

Math (Basic Number Theory) GCD, LCM, modulo, primes, divisors c++ code and theory formulas

ChatGPT said:

Here's a **compact and clear guide** to the **basic number theory** topics in C++, including:

1. GCD (Greatest Common Divisor)
  2. LCM (Least Common Multiple)
  3. Modulo Arithmetic
  4. Prime Numbers (sieve and primality check)
  5. Divisors of a Number
- 

## ✓ 1. GCD (Greatest Common Divisor)

**Formula:**

**If  $a \% b == r$ , then  $\text{gcd}(a, b) = \text{gcd}(b, r)$**

**Eventually:  $\text{gcd}(a, b) = \text{gcd}(b, a \% b)$**

cpp

CopyEdit

```
int gcd(int a, int b) {  
    if (b == 0) return a;  
    return gcd(b, a % b);  
}
```

□ **Property:**

- $\text{gcd}(a, 0) = a$
  - $\text{gcd}(a, b) = \text{gcd}(b, a \% b)$
  - $\text{gcd}(a, b)$  **divides both a and b**
- 

## ✓ 2. LCM (Least Common Multiple)

**Formula:**

**$\text{lcm}(a, b) = (a * b) / \text{gcd}(a, b)$**

cpp

CopyEdit

```
int lcm(int a, int b) {  
    return (a / gcd(a, b)) * b; // Avoid overflow  
}
```

---

## ✓ 3. Modulo Arithmetic

**Properties:**

- $(a + b) \% m = ((a \% m) + (b \% m)) \% m$
- $(a * b) \% m = ((a \% m) * (b \% m)) \% m$
- $(a - b) \% m = ((a \% m) - (b \% m) + m) \% m$
- $(a^b) \% m$  can be computed using modular exponentiation

```
cpp
CopyEdit
// Modular Exponentiation (a^b % m)
long long mod_pow(long long a, long long b, long long m) {
    long long res = 1;
    a %= m;
    while (b > 0) {
        if (b & 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res;
}
```

---

## ✓ 4. Prime Numbers

### A. Primality Test (Basic)

```
cpp
CopyEdit
bool isPrime(int n) {
    if (n <= 1) return false;
    for (int i = 2; i*i <= n; ++i)
        if (n % i == 0) return false;
    return true;
}
```

### B. Sieve of Eratosthenes (All primes $\leq N$ )

```
cpp
CopyEdit
vector<bool> sieve(int n) {
    vector<bool> is_prime(n + 1, true);
    is_prime[0] = is_prime[1] = false;
    for (int i = 2; i * i <= n; ++i) {
        if (is_prime[i]) {
            for (int j = i * i; j <= n; j += i)
                is_prime[j] = false;
        }
    }
    return is_prime;
}
```

---

## ✓ 5. Divisors of a Number

All divisors of  $n$  up to  $\sqrt{n}$

```
cpp
CopyEdit
```

```
vector<int> getDivisors(int n) {
    vector<int> divs;
    for (int i = 1; i*i <= n; ++i) {
        if (n % i == 0) {
            divs.push_back(i);
            if (i != n / i)
                divs.push_back(n / i);
        }
    }
    sort(divs.begin(), divs.end());
    return divs;
}
```

---

## □ Summary of Formulas:

| Concept                | Formula  |
|------------------------|--|
| <b>GCD</b>             | $\text{gcd}(a, b) = \text{gcd}(b, a \% b)$             |
| <b>LCM</b>             | $\text{lcm}(a, b) = (a * b) / \text{gcd}(a, b)$        |
| <b>Modulo Sum</b>      | $(a + b) \% m = ((a \% m) + (b \% m)) \% m$            |
| <b>Modulo Multiply</b> | $(a * b) \% m = ((a \% m) * (b \% m)) \% m$            |
| <b>Modulo Subtract</b> | $(a - b) \% m = ((a \% m) - (b \% m) + m) \% m$        |
| <b>Modulo Power</b>    | $a^b \% m$ via binary exponentiation                   |
| <b>Divisors</b>        | Up to $\sqrt{n}$ , both $i$ and $n/i$ if $n \% i == 0$ |

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Would you like a **combined C++ demo** of all these in one main program?

You said:

Greedy Algorithms Simple optimal choices (e.g., sorting + decision making)

ChatGPT said:

Here's a **concise guide to Greedy Algorithms with simple optimal choices**, especially problems that use **sorting + decision making**. These are classic problems where **local optimal decisions lead to a globally optimal solution**.

