

• Compiler Design

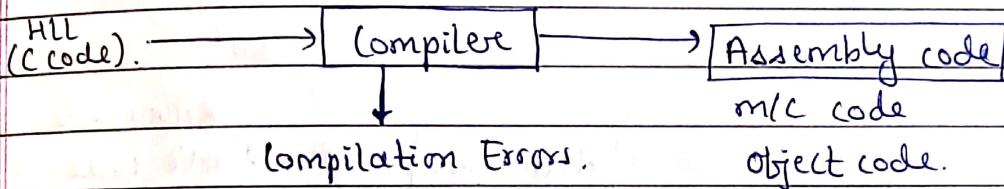
Weightage

(5-8 marks)

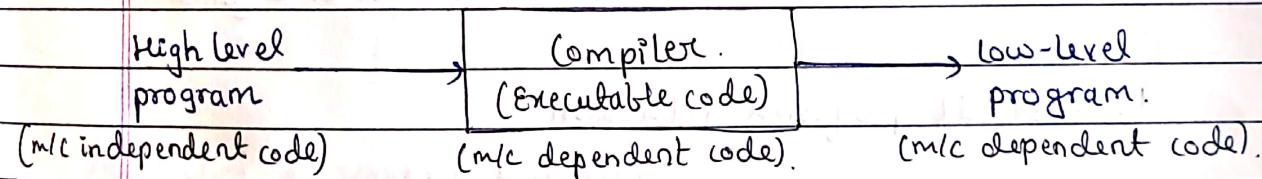
textbook: Dogon edition.

1. Introduction.
2. Lexical Analysis.
3. Syntax Analysis. (3-4 mks).
4. Syntax Directed Translations (SDTs).
5. Intermediate Code & Code optimization.

→ Compiler: - Translator.



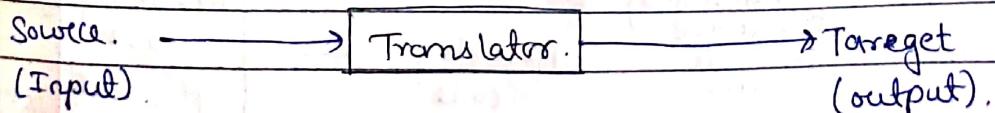
- It is translator.
- It is program.
- It is executable.



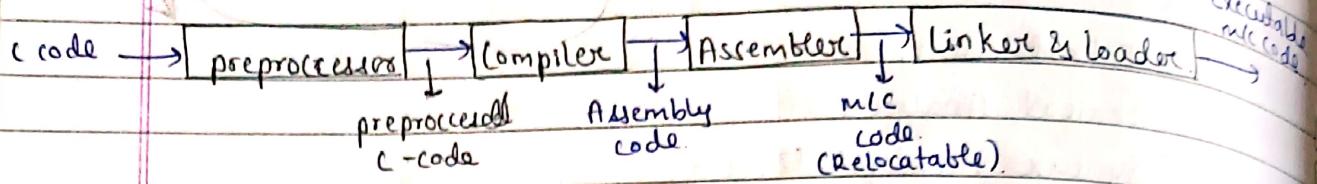
Portable.
m/c Independent.
ex- HLL, Intermediate
codes.

Not-Portable.
m/c dependent.
ex- Compiler, Assembly code,
m/c code.

• Translator



C language Translation -



Preprocessor -

#include<...>
define MAX 10
int a=MAX;
→ preprocessor → int a=10;

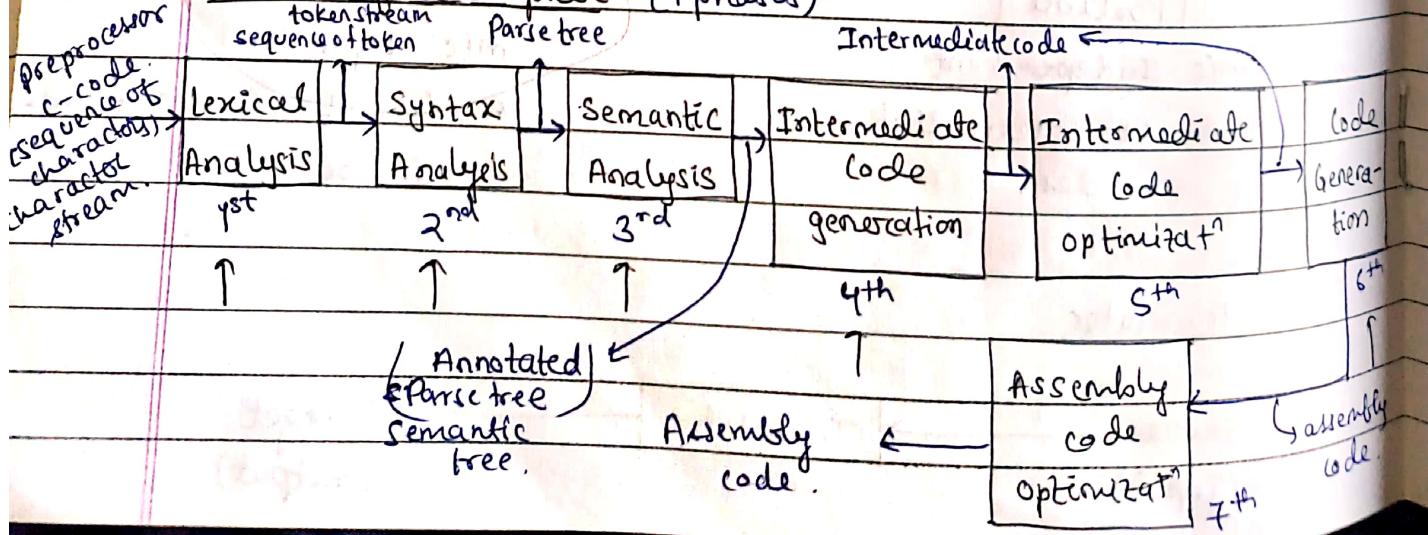
Assembler -

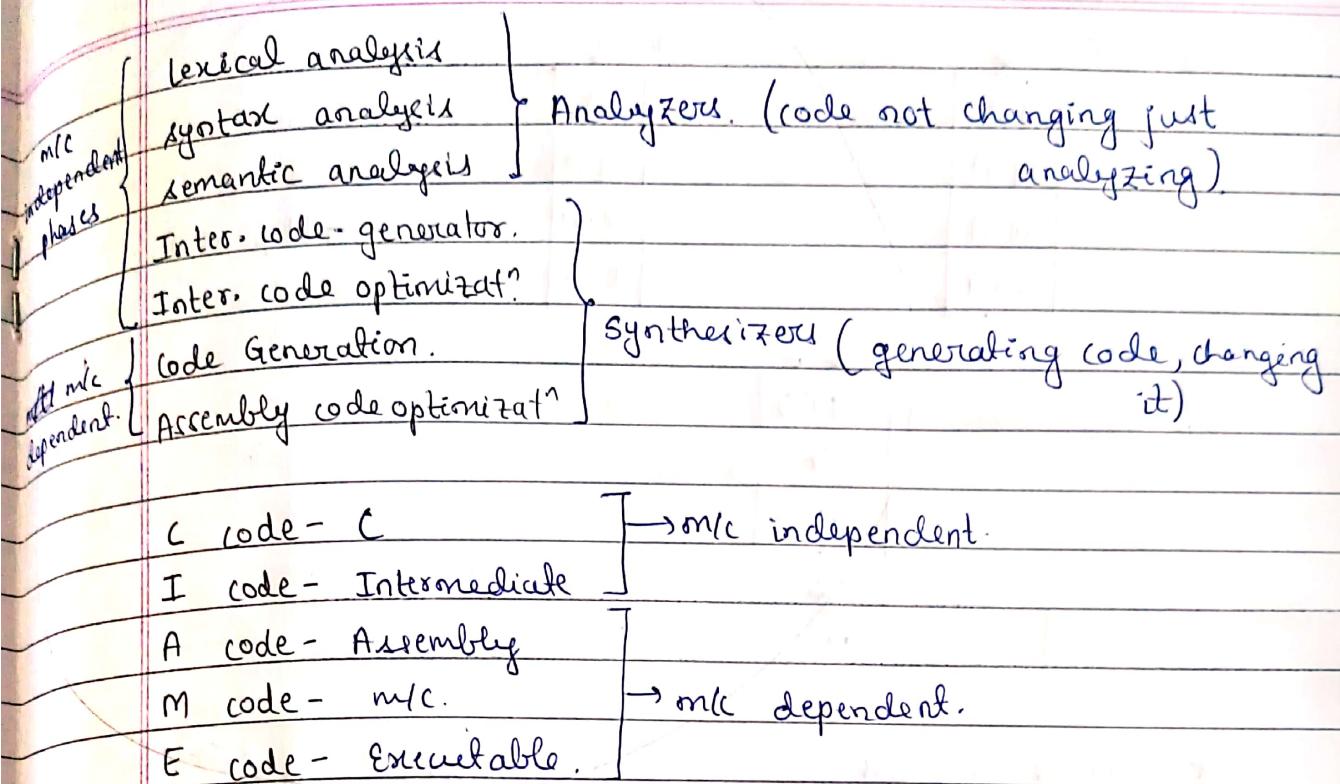
x=10;
Assembly code. → Assembly → m/c code.
MOV A, 10

Linker - To resolve all external references
(all external files)
(all external variables & functions)

Loader - It performs "relocation"
→ Altering address of code/Data

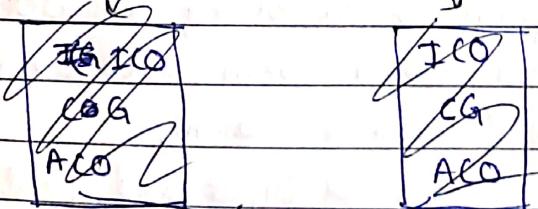
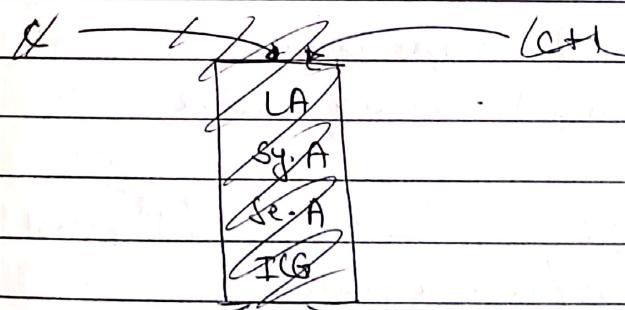
* Phases of a Compiler - (7 phases)





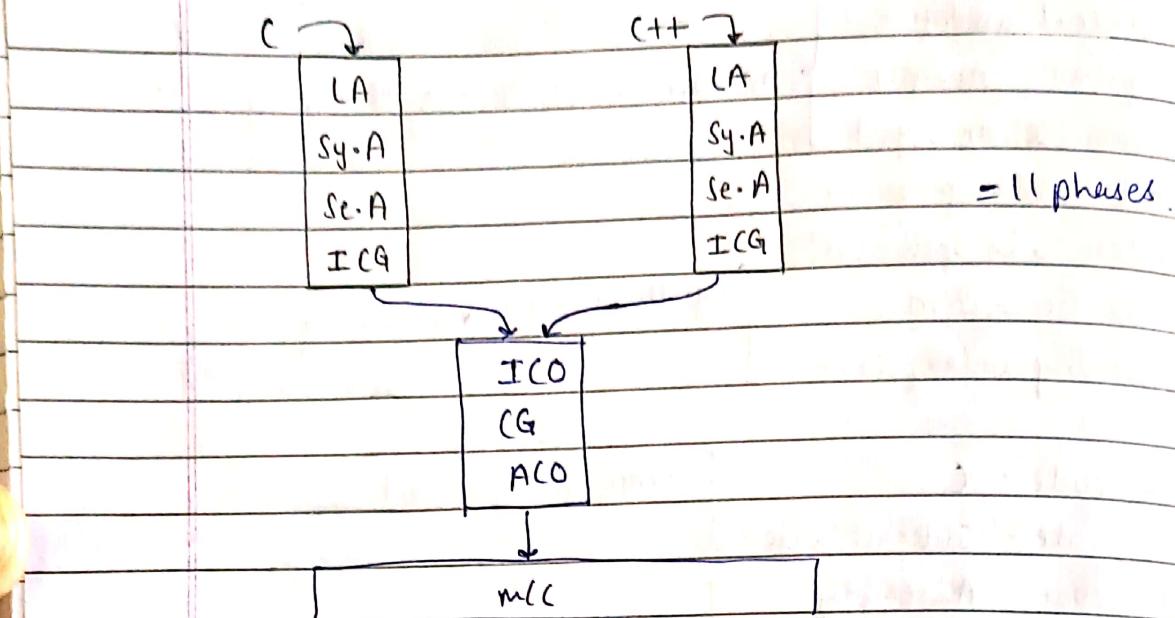
- ALL 7 phases can interact with 1 data structure i.e. Symbol Table.
- Symbol table is used in all the phases, throughout the process of compilation.

cost of one compiler = 7 (i.e. 7 phases).

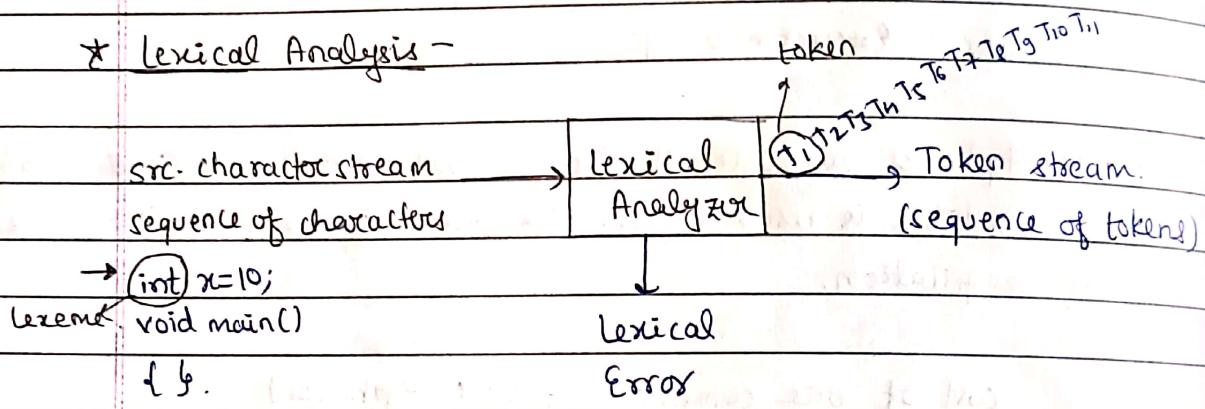


- "file no
need to have
main()
for
compilation"
- Execution begins from `begin main()`.
 - Compilation begins from 1st character of file & only ends at the last character of file.

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* Lexical Analysis -



- Compilation (static) - Before runtime
- Execution (Dynamic). - During runtime.

- Lexical Analysis / Scanner / Linear phase / 1st phase / Token Recognizer / token generator / lexical phase.
 - It scans whole program character by character.
 - It groups characters into a "Token" with the help of "longest prefix Rule" (maximal munch).
 - It takes character stream as input and produces token stream as output.
 - It may produce lexical errors if any.
 - It recognizes tokens, whitespaces, and comments & also newline character
 - It produces only tokens.



- * It may take help of parser to separate tokens.
- * It also uses/creates symbol table.

- symbol table -

```

void fun() {
    int x = 10;
    char y;
}
  
```

literal.

Literal table
10

lexeme	token	line	type	
fun	id ₁	1		→ symbol table.
x	id ₂	2	int	
y	id ₃	3	char.	

- It is a data structure
- It stores attribute information of each identifier.

- (logic)
- 1) Lexeme : It's smallest unit of program. int x;
 - 2) Token : It represents lexemes.
 - 3) Types of token.
 - 4) finding tokens.
 - 5) finding lexical errors.
- Internal representation of lexeme.
- | | | |
|--------|-----|----------------|
| Lexeme | int | T ₁ |
| | x | T ₂ |
| | ; | T ₃ |

- Types of Tokens - (KISCO)
- 1). Identifiers
- 2). Keywords.
- 3). Operators
- 4). literals/ constants.
- 5). Special.

- 1) Identifier - Name of variable/ function.
 - i). Contains ^{only} letters, digits, underscore.
 - ii). Should not start with digit.
 - iii). should not be a keyword.
- ex. x, x123, Int, int₂ ✓
 1x123X, int X

→ 32 keywords [Reserved words]

3) Keywords : int, float, char, double, short, long, void, auto, static, register, extern, signed, unsigned, const, volatile, struct, union, enum, typedef, if, else, switch, case, default, break, while, do, for, continue, return, goto, sizeof.

- 3) Operators :
- Arithmetic → +, -, *, /, %
 - Relational → <, ≤, >, ≥
 - Logical → (or, ||, !)
 - Bitwise → (&, |, ~, ^, <<, >>)
 - Unary → (+, -, ++, --, &, *, (type), !, ~, sizeof)
 - Assignment → (=, +=, -=, *=, /=, <<=, >>=)
 - Special operators → (., , , →)

4) Literals/ constants :

Integer: \rightarrow Decimal: 239, Octal: 0237
 Hexa: 0x2ad3, Octal: 0 to 7

Character: 'a' '\n' 'A'

Real: (float/double): 23.64f, 23.64, 10e-2.

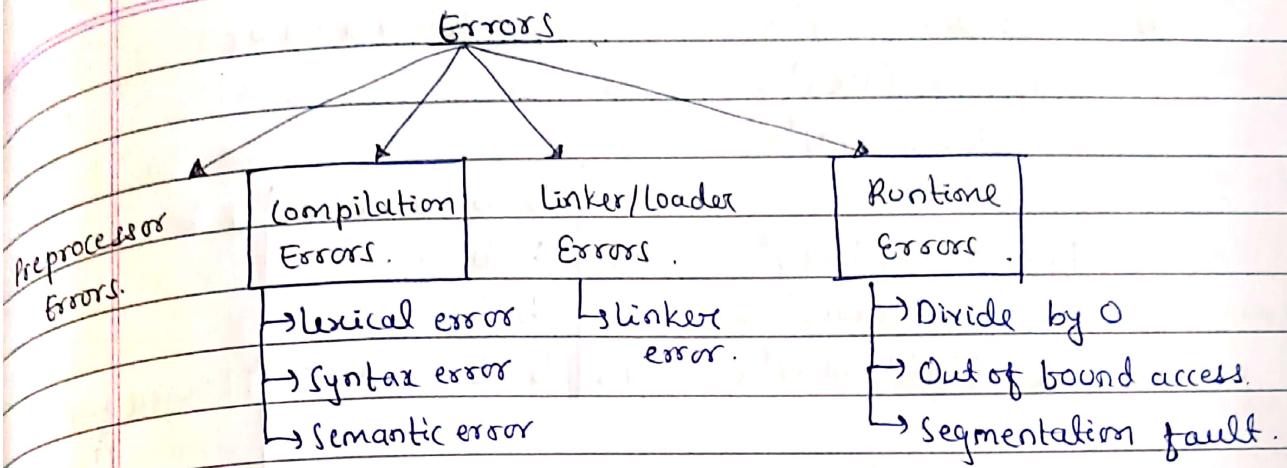
String: "gate"

5) Special tokens : [,], (,), {, }, ;

NOTE:- Whitespace & comments are not tokens.

* find the tokens -

- (1) int x=10; (5) {No error}
- (2) int x,y; (5) {No error}
- (3) int x=y=z; (7) {Semantic error, coz y, z not declared}
- (4) int float,char; (5) {Syntax error → compilation error}
- (5) x="gate"; (4) {Semantic error → x not declared}
- (6) int x="gate"; (5) {Semantic error} coz assigning addr. to int, sizeof addr. can be greater than size of integer.



(7) `int *p;` (4) No error

(8) `int *p=1000;` (6) {semantic error → p expecting address, u are giving integer}

(9) `int *p=(int*)1000;` (10) {No error}

(10) void main()
`printf("x=%od", x);` (13) {semantic error → x not declared}.

(11) void x=a+++++=*b; `x=a+++++=*b;` (11)

(12) `int **p;` (5)

(13) `x+j;` (4) (3).

(14) `int 12xy;` → lexical error {no meaning of 12x-y}

(15) `int x=02g;` → 0-octal, 2-octal, 9-not octal, so lexical error.

(16) `int x=0XA23;` A23 is not hexa digit of X, so lexical error.

(17) `in t x;` (4) tokens

(18) `intx;` (2) tokens

(19) `in /* comment */ t x;` in t x; (4) tokens.

(20) `in /* comment t x;` lexical error → comment not ended.

(21) `int /* comment */ /* t x;` } int a; (3) tokens → No error.

(22) `int /* xyz */ /a;`

(23) `x==+=<<=|===y;` (9) tokens

GATE-2018

(23)

$$T_1: a^? (b|c)^* a$$

$$i/p = bbaacabc.$$

$$T_2: b^? (a|c)^* b$$

$$T_3: c^? (b|a)^* c.$$

$$T_1 = (\epsilon + a) (b + c)^* a.$$

T_1

b b a a c | a b c
b b a.

$$T_2 = (\epsilon + b) (a + c)^* b$$

T_2

b b | a a c

$$T_3 = (\epsilon + c) (b + a)^* c.$$

T_3

b b a a c

Longest from

(T_3)

bb a a c a b c
i/p →

T_1 | in LA
 T_2
 T_3

$T_3 T_3$

T_1 | a
 T_2 | a b
 T_3 | a b c

a b c

(T_3)

(24) $T_1 = a^* bc$

$$i/p = aabc bcbbaac$$

$$T_2 = b^* ac$$

T_1 a a b c | b c | b

T_2 a | b | b b a c.

i/p → $T_1 T_1 T_2$

Q Which of the following is used in Lexical Analyzer?

(A) Regular expression

(B) Finite Automata.

(C) Regular Grammar.

(D) None.

(A)(B),(C)

* Syntax Analysis :-

Basics.

Syntax errors.

CFG → Types, elimination of left Recursion.

Left factoring.

first & follow computation (*)

Topics.

Top-down parser : LL(1)

Bottom-up parser → LR parsers

Operator precedence parser

Find syntax errors :-

- (1) int x; No syntax error & No Error.
- (2) int x, float y; syntax error (its expecting identifier after ,).
- (3) x, y; No syntax error but semantic error [x, y not declared]
- (4) x = y; No syntax error but semantic error [not declared]
- (5) IF(); } syntax error, condition/expression missing.
- (6) while();
- (7) for(); } No syntax error, No semantic error, ∞ loop.
- * (8) x y;] No syntax error, 2 semantic error exist.
- (9) if(23); No error.
- (10) typedef int x; } ≡ int y; No error at all.
x y;

Q. 202)

- Q. int main() { (A) Lexical Analysis
 Integer x; (B) Syntax Analysis
 return 0; (C) Semantic Analysis.
 } (D) m/c dependent optimized.

