Lecture: Deep Learning Fundamentals

KTH AI Student

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What we will cover today

- History of Deep Learning.
- Fundamentals of a Perceptron.
- Understanding and Implementing Gradient Descent.
- Introduction to Autograd and Backpropagation.
- Training a Multilayer Perceptron (MLP).
- Deep Dive into Convolutional Neural Networks (CNNs).

Early Beginnings

- 1943: McCulloch and Pitts propose the first mathematical model of a neuron.
- 1958: Rosenblatt introduces the Perceptron, an early neural network.

First AI Winter

- 1970s: AI research faces criticism and reduced funding.
- Limitations of Perceptrons highlighted by Minsky and Papert.

Revival and Second AI Winter

- 1980s: Backpropagation algorithm popularized by Rumelhart, Hinton, and Williams.
- 1990s: AI research faces another decline due to unmet expectations.

Milestones in AI

- 1997: IBM's Deep Blue defeats world chess champion Garry Kasparov.
- 2006: Hinton and colleagues introduce deep belief networks, sparking renewed interest in deep learning.

Modern Deep Learning Era

- 2012: AlexNet wins ImageNet competition, demonstrating the power of deep convolutional networks.
- 2014: GANs introduced by Goodfellow et al., enabling generative models.
- 2017: Transformers introduced by Vaswani et al., revolutionizing NLP.
- 2023: Large language models (LLMs) like GPT-3 and BERT achieve human-level performance on various tasks.

What is Linear Regression?

- Linear regression is a simple model used to predict continuous values.
- It assumes a linear relationship between input features and output.
- The model is represented as:

$$y = wx + b$$

- Where y is the output, x is the input, w is the weight, and b is the bias.

What is a Perceptron?

- A Perceptron is a simple neural network unit.
- It takes input features, applies weights, and produces an output.
- The output is passed through an activation function.
- The equation for a simple linear model is:

$$y = wx + b$$

- Then we apply an activation function to y.
- Example: Sigmoid, ReLU, Tanh.

Perceptron Example

- Consider a Perceptron with one input feature.
- Weights: $w = 2$, bias: $b = 1$.
- Input: $x = 3$.
- Output: $y = 2 \times 3 + 1 = 7$.
- Apply ReLU activation: $f(y) = \max(0, y) = 7$.

Perceptron

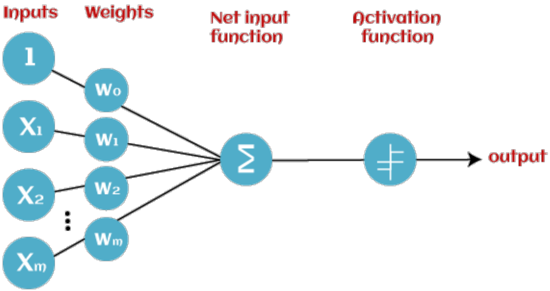


Figure: Perceptron

Linear Regression Assumptions

- Linearity: The relationship between dependent and independent variables is linear.
- Independence: Observations are independent of each other.
- Homoscedasticity: Constant variance of residuals.
- Normality: Residuals should be normally distributed.

How can we learn non-linear relationships?

- We can use **Multilayer Perceptrons (MLPs)**.
- MLPs consist of multiple layers of neurons.
- Hidden layers introduce non-linearity.
- Activation functions like ReLU are used to introduce non-linearity.

What is an MLP?

- A **Multilayer Perceptron (MLP)** consists of multiple layers of neurons.
- It includes an **input layer, hidden layers, and an output layer**.
- Uses **activation functions** to introduce non-linearity.

Common Activation Functions

- **Sigmoid:** Used in binary classification problems.

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

- **ReLU (Rectified Linear Unit):**

$$f(x) = \max(0, x)$$

- **Tanh:** Scales input to range $[-1, 1]$.

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

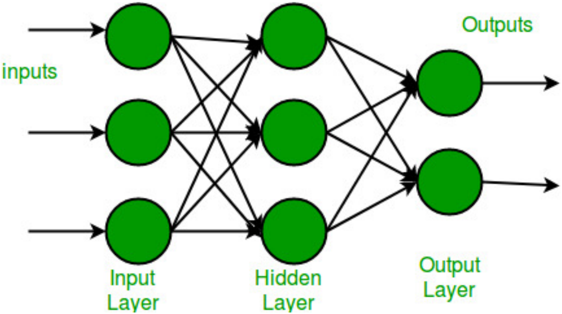


Figure: Multilayer Perceptron

MLP Architecture

- Input layer: Receives input features.
- Hidden layers: Perform computations and transformations.
- Output layer: Produces final predictions.
- Example: MLP for digit classification.

