HNDIT1032 Computer and Network Systems

Week4- Karnaugh Maps

Introduction

- So far we can see that applying Boolean algebra can be awkward in order to simplify expressions.
- The Karnaugh map provides a simple and straight-forward method of minimizing Boolean expressions

What is a Karnaugh map?

- A Karnaugh map provides a pictorial method of grouping together expressions with common factors and therefore eliminating unwanted variables.

Two Variable K Maps

- Two variable K Map is drawn for a boolean expression consisting of two variables.
- The number of cells present in two variable K Map = 2^2 = 4 cells.
- So, for a Boolean function consisting of two variables, we draw a 2 x 2 K Map.

Two Variable K Maps…

A	B	X
0	0	S_0
0	1	S_1
1	0	S_2
1	1	S_3

A \ B	0	1
0	S_0	S_1
1	S_2	S_3

Two Variable K Maps…

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table.

B \ A	0	1
0	0	1
1	1	1

F.

Three Variable K Maps

- Three variable K Map is drawn for a Boolean expression consisting of three variables.
- The number of cells present in three variable K Map = 2^3 = 8 cells.
- So, for a Boolean function consisting of three variables, we draw a 2 x 4 K Map.

Three Variable K Maps…

A	B	C	Minterm
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

A \ BC	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

Three Variable K Maps…

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

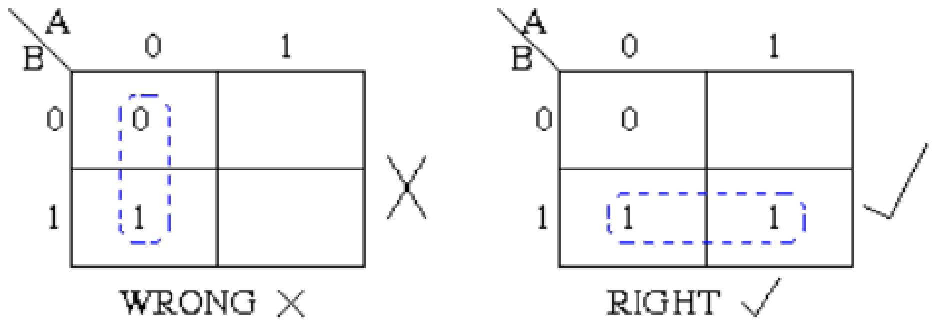
AB \ C	0	1
00	0	1
01	0	
11	1	
10	1	

Karnaugh Maps - Rules of Simplification

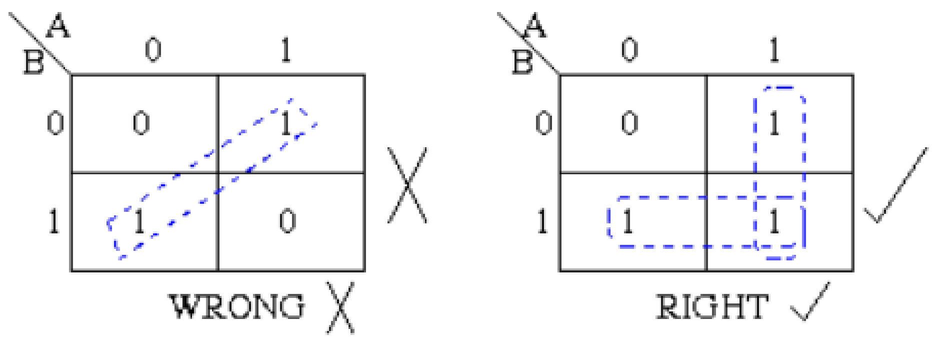
- No zeros allowed.
- No diagonals.
- Only power of 2 number of cells in each group.
- Groups should be as large as possible.
- Every one must be in at least one group.
- Overlapping allowed.
- Wrap around allowed.
- Fewest number of groups possible.