(C)

- (12) void main (→ Syntax Error.

{

}

- (13) for(x,y,z); → Syntax error, for had fixed syntax
 (missed ;)

- *** (14) foo(x,y,z);

- Void main () { → semantic error, fro not defined.

fro(2,3,4);

{

- (15) void main () {

- for(2;3;4); → syntax error No error, runtime-∞ loop.

{

(16) `#include <stdio.h>` Linker error
`void main() {` from Not in `#stdio.h`.
 `foo(2,3,4);`

y

(17) `#include <p2.c>`
`void main() {`
 `foo(2,3,4);`

y. { } p2.c file

`void foo(int x,int y,int z) {`
 `int a;`
 `a=x+y+z;`
`}`

compile later. } 1st compile this. } No errors. } v

* Context Free Grammar:-

Grammar (G) = (V, T, P, S) . \hookrightarrow start symbol.

→ To represent a language.

→ It is set of rules. $P \rightarrow$ set of productions.

$T \rightarrow$ terminals set of terminals

set of characters (lexical analysis).

set of tokens (syntax analysis)

$V \rightarrow$ set of variables (Non-terminal)

$$CFG = (V, T, P, S).$$

$V \rightarrow$ Any

$V \rightarrow (VUT)^*$

• Why CFG?

→ It is most suitable grammar to represent syntax of ^{most} programming languages.

`void main()`

{

`int x;`

y.

Program $\rightarrow R \text{ main()} B$

$R \rightarrow$ void | int

$B \rightarrow \{ \text{ D; } \}$

$D \rightarrow T \text{ Id}$

$T \rightarrow$ int | float.

$V = \{ \text{program, R, B, D} \}$

$T = \{ \text{main, (,), void, int, ;, id, float} \}$

$;, id, float \}$

- Derivations of a string:-

- 1) Left most Derivation (LMD). } linear.
- 2) Right most Derivation (RMD).
- 3) Parse tree. } Non linear.
↳ Derivation tree.

$$S \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

LMD :



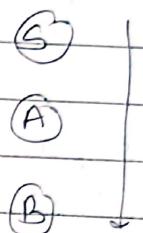
left most non-terminal $(\boxed{A})B$



$a(\boxed{B})$. left most non-terminal.

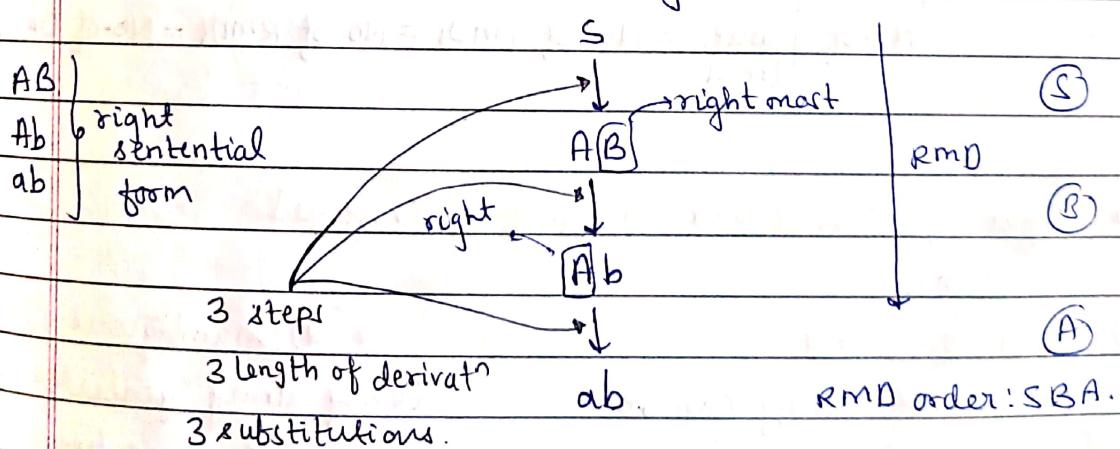


ab.



LMD order: SAB.

- Sentential form: Any sequence of symbols derived from START.
 S, AB, aB, ab } left sentential form.
- LMD: In every sentential form, left most non-terminal is substituted to derive a string.
- RMD: In every sentential form, right most non-terminal is substituted to derive a string.



→ for a string: LMD & RMD need not be same.

- Parse Tree (Derivation Tree) -

$$S \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

leaf nodes - a, b

non leaf nodes - A, B, S.

root - S.

NOTE * Length of derivation in LMD/RMD = no. of substitutions = no. of non-leaf nodes in parse tree = No. of steps.

No. of Parse trees?

$$S \rightarrow A|a$$

$$A \rightarrow a|b$$

$$w=a$$

$$\text{or}$$

$$\text{Length}=2$$

$$S$$

$$S$$

2 Parse trees

2 LMDs

2 RMDs.

Length=1.

* For any string:

No. of parse trees = No. of LMDs = No. of RMDs = No. of Derivations

- Types of CFGs [Based on no. of derivations]

- Ambiguous CFG.

Atleast one string can be derived with more than one parse tree.

- Unambiguous CFG

Every string generated has only 1 parse tree.

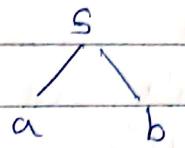
(1 LMD)

(1 RMD).

① $S \rightarrow ab$.

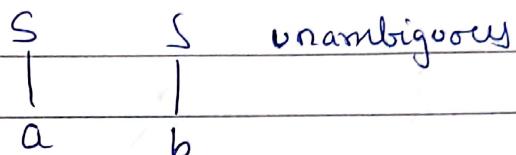
ab : 1 parse tree

Unambiguous (CFG).

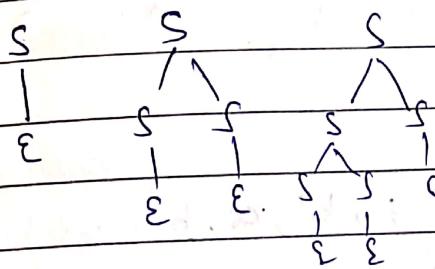


② $S \rightarrow a \mid b$.

$a \rightarrow \text{PT}$
 $b \rightarrow \text{PT}$



③ $S \rightarrow S \mid S \mid \epsilon$

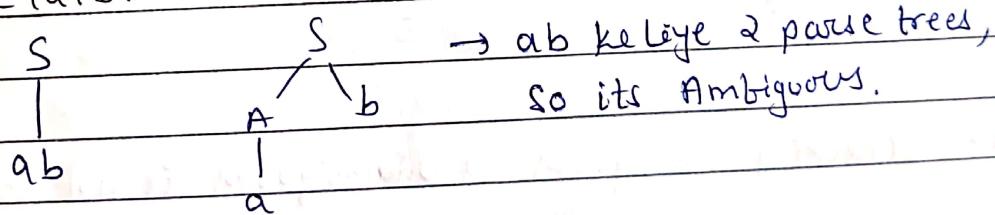


... infinite ways to generate
ε.
∴ Ambiguous.

④ $S \rightarrow A \mid b \mid \epsilon$

ϵ, ab, b, bb .

$A \rightarrow \epsilon \mid a \mid b$.

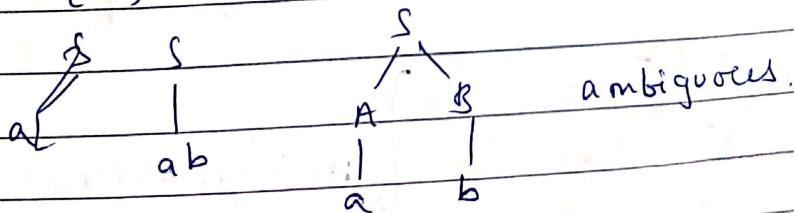


⑤ $S \rightarrow AB \mid ab$

ab, a^+b^+

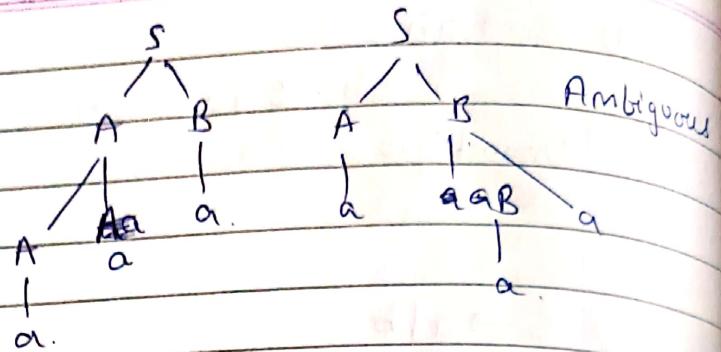
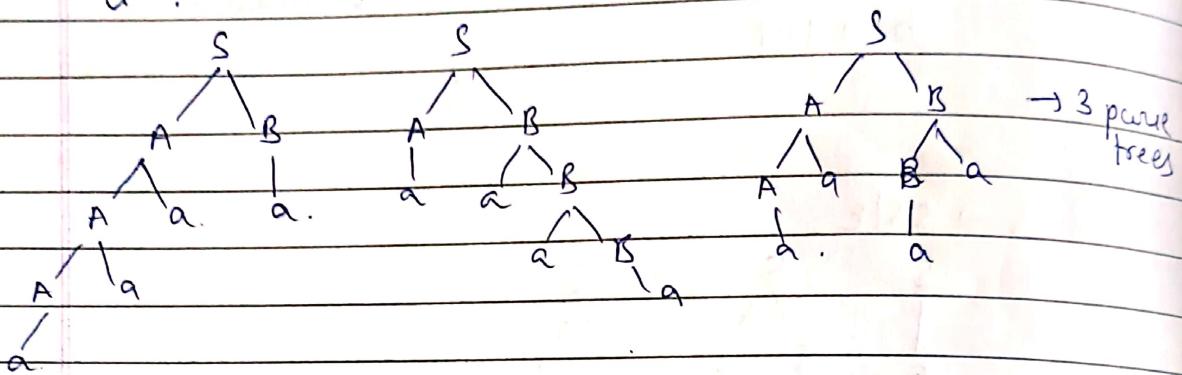
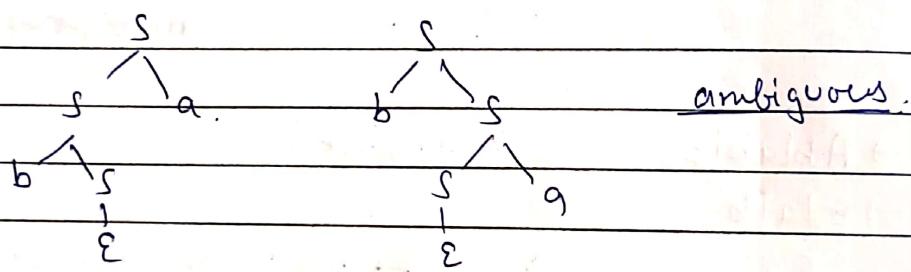
$A \rightarrow A \mid a \quad (a^+)$

$B \rightarrow B \mid b \quad (b^+)$



(6) $S \rightarrow AB$

$$\begin{array}{l} A \rightarrow Aala \quad (a^+) \\ B \rightarrow Ba\bar{a} \quad (\bar{a}^+) \end{array}$$

 $a^n ?$ (7) $S \rightarrow Sa|bS|\epsilon$ $a, b^+, a^+, b\dots$ 

NOTE!- Checking Ambiguous & Unambiguous is not having algorithm.
right recursion.

$S \rightarrow S a b S | \epsilon$
left recursion.

+ If same non terminal is having both
left & Right recursion, then CFG is
definitely AMBIGUOUS.

(8) $\boxed{S} \rightarrow \boxed{S} a \boxed{S} | \epsilon$ (9) $\boxed{S} \rightarrow \boxed{S} \boxed{S} | a$.(10) $S \rightarrow A b$ (11) $\boxed{A} \rightarrow \boxed{A} a | b \boxed{A} | c$

Ambiguous (FG).

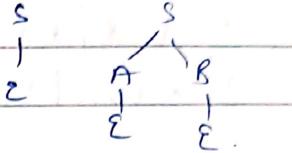
(12) $S \rightarrow (S) \mid \epsilon$ → unambiguous

(13) $S \rightarrow a S b \mid \epsilon$ → unambiguous

(14) $S \rightarrow A B \mid \epsilon$ → ambiguous

$A \rightarrow a A \mid \epsilon$

$B \rightarrow b B \mid \epsilon$



$$\begin{array}{ll}
 \textcircled{15} \quad S \rightarrow A\beta & a^+ b^+ \\
 & \hookrightarrow \text{unambi.} \\
 A \rightarrow aA' \mid \epsilon & \\
 \beta \rightarrow bB' \mid \epsilon &
 \end{array}$$

(16) $E \rightarrow E + E | a$
 (17) $E \rightarrow E + E | E * E | E - E | E / E | a$. ↳ Ambiguous
(by shortcut)

* Elimination of Left Recursion -

- Top down parser not possible if CFG has left Recursion.
- Conversion from left recursion to right recursion.

① $S \rightarrow S \alpha | b$, $L = b \alpha^*$

$S \rightarrow b X \quad | \quad X \rightarrow aX | \epsilon$

} Right recursion.

Left rec. *nonleft
rec*

(2) $s \rightarrow s_a [s_b | c d] s_c$
leftrec. nonleft

$$\begin{array}{l} S \rightarrow cX \mid dX \mid eSX \\ X \rightarrow aX \mid bX \mid \epsilon \end{array} \quad \left. \begin{array}{l} \text{Right recursion.} \end{array} \right\}$$

NOTE :- $A \rightarrow A\alpha_1 | A\alpha_2 | \dots | A\alpha_k | \beta_1 | \beta_2 | \dots | \beta_n$

K left rec. term. or nonleft rec. terms.

$$\begin{array}{l|l} \cancel{A \rightarrow \beta_1 | \beta_2} & A \rightarrow \beta_1 X | \beta_2 X | \dots | \beta_n X \\ & X \rightarrow \alpha_1 X | \alpha_2 X | \dots | \alpha_k X | \varepsilon \end{array}$$

③ $S \rightarrow Sab | cdS | Se f | gh$

$$\boxed{S \rightarrow cdSX | fX | ghX}$$

$$X \rightarrow abX | cX | \epsilon$$

④ ~~$S \rightarrow SABcdefg$~~
~~A $\rightarrow Sbf$~~

④ $S \rightarrow \underline{Sa} | \underline{Abc} | efg$
 direct left rec. indirect left rec.
 $A \rightarrow \underline{Sbf}$
 indirect left rec.

Direct left Rec.



Visible.

Indirect left Rec.



Substitute using Algo (only 1st place in RHS)

Direct left Rec.

	Algo 1	Algo 2	Algo 3
$S \rightarrow$	S	S	A
$A \rightarrow$	A	A	B
$B \rightarrow$	B	C	S
$C \rightarrow$	C	B	C

we follow

this one

1. algorithm.

2n answers.

Algo 1.

S step 1: Solve S using direct left Recursion elimination.

A step 2: Substitute S in A rules then apply Direct left Rec. elimination

B step 3: Substitute both S & A in B rules recursively, then do direct left

C step 4: Substitute S, A, & B in C productions recursively then apply direct left rec. elimination.

⑤ $S \rightarrow Sa | Abc | efg$
 $A \rightarrow Sbf$

Step 1: choose S,
 $S \rightarrow S a | A b c f e g$,
 left rec. Assume non-left rec.

$$S \rightarrow A b c X | e f g X$$

$$X \rightarrow a X | \epsilon$$

Step 2: choose A,
 $A \rightarrow [S] b S | f$.

↓ substitute S in 1st place.

$$A \rightarrow [A] b c X b S | e f g X b S | f$$

↓ Apply direct left rec. elimination?

$$A \rightarrow e f g X b S Y | f Y$$

$$Y \rightarrow b c X b S Y | \epsilon$$

$$X \rightarrow a X | \epsilon$$

$$\text{final ans.} \rightarrow S \rightarrow A b c X | e f g X$$

$$A \rightarrow e f g X b S Y | f Y$$

$$X \rightarrow a X | \epsilon$$

$$Y \rightarrow b c X b S Y | \epsilon$$

$$(5) E \rightarrow E + T | a$$

$$T \rightarrow T * F | b$$

$$F \rightarrow c$$

$$T \rightarrow T * F | b$$

$$T \rightarrow b Y$$

$$Y \rightarrow * F Y | \epsilon$$

$$E \rightarrow E + T | a$$



$$E \rightarrow a X$$

$$X \rightarrow * T Y | \epsilon \quad X \rightarrow + T X | \epsilon$$

final ans :- $E \rightarrow a X$

$$X \rightarrow + T X | \epsilon$$

$$T \rightarrow b Y$$

$$Y \rightarrow * F Y | \epsilon$$

$$F \rightarrow c$$

$$(6) \quad S \rightarrow Ab$$

$$A \rightarrow Sa | Ad | f$$

$$B \rightarrow Sg | Ah | e$$

$$S \rightarrow Ab$$

$$A \rightarrow Aba | Ad | f$$

$$A \rightarrow fX$$

$$X \rightarrow bax | dx | \epsilon$$

$$B \rightarrow Abg | Ah | e$$

$$B \rightarrow fXbg | fxh | e$$

$$\text{find ans. } S \rightarrow Ab$$

$$A \rightarrow fX$$

$$X \rightarrow bax | dx | \epsilon$$

$$B \rightarrow fXbg | fxh | e$$

* Left factoring -

Unambiguous CFG.

↓ left factoring

left factored CFG. [free from common prefixes]

$$(1) \quad S \rightarrow @b | @c$$

common prefixes.



$$S \rightarrow aX$$

Left

$$X \rightarrow b | c$$

factored CFG.

$$(2) \quad S \rightarrow (@) | @b | (@)bc | d$$

common.

$$S \rightarrow aX | d$$

$$X \rightarrow (@) | b | c | \epsilon$$

common.

$$X \rightarrow bY | \epsilon$$

$$Y \rightarrow c | \epsilon$$

$$S \rightarrow aX | d$$

$$X \rightarrow bY | \epsilon$$

$$Y \rightarrow c | \epsilon$$

(3) $S \rightarrow A b | a c A | d$ Indirect common prefix
 $A \rightarrow a b l f .$

$S \rightarrow [A] b | a c A | d .$
 non terminal, substitute.

$S \rightarrow [a] b b | a f b | [a] c A | d .$
 common.

$S \rightarrow a X | f b l d$
 $X \rightarrow b b | c A .$ ① final ans.
 $A \rightarrow a b | f .$

(h) $S \rightarrow S a b | b A$ firstly eliminate left Rec.

$A \rightarrow B c | d e B$

$B \rightarrow f | f l d$

$S \rightarrow b A X$

$X \rightarrow a b X | \epsilon$

$A \rightarrow B c | d e B$

$B \rightarrow f l d .$

$S \rightarrow b A X$

$X \rightarrow a b X | \epsilon$

$A \rightarrow f c | \overset{a}{d} e B$

$A \rightarrow f c | d \cancel{e} Y$

$Y \rightarrow c l e B , B \rightarrow f l d .$

$S \rightarrow b A X$

$X \rightarrow a b X | \epsilon$

$A \rightarrow d e B | f$

$B \rightarrow f l d .$

$A \rightarrow d e B | f$

$B \rightarrow f l d .$

• left factoring Algorithm

unambiguous CFG

step 1: Eliminate left Rec.

Non-left Recursive CFG

substitute non terminals if they appear in 1st place.

CFG which is free from "non-terminal that appears in 1st place"

Eliminate direct common prefixes

Left factored CFG.

* first Set & follow Set Computation -

$$X \rightarrow aYb \mid fYg \mid e \mid \epsilon$$

$\text{First}(X)$ = set of terminals (including ϵ) where every terminal is derived as 1st symbol from X .
 $= \{t \mid x \Rightarrow t \alpha, \text{ where } t \text{ is terminal or } \epsilon\}$

$$\text{FIRST}(X) = \{a, f, e, \epsilon\}$$

$\text{follow}(Y)$ = set of terminals (including ϵ) where every terminal is derived as 1st symbol after Y .

$$\text{follow}(Y) = \{b, g\}$$

FIRST(A)

Look at all

A production.

Follow(A)

Look at whole CFG

where RHS has A.

$$A \rightarrow$$

NOTE:- $\text{FIRST}(X)$ may contain ϵ , $\text{follow}(X)$ never contain ϵ .

① $S \rightarrow a \mid \epsilon \mid b \mid c \mid d$

$$\text{FIRST}(S) = \{a, \epsilon, b, c, d\}$$

② $S \rightarrow A \mid b \mid c$

$$A \rightarrow d$$

$$\text{FIRST}(S) = \{c, d\}$$

$$\text{FIRST}(A) = d$$

③ $S \rightarrow A \mid b \mid c$

$$A \rightarrow \epsilon$$

$$\text{FIRST}(S) = \{c, a\}$$

$$\text{FIRST}(A) = \{\epsilon\}$$

④ $S \rightarrow A$

$$A \rightarrow \epsilon$$

$$\text{FIRST}(S) = \{\epsilon\}$$

$$\text{FIRST}(A) = \{\epsilon\}$$

(5) $S \rightarrow AB$ $\text{FIRST}(S) = \{a\}$
 $A \rightarrow \epsilon$ $\text{FIRST}(A) = \{\epsilon\}$
 $B \rightarrow a$ $\text{FIRST}(B) = \{a\}$

(6) $S \rightarrow AB$ $\text{FIRST}(S) = \{a, c, \epsilon\}$
 $A \rightarrow ab\epsilon$ $\text{FIRST}(A) = \{a, \epsilon\}$
 $B \rightarrow cd|\epsilon$ $\text{FIRST}(B) = \{c, \epsilon\}$

• FOLLOW set :-

(1) $S \rightarrow a$ $\text{FOLLOW}(S) = \{\$\}$.

* If S is start symbol,
then $\$ \in \text{FOLLOW}(S)$

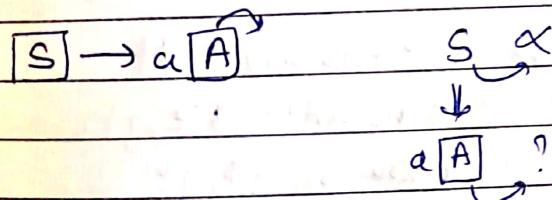
$\$$ \rightarrow special end terminal.

(2) $S \rightarrow Sa|sb|c$ $\text{FOLLOW}(S) = \{a, b, \$\}$.

