EEE3027 Integrated Circuit Design

Semester 1 Report

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Contents

Abstract

This is a placeholder for the abstract of the report.

INTRODUCTION

1.1 Background and Key Concepts

1.1.1 Transistors

A transistor is a semiconductor device used to amplify or switch electronic signals and electrical power. It is composed of semiconductor material, usually with at least three terminals for connection to an external circuit. A voltage or current applied to one pair of the transistor's terminals controls the current through another pair of terminals.

Transistor Structure:

- **Source (S):** The terminal through which carriers enter the channel.

- **Drain (D):** The terminal through which carriers leave the channel.

- **Gate (G):** The terminal that modulates the conductivity of the channel.

- **Body (B):** The substrate on which the transistor is built, often connected to the source.

- **Channel:** The region between the source and drain where current flows when the transistor is on.

- **Oxide Layer:** An insulating layer between the gate and the channel, typically made of silicon dioxide (SiO2).

Figure 1: Basic structure of an NMOS and PMOS transistor.

1.1.2 MOSFET Types

- **N-channel (NMOS):** electrons are charge carriers.

- **P-channel (PMOS):** holes are charge carriers.

1.1.3 Operating Regions of a MOSFET

Region	Condition (N-channel)	Description
Cutoff	$V_{GS} < V_{th}$	OFF (no current)
Triode (Linear)	$V_{GS} > V_{th}$, $V_{DS} < V_{GS} - V_{th}$	Acts as variable resistor
Saturation (Active)	$V_{GS} > V_{th}$, $V_{DS} \geq V_{GS} - V_{th}$	Current source (amplifier region)

1.2 Importance of MOSFET Physical Parameters

The physical dimensions of a MOSFET determine its performance in terms of speed, power, and reliability.

1.2.1 Key Parameters and Effects

Parameter	Symbol	Importance
Gate Length	L	Controls speed/gain; shorter L, more speed but causes leakage.
Gate Width	W	Controls current; wider W, more current and capacitance.
Oxide Thickness	t_{ox}	Thinner ox - improves gate control but increases tunneling.
Channel Depth	$-$	Affects channel formation and conductivity.
Threshold Voltage	V_{th}	Determines switching voltage.
Junction Depth	$-$	Impacts short-channel effects and capacitance.

1.2.2 Significance

- **Speed:** shorter channel $\Rightarrow$ faster switching.
- **Power Efficiency:** optimized V_{th} and t_{ox} reduce power loss.
- **Current Drive:** larger W gives more drive capability.
- **Reliability:** careful balance of parameters prevents breakdown and leakage.

1.3 Gain

1.3.1 Definition

Gain is the ratio of output signal to input signal, as shown by *Equations 1 and 2*:

$$\text{Gain} = \frac{\text{Output}}{\text{Input}} \tag{1}$$

Types include:

$$A_v = \frac{V_{out}}{V_{in}}, \quad A_i = \frac{I_{out}}{I_{in}}, \quad A_p = \frac{P_{out}}{P_{in}} \tag{2}$$

1.3.2 Gain in BJTs and MOSFETs

Equations 3 and 4 show gain definitions for BJTs and MOSFETs:

$$\text{BJT current gain} \beta = \frac{I_C}{I_B} \tag{3}$$

$$\text{MOSFET transconductance} g_m = \frac{\partial I_D}{\partial V_{GS}} = \beta(V_{GS} - V_{th}) \tag{4}$$

1.3.3 Gain in Decibels

Gain in decibels (dB) is given by *Equation 5*

$$\text{Voltage Gain (dB)} = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{5}$$

:

1.3.4 Importance

- Determines amplification strength.
- Critical in analogue amplifiers, control systems, and logic gates.

1.4 Gate Length vs. Channel Length

- **Gate Length (L_G):** physical distance between source and drain under the gate.
- **Channel Length (L_{CH}):** effective electrical distance current travels.

They are related by *Equation 6*:

$$L_{CH} = L_G - \Delta L \tag{6}$$

where ΔL is due to lateral diffusion of dopants.

