

# Deliverable 1: Final Year Dissertation

Sahil Pattni

BSc. Computer Science (Honours)

Supervisor: Neamat El Gayar

December 3, 2020

## Declaration

I, Sahil Manojkumar Pattni, confirm that this work submitted for assessment is my own and is expressed in my own words. Any uses made within it of the works of other authors in any form (e.g., ideas, equations, figures, text, tables, programs) are properly acknowledged at any point of their use. A list of the references employed is included.

Signed: Sahil Manojkumar Pattni

Date: December 3, 2020

## Abstract

In this digital age, data is being generated at an exponential rate, with data analytics being used by corporations and small businesses alike to produce insights, reduce costs, optimize operations and increase profits. Generating association rules allow us to find non-intuitive associations that bring new insights to the management, allowing them to leverage this information to maximize their profits. A prime example would be the '*Beers and Diapers*' case [1], where a company looked at their point-of-sale data and found a strong association between beers and diapers being co-purchased, which seems rather unintuitive.

In this study, we will extract a minimum spanning tree (MST) from a data set using machine learning techniques. We will then use this minimum spanning tree to segment products together, and produce association rules and extract the most interesting ones. The resulting ruleset will be compared to rules generated by the established Apriori Algorithm [2], which is the foundation of association rule mining. Additionally, we will study how the grouping of the MST compares to clustering algorithms, and how the structure of our MST would change before and after promotional events, leading to insights that may help the management in such firms make better informed decisions about the type of promotions they would like to run.

# Contents

<b>1</b>	<b>Introduction</b>	<b>4</b>
1.1	Context . . . . .	4
1.2	Aims . . . . .	4
1.3	Objectives . . . . .	5
<b>2</b>	<b>Background</b>	<b>6</b>
2.1	Core Concepts . . . . .	6
2.1.1	Graphs . . . . .	6
2.1.2	Minimum Spanning Trees . . . . .	6
2.1.3	Market Basket Analysis and Apriori Rule . . . . .	7
2.2	Related Work . . . . .	9

# 1 Introduction

## 1.1 Context

For business which deal with the sale of a heterogeneous physical assets - such as groceries, hypermarkets and select retail outlets - operations such as inventory management and product placement play an instrumental role in determining the business' financial success. These involve asking questions such as:

- Which products should be placed at the entrance of the store? Which should be placed closer to the exit?
- Which products will benefit the most by being placed at eye-level?
- Which products should be placed next to each other to maximize the purchase volume?

One way to find optimal solutions for such questions is to employ the use of Association Rule Mining (also known as Market Basket Analysis). This set of techniques assess frequent itemsets (e.g. from sales data) and generate association rules between products. Several algorithms and techniques exist for association rule mining, such as the Apriori Algorithm and FP-Growth [3]. One problem with these algorithms is that they tend to generate an enormous amount of rules, of which many of the rules themselves are large. This makes it grossly inconvenient for the end-user to retrieve any actionable information from the results.

What we will do is construct a network of products - where each vertice represents a product or product category, and an edge between two vertices represents the simultaneous occurrence of the the two products. The result is a graph (i.e. a network), whose architecture is visually informative of the products and their associations. The insights that can be gained from the MST can help the management of the mentioned businesses to maximize their profit.

## 1.2 Aims

The aim of this study is to study the effectiveness of a minimum spanning tree as a market basket analysis tool - from product clustering to association rule mining. Additionally, we will determine how the architecture of the MST will change during and after large events such as promotions. The model will be tested on a relatively large dataset of sales data.

### 1.3 Objectives

The research objectives for this project have been laid out below, in the order that they will be carried out.

1. Acquire a suitable dataset upon which the MST can be constructed.
2. Explore and evaluate MST extraction algorithms.  
(While Prim's [4][5] and Kruskal's [6] algorithms are the most commonly used methods to extract the MST from a graph, research has been conducted on more efficient ways to extract the MST using machine learning techniques such as Artificial Neural Networks and K-Nearest-Neighbors, and these will be discussed in the literature review (see Section 2.2)).
3. Generate an undirected graph  $G(E, V)$  where the edges  $E$  are the correlation values between the product vertices.
4. Extract a minimum spanning tree from this graph using the technique determined in step 2.
5. Analyze this MST and use its architecture to determine product clusters and generate association rules.
6. Generate association rules using the Apriori and FP-Growth algorithm and compare them in both their time complexity and their '*interestingness*' (i.e. the unintuitive rules they generate).
7. Validate the product grouping present in the MST against a clustering algorithm (e.g. such as the K-Means algorithm).