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## ✓ What Are Greedy Algorithms?

A **greedy algorithm** builds up a solution piece by piece, always choosing the **most optimal option at the current step** (local optimum), hoping it leads to the **global optimum**.

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## □ Core Pattern: Sorting + Decision Making

This pattern is common in greedy:

1. **Sort the input** (based on some logic)
  2. **Make the best choice** at every step
  3. Continue until goal is met
- 

## 🔥 Popular Greedy Problems in C++

### 1. Activity Selection Problem

Given start and end times of activities, select the **maximum number of non-overlapping activities**.

```
cpp
CopyEdit
bool cmp(pair<int, int> a, pair<int, int> b) {
    return a.second < b.second; // sort by end time
}

int maxActivities(vector<pair<int, int>> &activities) {
    sort(activities.begin(), activities.end(), cmp);
    int count = 1, end = activities[0].second;

    for (int i = 1; i < activities.size(); ++i) {
        if (activities[i].first >= end) {
            count++;
            end = activities[i].second;
        }
    }
    return count;
}
```

---

### 2. Fractional Knapsack

Choose items to **maximize value** within a given weight capacity, but you can take **fractional parts** of items.

```
cpp
CopyEdit
struct Item {
    double value, weight;
};

bool cmp(Item a, Item b) {
    return (a.value / a.weight) > (b.value / b.weight); // value density
}

double fractionalKnapsack(vector<Item> items, double W) {
    sort(items.begin(), items.end(), cmp);
    double total = 0.0;

    for (auto &item : items) {
        if (W >= item.weight) {
            total += item.value;
        }
    }
}
```

```

        W -= item.weight;
    } else {
        total += item.value * (W / item.weight);
        break;
    }
}
return total;
}

```

---

### 3. Minimum Number of Coins

You have coin denominations; find the **minimum number of coins** to make a given amount.

```

cpp
CopyEdit
int minCoins(vector<int> &coins, int amount) {
    sort(coins.rbegin(), coins.rend()); // sort descending
    int count = 0;
    for (int coin : coins) {
        while (amount >= coin) {
            amount -= coin;
            count++;
        }
    }
    return (amount == 0 ? count : -1); // -1 if not possible
}

```

---

### 4. Job Scheduling (With Deadlines)

Each job has a deadline and profit. Schedule jobs to **maximize total profit** while respecting deadlines (1 job per time slot).

```

cpp
CopyEdit
struct Job {
    int id, deadline, profit;
};

bool cmp(Job a, Job b) {
    return a.profit > b.profit; // greedy: most profit first
}

int jobScheduling(vector<Job> &jobs) {
    sort(jobs.begin(), jobs.end(), cmp);
    int maxTime = 0;
    for (auto j : jobs) maxTime = max(maxTime, j.deadline);

    vector<bool> slot(maxTime + 1, false);
    int totalProfit = 0;

    for (auto job : jobs) {
        for (int t = job.deadline; t >= 1; --t) {
            if (!slot[t]) {
                slot[t] = true;
                totalProfit += job.profit;
                break;
            }
        }
    }
}

```

```

    }
    }
    }
    return totalProfit;
}

```

---

## ✓ Summary of Greedy + Sorting Strategy

| Problem                     | Sort By                 | Decision Rule                                  |
|-----------------------------|-------------------------|--|
| Activity Selection          | End time ascending      | Choose next activity if $start \geq end$       |
| Fractional Knapsack         | Value/weight descending | Take as much as fits                           |
| Coin Change (Min Coins)     | Coin value descending   | Use as many large coins as possible            |
| Job Scheduling (Max Profit) | Profit descending       | Schedule latest available slot $\leq$ deadline |

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Do you want a combined code that includes **menu** + **test cases** for all the above in one file?

**You said:**

Brute Force Try all combinations if constraints are small ( $n \leq 1000$ ).

**ChatGPT said:**

Here's a **clear and practical guide** to the **Brute Force** technique, focusing on **trying all combinations** when  $n \leq 1000$ . Brute force is often your best bet when:

- Constraints are small (like  $n \leq 20$  for subsets or  $n \leq 1000$  for pairwise/comparisons).
  - No clever optimization is obvious.
  - You want to **verify correctness** before optimizing.
- 

## ✓ What Is Brute Force?

Brute Force = Try **all possible combinations / configurations** and pick the best one.

