COL 765: Introduction to Logic and Functional Programming
Quiz 1, 25.07.2024
(Direct proofs from definitions: Case Analysis)

Name: ____________________SOLUTION_____________ Entry No. _XXXX__

Q1 [5] Prove Left Distributivity of union over intersection: Show from the definitions of union and intersection that $S_1 \cup (S_2 \cap S_3) = (S_1 \cup S_2) \cap (S_1 \cup S_3)$**.**

Proof Let $x \in S_1 \cup (S_2 \cap S_3)$.

So by definition of union, $x \in S_1$ *or* $x \in (S_2 \cap S_3)$.

- If $x \in S_1$ then $x \in (S_1 \cup S_2)$ *and* similarly $x \in (S_1 \cup S_3)$. (Defn of union)
  So, $x \in (S_1 \cup S_2) \cap (S_1 \cup S_3)$. (Defn of intersection)
- If $x \in (S_2 \cap S_3)$, then $x \in S_2$ *and* $x \in S_3$ (Defn of intersection)
  So, $x \in (S_1 \cup S_2)$ *and* $x \in (S_1 \cup S_3)$ (membership of union, used twice)

So, in both cases, $x \in (S_1 \cup S_2) \cap (S_1 \cup S_3)$.

Conversely, let $x \in (S_1 \cup S_2) \cap (S_1 \cup S_3)$.

So, $x \in (S_1 \cup S_2)$ *and* $x \in (S_1 \cup S_3)$ (Defn of intersection)

- If $x \in S_1$ then $x \in S_1 \cup (S_2 \cap S_3)$ (Defn of union)
- If $x \notin S_1$ then $x \in S_2$ *and* $x \in S_3$ (membership of *both* of the unions)
  i.e., $x \in (S_2 \cap S_3)$. (Defn of intersection)
  So, $x \in S_1 \cup (S_2 \cap S_3)$ (Defn of union)

So in either case, $x \in S_1 \cup (S_2 \cap S_3)$.

Q2 [5] Prove Absorption of intersection into union: Show from the definitions of union and intersection that $S_1 \cap (S_1 \cup S_2) = S_1$**.**

Proof Let $x \in S_1 \cap (S_1 \cup S_2)$

So $x \in S_1$ *and* $x \in (S_1 \cup S_2)$ (by Defn of intersection)

So, trivially, $x \in S_1$

Conversely, let $x \in S_1$

So, $x \in S_1$ (*trivially*)

and $x \in (S_1 \cup S_2)$ (by Defn of union)

So, $x \in S_1 \cap (S_1 \cup S_2)$ (by Defn of intersection)