L.P.P

linear programming language.

Types

1. Basic solution
2. Standard form
3. Simplex method
x 4. penalty method / Big M
5. Dual form
x 6. Duality
x 7. Dual simplex method

Type-(I) Basic Solutions.

Ex 1. Find all basic solutions of

$$\left.\begin{array}{l} \max\ z = x_1 + 3x_2 + 3x_3 \\ x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 3x_2 + 5x_3 = 7 \end{array}\right\} x_1, x_2, x_3$$

दो values में से एक ZERO

$z_{max} = 5$
$x_1 = 2$
$x_2 = 1$
$x_3 = 0$

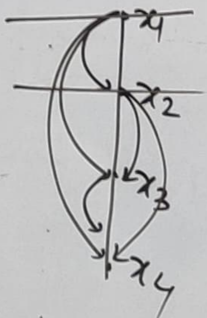
Sol^n

No. of Solution	Non Basic Variable	Basic Variable	Equations and values of B.V.	Is the sol^n feasible? (+ve)	Is the sol^n Degenerate? (Either value is 0)	value of Z	optimal Solution
1	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1,\ x_3 = 2$	N	–	–	–
2.	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1,\ x_3 = 1$	Y	N	4	No
3	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2,\ x_2 = 1$	Y	N	5	Yes

Type-(I) Basic Solutions.

Ex 2 Find all basic solutions of

$$x_1+2x_2+4x_3+x_4=7,\quad 2x_1-x_2+3x_3-2x_4=4.$$



Sol^n:

No. of sol^n	Non-Basic variable	Basic variable	Equations and values of B.V		Is the sol^n feasible?	Is the sol^n Degenerate?
1	$x_1, x_2=0$	x_3, x_4	$4x_3+x_4=7$, $3x_3-2x_4=4$	$x_3=1.6$, $x_4=0.4$	Y	N
2	$x_1, x_3=0$	x_2, x_4	$2x_2+x_4=7$, $-x_2-2x_4=4$	$x_2=6$, $x_4=-5$	N	—
3	$x_1, x_4=0$	x_2, x_3	$2x_2+4x_3=7$, $-x_2+3x_3=4$	$x_2=0.5$, $x_3=1.5$	Y	N
4	$x_2, x_3=0$	x_1, x_4	$x_1+x_4=7$, $2x_1-2x_4=4$	$x_1=4.5$, $x_4=2.5$	Y	N
5	$x_2, x_4=0$	x_1, x_3	$x_1+4x_3=7$, $2x_1+3x_3=4$	$x_1=-1$, $x_3=2$	N	—
6	$x_3, x_4=0$	x_1, x_2	$x_1+2x_2=7$, $2x_1-x_2=4$	$x_1=3$, $x_2=2$	Y	N.

Type-(II) standard form

z

1. Objective function (z) must be maximise,
   If not then multiply by (-1) & denoted it by z'
2. RHS must be Positive, If not then multiply by (-1)
3. For unrestricted variable (x_n), replace it by $x_n' - x_n''$
4. Inequality ($\leq \geq$) to equality ($=$)
   for less than equal ($\leq$) = Add slack variable.
   for greater than equal ($\geq$) = Sub slack variable.

Type-(II) Standard form

Ex. 3. Convert the L.P.P. to canonical form.

max $z = 2x_1 - x_2 + 3x_3$

$$\left.\begin{array}{l} 2x_1 + x_2 - 4x_3 \le 30 \\ 4x_1 - x_2 + x_3 \le 20 \\ x_1 - 5x_2 - 7x_3 \ge 2 \end{array}\right\} \begin{array}{l} +s_1 \\ +s_2 \\ -s_3 \end{array}$$

$x_1 \to x_1' - x_1''$

$x_3 \to x_3' - x_3''$

$x_2 \ge 0$, x_1 & x_3 unrestricted.

Soln: Max $z = 2(x_1' - x_1'') - x_2 + 3(x_3' - x_3'') + 0s_1 + 0s_2 + 0s_3$

$$2(x_1' - x_1'') + x_2 - 4(x_3' - x_3'') + s_1 + 0s_2 + 0s_3 = 30$$

$$4(x_1' - x_1'') - x_2 + (x_3' - x_3'') + 0s_1 + s_2 + 0s_3 = 20$$

$$(x_1' - x_1'') - 5x_2 - 7(x_3' - x_3'') + 0s_1 + 0s_2 - s_3 = 2$$

Type-(II) Standard form

Ex 4 Convert the L.P.P. to canonical form.

