Peer Monitoring in Microcredits: the Structure of Borrower Social Network Matters

Bahar Rezaei,* Sriram Dasu,† Reza Ahmadi‡

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Abstract

(1) Problem definition: Microfinance institutions (MFIs), ubiquitous in the developing world, have had noticeable successes lending to borrowers at the bottom of the pyramid. One of the most practiced lending methods by MFIs is group lending with joint liability. We study the sustainability of group lending under joint liability contracts when borrowers are subject to strategic default. The benevolent lender (a non-profit) optimizes the borrower welfare while covers her costs of lending, and social ties between borrowers serve as monitoring tools.

(2) Academic/Practical relevance: The observation that social relationships promote cooperation is long-standing in the socio-economic literature. Group lending implicitly leverages borrowers' social ties by having self-selected groups of borrowers enter into a joint liability contract. We model borrowers' social network structures as patterns of peer monitoring and explains how different network structures may affect the lending outcome. To the best of our knowledge, our work is the first theoretical study of this problem.

(3) Methodology: The methodology employed in this paper is the reverse game theory or mechanism design.

(4) Results: The results show that a higher average centrality and connectivity in the borrower network as well as a higher degree of borrower discount factor allow for more favorable contract terms (i.e., a higher loan ceiling and a lower repayment amount). Further, we argue

*Anderson School of Management, University of California, Los Angeles, Los Angeles, CA 90095, USA; rezaei@ucla.edu

†Marshall School of Business, University of Southern California, Los Angeles, CA 90089, USA; dasu@marshall.usc.edu

‡Anderson School of Management, University of California, Los Angeles, Los Angeles, CA 90095, USA; reza.ahmadi@anderson.ucla.edu

that while for smaller groups, the highly decentralized ring networks may have higher performance (i.e., a higher chance of game continuation and a higher repayment rate) than the highly centralized star networks, for larger groups, star networks outperform the rings. Intuitively, although decentralized networks are less susceptible to the failure of one member, they have a disadvantage in the level of monitoring they provide. This disadvantage manifests itself to a greater degree in larger groups.

(5) Managerial implications: The setting studied in this paper can emerge in a variety of team cooperation or social dilemma contexts that deal with monitoring, and thus the results are valid wherever the interests of the individual and of the group are not aligned and agents exert externalities on one another.

Keywords: Joint Liability, Strategic Default, Monitoring, Social Networks, Graph Theory

1 Introduction

The observation that social relationships promote cooperation is not a new one: the role of network structures and interpersonal relationships has been studied extensively in socioeconomics literature.[1] Social proximity might help to mitigate tendencies to renege; and thus, socially closer agents (e.g., friends, friends of friends) may be able to sustain a higher level of cooperation even without enforceable contracts. Individuals might have more incentives to cooperate with more central partners, maybe because central partners have better monitoring tools, can impose larger reputational punishments by dispersing information widely, or by abstaining from future interactions. Microfinance Institutions (MFIs), widespread in the developing world, are perceived to work based on this principle.[2] They leverage borrowers' social capital and local information to lend to borrowers that are at the bottom of the pyramid and are otherwise unbankable.

One of the most studied and practiced lending models used by MFIs is lending to self-selected groups of borrowers under joint liability contract.[3] In a jointly liable group, if one member does not repay his loan, the other members are held responsible to repay for him. The joint liability lending

[1]See, for example, Leider et al. (2009); Jackson et al (2012); Banerjee et al., (2013); and Breza et al (2019).

[2]Microfinance pioneers, Grameen Bank and its founder Mohammad Yunos, were awarded the Nobel Peace Prize in 2006 for their successful initiative in Bangladesh, aiming to "put poverty in museums".

[3]Recently, some of older MFIs have shifted from joint liability to individual lending. Nevertheless, group lending with joint liability remains the main channel for microcredit lending by most MFIs. Moreover, MFIs moving away from explicit joint liability have retained some of its traditions such as organizing borrowers into groups and holding group meetings on a regular basis (Haldar and Stiglitz, 2016; and De Quit et al., 2018).

scheme is believed to improve repayment rates in two ways: it incentivizes borrowers not only to help their group members repay their installments but also to avoid delinquent behavior that may trigger social sanctions (Besley and Coate, 1995). Microfinance literature argues that joint liability creates an effective way of screening, monitoring, and enforcement of contracts (see e.g., Ghatak, 1999; and Armendáriz and Morduch, 2010).

In this paper, we analyze a microcredit lending game that is similar to the lending model of Grameen Bank (a non-profit organization). A benevolent lender (she) provides loans to a self-selected group of micro-entrepreneur borrowers with joint liability. Then the loans are invested in projects that are disjointed and have an equal chance of success. Note that the social network theory literature believes that people tend to homophilous, meaning that they tend to have stronger ties with people who are similar to themselves (Borgatti and Halgin, 2011). Thus, it is not unrealistic to assume that the chance of project success for all members of a self-selected group is equal. We also assume that borrowers can monitor each others' project output at least to some extent, but the lender cannot. This is also a reasonable assumption considering that group members are self-selected and have better information about each other than the lender. The entire group will be deprived of future loans if the total repayment obligation is not met. Such a setting creates a risk-sharing arrangement that is subject to ex-post moral hazard or *strategic default*: a borrower who is expected to repay may prefer not to. We assume that, in the game played between group members, they play a grim-trigger strategy. Meaning that they stop repaying their loans, as well as the loans of their defaulting peers, as soon as they detect strategic default.[4]

Our study is primarily concerned with finding what kind of network structure is more reliable to return the loan successfully. For example, while a highly centralized network provides a high level of monitoring, it is susceptible to the behavior of its central member(s). In reverse, a highly decentralized network is not dependent on any single member but does not provide a good level of monitoring either, especially when groups grow large. If and how the lender could adjust the contract terms in response to differences in the structures of social networks in order to make the lending a success? What network structure(s) qualifies for more favorable contract terms and under

[4]This is a plausible assumption as both real-world experience and scientific evidence suggest that in social dilemma situations, many people have a strong aversion against being the "gullible". Fehr and Gächter (2000), for example, argue that "those who cooperate may be willing to punish free-riding, even if this is costly for them and even if they cannot expect future benefits from their punishment activities". For another example, see Carpenter (2007).

what circumstances?

We model the borrower's social network as a set of pre-existing relationships, such as friendships. Network links serve as a monitoring instrument ensuring that borrowers live up to their responsibilities. We assume that borrowers with a *direct* social tie can fully monitor the realized project output of each other while borrowers with an *indirect* social tie, who are connected through friends in common, can only partially monitor each other's project output. Having access to a higher level of monitoring allows a borrower to reveal his truthfulness and verify the truthfulness of his peers and thus receive and give support with a higher probability when needed. Such a setting allows us to investigate how variation in the structure of borrowers' social network (who is a friend of whom) relates to their collective levels of cooperation and consequently to the lending outcome.

We derive the optimal joint liability contract depending on the structure of the borrowers' social network that is captured by the chance of a successful group repayment, the expected repayment of the group, and the highest expected repayment among borrowers. In our first proposition, we show that the contract terms are positively affected by the chance of game continuation and the expected repayment of the group but negatively affected by the highest expected repayment of individual members. Now the question will be what kind of network structures may allow for a high chance of game continuation and high expected group repayment while non of its members has an overly high expected repayment.

Analyzing different network structures, we center our focus on two specific structures: the highly centralized *star* structure, in which a core borrower has direct links with all other group members while the others are only indirectly connected to each other through the core borrower, and the highly decentralized *ring* structure, in which every borrower has two direct links and two indirect links with other group members. The reason for this focus, apart from simplifying the analysis, is as follows. Many theoretical studies provide evidence for the formation of ring and star structures (or convergence to these structures) as likely evolving structures in strategic settings (see e.g. the seminal work of Bala and Goyal, 2000). Ring- and star-like topologies have also received strong support from empirical and experimental studies (see e.g., Baker and Faulkner, 1993; Goeree et al., 2009; Kumar et al., 2010; and Falk and Kosfield, 2012).

It is rather trivial and is also confirmed by our results that a complete borrower network, in which all borrowers are connected through direct social ties, provides full monitoring and is the best network to induce cooperation among borrowers and improve their collective welfare. However, in the real world, borrower networks can be less than complete, especially in urban areas that people have less reliable information about their neighbors and friends. Thus, it makes sense to look at the effect of different social network structures on the borrower's behavior and adjust the lending contracts accordingly.

Our results suggest that a higher average centrality and connectivity improves the performance of the network, both in terms of a higher chance of game continuation and a higher expected repayment. Such networks are more robust to the failure of a few members. In other words, both the number and the patterns of the links in a network can affect the outcome of the lending game. Moreover, connected networks, in which every member is fully monitored by at least one other member of the group, have a higher performance than disconnected networks. Thus, any network with some connections is better off removing a borrower, who is poorly monitored by other group members except when projects are highly risky.

Perhaps the most important result of our study is showing that both the highly centralized star structure and the highly decentralized ring structure have advantages and disadvantages. We prove that in larger groups, the star structure can maintain a higher level of performance than the same-sized ring structure, while in smaller groups, the ring structure performs better. The intuition behind this result can be explained as follows. In a star group, all members can monitor each other's project outcomes at least indirectly. Thus, they are able to verify the truthfulness of their defaulting members and maintain a higher level of cooperation. But the star group is highly dependent on the central member and falls apart if he fails on his repayment. In a ring group, no member is overly important in terms of robustness, and thus any individual player's default is less of a threat to coordination. But in a ring group, some members are out of each other's monitoring scope and cannot tell apart strategic defaults from non-strategic defaults. In larger groups, the monitoring disadvantage of a ring structure with respect to a star structure intensifies as no member is able to monitor more than four other members. However, in small groups, the monitoring disadvantage of a ring structure fades away and its robustness advantage comes into play.