Karnaugh Maps - Rules of Simplification…

- **Groups may not include any cell containing a zero**

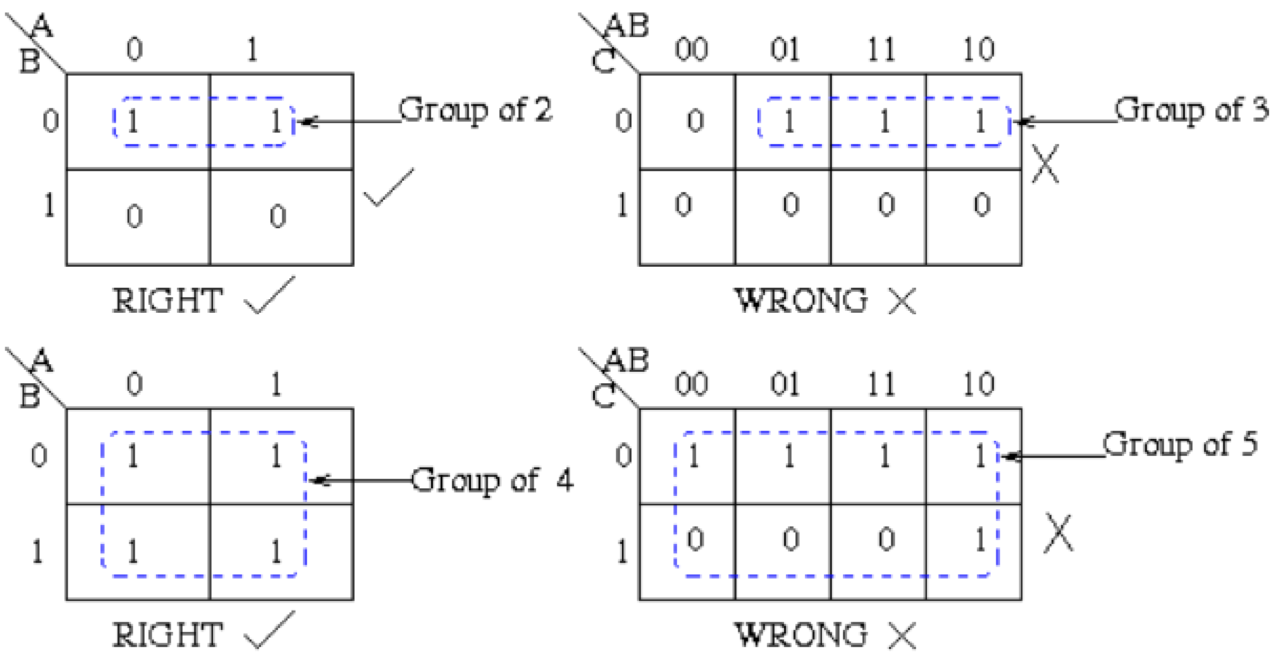


- **Groups may be horizontal or vertical, but not diagonal.**



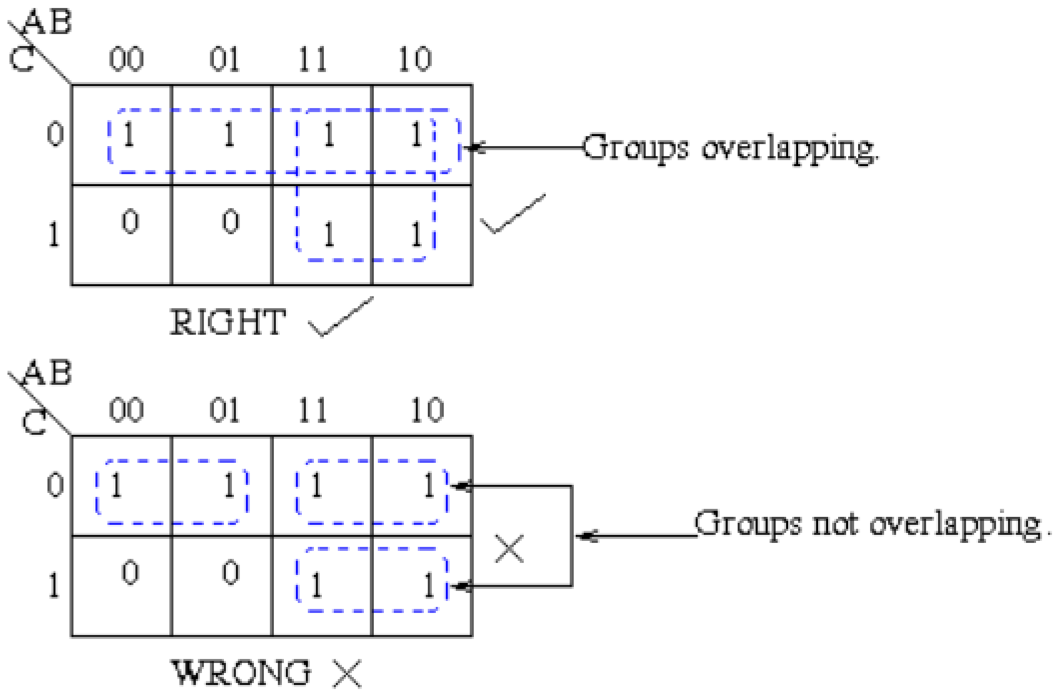
Karnaugh Maps - Rules of Simplification…

