COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)

END Semester Examination

Programme: B.Tech [TY]	**Semester: VI**
Course Code: IT-1707	**Course Name: Design & Analysis of Algorithm**
Branch: Information Technology	**Academic Year: 2017-2018**
Duration: 3Hrs	**Max Marks: 60**

Student PRN No. □□□□□□□□□

Instructions:

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of stationery, calculator etc. not allowed.
5. Write your PRN Number on Question Paper.

			Marks	CO	PO
Q 1	A	Find the time complexity for the following code	2M	1	a

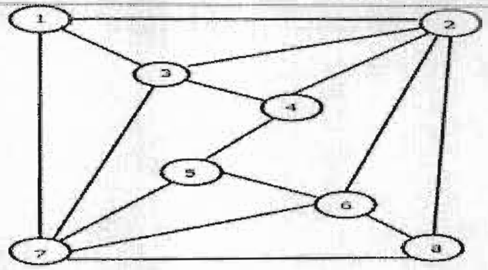
i)

```
input : n
  s ←0
  i ←0
   while i < n do
        j ←0
      while j < i do
         s ← s + 1
        j ← j + 1
    i ← i + 1
```

ii)

```
input : n
  if n = 1 then
     return 1
  else
     return n * factorial(n - 1)
```

			Marks	CO	PO
Q1	B	If we perform merge sort on the array of integer [5,3,8,9,1,7,0,2,6,4]. What is the number at the 6th position of partially sorted array [array start with index as 0] after the outermost two recursive calls have completed? [i.e. just before the very last merge step]	2M	2,4	a,d, g,i, k
Q1	C	Write out the table for these inputs, and find the longest common subsequence: S1= GCCCT S2= GCGCAAT	2M	2,3, 4	a,d, g,i, k
Q1	D	Consider a link-list of n elements which is pointed by an external pointer. What is the time taken to delete the element which is successor of the element pointed to by a given pointer?	2M	1,4	a, g, i, k
Q1	E	State which complexity can be executed in polynomial and non- polynomial time. i) $O(2^{n/2})$ ii) $O(n)$ iii) $O(n^2 2^n)$ iv) $O(n\log n)$	2M	4	g, i, k
Q1	F	Below graph can be colored in k colors. State the value of k.	2M	2,3, 4	a,d, g,i, k



Q 2 A Let A be an array containing n numbers (positive and negative). — 1,2, 3 — a, d

i) Develop an algorithm that finds two indices where $0 \leq i \leq j \leq n$ such that $S_{i,j} = \sum A[k]$ is maximized. — 4M

For example, in an array A = [2, -5, 6, -2, -3, 1, 5, -6], the sub-array A [2, 6] has the sum S2, 6 = 6 -2 -3 + 1+ 5= 7 and no other sub-array contains elements that sum to a value greater than 7, so for input A, the algorithm should give output as (2, 6). — 2M

ii) The time complexity of your algorithm should be O (nlogn). Justify briefly why your algorithm achieves this running time and why it gives the correct answer.

B Show what the time complexity [**use recursion tree**] are for each the algorithm.Let n be the number of bits in input — 1 — a

i) An algorithm that solves the problem by dividing problem into 2 sub-problem of size $\frac{9}{10}$ n and $\frac{1}{10}$ n, recursively solve each sub-problem and combine then into a solution in O(n) time. — 2M

ii) An algorithm that solves the problem by dividing problem into eight sub-problem of size $\frac{n}{2}$, recursively solve each sub-problem and combine then into a solution in O (n^2) time. — 2M

Q2 C Inventory operator maintain a collection of lists that support the following operations. — 2M — 1,4 — a, g,i, k

(a) insert (item, list): insert item into list (cost = 1).

(b) Sum (list): sum the items in list, and replace the list with a list containing one item that is the sum (cost = length of list).

i) Use the Potential Method to find that the amortized cost of an insert operation and sum operation

Q3 A Draw the schedule for summer camp in such way that you can schedule max activities in a single day. — 4M — 2,4 — a,d, g,i, k

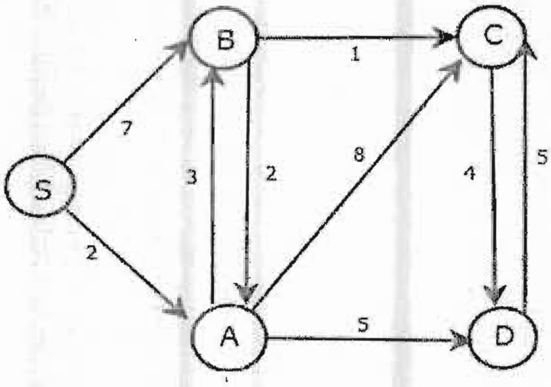
Activities	Timing [Starting time – Finish time]	Cost Rs:
Swimming	8.00 am – 9.30 am	1000/-
Horse Ridding	10.00 am – 11.00 am	1200/-
Canoeing	9.30 am – 11.30 am	1100/-
Kayaking	11.00 am - 1.30 pm	600/-
Lunch	1pm - 2.30pm	500/-
Drawing	12.00 noon – 1pm	1300/-
Pottery	2.00 pm - 3.30 pm	1500/-
Tea Break	11.30 am - 12.00 noon	50/-

Q3 B You are given a sorted array A of size n. For simplicity, assume that n is a power of three. You want to find whether a value a is present in A. You first compute the two indices M1 = n/3 and — 1 — a

M2 = 2n/3. If a < A[M1], recursively search for a in A[0 . . .M1 − 1], else if a < A[M2], recursively search for a in A[M1 . . .M2 − 1], else recursively search for a in A[M2 . . . n − 1]. This is called the ternary search algorithm.