We also prove that enlarging the group can contribute to its performance, but group size can be enlarged limitedly. Larger group size has two counteracting effects on borrower welfare. On one hand, a large group can ceteris paribus provide a borrower with a higher chance of being paid off and continue receiving loans in the next round. On the other hand, a larger group can put a successful borrower in charge of repaying for many other defaulting members. We find an upper bound for the size of the group depending on the chance of project success and the borrower discount factor.

2 Literature Review

This research is inspired by the growing management literature focused on issues arising from the execution of *socially and environmentally responsible operations* and *value chain innovations* in developing economies (see e.g. Sodhi and Tang, 2014; Lee and Tang, 2018; de Zegher et al., 2019).

The model in this paper builds on our previous work Rezaei et al. (2017). In our previous paper, we assumed an ideal situation, in which all members of the group can fully monitor each other's project returns. Here, we relax that condition and assume that group members have different levels of monitoring on each other's project outcomes depending on how close they are in their social network. Such a setting allows us to prove that for groups of the same size, different network structures result in different lending outcomes. Put it differently, Rezaei et al. (2017) is a special case of this research, in which borrowers are connected via a complete graph.

Our paper contributes to the microfinance literature that views borrowers' social ties as collateral that can be used to circumvent moral hazard problem and incentivize repayment (see e.g. Zeller, 1998; Ghatak and Guinnane, 1999; Wydick, 1999; Hermes et al., 2005; Cassar et al., 2007; Paal and Wiseman, 2011; Cason et al., 2012; Feigenberg et al., 2013; and Breza and Chandrasekhar, 2019). We differ from this literature by explicitly modeling social ties as a means of peer monitoring. In our model, a closer social tie between two borrowers allows them to better monitor each other's project outcome, which in turn induces cooperation because it provides a higher chance of both being backed by peers in case of non-strategic default and being punished by peers in case of strategic default.

The extant microfinance literature provides evidence that stronger social ties between group members can improve the repayment rate. Our research complements this literature by discussing that apart from the prevalence and the strength of the social ties in a borrowing group, the existing patterns of these ties (who can monitor whom) can also affect the borrowers' cooperative behavior and thus the lending outcome.[5]

Closer to our paper, Breza and Chandrasekhar (2019) experimentally study how peer monitoring improves cooperation in borrower groups in rural Karnataka, India. They focus their study on whether the monitor's effectiveness depends on her social network position. Apart from the methodology, we differ from this research by considering the higher commitment to repayment as a measure of performance; while they look at the higher commitment to saving.

Apart from the microfinance literature, our research relates to several streams of research. Our research is closely related to the literature that highlights the effect of network structures on team cooperation and performance. Examples include but are not limited to Rulke and Galaskiewicz (2000), Sparrowe et al., (2001), Reagans and Zuckerman (2001), Collins and Clark (2003), Cummings and Cross (2003), Carson et al. (2007), Gloor et al. (2008), and more recently Grund (2012) and Belavina and Girotra (2012a; 2012b). Closer to our paper, Belavina and Girotra (2012a) consider the supply chain with decentralized decision-making and continuing trade. They explain how more decentralized supply chains in emerging economies often outperform vertically integrated supply chains without employing any formal contracts and by relying on relationships between players. Similar to us, they argue that with decentralization, each player has a smaller effect on the system, and thus their opportunistic behavior is less of a risk to the coordination of the system. We differ from them by emphasizing the importance of monitoring and arguing that in larger groups, monitoring the disadvantage of a decentralized ring structure with respect to a centralized star structure intensifies as no member is able to monitor more than four other members.

A further relevant literature for this paper is the literature that explores the interplay between network structures and default risk. See, for examples, Cont et al. 2010, Allen et al. (2010), Elliot et al. (2014), Acemoglu et al. (2015), Huang et al. (2016), Chen et al. (2016), Gandy and Veraart (2017), Beaman and Dillon (2018), Schuldenzucker et al. (2019).

Finally, this paper is related to the literature on risk-sharing in social networks. For recent examples, we could mention Bramoullé and Kranton (2007b), Attanasio et al. (2012), Ambrus et al. (2014), Mobius and Rosenblat (2016). For a recent review of this line of research in the context

[5] A similar idea has been examined in the context of public good games by various researchers. For example, see the experimental works of Carpenter and Kariv (2012) and Leibbrandt et al. (2015). The former proves that a connected network with less than the complete structure can achieve a high level of cooperation; and the latter finds that the structure of the network significantly affects the contributions and punishment decisions of its members.

Notation	Description
n	group size
δ	borrower's discount factor
(L, R)	contract (L, R) determines the amounts of loan L and repayment R
α	the chance of project success
Y^H, Y^L	project yields either a high outcome $Y^H > 0$ or a low outcome $Y^L = 0$
S, F	true signals of success and failure
S^*, F^*	observed signals of success and failure
θ_1, θ_4	accuracy of signals
θ_2, θ_3	inaccuracy or noisiness of signals
σ	the probability that an indirect tie pays for j when he cannot pay
β	the probability that a peer with no tie with j pays for him when he cannot pay
$k_j R$	expected repayment of a successful borrower j
v_j^R	expected utility of a cooperative borrower j (he pays when he can)
Q	probability of a successful group repayment / game continuation
V^R	expected utility of a cooperative group (members pay when they can)

Table 1: Nomenclature

of developing countries see also Breza et al. (2019).

3 Model

Suppose borrowers of a lending group are connected through a social network. In this network, there are two types of connections that provide different levels of monitoring: direct and indirect. Borrowers with a direct social tie can fully monitor each other's project returns (friends); while borrowers with an indirect social tie can monitor each other's project return through other direct connections (friends of friends). An indirect tie provides a monitoring level that is a slightly less reliable than a direct tie. The sequential group lending under study can be modeled as a stationary infinitely repeated game between a benevolent lender and a group of n borrowers. Each period of the game has the following steps.

1. Each borrower receives an individual contract (L, R), specifying the amount of loan L, which is large enough to cover the costs of project, and the repayment R that consists of the primary loan L and interest ϵ. Interest ϵ is to cover the costs of lending and does not generate profit for the lender. We assume that borrower welfare is increasing in the amount of loan L; that is, the expected return of a larger loan is higher than the expected repayment.

2. Each borrower invests the entire L in his project that will either succeed with a chance of $\alpha \in [0, 1]$ and yields a high return $Y^H > 0$ or not succeed with a chance of $1 - \alpha$ and yields a low return $Y^L = 0$. We assume that projects do not differ in their riskiness (i.e., the chance of success α is the same for all borrowers) and each borrower always invests in the same project.

3. Each borrower learns about his project outcome.

4. Borrowers announce their project outcomes. Members announcing low are not willing to pay.

5. Each member receives a noisy private signal about other members' project return. The lender cannot monitor project returns.

6. Members make their repayment decisions based on whether or not their private signal is consistent with the announced outcomes.

7. If the total group repayment is equal to nR or more, the group receives future financing; otherwise, the lender will exclude the entire group from future loans. In periods in which no loans are extended, borrowers' utility will be zero.

Two types of defaults are possible: *strategic default,* in which the borrower does not repay although he had high outcome Y^H, and *non-strategic default* as a result of obtaining low outcome resulting from bad luck. Although, the lender is unable to monitor whether a borrower defaults strategically or non-strategically, borrowers are at least partially able to monitor strategic defaults of their peers.

We assume borrowers continue cooperating until they come to believe that someone defaults strategically. Borrowers play grim-trigger if they detect strategic default; that is, they stop paying for the defaulting peer,[6] and the game ends.

If a borrower j defaults non-strategically, then a successful direct connection of j, who can fully monitor j's outcome, would receive a perfect signal about j's truthfulness and would pay for him; while a successful indirect tie, who has a slightly worse level of monitoring, would receive a slightly noisy signal about j's truthfullness and pay for him with probability $\sigma < 1$. Accordingly, an indirect friend who is connected to j through a chain of k friends would receive a noisy signal

[6] This is a plausible assumption as both real world experience and scientific evidence suggest that in social dilemma situations, many people have a strong aversion against being the "gullible". Fehr and Gächter (2000), for example, argue that "those who cooperate may be willing to punish free-riding, even if this is costly for them and even if they cannot expect future benefits from their punishment activities". For another example, see Carpenter (2007).

about j's truthfullness and pay for him with probability $\sigma^k < 1$. This is a reasonable assumption as information achieved through longer chains of friends is more likely to be distorted. All other members of the group, who have neither direct nor indirect ties with j, would receice signals that are less informative and pay for him with a probability $\beta < \sigma$. For simplicity of our calculation, we assume that for any $k \geq 2$, $\sigma^k = \beta$.

Suppose true signals of success and failure are denoted by S and F; and observed signals of success and failure are denoted by S^* and F^*. If S is realized, it will be observed as S^* with probability θ_1 and as F^* with probability $\theta_2 = 1 - \theta_1$; respectively, if F is realized, it will be observed as S^* with probability θ_3 and as F^* with probability $\theta_4 = 1 - \theta_3$. In summary,

$$\begin{array}{ccc} & S^* & F^* \\ S & \theta_1 & \theta_2 = 1 - \theta_1 \\ F & \theta_3 & \theta_4 = 1 - \theta_3 \end{array}.$$

Assume signals are unbiased in the following sense: $\alpha\theta_1 + (1-\alpha)\,\theta_3 = \alpha$ and $\alpha\theta_2 + (1-\alpha)\,\theta_4 = (1-\alpha)$. Thus, given α and one of the θ_i, we could calculate the rest of θ_i's.