(3) $S \rightarrow Sac|Sbd|e$ $\text{FOLLOW}(S) = \{a, b, \$\}$

(4) $S \rightarrow aAb|bSd|c$ $\text{FOLLOW}(S) = \{\$, d, g\}$

$A \rightarrow Ae|Sg|f$ $\text{FOLLOW}(A) = \{e, g, d, \$\} = \{e\} \cup \text{FOLLOW}(S)$



NOTE!- $\text{FOLLOW}(A) =$ we should compute $\text{FOLLOW}(S)$

If $x \rightarrow a Y$ then $\text{FOLLOW}(Y) = \underline{\text{FOLLOW}(x)}$

(5) $S \rightarrow AB$ $\text{FOLLOW}(S) = \{\$\}$
 $A \rightarrow a$ $\text{FOLLOW}(A) = \{b\}$
 $B \rightarrow b$ $\text{FOLLOW}(B) = \{\$\}$

(6) $S \rightarrow AB$ $\text{Follow}(S) = \{\$\}$
 $A \rightarrow a$ $\text{Follow}(A) = \{\$\} = \text{Follow}(S)$
 $B \rightarrow \epsilon$ $\text{Follow}(B) = \{\$\} = \text{Follow}(S)$.

(7) $S \rightarrow AB$ $\text{Follow}(S) = \{\$\}$
 $A \rightarrow a$ $\text{Follow}(A) = \{\$, b\}$
 $B \rightarrow bc|\epsilon$ $\text{Follow}(B) = \{\$\}$

* Compute first & follow -

(1) $S \rightarrow SS|Cs|a$ $\text{FIRST}(S) = \{a, C, \$\}$, $\text{Follow}(S) = \{\$,)\}$
 $\text{Follow}(S) = \{\$, (, a,)\}$

(2) $E \rightarrow E+E|E*E|a$ $\text{FIRST}(E) = \{a\}$
 $\text{Follow}(E) = \{+, *\}, \$\}$

(3) $E \rightarrow E+T|a$ $\text{FIRST}(E) = \{a\}$ $\text{Follow}(E) = \{+, *,)\}$
 $T \rightarrow f*T|b$ $\text{FIRST}(T) = \{b, C, c\}$ $\text{Follow}(T) = \{\$, +,)\}$
 $F \rightarrow (E)|c$ $\text{FIRST}(F) = \{C, c\}$ $\text{Follow}(F) = \{\$, *\}$

(4) $S \rightarrow ABC$ $\text{FIRST}(S) = \{\epsilon, a, e, g\}$ $\text{Follow}(S) = \{\$\}$
 $A \rightarrow ab|\epsilon$ $\text{FIRST}(A) = \{\epsilon, a\}$ $\text{Follow}(A) = \{\$, e, g\}$
 $B \rightarrow ef|\epsilon$ $\text{FIRST}(B) = \{\epsilon, e\}$ $\text{Follow}(B) = \{\$, g\}$
 $C \rightarrow gh|\epsilon$ $\text{FIRST}(C) = \{\epsilon, g\}$ $\text{Follow}(C) = \{\$\}$

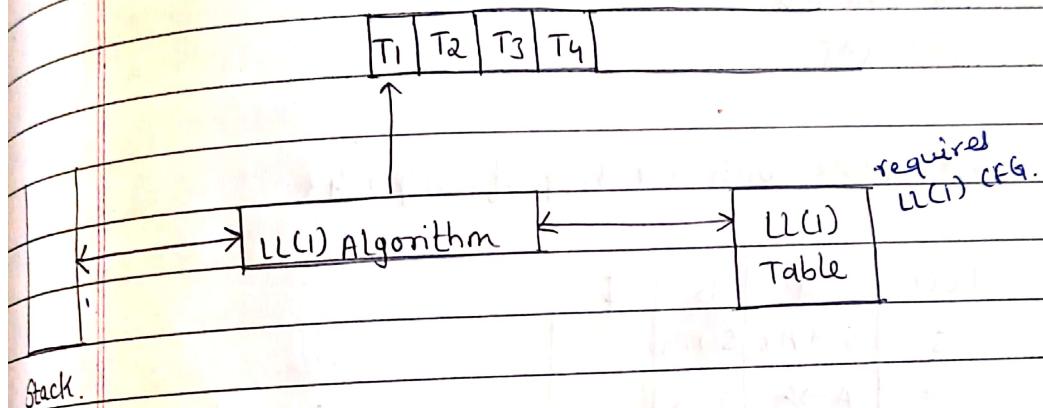
* LL(1) Parser :-

[] → one token prediction.
 [] → Left most derivation (LMD)
 [] → Left to Right scan.

- Top-down parser.
- Predictive parser.
- Recursive Descent (RD)

→ Recursive Descent OR Non-recursive Descent
 writing procedures
 for each production.
 ex:- $S \rightarrow A$ $S()$
 $A \rightarrow a$ a
 $ch = 'a'$ $S \mid S \rightarrow a$
 \downarrow

→ LL(1) parser configuration-



• LL(1) CFG -

→ How to write LL(1) CFG?

Step 1: Take unambiguous CFG.

↓ Eliminate left recursion.

Step 2: Unambiguous & Non Left Recursion CFG.

↓ Apply left factoring.

Step 3: Unambiguous, Non Left Recursion, & Left factored CFG.

↓ using first & follow sets, construct LL(1) Table.

Step 4: If no multiple productions in the same entry of table
 then CFG is LL(1).

- How to construct LL(1) Table?

(1) $S \rightarrow Aa$

$A \rightarrow b|\epsilon$

step 1: Compute FIRST set for every non-terminal.

$$\text{FIRST}(S) = \{a, b\}$$

$$\text{FIRST}(A) = \{\epsilon, b\}$$

Because of this ϵ , we require follow(A).

step 2: $\text{follow}(A) = \{\$, a\}$.

If any FIRST set contain ϵ then compute follow set only for that non-terminal.

$$\text{follow}(A) = \{a\}$$

step 3: construct table with the help of step 1 & 2

LL(1)	a	b	\$	
S	$S \rightarrow Aa$	$S \rightarrow Aa$		
A	$A \rightarrow \epsilon$	$A \rightarrow b$		$ N \times T + 1$

a

S fill this entry with an production of S, where they derive 'a' as 1st symbol.

ex:- $S \rightarrow a$

$S \rightarrow ab$

} for $S \rightarrow @|@b|@ab$

S we will have only S productions

A we will have only A productions

B we will have only B productions

*when we look
of follow set.*

S	t
	fill null productions of S.

ex :- $S \rightarrow AB, S \rightarrow \epsilon$.

$$S \rightarrow AB|\epsilon$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

* Shortcut

$$S \rightarrow Aa$$

$$A \rightarrow b|\epsilon$$

$$\{b\} \cap f_0(A) \neq \emptyset$$

$$\{b\} \cap \{\epsilon\} = \emptyset$$

$\rightarrow LL(1) \checkmark$

$$i) X \rightarrow \alpha_1 | \alpha_2$$

If $F_i(\alpha_1) \cap F_i(\alpha_2) \neq \emptyset$ then not LL(1).

$$ii) X \rightarrow \alpha | \epsilon$$

If $F_i(\alpha) \cap F_0(X) \neq \emptyset$ then not LL(1).

(2) $S \rightarrow aSb|\epsilon$

$$f_1(aSb) \cap f_0(\epsilon) \neq \emptyset$$

$$a \cap \{b, \$\} = \emptyset \rightarrow LL(1) \checkmark$$

$first(S) = \{a, (\epsilon)\}$

$follow(S) = \{b, \$\}$

a	b	$\$$
$S \rightarrow aSb$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

(3) $S \rightarrow Sa|b$.

* Every LL(1) CFG is -

I) Unambiguous.

II) Non Left Recursive.

III) Left factored.

→ Given CFG is having

left recursion, so not

LL(1) \checkmark .

* If CFG is unambiguous, Non left recursive, left factored then CFG need not be LL(1).

(4) $S \rightarrow aSa\epsilon$

$$f_i(aS\alpha) \cap f_0(S)$$

$$a \cap \{a, \epsilon\} \neq \emptyset$$

$a \neq \emptyset$ \Rightarrow Not LL(1), even if its unambi, Non left rec
Left factored

$$f_i(S) = \{a, \epsilon\}$$

$$f_0(S) = \{a, \$\}$$

	a	\$
S	$S \rightarrow aSa$	$S \rightarrow \epsilon$
	$S \rightarrow \epsilon$	

- Identify CFG is LL(1) or not.

(1) $S \rightarrow a$. LL(1).

(2) $S \rightarrow SS|\epsilon$. \neq LL(1)

(3) $S \rightarrow a|ab$. \neq LL(1) (coz common prefix)

(4) $S \rightarrow S|ab|\epsilon$ \neq LL(1)

(5) $S \rightarrow Ab|Ba$ $f_i(Ab) \cap f_i(Ba)$

$$A \rightarrow ab \quad a \cap a \neq \emptyset \rightarrow \neq \text{LL}(1)$$

$$B \rightarrow ac$$

(6) $S \rightarrow Aa|Bb|ac|cd|\epsilon$ $f_i(Aa) \cap f_i(Bb)$

$$A \rightarrow d$$

$$d \cap a = \emptyset$$

$$B \rightarrow ab$$

$$f_i(Aa) \cap f_i(acd)$$

$$f_i(Bb) \cap f_i(acd)$$

$$d \cap a \neq \emptyset$$

$$a \cap a \neq \emptyset$$

$$\hookrightarrow \neq \text{LL}(1)$$

LL(1) parsing Algorithm -

$\zeta \rightarrow \overline{AB}$

$$f_1(s) = t \neq b$$

$$A \rightarrow a A' \epsilon$$

$$f_1(s) = \{ \varnothing, b, a \}$$

$\beta \rightarrow b$

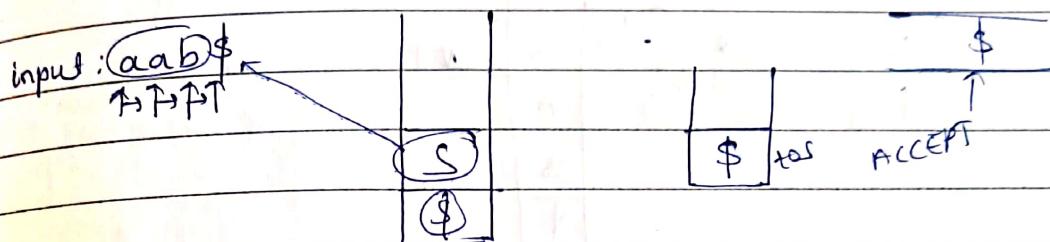
$$f_1(A) = \{\varepsilon, a\}$$

$$f_1(\{b\}) = \{b\}.$$

$$f_0(A) = \{ \$, b \}$$

		input		
		a	b	\$
S		S → AB	S → AB	
T	A	A → aA	A → ε	
A	C			
K.	B		B → b	

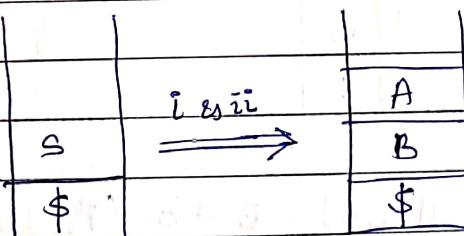
$$f_0(A) = \{ \$, b \}$$



I)		$a \ a \ b \ \$$	$m[s, a] = s \xrightarrow{\leftarrow} AB.$	{ PREDICTION (substitution) }
	s	\uparrow		

i) pop LHS.

ii) PUSH RHS in reverse order.



- $M[x, t] : x \rightarrow \alpha$

i) $\text{POP } X^{\text{ms}}$

ii) Push all in reverse order -
RHS

II).

a a b f

$$m[A, a] : A \rightarrow aA.$$

→ A

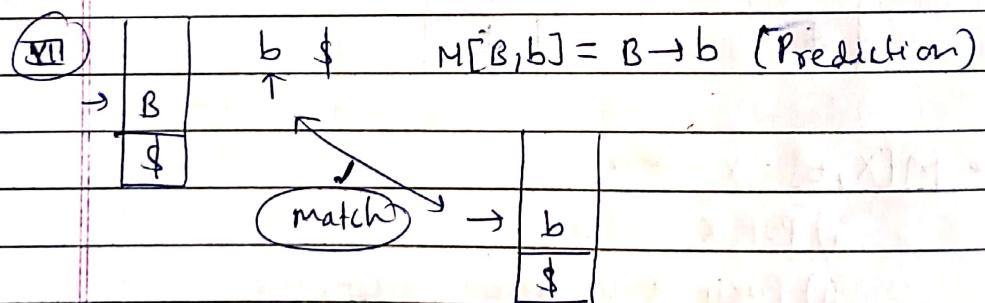
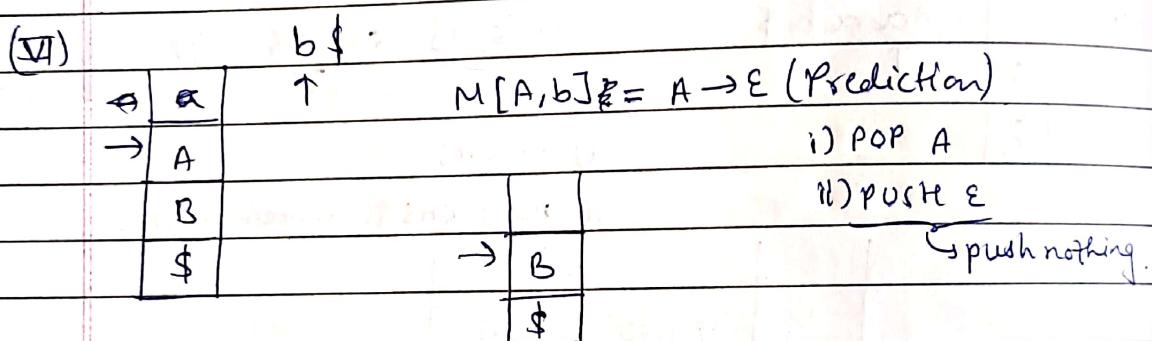
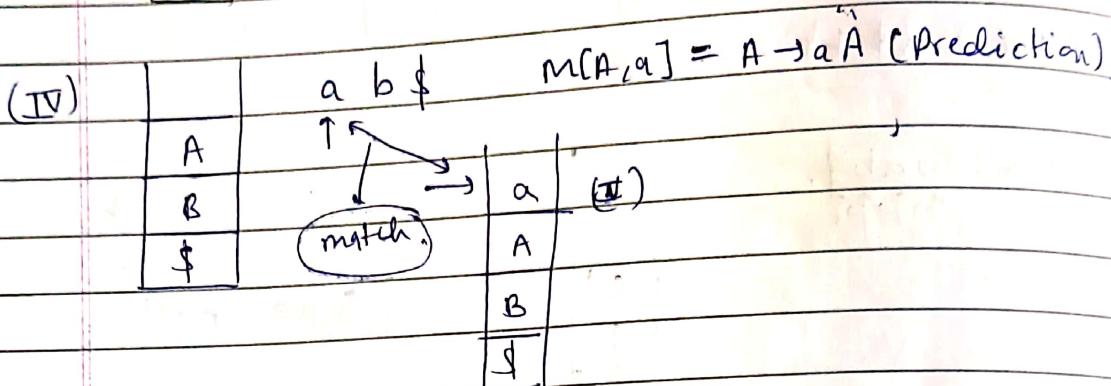
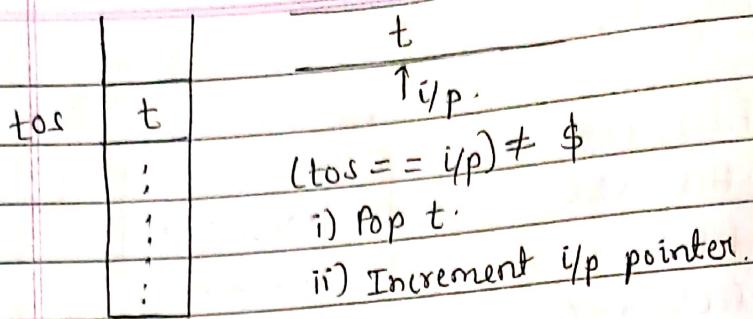
B

\$

match

i) Pop A

ii) push aA in reverse order



(S)

↓

(A)B

↓

a(A)B

↓

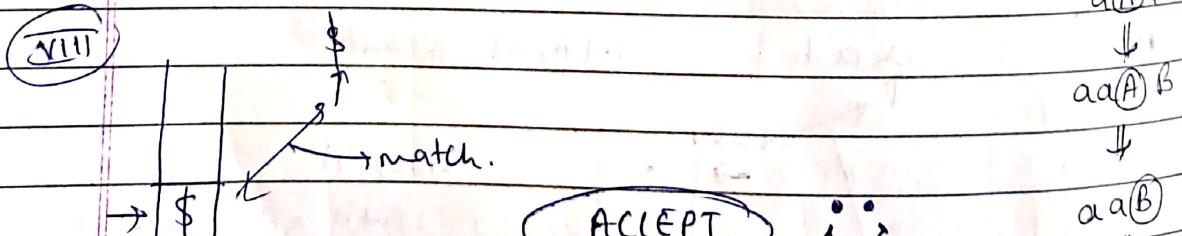
aa(A)B

↓

aa(a)B

↓

aab.



LMD order: S A A A B Y Y Y Y Y Y
 order: AB Y A Y A Y E Y Y

I) T-conflict [Terminal conflict]

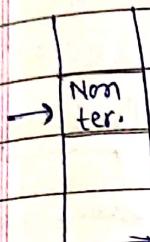
If tos is terminal

and not matching with i/p



T-conflict.

II) N-conflict [Non-terminal conflict]



terminal

If tos is non-terminal,
and there is no production
in table



N-conflict.

ERL		<u>a</u>
	B	↑
:		
:		
		a
	B	Blank.

• LL(1) Parsing Errors -

↳ (I) T-conflict. ($\text{tos} \neq \text{i/p}$)

(II) N-conflict. ($M[\text{tos}, \text{i/p}]$ is blank).

X

X

X

LL(0)

LL(1)

LL(2)

LL(3)

...

1)

$S \rightarrow \#a$

✓

✓

✓

2)

$S \rightarrow aA$

✓

✓

✓

$A \rightarrow b$

3)

$S \rightarrow aA$

✓

✓

✓

$A \rightarrow bB$

$B \rightarrow c$

	LL(0)	LL(1)	LL(2)	LL(3)
1) $S \rightarrow a \mid b$	X	✓	✓	✓
2) $S \rightarrow a \underline{a} \mid \underline{b}$	X	X	✓	✓
3) $S \rightarrow a$	✓	✓	✓	✓
4) $S \rightarrow a \mid b \mid abc \mid \text{left}$	X	X	X	✓

Bottom-up parser - (SR Parser)

Shift Reduce

LR parser.

Operator Precedence Parser

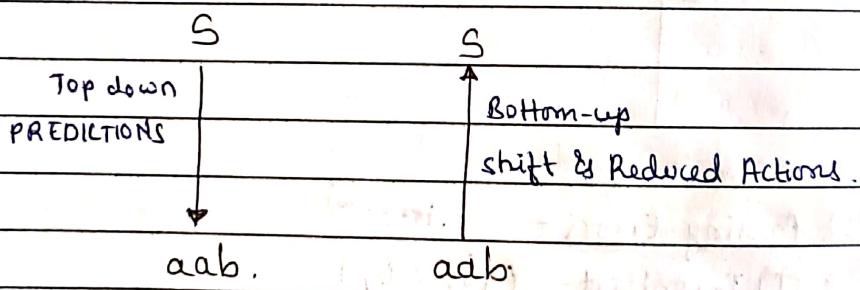
- LR(0) parser.
- Simple → (0LR / SLR(1))
- look ahead → (LALR / LALR(1))
- (canonical) → (0LR / LR(1))

for precedence relation

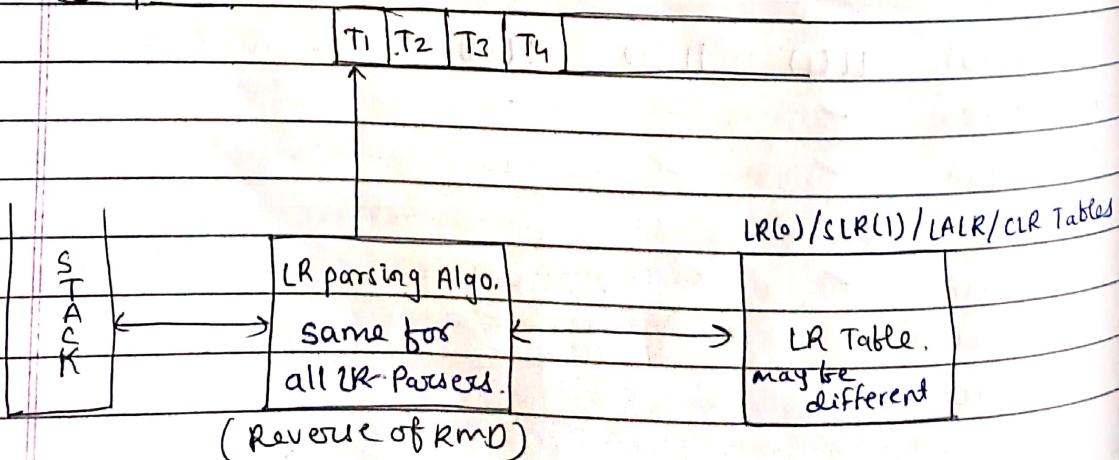
LR

→ RMD in reverse.

→ Left to right scan.



• LR parser



LR(0) Parsing DFA

LR(0) Grammar Identification.

SLR(1) Grammar Identification.

LR(1) Parsing DFA.

LR(1) Grammar Identification.

LALR(1) Parsing DFA.

IALR(1) Grammar Identification.

LR(0) Table \rightarrow SLR Table \rightarrow LALR Table \rightarrow CLR Table.

LR Parsing Algo.

LR(0) items required.

LR(1) items required.

Basics

→ Augmented CFG.

→ Item

→ Types of Items.

→ Closure ()
and goto ()

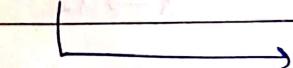
Helps to construct DFA.

1) Augmented CFG -

$$S \rightarrow S_a | S_b | C$$

$$\text{follow}(S) = \{a, b, \$\}$$

(FG)



$$S' \rightarrow S$$

append $S' \rightarrow S$

$$S \rightarrow S_a | S_b | C$$

Augmented CFG.

$$S \rightarrow S_a | S_b | C$$

I/p : cab.

DOT

S'

\uparrow → reduce.

S.

S

\uparrow

S.b

$\uparrow S \rightarrow S_a$

(S.a)b

$\uparrow S \rightarrow C$

(C)a.b

shift
 $S.b \Rightarrow (S.b)$

\uparrow → reduce.

$S.ab \Rightarrow (S.a).b$

\downarrow shift.

