SUMMER OF SCIENCE

MID TERM REPORT

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FURTHER PLAN OF ACTION

Following is a short summary of what I have studied and understood till now:

The Qubits

The quantum bits or qubits are the fundamental units of quantum computation and information.Two possible states for this (analogous to classical bits) are |0> and |1> and qubits not only exist in this state but also in the superposition of these (linear combination). Thus qubit's state is also called as a 2-D unit vector space.The following expression helps in geometric representation of qubit:

|ψ>= cos θ/2 |0> + eiϕ sin θ/2 |1>. In 3-D it is shown in Bloch sphere.

consider a system of n qubits. The computational basis states of this system are of the form |x1x2 ...xn, and so a quantum state of such a system is specified by 2^n amplitudes.

Quantum gates

A quantum computer is built from a quantum circuit containing wires and elementary quantum gates to carry around and manipulate the quantum information.The quantum NOT gate acts linearly as:

α|0> + β|1> —-->α|1 >+ β|0>.

Other two important gates which are the Z gate: Z ≡ [1 0 0 −1] which leaves |0> unchanged, and flips the sign of |1> to give −|1>, and the Hadamard gate, H ≡ 1 √ 2 [1 1 1 −1] . (1.14) This gate is sometimes described as being like a 'square-root of ' gate, in that it turns a |0>into (|0 + |1)/ √ 2 (first column of H), 'halfway' between |0> and |1>, and turns |1 into (|0−|1)/ √ 2 (second column of H), which is also 'halfway' between |0> and |1>.

Quantum Algorithm

There are three classes of quantum algorithms which provide an advantage over known classical algorithms:

- The class of algorithms based upon quantum versions of the Fourier transforms. The Deutsch–Jozsa algorithm is an example of this type of algorithm, as are Shor's algorithms for factoring and discrete logarithm.
- The second class of algorithms is quantum search algorithms.
- The third class of algorithms is quantum simulation, whereby a quantum computer is used to simulate a quantum system.

Experimental Quantum information processing:

Stern-Gerlach experiment:

This experiment gave evidence of existence of qubits in nature.A beam of hot (originally silver)atoms was passed through a non uniform magnetic field, which caused the atoms to split into two distinct paths. This splitting was observed on a screen and indicated that the atoms had only two possible orientations of their magnetic moments, aligned either with pr against the field.Thus this experiment provided evidence for the existence of quantised angular momentum or spin in particles.

Quantum information Processing:

A. **Quantum state tomography and quantum process tomography** are two elementary processes of great importance.

- Quantum tomography or quantum state tomography is the process by which a quantum state is reconstructed using measurements on an ensemble of identical quantum states.
- Quantum process tomography is a procedure to completely characterize the dynamics of a quantum system.

B. **Nuclear Magnetic Resonance**:

These schemes store quantum information in the nuclear spin of atoms in a molecule, and manipulate that information using electromagnetic radiation.

Quantum Information Theory:

applications–

- **Classical Information through quantum channels-**
- **Quantum Information through quantum channels:**
- **Quantum distinguishability**
- **Creation and transformation of entanglemt**

Further of what I learnt comprises of a great deal of linear algebra with matrices and linear independence, etc which was basically a recap of all the maths we have already learnt. Continuing after that:

Postulates of Quantum Mechanics:

1. **State space:**Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system's state space.
2. **Evolution:** The evolution of a closed quantum system is described by a unitary transformation. That is, the state |ψ of the system at time t1 is related to the state |ψ of the system at time t2 by a unitary operator U which depends only on the times t1 and t2, |ψ> = U|ψ>.

   2': The time evolution of the state of a closed quantum system is described by the Schrodinger equation, id|ψ>/ dt = H|ψ>. In this equation, is a physical constant known as Planck's constant whose value must be experimentally determined. The exact value is not important to us. In practice, it is common to absorb the factor into H, effectively setting = 1. H is a fixed Hermitian operator known as the Hamiltonian of the closed system

3. **Quantum measurement**: Quantum measurements are described by a collection {Mm} of measurement operators. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is |ψ immediately before the measurement then the probability that result m occurs is given by p(m) = ψ|M† mMm|ψ> and the state of the system after the measurement is Mm|ψ ψ|M† mMm|ψ>.The measurement operators satisfy the completeness equation, m M† mMm = I . The completeness equation expresses the fact that probabilities sum to one: 1 = m p(m) = m ψ|M† mMm|ψ>.
4. **Distinguishing quantum States:** To distinguish orthonormal states we can define measurement operators Mi ≡ |ψiψi|, one for each possible index i, and an additional measurement operator M0 defined as the positive square root of the positive operator I − i   = 0 |ψiψi|. These operators satisfy the completeness relation, and if the state |ψi is prepared then p(i) = ψi|Mi|ψi = 1, so the result i occurs with certainty.
5. **Projective measurements:**A projective measurement is described by an observable, M, a Hermitian operator on the state space of the system being

observed. The observable has a spectral decomposition, M = m mPm , (2.102) where Pm is the projector onto the eigenspace of M with eigenvalue m. The possible outcomes of the measurement correspond to the eigenvalues, m, of the observable. Upon measuring the state |ψ, the probability of getting result m is given by p(m) = ψ|Pm|ψ. (2.103) Given that outcome m occurred, the state of the quantum system immediately after the measurement is Pm|ψ √p(m) .