Borrowers with a direct social tie receive a perfect signal about each others project return, and thus $\theta_1 = \theta_4 = 1$ and $\theta_2 = \theta_3 = 0$. Borrowers with an indirect social tie receive a slightly noisy signal about each other's project return. If borrower j fails in his project and announces failure, he will be believed by his indirect ties with probabilty $\theta_4 = \sigma$; as signals are unbiased, if borrower j succeeds but announces failure, he will still be believed by his indirect ties with probability $\theta_2 = \frac{(1-\alpha)(1-\sigma)}{\alpha}$;

$$\begin{array}{ccc} & S^* & F^* \\ S & \theta_1 = 1 - \frac{(1-\alpha)(1-\sigma)}{\alpha} & \theta_2 = \frac{(1-\alpha)(1-\sigma)}{\alpha} \\ F & \theta_3 = 1 - \sigma & \theta_4 = \sigma \end{array}.$$

In a similar way, borrowers with no social tie receive a more noisy signal about each other's thrutfulness. Assuming that $\theta_4 = \beta$, we have

$$\begin{array}{ccc} & S^* & F^* \\ S & \theta_1 = 1 - \frac{(1-\alpha)(1-\beta)}{\alpha} & \theta_2 = \frac{(1-\alpha)(1-\beta)}{\alpha} \\ F & \theta_3 = 1-\beta & \theta_4 = \beta \end{array}.$$

The benevolent lender strives to maximize the payoff of each borrower contingent on the following criteria. First, each borrower must be willing to accept a loan (the repayment amount must be affordable); second, each successful borrower must have the incentive to repay for himself and for each defaulting peer (in the worst case that all other members default, he must still be willing to repay for the entire group); and third, the lender must break even, meaning that she must maintain a sustainable lending operation over the entire loan portfolio by charging the appropriate repayment amount. We restrict our attention to stationary contracts, in which the bank's choice variables are assumed to be independent of the past history and of the period.

Borrower Networks: We assume that information achieved through indirect ties has a lower value than the one achieved through a direct tie. This is a reasonable assumption as indirect monitoring can distort information. We further assume that the value of the information diminishes as the length of the chain of ties grows. Suppose, each arc reduces the value of information by a factor σ, then information transmitted over two arcs has a value $\sigma < 1$, and information transmitted over a path with $k+1$ arcs has a value σ^k. Information between two nodes that are neither directly and nor indirectly connceted has a value $\beta < \sigma$. For simplicity of our calculation, we assume that for any $k \geq 2$, $\sigma^k = \beta$.

In a network of n nodes, *centrality* of a node j is the sum of values of shortest paths to all other nodes divided by $n-1$, and average centrality of a network is the average over node centralities in the network. In our model, the shortest path between any two nodes can be of value 1, σ, or β. For example, in a ring network of four nodes (tetrad), centrality of each node is $\frac{2+\sigma}{3}$ as each node has two direct ties and an indirect tie. The average centrality for the ring network is also $\frac{2+\sigma}{3}$. For more examples, see Figures 2 and 3 (in Subsection 4.1).

In a network of n nodes, *connectivity* of a node j is the sum of link values that the network loses if j is removed divided by $n-1$, and average connectivity of a network is the average over node connectivities in the network. For example, connectivity of each node in a ring tetrad network is $\frac{2+2\sigma}{3}$ as removing a node would remove two direct and two indirect ties from the network. The average connectivity for a ring network is $\frac{2+2\sigma}{3}$. For more examples, see Figures 2 and 3 (in Subsection 4.1).

4 Simple Joint Liability (SJL) Contract

In this section, we characterize feasible simple joint liability contracts, assuming that borrowers play grim-trigger as punishment strategy. We examine how contract terms change in response to changes in the network structure. All group members receive the same contract.

The expected utility of a cooperative borrower j (he pays if he can) with a discount factor δ in a period in which he obtains financing is

$$v_j^R = \alpha\left(Y^H - k_j R\right) + Q\delta v_j^R,$$

where Q is the probability of a successfull group repayment and receiving a loan in the next round when all members of the group are cooperative, and $k_j R$ is the borrower j's expected repayment when he has a high project return. The above equation can be rewritten as

$$v_j^R = \frac{\alpha Y^H - \alpha k_j R}{1 - \delta Q}.$$

Accordingly, the total expected utility of a cooperative group in a period in which borrowers obtain financing will be

$$V^R = \sum_{j=1}^{n} v_j^R = \frac{n\alpha Y^H - \sum_{j=1}^{n} \alpha k_j R}{1 - \delta Q}.$$

A cooperative group successfully repays nR with probability Q. Thus, the total expected repayment of the group $\sum_{j=1}^{n} \alpha k_j R$ equals QnR; that is,

$$\sum_{j=1}^{n} \alpha k_j = nQ. \tag{1}$$

$$V^R = \frac{n\alpha Y^H - nQR}{1 - \delta Q}. \tag{2}$$

The benevolent lender seeks to maximize V^R, but she still needs to get her money back to sustain the lending over time. Each borrower succeeds with probability α and pays $k_j R$. Thus, the lender has to design the lending contract such that the expected repayment of the group, $\alpha \sum_{j=1}^{n} k_j R$, covers the total loan granted to borrowers and the costs of lending services; that is,

$$Q\,(nR) \quad \geq \quad nL\,(1+\epsilon)\,. \tag{3}$$

Moreover, the lender has to ensure that the expected repayment is affordable and is better than default for any successful borrower; that is,

$$k_j R \leq Y^H, \tag{4}$$

$$k_j R \leq \delta v_j^R. \tag{5}$$

In the following we discuss that the latter constraint quaranties that strategic default won't be attractive. The decision to strategically default has to be made at Step 4, when he knows he has succeeded but does not know of others' outcomes. Strategic default will be unattractive if and only if the expected payoff from strategically defaulting does not exceed the expected payoff from declaring a successful outcome.

$$Y^H + Q_{-j}\delta v_j^R \leq Y^H - k_j R + Q_j \delta v_j^R,$$

where Q_{-j} and Q_j, respectively, denote the odds of game continuation when the successful borrower j defaults strategically and when he decides to pay, assuming that every other group member cooperates. The equation can be rewritten as

$$\frac{Q_{-j}}{Q_j} \leq 1 - \frac{k_j R}{Q_j \delta v_j^R}. \tag{6}$$

Now consider Step 6 of the game when borrowers make their repayment decisions based on signals they receive. For the loan payments to continue, a successful borrower who announces success and finds that all others have failed and his private signal also indicates that all the others have failed,

#Case	Borrower 1	Borrower 2	Odds of a successful group repayment
1	ST	ST	α^2
2	ST	SL	$\alpha^2\theta_2$
3	ST	F	$\alpha\,(1-\alpha)\,\theta_4$
4	SL	ST	$\alpha^2\theta_2$
5	F	ST	$\alpha\,(1-\alpha)\,\theta_4$

Table 2: Possible successful group repayment scenarios for a group of two borrowers.

will pay if

$$nR \leq \delta v_j^R.$$

From the above two equations, we can assert that strategic default is unattractive if

$$\frac{Q_{-j}}{Q_j} \leq 1 - \frac{k_j}{nQ_j}. \tag{7}$$

Note that $0 < \frac{k_j}{nQ_j} < 1$ and the above equation requires that $\frac{Q_{-j}}{Q_j} < 1$. If signals are perfect (i.e., $\tau = 1$) then $\theta_2 = \theta_4 = 1$ and a strategic defaulter has zero chance to get away with his strategic default. In this case, Q_{-j} would be zero and equation (7) is always satisfied. If signals are imperfect, and thus $Q_{-j} \neq 0$, the equation (7) may still hold. Assume $\tau^* < 1$ is the infimum of such signals for all borrowers in the group; that is,

$$\tau^* = inf\{\tau : \frac{Q_{-j}}{Q_j} \leq 1 - \frac{k_j}{nQ_j}, \forall j\}. \tag{8}$$

In order to avoid strategic behavior, the group composition has to be such that $\tau \geq \tau^*$.

To develope some intuition about the equation (8) and verify that it is not overly restrictive, consider the following example and discussion. For a group of two borrowers, possible successful group repayment scenarios are summarized in Table 2, where S stands for success, F for failure, T for truth (announcing the high project outcome), and L for lie (pretending low project outcome).

Suppose, upon a successful outcome, borrower 1 decides whether to announce his high project outcome (truth) or announce failure (lie), given that borrower 2 always cooperates (pays if he can). From the point of view of the successful borrower 1, possible scenarios for a successful group repayment consist of cases 1 and 3, in which he tells the truth and pays as well as case 4, in which he defaults strategically. If he pays, then the chance of game continuation for the group will be sum of two cases: either borrower 2 succeeds and pays or borrower 2 fails and borrower 1 belives him and pays for him. Thus

$$Q_1 = \alpha + (1-\alpha)\,\theta_4.$$

In this case, the expected repayment of borrower 1 will be

$$k_1 R = \alpha R + (1-\alpha)\,\theta_4\,(2R)\,.$$

If borrower 1 defaults, the chance of game continuation will be the case that borrower 2 succeed and believes borrower 1 and pays; that is,

$$Q_{-1} = \alpha\theta_2,$$

and his expected repayment will be zero. Replacing Q_1, Q_{-1}, and k_j in the equation (8) and simplifing, borrower 1 should not find strategic defaulting attractive if for some parameter settings, the following equation holds,

$$\frac{\alpha + 2\,(1-\alpha)\,\theta_4}{\alpha + (1-\alpha)\,\theta_4 - \alpha\theta_2} \leq 2.$$

A network of two borrowers can be either connected or disconnected. For a connected diad, $\theta_4 = 1$ and $\theta_2 = 0$; and thus, the above equation will be trivial. For a disconnected diad, $\theta_4 = \beta$ and $\theta_2 = \frac{(1-\alpha)(1-\beta)}{\alpha}$. In this case, the above equation simplifies to

$$3\alpha + (1-\alpha)\,(2\beta) \geq 2,$$

which holds for any $\alpha \geq 0.5$ if $\beta \geq 0.5$.