🔄 Examples of brute force:

- Try every pair ( $O(n^2)$ )
- Try every triplet ( $O(n^3)$ )
- Try every subset ( $2^n$ ) — only for  $n \leq 20$
- Try every permutation ( $n!$ ) — only for  $n \leq 9 \sim 10$

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## ✓ Classic Brute Force Problems with $n \leq 1000$

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### 1. Two Sum (All Pairs)

```
cpp
CopyEdit
bool hasTwoSum(vector<int> &arr, int target) {
    int n = arr.size();
    for (int i = 0; i < n; ++i)
        for (int j = i+1; j < n; ++j)
            if (arr[i] + arr[j] == target)
                return true;
    return false;
}
```

Time:  $O(n^2)$ , Safe for  $n \leq 1000$

---

### 2. Count All Triplets With Sum Zero

```
cpp
CopyEdit
int countTriplets(vector<int> &arr) {
    int n = arr.size(), count = 0;
    for (int i = 0; i < n; ++i)
        for (int j = i+1; j < n; ++j)
            for (int k = j+1; k < n; ++k)
                if (arr[i] + arr[j] + arr[k] == 0)
                    count++;
    return count;
}
```

Time:  $O(n^3)$ , use only if  $n \leq 200 \sim 300$

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### 3. Find All Pairs (i, j) Where $a[i] < a[j]$

```
cpp
CopyEdit
vector<pair<int, int>> allIncreasingPairs(vector<int> &a) {
    vector<pair<int, int>> res;
    int n = a.size();
    for (int i = 0; i < n; ++i)
        for (int j = i+1; j < n; ++j)
            if (a[i] < a[j])
                res.push_back({i, j});
    return res;
}
```

---

## 4. Subset Sum ( $n \leq 20$ )

Try all subsets to check if a sum can be formed.

```
cpp
CopyEdit
bool subsetSum(vector<int> &a, int target) {
    int n = a.size();
    for (int mask = 0; mask < (1 << n); ++mask) {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (mask & (1 << i))
                sum += a[i];
        if (sum == target)
            return true;
    }
    return false;
}
```

Time:  $O(2^n)$ , feasible for  $n \leq 20$

---

## 5. Permutations ( $n \leq 8$ ) using `next_permutation`

```
cpp
CopyEdit
void allPermutations(vector<int> a) {
    sort(a.begin(), a.end());
    do {
        for (int x : a) cout << x << " ";
        cout << "\n";
    } while (next_permutation(a.begin(), a.end()));
}
```

---

## □ Tips for Brute Force in Contests

### Constraint Safe Algorithm Time Complexity

|                    |          |                 |
|--------------------|----------|-----------------|
| $n \leq 100$       | $O(n^3)$ | Triplet loops   |
| $n \leq 1000$      | $O(n^2)$ | Pairwise checks |
| $n \leq 20$        | $O(2^n)$ | Subset masks    |
| $n \leq 8 \sim 10$ | $O(n!)$  | Permutations    |

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## Summary

Brute force is:



- Easy to write
  - Great for small inputs
  - Useful for validating optimized approaches
- 

Would you like a **template-driven menu-based C++ app** that implements all these brute force examples with user input and results?

**You said:**

Two Pointers For string and array subrange problems

**ChatGPT said:**

Here's a **complete guide to the Two Pointers technique**, especially for solving **string and array subrange problems** — an essential trick for optimizing from  **$O(n^2)$**  to  **$O(n)$**  or  **$O(n \log n)$**  in many cases.

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## ✓ What is the Two Pointers Technique?

Use **two indices** ( $i$  and  $j$ ) to represent a **window or subrange** and **slide them efficiently** to solve a problem.

