PRACTICAL-14

Second Order Runge-Kutta Methods

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1. Modified Euler Method

Ques-1 Using **Runge-Kutta Method of second order [Modified Euler Method]**. Find approximate solution for the initial value problem

$x'(t) = 1 + \frac{x}{t}$ **,1 ≤ t ≤ 6, x(1)=1.** use **n=5** discrete points at equal space**.**

compare the approximate solution with the exact solution **x(t)=t(1+ln(t)).**

In[6]:=
```
ModifiedEulerMethod[a0_, b0_, n0_, f_, alpha_, actualSolution_] :=
  Module[{a = a0, b = b0, n = n0, h, ti, K1, K2},
   h = (b - a) / n;
   ti = Table[a + (j - 1) h, {j, 1, n + 1}];
   wi = Table[0, {n + 1}]; wi[[1]] = alpha;
   actualSol = actualSolution[ti[[1]]];
   difference = Abs[actualSol - wi[[1]]];
   OutputDetails = {{0, ti[[1]], alpha, actualSol, difference}};
   For[i = 1, i ≤ n, i++,
    K1 = h f[ti[[i]], wi[[i]]];
    K2 = h f[ti[[i]] + h / 2, wi[[i]] + K1 / 2];
    wi[[i + 1]] = wi[[i]] + K2;
    actualSol = actualSolution[ti[[i + 1]]];
    difference = Abs[actualSol - wi[[i + 1]]];
    OutputDetails = Append[OutputDetails,
      {i, N[ti[[i + 1]]], N[wi[[i + 1]]], N[actualSol], N[difference]}];];
   Print[NumberForm[
     TableForm[OutputDetails, TableHeadings →
       {None, {"i ", "ti ", "wi ", "actSol(ti) ", "Abs(wi-actSol(ti))"}}], 6]];
  ];
f[t_, x_] := 1 + x / t;
actualSolution[t_] := t (1 + Log[t]);
ModifiedEulerMethod[1, 6, 5, f, 1, actualSolution]
```

i	ti	wi	actSol(ti)	Abs(wi-actSol(ti))
0	1	1	1	0
1	2.	3.33333	3.38629	0.052961
2	3.	6.2	6.29584	0.0958369
3	4.	9.40952	9.54518	0.135654
4	5.	12.873	13.0472	0.174174
5	6.	16.5385	16.7506	0.212029

Ques-2 Using **Runge-Kutta Method of second order [Modified Euler Method]**. Find approximate solution for the initial value problem

$x'(t) == t^2 - x$ **,** $0 \le t \le 0.8$**, x(0)=1.** use **n=8** discrete points at equal space**.**
compare the approximate solution with the exact solution **$x(t)=2-e^{-t}-2\,t+t^2$**

In[14]:=
```
Clear[f]
f[t_, x_] := t^2 - x;
actualSolution[t_] := 2 - Exp[-t] - 2 t + t^2;
ModifiedEulerMethod[0, 0.8, 8, f, 1, actualSolution]
```

i	ti	wi	actSol(ti)	Abs(wi-actSol(ti))
0	0.	1	1.	0.
1	0.1	0.90525	0.905163	0.000087418
2	0.2	0.821451	0.821269	0.000182003
3	0.3	0.749463	0.749182	0.000281602
4	0.4	0.690064	0.68968	0.000384406
5	0.5	0.643958	0.643469	0.000488906
6	0.6	0.611782	0.611188	0.000593849
7	0.7	0.594113	0.593415	0.000698206
8	0.8	0.591472	0.590671	0.000801141

2. Heun Method

Ques-3 Using **Runge-Kutta Method of second order [Heun Method]**. Find approximate solution for the initial value problem

$x'(t) = 1 + \frac{x}{t}$, **$1 \le t \le 6$, x(1)=1.** use **n=10** discrete points at equal space.

compare the approximate solution with the exact solution **x(t)=t(1+ln(t)).**

In[10]:=
```
HeunMethod[a0_, b0_, n0_, f_, alpha_, actualSolution_] :=
  Module[{a = a0, b = b0, n = n0, h, ti, K1, K2},
   h = (b - a) / n;
   ti = Table[a + (j - 1) h, {j, 1, n + 1}];
   wi = Table[0, {n + 1}]; wi[[1]] = alpha;
   actualSol = actualSolution[ti[[1]]];
   difference = Abs[actualSol - wi[[1]]];
   OutputDetails = {{0, ti[[1]], alpha, actualSol, difference}};
   For[i = 1, i ≤ n, i++,
    K1 = h/2 * f[ti[[i]], wi[[i]]];
    K2 = h/2 * f[ti[[i]] + h, wi[[i]] + 2 * K1];
    wi[[i + 1]] = wi[[i]] + K1 + K2;
    actualSol = actualSolution[ti[[i + 1]]];
    difference = Abs[actualSol - wi[[i + 1]]];
    OutputDetails = Append[OutputDetails,
      {i, N[ti[[i + 1]]], N[wi[[i + 1]]], N[actualSol], N[difference]}];];
   Print[NumberForm[
     TableForm[OutputDetails, TableHeadings →
       {None, {"i ", "ti ", "wi ", "actSol(ti} ", "Abs(wi-actSol(ti))"}}], 6]];
  ];
f[t_, x_] := 1 + x / t;
actualSolution[t_] := t (1 + Log[t]);
ModifiedEulerMethod[1, 6, 10, f, 1, actualSolution]
```

i	ti	wi	actSol(ti}	Abs(wi-actSol(ti))
0	1	1	1	0
1	1.5	2.1	2.1082	0.00819766
2	2.	3.37143	3.38629	0.0148658
3	2.5	4.76984	4.79073	0.0208856
4	3.	6.26926	6.29584	0.0265728
5	3.5	7.8526	7.88467	0.0320674
6	4.	9.50774	9.54518	0.0374407
7	4.5	11.2256	11.2683	0.0427327
8	5.	12.9992	13.0472	0.0479676
9	5.5	14.823	14.8761	0.0531608
10	6.	16.6922	16.7506	0.0583228

Ques-4 Using **Runge-Kutta Method of second order [Heun Method]**. Find approximate solution for the initial value problem

$x' = tx^3 - x$, $0 \le t \le 1$, x(0)=1. use **n=4** discrete points at equal space.

In[21]:=
```
HeunMethod[a0_, b0_, n0_, f_, alpha_] :=
  Module[{a = a0, b = b0, n = n0, h, ti, K1, K2},
   h = (b - a) / n;
   ti = Table[a + (j - 1) h, {j, 1, n + 1}];
   wi = Table[0, {n + 1}]; wi[[1]] = alpha;
   actualSol = actualSolution[ti[[1]]];
   difference = Abs[actualSol - wi[[1]]];
   OutputDetails = {{0, ti[[1]], alpha}};
   For[i = 1, i ≤ n, i++,
    K1 = h/2 * f[ti[[i]], wi[[i]]];
    K2 = h/2 * f[ti[[i]] + h, wi[[i]] + 2 * K1];
    wi[[i + 1]] = wi[[i]] + K1 + K2;
    OutputDetails = Append[OutputDetails,
      {i, N[ti[[i + 1]]], N[wi[[i + 1]]]}];];
   Print[NumberForm[
     TableForm[OutputDetails, TableHeadings → {None, {"i ", "ti ", "wi "}}], 8]];
  ];
f[t_, x_] := t x^3 - x;
HeunMethod[0, 1, 4, f, 1]
```

i	ti	wi
0	0	1
1	0.25	0.79443359
2	0.5	0.64782017
3	0.75	0.532024
4	1.	0.4359812