## 2 Background

### 2.1 Core Concepts

#### 2.1.1 Graphs

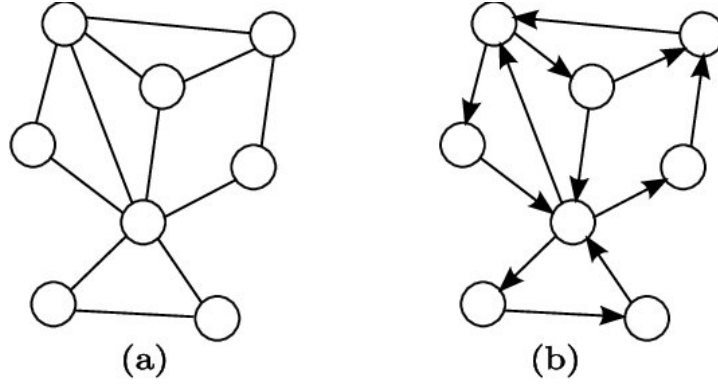


Figure 1: Undirected and Directed Graphs

In discrete mathematics and more specifically - graph theory, a graph is a data structure that contains a set of nodes (i.e. vertices) connected by lines (i.e. edges). These edges may be directed - such as in Figure 1:(a), or undirected - such as in Figure 1:(b). A graph  $G$  with a set of vertices  $V$  and a set of edges  $E$  can be represented via the notation  $G = (V, E)$ . For the purposes of our research, we will be focusing on undirected graphs, where the weight between two vertices is the same in both directions.

#### 2.1.2 Minimum Spanning Trees

Given an undirected  $G = (E, V)$ , a *spanning tree* can be described as a subgraph that is a tree which includes all the vertices  $V$  of  $G$  with the minimum number of edges required. A *minimum spanning tree* (MST) is the spanning tree with the smallest sum of edge weights. This means that if the graph has  $n$  vertices, each spanning tree - including the minimum spanning tree - will have  $n - 1$  edges.

### 2.1.3 Market Basket Analysis and Apriori Rule

Market Basket Analysis (MBA), also known as Affinity Analysis, is a data mining and analysis technique that identifies co-occurrence patterns between products purchased together, and produces association rules for these products as such:  $\{A\} \rightarrow \{B\}$  which implies a strong relationship between the purchase of product  $A$  and product  $B$ . In the case of our *Beers and Pampers* example in [1], the association rule could be represented as  $\{Beers\} \rightarrow \{Pampers\}$ . The contents of a purchase basket (i.e. the contents of a customer's basket when they check out) is called a *itemset*, which - as the name suggests - is a set of all the items in the basket. For example, if a customer had bought detergent, bread and soda, the itemset would be  $\{bread, detergent, soda\}$ . Association rules are generated by looking at different combinations of the itemset (e.g.  $\{bread, soda\} \rightarrow \{detergent\}$  and  $\{soda\} \rightarrow \{bread\}$ ). The equation [7] to calculate the number of rules for an itemset of length  $d$  is as follows :

$$number\ of\ rules = \sum_{k=1}^{d-1} \left( \binom{d}{k} * \sum_{j=1}^{d-k} \binom{d-k}{j} \right) \quad (1)$$

Analyzing the equation, it becomes apparent that the problem with association rule generation from itemsets is that the number of rules produced grows exponentially with the size of the itemset, as we can see in Figure 2, using Equation 1.