Write the recurrence equation and the time complexity of this algorithm. 2M

Q3 C Find the all pair shortest path for the beilow graph 6M 2,4 a,d, g,i, k



Q4 A State whether this is true or false with justification 4M 4 g,i, k

i) The problems 3-SAT and 2-SAT are NP-complete and in P, respectively.
ii) If a new problem is in NP and we can reduce a known NP-Complete Problem to it, then the new problem is NP -Complete
iii) The problem of determining whether there exists a cycle in an undirected graph is in NP.
iv) Hamiltonian Cycle problem belongs to P set.

Q4 B Apply the branch and bound algorithm to solve TSP for following cost 8M 2,4 a,d, g,i, k

	V1	V2	V3	V4	V5	V6
V1	∞	5	∞	6	5	4
V2	5	∞	2	4	3	∞
V3	∞	2	∞	1	∞	∞
V4	6	4	1	∞	7	∞
V5	5	3	∞	7	∞	3
V6	4	∞	∞	∞	3	∞

Q5 A **Problem:** You are given infinite coins of denominations v1, v2, v3,.....,vn and a sum S. The coin change problem is to find the minimum number of coins required to get the sum S. 2,3, 4 a,d, g,i, k

For example: Say you went to a shop and bought 4 toffees. It cost you Rs. 5 in total. So, You gave Rs. 10 to the shopkeeper. The shopkeeper had enough number of coins Rs. 1 coins, Rs. 2 coins and Rs. 3 coins with him.
Now, the goal is: The shopkeeper has to make change for Rs. 5 using least number of coins from the available denominations coins (1, 2 and 3)
Output: 2 [Minimum 2 coins of Rs 2 and Rs 3]

i) This problem can be best solved using which design paradigm? Why 1M

ii) Suppose you have coins of denominations 1, 3 and 4. You use a greedy method, in which you choose the largest denomination coin which is not greater than the remaining sum. For which of the following sums, will the algorithm produce an optimal answer? 2M

a) 14 b) 10 c) 6 d) 100

iii) Suppose you have coins of denominations [n=3] → Rs1, Rs 2 and Rs 5. Need to make change for an amount Rs. 6 So, we can write A = 6

Array index →	1	2	3
Di → [Rs]	1	2	5

4M

Complete the below table using the following formula

$$C[p] = \begin{cases} 0 & \text{if } p = 0 \\ \min_{i:d_i \le p}\{1 + C[p - d_i]\} & \text{if } p > 0 \end{cases}$$

where C[p] denotes the minimum number of coins required to make change for an amount p. where p = 1, 2, ..., A

P →	0	1	2	3	4	5	6
Cp → [Rs]	0	1	1				

To make change of amount Rs. 6 You will need minimum ___ coins and the coins will be ________ 1M

Q5 B Consider primality testing code fragment — 1,4 — a, g,i, k

```
Prime(int n)
{
 int i;
 for(int i=2; i<number; i++)
    if(number % i==0)
    {
     printf (“not prime\n”);
     return;
    }
  printf (“prime\n”);
}
```

i) Stepwise calculate the time complexity of the algorithm and state with appropriate reason whether algorithm in P or NP. 2M

ii) Show that if any NP-complete problem is not in P then no NP-complete problem is in P. 2M

Q5 OR Q6

Q6 A SUDOKU Puzzle : The objective is to fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids that compose the grid (also called "boxes", "blocks", "regions", or "sub-squares") contains all of the digits from 1 to 9. — 2,3, 4 — a,d, g,i, k

i) Design an algorithm for this puzzle. 2M

ii) Write recurrence equation for puzzle and find time and space complexity 2M

iii) Define a state representation for Sudoku answer search. How do you check a Sudoku game to make sure all the rules have been respected? 2M

		iv) If the puzzle begins with 28 digits filled, what is L, the length of the shortest path to goal using your representation?	2M		
Q6	B	Suppose code was implemented using stack and new updated wants it to be implemented using queue. Solution to this was to implement two stack S1 and S2. To insert into queue, push element into stack S1. To remove from queue first check if S2 is empty and if so dump S1 into S2. Then pop from S2, Suppose each push and pop costs 1 unit of work, so that performing a dumb when S1 has n elements cost 2n units i) Suppose that (starting from an empty queue) we do 4 insertion, then 2 removals, then 4 more insertions and then 2 more removal. Show what the total cost of these 12 operations is, and how many elements are in each stack at the end? ii) Suppose we perform an arbitrary sequence of insertions and removals, starting from an empty queue. Use the accounting method to find the amortized cost of each operation? That is, charge $x for insertion and $y for deletion. What are x and y? Prove your answer.	2M 2M	1,4	a g,i, k
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