CS 261: Project Report
Dynamic Formation of Credit Networks

Ana Carolina Queiroz, Sahasrajit Sarmasarkar, Hoden Abdi-Yusuf

May 2022

1 Updates/ Progress on the project

Progress and possible simplifications in the model:

- We update our belief of every node u about the overall network as follows. Let us denote the number of nodes which defaulted after a transaction with u at round i by D_u^i and total number of transactions of node u at round t by T_u^i.

  We update the overall belief of node u about the network at time t by $\hat{\mu}_{(u,V)} = \frac{\mu+\alpha(\sum_{i=1}^{t} D_u^i)}{1+\alpha(\sum_{i=1}^{t} D_u^i)}$ for some pre-defined constant α.

  We may obtain this belief update method by using a prior of Beta distribution as described in [2, Sec 2.4.3].

  Also, the belief of a node u about a node v at time-step t is given as follows:

  Suppose node u has $T_{u,v}$ transactions with node v till round t.

$$\mu_{(u,v)} = \begin{cases} 0 \text{ if node } v \text{ has defaulted} \\ (1-\alpha_t).\hat{\mu}_{(u,V)} + \alpha_t \text{ otherwise} \end{cases}$$

  Note that α_t is a decreasing function of t.

- Note that we define the utility as a concave function of the credit values assigned. We currently plan to model the utilities as described by the transition probabilities in [2]. Also we plan to do edge transactions proportional to the credit values assigned to every edge (current plan). Maybe later we may shift to diminishing utilities (not completely sure yet?) Also, we plan to use some software to solve the constrained maximisation problem which should be feasible as the objective function is concave, hence plan to use solvers like cvxpy for the same.

- We also plan to change the defaulting strategies of the unreliable nodes. We plan to make a start by making the un-relaible nodes default with a constant probability and also proportional to the values that they owe to their neighbours. We are slightly shifting from the model described above as it is difficult to compute the future expected marginality based on the history till round t.

2 Motivation

Credit networks are a useful abstraction to model trust in a network of individuals [1, 2]. Transactions between two individuals u and v who don't trust each other directly can occur as long as there is a directed path of users who trust each other, connecting u and v [2].

Our project focuses on the strategic formation of credit networks. In their seminal work on the topic, Dandekar et al. [2] study a one-shot game in which self-interested agents belonging to an exogenous network H decide how much trust they will assign to other agents in the resulting network G. We build on their work to study network formation with repeated games, in which agents have Bayesian beliefs about the *trustworthiness* of other agents in the network.

We adopt an externalist position of trust: the person who assigns trust to someone else has rationally estimated this amount, and updates this belief as new information becomes available [3]. From this perspective, it becomes important to investigate how trust networks arise in a dynamic setting, in which beliefs about the trustworthiness of other agents can evolve.

3 Current Model Outline

The first part of the project will set up the dynamic model of strategic formation of credit networks. Some sketches of elements of the model are as follow:

We have an initial network $H = (V, E)$, with agents partitioned into two types: trustworthy (T) and untrustworthy (U). There are $\mu \cdot |V|$ trustworthy agents and $(1-\mu) \cdot |V|$ untrustworthy agents. The true μ is not observed by any of the agents, but it is common knowledge that μ is drawn from a uniform distribution $\mu \sim \mathcal{U}_{[a,b]}$ for some $0 \leq a < b \leq 1$. Each agent also knows their true type.

Term	Definition
$d^{(i)}_{(u,v)}$	Node u honours IOU to node v at round i
$\tilde{\mu}^{(i)}_{(u,v)}$	Belief of trustworthiness of node u about node v at round i
$\hat{\mu}^{(i)}_{(u,V)}$	Belief of trustworthiness of node u about all nodes V in graph $G^{(i)}$ at round i
$c^{(i)}_{(u,v)}$	Credit extended to node v by node u at round i
B_u	Credit issuing limit of node u
$x^{(i)}_{(u,v)}$	IOU's that node u owes node v at round i

The following game is played repeatedly. In the first step of the i^{th} game, as in [2] each node decides how much credit (trust) to assign to each of the other nodes, and is also bounded by some credit limit it can extend to others. This process creates a graph $G^{(i)}$. We then allow for a series of transactions to occur in $G^{(i)}$, in which IOUs are exchanged between neighbors. In the second step, each node u "reclaims" for each neighbor v the IOUs issued by v that are in u's possession at the end of the transaction stage. At this point, each node u decides whether to default and not honor the IOUs it issued for v (i.e. $d^{(i)}_{(u,v)} = 1$) or to honor the IOUs (i.e. $d^{(i)}_{(u,v)} = 0$).

Step 0: Agents are Bayesian: The beliefs of agent u towards the trustworthiness of an agent v, $\tilde{\mu}_{(u,v)}$, is a function of u's belief about the overall trustworthiness of all agents in the network, $\hat{\mu}_{(u,V)}$, and the past behaviors of v that have been observed by u, $d_{(v,u)}$. After one round of transactions in a network $G^{(i)}$, u updates their belief about v based on their new belief of the overall trustworthiness of the network and on the newly-observed behavior of v. Our goal is to incorporate positive and negative reinforcement [4] on the beliefs agent u has about the other agents: A positive interaction with v (i.e. v honor IOUs issued to u) marginally increases $\hat{\mu}_{(u,V)}$ (belief of overall trustworthiness in network) and $\tilde{\mu}_{(u,v)}$ (belief of trustworthiness of node v). A negative interaction decreases both values, and we assume that once a node v defaults on u, u never extends credit to v again. Even if one node z did not default on a transaction with u, the belief of u towards z's trustworthiness might decrease if u has other negative interactions, since $\tilde{\mu}_{(u,z)}$ is also a function of the overall belief $\hat{\mu}_{(u,V)}$.