C reduced to S

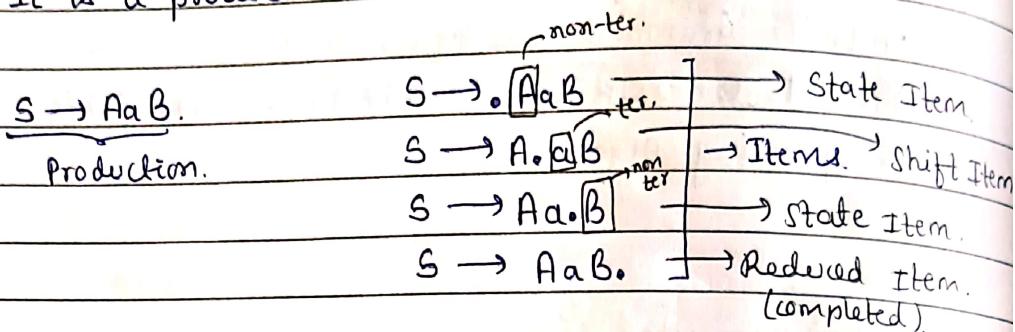
$\cdot cab \Rightarrow (C).ab$

shift
dot.

- Why we need a DOT?
 - To perform shift & Reduced Action.
 - To keep track of program.

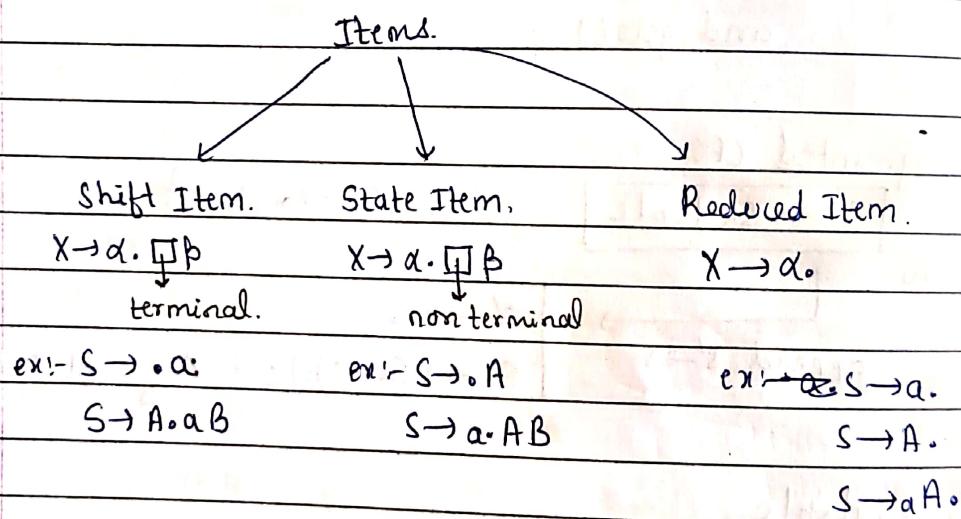
- Item. (3 types)

↳ It is a production which has a dot.



- Why we need a symbol which is immediately after DOT?

i.e. $X \rightarrow a. \boxed{\text{symbol}} B.$

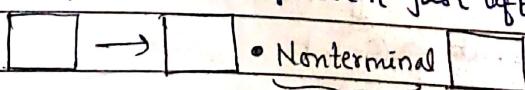


$S \rightarrow ab.Ac \rightarrow \text{State item.}$

* closure(): → It is "set of items" state.

↳ It should be applied on every item in DFA.

II) If Nonterminal present just after dot then.



X in same set
we should add all X productions by placing dot at begin.

• LR(0) DFA:

$$S \rightarrow AaB.$$

$$A \rightarrow b$$

$$B \rightarrow c.$$

I_0

$$S' \rightarrow .S$$

$$S \rightarrow .AaB$$

$$A \rightarrow .b$$

$$\text{closure } (S' \rightarrow .S) = \{ S' \rightarrow .S, \\ S \rightarrow .AaB, A \rightarrow .b \} \\ = I_0$$

S

A

b

goto(I_0, S)

$$S' \rightarrow S.$$

I_1

goto(I_0, A)

$$S \rightarrow A.aB$$

I_2

goto(I_0, b)

$$A \rightarrow b.$$

I_3

* It is LR(0) CFG

so, SLR(1)

$$S \rightarrow Aa.B$$

$$B \rightarrow .c.$$

B

I_4

c

goto(

I_f

$$S \rightarrow AaB.$$

$$B \rightarrow C.$$

I_5

(7 states
in LR(0)
DFA).

$$2) S \rightarrow \epsilon.$$

\rightarrow

$$S' \rightarrow .S.$$

I_0

$$\bullet \epsilon = \epsilon_0 = \epsilon$$

$$S \rightarrow .\epsilon$$

S

$$S' \rightarrow S.$$

$$3) S \rightarrow SS|a.$$

$$S' \rightarrow .S$$

$$S \rightarrow .SS$$

$$S \rightarrow .a$$

S

a

$$S \rightarrow a.$$

SR
reject

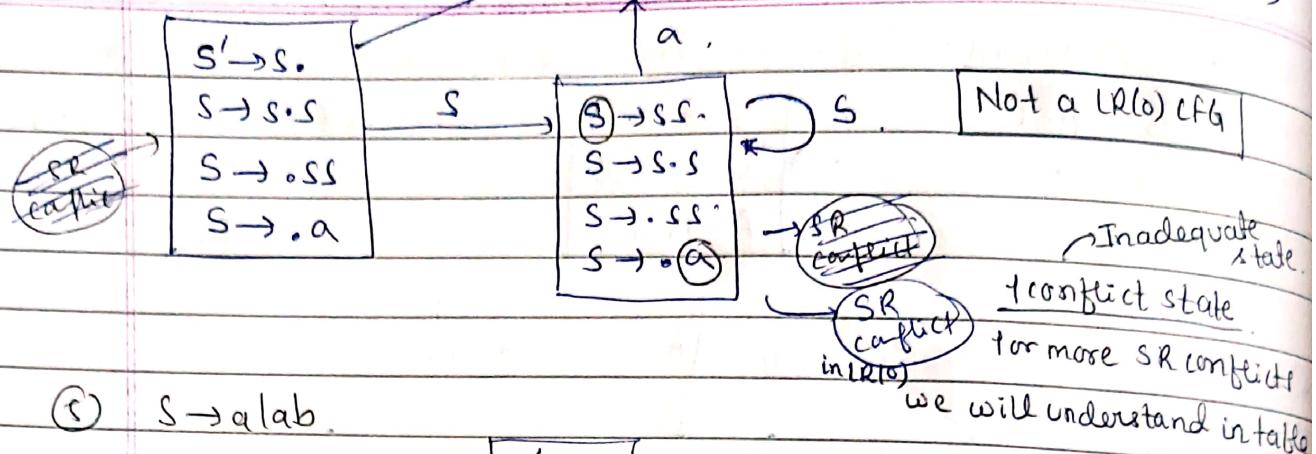
$$S' \rightarrow S.$$

$$S \rightarrow S.S$$

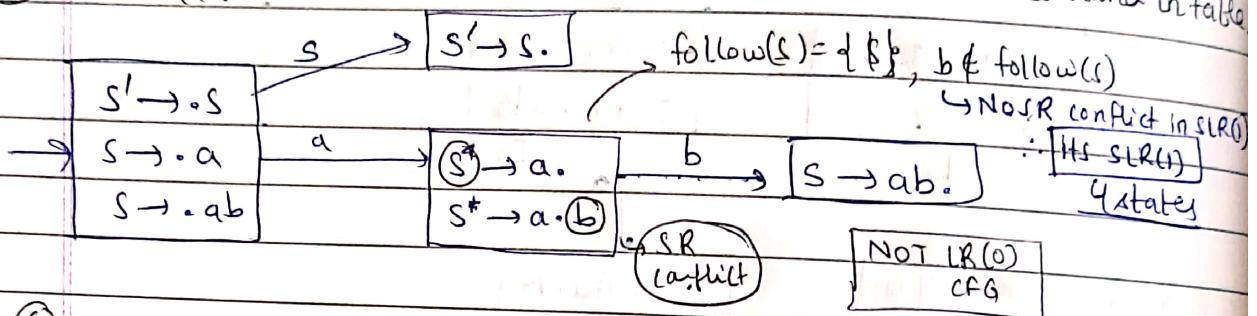
$$S \rightarrow .SS$$

$$S \rightarrow .a$$

$\text{follow}(S) = \{\$, a\}$
 $a \in \text{follow}(S)$
 So its SR conflict in $\text{SLR}(1)$.
 $\therefore \text{It is not } \text{SLR}(1) \text{ CFG.}$
 (same state present on previous page, no need to make it again, just loop it to previous one)



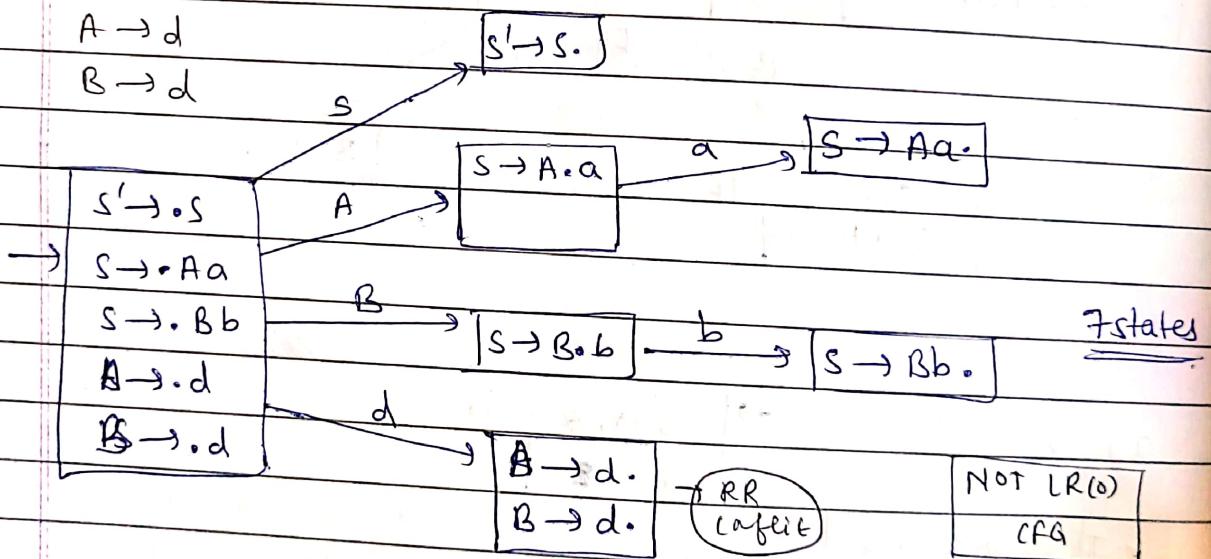
(5) $S \rightarrow a \text{ lab.}$



(6) $S \rightarrow Aa | Bb$

$A \rightarrow d$

$B \rightarrow d$

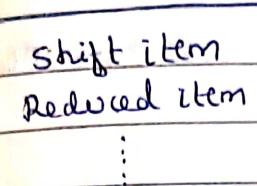


• How to check CFG is LR(0) or not?

\rightarrow construct LR(0) DFA. (step 1)
 \rightarrow If no conflict is present on DFA, (step 2).
 then CFG is LR(0)

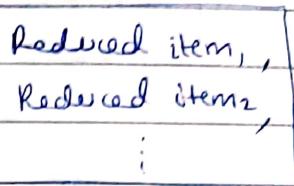
\rightarrow follow(A) = {a}
 \rightarrow follow(B) = {b}
 \rightarrow fol(A) \cap fol(B) = \emptyset
 No RR conflict \rightarrow its SLR

SR conflict in LR(0).



If shift item & reduced item are present in same state then it produces SR conflict.

RR conflict in LR(0)



If 2 reduced items are present in same state then it produces RR conflicts.

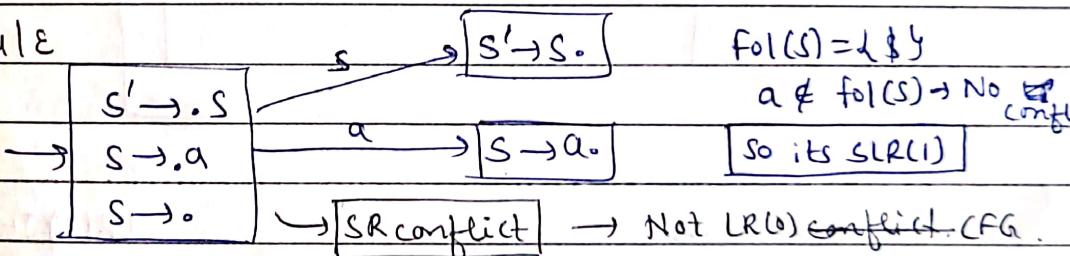
NOTE:- **I** If state has single item then it never produces conflicts.

II State item will not participate in any conflict.

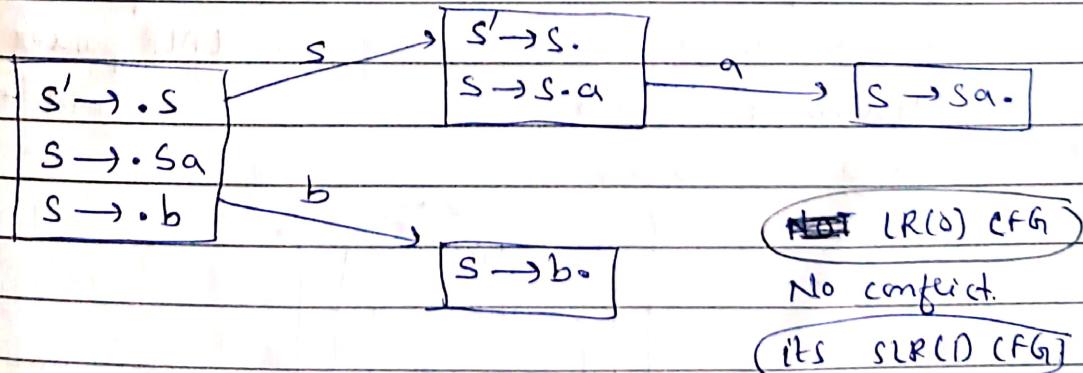
III Acceptance item ($S' \xrightarrow{S \cdot} \$$) not participates in any conflict.

IV If state is not having reduced item then it never produces any conflict.

(7) $S \rightarrow a \mid \epsilon$



(8) $S \rightarrow Sa \mid b$



• How to check CFG is SLR(1) or Not?

→ step 1: construct LR(0) DFA.

→ step 2: If no SLR(1) conflicts in DFA then CFG is SLR.

SR conflict in SLR(1)

shift item $X \rightarrow a \cdot B$

Reduced item $\boxed{Y} \rightarrow a \cdot$

If $t \in \text{Follow}(Y)$

then SR conflict

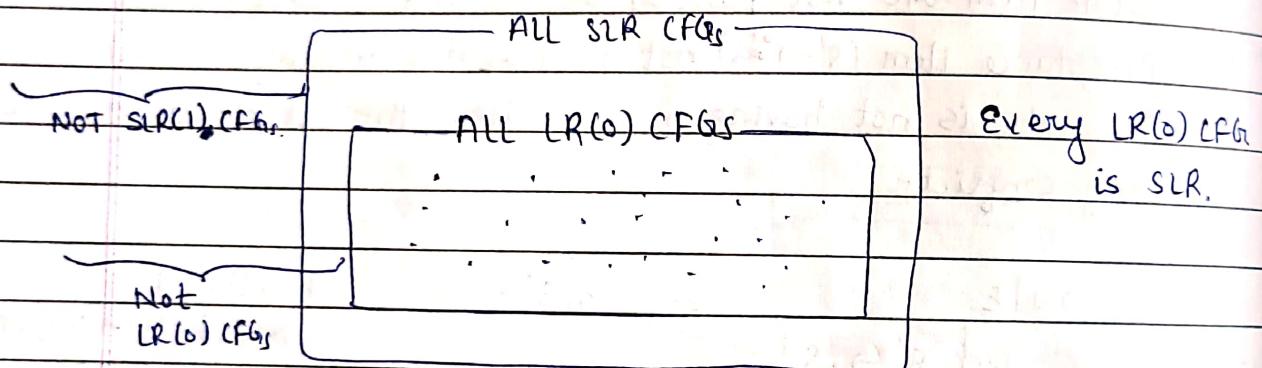
RR conflict in SLR(1)

Red₁ $X \rightarrow a \cdot$

Red₂ $Y \rightarrow x \cdot$

If $\text{follow}(X) \cap \text{follow}(Y) \neq \emptyset$

then RR conflict.



LR(0) Items



LR(0) parser.

SLR(1) parser.

follow set computed

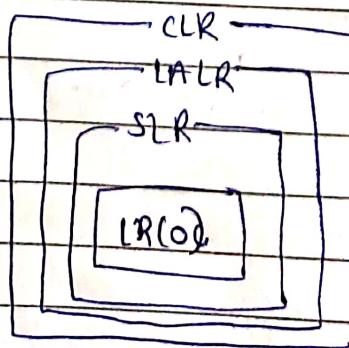
based on whole CFG

LR(1) Items.



CLR parser [LRC(1) parser]

LALR parser



* CLR & LALR -

• How to check given CFG is CLR or not?

→ step 1: construct CLR DFA.

CLR aka. LR(1)

→ step 2: check SR & RR conflicts.

If no conflicts, then CFG is CLR.

• How to construct CLR(1) parsing DFA?

↳ Use LR(1) items.

LR(0) item

$X \rightarrow \alpha \cdot \beta$

LR(1) Item

$X \rightarrow \alpha \cdot \beta, Y^*$

Actual Rule.

↳ Look-a-heads.

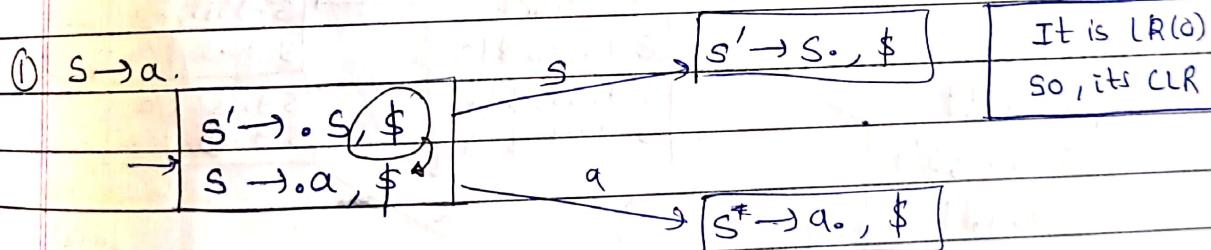
• How to compute look-a-heads in LR(1) item?

~~X → α · β~~ $X \rightarrow \alpha \cdot Y \beta, L_1$ → First(βL_1)

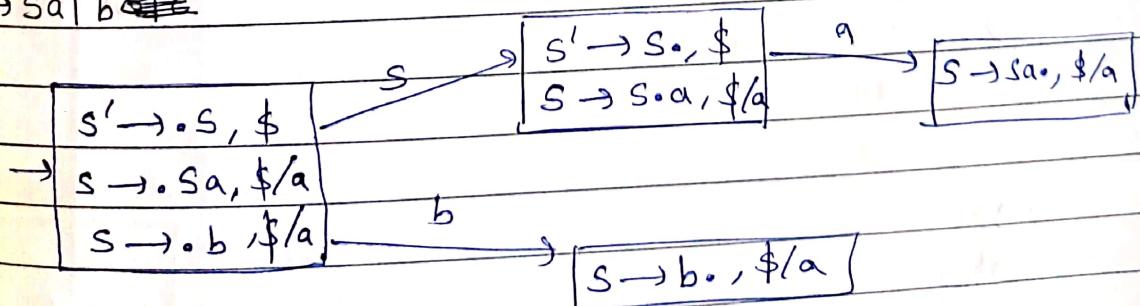
$Y \rightarrow \boxed{\quad}$, look-a-heads of Y .

$Y \rightarrow$

$Y \rightarrow$



② $S \rightarrow Sa | b$



it is LR(0)
so its CLR, LALR,
SLR

LR(0)
Thus SLR, CLR,
LALR

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(3) $S \rightarrow S a | S b | c$.

$S' \rightarrow \cdot S, \$$
 $S \rightarrow \cdot S a, \$ / a / b$
 $S \rightarrow \cdot S b, \$ / a / b$
 $S \rightarrow \cdot c, \$ / a / b$

$S' \rightarrow S \cdot, \$$
 $S \rightarrow S \cdot a, \$ / a / b$
 $S \rightarrow S \cdot b, \$ / a / b$

$a \rightarrow S \rightarrow S a \cdot, \$ / a / b$
 $b \rightarrow S \rightarrow S b \cdot, \$ / a / b$

c

$S \rightarrow c \cdot, \$ / a / b$

(4) $S \rightarrow S a S | b$

$S' \rightarrow \cdot S, \$$
 $\rightarrow S \rightarrow \cdot S a S, \$ / a$
 $S \rightarrow \cdot b, \$ / a$

$S' \rightarrow S \cdot, \$$
 $S \rightarrow S \cdot a S, \$ / a$

$a \rightarrow S \rightarrow \cdot S a S, \$ / a$
 $b \rightarrow S \rightarrow \cdot b, \$ / a$

$S \rightarrow b \cdot, \$ / a$

$S \rightarrow \cdot b, \$ / a$

NOT CLR(1)

LR(0)*, SLR*, LALR*

$S \rightarrow S a \cdot S, \$ / a$

SR conflict
in CLR & LALR

(5) $S \rightarrow S S | a$.