- **Groups must contain 1, 2, 4, 8, or in general 2^n cells.**
  That is if n = 1, a group will contain two 1's since $2^1 = 2$.
  If n = 2, a group will contain four 1's since $2^2 = 4$.



Karnaugh Maps - Rules of Simplification…

- **Groups may overlap.**



Consider the following map. The function plotted is: Z = f(A,B) = A B + AB
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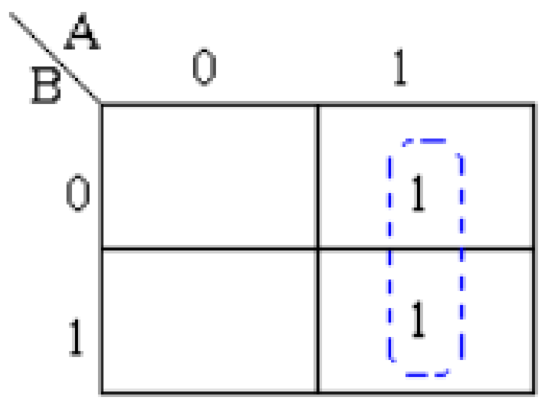
Example 01

- $F(A,B) = A\bar{B} + AB$



$Z = A$

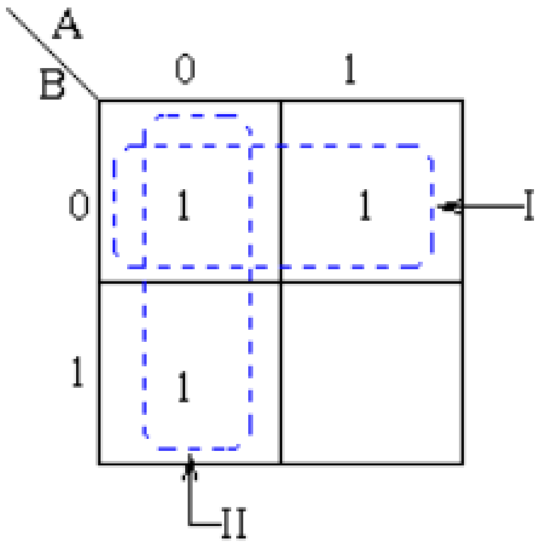
Example 01…

- The two adjacent 1's are grouped together. Through inspection it can be seen that variable B has its true and false form within the group.
- This eliminates variable B leaving only variable A which only has its true form. The minimized answer therefore is Z = A.