Min $Z = 2x_1 + x_2 + 4x_3$

$-2x_1 + 4x_2 \le 4 \rightarrow +s_1$

$x_1 + 2x_2 + x_3 \ge 5 \rightarrow -s_2$

$2x_1 + 3x_3 \le 2 \rightarrow +s_3$

$x_1, x_2 \ge 0$, x_3 is unrestricted.

max $Z' = -Z = -2x_1 - x_2 - 4x_3$

Sol^n: Max $Z' = -Z = -2x_1 - x_2 - 4(x_3' - x_3'') + 0s_1 + 0s_2 + 0s_3$

$-2x_1 + 4x_2 + 0(x_3' - x_3'') + s_1 + 0s_2 + 0s_3 = 4$

$x_1 + 2x_2 + x_3' - x_3'' + 0s_1 - s_2 + 0s_3 = 5$

$2x_1 + 0x_2 + 3(x_3' - x_3'') + 0s_1 + 0s_2 + s_3 = 2$

100 %.

Type-III Simplex Method

Steps 1. Convert the L.P.P. into standard form.

2. Simplex Table

- i) Key column: Most negative value in Z
- ii) Ratio: Divide RHS by key column.
- iii) Key Row: minimum Positive value
- iv) Key element: common element
- v) Incoming variable: column

OR

- Outgoing variable = Row

2
1
1
3

III Matrix calculation

$AX = B$

i) Make key element 1 by direct division.

ii) Make other elements of key column ZERO

IV Repeat the procedure till Z is POSITIVE

Type-III Simplex Method

Ex 5. Solve the L.P.P. by simplex method

maximise: $Z = x_1 + 9x_2 + x_3$

$x_1 + 2x_2 + 3x_3 \le 9$

$3x_1 + 2x_2 + 2x_3 \le 15$

$x_1, x_2, x_3 \ge 0$

Sol^n: a) standard form

max $Z = x_1 + 9x_2 + x_3 + 0s_1 + 0s_2$

$1x_1 + 2x_2 + 3x_3 + s_1 + 0s_2 = 9$ ✓

$3x_1 + 2x_2 + 2x_3 + 0s_1 + s_2 = 15$ ✓

$Z - x_1 - 9x_2 - x_3 - 0s_1 - 0s_2 = 0$

(b) Simplex Table

No. of Interation	Basic Variable	Coefficients of x_1	x_2	x_3	s_1	s_2	RHS	Ratio
0	Z	-1	$\boxed{-9}$	-1	0	0	0	—
s_1 leaves	s_1	1	2	3	1	0	9	$9/2 = 4.5$
x_2 enters.	s_2	3	2	2	0	1	15	$\frac{15}{2} = 7.5$

Key column

Type-III Simplex Method

(c) MATRIX calculation

$$AX = B$$

same = −
diff = +

$$\begin{array}{c} \\ R_2 \\ \\ \end{array}\begin{bmatrix} -1 & -9 & -1 & 0 & 0 \\ 1 & \boxed{2} & 3 & 1 & 0 \\ 3 & 2 & 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 15 \end{bmatrix}$$

$$\frac{R_2}{2}$$

$$\begin{array}{c} R_1 \\ H \\ R_3 \end{array}\begin{bmatrix} -1 & -9 & -1 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 3 & 2 & 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{9}{2} \\ 15 \end{bmatrix}$$

$$1R_1 + 9R_2,\ 1R_3 - 2R_2$$

$$\begin{bmatrix} \frac{7}{2} & 0 & \frac{25}{2} & \frac{9}{2} & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 2 & 0 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \frac{81}{2} \\ \frac{9}{2} \\ 6 \end{bmatrix}$$

(b) Simplex Table

No. of Interation	Basic Variable	coefficients of x_1	x_2	x_3	s_1	s_2	RHS
1	Z	$\frac{7}{2}$	0	$\frac{25}{2}$	$\frac{9}{2}$	0	$\frac{81}{2}$
	x_2	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{1}{2}$	0	$\frac{9}{2}$
	s_2	2	0	−1	−1	1	6

$$Z_{max} = \frac{81}{2}$$

$$x_1 = 0$$

$$x_2 = \frac{9}{2}$$

$$x_3 = 0$$

Ex: 6. Solve by simplex method

min $z = x_1 - 3x_2 + 2x_3$

$3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$.