$S' \rightarrow \cdot S, \$$
 $\rightarrow S \rightarrow \cdot S S, \$ / a$
 $S \rightarrow \cdot a, \$ / a$

$S' \rightarrow S \cdot, \$$
 $S \rightarrow S \cdot S, \$ / a$
 $S \rightarrow \cdot S S, \$ / a$
 $S \rightarrow \cdot a, \$ / a$

$S \rightarrow S \cdot, \$ / a$
 $S \rightarrow S \cdot S, \$ / a$
 $S \rightarrow \cdot S S, \$ / a$
 $S \rightarrow \cdot a, \$ / a$

↳ SR conflict
in CLR & LALR

$S \rightarrow a \cdot, \$ / a$

NOT CLR*, LALR*, LR(0)*
SLR*

(6) $S \rightarrow A a | B b | c A b$

$A \rightarrow d$

$B \rightarrow d$

$S' \xrightarrow{A} S a \$$

$S \xrightarrow{\cdot} \cdot A a \$ / d$

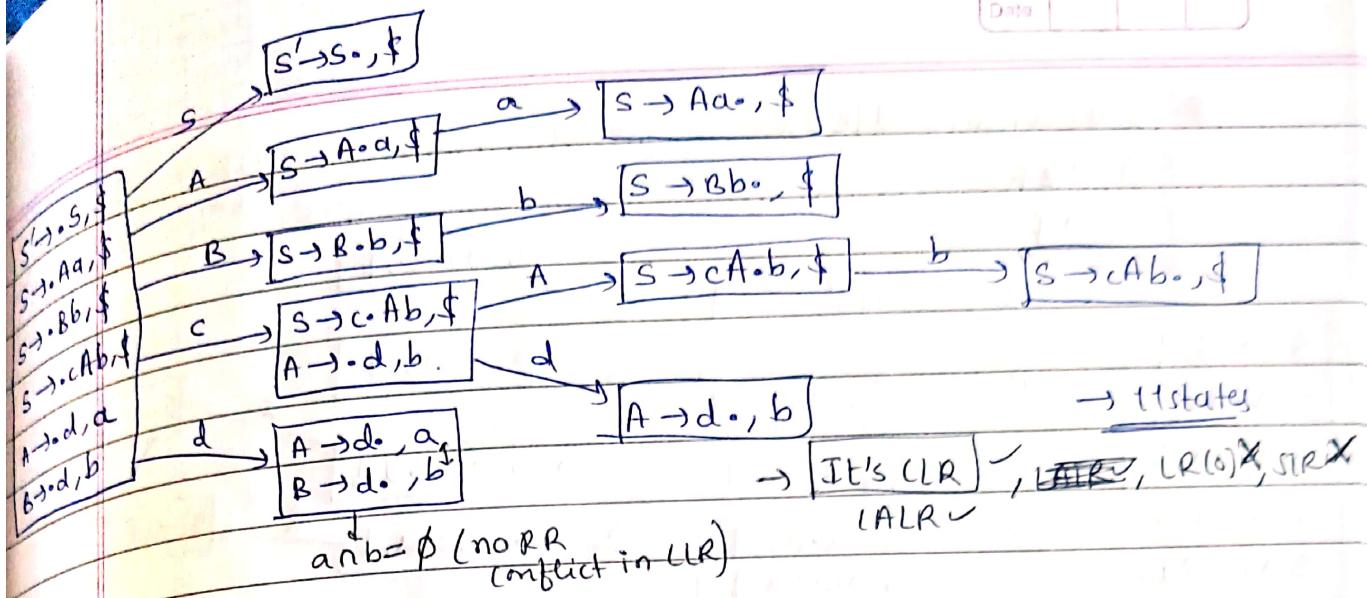
$S \xrightarrow{\cdot} \cdot B b \$ / d$

$S \xrightarrow{\cdot} \cdot c A b \$ / d$

$S \xrightarrow{\cdot} \cdot c \cdot A b \$ / d$

$S \xrightarrow{\cdot} \cdot c \cdot d \$ / d$





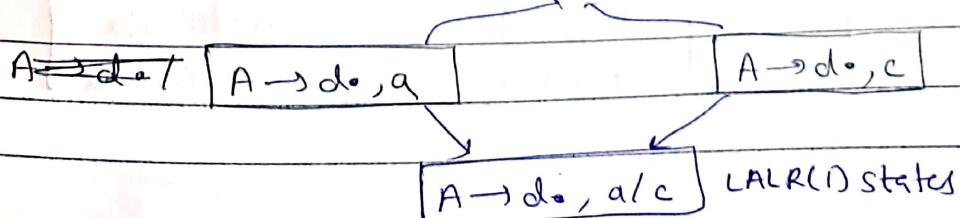
LR(0).		SLR.	CLR & LALR.
conflicts.			
$X \rightarrow \alpha \cdot t \beta$ SR conflict	$\gamma \rightarrow \alpha \cdot$;	IF $t \in \text{follow}(\gamma)$ then SR conflict	$X \rightarrow \alpha \cdot (\underbrace{t \beta}_{L_1})$ $\gamma \rightarrow \alpha \cdot, L_2$ If $t \in L_2$, then SR conflict.
$X \rightarrow \alpha \cdot$ RR conflict	$\gamma \rightarrow \alpha \cdot$;	If $f_0(X) \cap f_0(\gamma)$ is not empty then RR conflict	$X \rightarrow \alpha \cdot, L_1$ $\gamma \rightarrow \alpha \cdot, L_2$ If $L_1 \cap L_2 \neq \emptyset$ then RR conflict

• How to check given (FG) is LALR(1) or not?

Step 1: → construct CLR(1) DFA.

Step 2: → we can construct LALR(1) DFA by merging states

if they have same items where look-a-heads may be different.

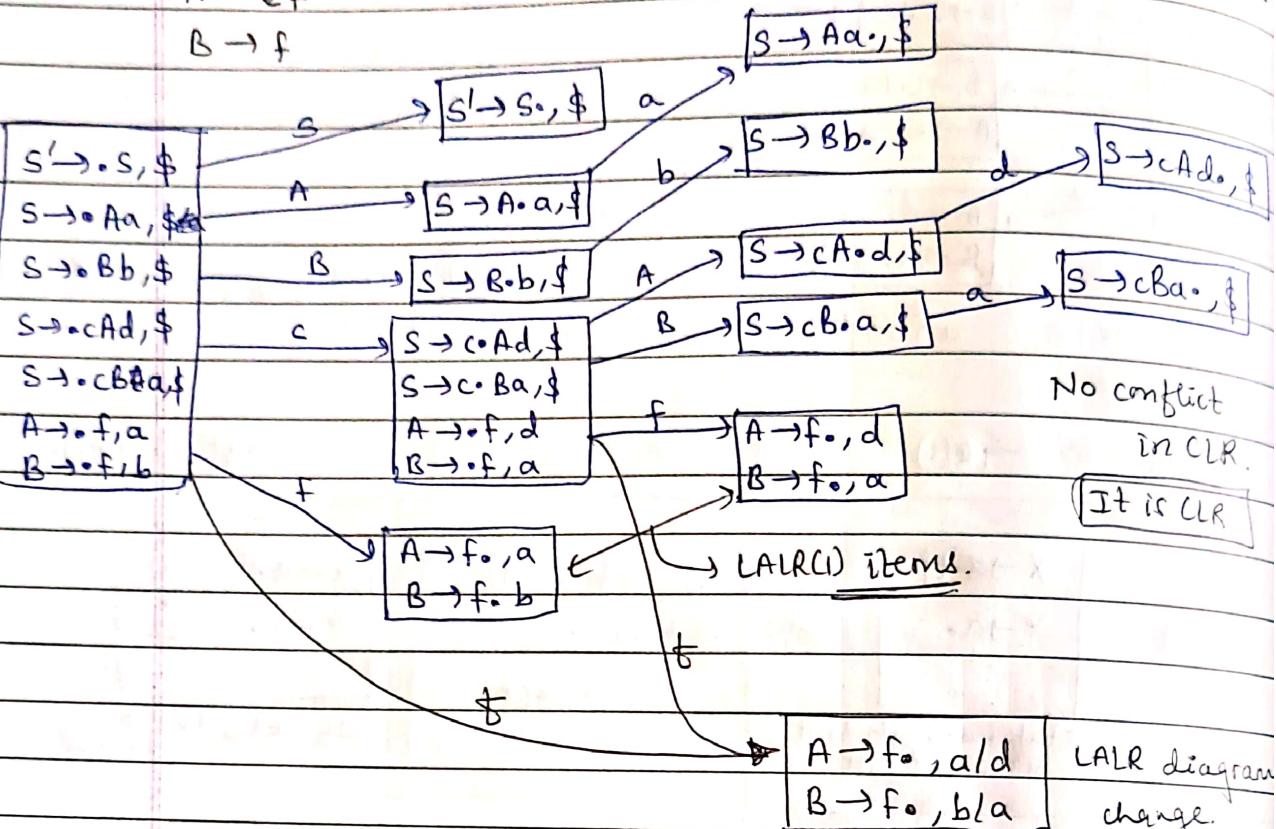


Q. $S \rightarrow Aa \mid Bb \mid cAd \mid cBa$.

CLR \rightarrow 13 states
LALR \rightarrow 12 states

$A \rightarrow e_f$

$B \rightarrow f$



No conflict
in CLR.

It is CLR

t

A -> f, a/d
B -> f, b/a

LALR diagram
change.

da, d ∩ db, a ≠ ∅
a ≠ ∅

Not a LALR

RR in LALR.
conflict

* No. of states:

$$n(LR(0)) = n(SLR) = n(LALR) \leq n(CLR)$$

* Power: $LR(0) < SLR < LALR < CLR$

less powerful.

more powerful.

② Entries in LR(1) Table:

- Production.
- Blank.

LR(0) Table

SLR(1) Table

LALR(1) Table

CLR(1) Table

Entries

- Shift entry
- Reduced entry
- Go to entry.
- Acceptance entry
- Blank entry.

- EL(LR(0)) Table construction -

$$S \xrightarrow{I} Aa \mid Bb \mid cAb.$$

$$A \xrightarrow{II} f$$

$$B \xrightarrow{IV} f$$

$$S' \xrightarrow{S} S \cdot, \$$$

$$\text{follow}(S) = \{ \$ \}$$

$$\text{follow}(A) = \{ a, b \}$$

$$\text{follow}(B) = \{ b \}$$

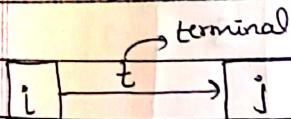
		1	2	6	
$S' \xrightarrow{S} S \cdot, \$$	$A \xrightarrow{I} S \rightarrow A \cdot a, \$$	$\xrightarrow{a} S \rightarrow Aa \cdot, \$$	(I)		
$S \rightarrow \cdot Aa, \$$	$B \xrightarrow{II} S \rightarrow B \cdot b, \$$	$\xrightarrow{b} S \rightarrow Bb \cdot, \$$	(II)		
$S \rightarrow \cdot Bb, \$$	$S \rightarrow \cdot cAb, \$$				
$S \rightarrow \cdot cAb, \$$	$c \xrightarrow{III} S \rightarrow c \cdot Ab, \$$	$\xrightarrow{A} S \rightarrow cAb \cdot, \$$	$\xrightarrow{b} S \rightarrow cAb \cdot, \$$	(III)	
$A \xrightarrow{IV} f, a$	$A \xrightarrow{IV} f, b$				
$B \xrightarrow{IV} f, b$			$\xrightarrow{f} A \rightarrow f \cdot, b$	(IV)	
		$\xrightarrow{S} A \rightarrow f \cdot, b$			= 11 states
	$f \xrightarrow{V} A \rightarrow \cdot f, a$	(V)			
	$B \xrightarrow{VI} \cdot f, b$	(VI)			

no. of Rows X No. of columns.

no. of states X $\frac{(|T| + 1 + 1 \times 1)}{\text{action}} \downarrow \text{gotu.}$

- Rule for shift entry

in all LR Tables :



action col^m

i	t	j
i	S_j	

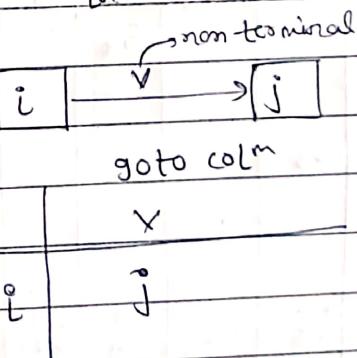
How many shift entries in table?

→ (6).

= no. of terminal transition in DFA.

- Rule for goto entry

in all LR Tables :



How many goto entries in table?

→ (4)

= no. of non-terminal transition in DFA.

- Acceptance Entry Rule in all LR Tables :-

i		\$	
$S' \rightarrow S_0$	i	Accept	
⋮			

- Reduced Entry Rule in LR(0) Table:

- Reduced Entry in SLR Table.

- Reduced Entry Rule in LALR & CLR

P: X $\rightarrow \alpha_0$
⋮

P : X $\div \alpha_0$	i
number	state

p: X $\rightarrow \alpha_0, t_1/t_2$	i	d
⋮		

$$\text{follow}(X) = \{t_1, t_2\}$$

ACTION				
i	R _p	R _p	R _p	R _p

i	t ₁		t ₃
i	R _p		R _p

i	t ₁	t ₂
i	R _p	R _p

• LR(0) Table Construction -

← ACTION TABLE → ← GOTO TABLE →

	a	b	c	f	\$	s	A	B
0			S ₄	S ₅		1	2	3
1					Accept			
2	S ₆							
3		S ₇						
4				S ₉		8		
5	R _{II} R _{IV}	R _{IV} R _{II}	R _{II} R _{IV}	R _{II} R _{IV}	R _{II} R _{IV}			
6	R _I							
7	R _{II}							
8		S ₁₀						
9	R _{IV}							
10	R _{III}							

• SLR(1) Table construction -

	a	b	c	f.	\$	S	A	B
0					s_4 s_5	1	2	3
1						Accept		
2			s_6					
3			s_7					
4					s_9		8	
5				R^{IV}	R^{IV}			
6						RI		
7						RII		
8					s_{10}			
9				R^{IV}	R^{IV}			
10						R^{III}		

• LALR(1) Table construction -
& CLR(1)

	a	b	c	f	\$	S	A	B
0					s_4 s_5	1	2	3
1						Accept		
2			s_6					
3			s_7					
4					s_9		8	
5				R^{IV} R^{IV}				
6						RF		
7						RII		
8					s_{10}			
9				R^{IV}				
10						R^{III}		

No. of states Table size	$\# \text{states}(\text{LR}(0)) = \# \text{states}(\text{SLR}) = \# \text{states}(\text{LALR}) \leq \# \text{states}(\text{CLR})$ $ \text{LR}(0) \text{ Table} = \text{SLR Table} = \text{LALR Table} \leq \text{CLR Table} $
No. of shift entries	$[\text{no. of shift entries in LR}(0), \text{SLR}, \text{LALR are same}] \leq [\text{no. of shift entries in CLR}]$
No. of goto entries	$[\text{no. of goto entries in LR}(0), \text{SLR}, \text{LALR are same}] \leq [\text{no. of goto entries in CLR}]$
No. of reduced entries	$\# \text{reduced entries in LR}(0) \geq [\text{in SLR}] \geq [\text{in LALR}] \geq [\text{in CLR}]$
No. of conflicts.	$\text{in LR}(0) \geq \text{in SLR} \geq \text{in LALR} \geq \text{in CLR}$

NOTE:- If CFG is CLR and containing states to get LALR then RR conflict possible, SR conflict is impossible

\xrightarrow{a}	\xrightarrow{b}	no SR conflict	no SR conflict
$X \rightarrow \alpha_0, L_1$	β_1	$X \rightarrow \alpha_0, L_3$	$X \rightarrow \alpha_0 \oplus \beta, L_1$
$\gamma \rightarrow \alpha_0, L_2$	$\gamma \rightarrow \alpha_0, L_4$	$\gamma \rightarrow \alpha_0, L_2$	$\gamma \rightarrow \alpha_0, L_4$
$\downarrow b$	$\downarrow c$.	$t \notin L_2$	$t \notin L_4$
No RR conflict.	No RR conflict.		combine to get LALR.
	combine to get LALR.		
	$\downarrow \alpha, \beta, \gamma$	$X \rightarrow \alpha_0 \oplus \beta, L_1 \cup L_3$	impossible to get
	$X \rightarrow \alpha_0, L_1 \cup L_3$	$\gamma \rightarrow \alpha_0, L_2 \cup L_4$	SR conflict.
	possibility of RR conflict	$t \notin L_2 \cup L_4$	
	$\downarrow \{b, c\}$		

ALL CLR grammars

ALL LALR grammars

why these CFGs
are not LALR?

If we construct CLR, no conflicts.

If we construct LALR, RR conflicts exist

input: fa . Table : LR

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step 1:

Input

f	a	\$
0		

$$M[0, f] = S_5.$$

step 2:

a	\$
0	

$$M[S, a] : R_{IV}$$

$$A \rightarrow f.$$

i) Pop 2 * [f]

ii) Push A

iii) Push $M[0, A] = 2.$

step 3:

2	a	\$
A		
0		

$$m[2, a] = S_6.$$

step 4:

\$

↑

steps:

\$

↑

$$m[6, \$] = R_I$$

$$S \rightarrow Aa.$$

1

↑

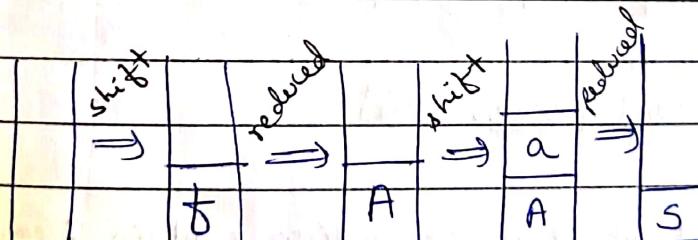
S

$$m[1, \$]$$

ACCEPT

A

0



Reverse of RMD

S
↑

Aa
↑

fa.

S
↑

Aa
↑

fa.

RMD.

• fa $\xrightarrow{\text{shift}} f \cdot a \xrightarrow{\text{reduced}} A \cdot a \xrightarrow{\text{shift}} Aa \xrightarrow{\text{reduced}} S.$

- I) Every LL(1) is LR(1), \rightarrow CLR
- II) Every LR(0) is SLR
- III) Every SCR is LALR.
- IV) Every LALR is CLR, \rightarrow LR(1).
- V) Every ambiguous CFG is not CLR.
- VI) Every not CLR is not LALR.
- VII) Every not LALR is not SLR.
- VIII) Every not SLR is not LR(0).
- IX) Every LR(0)
- } SLR
- } CLR
- } LALR
- } LL(1)
- is unambiguous.

* Operator precedence parsing -

- \rightarrow operator Grammar.
- \rightarrow operator precedence Relations.
- \rightarrow How to compute operator precedences.

\rightarrow Operator Grammar [operator CFG]

\rightarrow It is CFG where no production contain 2 consecutive non terminals. and no null production in CFG

- | | | | | |
|---------------------|-----------------------|----------------------|-------------------------|-------------------------|
| ① $S \rightarrow a$ | ② $E \rightarrow E+E$ | ③ $E \rightarrow EE$ | ④ $E \rightarrow E+E E$ | ⑤ $E \rightarrow EE aE$ |
| OG. | $E \rightarrow a$ | $E \rightarrow a$ | | Not OG. |
| OG. | XOG. | | | Not OG. |

\rightarrow Operator precedence Relations :-

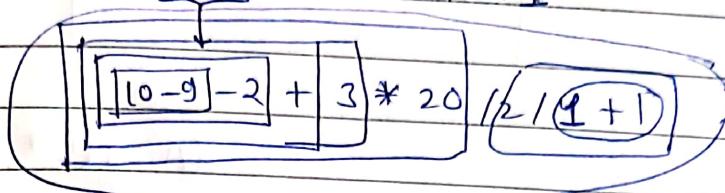
- 1) Highest precedence
- 2) lowest precedence.
- 3) Equal precedence } Associativity.

\hookrightarrow (i) Left Associativity.
 (ii) Right Associativity

	order	Associativity
-	Highest	left-to-right
+, *		left-to-right
/	lowest	right-to-left

Consider this precedence table.

Find O/P for $10 - 9 - 2 + 3 * 20 / 2 / 4 + 1$



$$= 40 / 1 = (40)$$

② - is highest, * is lowest

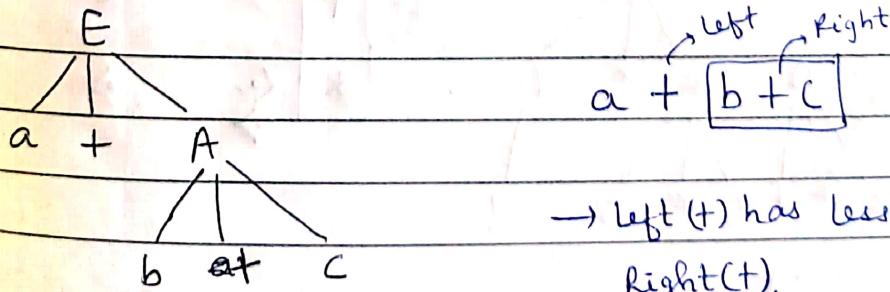
- follows left associative, Remaining follows Right to Left.

Find output for $3 * 2 + 3 - 1 + 4 - 2 - 1$.

-	left to Right	$3 * 2 + (3 - 1) + \underbrace{4 + 2 - 1}$
+	Right to left	$\rightarrow \cancel{3} - 1$
*	Right to left	$3 * 2 + \cancel{2 + 1} - \cancel{4 - 1}$

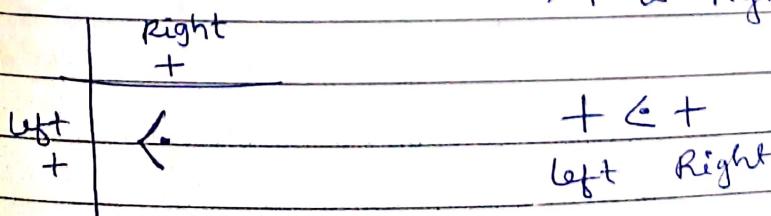
$$3 * 2 + 3 = (15)$$

③

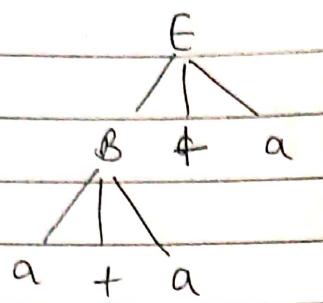


→ left (+) has less precedence than Right(+).

→ + is right associative.