1.4.1 Difference and Importance

Shorter effective channel length causes:

- Increased leakage and short-channel effects.
- Lower threshold voltage.
- Reduced control by the gate.

1.5 Beta (β) Equations for MOSFETs

Shown by *Equations 7 and 8*:

1.5.1 Definition

$$I_D = \frac{1}{2}\beta(V_{GS} - V_{th})^2 \quad \text{(saturation region)} \tag{7}$$

$$I_D = \beta\left[(V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right] \quad \text{(linear region)} \tag{8}$$

1.5.2 Beta Expression

Beta can also be expressed by *Equation 9*:

$$\boxed{\beta = \mu_n C_{ox} \frac{W}{L}} \tag{9}$$

where:

- μ_n = carrier mobility
- $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$ = oxide capacitance per unit area
- W = gate width
- L = gate length

1.5.3 Related Terms

$$k' = \mu_n C_{ox}, \quad \beta = k' \frac{W}{L}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \beta(V_{GS} - V_{th})$$

1.5.4 Importance

- Determines how strongly a MOSFET amplifies a signal.
- Affects gain, speed, and current drive.
- Critical for matching and biasing in analogue ICs.

FORMATIVE ASSESSMENT 1 REPORT

A MOS transistor operates in two main regions depending on the drain-source voltage (V_{DS}) and the gate-source voltage (V_{GS}). In *Figure 2*, we can see the NMOS Sweep for a few specified V_{GS} values. The explanation of the code will be explained below

2.1 NMOS I-V Characteristics (First Task - Lab 1 Part 1)

Linear Region

This occurs when:

$$V_{DS} < V_{GS} - V_T \tag{10}$$

In this region, the transistor behaves like a voltage-controlled resistor. The drain current is given by *Equation 11*:

$$I_D = \beta \left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2 \right] \tag{11}$$
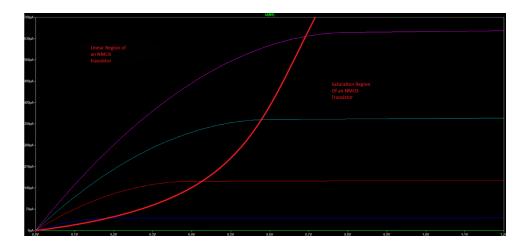
Figure 2: NMOS I-V Characteristics showing Linear and Saturation regions for different V_{GS} values.

where, if we refer to *Equation 10* again:

$$\beta = \frac{W}{L}\mu_n C_{ox} = \frac{W}{L}K_P$$

Initially, the current I_D increases approximately linearly with V_{DS}.

Saturation Region

This occurs when:

$$V_{DS} \geq V_{GS} - V_T \tag{12}$$

In this region, the channel is *pinched off* near the drain, and the drain current becomes almost independent of V_{DS}:

$$I_D = \frac{\beta}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \tag{13}$$

The slight upward slope in the I_D–V_{DS} characteristics is due to **channel-length modulation**, represented by the parameter λ (typically $\lambda = 0.02$). Parameters have been defined below and in the Background section. *Listing 1* shows the code used to generate the graph (next page).

