

1. Find the GCD of two given numbers using Euclid's Algorithm.
2. Compute the LCM of two numbers using their GCD.
3. Find GCD of three numbers efficiently.
4. Check if two numbers are co-prime using GCD.
5. Count pairs (i, j) in an array such that $\text{GCD}(i, j) = 1$.
6. Find the largest divisor of N that divides another number M .
7. Compute GCD of all elements in an array.
8. Reduce a fraction a/b to its simplest form using GCD.
9. Find GCD of large numbers represented as strings.
10. Compute GCD recursively without using loops.
11. • Compute GCD of two large integers using Euclid's algorithm.
12. • Compute LCM of two large integers efficiently.
13. • Find modular exponentiation: compute $a^b \bmod m$ for large a, b, m .
14. • Find modular inverse of an integer a modulo m .
15. • Determine if a number is prime using Sieve of Eratosthenes.
16. • Generate all prime numbers up to n .
17. • Factorize a number n into its prime factors.
18. • Compute Euler's Totient function $\phi(n)$ for a given n .
19. • Solve linear congruence equations $ax \equiv b \pmod{m}$.
20. • Apply Fermat's Little Theorem to compute $a^{p-1} \bmod p$.
21. • Solve system of congruences using Chinese Remainder Theorem.
22. • Count divisors of a number n .
23. • Sum all multiples of a number n up to m .
24. • Compute power mod avoiding integer overflow for large numbers.
25. • Find all numbers x ($1 \leq x \leq n$) such that $x^k \equiv 1 \pmod{m}$.
26. • Count integers co-prime with n in range $[1, n]$.
27. • Find smallest x such that $x^k \equiv a \pmod{m}$.
28. • Find number of integers divisible by exactly k distinct primes $\leq n$.
29. • Determine if a number can be expressed as a sum of two squares.
30. • Solve Diophantine equations of type $ax+by=c$ for integers.

♻ Modular Arithmetic

11. Compute $(a * b) \% m$ for large a and b .
12. Compute $(a^b) \% m$ efficiently using modular exponentiation.
13. Check if $(a * b) \% m == (a \% m * b \% m) \% m$ for all integers.
14. Find the modular inverse of a under modulo m .
15. Solve $(a / b) \% m$ using modular inverse when m is prime.
16. Compute $(a + b - c) \% m$ ensuring non-negative result.
17. Check if $(a^b) \% m == (b^a) \% m$.
18. Compute $(a^b + b^a) \% m$ for given integers.
19. Find the smallest positive x such that $(a * x) \% m = b$.
20. Compute $(n!) \% p$ for large n and prime p .

Prime Numbers

21. Generate all prime numbers up to N using the Sieve of Eratosthenes.
22. Find the count of primes in a given range $[L, R]$.
23. Find the sum of all primes less than N .
24. Check if a number is prime using optimized trial division.
25. Find the k th prime number.
26. Find the smallest prime factor (SPF) for all numbers up to N .
27. Compute prime factorization of a number using SPF.
28. Count distinct prime factors of N .
29. Find the highest power of a prime p dividing $N!$.
30. Find the number of divisors of N using prime factorization.

Euler's Totient Function ($\phi(n)$)

31. Compute $\phi(n)$ using prime factorization.
32. Find the sum of $\phi(i)$ for all i from 1 to N .
33. Count numbers $\leq N$ that are co-prime with N .
34. Compute $\phi(n!)$ for a given n .
35. Find the smallest n such that $\phi(n) = k$.
36. Compute $a^{\phi(m)} \% m$ for given a and m .
37. Verify Euler's theorem: $a^{\phi(m)} \equiv 1 \pmod{m}$ for co-prime a, m .
38. Compute modular inverse using Euler's theorem.
39. Find all values of $\phi(n)$ for $n \leq N$ using sieve method.
40. Check if $\phi(n)$ divides $n-1$ for given n .

Fermat's Little Theorem

41. Verify Fermat's theorem for given prime p and integer a .
42. Compute modular inverse using Fermat's theorem when m is prime.
43. Compute $a^{(p-1)} \% p$ for given prime p .
44. Find the smallest base a violating Fermat's theorem for composite n .
45. Implement Fermat's primality test for large integers.
46. Use Fermat's theorem to compute large modular powers quickly.
47. Check if $a^{(p-1)} \equiv 1 \pmod{p}$ for multiple values of a .
48. Find modular inverse of factorial $(n!)^{-1} \% p$ using Fermat.
49. Compute binomial coefficient $C(n, k) \% p$ using Fermat's theorem.
50. Compute $(a/b) \% p$ where p is prime using modular inverse.

Would you like me to give **input/output examples** for each of these 50 problems (like a practice set format for coding)?

You said:

Chinese Remainder Theorem (CRT) • Power Mod / Modular Multiplication without overflow • Divisors, Multiples, and Factorization problems 50 problem in one line

ChatGPT said:

Here are **50 one-line problem statements** covering **Chinese Remainder Theorem (CRT)**, **Power Mod / Modular Multiplication without overflow**, and **Divisors–Multiples–Factorization** problems ↵

📖 Chinese Remainder Theorem (CRT)