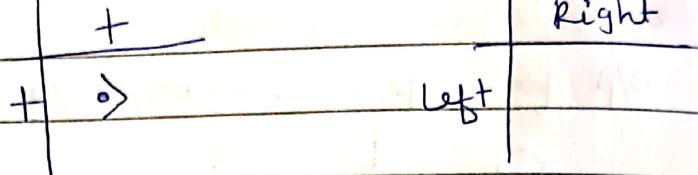
(4)



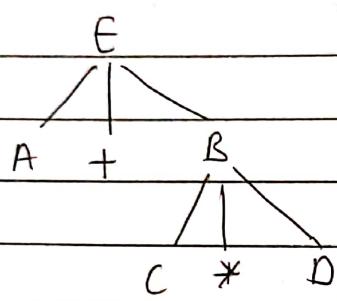
$$(a + a) + a$$

left \Rightarrow Right. $[+ \Rightarrow +]$

left to right associativity



(5)



$$(A + (C * D))$$

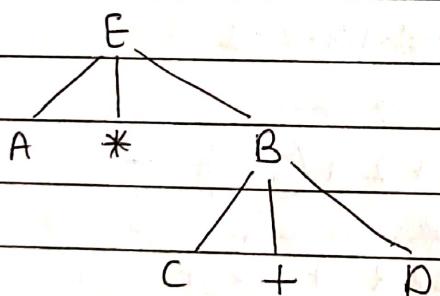
~~* $\Rightarrow +$~~

highest $\stackrel{\text{left}}{+} \stackrel{\text{right}}{*}$, lowest $- (+)$

I) $+ < *$ ✓

II) $* \Rightarrow +$ ✗
left right

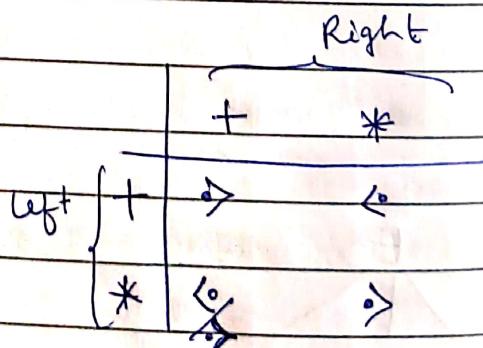
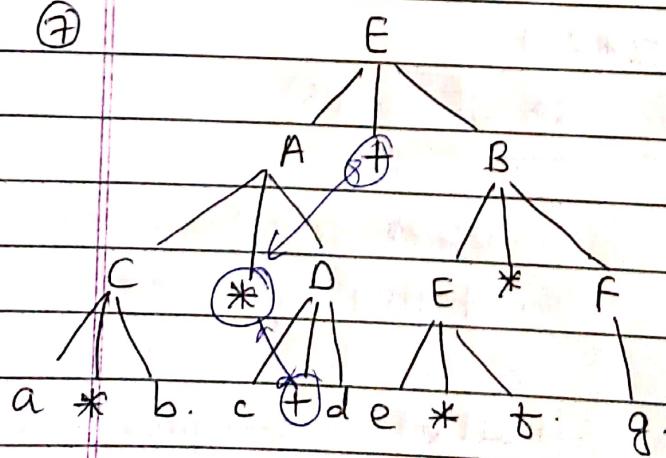
(6)



$$(A * (C + D))$$

$* < +$

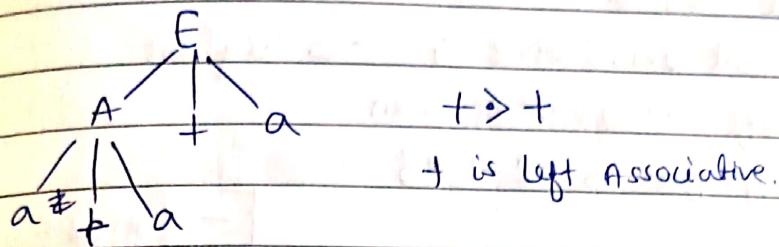
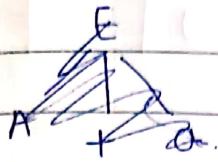
(7)



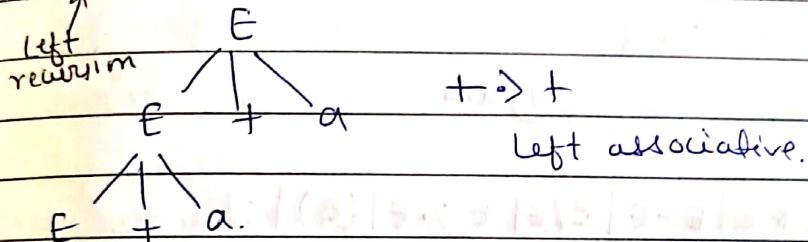
$* \Rightarrow *$

$$⑧ E \rightarrow A + B | a$$

$$A \rightarrow a \alpha a$$



$$⑨ E \rightarrow (E) + a | a$$



* + is left Associated because + associated with left Recursion.

$$⑩ E \rightarrow a * E | a \rightarrow \text{Right associative coz Right recursion.}$$

$$⑪ E \rightarrow a * A | a \rightarrow * \text{ is Right associative}$$

$$A \rightarrow a * a$$

$$⑫ S \rightarrow A + B | a \quad a + a * a.$$

$$\therefore A \rightarrow a$$

$$B \rightarrow a * a.$$

+ < *	+	*
A + B	+	<
a * a	*	

$$⑬ E \rightarrow (E) + T | a$$

$$T \rightarrow F * T | b$$

$$F \rightarrow c. \quad \text{Right.}$$

w	+	+ *	
		① >	② <
*	*	③ * >	+
		④ * >	⑤ <

① + is left associative.

② + < *

③ * > +

④ * is right associative.
coz right recursion.

(14) $E \rightarrow (E) \mid a.$

	()
	($\hat{=}$

$(\hat{=})$.

Left Right

(15) $E \rightarrow E+E \mid E * E \mid id$

If $*$ is right associative & $-$ is highest

$*$ is left associative then

find %p for $5-2-1 * 3$

$5-2-1 * 3$

$\underbrace{5-}_{\downarrow} 2-1 * 3$

$\underbrace{4}_{(12)}$ and.

-	Right to left
*	left to right

• $E \rightarrow E+E \mid E * E \mid E-E \mid E/E \mid E \% E \mid (E) \mid id$.

It will generate all arithmetic expressions.

NOTE:- Operator precedence Relations can be computed using -

- 1) Parse tree.
- 2) Operator precedence table.
- 3) Unambiguous operator Grammar.
- 4) Information provided in English statements.

• Semantic Analysis

↳ It uses SOT to perform type checking, function compatibility, variable declaration etc.

VS. Syntax Directed Translation

- ↳ It is more powerful program or tool that can be used for
- 1) Semantic Analysis.
 - 2) Parse tree Generation.
 - 3) Intermediate code.
 - 4) Evaluations of expressions
 - 5) Conversions.

- SDT

SDT = Syntax + translation.
(structure),

- semantic.
- Evaluations.
- Actions.
- Specific Requirement.
- Program.

= CFG + Translation

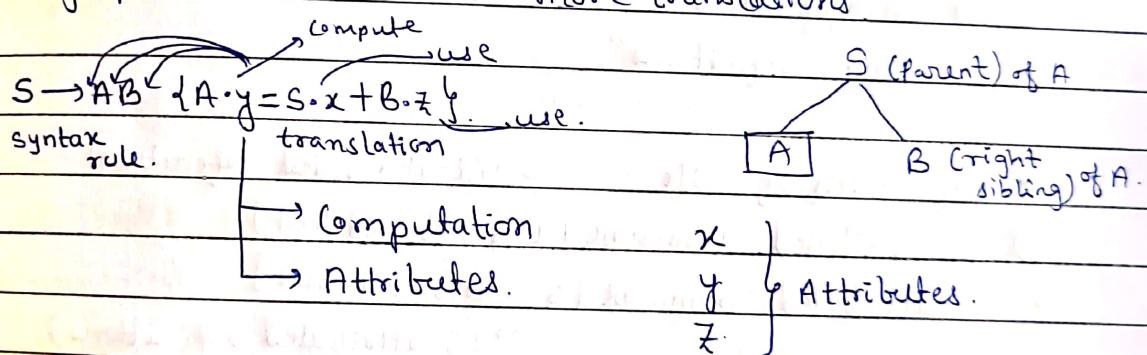
$\vdash \dots \vdash$

ex:-	$S \rightarrow AB$	$\{S.x = A.x * 2\}$
	$A \rightarrow a$	$\{A.x = a.val + 100\}$
	$B \rightarrow b$	$\{B.x = b.y\}$

Syntax.

translation.

→ Every production has 0 or more translations



- Attributes -

① Inherited Attribute.

$$S \rightarrow A a [B]$$

$\{B.val = S.val * A.val\}$

computation.

→ Computation depends on parent/siblings.

② Synthesized Attribute.

$$[S] \rightarrow A a B$$

$\{S.val = A.val + a.val\}$

computation.

→ Computation depends on children.

→ Find Attribute type -

① $S \rightarrow a \{S.x = a.val\}$ (synthesized)

② $S \rightarrow Sa \{S_1.x = S.x + 1; S.y = S_1.y * 2\}$ $x \rightarrow$ Inherited

$S \rightarrow b \{S.x = b.val - 1; S.y = S.x + 9\}$ $x \rightarrow$ synthesis.

x is neither inherited nor synthesis.

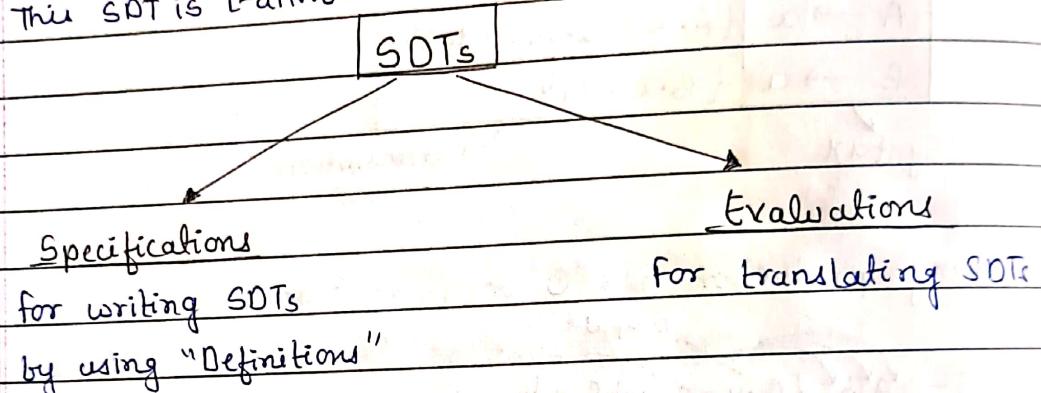
$y \rightarrow \text{synthesized}$ } y is synthesized attribute.
 $y \rightarrow \text{synthesized}$

③ $E \rightarrow E_1 + E_2 \quad \{E \cdot x = E_1 \cdot x + E_2 \cdot x\}$ SOT is S-attributed,
 $E \rightarrow a \quad \{E \cdot x = a \cdot \text{val}\}$ L-attributed.

x is synthesized attribute.

④ $D \rightarrow T[I]; \quad \{I \cdot \text{type} = T \cdot \text{type}\}$ type is neither
 $T \rightarrow \text{int} \quad \{T \cdot \text{type} = \text{int}\}$ inherited nor
 $I \rightarrow a \{ \}$ synthesized.

This SOT is L-attributed but not S-attributed.



→ Definitions of SOTs :- → (L-attributed definition)

① L-attributed Grammar [L-attributed SOT]

② S-attributed Grammar [S-attributed SOT]

↳ (S-attributed definitions)

① L-attributed SOT.

→ Computation depends on parent/left siblings/children

$S \rightarrow a \quad \{S \cdot \text{val} = a \cdot \text{num}\}$

$S \rightarrow AB \quad \{B \cdot \text{val} = S \cdot \text{val}\}$

$\{A \cdot \text{val} = S \cdot \text{val}\}$

$\{B \cdot x = S \cdot x + A \cdot y\}$

$\{S \cdot x = A \cdot x + B \cdot y\}$

② S-attributed SOT

→ Computation depends on only children.

[It uses synthesized attribute]

→ Translation must appear only at end of production.

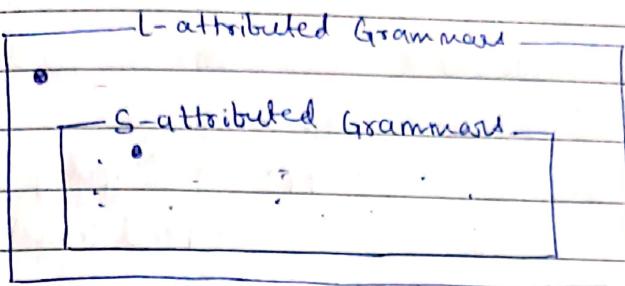
$S \rightarrow a \dots y$

NOTE:- Except right sibling,

it can depend on any.

→ Translations can appear anywhere. $S \rightarrow \dots y a$

→ Every S-attributed SDT is always L-attributed.



- $E \rightarrow E_1 \quad \{ E \cdot \text{val} = E_1 \cdot \text{val} + E_2 \cdot \text{val} \} + E_2$.

$E \rightarrow a \quad \{ E \cdot \text{val} = a \cdot \text{num} \}$.

not at the end of production.

↳ SDT is not S-attributed

SDT is L-attributed.

- L-attributed Grammar -

Computation can depend on parent/Left sibling/children.

evaluated in

top-down approach

evaluated in

bottom-up

approach.

- S-attributed Grammar -

Computation depends only on children.

↳ (Bottom up approach)

(1) $S \rightarrow S_1 S_2 \quad \{ S \cdot x = S_1 \cdot x + S_2 \cdot x \}$

$S \rightarrow (S_1) \quad \{ S \cdot x = S_1 \cdot x + \$ \}$

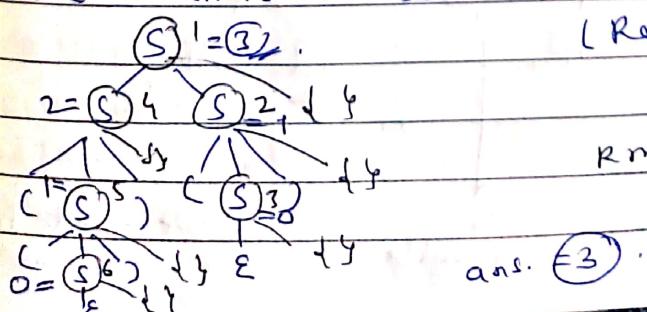
$S \rightarrow \epsilon \quad \{ S \cdot x = 0 \}$

(Q1) What is attribute type? (x) → x is synthesized attribute.

(Q2) What is SDT definition? → Both S-attributed & L-attributed

(Q3) Find attribute value at root for $((())())$.

If we use S-attributed evaluation: (Bottom up parsing)



RMD Numbering: 1st, 2nd, 3rd, 4th, 5th, 6th
Reverse.

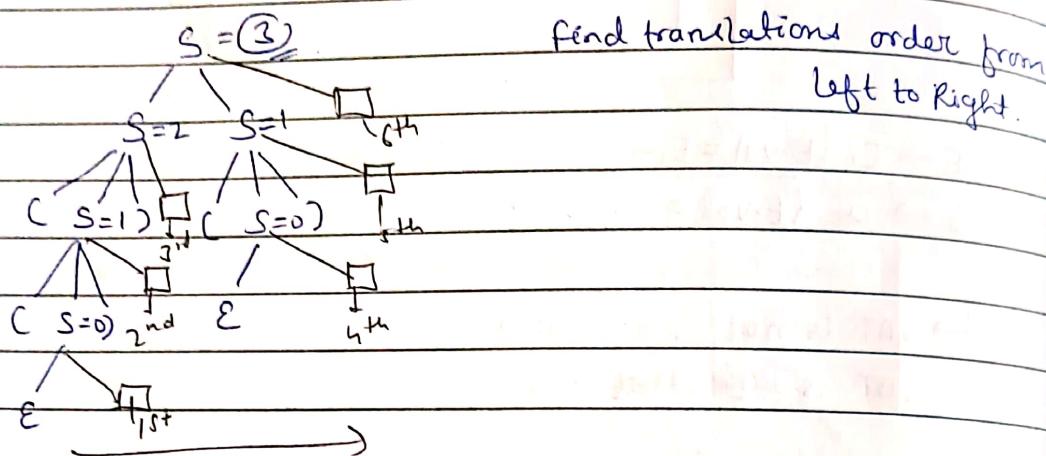
ans. = 3.

(Q4) what is functionality of this SBT? \Rightarrow computing no. of balanced parentheses.

\rightarrow If we use L-attributed evaluation:

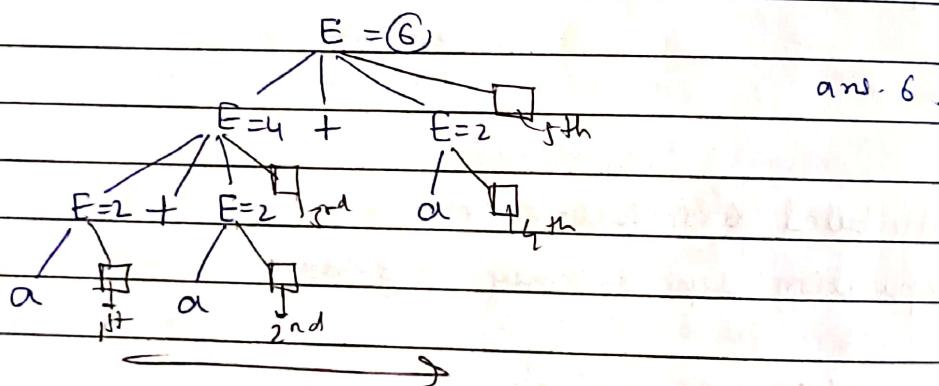
(Depth first, left to right).

(Inorder approach) (Topological order).



$$?) \quad E \rightarrow E_1 + E_2 \quad \{E \cdot x = E_1 \cdot x + E_2 \cdot x\}.$$

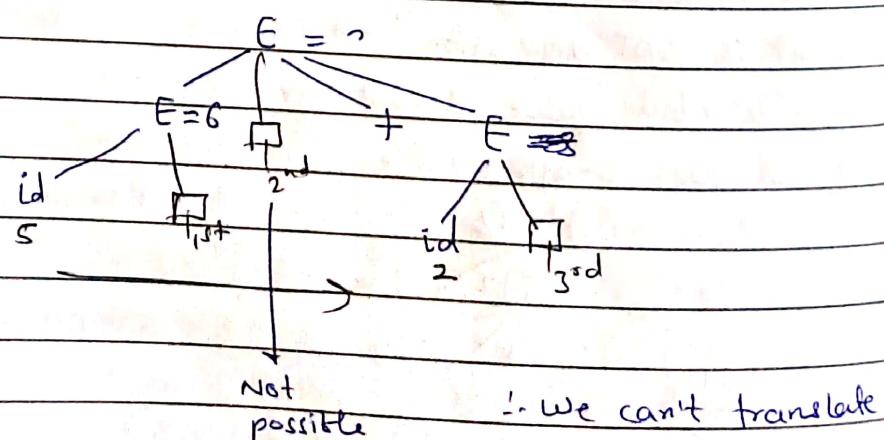
$E \rightarrow a \quad \{E \cdot x = 2y\}$, find translation for $a+a+a$.



$$3). \quad E \rightarrow E_1 \quad \{E \cdot x = E_1 \cdot x - E_2 \cdot x\} + E_2$$

$E \rightarrow id \quad \{E \cdot x = id \cdot val + 1\}$.

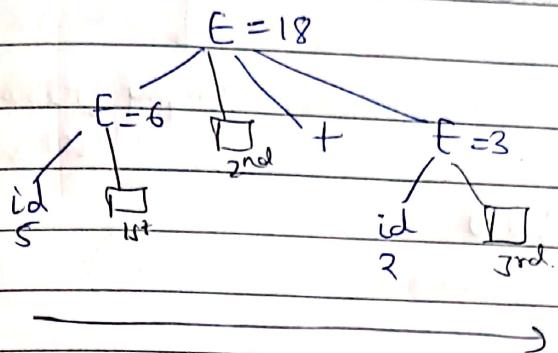
Find translation for $id + id \cdot 5 + 2$.



(4) $E \rightarrow E_1 \{ E \cdot x = E_1 \cdot x \# 3 \} + E_2$.

$E \rightarrow id \{ E \cdot x = id \cdot val + \}$.

find translation for $5+3$.



(5) $E \rightarrow E + E \{ print + \}$

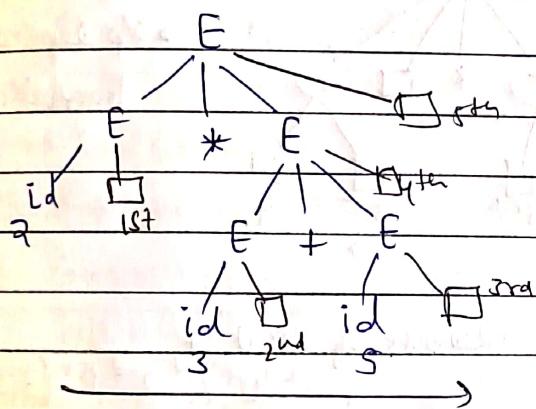
+ is high precedence.

$E \rightarrow E * E \{ print * \}$

+ and * are left associate.

$E \rightarrow id \{ print id \cdot val \}$

find o/p for $2 * 3 + 5$.



o/p: ~~2 3 5 + *~~

(6) $t \rightarrow EFT \{ p$

$E \rightarrow \{ print + \} E + T$

$E \rightarrow F E + T$

$E \rightarrow id \{ print id \cdot val \}$

$E \rightarrow id^F$

$T \rightarrow T * \{ print * \} F$.

$T \rightarrow T * F$

$T \rightarrow id \{ print id \cdot val \}$

$T \rightarrow id^F$

$F \rightarrow \{ print id \cdot val \} id$

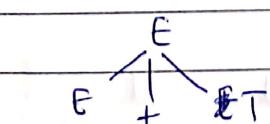
$F \rightarrow id^F$

(Q1) find o/p for $2 + 3 * 4$

(Q4) find o/p for $2 + 3 * 4$ using TDPL (LMD)(LL(1)).

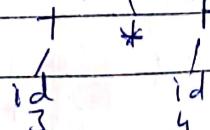
(Q2) find o/p for $2 + 3 * 4$

using LR parsing

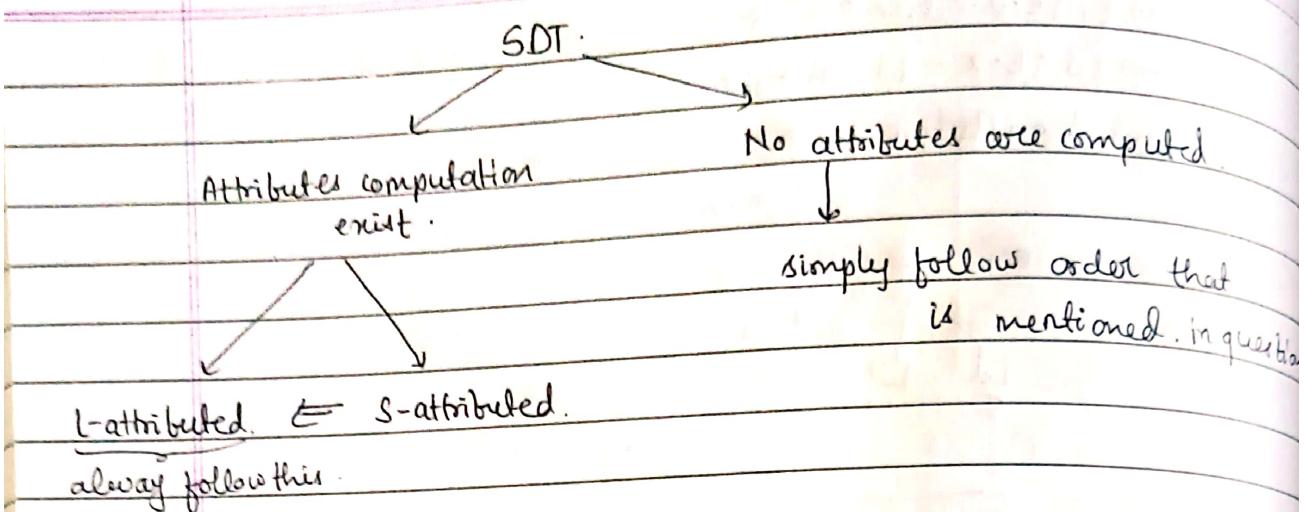


(Q5) find o/p for $2 + 3 * 4$ using reverse of LMD.