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```
1 * Task 1 Extended: Sweep VDS and VGS (multiple curves)
2 * nmos_DC_sweep.cir is a netlist.
3
4 * Sets the VGS value to 1.2V (can be changed to sweep different values)
5 .param VGSVAL=1.2
6
7 * Tells us the voltages that we will use and the connections (so V_GS
      is a DC source and it has a gate connection (0V DC), same with the
      Drain)
8 VGS G 0 DC {VGSVAL}
9 VDS D 0 DC 0
10 * Tells us the transistor dimensions (gate width (W), channel length (L
      )) along with source and body voltage.
11 M1 D G 0 0 NLEVEL1 W=10u L=1u
12
13 .model NLEVEL1 NMOS LEVEL=1 VTO=0.4 KP=200u LAMBDA=0.02
14 * VTO is V_th, KP = transconductance parameter, LAMBDA is the channel-
      length modulation factor.
15
16 .dc VDS 0 1.2 0.01 VGS 0.4 1.2 0.2
17 * plots the drain current (I(M1)s) against VDS for different VGS values
       from 0.4V to 1.2V in steps of 0.2V
18 .plot DC I(M1)s
19 .end
20
21 * Comments start with asterisk
22 * First line is title (can be anything)
23 * Last line must be .END (or .end as SPICE is case insensitive)
24 * Node names: numbers or alphanumeric
25 * Node 0 is always ground
26 * Component values: scientific notation (1e-12) or units (1p, 1n, 1u, 1
      m, 1k, 1meg)
```

Listing 1: NMOS Sweep Simulation Code

Parameter Definitions

- I_D : Drain current
- V_{GS} : Gate-to-source voltage
- V_T : Threshold voltage
- V_{DS} : Drain-to-source voltage
- μ_n : Electron mobility
- C_{ox} : Oxide capacitance per unit area
- W : Channel width
- L : Channel length
- $\beta = \dfrac{W}{L}\mu_n C_{ox}$: Process transconductance parameter
- λ : Channel-length modulation parameter (typically $\lambda = 0.02$)
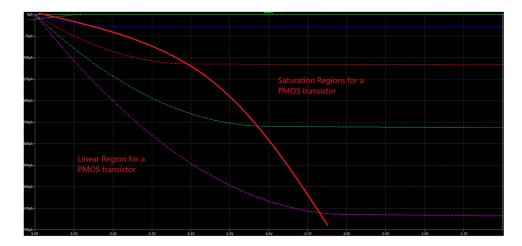
Figure 3: PMOS I-V Characteristics showing Linear and Saturation regions for different V_{SG} values.

2.2 PMOS I-V Characteristics (Second Task - Lab 1 Part 1)

We can see the PMOS Sweep for a few specified V_{SG} values in *Figure 3*. The explanation of the code will be shown below:

For a PMOS transistor, all voltages are **negative** relative to the NMOS device. The key differences are:

- V_{SG} (source-to-gate voltage) replaces V_{GS}
- V_{SD} (source-to-drain voltage) replaces V_{DS}
- The drain current I_D flows from the **source to the drain**, opposite to that of the NMOS

Linear Region

This occurs when:
$$V_{SD} < V_{SG} - |V_T|$$