In general, consider a network of n cooperative members (they pay if they can). Assume borrower j has a high return from his project and is considering to pay or not to pay. If he decides to pays, there are $\binom{n-1}{i}$ number of cases, in which $i = 0, \ldots, n-1$ members default non-strategically on their repayments and the other $n-i$ members repay. For a successful group repayment, each successful member of the group should believe the i defaulting members and pay for them; this would happen with probability θ_4^i. Thus, for a network of n cooperative members, the chance of a successful group repayment will be

$$Q_j \quad = \quad \sum_{i=0}^{n-1} \binom{n-1}{i} \alpha^{n-1-i} (1-\alpha)^i \, \theta_4^{i(n-i)}. \tag{9}$$

Accordingly, upon a successful project, borrower j's expected repayment will be

$$k_j R = \sum_{i=0}^{n-1} \binom{n-1}{i} \alpha^{n-1-i} (1-\alpha)^i \, \theta_4^i \left(R + \frac{iR}{n-i} \right),$$

that further simplifies to

$$k_j = \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-1-i} (1-\alpha)^i \, \theta_4^i. \tag{10}$$

Note that when the successful borrower j is repaying, the chance of a successful group repayment will be the sum of the cases, in which the other $n-1$ members of the network cooperate.

Now consider the case that borrower j decides to default strategically while other group members cooperate. In this case, for a successful repayment, the rest of the network needs to believe that j's is due to bad luck and not having the money. For such a network the chance of a successful group repayment will be

$$Q_{-j} \quad = \quad \sum_{i=0}^{n-2} \binom{n-1}{i} \alpha^{n-1-i} (1-\alpha)^i \, \theta_4^{i(n-1-i)} \theta_2^{n-1-i}. \tag{11}$$

In the following, using equations (9), (10), and (11), we verify that equation (8) can hold for a range of parameter value. That is, under the given paramer values, strategic default will not attractive even in empty networks that monitoring is minimal. Tables 3 and 4 demonstrate how the ratio $\frac{Q_{-j}}{Q_j}$ changes with θ_4, given α and n. More specifically, Table 3 records the changes of $\frac{Q_{-j}}{Q_j}$ with respect to changes in θ_4 when $\alpha = 0.8$ and $n = 2, \ldots, 10$; and Table 4 looks at the changes of $\frac{Q_{-j}}{Q_j}$ with respect to changes in θ_4 when $n = 7$ and $\alpha = 0.5$, 0.55, $0.6, ...,$ 0.9. From these Tables, it can be verified that strategic default is less attractive in larger groups with less risky projects. Intuitively, in larger groups, it is increasingly harder to survive a strategic default, as more people should be convinced of a defaulting member's truthfulness.

All in all, from the above discussion and equations (2), (3), (4), and (5), the lender faces the

	$n=2$	$n=3$	$n=4$	$n=5$
$\theta_4 = 0.98$	0.004016064	0.001606908	0.000485073	0.00013092
$\theta_4 = 0.95$	0.01010101	0.00404187	0.001228727	0.000336209
$\theta_4 = 0.90$	0.020408163	0.008158008	0.002493629	0.000689847
$\theta_4 = 0.85$	0.030927835	0.012331963	0.003759297	0.001035492
$\theta_4 = 0.80$	0.041666667	0.016544118	0.004987064	0.001346265

	$n=6$	$n=7$	$n=8$	$n=9$	$n=10$
$\theta_4 = 0.98$	3.33186E-05	8.18683E-06	1.9668E-06	4.65445E-07	1.09025E-07
$\theta_4 = 0.95$	8.72969E-05	2.20167E-05	5.45986E-06	1.34086E-06	3.27554E-07
$\theta_4 = 0.90$	0.000181916	4.67587E-05	1.18463E-05	2.97614E-06	7.43939E-07
$\theta_4 = 0.85$	0.000271095	6.89015E-05	1.71759E-05	4.22197E-06	1.02632E-06
$\theta_4 = 0.80$	0.00034203	8.35296E-05	1.98269E-05	4.60472E-06	1.0511E-06

Table 3: Changes of the ratio $\frac{Q_{-j}}{Q_j}$ with respect to changes in the strength of signal θ_4, given $\alpha = 0.8$.

	$\alpha = 0.5$	$\alpha = 0.55$	$\alpha = 0.6$	$\alpha = 0.65$
$\theta_4 = 0.98$	0.002194146	0.001156826	0.000564347	0.000249721
$\theta_4 = 0.95$	0.006873239	0.003574027	0.00171083	0.000739255
$\theta_4 = 0.90$	0.019486818	0.009838343	0.004528935	0.001866784
$\theta_4 = 0.85$	0.039929618	0.019363693	0.008476268	0.00330021
$\theta_4 = 0.80$	0.069388994	0.031874704	0.013108467	0.004782275

	$\alpha = 0.7$	$\alpha = 0.75$	$\alpha = 0.8$	$\alpha = 0.85$	$\alpha = 0.9$
$\theta_4 = 0.98$	9.73483E-05	3.19539E-05	8.18683E-06	1.4202E-06	1.21208E-07
$\theta_4 = 0.95$	0.000280171	8.90526E-05	2.20167E-05	3.67456E-06	3.00978E-07
$\theta_4 = 0.90$	0.000670588	0.000201086	4.67587E-05	7.32784E-06	5.63325E-07
$\theta_4 = 0.85$	0.001115915	0.000314716	6.89015E-05	1.01903E-05	7.41547E-07
$\theta_4 = 0.80$	0.001517358	0.000403081	8.35296E-05	1.17534E-05	8.17757E-07

Table 4: Changes of the ratio $\frac{Q_{-j}}{Q_j}$ with respect to changes in the strength of signal θ_4, given $n = 7$.

following optimization problem,

$$
\begin{aligned}
\underset{L,R}{maximize} \quad V^R \quad &= \frac{n\alpha Y^H - nQR}{1-\delta Q} \\
s.t. \qquad\qquad & QR \geq L(1+\epsilon) \\
& k_j R \leq Y^H \\
& k_j R \leq \delta v_j^R .
\end{aligned}
$$

Proposition 1 solves the lender's optimization problem and specifies the feasible contract terms for a network of n members depending on the structure of the borrower network captured by Q.

Proposition 1. *A group lending contract with simple joint liability is feasible if and only if*

$$
L_{SJL} \leq \mathscr{L} = \begin{cases} \frac{\alpha Y^H}{(1+\epsilon)} \left(\frac{\alpha\delta}{1-\delta Q+\alpha\delta} \right) & if \quad \delta \leq \frac{1}{Q} \\ \frac{\alpha Y^H}{(1+\epsilon)} & if \quad \delta \geq \frac{1}{Q} \end{cases},
$$

$$
R_{SJL} = \frac{L_{SJL}(1+\epsilon)}{Q}.
$$

The total expected lifetime utility for borrowers receiving the contract (L_{SJL}, R_{SJL}) *will amount to* $V_{SJL}^R = \frac{n\alpha Y^H - nL_{SJL}(1+\epsilon)}{1-\delta Q}$.

From Proposition 1, it can be inferred that the loan ceiling $\mathscr{L}$, and consequently the total borrower welfare V_{SJL}^R, is increasing in Q. It can also be inferred that the repayment amount R_{SJL} is decreasing in Q. Intuitively, a network with a higher chance of game continuation Q may qualify for a more favorable contract (a larger loan and a lower repayment). A better contract terms, in turn, enhances the borrower welfare.

This proposition also highlights that the loan ceiling depends positively on the borrower discount factor, as $\frac{\delta\alpha}{n-n\delta Q+\delta\alpha k_m}$ is increasing in δ. Intuitively, if borrowers highly value receiving future loans, they would have a higher incentive to repay their loans and thus could be trusted to repay larger loans.

4.1 Network Structures

In this section, we look for ways we can order the chance of game continuation and the expected repayment of individuals. We focus our attention mainly on symmetric networks especially ring and star structure networks, which respectively represent the most decentralized and the most centralized networks. Before we go further with our analysis, let us take a closer look at the notation that capture the structure of the network; i.e., Q.

Using Table 2, for a group of two borrowers, the chance of game continuation when borrowers cooperate (they pay if they can) consists of cases 1, 3, and 5; that is,

$$Q = \alpha^2 + 2\alpha\,(1-\alpha)\,\theta_4.$$

We recall that θ_4, the probability that one defaults non-strategically and be backed by peers, depends on the type of connection between borrowers. For a diad group, borrowers are either connected with $\theta_4 = 1$ or disconnected with $\theta_4 = \beta$. Thus, for a connected diad group, we have

$$Q = \alpha^2 + 2\alpha\,(1-\alpha)\,;$$

and for a disconnected diad group, we have

$$Q = \alpha^2 + 2\alpha\,(1-\alpha)\,\beta.$$

Similarly, one could verify that for a network of three cooperative borrowers, the chance of game continuation and the expected repayment of each successful borrower j are

$$Q = \alpha^3 + 3\alpha^2\,(1-\alpha)\,\theta_4^2 + 3\alpha\,(1-\alpha)^2\,\theta_4^2.$$

A triad group allows for more patterns of social ties (See Figure 1). In empty and complete triad networks (Figures 1a and 1d), θ_4 is the same for all borrowers and thus for an empty triad,

$$Q = \alpha^3 + 3\alpha^2\,(1-\alpha)\,\beta^2 + 3\alpha\,(1-\alpha)^2\,\beta^2,$$

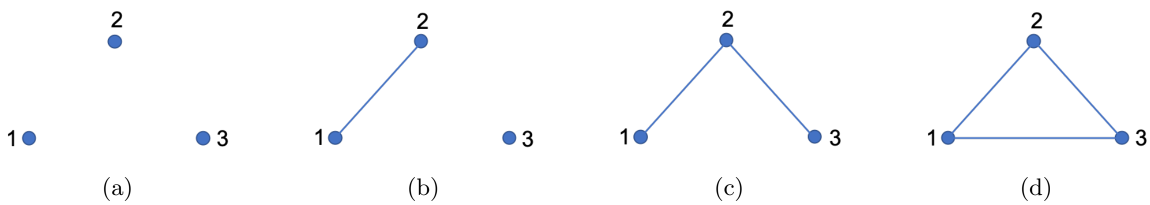


Figure 1: Different network structures possible for a group of three borrowers.