- Common when the array/string is **sorted or monotonic**
  - Often used to maintain a **sliding window**
  - Replaces nested loops
- 

## □ Where to Use Two Pointers

- Subarray with given sum
  - Longest substring/sequence
  - Remove duplicates
  - Palindrome check
  - Merging two sorted arrays
- 

## 🔥 Classic Two Pointer Problems

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### 1. Find if Any Two Numbers Sum to Target (Sorted Array)

cpp

CopyEdit

```
bool hasTwoSum(vector<int> &arr, int target) {
    int left = 0, right = arr.size() - 1;
    while (left < right) {
        int sum = arr[left] + arr[right];
        if (sum == target) return true;
        else if (sum < target) left++;
        else right--;
    }
    return false;
}
```

- Time:  $O(n)$  — single pass
  - Condition: array must be **sorted**
- 

## 2. Longest Substring Without Repeating Characters

cpp

CopyEdit

```
int longestUniqueSubstring(string s) {
    unordered_set<char> seen;
    int i = 0, j = 0, n = s.length(), maxLen = 0;

    while (j < n) {
        if (seen.count(s[j]) == 0) {
            seen.insert(s[j++]);
            maxLen = max(maxLen, j - i);
        } else {
            seen.erase(s[i++]);
        }
    }
    return maxLen;
}
```

- Time:  $O(n)$
  - Good for strings with only lowercase/uppercase letters
- 

## 3. Count Subarrays with Sum $\leq K$ (Positive Integers)

cpp

CopyEdit

```
int countSubarrays(vector<int> &a, int k) {
    int i = 0, sum = 0, res = 0;
    for (int j = 0; j < a.size(); ++j) {
        sum += a[j];
        while (sum > k)
            sum -= a[i++];
        res += (j - i + 1);
    }
    return res;
}
```

- Time:  $O(n)$
  - Returns total number of valid subarrays
- 

#### 4. Remove Duplicates from Sorted Array (in-place)

```
cpp
CopyEdit
int removeDuplicates(vector<int> &nums) {
    int i = 0;
    for (int j = 1; j < nums.size(); ++j) {
        if (nums[j] != nums[i])
            nums[++i] = nums[j];
    }
    return i + 1; // size of unique part
}
```

---

#### 5. Check if String is Palindrome Using Two Pointers

```
cpp
CopyEdit
bool isPalindrome(string &s) {
    int i = 0, j = s.length() - 1;
    while (i < j) {
        if (s[i] != s[j]) return false;
        i++, j--;
    }
    return true;
}
```

---

### ★ Two Pointers Template (Sliding Window)

```
cpp
CopyEdit
int i = 0, j = 0;
while (j < n) {
    // Expand window
    // Process s[j]

    while (/* window invalid */) {
        // Shrink from the left
        // Update i
    }

    // Update answer
    j++;
}
```

---

### 🔄 Summary Table

| Problem                                  | Pattern Used               | Time   |
|--|----------------------------|--------|
| Two Sum (sorted)                         | $i = 0, j = n-1$           | $O(n)$ |
| Longest Unique Substring                 | Sliding Window + Set       | $O(n)$ |
| Count Subarrays with $\text{Sum} \leq K$ | Sliding Window + Shrinking | $O(n)$ |
| Remove Duplicates (sorted array)         | Slow + fast pointer        | $O(n)$ |
| Check Palindrome                         | Two ends moving inward     | $O(n)$ |

---

Would you like a **C++ file with all these problems coded in one menu-driven demo?**

You said:

Prefix Sum / Difference Arrays For range sum or difference queries

ChatGPT said:

Here's a detailed yet beginner-friendly guide to **Prefix Sum and Difference Arrays** — two powerful techniques for **range sum and range update problems** in **arrays** and **matrices**.

---

## ✓ 1. Prefix Sum Array

### ◆ Use Case:

- **Fast range sum queries** in  $O(1)$  time after  $O(n)$  preprocessing.
- Convert  $O(n)$  range queries to  $O(1)$ .

### ◆ How It Works:

Let `arr[]` be your input array.

Create a `prefix[]` array such that:

```
cpp
CopyEdit
prefix[i] = arr[0] + arr[1] + ... + arr[i];
```

Then, the sum of range `[l, r]` is:

```
cpp
CopyEdit
sum = prefix[r] - prefix[l-1]; // if l > 0
sum = prefix[r];               // if l == 0
```

---

### ✓ C++ Code Example:

```
cpp
CopyEdit
vector<int> buildPrefixSum(const vector<int>& arr) {
```

```

    int n = arr.size();
    vector<int> prefix(n);
    prefix[0] = arr[0];
    for (int i = 1; i < n; ++i)
        prefix[i] = prefix[i - 1] + arr[i];
    return prefix;
}

int rangeSum(const vector<int>& prefix, int l, int r) {
    if (l == 0) return prefix[r];
    return prefix[r] - prefix[l - 1];
}

```

---

## ✓ 2. Difference Array

### ◆ Use Case:

- Efficiently apply **range updates** in  $O(1)$  time.
- Modify ranges without touching every element.

