COL 765: Introduction to Logic and Functional Programming
Quiz 0, 22.07.2024
(Direct proofs from definitions)

Name: _____ Entry No. _____

Q1 [5] From the definition (of set equality), show that the notion of set equality is transitive. That is, if $S_1 = S_2$ and $S_2 = S_3$ then $S_1 = S_3$.

Proof Let $S_1 = S_2$ (i) and $S_2 = S_3$ (ii).
So, by Def of set equality,
$x \in S_1$ if and only if $x \in S_2$ from (i), and
$x \in S_2$ if and only if $x \in S_3$ from (ii)
Therefore, $x \in S_1$ if and only if $x \in S_3$
i.e., by Def of set equality, $S_1 = S_3$.

Q2 [5] From the definition (of subset), show the subset relationship is anti-symmetric. That is, if $S_1 \subseteq S_2$ and $S_2 \subseteq S_1$ then $S_1 = S_2$.

Proof Let $S_1 \subseteq S_2$ (i) and $S_2 \subseteq S_1$ (ii).
So, by Def of subset,
if $x \in S_1$ then $x \in S_2$ from (i), and
if $x \in S_2$ then $x \in S_1$ from (ii).
Therefore, $x \in S_1$ if and only if $x \in S_1$
i.e., by Def of set equality, $S_1 = S_3$.