and for a complete triad,

$$Q = \alpha^3 + 3\alpha^2 (1-\alpha) + 3\alpha (1-\alpha)^2 .$$

In the triad network in Figure 1b, each of the borrowers 1 and 2 have only one direct tie and borrower 3 has non, thus

$$Q = \alpha^3 + 2\alpha^2 (1-\alpha) \beta + \alpha^2 (1-\alpha) \beta^2 + \alpha (1-\alpha)^2 \beta^2 + 2\alpha (1-\alpha)^2 \beta.$$

Finally, in the triad in Figure 1c, each of the borrowers 1 and 3 have a direct and an indirect connections, and borrower 2 has two direct connections, thus

$$Q = \alpha^3 + 2\alpha^2 (1-\alpha) \sigma + \alpha^2 (1-\alpha) + 2\alpha (1-\alpha)^2 \sigma + \alpha (1-\alpha)^2 .$$

In general, for a network of n members, there are $\binom{n}{i}$ number of cases, in which i members default non-strategically on their repayments and the other $n-i$ member repay. A successful group repayment would be possible when each successful member believes and pay for the i defaulting members; and this would happen with probability θ_4^i, where θ_4 takes one of the values $1, \sigma, \beta$ depending on the type of connections between the repaying borrower with each of the i defaulting borrowers. Thus, for a network of n cooperative members (they pay if they can), the chance of a successful group repayment will be

$$Q \;=\; \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-i} (1-\alpha)^i \left(\theta_4^i\right)^{n-i}, \tag{12}$$

Equation (12) suggests that for an empty network of n borrowers we should have

$$Q^e = \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-i} (1-\alpha)^i \beta^{i(n-i)},$$

and for a complete network, in which $\theta_4 = 1$, we should have

$$Q^c = \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-i} (1-\alpha)^i.$$

The empty and the complete networks of n borrowers, respectively, set the lower and the upper bounds for Q for all networks of the same number of borrowers n. Calculatin of Q for networks other than empty and complete is a bit tedious. See Claim 1 in the Appendix for the chance of game continuation in ring, star, and line networks.

It can be verified that the chance of game continuation Q increases with the chance of project success α. We will discuss in Proposition 2 that Q also increases with the centrality and connectivity of the network.

Proposition 2. *A higher average centrality and average connectivity of the network increase the chance of game continuation Q of the network.*

Proposition 2 suggests that not only the number but also the distribution of direct connections affect the chance of game continuation of a network. A well-connected borrower can contribute to the chance of game continuation of the network in two ways: by adding indirect connections between borrowers that are not otherwise connected, and by adding a second indirect connections between borrowers that are already indirectly connected. Such a borrower is likely to have a higher contribution to the total repayment of the group and is also likely to receive support from the network when needed.

Proposition 2 can help us sort networks of the same size and total degree according to their performance in terms of the chance of game continuation. Let us for example look at the pentad networks with four links. Structures including four connected nodes and an isolated node (Figures 2b and 2c) have a higher performance than the one including two components (Figure 2a) because the formers have a higher number of second-hand connections and thus a higher average centrality. Similarly, network structures that include five connected nodes (Figures 2d and 2e and 2f) have a

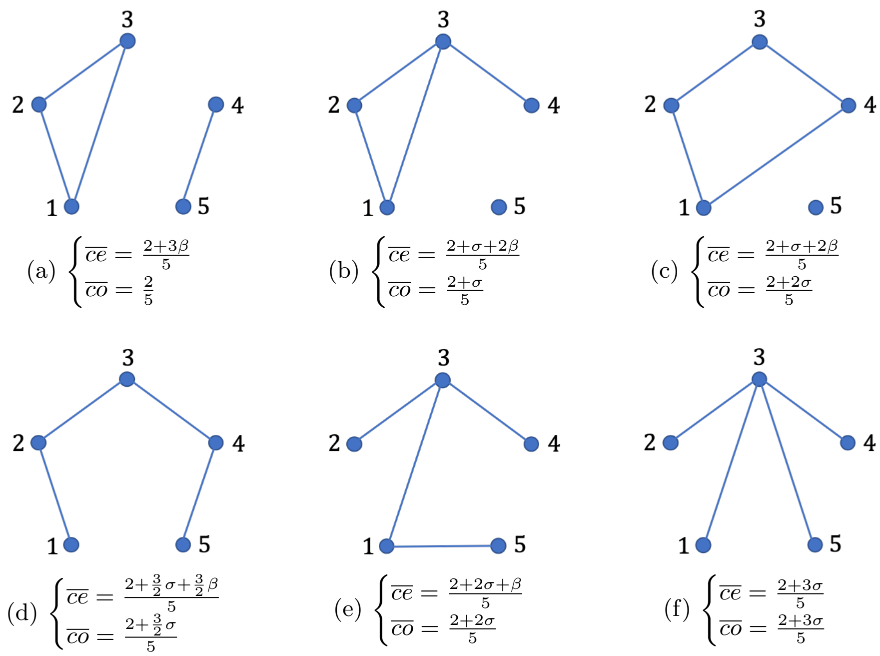


Figure 2: Pentad networks of total degree eight sorted with their average centrality.

higher overall performance than the ones with four connected nodes and an isolated node (Figures 2b and 2c).

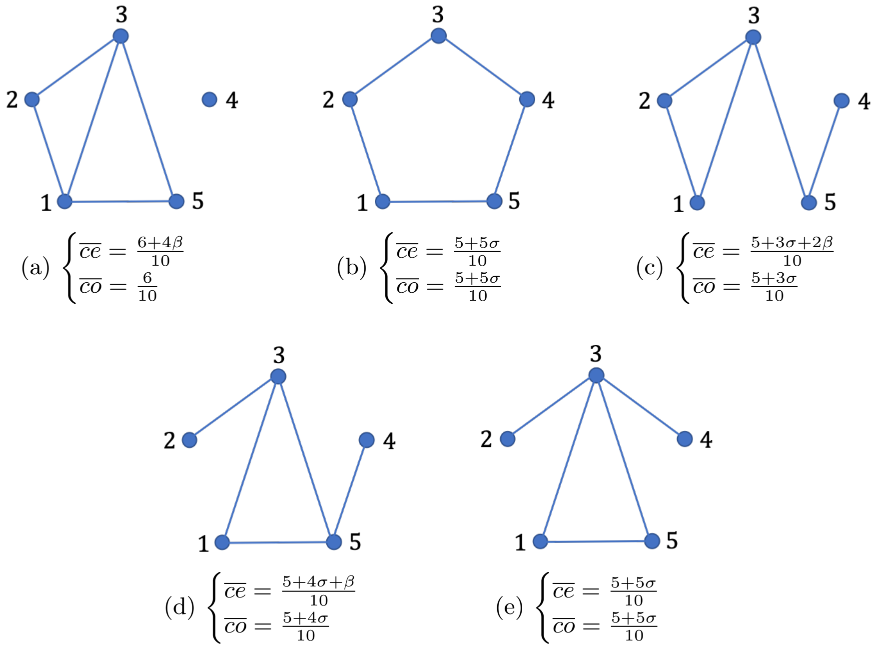
Moreover, among the two structures with four connected nodes (Figures 2b and 2c), the one which includes a ring structure (Figure 2c) has a higher performance than the one which includes a core-periphery (Figure 2b) despite both networks having the same average centrality. The reason for this is that the pentad including a tetrad ring provides some indirectly connected nodes with more than one friend in common and thus has a higher connectivity than the pentad network including a tetrad core-periphery.

Further, the star network (Figure 2f) has the highest performance among the structures of the same total degree; but the ring pentad (Figure 3b) has a lower performance than all the core-periphery structures of the same total degree (Figures 3c and 3d and 3e).

Corollary 1 is a direct result of Proposition 2.

Corollary 1. *Removing an isolated borrower from a network may increase its chance of game continuation Q if the remaining group forms a connected network.*

Corollary 1 implies that a network, in which every member is fully monitored by at least one other member of the group, should be preferred to disconnected networks even at the cost of losing



(a) $\begin{cases} \overline{ce} = \frac{6+4\beta}{10} \\ \overline{co} = \frac{6}{10} \end{cases}$

(b) $\begin{cases} \overline{ce} = \frac{5+5\sigma}{10} \\ \overline{co} = \frac{5+5\sigma}{10} \end{cases}$

(c) $\begin{cases} \overline{ce} = \frac{5+3\sigma+2\beta}{10} \\ \overline{co} = \frac{5+3\sigma}{10} \end{cases}$

(d) $\begin{cases} \overline{ce} = \frac{5+4\sigma+\beta}{10} \\ \overline{co} = \frac{5+4\sigma}{10} \end{cases}$

(e) $\begin{cases} \overline{ce} = \frac{5+5\sigma}{10} \\ \overline{co} = \frac{5+5\sigma}{10} \end{cases}$

Figure 3: Pentad networks of total degree five sorted with their average centrality.

a member. However, removing an isolated node benefits different networks differently, depending on how connected the rest of the network is. For example, consider a pentad network that includes a ring tetrad (Figures 2c) and a pentad network that includes a core-periphery tetrad (Figures 2b). Removing the isolated node from the former increases the chance of game continuation for the remaining tetrad more than removing the isolated node from the latter. In other words, the better connected the network is, the more it benefits such a contraction.

From Corollary 1, it can also be inferred that adding a member with a high centrality and connectivity may improve the network performance. However, enlarging the group should be done more carefully because a new node can be connected to the existing nodes in different ways.

Apart from centrality and connectivity, there is another factor that affects the chance of game continuation. Proposition 3 highlights the importance of the group size in improving the performance of some structures.

Proposition 3. *A star structure outperforms the same-sized ring structure if*

$$\left(n^2 - 5n + 2\right)\sigma > n\left(n - 5\right)\beta + 2.$$

Proposition 3 shows that having many second-hand links may well compensate lacking a few direct links if the second-hand connections are relatively well trusted by borrowers. This proposition also highlights that a larger group size adds to the relative attraction of the star structure while reduces the relative appeal for the ring structure, so that for a larger group size, the star structure has a higher performance than the ring structure.