Training an MLP

- Initialize weights and biases.
- Forward pass: Compute predictions.
- Compute loss: Measure prediction error.
- Backward pass: Compute gradients.
- Update weights: Apply gradient descent.
- Repeat until convergence.

Loss Functions

- **Mean Squared Error (MSE):** Used for regression problems.

$$\mathrm{MSE} = \frac{1}{N}\sum (y_i - \hat{y}_i)^2$$

- **Cross-Entropy Loss:** Used for classification problems.

$$\text{Cross-Entropy} = -\frac{1}{N}\sum y_i \log(\hat{y}_i)$$

Gradient Descent Algorithm

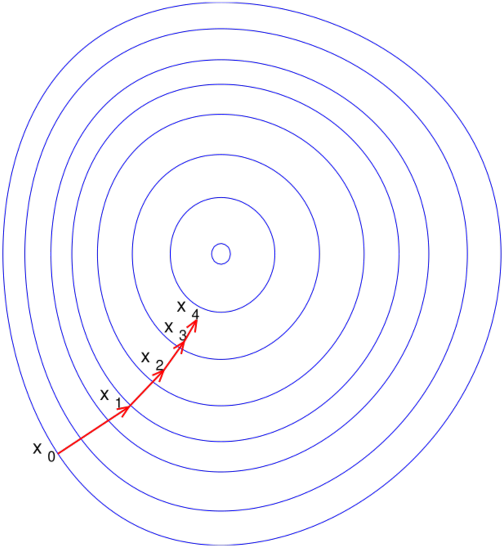
- An optimization technique used to minimize the loss function.
- The parameter updates are:

$$w := w - \alpha \frac{\partial \text{MSE}}{\partial w}$$

$$b := b - \alpha \frac{\partial \text{MSE}}{\partial b}$$

- α is the learning rate, which controls step size.

Gradient Descent Algorithm



Challenges in Gradient Descent

- Choosing the correct learning rate is critical.
- Too small α results in slow convergence.
- Too large α may cause divergence.
- Local minima vs. global minima.

Variants of Gradient Descent

- **Batch Gradient Descent:** Uses the entire dataset to compute gradients.
- **Stochastic Gradient Descent (SGD):** Uses one sample at a time.
- **Mini-batch Gradient Descent:** Uses a subset of the dataset.
- **Momentum:** Accelerates convergence by considering past gradients.
- **Adam:** Adaptive learning rate method combining momentum and RMSProp.

Mathematical Derivation of Gradient Descent

- The gradient of MSE with respect to w is:

$$\frac{\partial \text{MSE}}{\partial w} = -\frac{2}{N} \sum x_i (y_i - (wx_i + b))$$

- The gradient of MSE with respect to b is:

$$\frac{\partial \text{MSE}}{\partial b} = -\frac{2}{N} \sum (y_i - (wx_i + b))$$

- Update rules:

$$w := w - \alpha \left(-\frac{2}{N} \sum x_i (y_i - (wx_i + b)) \right)$$

$$b := b - \alpha \left(-\frac{2}{N} \sum (y_i - (wx_i + b)) \right)$$

Backpropagation: The Core of Neural Networks

- Backpropagation is used to adjust weights to minimize loss.
- Steps:
  1. Compute forward pass (prediction).
  2. Calculate loss function.
  3. Compute gradients of loss with respect to parameters.
  4. Update weights using gradient descent.

Chain Rule in Backpropagation

- The derivative of a composite function:

$$\frac{df}{dx} = \frac{df}{du} \times \frac{du}{dx}$$

- Used to efficiently propagate gradients in deep networks.

Example of Backpropagation

- Consider a simple neural network with one hidden layer.
- Forward pass:

$$z_1 = w_1 x + b_1$$

$$a_1 = \sigma(z_1)$$

$$z_2 = w_2 a_1 + b_2$$

$$\hat{y} = \sigma(z_2)$$

- Loss function:

$$L = \frac{1}{2}(\hat{y} - y)^2$$

Backpropagation Steps

- Compute gradients:

$$\frac{\partial L}{\partial \hat{y}} = \hat{y} - y$$

$$\frac{\partial \hat{y}}{\partial z_2} = \sigma'(z_2)$$

$$\frac{\partial z_2}{\partial w_2} = a_1$$

- Chain rule:

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_2}$$

- Update weights:

$$w_2 := w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Regularization Techniques

- **L2 Regularization:** Adds penalty for large weights.

$$L = \text{MSE} + \lambda \sum w^2$$

- **Dropout:** Randomly drops neurons during training.
- **Early Stopping:** Stops training when validation loss increases.

What is a CNN?

- CNNs are specialized for image recognition and processing.
- Use **convolutional layers** to extract important features.
- Pooling layers reduce dimensionality while retaining information.