Step 1: In the first step of each round, agent u decide whether to form a tie with a given v, and if so, how much credit to extend to v, $c^{(i)}_{(u,v)}$, such that it maximizes its expected gains from trade $g_u(.)$ minus the losses $l(.)$ from potential defaults of neighbors. Note that we define $g_u(.)$ (gain of node u from trading) as a function of the credit extended by various nodes. The losses are a function of the belief u has of each v defaulting, given the amount of credit u extends to v and the (estimated) trustworthiness of v. The variables in bold are vectors with the corresponding values for each v, for a given u:

$$\gamma^{(i)}_u(G^{(i)}) = \mathbb{E}\left[g_u(\{\boldsymbol{c^{(i)}_{(a,b)}}\}_{(a,b)\in E})\right] - \sum_{v:(u,v)\in E} \mathbb{E}\left[l(\boldsymbol{\tilde{d}^{(i)}_{(v,u)}} | \boldsymbol{c^{(i)}_{(u,v)}}, \boldsymbol{\tilde{\mu}^{(i)}_{(u,v)}})\right]$$

We also define $\gamma^{(i)}_{(u,v)}$ as the difference between the expected utility of node u when credit $c^{(i)}_{(u,v)}$ is extended to node v for every v s.t $(u,v) \in E$ and the case when 0 credit is extended by node v to node u but credit extended to all other nodes remain same.

Let us now define $\tilde{c}^{(i),(u \not\leftarrow v)}_{(a,b)}$ which defines the credit profile of the credit network when no credit is extended by node v to node u.

$$\tilde{c}^{(i),(u \not\leftarrow v)}_{(a,b)} = \begin{cases} 0 \text{ if } a = v \text{ and } b = u, \\ c^{(i)}_{(a,b)} \text{otherwise} \end{cases}$$

$$\gamma^{(i)}_{(u,v)}(G^{(i)}) = \left[g_u(\{\boldsymbol{c^{(i)}_{a,b}}\}_{(a,b)\in E}) \right] - \left[g_u(\{\boldsymbol{\tilde{c}^{(i),(u \not\leftarrow v)}_{(a,b)}}\}_{(a,b)\in E}) \right]$$

Subject to u's own credit-issuing constraint, B_u:

$$\sum_{v:(u,v)\in E} c^{(i)}_{(u,v)} \leq B_u$$

Step 2: In the second step, each u decides whether to default or honor a transaction with v. Note that a node u can only observe whether v has defaulted or honored a transaction if they are neighbors and have transacted. Recall we have two types of agents: trustworthy (T) and untrustworthy (U). If u is of type T, u will always honor a transaction. For u of type U, defaulting is contingent on whether the gain from defaulting on the previous transactions of round i, $x^{(i)}_{(u,v)}$, is greater than the future expected (marginal) utility of maintaining a tie with v ($\mathbb{E}[\sum_{k=i+1}^{\infty} r^{k-i} \cdot \gamma^{(k)}_{(u,v)} | \mathcal{H}_i]$), with some discount rate $r \in [0, 1]$.

$$d^{(i)}_{u,v} = \begin{cases} 0 & \text{if } u \in T \text{ or } \sum\limits_{k=i+1}^{\infty} r^{k-i} \cdot \mathbb{E}[\gamma^{(k)}_{(u,v)}) | \mathcal{H}_i] > x^{(i)}_{(u,v)} \\ 1 & \text{otherwise} \end{cases}$$

Note $\mathcal{H}_k$ denotes the set of all transactions and defaults by different nodes upto round k.

4 Analysis Plan

Some questions we may explore:

- Whether trust levels converge and the network becomes stable— that is, it does not change between two rounds.

We might simplify our model, and will produce some toy models. We hope be conducting simulations to understand the models behavior at a larger scale.

References

[1] Pranav Dandekar et al. “Liquidity in credit networks: a little trust goes a long way”. In: *Proceedings of the 12th ACM conference on Electronic commerce*. EC ’11. New York, NY, USA: Association for Computing Machinery, June 2011, pp. 147–156. ISBN: 978-1-4503-0261-6. DOI: 10.1145/1993574.1993597. URL: https://doi.org/10.1145/1993574.1993597 (visited on 05/13/2022).

[2] Pranav Dandekar et al. “Strategic Formation of Credit Networks”. In: *ACM Transactions on Internet Technology* 15.1 (Mar. 2015), 3:1–3:41. ISSN: 1533-5399. DOI: 10.1145/2700058. URL: https://doi.org/10.1145/2700058 (visited on 05/11/2022).

[3] Carolyn McLeod. “Trust”. In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2021. Metaphysics Research Lab, Stanford University, 2021. URL: https://plato.stanford.edu/archives/fall2021/entriesrust/ (visited on 05/11/2022).

[4] Brian Skyrms and Robin Pemantle. “A Dynamic Model of Social Network Formation”. en. In: *Adaptive Networks: Theory, Models and Applications*. Ed. by Thilo Gross and Hiroki Sayama. Understanding Complex Systems. Berlin, Heidelberg: Springer, 2009, pp. 231–251. ISBN: 978-3-642-01284-6. DOI: 10.1007/978-3-642-01284-6_11. URL: https://doi.org/10.1007/978-3-642-01284-6_11 (visited on 05/13/2022).