This relative advantage of star networks may be explained as follows. A star network with $n+1$ members compared to a star network with n members has one extra direct link and $n-1$ extra indirect links; while a ring network of $n+1$ members compared to a ring network with n members has one extra direct link and one extra indirect link. Thus, larger group size benefits the star structure more than the ring structures.

Intuitively, the star network, compared to the ring network, has the advantage of providing a high level of monitoring as every one is at least friend of friend of every other group member. When the group size grows larger, ring cannot sustain a high level of monitoring but star network can.

4.2 Group Size

Now, it may be time to ask the question whether a larger group size is at all desirable and how large a group can become. Proposition 4 sheds light on this question.

Proposition 4. *For a borrower network of n members, the loan ceiling of the SJL contract can increase in the group size n if and only if $\frac{\partial Q}{\partial n} \geq 0$; borrower welfare increases in group size and in the chance of project success where loan ceiling increases in group size and the chance of project success.*

Proposition 4 proves that depending on the network structure, a larger group size may allow an increase in the loan ceiling, and consequently in the borrower welfare. Lemma 1 argues that this holds at least for complete networks. .

Lemma 1. *The loan ceiling of the SJL contract can increase in the group size n at least for complete networks.*

5 Less Strict Borrowers

The grim-trigger strategy in SJL contarct discussed previously assumes that borrowers punish free-riding even at their own cost. In the real world, it may well benefit the repaying borrowers to repay for their strategically defaulting group members and proceed to the next round of the lending game. In this section, we examine joint liability under a strategy that is less severe than grim-trigger. We call this new contract Flexible Joint Liability (FJL). Players start in the lending phase and cooperate until someone defaults strategically. They then go to the punishment phase and exclude the strategically defaulting member for T periods of receiving loans. They allow him to re-enter the game after T periods of punishment. Timing remains the same as in the SJL contract.

The lender's optimization problem in this case is similar to the one of the SJL contract, except for the incentive constraint (6). Successful borrowers must be incentivized by the lender not to default strategically. Each successful borrower have to be willing to repay not only for himself, but also for all of the defaulting group members (even in the worst case that $n-1$ members default). Thus we must have,

$$\delta^{T+1} v_j^R \leq -nR_{FJL} + \delta v_j^R.$$

The above inequality can be rewritten as

$$nR \leq \left(1-\delta^T\right)\delta v_j^R. \tag{13}$$

If $T \to \infty$, then $1-\delta^T \to 1$, and equation (13) will equal equation (6). Although with FJL, borrowers could be more forgiving to strategic default and repay more often than they do with SJL, the punishment period T should still be chosen large enough so that strategic default is not desirable. Clearly, equation (13) is more difficult to satisfy than equation (6) as it is less costly to default strategically under the FJL contract.

Proposition 5. *A group lending contract with flexible joint liability (FJL) is feasible if and only if*

$$L_{FJL} \leq \begin{cases} \frac{QY^H}{n(1+\epsilon)} \equiv \tilde{L}_{FJL} & \text{if} \quad \delta \geq \frac{n}{n\alpha(1-\delta^T)+nQ-\alpha k_m(1-\delta^T)} \\ \frac{QY^H}{(1+\epsilon)}\left(\frac{\delta\left(1-\delta^T\right)\alpha}{n-n\delta Q+\delta(1-\delta^T)\alpha k_m}\right) \equiv \hat{L}_{FJL} & \text{if} \quad \delta \leq \frac{n}{n\alpha(1-\delta^T)+nQ-\alpha k_m(1-\delta^T)} \equiv \tilde{\delta}_{FJL} \end{cases},$$

$$R_{FJL} = \frac{L_{FJL}\,(1+\epsilon)}{Q}.$$

The total expected lifetime utility for borrowers receiving the contract (L_{FJL}, R_{FJL}) *will amount to* $V^R_{FJL} = \frac{n\alpha Y^H - nL_{FJL}(1+\epsilon)}{1-\delta Q}$.

Proposition 5 suggests that with the FJL, the loan ceiling, and consequently the total borrower welfare, is increasing in Q but decreasing in k_m. It also suggests that the repayment amount is decreasing in Q. A comparison between the results of Propositions 5 and 1 shows that although the amount of loan and repayment are affected by the length of the punishment, but the changing trend of the loan and the repayment with respect to changes of network structure stay the same under both types of contracts. In other words, with FJL contracts, similar to the SJL contracts, a network with a higher chance of game continuation Q may qualify for a more favorable contract (a larger loan and a lower repayment), and a network with an overly high k_m receives a less favorable contract.

This proposition also highlights that the loan ceiling with the FJL contract, similar to the case with the SJL contract, depends positively on the borrowers' discount factor. Intuitively, if borrowers highly value receiving future loans, they would have a higher incentive to repay their loans even at the prospect of a more forgiving punishment phase.

We continue this section by examining whether and how the length of the punishment phase T affects the loan ceiling of the FJL contract. First, we need to know how $\tilde{L}_{FJL}$, $\hat{L}_{FJL}$, and $\tilde{\delta}_{FJL}$ respond to changes in length of punishment period T. This is what we discuss in Lemma 2. Note that $\tilde{L}_{FJL}$ does not depend on T and thus $\tilde{L}_{FJL} = \tilde{L}_{SJL}$.

Lemma 2. *Assume* $\hat{L}_{FJL}$ *and* $\tilde{\delta}_{FJL}$ *are functions defined in Proposition 5.* $\hat{L}_{FJL}$ *increases and* $\tilde{\delta}_{FJL}$ *decreases as* T *grows.*

Lemma 2 proves that decreasing the length of the punishment period has two counteracting effects on the loan ceiling of the FJL contract. Although reducing the length of the punishment period allows for an increase in the loan ceiling, it decrease the range of discount factors that allow for such increase.

In other words, when the punishment phase is not that long—and thus it is less costly for members to default strategically—groups may be able to continue the game more often but only if members highly value future gains.

Proposition 6 builds on the findings of Proposition 5 and compares the FJL and SJL contracts.

Proposition 6. *Assume* $\hat{L}_{SJL}(n, \alpha, \delta, H)$ *and* $\tilde{\delta}_{SJL}(n, \alpha, H)$ *are functions defined in Proposition 1, and* $\hat{L}_{FJL}(n, \alpha, \delta, T, H)$ *and* $\tilde{\delta}_{FJL}(n, \alpha, \delta, T, H)$ *are functions defined in Proposition 5. The following statements hold when both of the SJL and the FJL contracts are feasible.*

1) $\hat{L}_{FJL}(n, \alpha, \delta, T, H) \leq \hat{L}_{SJL}(n, \alpha, \delta, H)$.

2) $\tilde{\delta}_{FJL}(n, \alpha, \delta, T, H) \geq \tilde{\delta}_{SJL}(n, \alpha, H)$.

According to the first part of Proposition 6, reducing the length of the punishment period creates a disadvantage for joint liability contracts, in terms of the loan ceiling. The FJL contract is feasible for, at most, the same loan ceiling as the SJL contract. Intuitively, the more forgiving the lender is towards strategic default, the less likely the group members are to fulfill their repayment obligations.

However, the second part of Proposition 6 proves that this disadvantage could be offset by forming larger groups. More specifically, the interval $\left(\hat{\delta}_{FJL}, \tilde{\delta}_{FJL}\right)$ is wider than the interval $\left(\hat{\delta}_{SJL}, \tilde{\delta}_{SJL}\right)$; that is, under the FJL contract, a larger range of discount factors allow the loan ceiling to increase in group size, as compared with the SJL contract. In other words, the FJL contract accommodates larger group sizes, as compared with the SJL contract, and larger group sizes enhance the loan ceiling of the FJL contract.

6 Less Strict Lender

In coalitional/cooperative game theory, we model a group of agents as a single decision maker. The aim here is to figure what kind of payoffs the group as a whole is able to achieve. Assumption: utility is transferable; that is, the utility achieved by the coalition can be arbitrarily redistributed among members. Questions: which coalition will form? How should the coalition divide the payoff?

Assuming that the payoff of the coalition is larger than the sum of the payoffs of individuals (i.e., the game is superadditive), a grand coalition (including all agents) will be formed if the division of payoff is "fair". What is fair?

"Shaply Value" is one way for fair division of the utility between members of a coalition: Members should receive shares proportional to their contributions (everyone gets their outside option plus their fair share of the surplus generated by the coalition). SV is fair but it allows for the formation of sub-coalitions that are more profitable for their members.

Under what payment scheme, agents would be forming the grand coalition? if and only if the payment scheme belongs to "the Core"; that is, for any possible sub-coalition, the total paid to individuals exceeds their value of forming that sub-coalition. You could think of this as a Nash Equilibrium of the grand coalition in which no subgroup benefits a deviation.

Does such a payment scheme always exists and if yes is it unique? No and no, but it becomes possible under some conditions (it's possible to design a game for which such payment schemes exist). For exmaple, for a simple game (the payoff is binary) with some veto players (players without whom a coalition won't be formed), the payment scheme could be: every non-veto player gets nothing and veto players divide the payoff among themselves.

In our setting, the game between the lender and the joint liability group can be seen as a simple game as they either get the next loan or they don't. Assume the lender allows the group to proceed to the next round with majority repaying (let's say 80% repay). This would allow then for forming coalitions among borrowers with high-centrality borrowers (i.e. the core in star network) being the veto-players.

In our current version, we losen the grim-trigger played by borrowers against the free-rider but not the grim-trigger played by the lender. Loosening the latter grim-trigger may also be interesting if we are seeking more theory results.

7 Conclusion

This paper presents a study of how the structure of borrowers' social networks may affect the outcome of group lending under joint liability and what characteristics a potential optimal network structure should have. Extant microfinance literature argues that borrowers' social collateral may facilitate cooperation and substitute contract enforcement (e.g., Besley and Coate, 1995). Aligned with this literature, we provide evidence that stronger social connectedness can improve the repayment rate. We differ from this literature by explicitly modeling social ties as a means of peer monitoring. In our model, a borrower that is directly connected to the network better monitors and is better monitored. Thus, he is more likely to back the non-strategic defaulting and punish the strategically defaulting peers. Similarly, such a borrower has a higher chance of both being backed by peers in case of non-strategic default and being punished by peers in case of strategic default.