The transistor behaves like a voltage-controlled resistor. The drain current is given by:

$$I_D = \beta \left[(V_{SG} - |V_T|)V_{SD} - \frac{1}{2}V_{SD}^2 \right]$$

where:
$$\beta = \frac{W}{L}\mu_p C_{ox} = \frac{W}{L}K_P$$

Initially, I_D increases approximately linearly with V_{SD}.

Saturation Region

This occurs when:
$$V_{SD} \geq V_{SG} - |V_T|$$

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In this region, the channel is *pinched off* near the drain, and the current becomes almost independent of V_{SD}:

$$I_D = \frac{\beta}{2}(V_{SG} - |V_T|)^2(1 + \lambda V_{SD})$$

The small increase in I_D with V_{SD} is due to **channel-length modulation**, represented by λ.

Calculation with Example Values from PMOS Sweep

For a PMOS transistor with:

$$V_{SG} = 1.2 \text{ V}, \quad |V_T| = 0.4 \text{ V}$$

the boundary between the linear and saturation regions occurs at:

$$V_{SD} = V_{SG} - |V_T| = 1.2 - 0.4 = 0.8 \text{ V}$$

On an I_D–V_{DS} plot, this corresponds to:

$$V_{DS} = -0.8 \text{ V}$$

indicating the same boundary but with reversed voltage polarity compared to NMOS operation.

For $V_{GS} = -1.2\,\text{V}$:

$$\text{Saturation occurs when } V_{DS} \leq -0.8\,\text{V}$$
$$\text{Linear region: } -0.8\,\text{V} < V_{DS} < 0\,\text{V}$$
$$\text{Saturation region: } V_{DS} \leq -0.8\,\text{V}$$

$$\text{Linear Region } (V_{SD} < V_{SG} - |V_T|):$$
$$I_D = -\beta \left[(V_{SG} - |V_T|)V_{SD} - \tfrac{1}{2}V_{SD}^2\right]$$

$$\text{Note: Current is negative (flows from source to drain)}$$

$$\text{Example calculation for } V_{SG} = 1.2\,\text{V}, \ V_{SD} = 0.4\,\text{V}:$$
$$\beta = \frac{W}{L} \cdot K_P = \frac{10}{1} \cdot 200\,\mu = 2\,\text{mA/V}^2$$
$$I_D = -2\,\text{mA/V}^2 \times \left[(1.2 - 0.4) \times 0.4 - 0.5 \times 0.4^2\right]$$
$$I_D = -2\,\text{mA/V}^2 \times [0.32 - 0.08]$$
$$I_D = -2\,\text{mA/V}^2 \times 0.24$$
$$I_D = -0.48\,\text{mA} = -480\,\mu\text{A}$$

Saturation Region $(V_{SD} \geq V_{SG} - |V_T|)$:

$$I_D = -\frac{\beta}{2}(V_{SG} - |V_T|)^2(1 + \lambda V_{SD})$$

Example for $V_{SG} = 1.2\,\text{V},\ V_{SD} = 1.2\,\text{V}$:

$$I_D = -\left(\frac{2\,\text{mA/V}^2}{2}\right)(0.8)^2(1.024)$$

$$I_D = -1\,\text{mA/V}^2 \times 0.64 \times 1.024$$

$$I_D = -0.655\,\text{mA} = -655\,\mu\text{A}$$

Current is negative (opposite polarity to NMOS) and the curves mirror NMOS behavior, with saturation boundaries at $V_{DS} = -(V_{SG} - |V_T|)$. This is shown in *Figure 3*.

