Function	**Name**
1	Constant
logn	Logarithmic
n	Linear
nlogn	nlogn
n^2	Quadratic
n^3	Cubic
2^n	Exponential
n!	factorial

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2) 2^n $\boxed{\Omega}$ n^5.

Ans. $\log 2^n = \log n^5$

$\Rightarrow n \cancel{\log 2} = \cancel{5} \log n$ Cancel constant term

$\Rightarrow n \geq \log(n) \Rightarrow n = \Omega(\log n)$

2) $n^{\log n}$ $\boxed{O}$ 2^n

Ans. $\log(n^{\log n})$ $\log(2^n)$

$\Rightarrow \log n \log n$ $n \cancel{\log 2}$.

$\Rightarrow (\log n)^2 \leq n. \Rightarrow (\log n)^2 = O(g(n))$

3) n^3 $\boxed{O}$ n^4

$\log(n^3) = \log(n^4)$

$\cancel{3} \log n \cancel{=} \cancel{4} \log n. \Rightarrow O.$

4) $\log n$ $\boxed{\Omega}$ $\log \log n$

let $\log n = k \Rightarrow k \geq \log k.$

5) n^t $\boxed{\Omega}$ $(\log n)^k$

$t > 0$ $k \geq 0.$

we have already proved that $n^k \geq (\log n)^k.$

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* $n^{\log n}$ $\boxed{O}$ $(\log n)^n$

$\Rightarrow \log(n^{\log n})$ $\log((\log n)^n)$

$\Rightarrow (\log n)^2 = n \log \log n.$

it is itself $\geq \log n.$

$(\log n)^2 \leq n$

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$3n^{\sqrt{n}} \ \boxed{\Theta} \ 2^{\sqrt{n}\log_2 n}$.

Ans. $\cancel{\log}(3n^{\sqrt{n}}) = \cancel{\log}(2^{\sqrt{n}\log_2 n})$

$\boxed{2^{\log_2 x} = x.}$

$\cancel{\sqrt{n}} \Rightarrow 2^{\log_2(n^{\sqrt{n}})}$

$3n^{\sqrt{n}} \Rightarrow \cancel{O}\ n^{\sqrt{n}}$.

$\boxed{\Theta}$

* $n! \ \boxed{\Omega} \ 3n^{\sqrt{n}}$

$\Rightarrow \log(n!) = \log(3n^{\sqrt{n}})$ $\boxed{\log n! = n\log n.}$

$\Rightarrow n\log n = \sqrt{n}\log n.$

$>$

* $2^n \ \boxed{\Omega} \ n^{\log n}$.

$\Rightarrow \log(2^n) = \log(n^{\log n})$

$\Rightarrow n\cancel{\log 2} = (\log n)^2$. Cancel constant term.

$\Rightarrow n > (\log n)^2$.

* $2^n \ \boxed{\Omega} \ n!$

$n! > c^n \rightarrow$

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↑ order arrange – 2^n $n^{3/2}$ $n^{\log_2 n}$ $n\log_2 n$

2^n $n^{\log_2 n}$ n $\log n$ $(\log n)^2$ $n\log n$.

$\log(2^n) = \log(n^{\log n})$

$\log n \rightarrow (\log n)^2 \rightarrow n \rightarrow n\log n$

$\log n < (\log n)^2 < n < n\log n$

$n \log 2 = (\log n)^2$

$n > (\log n)^2$.

$2^n = \Omega\ n^{\log n}$

$n^{3/2} = n\log n$.

$\cancel{\log(n^{3/2})}$ $\cancel{\log(n}$

$n^{1.5} = n\log n$.

$\Rightarrow$ $n^{0.5} = \log n$.

$n^{0.5} > \log n$

$\Rightarrow$ $n^{3/2} = \Omega(n\log n)$

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Q- which is false ?

(1) $100n \log n = O\left(\frac{n \log n}{100}\right)$ ✓

(2) $\sqrt{\log n} = O(\log \log n)$ ✗

(3) $n^x = O(n^y)$ if $0 < x < y$. ✓

(4) $2^n \neq O(n^k)$. ✗

Ans. (1) we know that — $f(n) \le c \cdot g(n)$ for $O(g(n))$.

so $100\, n \log n = 10000 \times \left(\frac{n \log n}{100}\right)$
$\downarrow$
c.

It is True.

(2) $(\log n)^{1/2} = (\log \log n)$

$\Rightarrow \frac{1}{2} \log \log n = \log(\log \log n)$

$\log \log n \ge \log(\text{—"—})$ ~~so~~ ~~false~~.

So false.

(c) True.

(d) for $k=2$, $2n = 2n$ ✓
$= O(2n)$

So false.

Q. $\sum i^3 = x$.

choices for x -

$\Theta(n^4)$, $\cancel{\theta(n^5)}$, $O(n^5)$, $\Omega(n^3)$

Ans. $\sum i^3 = 1^3 + 2^3 + 3^3 + \cdots n^3$

$$= \frac{n^2(n+1)^2}{4} = O(n^4)$$

$$= \Theta(n^4)$$

$$= O(n^5)$$

$$= \Omega(n^3)$$

$$\neq \Theta(n^5)$$