We prove that the structure of a borrower network together with the borrower discount factor affect the lending outcome. A borrower network with a higher average centrality and connectivity provides higher security both in terms of the chance of game continuation Q and the total expected repayment K. Such a network may qualify for a more favorable contract (i.e., a larger loan and a lower repayment) under the condition that the expected repayment of no individual borrower is too high; otherwise, the borrower network will be susceptible to the default of the members with a high expected contribution. We also prove that a higher borrower discount factor allows for a better contract. Intuitively, the more the borrowers value receiving future loans, the more likely they are to repay their loans. Such borrowers could be trusted to repay larger loans. But there is a limit on the amount of loan they can be granted, and the loan size cannot grow larger after some point.

Our model confirms that the borrower welfare is maximized in a complete borrower network, in which borrowers can fully monitor each other's project output and are able to differentiate strategic defaults from non-strategic defaults.

We focus our study on the star and the ring structure networks, which represent highly centralized and highly decentralized networks, respectively. We prove that in smaller groups, the ring structure can maintain a higher level of performance than the same-sized star structure; however, in larger groups, the star structure performs better. A star group has a disadvantage and an advantage to a ring group. The success of a star group depends highly on the central member, and the group may fall apart if the central member fails on his repayment; while in a ring group, no member is overly important; and thus, any individual player's default is less of a threat to coordination. However, a star group provides a good level of monitoring for its members, as all members can monitor each other's project outcomes at least indirectly through a friend of a friend. Thus, they are able to verify the truthfulness of their defaulting members and maintain a higher level of cooperation; while in a ring group, some members are out of each other's monitoring scope and cannot tell apart strategic defaults from non-strategic defaults.

In a smaller group, the monitoring disadvantage of a ring group fades away, and thus a ring structure wins on performance because of its higher robustness to the failure of one or a few of its members. However, in a larger group, the monitoring issue in ring networks intensifies as no member is able to monitor more than four other members.

The stylized model in this paper could be generalized by loosening different assumptions. For example, we assume that the group must pay the total loan to qualify for the next loan. This constraint may be too limiting. There may be cases where repaying borrowers can repay a portion of the total loan but not the entire total loan, while the net present value of the lender is still positive.

Moreover, we require in our optimization problem that a successful member pays the total loan if it happens that other members don't have the money to pay. In real life, it doesn't happen very often that a borrower has to pay for the entire group. Many MFI's allow partial repayment for defaulting peers or have a compulsory saving program that borrowers keep safe for bad days. This would, in turn, allow for larger borrower groups and adds to the appeal of star-like networks.

One could also extend this work by looking at contracts that may vary between group members. The contract could depend on the level of social capital each individual borrower brings to the group (his centrality in our case) instead of the collective social capital of the group. Such an extension is also in line with the recent shift from explicit joint liability to individual lending by some MFIs, which interestingly preserve some of the group lending traditions.

Another interesting extension of our research is examining a longer chain of friendships as channels of information instead of considering only friends of friends. Under such an assumption, line- and tree-like networks may have an additional attraction. In this case, we might also end up with a combination of ring and star networks as the best performing network. A core-periphery structure with a ring at its core can provide the high-level monitoring of a star structure and the high-level robustness of a ring structure.

The setting under study in this paper, apart from the group lending with joint liability, can emerge in other social dilemma contexts that deal with monitoring issues. Thus, the results are valid whenever the interests of the individual and of the group are not aligned and agents exert externalities on one another.

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Appendix

Proposition 1.

Proof. The lender's optimization problem can be written as

$$\begin{aligned}
\underset{L,R}{maximize} \quad V^R \quad &= \frac{n\alpha Y^H - nQR}{1-\delta Q} \\
s.t. \qquad\quad &QR \geq L(1+\epsilon) \\
&\sum_{j=1}^{n} k_j R \leq nY^H \\
&\sum_{j=1}^{n} k_j R \leq \delta V^R,
\end{aligned}$$

We know from equation (1) that $\sum_{j=1}^{n} k_j = \frac{nQ}{\alpha}$. Thus the optimization problem can be further rewritten as

$$\begin{aligned}
\underset{L,R}{maximize} \quad V^R \quad &= \frac{n\alpha Y^H - QnR}{1-\delta Q} \\
s.t. \qquad\quad &QR \geq L(1+\epsilon) \\
&QR \leq \alpha Y^H \\
&QR \leq \frac{\alpha \delta V^R}{n}.
\end{aligned}$$

Since V^R is decreasing in R, the lender sets the lowest R possible; that is, $R = \frac{L(1+\epsilon)}{Q}$. Replacing R into the objective function and constraints, we will have

$$\begin{aligned}
\underset{L,R}{maximize} \quad V^R \quad &= \frac{n\alpha Y^H - nL(1+\epsilon)}{1-\delta Q} \\
s.t. \qquad\quad &L \leq \frac{\alpha Y^H}{(1+\epsilon)} \equiv \tilde{L} \\
&L \leq \frac{\alpha Y^H}{(1+\epsilon)} \left(\frac{\alpha\delta}{1-\delta Q+\alpha\delta} \right) \equiv \hat{L}.
\end{aligned}$$

From the above constraints, a feasible loan L must satisfy $L \leq \min\left\{\tilde{L}, \hat{L}\right\}$. It is not difficult to verify that $\hat{L} \leq \tilde{L}$ if and only if

$$\frac{\alpha\delta}{1-\delta Q+\alpha\delta} \leq 1,$$

or

$$\delta \leq \frac{1}{Q}.$$

Thus, $L \leq \hat{L}$ if $\delta \leq \frac{1}{Q}$ and $L \leq \tilde{L}$ if $\delta \geq \frac{1}{Q}$. □

Claim 1. The chance of game continuation and the expected repayment of individual borrowers in ring and star networks are as follows:

1. For a star network of $n \geq 5$,

$$Q^s = \alpha^n + \sum_{i=1}^{n-1} \alpha^{n-i} (1-\alpha)^i \left[\binom{n-1}{i} \left(1\sigma^i\right)^{n-1-i} + \binom{n-1}{i-1} \left(1\beta^{i-1}\right)^{n-i} \right].$$

2. For a ring network of size $n \geq 5$,

$$Q^r = Q^{r_0} + Q^{r_1} + Q^{r_2} + \cdots + Q^{r_{n-1}},$$

where

$$\begin{aligned}
Q^{r_0} &= \alpha^n \\
Q^{r_1} &= n\alpha^{n-1} (1-\alpha) \left(1^2 \sigma^2 \beta^{n-5}\right), \\
Q^{r_2} &= \sum_{h_1=0}^{n-2} n\alpha^{n-2} (1-\alpha)^2 p(h_1)\, p(n-2-h_1), \\
Q^{r_3} &= \sum_{h_1=0}^{n-3} \sum_{h_2=0}^{n-3-h_1} n\alpha^{n-3} (1-\alpha)^3 p(h_1)\, p(h_2)\, p(n-3-h_1-h_2), \\
\vdots &\;\vdots\; \vdots \\
Q^{r_{n-1}} &= n\alpha (1-\alpha)^{n-1} \left(1^2 \sigma^2 \beta^{n-5}\right),
\end{aligned}$$

and

$$p(h_i) = \begin{cases} 1 & h_i = 0, 1 \\ \sigma^2 & h_i = 2 \\ \sigma^2 \beta^{2h_i - 4} & h_i \geq 3 \end{cases}.$$

3. For a line structure network of size $n \geq 5$,

$$Q^l = Q^{l_0} + Q^{l_1} + Q^{l_2} + \cdots + Q^{l_{n-1}},$$

where

$$\begin{aligned}
Q^{l_0} &= \alpha^n, \\
Q^{l_1} &= 2\alpha^{n-1}(1-\alpha)\left(1\sigma\beta^{n-3}\right) + 2\alpha^{n-1}(1-\alpha)\left(1^2\sigma\beta^{n-4}\right) + (n-4)\,\alpha^{n-1}(1-\alpha)\left(1^2\sigma^2\beta^{n-5}\right), \\
&\vdots \\
Q^{l_{n-1}} &= 2\alpha(1-\alpha)^{n-1}\left(1\sigma\beta^{n-3}\right) + 2\alpha(1-\alpha)^{n-1}\left(1^2\sigma\beta^{n-4}\right) + (n-4)\,\alpha(1-\alpha)^{n-1}\left(1^2\sigma^2\beta^{n-5}\right).
\end{aligned}$$

Proof. From equations 10 and 10, we know that

$$Q = \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-i} (1-\alpha)^i \left(\theta_4^i\right)^{n-i},$$
$$k_j = \sum_{i=0}^{n-1} \binom{n-1}{i} \alpha^{n-1-i} (1-\alpha)^i \,\theta_4^{i(n-i)} \left(1 + \frac{i}{n-i}\right),$$

where θ_4 could take one of the values 1, σ, or β, depending on the type of the connection between the repaying borrowers with the defaulting borrowers.