1. Solve a system of two congruences using the Chinese Remainder Theorem.
 2. Solve three simultaneous modular equations with pairwise co-prime moduli.
 3. Find the smallest x that satisfies $(x \equiv 2 \pmod{3})$, $(x \equiv 3 \pmod{5})$, $(x \equiv 2 \pmod{7})$.
 4. Determine if a given set of congruences has a valid solution.
 5. Implement CRT for non-coprime moduli using extended Euclid's algorithm.
 6. Find x such that $(x \equiv a_1 \pmod{m_1})$ and $(x \equiv a_2 \pmod{m_2})$ for large values.
 7. Compute minimal non-negative solution for multiple modular equations.
 8. Apply CRT to compute $(a^b) \% m$ where $m = \text{product of coprime moduli}$.
 9. Use CRT to reconstruct a number from its remainders modulo 3, 5, and 7.
 10. Solve CRT when one modulus divides another.
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⚙️ Power Mod / Modular Multiplication without Overflow

11. Compute $(a * b) \% m$ without overflow for very large a and b .
 12. Implement fast modular multiplication using addition and doubling.
 13. Compute $(a^b) \% m$ for large a and b using binary exponentiation.
 14. Calculate $(a^{(b^c)}) \% m$ efficiently using modular exponentiation.
 15. Find last digit of a^b for very large a and b .
 16. Compute $(a^b * c^d) \% m$ efficiently using modular properties.
 17. Compute $(a^b + b^a) \% m$ for large inputs.
 18. Implement modular exponentiation using iterative method.
 19. Compute $(a^b - c^d) \% m$ ensuring non-negative result.
 20. Compute $(a^{(b!)}) \% m$ efficiently for small b .
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📖 Divisors Problems

21. Find all divisors of a number N .
22. Count total number of divisors of N .
23. Find the sum of all divisors of N .
24. Find the product of all divisors of N .
25. Check if N is a perfect number (sum of divisors equals $2N$).
26. Find all numbers $\leq N$ having exactly k divisors.

27. Find the largest divisor of N less than N .
 28. Count divisors of $N!$ efficiently.
 29. Find all pairs (a, b) such that $a * b = N$.
 30. Check if N is a highly composite number.
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Multiples Problems


31. Find all multiples of k up to N .
 32. Count numbers $\leq N$ divisible by a or b .
 33. Find the smallest common multiple of an array of numbers.
 34. Find the smallest number divisible by all integers from 1 to n .
 35. Count numbers $\leq N$ that are multiples of both a and b .
 36. Find the least number greater than N divisible by k .
 37. Find all numbers $\leq N$ that are multiples of exactly two given numbers.
 38. Compute sum of all multiples of k up to N .
 39. Find smallest x such that $(x \bmod a = b \bmod a)$ and $(x \bmod c = d \bmod c)$.
 40. Determine if a given number is a multiple of another using modular arithmetic.
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Factorization Problems

41. Find the prime factorization of N .
 42. Count total prime factors (with repetition) of N .
 43. Count distinct prime factors of N .
 44. Find largest prime factor of N .
 45. Compute smallest prime factor (SPF) for all numbers up to N using sieve.
 46. Find all numbers $\leq N$ having exactly two distinct prime factors.
 47. Find number of divisors using factorization formula.
 48. Compute number of trailing zeros in $N!$ using factorization of 5.
 49. Factorize a large composite number using Pollard's Rho algorithm (conceptually).
 50. Find the sum of exponents in the prime factorization of N .
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Would you like me to expand these into a **coding practice set** (each with input/output examples and constraints, ready for implementation in C++ or Java)?

You said:

 2. Combinatorics Used in counting arrangements, selections, and ways. • Factorials and nCr computation (mod M) • Pascal's Triangle • Inclusion–Exclusion Principle • Catalan Numbers (used in valid bracket / tree problems) • Permutations & Combinations under constraints • Pigeonhole Principle 70 problem statement in one line

ChatGPT said:

Here are **70 one-line combinatorics problem statements** covering **Factorials, nCr, Pascal's Triangle, Inclusion–Exclusion, Catalan Numbers, Permutations & Combinations**, and **Pigeonhole Principle** 🐦

📦 Factorials and nCr Computation (mod M)

1. Compute $n!$ for a given n .
 2. Compute $(n!) \% M$ where M is a large prime.
 3. Find the number of trailing zeros in $n!$.
 4. Compute the number of digits in $n!$.
 5. Compute nCr for given n and r .
 6. Compute $(nCr) \% M$ using modular inverse.
 7. Find the largest r such that nCr is divisible by a given number k .
 8. Compute $(nPr) \% M$ using factorials mod M .
 9. Precompute factorial and inverse factorial arrays for $nCr \bmod M$.
 10. Compute sum of all combinations $C(n, 0) + C(n, 1) + \dots + C(n, n)$.
 11. Compute $C(n, r)$ without overflow using iterative formula.
 12. Compute $C(n, r)$ for large n using Lucas Theorem.
 13. Find the largest n for which $n!$ fits in 64-bit integer.
 14. Compute $(nCr)^2 \bmod M$ for given inputs.
 15. Find smallest n such that $n!$ has at least x trailing zeros.
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▲ Pascal's Triangle

16. Print the first n rows of Pascal's Triangle.
 17. Find the element at row r and column c in Pascal's Triangle.
 18. Compute sum of elements in the n th row of Pascal's Triangle.
 19. Check if a given number exists in Pascal's Triangle.
 20. Generate Pascal's Triangle modulo M .
 21. Compute diagonal sums of Pascal's Triangle.
 22. Count odd numbers in the first n rows of Pascal's Triangle.
 23. Print the middle element of each row in Pascal's Triangle.
 24. Find maximum element in the n th row of Pascal's Triangle.
 25. Print Pascal's Triangle in a right-aligned format.
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📦 Inclusion–Exclusion Principle

26. Count numbers $\leq N$ divisible by at least one of a, b , or c .
27. Count numbers $\leq N$ divisible by exactly one of a, b, c .
28. Count numbers $\leq N$ not divisible by any of a, b, c .
29. Count integers $\leq N$ divisible by at least two of three given numbers.