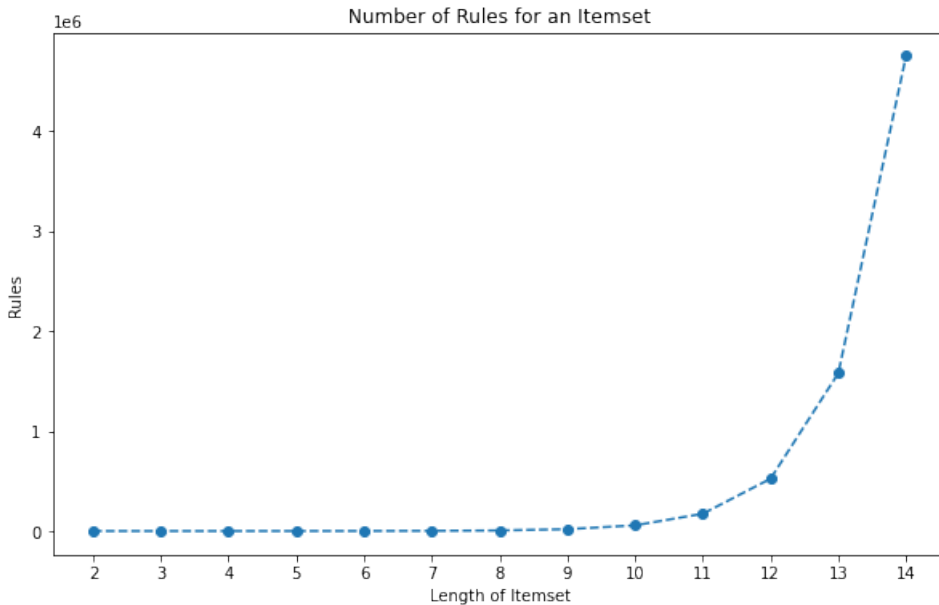


Figure 2: Number of Association Rules for an Itemset

Clearly, checking all itemsets in a database would be computationally expensive, and so we

only look at *frequent itemsets* - itemsets that have a support value above a minimum threshold  $min_{support}$ . Even so, checking each itemset's support score via brute force is unacceptably time consuming, therefore we can optimize the search for frequent itemsets using the Apriori Principle [2] which states that *all subsets of a frequent itemset must be frequent*. To understand how the Apriori Principle can be applied here, we first look at three key metrics:

### Support

The support of a set of products is the fraction of transactions in which these set of products are present. For example, for a list of transactions  $T$ , for the product  $X$  where  $T(X)$  denotes that the set of transactions in which  $X$  was present:

$$support(\{A\}) = \frac{T(A)}{T}$$

and similarly,

$$support(\{A\} \rightarrow \{B\}) = \frac{T(A, B)}{T}$$

denotes the support for the rule  $\{A\} \rightarrow \{B\}$ . We can prune rules with a low support as they indicate a rule does not occur enough to draw any reasonable conclusions from.

### Confidence

Confidence is the measure of how likely a product will be in a basket given that another product is in it. That is to say, the confidence of a rule  $\{A\} \rightarrow \{B\}$  is the conditional probability that  $\{B\}$  occurs in the basket given that  $\{A\}$  is present. The confidence of a rule can be denoted as:

$$confidence(\{A\} \rightarrow \{B\}) = \frac{T(A, B)}{T(A)} \equiv \frac{support(\{A\} \rightarrow \{B\})}{support(\{A\})}$$

### Lift

The lift of a rule  $\{A\} \rightarrow \{B\}$  is the rise (i.e. **lift**) that  $\{A\}$  gives to our  $confidence(\{A\} \rightarrow \{B\})$ .

$$lift(\{A\} \rightarrow \{B\}) = \frac{confidence(\{A\} \rightarrow \{B\})}{support(\{B\})}$$

To make this easier to understand, imagine that  $confidence(\{A\} \rightarrow \{B\}) = 0.5$ , and  $support(\{B\}) = 0.4$ . This means that the presence of  $\{A\}$  increases the probability of  $\{B\}$  being in the same basket by 25% ( $\frac{0.5}{0.4} = 1.25$ ), therefore providing us with a lift value of 1.25. A lift value below 1 would indicate that the occurrence of  $\{A\}$  in a basket decreases the likelihood of  $\{B\}$  occurring



in the same basket (i.e. a low product association).

As we established above, the Apriori Principle states that all subsets of a frequent itemset must be frequent. The Apriori Principle is a result of the *anti-monotone property of support* [8], which means  $\{A, B, C\}$ ,  $\text{support}(\{A, B\}) \geq \text{support}(\{A, B, C\})$ . We can use this to prune the frequent itemsets much more efficiently, because if  $\text{support}(\{A, B\}) < \text{min}_{\text{support}}$ , then any itemset containing the set  $\{A, B\}$  will also fall below  $\text{min}_{\text{support}}$ . Once the frequent itemsets have been pruned, association rules can be generated from the remaining itemsets. The resulting association rules can be even further pruned by removing those that fall below a confidence threshold  $\text{min}_{\text{confidence}}$ . Finally, the remaining rules can be ranked according to their lift to find the rules with the highest associations.