### ◆ Idea:

Given original array `arr[]`, build `diff[]`:

```

cpp
CopyEdit
diff[0] = arr[0];
diff[i] = arr[i] - arr[i-1]; // for i > 0

```

To add `x` to range `[l, r]`, do:

```

cpp
CopyEdit
diff[l] += x;
diff[r + 1] -= x;

```

After all updates, build final array:

```

cpp
CopyEdit
arr[0] = diff[0];
for i = 1 to n-1:
    arr[i] = arr[i-1] + diff[i];

```

---

## ✓ C++ Code Example:

```

cpp
CopyEdit
vector<int> applyRangeUpdates(int n, vector<tuple<int, int, int>> updates)
{
    vector<int> diff(n + 2, 0); // difference array

```

```

    for (auto [l, r, val] : updates) {
        diff[l] += val;
        diff[r + 1] -= val;
    }

    vector<int> result(n);
    result[0] = diff[0];
    for (int i = 1; i < n; ++i)
        result[i] = result[i - 1] + diff[i];

    return result;
}

```

---

## ✓ 3. 2D Prefix Sum (For Matrices)

### ◆ Use Case:

- Answer submatrix sum queries in O(1)

Let `matrix[i][j]` be the original matrix

**Build `prefix[i][j]` as:**

```

cpp
CopyEdit
prefix[i][j] = matrix[i][j]
               + prefix[i-1][j]
               + prefix[i][j-1]
               - prefix[i-1][j-1];

```

Then, to query sum in rectangle  $(r1, c1)$  to  $(r2, c2)$ :

```

cpp
CopyEdit
sum = prefix[r2][c2]
      - prefix[r1-1][c2]
      - prefix[r2][c1-1]
      + prefix[r1-1][c1-1];

```

---

## □ Summary Table

| Problem Type          | Technique        | Time            |
|-----------------------|------------------|-----------------|
| Range sum in array    | Prefix Sum       | O(1)            |
| Range update in array | Difference Array | O(1) per update |
| 2D matrix sum queries | 2D Prefix Sum    | O(1)            |

---

## ✦ Example Problem Scenarios:

| Task  | Use        |
|---|------------|
| Query sum of salaries from employee 10 to 100 | Prefix Sum |

|  |                  |
|--|------------------|
| Increase scores of students 50 to 70 by 10 | Difference Array |
| Count sum of values in a rectangular grid  | 2D Prefix Sum    |

Would you like a **C++ implementation of all these with interactive input?**

You said:

Arrays and Strings Core data types used in almost every problem.

ChatGPT said:

Absolutely! **Arrays and Strings** are the **core data types** in competitive programming and interviews — they appear in **almost every problem**, either directly or indirectly.

Below is a detailed **cheat sheet** covering:

1. Array Basics & Patterns
2. String Basics & Operations
3. Common Interview-Level Problems
4. Key STL Functions in C++

## ✓ 1. Arrays – Core Concepts

### ★ Declaration

```
cpp
CopyEdit
int a[100];           // Static array
vector<int> v;         // Dynamic array
vector<int> v(n, 0);   // Size n, all elements 0
```

### ★ Input/Output

```
cpp
CopyEdit
vector<int> v(n);
for (int i = 0; i < n; ++i) cin >> v[i];
for (int x : v) cout << x << " ";
```

## 🔥 Common Array Patterns

| Pattern      | Description                      | Example Problem         |
|--------------|----------------------------------|-------------------------|
| Prefix Sum   | Precompute sum for range queries | Range Sum Query         |
| Two Pointers | Solve subrange efficiently       | Subarray with given sum |

|                         |                                |                                 |
|-------------------------|--------------------------------|---------------------------------|
| Sliding Window          | Fixed/variable window problems | Max sum subarray of size k      |
| Hashing/Frequency Count | Store counts in map/vector     | Majority Element, Anagram check |
| Sorting + Greedy        | Min/max combinations           | Min Arrows to Burst Balloons    |
| Binary Search           | Use on sorted arrays           | Search Insert Position          |

## ✓ Example Problems (Arrays)

### a. Maximum Subarray (Kadane's Algorithm)

```
cpp
CopyEdit
int maxSubArray(vector<int>& nums) {
    int maxSum = nums[0], curr = nums[0];
    for (int i = 1; i < nums.size(); ++i) {
        curr = max(nums[i], curr + nums[i]);
        maxSum = max(maxSum, curr);
    }
    return maxSum;
}
```