Example 02

- $F(A, B) = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$

Example 02



Example 03

- $F(A, B) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$

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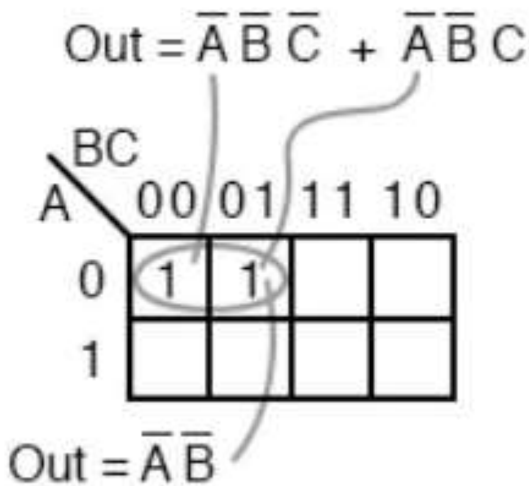

Example 04

- $F(A,B) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$

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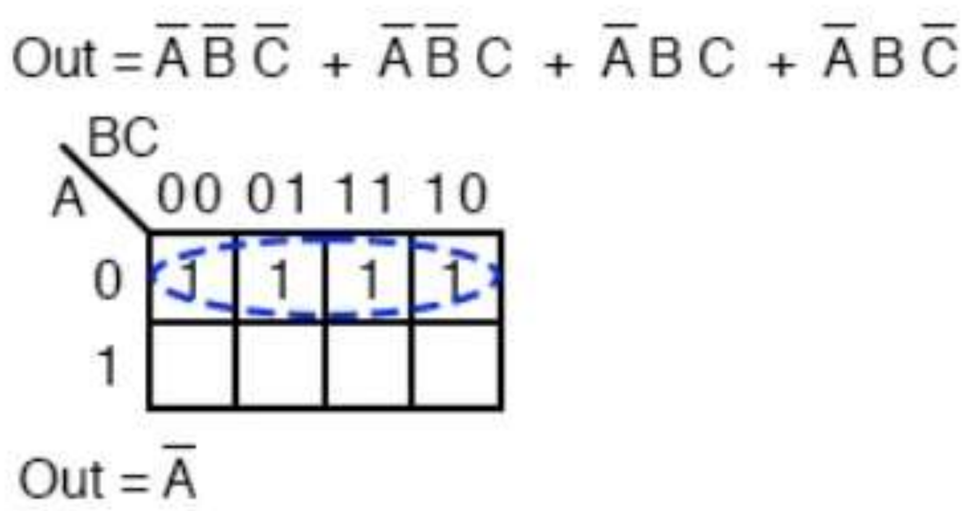
- $F(A,B) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$

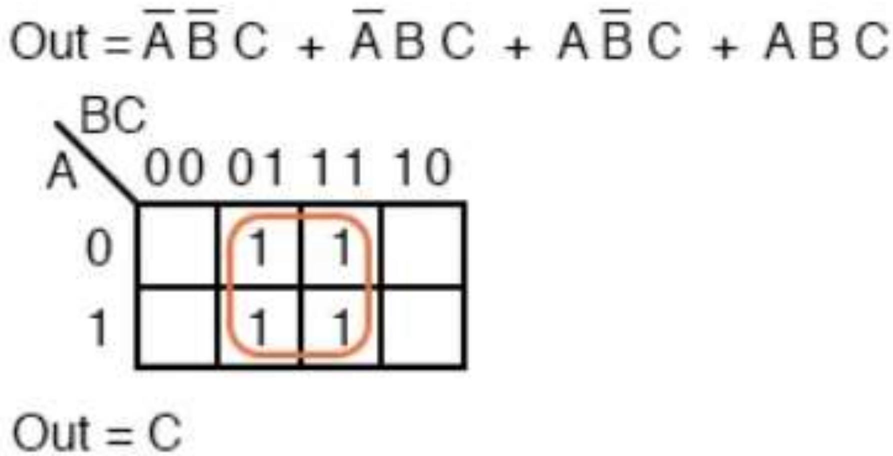


Example 05

- $F(A,B) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

Example 05

- $F(A,B) = \bar{A}\bar{B}C + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$



3. $F(A,B,C) = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$

4. $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$

5. $F(A,B,C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

Next Week Discussion

- How to draw circuits?