30. Find count of integers $\leq N$ divisible by 2 or 3 or 5.
 31. Count numbers ≤ 1000 divisible by 3 or 7 but not both.
 32. Count numbers $\leq N$ having gcd with N equal to 1 (using inclusion–exclusion).
 33. Count numbers $\leq N$ coprime to any of a_1, a_2, \dots, a_k .
 34. Compute number of onto functions from set A to set B using inclusion–exclusion.
 35. Count permutations where at least one element is fixed (derangements using inclusion–exclusion).
 36. Count numbers $\leq N$ that are multiples of prime factors of M .
 37. Count numbers divisible by any element in an array of divisors.
 38. Count integers $\leq N$ that share a divisor with given set S .
 39. Count integers $\leq N$ divisible by 2 or 3 but not by 5.
 40. Use inclusion–exclusion to count subsets with $\gcd = 1$.
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Catalan Numbers (used in valid brackets, trees, etc.)

41. Compute the n th Catalan number.
 42. Count number of valid parenthesis sequences of length $2n$.
 43. Count the number of possible Binary Search Trees with n nodes.
 44. Count number of non-crossing handshakes among $2n$ people sitting in a circle.
 45. Count number of ways to triangulate an n -sided polygon.
 46. Count the number of full binary trees with n internal nodes.
 47. Count ways to correctly arrange n pairs of parentheses.
 48. Find n th Catalan number modulo $1e9+7$.
 49. Count Dyck paths of length $2n$.
 50. Count mountain ranges that can be drawn with n upstrokes and n downstrokes.
 51. Count non-intersecting chord arrangements in a circle.
 52. Count ways to merge n sorted sequences using Catalan number formula.
 53. Count ways to pair brackets so that no two pairs overlap.
 54. Count distinct binary tree structures with n nodes.
 55. Count possible stack permutations of $1\dots n$.
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Permutations & Combinations under Constraints

56. Count total permutations of n distinct objects.
57. Count permutations of a string with repeated characters.
58. Count number of ways to arrange n objects where certain items must be together.
59. Count ways to distribute r identical balls into n boxes.
60. Count ways to distribute r distinct balls into n boxes.
61. Count arrangements of the word “SUCCESS”.
62. Count number of ways to choose r elements from n distinct elements.
63. Count number of circular permutations of n items.
64. Count number of derangements of n objects (no item in original position).
65. Count ways to seat n people in a circle such that two specific people sit together.
66. Count arrangements of digits 1–9 such that no two even digits are adjacent.
67. Count ways to form committees from a group under gender constraints.
68. Count ways to select teams from two different groups with limits per group.

69. Count number of arrangements where vowels and consonants alternate.
70. Count permutations of 1 to N where exactly K elements are fixed.

● Pigeonhole Principle (Bonus if needed for extra 10 problems)

71. Prove that in any group of n people, two have the same number of friends.
72. Show that in a set of 13 integers, at least two have the same remainder mod 12.
73. Find smallest number of socks to pick to ensure a matching pair.
74. Determine minimum people needed to ensure two share a birthday (Birthday Paradox).
75. Show that in any sequence of $n+1$ integers, two have the same remainder mod n .
76. Given 11 points inside a unit square, prove two are within distance $< \sqrt{2}/3$.
77. Show that in any set of 367 people, two share a birthday.
78. Prove that among 6 people, there are 3 mutual friends or 3 mutual strangers.
79. Count minimum number of items required to ensure at least 3 duplicates.
80. Determine number of pigeonholes needed for given distribution problem.

Constraint-based Counting Problems (20)

1. Count the number of strings of length nnn using kkk characters with no two adjacent characters equal.
2. Count the number of binary strings of length nnn with no consecutive 1s.
3. Count the number of ternary strings $(0,1,2)$ of length nnn with no adjacent digits equal.
4. Count number of sequences of length nnn of numbers $1..k$ with no increasing adjacent pair.
5. Count number of strings of length nnn with exactly kkk occurrences of a given character, no two adjacent.
6. Count number of arrangements of nnn people in a line such that no two friends are adjacent.
7. Count number of ways to color a path of length nnn with kkk colors, no two adjacent nodes same color.
8. Count number of valid parentheses sequences of length $2n$ (Catalan Numbers).
9. Count number of sequences of length nnn using numbers $1..k$ such that adjacent numbers differ by at least 2.
10. Count number of ways to arrange nnn chairs with mmm reserved seats such that no two reserved are adjacent.
11. Count number of binary strings of length nnn with no substring "11".
12. Count number of sequences of length nnn over alphabet $\{A,B,C\}$ avoiding substring "AB".
13. Count number of ways to assign tasks to nnn people such that no two same tasks are adjacent.
14. Count number of ways to place nnn rooks on a $1 \times n$ board so that no two are adjacent.
15. Count number of bitstrings of length nnn with exactly kkk ones and no consecutive ones.
16. Count number of ways to fill an array of length nnn with numbers $1..k$ avoiding repeated neighbors.

17. Count number of necklaces of length nnn using kkk colors with no adjacent beads same (circular).
18. Count number of sequences of length nnn with sum SSS and no two equal consecutive numbers.
19. Count number of ways to place nnn books on a shelf so that no two books of the same type are together.
20. Count number of ways to seat nnn people around a table such that no two men sit together.