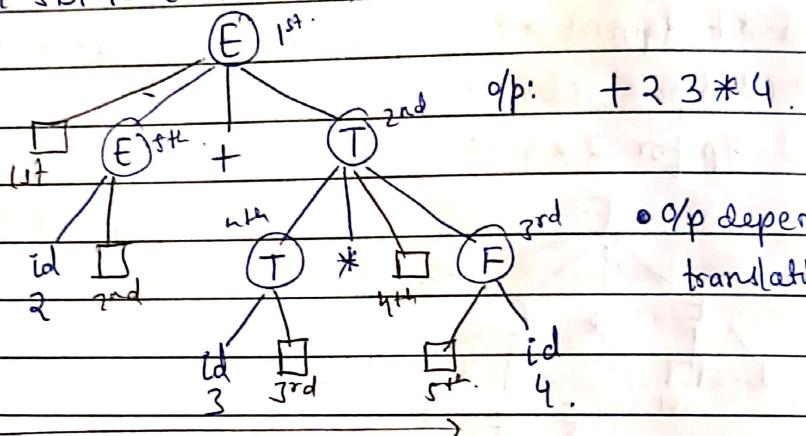
(Q3) find o/p for $2 + 3 * 4$ using RMD.



(Q6) find o/p for $2 + 3 * 4$ using reverse of L-attributed order.



Q1) Given SDT is L-attributed,



Q2) LR parsing \rightarrow bottom-up \rightarrow Reverse of RMD \rightarrow

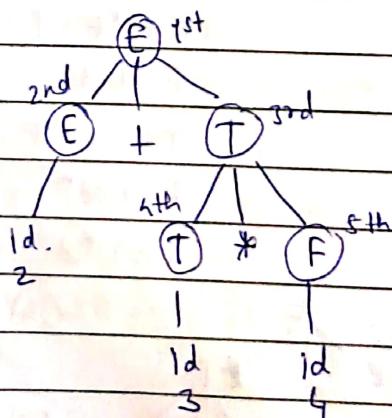
5th 4th 3rd 2nd 1st (2 3 4 * +).

Q3) RMD order: 1st 2nd 3rd 4th 5th

+ * 4 3 2.

o/p only depends on non-terminal n

Q4)



E
↓
(E)+T

↓
id+T

↓
id+(T)*F.

o/p: LRD numbering order: 1st 2nd 3rd 4th 5th
+ 2 * 3 4.

(Q5) Reverse of LMP: $43 * 2 +$

(Q6) $4 * 32 +] \rightarrow$ Reverse of L-attributed o/p.

(Q7) $S \rightarrow a A \{ \text{point 1}\}$

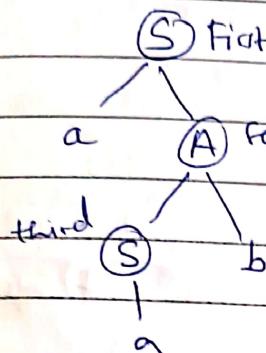
$$O/P = aab$$

$S \rightarrow a \{ \text{point 2}\}$

What is o/p using bottom up parser?

$A \rightarrow Sb \{ \text{point 3}\}$

reverse of RMD



Reverse of : Third Second First
RMD 2 3 1.

(Q8) $E \rightarrow E_1 + E_2 \{ E \cdot x = E_1 \cdot x + E_2 \cdot x + \text{①}\}$

$E \rightarrow E_1 * E_2 \{ E \cdot x = E_1 \cdot x + E_2 \cdot x + \text{②}\}$.

$E \rightarrow id \{ E \cdot x = \text{③}\}$

What is functionality of SDT?

→ It counts no. of operators

• Intermediate code.

Q) What is Intermediate code?

→ Postfix code.

HLL $\xrightarrow[\text{Easy}]{\text{ICG}}$ I.C. $\xrightarrow[\text{Easy}]{\text{CG}}$ Ass. Code.

→ 3 address code.

→ It's a machine independent code.

→ SSA code.

→ Portable.

→ Syntax tree

Analyzed code \longrightarrow [ICG] \longrightarrow Intermediate code.

→ DAG

(Lexically,
Syntactically,
Semantically
verified).

ICG \rightarrow Inter. code Generator

CG \rightarrow Code Generator.

$$\text{ex:- } x = y + z / c; \quad t_1 = z / c \\ t_2 = y + t_1$$

$$x = t_2$$

Intermediate code has nearest relation to assembly code.

← easily convertible to assembly code.

* Intermediate Code Representations :-

I) Linear. $\xrightarrow{\text{Postfix}}$ 3AC/TAC (Three Address Code).
 $\xrightarrow{\text{SSA}} \text{SSA. (static single Assignment)}$

II) Non linear. $\xrightarrow{\text{Syntax Tree}}$
 $\xrightarrow{\text{DAG (Directed Acyclic Graph)}}$
 $\xrightarrow{\text{CFG (Control flow graph)}}$

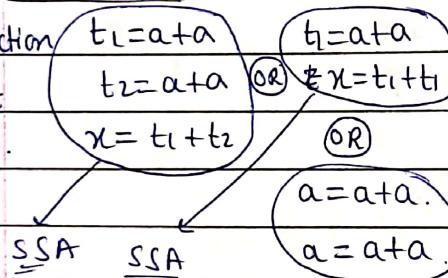
$$x = a + a + a + a$$

I) Postfix code

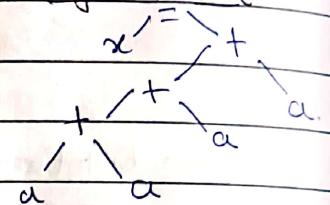
$$x a a + a + a =$$

** II) Intermediate code (TAC/3AC) -

Every instruction
has atmost
3 address.



I) Syntax Tree



Leaf: operand,
non-leaf: operator.

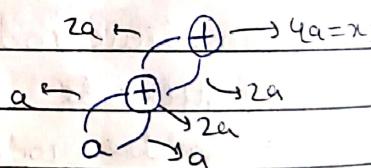
III) SSA code -

NotSSA

II) DAG.

→ Every variable is having single assignment.

→ It is 3 address code but, every var. is having single assignment.



Eliminates common
subexpression
→ 3 nodes, 4 edges

ex(I) $x = a * b$.

postfix: $x a b * =$

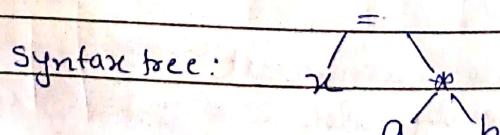
3AC: $x = a * b$.

(OR) $a = a * b$ \rightarrow a has 2 defⁿ \rightarrow they are
 $b = a * b$. \rightarrow b has 2 defⁿ \rightarrow not SSA code.

DAG

SSA: $x = a * b$.

Syntax tree:



② $a + a + a$.

Postfix: $x a a + a + =$

$$3AC: t_1 = a + a$$

(OR)

$$t_1 = t_1 + a$$

$$t_1 = a + a$$

(OR)

$$a = t_1 + a$$

$$t_1 = a + a$$

2 variables.

Best 3AC.

$$t_2 = t_1 + a$$

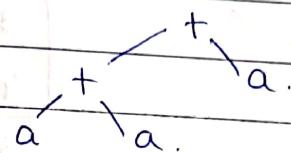
3 variables.

SSA: not SSA

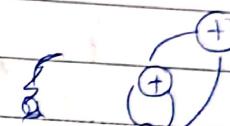
, notSSA,

SSA ✓

Syntax tree:



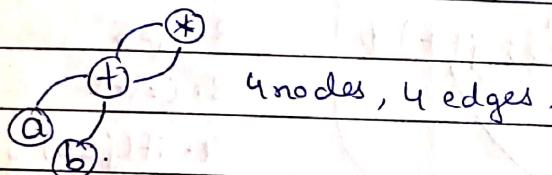
DAG.



3 nodes,
4 edges.

③ find no. of nodes & edges in DAG for,

$$x = (a+b)*(a+b)$$



④ find no. of min. variables in 3AC for $x = (a+b)*(a+b)$

$$t_1 = a + b$$

$$a = a + b$$

$$t_2 = t_1 * t_1$$

$$a = a * a$$

3 variables

(minimum)

(OR)

$$b = a + b$$

$$b = b * b$$

SSA code:

$$t_1 = a + b$$

← 4 variables (minimum)

$$t_2 = t_1 * t_1$$

⑤ $x = a + b + c - a$? How many min. variables in 3AC?

~~$$a = b + a$$~~

$$x = b + c$$

$$b = b - a$$

$$b = b + c \rightarrow 2 \text{ variables}$$

~~$$b = b + c$$~~

(3AC)

~~independent.~~

~~2nd~~ ~~1st~~

$x = (a * b) + b + (a * c)$. How many min. variables in 3AC?

$t_1 = a * b$

$t_1 = t_1 + b$. 4 variables.

$c = a * c$.

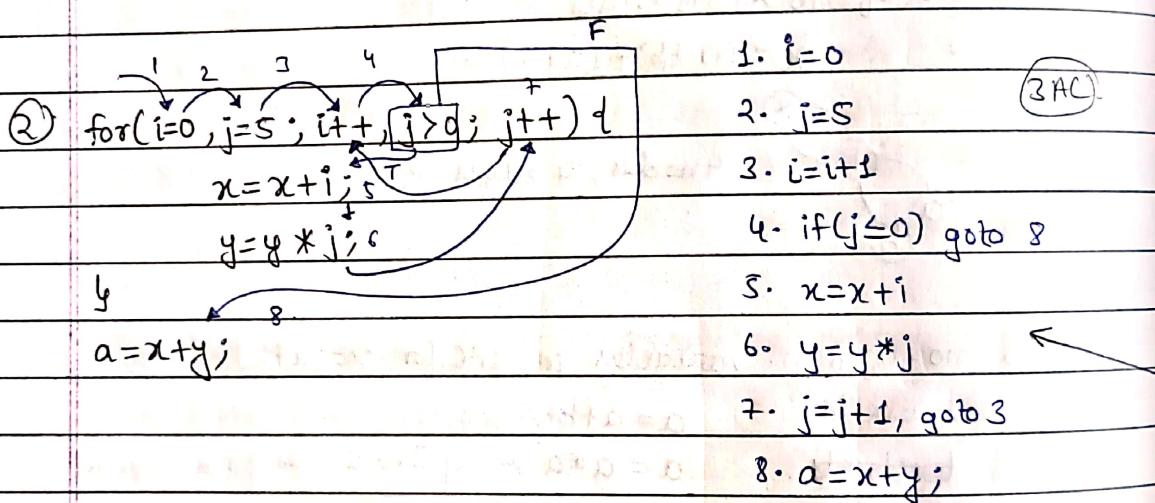
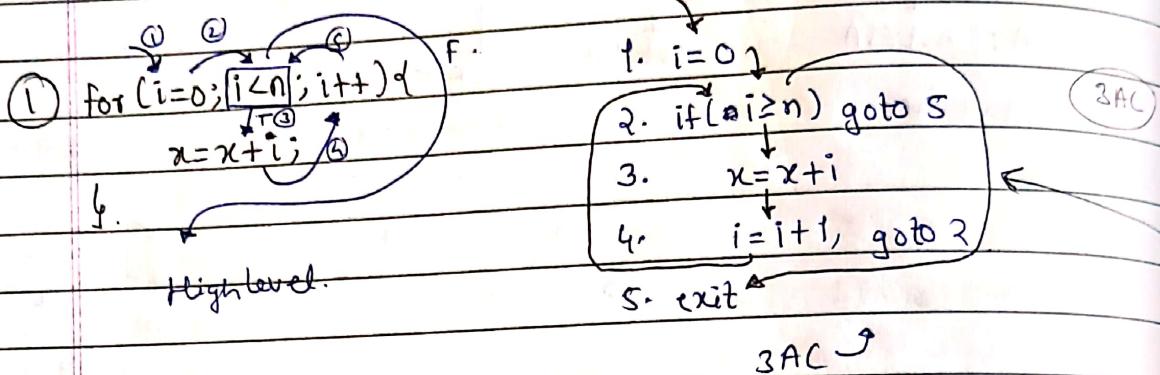
$t_1 = t_1 + c$.

$c = a * c$

$a = a * b$

$a = a + b$

$a = a + c$

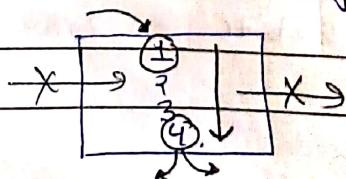


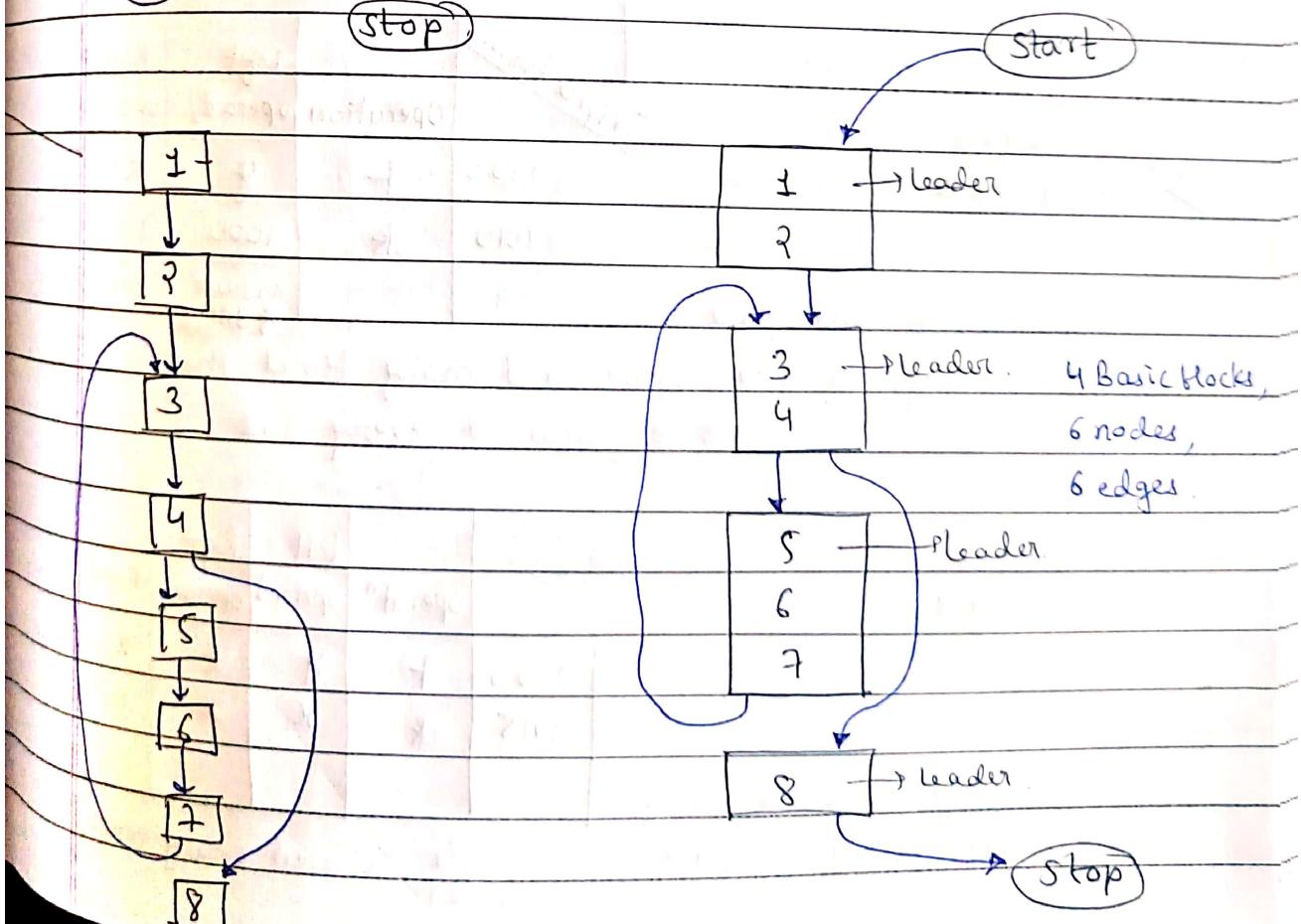
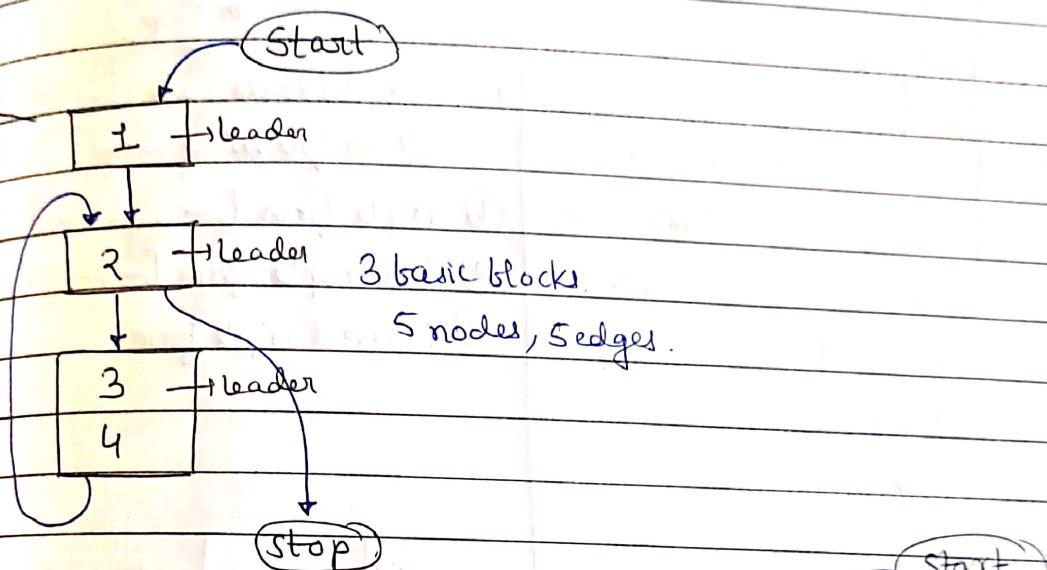
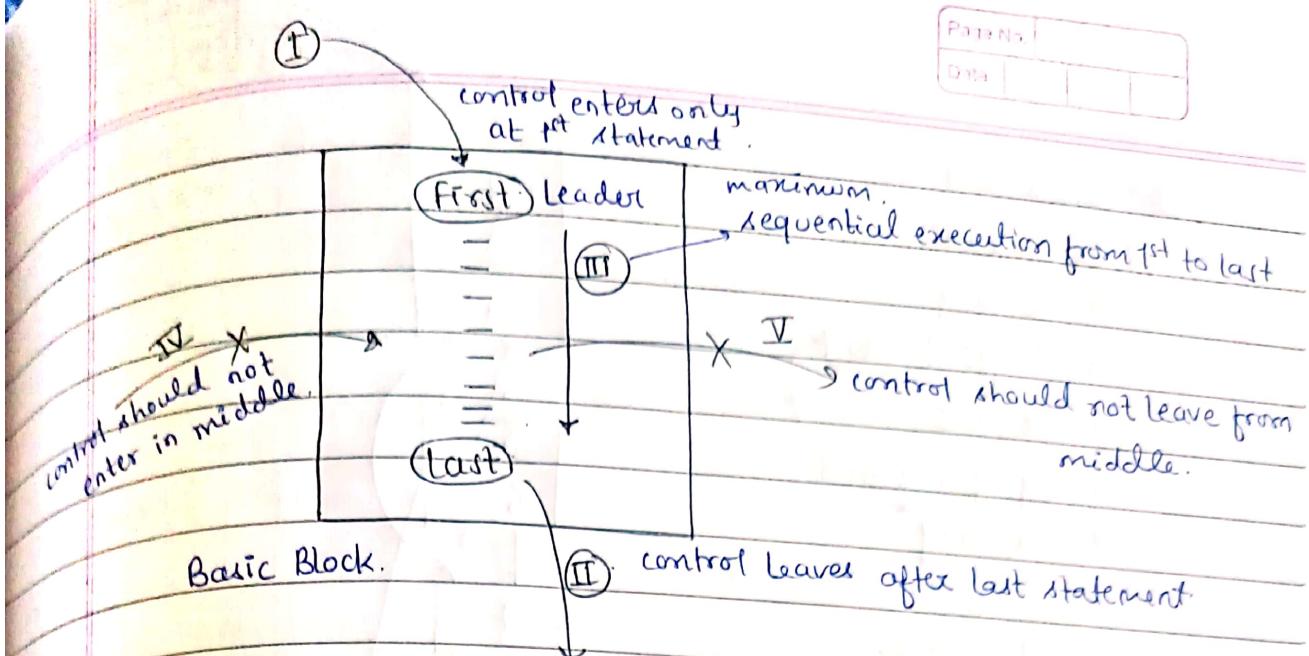
* Control Flow Graph (CFG) -

- It is "Flow Graph"
- Collection of nodes & edges.
- Collection of "basic blocks".

→ What is Basic Block? (sequence)

↳ It is maximum set of statements where control enters only at 1st statement and only leaves after last statement.





3. $y, x=a+1$

2. $y=x*z$

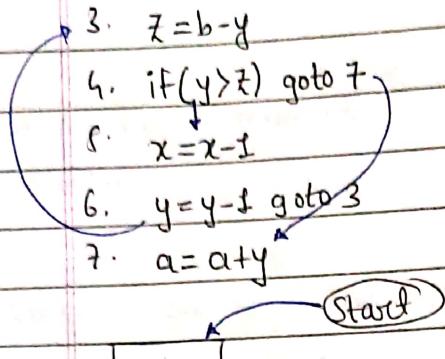
3. $z=b-y$

4. if($y > z$) goto 7

5. $x=x-1$

6. $y=y-t$ goto 3

7. $a=a+y$



1 → leader.

2

3 → leader. 6 nodes,

4 4 basic blocks,

6 edges.

5 → leader.

6

7 → leader.

Stop

• Three Address code : [we can store 3AC in following ways]

① Triple form. [-, -, -]

② Quadruple form. [-, -, -, -]

③ Indirect triple form.

3AC:

$x = y + z$

$a = x * x$

Triple form

Operation

left operand

Right operand

1000

+

y

z

1010

*

1000

1000

Advantage:- less space

Address of 3AC code

Address of 3AC code

Disadvantage:- If any value used many times then it has to be calculated every time.