Convolutional Layers

- Apply filters (kernels) to detect features like edges.
- Example: Using a **3x3 kernel** for edge detection.
- Convolution operation:

$$(I * K)(i, j) = \sum_m \sum_n I(i + m, j + n) K(m, n)$$

- Where I is the input image and K is the kernel.

Pooling Layers

- Reduce the size of feature maps while preserving important features.
- **Max pooling** selects the highest value in a region.
- **Average pooling** computes the average value in a region.

Fully Connected Layer

- After convolution and pooling, a fully connected layer is used for classification.
- Example: Final layer in **image classification** predicts categories.

CNN Architecture

- Input layer: Receives image data.
- Convolutional layers: Extract features.
- Pooling layers: Reduce dimensionality.
- Fully connected layers: Perform classification.
- Example: LeNet-5 for digit recognition.

Training a CNN

- Initialize filters and weights.
- Forward pass: Compute feature maps and predictions.
- Compute loss: Measure prediction error.
- Backward pass: Compute gradients.
- Update weights: Apply gradient descent.
- Repeat until convergence.

CNN

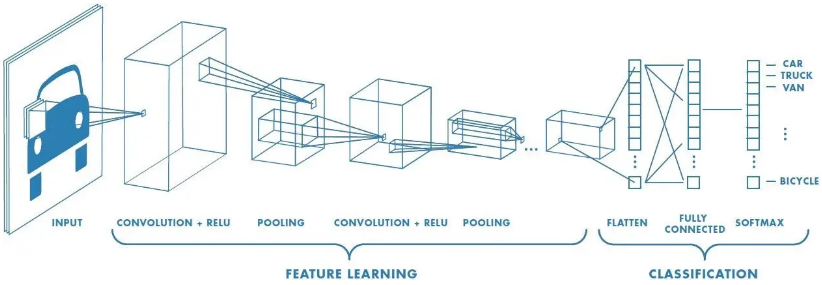


Figure: Convolutional Neural Network

Data Augmentation

- Technique to increase the diversity of training data.
- Examples: Rotation, scaling, flipping, and cropping.
- Helps prevent overfitting and improves generalization.

Transfer Learning

- Use pre-trained models on large datasets.
- Fine-tune the model on a specific task.
- Example: Using VGG16 for image classification.

Recurrent Neural Networks (RNNs)

- Designed for sequential data.
- Maintain hidden states to capture temporal dependencies.
- Applications: Time series prediction, language modeling.

Long Short-Term Memory (LSTM)

- A type of RNN designed to handle long-term dependencies.
- Uses gates to control the flow of information.
- Applications: Speech recognition, text generation.

Generative Adversarial Networks (GANs)

- Consist of a generator and a discriminator.
- Generator creates fake data, discriminator distinguishes real from fake.
- Applications: Image generation, data augmentation.

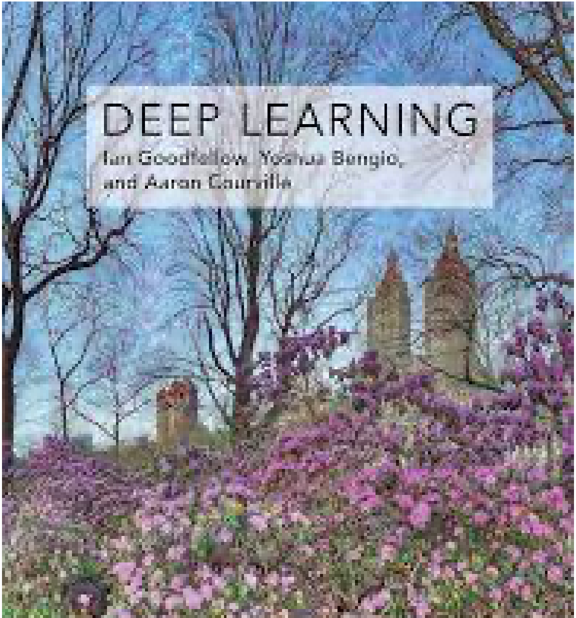
Transformers

- Use self-attention mechanisms to process sequences in parallel.
- Revolutionized NLP tasks.
- Applications: Machine translation, text summarization.

Large Language Models (LLMs)

- Built on transformer architecture.
- Trained on vast amounts of text data.
- Applications: Chatbots, content generation, question answering.

Resources


DEEP LEARNING
Ian Goodfellow, Yoshua Bengio,
and Aaron Courville

Summary

- **Linear Regression** is a fundamental prediction method.
- **Gradient Descent** optimizes model parameters.
- **Autograd** automates differentiation for neural networks.
- **MLPs** introduce complexity beyond linear models.
- **CNNs** excel in image-related tasks.
- **RNNs, LSTMs, GANs, Transformers, and LLMs** represent current advancements in deep learning.