Would you like me to give **input/output examples** and **formula hints** for each of these 70 combinatorics problems (like a coding practice workbook)?

You said:

Linear and Quadratic Equations • Polynomials & Roots • Arithmetic and Geometric Progressions (AP/GP) • Modular Linear Equations • Matrix Operations (addition, multiplication, inverse) • Matrix Exponentiation (for fast Fibonacci / recurrence DP)

ChatGPT said:

Here's a **set of 50 one-line problem statements** covering **Linear & Quadratic Equations, Polynomials & Roots, AP/GP, Modular Linear Equations, Matrix Operations, and Matrix Exponentiation**:

1 Linear and Quadratic Equations

1. Solve a linear equation $ax + b = 0$.
2. Solve a quadratic equation $ax^2 + bx + c = 0$.
3. Find real roots of a quadratic equation.
4. Find complex roots of a quadratic equation.
5. Determine discriminant and nature of roots of a quadratic.
6. Solve two linear equations with two variables using substitution.
7. Solve two linear equations using elimination method.
8. Solve system of three linear equations with three variables.
9. Find intersection point of two lines.
10. Check if three points are collinear using linear equations.

12 Polynomials & Roots

11. Evaluate a polynomial $P(x)$ at a given x .
12. Find sum of roots of a quadratic equation.
13. Find product of roots of a quadratic equation.
14. Compute sum of squares of roots of a quadratic.

15. Find cubic roots of a cubic polynomial.
16. Check if a given number is a root of a polynomial.
17. Factorize a quadratic polynomial.
18. Factorize a cubic polynomial using rational root theorem.
19. Find roots of polynomial using Newton-Raphson method.
20. Find multiplicity of roots in a polynomial.

Arithmetic Progressions (AP)

21. Find n th term of an AP.
22. Find sum of first n terms of an AP.
23. Find number of terms in an AP given first term, last term, and common difference.
24. Determine common difference of an AP from first and last term.
25. Find arithmetic mean between two numbers.
26. Insert k arithmetic means between two numbers.
27. Check if a given sequence is an AP.
28. Find sum of even/odd terms in an AP.
29. Solve word problems involving AP (e.g., age problems).
30. Find sum of terms of AP using formula without iteration.
31. • Find the 20th term of an AP whose first term is 5 and common difference is 3.
32. • Sum the first 50 terms of an AP where $a_1=2$, $a_n=2$ and $d=4$.
33. • Find the common difference if the 10th term is 35 and the 20th term is 75.
34. • Determine the number of terms if the AP 3, 7, 11, ..., 833, 7, 11, ..., 833, 7, 11, ..., 83 is given.
35. • Find the 15th term of an AP whose sum of first 15 terms is 360.
36. • If the sum of first n terms is $n^2 + 2n$, find the n th term.
37. • Find the sum of all odd numbers from 1 to 99 using AP formula.
38. • Determine the first term if the 5th term is 20 and the common difference is 4.
39. • Find the middle term of an AP 7, 13, 19, ..., 797, 13, 19, ..., 797, 13, 19, ..., 79.
40. • Sum of first 30 natural numbers in an AP.
41. • Find the 25th term when $a_1=-2$, $a_n=-2$ and $d=5$.
42. • Determine the sum of all terms of an AP from 10 to 100 with difference 5.
43. • Find the term number if $a_n=127$ in AP 2, 5, 8, ..., 2, 5, 8, ...
44. • Find the sum of even numbers between 50 and 200 using AP.
45. • Determine the 10th term from the end of an AP with 30 terms, $a_1=3$, $d=2$.
46. • Find the sum of all multiples of 7 below 500 using AP.
47. • If the sum of first 10 and first 20 terms are 100 and 300, find the common difference.
48. • Find the first negative term in AP 15, 10, 5, ..., 15, 10, 5, ...
49. • Determine the 12th term of an AP whose sum of first 12 terms is 180.
50. • Find the sum of the last 10 terms of 50-term AP where $a_1=2$, $d=3$.
51. • If $a_5=20$ and $a_{10}=40$, find the first term of the AP.
52. • Find the sum of first n terms if $a_1=3$, $a_n=99$ and n is unknown.

53. • Determine the 7th term if sum of first 7 terms is 77 and $d=3$.
54. • Find the sum of an AP 12,17,22,...,10712, 17, 22, ..., 10712,17,22,...,107.
55. • Find the 50th term of an AP with $a_1=1$ and $d=2$.
56. • If the sum of first n terms is 210 and $a_1=5$, $d=4$, find n .
57. • Determine the sum of all numbers divisible by 5 between 100 and 500.
58. • Find the arithmetic mean between 10 and 40 using AP.
59. • Find the 20th term from the end in AP 2,6,10,...,2022, 6, 10, ..., 2022,6,10,...,202.
60. • Sum of all integers between 1 and 100 that leave remainder 1 when divided by 3.
- 61.

▲ Geometric Progressions (GP)

31. Find n th term of a GP.
32. Find sum of first n terms of a GP.
33. Find sum to infinity of a GP with $|r|<1$.
34. Determine common ratio of a GP given first and last term.
35. Find geometric mean between two numbers.
36. Insert k geometric means between two numbers.
37. Check if a sequence forms a GP.
38. Find product of first n terms of a GP.
39. Solve word problems involving GP (e.g., population growth).
40. Find sum of terms at odd/even positions in GP.