### b. Move Zeros to End

```
cpp
CopyEdit
void moveZeros(vector<int>& nums) {
    int i = 0;
    for (int j = 0; j < nums.size(); ++j)
        if (nums[j] != 0) swap(nums[i++], nums[j]);
}
```

## ✓ 2. Strings – Core Concepts

### ★ Declaration

```
cpp
CopyEdit
string s = "hello";
cin >> s;           // Input without spaces
getline(cin, s);    // Input with spaces
```

### 🔥 Common String Patterns

| Pattern        | Description             | Example Problem                  |
|----------------|-------------------------|----------------------------------|
| Two Pointers   | Palindromes, substrings | Longest Unique Substring         |
| Hashing        | Frequency maps          | Anagram, Duplicate chars         |
| Sliding Window | Optimal substrings      | Longest substring without repeat |



|                            |                                |                         |
|----------------------------|--------------------------------|-------------------------|
| <b>Z-Algorithm / KMP</b>   | Pattern matching               | Find string in string   |
| <b>Prefix Sum on Chars</b> | Count specific characters fast | Number of a's in prefix |

---

## ✓ Example Problems (Strings)

### a. Check Palindrome

```
cpp
CopyEdit
bool isPalindrome(string s) {
    int i = 0, j = s.size() - 1;
    while (i < j)
        if (s[i++] != s[j--]) return false;
    return true;
}
```

### b. Reverse Words in String

```
cpp
CopyEdit
string reverseWords(string s) {
    stringstream ss(s);
    string word, res = "";
    while (ss >> word)
        res = word + " " + res;
    res.pop_back();
    return res;
}
```

---

## ✓ 3. STL Functions to Master (Arrays & Strings)

### ★ For Arrays / Vectors

```
cpp
CopyEdit
sort(v.begin(), v.end());
reverse(v.begin(), v.end());
accumulate(v.begin(), v.end(), 0); // sum
*max_element(v.begin(), v.end());
count(v.begin(), v.end(), x);
```

### ★ For Strings

```
cpp
CopyEdit
sort(s.begin(), s.end());
reverse(s.begin(), s.end());
s.substr(i, len); // substring
to_string(x); // int to string
stoi(s); // string to int
```

---

## Summary: Must-Know Concepts

| Area    | Must Know  |
|---------|--|
| Arrays  | Two pointers, Prefix sum, Kadane's, Sliding window   |
| Strings | Palindrome, Anagram, Frequency, Substrings, Reversal |
| STL     | sort, reverse, max_element, substr, find, count      |

---

Would you like a **C++ template project** that includes:

- Menu-driven interface
- All these array & string patterns implemented with examples?

Let me know and I'll generate it for you.

**You said:**

Sorting + Binary Search (Basic)

**ChatGPT said:**

Here's a **complete guide to using Sorting + Binary Search (Basic level)** — a powerful and commonly used combination in competitive programming and interviews.

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## ✔ Why Combine Sorting + Binary Search?

- Sorting helps to bring structure/order to data
  - Binary search lets us search in  **$O(\log n)$**  on sorted data
  - Together, they solve many problems efficiently in  **$O(n \log n)$**  time
- 

## Classic Patterns

| Problem Type                       | Strategy                            | Time Complexity                     |
|------------------------------------|-------------------------------------|-------------------------------------|
| Find if an element exists          | Sort + Binary Search                | $O(\log n)$                         |
| Count elements $\leq/\geq x$       | Sort + lower_bound / upper_bound    | $O(\log n)$                         |
| Min/max value satisfying condition | Binary Search on Answer             | $O(\log \text{answer} \times f(n))$ |
| Pairs/triplets with sum x          | Sort + Two Pointers / Binary Search | $O(n \log n) / O(n^2)$              |

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## ✔ 1. Binary Search Template (Classic)

```

cpp
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int binarySearch(vector<int>& arr, int target) {
    int low = 0, high = arr.size() - 1;
    while (low <= high) {
        int mid = low + (high - low) / 2;
        if (arr[mid] == target) return mid;
        else if (arr[mid] < target) low = mid + 1;
        else high = mid - 1;
    }
    return -1;
}