3AC:

$x = y + z$

$a = x * x$

Quadruple form

operatⁿ

left operand

Right operand

Result

1000

+

y

z

x

1015

*

x

x

a

Advantage: Saves time if any

Disadvantage: More space

variable is used many times.
(value)

$x = y + z$
 $a = x * x.$

Indirect table -

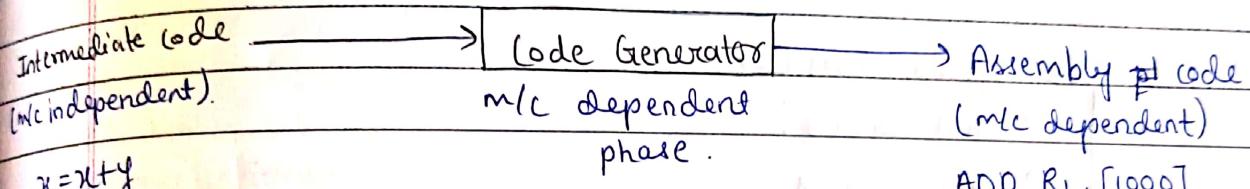
5000	1000
5010	1010.

Indirect triple form

operat ⁿ	left operand	right operand
1000	+	y z.
1010	*	5000 5000

↳ Address of
address of 3AC code.

• Code Generator -



$$x = y + z$$

x & y are variables.

x & y are registers/
memory addressed.

$$x = z + 15$$

$$ADD R1, IS$$

CG

Assembly instruction.

- Type of instruction.
 - Type of operation.
 - Type of addressing mode.
 - Data.
 - Memory Address / Register.
- operands { depends on m/c architecture.

Code Generator

↳ Register Allocation.

↳ Graph coloring

↳ To allocate min. no. of registers.

- we have only 1 register. How many mem. memory spills required?
 $a = a + b$
 $b = a * c$.

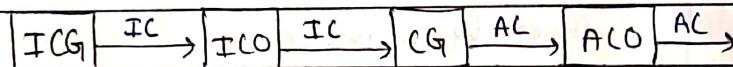
3AC

$R \leftarrow a$
 $\text{mem}_1 \leftarrow b$
 $R \leftarrow R + \text{mem}_1$
 $\text{mem}_1 \leftarrow c$
 $R \leftarrow R * \text{mem}_1$

} one memory spill.

- Code optimization [CO].

- what is CO?
- CO Techniques
- Data Flow Analysis.
 - Live variable analysis
 - Reaching Definition Analysis.



Code Optimization?

→ It may save time / space.

$$\underbrace{x = y * z}_{\text{costlier}} \longrightarrow \underbrace{x = y + y}_{\text{cheaper}}$$

Code Optimization -

At High Level.

In IDEs,
In text editors.

At Intermediate Level.

In compiler.

At assembly level.

Code optimization.

- At Statement Level.
 - At Basic Block Level.
 - At Loop Level.
 - Intra-procedural.
 - Inter-procedural.
- } local optimization.
- } global optimization.

→ Code optimisation techniques -

i) Constant folding -

$$x = \underbrace{2 * 3}_\text{folding} + y \Rightarrow x = 6 + y.$$

ii) Cancellations -

$$x = a + b * c - a \Rightarrow x = b * c.$$

iii) Identity -

$$\begin{aligned} x &= y + 0 - z \Rightarrow x = y - z \\ a &= b * 1 + e \Rightarrow x = b + e. \end{aligned}$$

iv) Strength Reduction -

$$\begin{array}{ccc} x = y * 2 & \xrightarrow{\text{costlier.}} & \begin{array}{l} x = y \ll 1 \text{ (OR)} \\ x = y + y \end{array} \\ & & \downarrow \text{cheaper.} \end{array}$$

v) Common sub-expression elimination -

↳ we can use DAG.

$$x = (a * b) + (a * b) - c.$$

↓

$$t_1 = a * b.$$

$$x = t_1 + t_1 - c.$$

vii) Copy propagation (constant/variable)

$x = 20$ → constant

$y = x + a$. Constant propagation

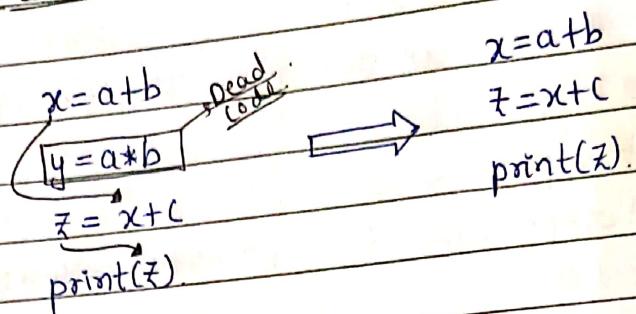
$$y = 20 + a$$

$$x = b$$

$y = x + a$. variable propagation

$$y = b + a$$

⑦ Dead Code Elimination -



⑧ Loop optimizations

- Code motion.
- Induction variables elimination.
- Loop merging.
- Loop unrolling.

• Code motion -

```
for(i=0; i<n; i++) {
```

$x = a + i;$

$y = c * b;$ Loop invariant code.



```
for(i=0; i<n; i++) {
```

$x = a + i;$

y

$y = c * b;$

• Induction Variables Elimination:-

Variables -

$j = 0;$

i

for($i=0; i < n; i++$) {

j

$x = x + i;$

n

$y = y * j;$

x

$z = x - a;$

y

$j++;$

a

b

Induction variables

(i)

for($i=0, i < n; i++$) {

(~~i~~) j

$x = x + i;$

x

$y = y * i;$

y

$z = x - a;$

z

• Loop Merge / Loop Combine / Loop fusion:

```
for(i=0; i<n; i++) {
    A[i] = i+1;
```

```
↓
for(j=0; j<n; j++) {
    B[j] = j*3;
```

```
for (i=0; i<n; i++) {
```

```
A[i] = i+1;  
B[i] = i*3;
```

• Loop unrolling - half.

```
for(i=0; i<(4n); i++) {
```

```
printf("a");
```

```
↓
```

```
for (i=0; i<(2n); i++) {
```

```
printf ("a");
printf ("a");
```

```
↓
```

```
for (i=0; i<=n; i++) {
```

```
printf("a");
```

```
printf("a");
```

```
printf("a");
```

```
printf("a");
```

```
↓
```

* Data Flow Analysis -

→ (i) Forward Analysis.

 → Reaching Definitions

 → Available Expressions.

→ (ii) Backward Analysis.

 → Live variable Analysis.

• Data Flow Analysis -

→ What is Live Variable?

1. $x = a + b$
 2. $y = c * d$
 3. $z = x - c$
 4. $w = y * z$
- IS x live at statement 2?
- Ans. Yes.

$s_i \quad x$ is live.

(III) There should be no write into x before s_j .

(II) path exist from s_i to s_j .

reads x . (I) s_j uses x .
(reads)

• x is live variable at statement s_i
iff

(Read) I) Some statement s_j reads x .

(path) II) Path exist from s_i to s_j .

(No write) III) No write into x in between s_i & s_j before reading x .

Q.1. find live variables at statement 1, 2, 3, 4.

1. $x = a + b$ ← a ✓ b ✓ c ✓ x y z. (a, b, c) live at ①.

2. $y = x * c$ 1 → 1 1 → 2 1 → 3
 1 → 2 → 3

3. $z = x + y$ → 4.

4. $a = b * z$. a b' c ✓ x' y z. (b, c, x) live at ②
 x 2 → 3 → 2 → 2 → 2 → 2 x x

a b' c x' y z. (x, b, y) live at ③.
x 3 → 4 x 3 → 3 3 → 3 x

a b c x y z.

X ✓ X X X ✓ (b, z, z) live at ④

$c = x - c$ ← Is c live?

Yes.

Q5 find live variables at all statement?

$1. x = a$	$1. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad \checkmark \quad x \quad x \quad x \quad x$ (a, c) live at ① ✓
$2. y = x + c$	$2. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad \checkmark \quad x \quad \checkmark \quad x \quad x$ (a, c, x) live at ② ✓
$3. z = a * y$	$3. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad \checkmark \quad x \quad \checkmark \quad \checkmark \quad x$ (a, c, x, y) live at ③ ✓
$4. x = a + z$	$4. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad \checkmark \quad x \quad x \quad x \quad \checkmark$ (a, c, z) live at ④ ✓
$5. y = x - c$	$5. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad \checkmark \quad x \quad \checkmark \quad x \quad x$ (a, x, y, z) live at ⑤ ✓
$6. a = x - z$	$6. a \ b \ c \ d \ x \ y \ z$ $x \quad x \quad x \quad x \quad \checkmark \quad \checkmark \quad \checkmark$ (x, y, z) live at ⑥ ✓
$7. b = x + a$	$7. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad x \quad x \quad \checkmark \quad \checkmark \quad x$ (a, x, y) live at ⑦ ✓
$8. c = x + a$	$8. a \ b \ c \ d \ x \ y \ z$ $\checkmark \quad x \quad x \quad x \quad \checkmark \quad \checkmark \quad x$ (a, x, y) live at ⑧ ✓
$9. d = c - y$	$9. a \ b \ c \ d \ x \ y \ z$ $x \quad x \quad \checkmark \quad x \quad x \quad \checkmark \quad x$ (c, y) live at ⑨ ✓

* GEN Set & KILL Set for Basic Block:
 (use) (definition)

$$KILL_K = \{x, c, z\}$$

Basic Block	$x = a + b$	= set of variables which are defined in Basic Block.
	$c = x - c$	
	$z = x * y$	= $\{v v \text{ is defined in BB}\}$

$$GEN_K = \{a, b, c, y\}$$

= set of variables in Basic

block where every variable used at some statement in Basic Block (BB) but no write into it before read.

- Find GEN set and KILL set for every Basic Block

			GEN set	KILL set
i = m - 1	BB ₁	BB ₁	{m, n, u}	{i, j, a}
j = n		BB ₂	{i, j}	{i, j}
a = u		BB ₃	{v}	{a}
		BB ₄	{a, j}	{i}

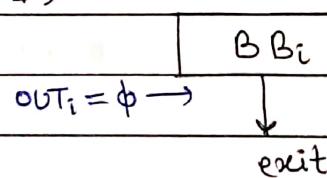
```

graph TD
    Exit(( )) --> BB1[BB1]
    BB1 --> BB2[BB2]
    BB2 --> BB3[BB3]
    BB3 --> BB4[BB4]
    BB4 --> BB1
  
```

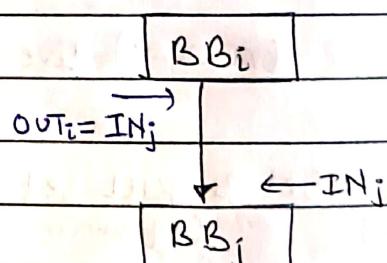
- IN set & OUT set for Basic Blocks!

→ How to compute OUT set for BB_i?

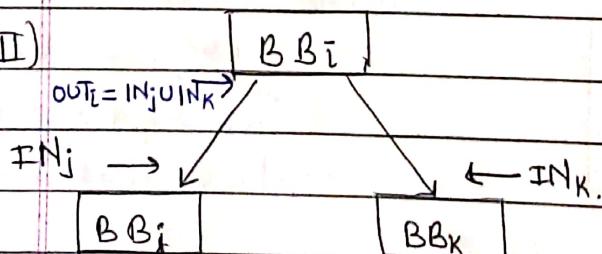
I).



II).



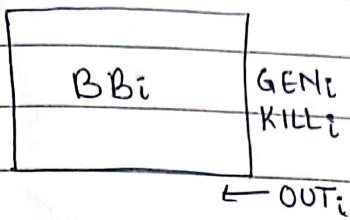
III)



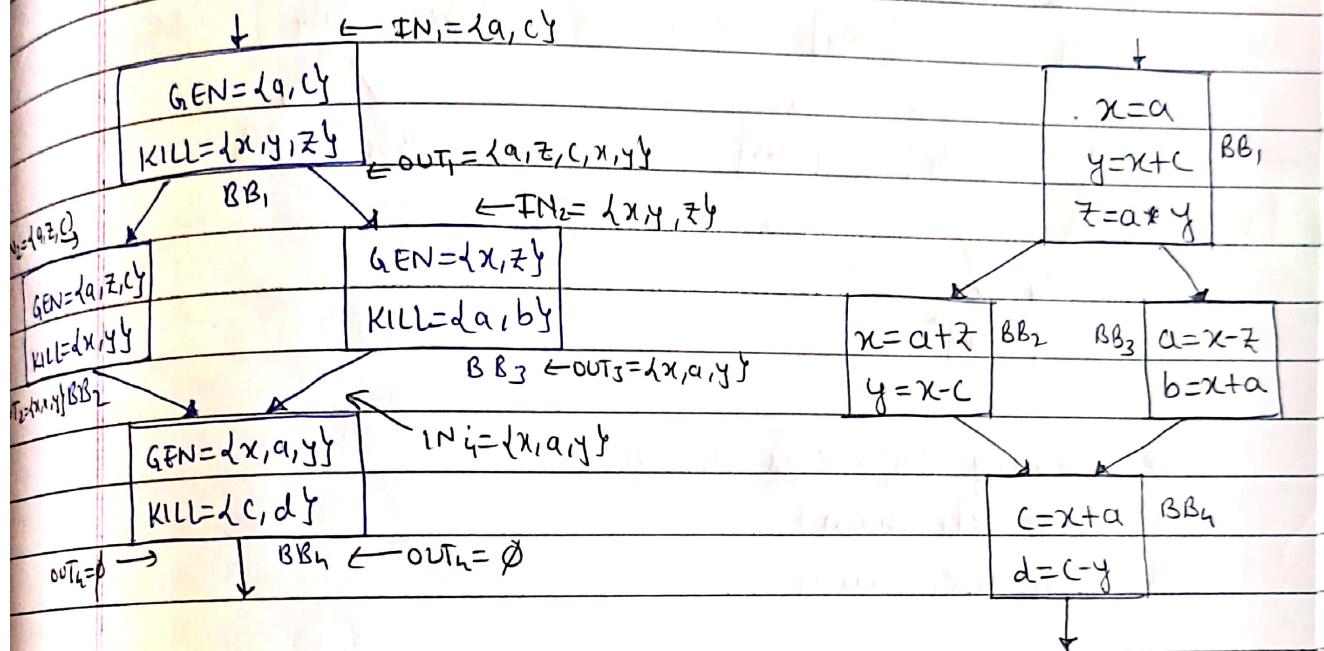
$$OUT_i = IN_j \cup IN_k.$$

How to compute IN set for BB_i?

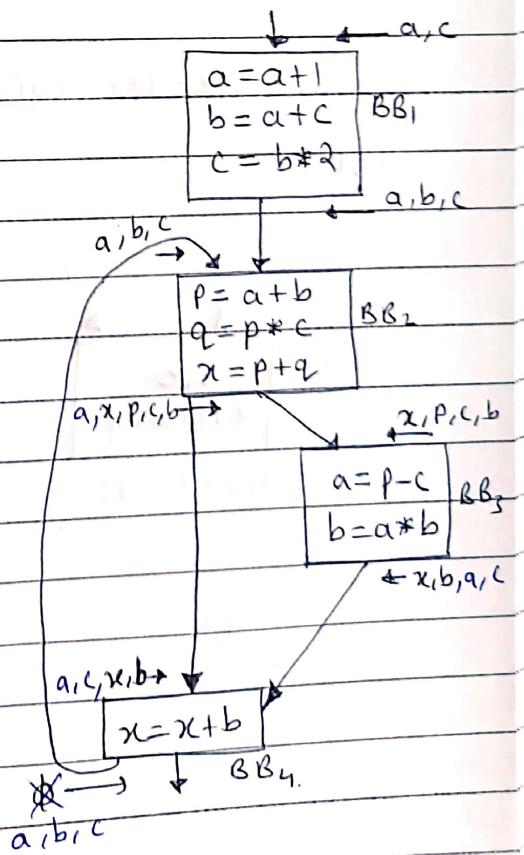
$$IN_i = (OUT_i - KILL_i) \cup GEN_i$$



↳ shortcut OK ~~H~~ GDB.

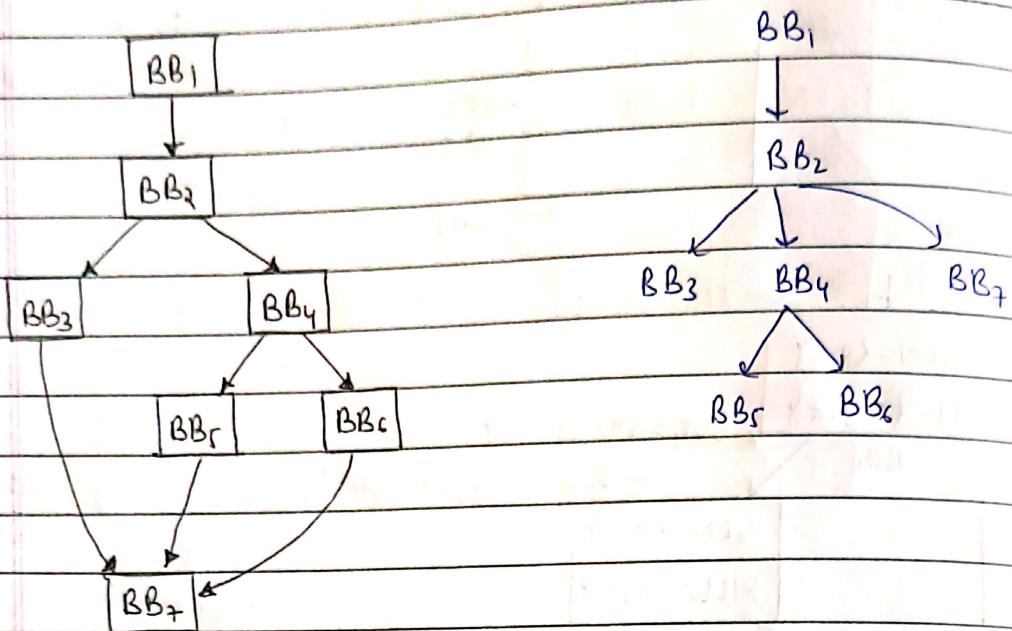


	GEN Set	KILL set.	OUT Set.	IN Set.
B ₁	a, c	a, b, c	a, b, c	a, c
B ₂	a, b, c	p, q, x	i) x, p, c, b ii) a, x, p, c, b	i) a, b, c ii) a, c, b
B ₃	p, q b	a, b	i) x, b ii) x, b, a, c	i) x, p, c, b ii) x, p, c, b
B ₄	x, b	x	i) \emptyset ii) a, b, c	i) \emptyset , c, b ii) a, c, x, b



→ Control Flow Graph

Dominator Tree



• Reaching Definitions Analysis-

- I) Use - Defs chain
- II) Def - use, chain

• Use-def chain

$x_1 = \dots$
Def₁

Use
 x reading

$\phi(x) = \{x_1, x_2\}$

• Def-use chain

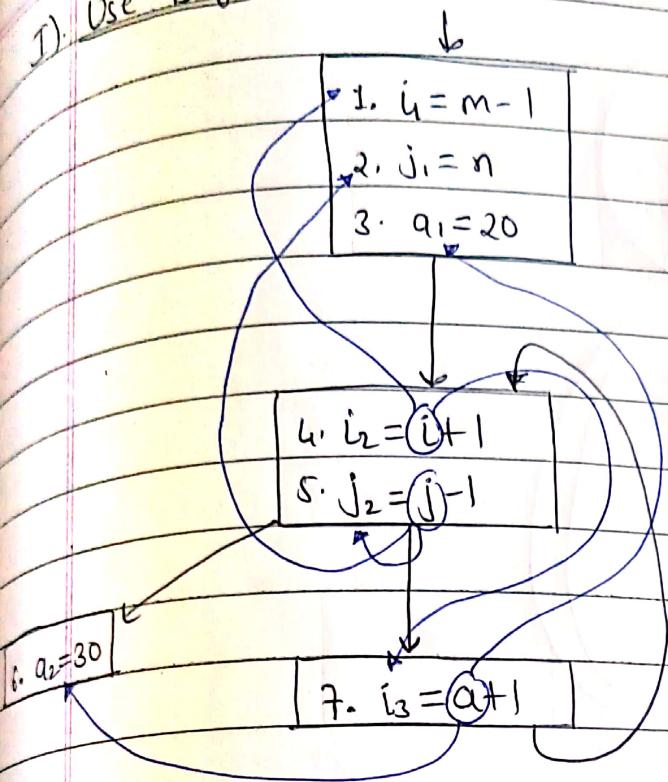
use
reading x .

Def
 $x = \dots$

use

ready x .
reading x

I). Use -Def chains -



$$\Phi(j) = \{j_1, j_2\}$$

at statement 5.

$$\Phi(i) = \{i_1, i_3\}$$

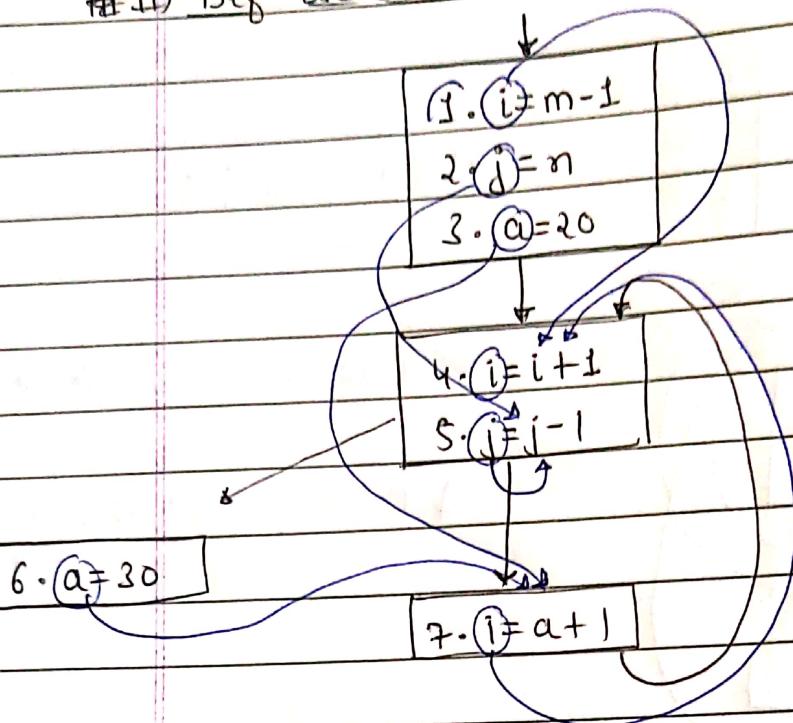
at statement 4.

$$\Phi(a) = \{a_1, a_2\}$$

at statement 7.

statement	USE	Definitions.
1	m	-
2	n	-
4	i	i_1, i_3
5	j	j_1, j_2
7	a	a_1, a_2

II) Def-Use chain



Statement	Def	Used
1	$i = m - 1$	i
2	$j = n$	j
3	$a = 20$	a
4	$i = i + 1$	i
5	$j = j - 1$	j
6	$a = 30$	a
7	$i = a + 1$	i