Here's a list of **30 one-line problem statements on Geometric Progressions (GP)** for practice:

41. Find the 10th term of a GP with $a_1=2$ and $r=3$.
42. Find the sum of first 8 terms of a GP where $a_1=5$ and $r=2$.
43. Determine the common ratio if $a_3=24$ and $a_6=192$.
44. Find the number of terms in GP 3,6,12,...,7683, 6, 12, ..., 7683,6,12,...,768.
45. Find the 12th term of a GP whose sum of first 12 terms is 4095 and $a_1=1$.
46. If the sum of first n terms is $a_1(r^n-1)/(r-1)$, find n for a given sum.
47. Determine the sum of all terms of GP 1,2,4,...,5121, 2, 4, ..., 5121,2,4,...,512.
48. Find the first term if $a_5=162$ and $r=3$.
49. Find the geometric mean between 4 and 64.
50. Determine the sum of first 20 powers of 2.
51. Find the 8th term in GP 5,15,45,...,5, 15, 45, ...5,15,45,....
52. If $a_1=2$ and sum to infinity is 8, find r .
53. Find the term number if $a_n=729$ in GP 3,9,27,...3, 9, 27, ...3,9,27,....
54. Find the sum of all terms in GP 3,6,12,...,3843, 6, 12, ..., 3843,6,12,...,384.
55. Determine the 5th term from the end of a GP with 10 terms, $a_1=2$, $r=3$.
56. Find the product of first 6 terms of GP 2,4,8,...2, 4, 8, ...2,4,8,....
57. If the sum of first 3 terms is 14 and product is 24, find a GP.

58. Find the sum of all terms in a GP with negative ratio $a_1=5, r=-2, n=6$ $a_1 = 5, r = -2, n = 6$.
59. Determine the 7th term if sum of first 7 terms is 127 and $a_1=1, r=2$ $a_1 = 1, r = 2$.
60. Find the sum of last 5 terms of a 10-term GP 3,6,12,... $3, 6, 12, \dots$
61. If $a_3=12$ $a_3 = 12$ and $a_5=48$ $a_5 = 48$, find a_1 and r .
62. Find the sum of a GP 1,3,9,...,729 $1, 3, 9, \dots, 729$.
63. Determine the 9th term in GP 7,14,28,... $7, 14, 28, \dots$
64. Find the sum to infinity of GP 5,2.5,1.25,... $5, 2.5, 1.25, \dots$
65. Find the 15th term if $a_1=1$ and $r=1.5$ $a_1 = 1$ and $r = 1.5$.
66. If the sum of first n terms is 364 and $a_1=1, r=2$ $a_1 = 1, r = 2$, find n .
67. Determine the geometric mean between 8 and 128.
68. Find the 20th term from the end in GP 2,6,18,...,2³⁰ $2, 6, 18, \dots, 2^{30}$.
69. If the sum of first 4 terms is 30 and the product is 216, find the GP.
70. Sum of all terms of GP $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ up to 10 terms.
- 71.

✂ Modular Linear Equations

41. Solve $ax \equiv b \pmod{m}$ for given integers.
42. Determine if modular equation has no solution, one solution, or multiple solutions.
43. Find smallest positive solution of modular linear equation.
44. Solve system of modular equations using extended Euclid.
45. Count number of solutions for $ax \equiv b \pmod{m}$.
46. Solve $ax + by \equiv c \pmod{m}$ using linear Diophantine approach.
47. Solve modular inverse problem $ax \equiv 1 \pmod{m}$.
48. Solve $a^x \equiv b \pmod{m}$ for small exponents.
49. Verify solution of modular linear equation programmatically.
50. Apply modular linear equations in CRT problems.

🔢 Matrix Operations

51. Add two matrices of same dimensions.
52. Subtract two matrices.
53. Multiply two matrices.
54. Compute transpose of a matrix.
55. Find determinant of a square matrix.
56. Compute inverse of a 2x2 matrix.
57. Compute inverse of a 3x3 matrix using adjoint method.
58. Check if a matrix is symmetric.
59. Solve system of linear equations using matrix inverse.
60. Find rank of a matrix.

Matrix Exponentiation (for Fibonacci / Recurrence DP)

61. Compute A^n for a square matrix A using fast exponentiation.
62. Compute n th Fibonacci number using matrix exponentiation.
63. Solve linear recurrence $F(n) = F(n-1) + F(n-2)$ using matrices.
64. Compute sum of first n Fibonacci numbers using matrix exponentiation.
65. Compute n th term of generalized linear recurrence using matrices.
66. Solve recurrence with constant coefficients using matrix exponentiation.
67. Compute power of matrix modulo M for large n .
68. Multiply two matrices efficiently for DP transitions.
69. Count number of ways to climb stairs using matrix method.
70. Solve tiling problem using matrix exponentiation approach.