## 2.2 Related Work

**M. A. Valle** [9] proposed a methodology to study the structure and behavior of consumer market baskets from the topology of a minimum spanning tree which represented the interdependencies between products. The input to their proposed methodology was a correlation matrix between the set of all one-hot encoded purchase vectors such that each vector denoted the presence or non-presence of each product from the dataset in that vector. The dataset used for the MST construction was a list of transactions from a large supermarket chain branch in Santiago, Chile. When building this correlation matrix, the authors opted to use the Pearson's Coefficient (which is equivalent to the coefficient  $\phi$  for binary data such as theirs) over the traditionally used Jaccard distance to compute the similarity between the binary product vectors, as the former provides both a positive and negative association between products. Additionally, they used the distance function  $d_{ij} = \sqrt{2(1 - \phi_{ij})}$  to transform the correlation matrix into a distance measurement (i.e. the weight of the edges). The authors constructed a MST for 220 product subcategories, and noted that there was a significant level of grouping between product sub-categories that belonged to the same parent category. To remove edges from the MST that were not statistically significant, the authors used the mutual information [10] measure  $\sum_{x,y} \log_2 \frac{r(x,y)}{p(x)q(y)}$  between product subcategories  $p$  and  $q$ , and were able to prune 14 edges, all of which were connected to a terminal node, therefore effectively pruning 14 vertices from the MST too. To identify the most influential regions of the MST, the authors defined an influence zone of distances that were in the  $10^{th}$  percentile. To generate meaningful association rules, for each MST product  $i$ , the authors ran a search for the set of all association rules  $R_i$  such

that  $P_i \rightarrow P_j (i \neq j)$ . Then from the resulting set of rules, they searched for rules that obeyed  $P_i \rightarrow P_m$  where  $m$  a product node connected to the product  $i$  in the minimum spanning tree. For both resulting sets of rules for each product, the mean of their lift scores were observed, and the authors determined that the rules that were reinforced by the MST had a higher mean, and that a majority of these rules had a lift score above the 90<sup>th</sup> percentile.

To identify the clusters each of the products should be identified under, the authors constructed a hierarchical tree using the average linkage clustering method, and by using an unspecified cut distance, they were able to produce 17 taxonomic groups (i.e. clusters).

### **Critical Analysis**

(To Be Completed) (talk about how the MST can potentially only do 1 → 1 association rules?)

## References

- [1] D. J. Power, Ed., *Ask Dan! What is the "true story" about data mining, beer and diapers?* Vol. 3 Nov. 2002, ISSN: 23. [Online]. Available: <http://www.dssresources.com/newsletters/66.php>.
- [2] R. Agrawal and R. Srikant, "Fast algorithms for mining association rules," *Proceedings of the 20th International Conference on Very Large Databases*, pp. 487–489, 1994.
- [3] J. Han, J. Pei, and Y. Yin, "Mining frequent patterns without candidate generation," *Proceedings of the 2000 ACM SIGMOD International Conference on Management of Data*, pp. 1–12, 2000. DOI: 10.1145/342009.335372.
- [4] R. C. Prim, "Shortest connection networks and some generalizations," *Bell System Technical Journal*, vol. 6, pp. 1389–1401, 6 1957. DOI: 10.1002/j.1538-7305.1957.tb01515.x.
- [5] V. Jarník, "O jistém problému minimálním," *Práce Moravské Přírodovědecké Společnosti*, pp. 57–63, 1930.
- [6] J. B. K. Jr., "On the shortest spanning subtree of a graph and the traveling salesman problem," *Proceedings of the American Mathematical Society*, vol. 7, pp. 48–50, 1956. DOI: 10.1090/S0002-9939-1956-0078686-7.
- [7] J. C. Pennsylvania. (2020). "Generating association rules," Juniata College Pennsylvania, [Online]. Available: <http://faculty.juniata.edu/rhodes/ml/assocRules.html>.
- [8] P.-N. Tan, M. Steinbach, A. Karpatne, and V. Kumar, "Definition 6.2," in *Introduction to Data Mining*, 2nd ed. 2019, ch. 6, p. 334, ISBN: 9780134080284.
- [9] M. A. Valle, G. A. Ruz, and R. Morr s, "Market basket analysis: Complementing association rules with minimum spanning trees," *Expert Systems with Applications*, vol. 97, May 2018. DOI: 10.1016/j.eswa.2017.12.028.
- [10] T. M. Cover and J. A. Thomas, *Elements of information theory*. 2006.