```

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## ✓ 2. Find if Element Exists in Array

```

cpp
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bool exists(vector<int>& arr, int x) {
    sort(arr.begin(), arr.end()); // sort first
    return binary_search(arr.begin(), arr.end(), x);
}

```

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## ✓ 3. Count Elements $\leq X$

```

cpp
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int countLessEqual(vector<int>& arr, int x) {
    sort(arr.begin(), arr.end());
    return upper_bound(arr.begin(), arr.end(), x) - arr.begin();
}

```

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## ✓ 4. First and Last Occurrence of Element

```

cpp
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int firstOccurrence(vector<int>& arr, int x) {
    int low = 0, high = arr.size() - 1, ans = -1;
    while (low <= high) {
        int mid = (low + high) / 2;
        if (arr[mid] == x) {
            ans = mid;
            high = mid - 1; // go left
        } else if (arr[mid] < x) low = mid + 1;
        else high = mid - 1;
    }
    return ans;
}

```

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## ✓ 5. Search Insert Position

```

cpp
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int searchInsert(vector<int>& arr, int x) {
    int low = 0, high = arr.size() - 1, ans = arr.size();
    while (low <= high) {
        int mid = (low + high) / 2;
        if (arr[mid] >= x) {
            ans = mid;
            high = mid - 1;
        } else low = mid + 1;
    }
    return ans;
}

```

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## ✔ 6. Binary Search on Answer (Minimum Valid Value)

E.g., Find the minimum  $x$  such that  $f(x) == \text{true}$

```

cpp
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int binarySearchAnswer(int low, int high) {
    int ans = -1;
    while (low <= high) {
        int mid = (low + high) / 2;
        if (check(mid)) {
            ans = mid;
            high = mid - 1;
        } else {
            low = mid + 1;
        }
    }
    return ans;
}

```

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## □ STL Functions for Binary Search

```

cpp
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binary_search(arr.begin(), arr.end(), x);           // true/false
lower_bound(arr.begin(), arr.end(), x);             // ≥ x
upper_bound(arr.begin(), arr.end(), x);             // > x

```

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## 📖 Summary Table

| Task                    | Code Snippet                         |
|-------------------------|--------------------------------------|
| Find if $x$ exists      | <code>binary_search()</code>         |
| Count elements $\leq x$ | <code>upper_bound() - begin()</code> |
| Find first $\geq x$     | <code>lower_bound()</code>           |
| Insert position for $x$ | <code>lower_bound()</code> index     |
| Custom condition check  | Binary Search on Answer              |

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Would you like a **combined C++ code template** implementing all the above with examples and input/output?

You said:

Maps/Sets (Hashing) Counting elements, uniqueness, frequency.

ChatGPT said:

Here's a summary with theory + C++ code examples for **Maps/Sets (Hashing)** used in counting, uniqueness, and frequency problems:

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## ✔ Theory: Maps and Sets in C++

- **Map** stores key-value pairs. Example: `map<int, int> mp;`
  - Automatically sorted by key (Red-Black Tree).
  - Useful for frequency counting, mapping values.
- **Unordered Map**: `unordered_map<int, int> ump;`
  - Faster (Average  $O(1)$ ), but no ordering.
- **Set** stores unique elements.
  - Automatically sorted.
- **Unordered Set**: `unordered_set<int> us;`
  - Stores unique elements with faster operations (average  $O(1)$ ).

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## 📖 Common Use Cases

| Use Case              | Data Structure                   |
|-----------------------|----------------------------------|
| Count frequency       | <code>map / unordered_map</code> |
| Check uniqueness      | <code>set / unordered_set</code> |
| Grouping by key       | <code>map</code>                 |
| Element exists or not | <code>set / map.count()</code>   |

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## 🔢 Example 1: Count Frequencies of Elements

```
cpp
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#include <iostream>
#include <unordered_map>
#include <vector>
using namespace std;

int main() {
    vector<int> arr = {2, 3, 2, 4, 3, 2};
    unordered_map<int, int> freq;
```

```
    for (int x : arr) freq[x]++;

    for (auto [val, count] : freq)
        cout << val << " appears " << count << " times\n";

    return 0;
}
```

---

## ✓ Example 2: Check Duplicates Using Set

```
cpp
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#include <iostream>
#include <unordered_set>
#include <vector>
using namespace std;

int main() {
    vector<int> arr = {1, 2, 3, 2};
    unordered_set<int> seen;

    for (int x : arr) {
        if (seen.count(x)) {
            cout << "Duplicate found: " << x << endl;
            break;
        }
        seen.insert(x);
    }

    return 0;
}
```