Sample Problems Using Matrix Exponentiation

1. Fibonacci sequence: Find $F(n)$ modulo 10^9+7 for $n \leq 10^{18}$.
2. Tribonacci sequence: $F(n) = F(n-1) + F(n-2) + F(n-3)$, find $F(n) \% M$.
3. Linear recurrence: $F(n) = 2F(n-1) + 3F(n-2)$, compute $F(n)$ for large n .
4. Count number of strings of length n with no two consecutive zeros.
5. Count ways to climb stairs of n steps, taking 1, 2, or 3 steps at a time.
6. Number of binary strings of length n with no two 1s adjacent.
7. Number of sequences of length n where each term is sum of previous two terms multiplied by a factor.
8. Solve $F(n) = 3F(n-1) + 4F(n-2) + 2F(n-3)$ modulo 10^9+7 .
9. Count tilings of $2 \times n$ board with 2×1 dominos.
10. Count tilings of $3 \times n$ board with 2×1 dominoes and 2×2 squares.
11. Solve linear recurrence: $F(n) = 5F(n-1) - 6F(n-2)$ for large n .
12. Find n th Catalan number modulo M using recurrence.
13. Number of ways to form valid bracket sequences of length $2n$ using recurrence.
14. Count number of ways to reach n th cell in a board game with dice rolls (sum of previous 6 terms).
15. Solve $F(n) = F(n-1) + F(n-3)$, given $F(0), F(1), F(2)$.
16. Count number of binary strings of length n with even number of 1s.
17. Compute n th term of Padovan sequence: $P(n) = P(n-2) + P(n-3)$.
18. Count number of ways to tile a $4 \times n$ board with 1×4 and 4×1 tiles.
19. Number of sequences of length n avoiding certain consecutive patterns using recurrence.
20. General linear recurrence: $F(n) = \sum_{i=1}^k c_i F(n-i)$ for $i=1..k$, find $F(n) \% M$ efficiently.

If you want, I can **merge all your combinatorics, modular arithmetic, and linear algebra problem statements into a single “100–200 problem workbook”** with input/output examples and hints for coding in **C++ or Java**. This would be like a full practice set for competitive programming.

Do you want me to do that?

You said:

Distance between points, midpoint, slope • Convex Hull (Graham Scan / Jarvis March) • Orientation test (cross product sign) • Line intersection • Area of polygon / triangle (Shoelace Theorem) • Circle equations, tangents, and intersections • Closest pair of points problem

ChatGPT said:

Here's a **set of 50 one-line problem statements** covering **Geometry topics** like distance, slope, convex hulls, polygon areas, circle equations, and closest pair of points:

Distance, Midpoint, Slope

1. Compute the Euclidean distance between two points (x_1, y_1) and (x_2, y_2) .
 2. Find the midpoint of a line segment joining two points.
 3. Compute the slope of the line passing through two points.
 4. Check if three points are collinear using slope formula.
 5. Find distance of a point from a line $ax + by + c = 0$.
 6. Determine if two line segments intersect using distance and slope.
 7. Compute the equation of a line given two points.
 8. Find the equation of a line passing through a point with given slope.
 9. Determine if a point lies on a given line segment.
 10. Compute distance between a point and midpoint of a segment.
-

Convex Hull (Graham Scan / Jarvis March)

11. Compute convex hull of a set of 2D points using Graham Scan.
12. Compute convex hull using Jarvis March (Gift Wrapping).
13. Find perimeter of convex hull for given points.
14. Compute area of convex hull polygon.
15. Determine if a given point lies inside the convex hull.
16. Count number of points on the convex hull.
17. Remove points inside the convex hull and return hull vertices.
18. Compute convex hull for collinear points.
19. Find farthest pair of points on the convex hull.
20. Compute convex hull for points with duplicate coordinates.

Convex Hull / Geometric Enclosure Problems (20)

1. Given N points on a plane, find the vertices of the convex hull enclosing all points in counterclockwise order.
2. Given N points, compute the **area** of the smallest convex polygon enclosing all points.
3. Given N points, compute the **perimeter** of the convex hull.
4. Given N points, determine the **number of points lying on the convex hull**.
5. Given N points, find the **pair of points farthest apart** on the convex hull (diameter of the set).

6. Given N points, count the number of **triangles formed by convex hull vertices**.
 7. Given N points, find the **smallest rectangle (axis-aligned)** enclosing all points.
 8. Given N points, find the **smallest area rectangle (any orientation)** enclosing all points.
 9. Given N points, determine the **largest distance between a point inside the hull and hull boundary**.
 10. Given N points, compute the **number of points strictly inside the convex hull**.
 11. Given N points, find the **convex hull after removing one point** to minimize area.
 12. Given N points, check if a given polygon contains **all points**.
 13. Given N points, compute **convex hull and then reflect it over x-axis**.
 14. Given N points, find **minimum bounding circle** containing all points (related to convex hull).
 15. Given N points, compute **minimum width of convex hull** (distance between two parallel lines enclosing hull).
 16. Given N points, find **convex hull and count collinear points on each edge**.
 17. Given N points, compute the **convex hull after translating all points by a vector**.
 18. Given N points, determine if **all points lie on the convex hull** (i.e., no internal points).
 19. Given N points, compute the **convex hull and rotate it to maximize area along x-axis**.
 20. Given N points, find the **number of integer grid points inside the convex hull**.
-

✕ Orientation Test (Cross Product Sign)

21. Determine orientation (clockwise, counterclockwise, collinear) of three points.
 22. Check if a point lies on a line segment using cross product.
 23. Detect if two line segments intersect using orientation tests.
 24. Determine if polygon vertices are given in clockwise or counterclockwise order.
 25. Check if a polygon is convex using orientation of consecutive triples.
 26. Find the area of a triangle using cross product formula.
 27. Determine relative position of a point to a directed line.
 28. Check if two vectors are parallel using cross product.
 29. Compute signed area of a triangle using cross product.
 30. Determine if a point lies inside a convex polygon using orientation tests.
-

📐 Line Intersection

31. Find intersection point of two lines given in slope-intercept form.
32. Find intersection of two lines given in general form $ax + by + c = 0$.
33. Check if two line segments intersect.
34. Find intersection of a line and a circle.
35. Find intersection of two circles.
36. Determine if lines are parallel or coincident.
37. Compute intersection of a line with polygon edges.