```
1  * TASK 2: PMOS I-V DC Sweep with Multiple VSG Curves
2
3  * Define gate-source voltage as parameter (negative for PMOS)
4  .param VGSVAL=-1.2
5
6  * Gate voltage source: negative voltage for PMOS operation
7  * Connected between gate (G) and ground (0)
8  VGS G 0 DC {VGSVAL}
9
10 * Drain voltage source: starts at 0V, swept negative
11 VDS D 0 DC 0
12
13 * PMOS transistor M1:
14 * Connections: Drain=D, Gate=G, Source=0(GND), Bulk=0(GND)
15 * In normal PMOS, bulk connects to highest potential (VDD)
16 * Here, 0V used  as reference for ease of simulation, refer to source
      in report.
17 * Geometry: Width=10um (same as NMOS), Length=1um
18 M1 D G 0 0 PLEVEL1 W=10u L=1u
19
20 * Level 1 PMOS model parameters:
21 * VTO   = -0.4V   : Threshold voltage (negative for PMOS)
22 * KP    = 200uA/V^2: Transconductance (typically half of NMOS in
      reality)
23 * LAMBDA= 0.02V^-1: Channel-length modulation (same as NMOS)
24 .model PLEVEL1 PMOS LEVEL=1 VTO=-0.4 KP=200u LAMBDA=0.02
25
26 * DC Sweep Analysis:
27 * Primary sweep: VDS from 0V to -1.2V in -0.01V steps (negative
      direction)
28 * Secondary sweep: VGS from -0.4V to -1.2V in -0.2V steps
29 * This generates 5 curves (VGS = -0.4, -0.6, -0.8, -1.0, -1.2V)
30 .dc VDS 0 -1.2 -0.01 VGS -0.4 -1.2 -0.2
31
32 * Plot drain current (will be negative for PMOS)
33 .plot DC I(M1)
34
35 * End of netlist
36 .end
```

Listing 2: PMOS Sweep Simulation Code

2.3 Beta Sweep

2.3.1 Question 1: Identification

- VGS = 1.2V (from .param VGSVAL=1.2)

- VDS = 1.2V (from .param VDSVAL=1.2)

- L = 1um (from M1 line: L=1u)

2.3.2 Question 2A Model Parameter Definitions

VTO (Threshold Voltage): Symbol: V_T or V_{th}. Units: Volts (V).

Zero-bias threshold voltage - the minimum gate-source voltage required to form a conducting inversion channel. Below this voltage, the transistor is in cutoff. Determined by gate material work function, oxide thickness, substrate doping, and interface charges. In our simulation: VTO = 0.4V (NMOS), -0.4V (PMOS).

KP (Transconductance Parameter): Symbol: k' or μC_{ox}. Units: A/V^2.

Intrinsic transconductance parameter representing the product of carrier mobility (μ) and gate oxide capacitance per unit area (C_{ox}):

$$KP = \mu \cdot C_{ox} = \mu \cdot \frac{\varepsilon_{ox}}{t_{ox}} \tag{14}$$

Determines current drive capability. The gain factor is $\beta = (W/L) \cdot KP$, so $I_D \propto KP$. NMOS typically has 2-3× higher KP than PMOS due to higher electron mobility. In our simulation: KP = $200\mu A/V^2$ for both types.

LAMBDA (Channel-Length Modulation): Symbol: λ. Units: V^{-1}. Default: 0.0.

Channel-length modulation coefficient accounting for effective channel shortening as V_{DS} increases in saturation. Without LAMBDA, saturation curves would be perfectly flat (infinite output resistance). With LAMBDA:

$$I_D(sat) = \frac{\beta}{2}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) \tag{15}$$

Output resistance: $r_o = 1/(\lambda I_D)$. Inversely proportional to channel length: $\lambda \propto 1/L$. Analogous to inverse Early voltage in BJTs. In our simulation: $\lambda = 0.02$ V^{-1}, causing $\sim 2.4\%$ current increase across 1.2V V_{DS} range.

2.3.3 Question 2B Model Assumptions

The standard textbook equation for saturation current is:

$$I_D = \frac{\beta}{2}(V_{GS} - V_T)^2 \quad \text{where } \beta = \frac{W}{L} \cdot \mu_n C_{ox} \tag{16}$$

The complete SPICE Level 1 equation is:

$$I_D = \frac{KP}{2} \cdot \frac{W_{eff}}{L_{eff}} \cdot (V_{GS} - VTO)^2 \cdot (1 + LAMBDA \cdot V_{DS}) \tag{17}$$

To simplify the SPICE model to match textbook equations, the following assumptions are made:

1. LAMBDA = 0 (or very small): The term $(1 + \lambda V_{DS})$ is set to 1, neglecting channel-length modulation. This assumes perfectly flat I-V curves in saturation with infinite output resistance. In our simulation, $\lambda = 0.02$ causes only a $\sim 2.4\%$ variation, which is small but non-zero.