→ (a) Standard form

max $z' = -z = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

$3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 7$

$-2x_1 + 4x_2 + 0x_3 + 0s_1 + s_2 + 0s_3 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + s_3 = 10$

(b)

No. of iteration	Basic variable	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
0	z'	1	-3	2	0	0	0	0	0
	s_1	3	-1	2	1	0	0	7	-7
s_2 leaves	s_2	-2	4	0	0	1	0	12	3
x_2 enter	s_3	-4	3	8	0	0	1	10	3.33

III Matrix calculation

$$AX = B$$

same = −

diff = +

$$\begin{bmatrix} 1 & -3 & 2 & 0 & 0 & 0 \\ 3 & -1 & 2 & 1 & 0 & 0 \\ -2 & \boxed{4} & 0 & 0 & 1 & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 12 \\ 10 \end{bmatrix}$$

$$\frac{R_3}{4}$$

$$\begin{array}{c} R_1 \\ R_2 \\ H \\ R_4 \end{array} \begin{bmatrix} 1 & -3 & 2 & 0 & 0 & 0 \\ 3 & -1 & 2 & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 \\ -4 & 3 & 8 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 3 \\ 10 \end{bmatrix}$$

$1R_1 + 3R_3$, $R_2 + R_3$, $1R_4 - 3R_3$

$$\begin{bmatrix} -\frac{1}{2} & 0 & 2 & 0 & \frac{3}{4} & 0 \\ \frac{5}{2} & 0 & 2 & 1 & \frac{1}{4} & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 \\ -\frac{5}{2} & 0 & 8 & 0 & -\frac{3}{4} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 3 \\ 1 \end{bmatrix}$$

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II Simplex Table.

Iteration No	Basic variable	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
1	Z'	$-\frac{1}{2}$	0	2	0	$\frac{3}{4}$	0	9	
s_2 leaves	s_1	$\frac{5}{2}$	0	2	1	$\frac{1}{4}$	0	10	
	x_2	$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3	
	s_3	$-\frac{5}{2}$	0	8	0	$-\frac{3}{4}$	1	1	

III Matrix calculation

$$AX = B$$

$$\begin{array}{c} \\ R_2 \\ \\ \\ \end{array}\begin{bmatrix} -\frac{1}{2} & 0 & 2 & 0 & \frac{3}{4} & 0 \\ \frac{5}{2} & 0 & 2 & 1 & \frac{1}{4} & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 \\ -\frac{5}{2} & 0 & 8 & 0 & -\frac{3}{4} & 1 \end{bmatrix}\begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 3 \\ 1 \end{bmatrix}$$

$$\frac{R_2}{5/2}$$

$$\begin{array}{c} R_1 \\ H \\ R_3 \\ R_4 \end{array}\begin{bmatrix} -\frac{1}{2} & 0 & 2 & 0 & \frac{3}{4} & 0 \\ 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{1}{10} & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 \\ -\frac{5}{2} & 0 & 8 & 0 & -\frac{3}{4} & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 3 \\ 1 \end{bmatrix}.$$

$$2R_1 + R_2,\quad 2R_3 + R_2,\quad \frac{2R_4}{5} + R_2$$

$$\begin{bmatrix} 0 & 0 & \frac{24}{5} & \frac{2}{5} & \frac{8}{5} & 0 \\ 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{1}{10} & 0 \\ 0 & 2 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 4 & \frac{2}{5} & -\frac{1}{5} & \frac{2}{5} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 4 \\ 10 \\ 22/5 \end{bmatrix}.$$

II Simplex Table.

Iteration No	Basic variable	x_1	x_2	x_3	s_1	s_2	s_3	RHS	Ratio
1	Z'	0	0	24/5	2/5	8/5	0	22	
s_1 leaves	x_1	1	0	4/5	2/5	1/10	0	4	
x_1 enters.	x_2	0	2	4/5	2/5	3/5	0	10	
	s_3	0	0	4	2/5	-1/5	2/5	22/5	

$$Z'_{max} = 22$$

$$-Z_{min} = 22$$

$$\boxed{Z_{min} = -22}$$

$x_1 = 4$

$x_2 = 10$

$x_3 = 0$