38. Determine number of intersection points among multiple lines.
 39. Compute intersection of line and ray.
 40. Solve line intersection for integer coordinates.
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▲ Area of Polygon / Triangle (Shoelace Theorem)

41. Compute area of a triangle using Shoelace formula.
 42. Compute area of a convex polygon given its vertices.
 43. Compute area of a concave polygon using Shoelace formula.
 44. Compute area of a triangle given coordinates of three points.
 45. Find centroid of a polygon using vertex coordinates.
 46. Check if a polygon is simple (non-self-intersecting) before computing area.
 47. Compute area of polygon using triangulation method.
 48. Compute perimeter and area together for a polygon.
 49. Compute area of polygon after rotation/translation.
 50. Compute area of triangle using vector cross product method.
-

○ Circle Equations, Tangents, Intersections

51. Find equation of a circle given center and radius.
 52. Find equation of a circle passing through three points.
 53. Determine if a point lies inside, outside, or on a circle.
 54. Find tangent to a circle from an external point.
 55. Find intersection points of two circles.
 56. Find intersection points of a line and a circle.
 57. Compute distance between two circle centers.
 58. Determine if two circles touch each other (externally or internally).
 59. Find common tangents between two circles.
 60. Compute radius of circle given chord length and distance from center.
-

🔍 Closest Pair of Points

61. Find the closest pair of points in a set using brute force.
 62. Find the closest pair of points using divide and conquer.
 63. Compute distance between closest pair.
 64. Count number of point pairs with minimum distance.
 65. Find points forming the closest pair.
 66. Solve closest pair problem in 3D coordinates.
 67. Find farthest pair of points as a comparison problem.
 68. Find closest pair with Manhattan distance metric.
 69. Find closest pair inside a given rectangle.
 70. Compute closest pair distance with floating-point precision.
-

If you want, I can **combine all your previous problem sets** (modular arithmetic, combinatorics, linear algebra, geometry) into a **single 300–400 problem competitive programming workbook** with **input/output examples and hints for coding in C++/Java**.

Do you want me to do that?

You said:

5. Probability and Expected Value Appears in medium-hard rounds. • Expected value formula: $E[X] = \sum p_i * x_i$ • Conditional probability • Combinatorial probability counting • Expected number of steps / trials (Markov chains)

ChatGPT said:

Here's a **set of 50 one-line problem statements** covering **Probability and Expected Value** including expected value formula, conditional probability, combinatorial probability, and expected steps/trials (Markov chains):

 Expected Value ($E[X] = \sum p_i * x_i$)

1. Compute expected value of a fair dice roll.
2. Compute expected value of rolling two dice.
3. Compute expected value of a random card drawn from a deck.
4. Find expected value of a lottery payout given probabilities.
5. Compute expected value of sum of numbers drawn from a bag of balls.
6. Find expected value of a spinner with unequal sections.
7. Compute expected number of heads in n coin tosses.
8. Compute expected value of selecting a random integer from a range.
9. Find expected value when picking a ball with weight from a weighted bag.
10. Compute expected score in a board game with probabilistic moves.
11. Expected number of dice rolls to get a six.
12. Expected number of tries to get the first success in Bernoulli trials.
13. Expected number of defective items in a sample from a batch.
14. Compute expected sum when drawing numbers with replacement.
15. Expected value of a random variable defined by a probability table.

 Conditional Probability

16. Compute probability of drawing a red card given that it is a face card.
17. Find probability of event A given event B occurs.
18. Compute probability of at least one success given previous failures.
19. Compute conditional probability in rolling two dice: $\text{sum} > 8$ given one die is 5.
20. Probability of drawing a number divisible by 3 given it is even.
21. Compute probability of rain given cloudy day using Bayes' theorem.
22. Find probability of drawing two aces consecutively without replacement.
23. Compute probability of drawing a king given a queen was drawn first.
24. Conditional probability of hitting target given first shot missed.

25. Probability that a student passed math given they passed physics.

Combinatorial Probability Counting

26. Probability of drawing k red balls from n total balls.
 27. Probability of selecting a committee with at least 2 men from a group.
 28. Probability of getting exactly 3 heads in 5 coin tosses.
 29. Probability of forming a valid hand in poker (flush, straight).
 30. Probability of at least one common element in two random subsets.
 31. Probability of drawing 3 consecutive numbers from 1–10.
 32. Probability of getting sum of 7 when rolling two dice.
 33. Probability of choosing two numbers with sum divisible by 5.
 34. Probability of forming a palindrome from given letters.
 35. Probability of selecting numbers with even sum from 1– n .
 36. Probability of drawing a pair in 5-card poker hand.
 37. Probability of selecting team members from different groups.
 38. Probability of rolling a prime number on a die.
 39. Probability that a randomly selected student studies both subjects.
 40. Probability that at least 2 students share the same birthday (birthday problem).
-

Expected Number of Steps / Trials (Markov Chains)

41. Expected number of steps to reach end in a board game.
42. Expected number of coin tosses until two consecutive heads appear.
43. Expected number of rolls to get a sum of 7 with two dice.
44. Expected number of trials to draw a specific card from a deck.
45. Expected number of moves to reach a target in a random walk on a line.
46. Expected number of throws to see all faces of a die (coupon collector).
47. Expected number of steps for a token to exit a Markov chain state.
48. Expected number of attempts to guess a password correctly.
49. Expected number of steps for a gambler to reach a target fortune.
50. Expected number of draws until first red ball appears in a bag.

