameter k_{max} which represents the maximum number of change points to find. Its computational complexity is quadratic in the series length *n*. Finally, we compare to the early Binary Segmentation baseline [8, 19] (BinSeg in Figure 9).

Importantly, all of these approaches consider the distribution of the *complete* numerical values in question. BenFound, in contrast, focuses on a particular kind of change event: a change in its leading digit distribution. This is achieved by considering BL and purposefully *ignoring the magnitude information* of the temporal measurements. Fundamentally then, BenFound distances itself from the discussed state-of-the-art in that it does not look for changes to the underlying distribution of *complete* values, but rather that of only the *leading digits*. For this reason, BenFound has been able to correctly identify events in synthetic and real data (Sections 7, 8 and 9) that are not found by any of the comparison techniques.

Although BL is typically illustrated by considering leading-digit histograms, BenFound is based on a robust conformity test that uses the logarithmic basis of BL. The leading-digit histograms can still be useful in the post-processing stage, helping to find the cause for any "red flags" raised by BenFound (as was done in Section 4). Future work should look into using the ECDF as an alternative tool for this same goal (for example, searching for regions of the ECDF with the largest deviation in gradient from the uniform CDF).

Finally, our technique has **limitations**. Firstly, it is clearly only applicable to numeric data that obeys BL. It is thus not able to find *all* conceivable types of anomalies and events in numerical data (BenFound focuses only on departures from Benfordness). Secondly, it requires a manual inspection step to analyze the cause for found events. Finally, it is not immune to false negatives. That is, if parties that artificially manipulate a system are aware of BL, they may tune their interactions such that the system metrics are not significantly affected from a BL perspective. In practice, BenFound can be deployed as a *complement* to other anomaly-detection techniques to help mitigate the latter limitation (perhaps as part of an ensemble approach).

11 CONCLUSION

Various metrics from online services such as Twitter, Wikipedia, YouTube and GitHub naturally obey BL. When these metrics violate BL in real-time, it is often a sign of significant "unnatural" behavior. In this work we have proposed BenFound, a real-time event-detection approach for "red-flagging" such violations. In the case of online services, the interactions may be anti-social or malicious in nature, like non-permitted bot activity or hashtag-hijacking. We have shown how BenFound exploits a novel BL-conformity test based on the Kolmogorov-Smirnov test, and compared BenFound to state-of-the-art event-detection techniques in controlled settings with synthetic data. Finally, we have deployed our technique to various real-world settings and demonstrated practical knowledge discovery. BenFound is non-parametric, robust, easily implemented and can be deployed efficiently in real-time on numerical data streams that obey BL.

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