2. No body effect ($\gamma = 0$ or $V_{SB} = 0$): The body effect modifies threshold voltage as:

$$V_T = V_{T0} + \gamma(\sqrt{|2\phi_F + V_{SB}|} - \sqrt{|2\phi_F|}) \tag{18}$$

When bulk and source are both at ground ($V_{SB} = 0$), this term vanishes and $V_T = V_{T0}$, which is our case.

3. Long-channel approximation: Velocity saturation is neglected, assuming drift velocity remains proportional to electric field. Valid for $L \geq 1\mu$m. Short-channel effects (DIBL, threshold roll-off) are ignored.

4. No mobility degradation: Carrier mobility μ is assumed constant, independent of V_{GS}. In reality, high vertical electric fields reduce mobility, but Level 1 ignores this.

5. Ideal effective dimensions: $W_{eff} = W$ and $L_{eff} = L$ (no parasitic effects). When $LD = WD = XL = XW = DEL = 0$, drawn and effective dimensions are equal.

6. Subthreshold conduction ignored: For $V_{GS} < V_T$, Level 1 assumes $I_D = 0$. In reality, exponential subthreshold current exists: $I_D \propto \exp(V_{GS}/nV_t)$.

Verification of equivalence: If $KP = \mu_n C_{ox}$, then:

$$\beta_{textbook} = \frac{W}{L} \cdot \mu_n C_{ox} = \frac{W}{L} \cdot KP = \beta_{SPICE} \tag{19}$$

With $LAMBDA = 0$, $V_{SB} = 0$, and ideal geometry, the equations become identical.

2.3.4 Simulation Verification

VTO verification: DC sweep shows $I_D \approx 0$ at $V_{GS} = 0.4$V, confirming threshold. Current increases dramatically for $V_{GS} > 0.4$V.

KP verification: From beta_w_sweep.cir:

$$\frac{dI_D}{dW} = 65.54\,\mu\text{A}/\mu\text{m (measured)} = \frac{KP}{2L}(V_{GS} - VTO)^2(1 + \lambda V_{DS}) = 65.54\,\mu\text{A}/\mu\text{m (theory)} \tag{20}$$

Perfect agreement validates KP = 200 μA/V^2.

LAMBDA verification: In saturation at $V_{GS} = 1.2$V:

- $V_{DS} = 0.8$V: $I_D = 642\ \mu$A, $(1 + 0.02 \times 0.8) = 1.016 \rightarrow +1.6\%$
- $V_{DS} = 1.2$V: $I_D = 656\ \mu$A, $(1 + 0.02 \times 1.2) = 1.024 \rightarrow +2.4\%$

Measured current increase: $(656 - 642)/642 = 2.2\% \approx 2.4\%$ predicted. Slight upward slope in saturation curves confirms $\lambda = 0.02$V^{-1}.

Body effect assumption: $V_{SB} = 0$ throughout (source and bulk both grounded), so $V_T = VTO = 0.4$V constant. Confirmed by consistent threshold across all simulations.

Subthreshold assumption: Setting $V_{GS} = 0.3$V $< V_T$ yields $I_D \approx 0$ as Level 1 predicts, though real devices show exponential subthreshold current ($\sim$pA range).

2.4 How Does SPICE Work?

SPICE is the circuit simulator that we have been using so far. It models electronic circuits by solving nonlinear differential-algebraic equations derived from Kirchhoff's laws. The general form of the circuit equations is:

$$\mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}, t) = 0 \tag{21}$$

where $\mathbf{x}$ represents the vector of unknown node voltages and branch currents.

Using modified nodal analysis (MNA), SPICE forms equations of the type:

$$\mathbf{C}\,\dot{\mathbf{v}} + \mathbf{G}(\mathbf{v}) = \mathbf{i}_s(t) \tag{22}$$

where: - $\mathbf{C}$ is the capacitance matrix, - $\mathbf{G}(\mathbf{v})$ represents nonlinear conductances (resistors, diodes, transistors, etc.), - $\mathbf{i}_s(t)$ represents source currents.

For transient analysis, SPICE discretizes the time derivative using a numerical integration method such as Backward Euler [source]:

$$\dot{\mathbf{v}}(t_n) \approx \frac{\mathbf{v}_n - \mathbf{v}_{n-1}}{\Delta t}$$

This transforms the equations into a system of nonlinear algebraic equations:

$$\mathbf{F}(\mathbf{v}_n) = \mathbf{G}(\mathbf{v}_n) + \frac{\mathbf{C}}{\Delta t}(\mathbf{v}_n - \mathbf{v}_{n-1}) - \mathbf{i}_s(t_n) = 0 \tag{23}$$

To solve this system, SPICE uses the **Newton–Raphson method**, an iterative root-finding algorithm for nonlinear equations. Given a function $\mathbf{F}(\mathbf{v})$, the goal is to find $\mathbf{v}$ such that:

$$\mathbf{F}(\mathbf{v}) = 0 \tag{24}$$

Starting from an initial guess $\mathbf{v}^{(0)}$, the method iteratively refines the estimate using:

$$\mathbf{J}(\mathbf{v}^{(k)})\,\Delta\mathbf{v}^{(k)} = -\mathbf{F}(\mathbf{v}^{(k)}), \quad \mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \Delta\mathbf{v}^{(k)}$$

Here is a simple worked example of the Newton-Raphson method:

We want to solve the nonlinear equation

$$f(x) = x^2 - 2 = 0 \tag{25}$$

whose root is $\sqrt{2}$.

The Newton–Raphson iteration formula is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For our function $f(x) = x^2 - 2$,

$$f'(x) = 2x$$

so the iteration becomes:

$$x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n}$$

Let the initial guess be $x_0 = 1$. Then:

$$x_1 = 1 - \frac{1^2 - 2}{2(1)} = 1.5$$

$$x_2 = 1.5 - \frac{1.5^2 - 2}{2(1.5)} = 1.4167$$

$$x_3 = 1.4167 - \frac{1.4167^2 - 2}{2(1.4167)} = 1.4142$$

After three iterations, $x_3 \approx 1.4142$, which is very close to the true value of $\sqrt{2}$.

Here, $\mathbf{J}(\mathbf{v})$ is the **Jacobian matrix** of $\mathbf{F}$, defined as:

$$\mathbf{J}(\mathbf{v}) = \begin{bmatrix} \frac{\partial F_1}{\partial v_1} & \frac{\partial F_1}{\partial v_2} & \cdots & \frac{\partial F_1}{\partial v_n} \\ \frac{\partial F_2}{\partial v_1} & \frac{\partial F_2}{\partial v_2} & \cdots & \frac{\partial F_2}{\partial v_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial v_1} & \frac{\partial F_n}{\partial v_2} & \cdots & \frac{\partial F_n}{\partial v_n} \end{bmatrix} \tag{26}$$

The Jacobian represents how each equation in $\mathbf{F}$ changes with respect to each variable in $\mathbf{v}$. In circuit terms, it corresponds to the small-signal conductance matrix, which describes the local linear behavior of the nonlinear circuit.

The Newton–Raphson method repeats this process until the solution converges, meaning:

$$\|\Delta \mathbf{v}^{(k)}\| < \epsilon \tag{27}$$

for some small tolerance ϵ.

In summary: 1. SPICE forms nonlinear equations via modified nodal analysis. 2. It discretizes time derivatives for transient analysis. 3. It applies the Newton–Raphson method to iteratively solve the nonlinear system. 4. The Jacobian matrix captures how circuit equations respond to voltage and current changes, enabling efficient convergence to the correct solution.

2.4.1 Actual Example of Newton Raphson Method with a Circuit

Consider the series circuit: a voltage source V_s, a resistor R and a diode D in series. We wish to find the diode voltage V_d at DC (steady state).

Kirchhoff's Voltage Law (KVL) around the loop gives

$$V_s - IR - V_d = 0 \tag{28}$$

and the diode follows the Shockley equation

$$I = I_s \left(e^{\frac{V_d}{nV_t}} - 1 \right) \tag{29}$$

Substituting the diode law into KVL yields the nonlinear scalar equation for V_d:

$$f(V_d) \equiv V_s - R\, I_s \left(e^{\frac{V_d}{nV_t}} - 1 \right) - V_d = 0. \tag{30}$$

To solve $f(V_d) = 0$ with Newton–Raphson, form the iteration

$$V_d^{(k+1)} = V_d^{(k)} - \frac{f\left(V_d^{(k)}\right)}{f'\left(V_d^{(k)}\right)}. \tag{31}$$

We need the derivative $f'(V_d)$. Differentiating (3) gives

$$f'(V_d) = -RI_s \frac{1}{nV_t} e^{\frac{V_d}{nV_t}} - 1. \tag{32}$$

Thus each Newton step can be written explicitly as

$$V_d^{(k+1)} = V_d^{(k)} - \frac{V_s - RI_s\left(e^{\frac{V_d^{(k)}}{nV_t}} - 1\right) - V_d^{(k)}}{-RI_s \frac{1}{nV_t} e^{\frac{V_d^{(k)}}{nV_t}} - 1}. \tag{33}$$

A practical step-by-step procedure:

1. Choose device and circuit parameters V_s, R, I_s, n, V_t and an initial guess $V_d^{(0)}$ (a typical choice is $0.6-0.8$ V for silicon diodes, or $V_s/2$ if unsure).

2. Evaluate $f(V_d^{(k)})$ using (3) and $f'(V_d^{(k)})$ using (4).

3. Compute the Newton update by (5) to obtain $V_d^{(k+1)}$.

4. Check convergence: accept $V_d^{(k+1)}$ if