Expected Value / Probability Sequence Problems (20)

1. Expected number of coin tosses until getting k consecutive heads.
2. Expected number of dice rolls until a 6 appears for the first time.
3. Expected number of dice rolls until all numbers 1–6 appear at least once (Coupon Collector).
4. Expected number of coin tosses until exactly n heads are obtained.
5. Expected number of coin tosses until a specific sequence (e.g., HTH) appears.
6. Expected number of rolls of a fair die until the sum reaches or exceeds S .
7. Expected number of cards drawn from a shuffled deck until an Ace appears.
8. Expected number of coin tosses until the number of heads equals the number of tails.
9. Expected number of steps to reach the end in a simple 1D random walk.
10. Expected number of rolls until two consecutive sixes appear.

11. Expected number of dice rolls until three consecutive odd numbers appear.
12. Expected number of draws without replacement until all suits are drawn.
13. Expected number of coin tosses until exactly k heads appear consecutively at least once.
14. Expected number of rolls to get a double in two dice.
15. Expected number of coin tosses until the difference between heads and tails reaches n .
16. Expected number of draws from a bag with replacement until drawing all colors at least once.
17. Expected number of coin tosses until a given pattern (e.g., HTHH) occurs.
18. Expected number of steps for a gambler to reach zero starting with initial capital C (simple random walk).
19. Expected number of rolls until a sum of dice is divisible by k .
20. Expected number of coin tosses until at least m heads appear with no two consecutive tails.

If you want, I can **combine this Probability set with all your previous problem sets (Combinatorics, Modular Arithmetic, Linear Algebra, Geometry, etc.)** into a **single 400–500 problem competitive programming workbook** with **input/output examples and coding hints for C++/Java**.

Do you want me to do that?

You said:

✂ 6. Discrete Mathematics Used in graph counting, combinatorial structures, and recurrence relations. • Recurrence relations • Generating Functions (sometimes advanced) • Graph theory (adjacency, degree, Euler's formula) • Set theory / Inclusion–Exclusion • Pigeonhole and Matching principles

ChatGPT said:

Here's a **set of 50 one-line problem statements** covering **Discrete Mathematics** topics: recurrence relations, generating functions, graph theory, set theory, inclusion–exclusion, pigeonhole, and matching principles:

Recurrence Relations

1. Solve $F(n) = F(n-1) + F(n-2)$ with given initial values.
2. Solve $F(n) = 2 \cdot F(n-1) + 3 \cdot F(n-2)$.
3. Find n th term of a linear homogeneous recurrence relation.
4. Find n th term of a non-homogeneous recurrence relation.
5. Solve recurrence $F(n) = F(n-1) + n$.
6. Solve recurrence $F(n) = 2 \cdot F(n/2) + n$ (divide & conquer type).
7. Solve $F(n) = F(n-1) + F(n-3)$ with initial terms.
8. Solve recurrence using characteristic equation method.

9. Find sum of first n terms of a recurrence sequence.
 10. Compute n th term of recurrence using matrix exponentiation.
-

Generating Functions

11. Find generating function for sequence $1, 1, 1, 1, \dots$
 12. Find generating function for sequence of natural numbers.
 13. Compute coefficient of x^n in a given generating function.
 14. Use generating function to solve recurrence relation.
 15. Find closed form of sequence using generating function.
 16. Find generating function for geometric progression.
 17. Use generating function to count partitions of a number.
 18. Find number of ways to reach sum n using coins of given denominations.
 19. Compute sum of sequence using generating function approach.
 20. Use generating functions to solve combinatorial distribution problems.
-

Graph Theory (Adjacency, Degree, Euler's Formula)

21. Compute degree of each vertex in a graph.
 22. Represent a graph using adjacency matrix.
 23. Represent a graph using adjacency list.
 24. Compute number of edges from adjacency matrix.
 25. Determine if a graph is connected.
 26. Compute Euler characteristic ($V - E + F$) of a planar graph.
 27. Determine if a graph has an Eulerian path.
 28. Determine if a graph has an Eulerian circuit.
 29. Count number of isolated vertices in a graph.
 30. Compute in-degree and out-degree for directed graph vertices.
 31. Find neighbors of a given vertex.
 32. Count number of connected components in an undirected graph.
 33. Check if a graph is bipartite.
 34. Determine if a graph is complete or regular.
 35. Find adjacency of two given vertices.
-

Set Theory / Inclusion–Exclusion

36. Find union of two sets.
37. Find intersection of three sets.
38. Find difference between two sets.
39. Count number of elements in union of n sets using inclusion–exclusion.
40. Count elements in exactly two of three sets.
41. Count elements not belonging to any of given sets.
42. Solve word problems using inclusion–exclusion principle.
43. Count integers $\leq N$ divisible by a set of numbers.

44. Count arrangements avoiding forbidden positions (derangement).
 45. Count students studying exactly one subject using set formula.
-

● Pigeonhole and Matching Principles

46. Prove at least two students have same number of friends in a group.
 47. Determine minimum items needed to ensure at least one duplicate.
 48. Find maximum number of items in bins without violating pigeonhole principle.
 49. Prove existence of perfect matching in bipartite graph using Hall's theorem.
 50. Count minimum number of matches needed to pair people with constraints.
-

If you want, I can **merge all your previous problem sets (Modular Arithmetic, Combinatorics, Linear Algebra, Geometry, Probability, and Discrete Math)** into a **single 500–600 problem competitive programming workbook** with **input/output examples, formula hints, and coding tips in C++/Java.**

Do you want me to do that?