Function	Name
1	Constant
logn	Logarithmic
n	Linear
nlogn	nlogn
n^2	Quadratic
n^3	Cubic
2^n	Exponential
n!	factorial

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(2) $2^n \quad \boxed{\Omega} \quad n^5$

Ans. $\log 2^n = \log n^5$

$\Rightarrow n \cancel{\log 2} = \cancel{5} \log n$ Cancel constant term

$\Rightarrow n \geq \log(n) \Rightarrow n = \Omega(\log n)$.

(2) $n^{\log n} \quad \boxed{O} \quad 2^n$

Ans. $\log(n^{\log n}) \qquad \log(2^n)$

$\Rightarrow \log n \log n \qquad n \cancel{\log 2}$.

$\Rightarrow (\log n)^2 \leq n. \Rightarrow (\log n)^2 = O(g(n))$

(3) $n^3 \quad \boxed{O} \quad n^4$

$\log(n^3) = \log(n^4)$

$\cancel{3} \log n \neq \cancel{4} \log n. \Rightarrow O.$

(4) $\log n \quad \boxed{\Omega} \quad \log \log n$

let $\log n = k \Rightarrow k \geq \log k.$

(5) $n^t \quad \boxed{\Omega} \quad (\log n)^k$

$t>0 \qquad k \geq 0.$

we have already Proved that $n^k \geq (\log n)^{\cancel{k}}.$

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* $n^{\log n}$ $\boxed{O}$ $(\log n)^n$

$\Rightarrow \log(n^{\log n})$ $\log((\log n)^n)$

$\Rightarrow (\log n)^2 = n \log \log n.$

it is itself $\geq \log n.$

$(\log n)^2 \leq \underline{\underline{n}}$

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$3n^{\sqrt{n}}$ $\boxed{\Theta}$ $2^{\sqrt{n}\log_2 n}$.

Ans.

$\log(3n^{\sqrt{n}}) = \log(2^{\sqrt{n}\log_2 n})$

$\boxed{2^{\log_2 x} = x.}$

$\Rightarrow 2^{\log_2(n^{\sqrt{n}})}$

$3n^{\sqrt{n}} \Rightarrow n^{\sqrt{n}}$.

$\boxed{\Theta}$

* $n!$ $\boxed{\Omega}$ $3n^{\sqrt{n}}$

$\Rightarrow \log(n!) = \log(3n^{\sqrt{n}})$

$\boxed{\log n! = n\log n.}$

$\Rightarrow n\log n = \sqrt{n}\log n.$

$>$

* 2^n $\boxed{\Omega}$ $n^{\log n}$.

$\Rightarrow \log(2^n) = \log(n^{\log n})$

$\Rightarrow n\cancel{\log 2} = (\log n)^2$. Cancel constant term.

$\Rightarrow n > (\log n)^2$.

* 2^n $\boxed{\Omega}$ $n!$

$n! > c^n \Rightarrow$

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$\uparrow$ order arrange $-$ 2^n $\quad n^{3/2}$ $\quad n^{\log_2 n}$ $\quad n\log_2 n$

$2^n \quad n^{\log_2 n} \quad n \quad \log n \quad (\log n)^2 \quad n\log n.$

$\log(2^n) = \log(n^{\log n})$

$n\cancel{\log 2} = (\log n)^2$ $\qquad \log n < (\log n)^2 < n < n\log n$

$n > (\log n)^2.$

$2^n = \Omega\, n^{\log n}$

$n^{3/2} = n\log n.$

$\cancel{\log(n^{3/2})} \quad \cancel{\log(n}$

$n^{1.5} = n\log n.$

$\Rightarrow n^{0.5} = \log n.$

$n^{0.5} \geq \log n$

$\Rightarrow n^{3/2} = \Omega(n\log n)$

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Q. which is false ?

1. $100n \log n = O\left(\frac{n \log n}{100}\right)$ ✓

2. $\sqrt{\log n} = O(\log \log n)$ ✗

3. $n^x = O(n^y)$ if $0 < x < y$. ✓

4. $2^n \neq O(n^k)$. ✗

Ans. (1) we know that — $f(n) \le c \cdot g(n)$ for $O(g(n))$.

So $100\, n\log n = 10000 \times \left(\frac{n \log n}{100}\right)$

$\downarrow$ c.

It is True.

(2) $(\log n)^{1/2} = (\log \log n)$

$\Rightarrow \frac{1}{2} \log \log n = \log(\log \log n)$

$\log \log n \ge \log(\text{—"—})$ ~~so~~ ~~false~~.

So false.

(c) True.

(d) for $k=2$, $2n = 2n$ ✓

$= O(2n)$

So false.

Q. $\sum i^3 = x$.

choices for x -

$\Theta(n^4)$, $\Theta(n^5)$, $O(n^5)$, $\Omega(n^3)$

Ans. $\sum i^3 = 1^3 + 2^3 + 3^3 + \dots n^3$

$$= \frac{n^2(n+1)^2}{4} = O(n^4)$$

$$= \Theta(n^4)$$

$$= O(n^5)$$

$$= \Omega(n^3)$$

$$\neq \Theta(n^5)$$