6. **POVM measurements:(Positive Operator-Valued Measure)-**Suppose a measurement described by measurement operators Mm is performed upon a quantum system in the state |ψ. Then the probability of outcome m is given by p(m) = ψ|M† mMm|ψ. Suppose we define Em ≡ M† mMm. The set of operators Em are sufficient to determine the probabilities of the different measurement outcomes.
7. **Phase:** There are two types of phase factors : relative and global as described.
8. **Composite system** :The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state |ψi>, then the joint state of the total system is |ψ1>⊗|ψ2>⊗···⊗|ψn>.

Superdense coding

Superdense coding is a mechanism through which two bits of information can be sent in single qubit. It utilises the technique of quantum entanglement to do so. The protocol involves two qubits but the sender has to interact only with one and can apply gates like Z, phase flip or NOT to modify and send the second one.

The process of superdense coding involves a sender (Alice) and a receiver (Bob) who share a pair of entangled qubits. These qubits are typically in an entangled state known as a Bell state. Alice possesses one qubit, and Bob possesses the other. The steps involved in superdense coding are as follows:

1.Preparation: Alice and Bob initially create an entangled pair of qubits, with one qubit each.

2.Encoding: Alice wants to send a classical two-bit message to Bob. By applying specific quantum gates to her qubit, Alice can encode the two-bit message onto her qubit, modifying the joint state of the two qubits.

3.Transmission: Alice sends her qubit to Bob through a quantum communication channel. This can be done using various methods, such as optical fibers or superconducting circuits.

4.Decoding: Upon receiving Alice's qubit, Bob applies specific quantum operations to his qubit, allowing him to decode the two-bit message encoded by Alice.

5.Measurement: Finally, Bob performs measurements on his qubit, extracting the classical information encoded by Alice.

Through superdense coding, Alice can transmit two classical bits of information by manipulating a single qubit and utilizing the shared entanglement with Bob. This process takes advantage of the inherent correlations between the entangled qubits, enabling efficient communication of classical information.

The density Operator

It is a mathematical tool used to formulate quantum mechanics.The density operator language provides a convenient means for describing quantum systems whose state is not completely known. More precisely, suppose a quantum system is in one of a number of states |ψi, where i is an index, with respective probabilities pi. We shall call {pi, |ψi} an ensemble of pure states. The density operator for the system is defined by the equation ρ ≡ i pi|ψiψi.

Properties of the density operator:

An operator ρ is the density operator associated to some ensemble {pi, |ψi} if and only if it satisfies the conditions:

(1) (Trace condition) ρ has trace equal to one.

(2) (Positivity condition) ρ is a positive operator.

Using the density operator all the four fundamental postulates can be reformulated which makes it easier to approach problems.

The Schmidt decomposition and purifications

Schmidt decomposition:

Suppose |ψ is a pure state of a composite system, AB. Then there exist orthonormal states |iA for system A, and orthonormal states |iB of system B such that |ψ = i λi|iA|iB, (2.202) where λi are non-negative real numbers satisfying i λ2 i = 1 known as Schmidt co-efficients.

The bases |iA and |iB are called the Schmidt bases for A and B, respectively, and the number of non-zero values λi is called the Schmidt number for the state |ψ>.The Schmidt number is an important property of a composite quantum system, which in some sense quantifies the ‘amount’ of entanglement between systems A and B.

Purification:

Given a state ρA of a quantum system A. It is possible to introduce another system, which we denote R, and define a pure state |AR for the joint system AR such that ρA =

trR(|ARAR|). That is, the pure state |AR reduces to ρA when we look at system A alone. This is a purely mathematical procedure, known as purification, which allows us to associate pure states with mixed states. For this reason we call system R a reference system: it is a fictitious system, without a direct physical significance.

EPR and Bell inequality:

$$E(QS) + E(RS) + E(RT) - E(QT) \leqslant 2.$$

The violation of the Bell inequality showcases the potential of quantum systems for information processing beyond classical limits. It enables quantum communication protocols such as quantum key distribution and quantum teleportation, revolutionizing the field of quantum information theory.

Revised Plan of action:

- 28 June-5 July: Chapter 4.
- 6 July -12 July : Chapter 5
- 13 July -20 July: CHapter 6