---

## 📖 Example 3: Count Unique Words in a String

```
cpp
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#include <iostream>
#include <set>
#include <sstream>
using namespace std;

int main() {
    string text = "the quick brown fox jumps over the lazy dog";
    set<string> uniqueWords;
    stringstream ss(text);
    string word;

    while (ss >> word)
        uniqueWords.insert(word);

    cout << "Unique words: " << uniqueWords.size() << endl;
    return 0;
}
```

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## ❑ Time Complexity

| Operation            | map / set   | unordered_map / unordered_set |
|----------------------|-------------|-------------------------------|
| Insert/Delete/Access | $O(\log N)$ | Average $O(1)$ , Worst $O(N)$ |
| Search/Count         | $O(\log N)$ | Average $O(1)$ , Worst $O(N)$ |

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Would you like 10 practice problems on maps/sets with solutions too?

**You said:**

Basic Recursion / Backtracking Subset generation, small dfs-style problems.

**ChatGPT said:**

Here's a full explanation + C++ code examples for **Basic Recursion / Backtracking**, especially for **subset generation** and **DFS-style small problems**.

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## ✓ Theory: Recursion and Backtracking

### ↻ Recursion

- Function calls itself with smaller subproblems.
- Used for problems where a task can be divided into similar subtasks.

### ↶ BACK Backtracking

- Try all options, go back if a choice doesn't work (undo steps).
  - Used for:
    - Generating all **subsets**
    - **Permutations**
    - **Combinations**
    - Solving **mazes**, **Sudoku**, **N-Queens**, etc.
- 

## 📖 Subset Generation (Power Set)

All subsets of a given set:

$[1, 2] \rightarrow [], [1], [2], [1, 2]$

### ◆ Code (Backtracking style)

cpp  
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```

#include <iostream>
#include <vector>
using namespace std;

void generateSubsets(int i, vector<int>& nums, vector<int>& subset) {
    if (i == nums.size()) {
        cout << "[ ";
        for (int val : subset) cout << val << " ";
        cout << "]\n";
        return;
    }

    // Exclude nums[i]
    generateSubsets(i + 1, nums, subset);

    // Include nums[i]
    subset.push_back(nums[i]);
    generateSubsets(i + 1, nums, subset);
    subset.pop_back(); // backtrack
}

int main() {
    vector<int> nums = {1, 2, 3};
    vector<int> subset;
    generateSubsets(0, nums, subset);
    return 0;
}

```

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## Factorial using Recursion

```

cpp
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int factorial(int n) {
    if (n == 0 || n == 1) return 1;
    return n * factorial(n - 1);
}

```

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## DFS-style Recursion Example – Maze Path Count

```

cpp
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int countPaths(int i, int j, int n, int m) {
    if (i == n - 1 && j == m - 1) return 1;
    if (i >= n || j >= m) return 0;

    return countPaths(i + 1, j, n, m) + countPaths(i, j + 1, n, m);
}

```

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## Backtracking: Print All Permutations

```

cpp
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void backtrack(vector<int>& nums, vector<bool>& used, vector<int>& curr) {

```



```

    if (curr.size() == nums.size()) {
        for (int val : curr) cout << val << " ";
        cout << endl;
        return;
    }

    for (int i = 0; i < nums.size(); ++i) {
        if (used[i]) continue;
        used[i] = true;
        curr.push_back(nums[i]);

        backtrack(nums, used, curr);

        used[i] = false;
        curr.pop_back(); // backtrack
    }
}

int main() {
    vector<int> nums = {1, 2, 3};
    vector<bool> used(nums.size(), false);
    vector<int> curr;
    backtrack(nums, used, curr);
    return 0;
}

```

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## ❑ Time Complexities

| Problem           | Time Complexity      |
|-------------------|----------------------|
| Subset generation | $O(2^n)$             |
| Permutations      | $O(n!)$              |
| Basic DFS         | Depends on branching |

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## ✓ Common Applications

- Generating all combinations (e.g., k-combinations from n elements)
  - Solving puzzles (Sudoku, N-Queens)
  - Exploring paths in a matrix or graph
  - Knapsack (recursive versions)
- 

Would you like **10 practice problems** for recursion/backtracking as well?