$$\left|V_d^{(k+1)} - V_d^{(k)}\right| < \varepsilon \quad \text{or} \quad \left|f(V_d^{(k+1)})\right| < \delta, \tag{34}$$

   where ε, δ are tolerances (e.g. 10^{-6} V or a small current error).

5. If not converged, set $k \leftarrow k+1$ and repeat from step 2. Include a maximum iteration limit to avoid infinite loops.

Notes and practical tips:

- Because the diode exponential can grow very rapidly, the term $RI_s e^{V_d/(nV_t)}$ may dominate for moderate V_d; choose the initial guess to avoid overflow in the exponential (or use scaled arithmetic / libraries that handle large arguments).

- If $f'(V_d^{(k)})$ is nearly zero, the Newton step can be large or unstable; in practice a damping (line search) factor $\alpha \in (0, 1]$ is sometimes used:

$$V_d^{(k+1)} = V_d^{(k)} - \alpha \frac{f(V_d^{(k)})}{f'(V_d^{(k)})}.$$

- For comparison, the same nonlinear equation can be solved in closed form using the Lambert W function to obtain

$$V_d = V_s + RI_s - nV_t\, W\left(\frac{RI_s}{nV_t} e^{\frac{V_s + RI_s}{nV_t}}\right), \tag{35}$$

   which gives an analytical reference solution (principal branch W_0) to verify the Newton result.

Finally, once V_d is found compute the diode current from the Shockley law:

$$I = I_s\left(e^{\frac{V_d}{nV_t}} - 1\right). \tag{36}$$

- Lab 1 Part 1

- N-Type characteristics

- P-Type characteristics

- MOSFET operation

- Parts that constitute a MOSFET and what altering them does to the operation of the MOSFET.

- Explanation of the PMOS and NMOS operation regions.

- What measurements were taken and how they were taken.

- Explanation of the graphs plotted and what they signify.

- Include code snippets of the simulation code used.

- Include graphs plotted.

- Analysis of the results.

  1. DC Analysis of NMOS and PMOS and explanation of what DC analysis is.

  2. Task 1: NMOS DC I-V sweep with annotations.

  3. Task 2: PMOS DC I-V sweep with annotations.

  4. Gain factor calculation and explanation.

  5. Beta explanation with Python example.

  6. Task 4 Iterative solution

- Lab 1 Part 2

- Explain what static analysis is.

- Draw an annotated schematic of the inverter circuit.

- Explain the operation of the inverter circuit.

- 1B, Annotate the file (All 7 lines)

- TASK 1C: DC Modes You have simulated the DC transfer characteristic of a CMOS inverter by sweeping the input voltage Vin from 0 V to 5 V. Plot the output voltage Vout against Vin. On your plot of Vout vs. Vin, clearly mark and label the following five operating points: A: Vin=0, B: Vin=1, C: Vin=2.5, D: Vin=4, E: Vin=5, For each point (A–E), identify the operating region of both the NMOS and PMOS transistors: Cutoff Triode (Linear) or Saturation, showing it as a table, and approximate Vout at each point.

- TASK 1D, NMOS pull up example, show the sweep as well, then answer these 5 questions:

- Q1. (1 mark) What is the highest output voltage reached in the NMOS pull-up version? Why does it not reach 5 V?

- Q2. (1 mark) Explain what happens to the NMOS pull-up transistor when Vin=0. Is it conducting?

- Q3. (1 mark) Describe what would happen if this NMOS-only inverter drove another CMOS logic gate. What are the risks?

- Q4. (1 mark) Why does a PMOS transistor avoid this issue? How does it behave differently from NMOS in pull-up?

- Q5. (2 marks) On your NMOS-only inverter plot, annotate the output voltage limit and mark the region where a 1 cannot be produced.

- TASK 2: TASK 2: CMOS Inverter Design for Delay Specification Use LTspice and theoretical models to design an inverter that meets timing specifications. Analyze performance using simulation and transistor-level delay theory (including propagation delay, rise/fall time, capacitive loading).

- TASK 2 Question 1 Transient Simulation and Analysis.

- TASK 2 Question 2 RC Delay Model.

- TASK 2 Question 3 Design and New Alternatives for performance.

- TASK 2 Question 4 Selection of questions to be answered.