Part 1. In a star network, if more than one member default, it matters whether the core member is one of them. Thus, the chance of game continuation for a star network can be written as follows.

$$\begin{aligned}
Q^s = {} & \alpha^n + \alpha^{n-1}(1-\alpha)\left[\binom{n-1}{1}\left(\sigma^1\right)^{n-2} + \binom{n-1}{0}(1)^{n-1}\right] \\
& + \alpha^{n-2}(1-\alpha)^2\left[\binom{n-1}{2}\left(\sigma^2\right)^{n-3} + \binom{n-1}{1}\left(\beta^1\right)^{n-2}\right] \\
& \vdots \\
& + \alpha^1(1-\alpha)^{n-1}\left[\binom{n-1}{n-1}\left(\sigma^{n-1}\right)^0 + \binom{n-1}{n-2}\left(\beta^{n-2}\right)^1\right],
\end{aligned}$$

or more simplified

$$Q^s = \alpha^n + \sum_{i=1}^{n-1} \alpha^{n-i}(1-\alpha)^i \left[\binom{n-1}{i}\left(\sigma^i\right)^{n-1-i} + \binom{n-1}{i-1}\left(\beta^{i-1}\right)^{n-i}\right].$$

Part 2. For a ring network, the chance of game continuation can be written as

$$Q^r = Q^{r_0} + Q^{r_1} + Q^{r_2} + \cdots + Q^{r_{n-1}},$$

in which Q^{r_i} is the probability of a successful group repayment when $i = 0, 1, ..., n-1$ members fail to repay. In such a network, any member who fails to repay has at most two friends who pay for him for sure, at most two friends of friends who pay for him with probability σ, and the rest of the group who pay for him with probability β.

When no one fails to repay, the probability of a successful group repayment is $Q^{r_0} = \alpha^n$. When only one member fails to repay, the probability of a successful group repayment will be $Q^{r_1} = n\alpha^{n-1}(1-\alpha)\left(1^2\sigma^2\beta^{n-5}\right)$. When two members fail, the network will be divided into two sections that consist of h_1 and $h_2 = n - 2 - h_1$ repaying members. In such a case, each of the two sections repays with probability

$$p(h_i) = \begin{cases} 1 & h_i = 0 \\ 1 & h_i = 1 \\ \sigma\sigma & h_i = 2 \\ \beta\sigma^2\beta & h_i = 3 \\ \beta(\sigma\beta)\left(\beta^2\right)^{h_i-4}(\sigma\beta)\beta & h_i \geq 4 \end{cases},$$

or more simplified

$$p(h_i) = \begin{cases} 1 & h_i = 0, 1 \\ \sigma^2 & h_i = 2 \\ \sigma^2\beta^{2h_i-4} & h_i \geq 3 \end{cases}.$$

Note that if one of the h_1 or $h_2 = n-2-h_1$ is zero, it means that two consecutive nodes have failed, and the graph consists of only one repaying section. Thus, when two members fail, the probability of a successful group repayment will be

$$Q^{r_2} = \sum_{h_1=0}^{n-2} n\alpha^{n-2}(1-\alpha)^2 p(h_1)\, p(n-2-h_1).$$

When three members fail, the probability of a successful group repayment will be

$$Q^{r_3} \quad = \quad \sum_{h_1=0}^{n-3} \sum_{h_2=0}^{n-3-h_1} n\alpha^{n-3} (1-\alpha)^3 p(h_1) p(h_2) p(n-3-h_1-h_2),$$

and so on for any $2 < i < n-2$.

When $n-2$ members fail, the two successful members may be direct friends, indirect friends, or not connected, thus the probability of a successful group repayment will be

$$\begin{aligned} Q^{r_{n-2}} &= n\alpha^2 (1-\alpha)^{n-2} \left(1\sigma\beta^{n-4}\right)^2 + n\alpha^2 (1-\alpha)^{n-2} \left(1^2\sigma\beta^{n-5}\right)^2 \\ &+ \left[\binom{n}{2} - 2n\right] \alpha^2 (1-\alpha)^{n-2} \left(1^2\sigma^2\beta^{n-6}\right)^2. \end{aligned}$$

When $n-1$ members fail, the only successful member will pay with probability

$$Q^{r_{n-1}} \quad = \quad n\alpha (1-\alpha)^{n-1} \left(1^2\sigma^2\beta^{n-5}\right).$$

Part 3. A line network is similar to a ring network, in which the link between nodes 1 and n is dropped. Thus, for calculating Q^l, we may be able to use some of the results of the previous part.

For a line network, the chance of game continuation for a line network can be written as

$$Q^l \quad = \quad Q^{l_0} + Q^{l_1} + Q^{l_2} + \cdots + Q^{l_{n-1}},$$

in which Q^{l_i} is the probability of a successful group repayment when $i = 1, ..., n-1$ members fail to repay. In such a network, any failing member has at least one and at most two close friends who are willing to pay for him, and at least one and at most two friend of friend who pay for him with probability σ.

When no one fails to repay, the probability of a successful group repayment is $Q^{l_0} = \alpha^n$. When one member fails, the network will be divided into two sections (similar to the case, in which two members fail in a ring) that consist of h_1 and $h_2 = n-1-h_1$ repaying members, respectively. In such a case, each of the two sections repays with probability

$$P(h_i) = \begin{cases} 1 & h_i = 0, 1 \\ \sigma^2 & h_i = 2 \\ \sigma^2 \beta^{2h_i - 4} & h_i \geq 3 \end{cases}.$$

If one of the h_1 or $h_2 = n - 1 - h_1$ is zero, then the failing member must be one of the nodes 1 or n. In such a case, the graph consists of only one repaying section and we simply assume $P(0) = 1$.

If one member defaults, for example in a group of $n = 5$ borrowers, then (h_1, h_2) can become one of the pairs $(0, 4)$, $(1, 3)$, $(2, 2)$, $(3, 1)$, or $(4, 0)$. Therefore, for $n = 5$, when one member defaults, the probability of a successful group repayment will be

$$\alpha^4 (1-\alpha) P(0) P(4) + \alpha^4 (1-\alpha) P(1) P(3) + \alpha^4 (1-\alpha) P(2) P(2)$$
$$+ \alpha^4 (1-\alpha) P(3) P(1) + \alpha^4 (1-\alpha) P(4) P(0).$$

In general, for any $n \geq 5$, if one member fails, then (h_1, h_2) can become $(0, n-1)$, $(1, n-2)$, $(2, n-3)$, ..., $(n-1, 0)$. In such a case, the probability of a successful group repayment will be

$$Q^{l_1} = \sum_{h=0}^{n-1} \alpha^{n-1} (1-\alpha) P(h_1) P(n-1-h_1).$$

Q^{l_i} can be calculated for any $2 \leq i \leq n-2$ in a similar way.

When $i = n-1$ members fail, the only successful member can be one of the nodes 1 and n or one the nodes 2 and $n-1$ or could be among the other $n-4$ nodes, thus, the probability of a successful group repayment will be

$$\begin{aligned} Q^{l_{n-1}} &= 2\alpha (1-\alpha)^{n-1} \left(1\sigma\beta^{n-3}\right) + 2\alpha (1-\alpha)^{n-1} \left(1^2\sigma\beta^{n-4}\right) \\ &\quad + (n-4)\,\alpha (1-\alpha)^{n-1} \left(1^2\sigma^2\beta^{n-5}\right). \end{aligned}$$

□

Proposition 2.

Proof. First, we should recal from equation (1) that $\sum_{j=1}^{n} \alpha k_j R = Q(nR)$, which can be rewritten as

$$Q = \frac{\sum_{j=1}^{n} \alpha k_j}{n}.$$

That is, the average of expected repayment of individual borrowers determines the chance of continuation of the network.

Using equation 10, the expected repayment of the successful borrower j when i members default non-strategically is as follows:

$$k_j R = \sum_{i=0}^{n-1} \binom{n-1}{i} \alpha^{n-1-i} (1-\alpha)^i \theta_4^i \left(R + \frac{iR}{n-i} \right),$$

where θ_4 accepts one of the values 1, σ, or β depending on the type of social connections that defaulting borrowers have with the successful borrower j. Suppose i members of the group default while borrower j is paying. Assume borrower j has m_1 direct and m_2 indirect ties with the defaulting borrowers, and all other $m_3 = i - (m_1 + m_2)$ of the defaulting members are not connected to j. In such a case, borrower j pays for the i defaulting members with probability $\theta_4^i = (1)^{m_1} (\sigma)^{m_2} (\beta)^{m_3}$. Again, the more central the borrower j is, the larger are m_1 and m_2 and the smaller is m_3; and thus, the more likely he is to help his non-strategically defaulting group members.

In turn, a network that consists of central and connected borrowers has a higher average centrality and connectivity and has a higher chance of succefully repaying and continuing to the next round of lending. □

Corollary 1.

Proof. From Proposition 2, we know that a more central and connected borrower has a higher expected repayment; we also know that a higher average centrality and connectivity of a network affect its performance positively.

Consider a network with some connections and an isolated node. The isolated node has a centrality and connectivity lower than the average of the network as the strength of all of his connections are β. Such a member negatively affects the average centrality and connectivity of the network. Intuitively, such a node is not monitored by peers and receives little support from them when he defaults. In turn, an isolated node has a poor monitoring on other group member's project returns, and thus his expected repayment is low when they default. The network is better off without such a member. □

Proposition 3.

Proof. For a network with a star structure, both the centrality and the connectivity of each periphery node are $\frac{1+(n-2)\sigma}{n-1}$ and of the core node are $\frac{n-1}{n-1}$. Thus, for a star network, the average centrality and the average connectivity will be

$$\begin{aligned}\overline{ce}^s = \overline{co}^s &= \frac{(n-1)\left(\frac{1+(n-2)\sigma}{n-1}\right)+\left(\frac{n-1}{n-1}\right)}{n} \\ &= \frac{2+(n-2)\,\sigma}{n}.\end{aligned}$$

For a network with a ring structure, the centrality of each node is $\frac{2+2\sigma+(n-5)\beta}{n-1}$, and the connectivity of each node is $\frac{2+2\sigma}{n-1}$. That is, the average centrality and connectivity of the network are

$$\begin{aligned}\overline{ce}^r &= \frac{2+2\sigma+(n-5)\,\beta}{n-1}, \\ \overline{co}^r &= \frac{2+2\sigma}{n-1}.\end{aligned}$$

Knowing centralities and connectivities, we could now verify which one has the highest and lowest performance. The star structure has a higher performance than the same-sized ring structure except for smaller groups.

$$\begin{aligned}\overline{ce}^r < \overline{ce}^s &\Leftrightarrow \frac{2+2\sigma+(n-5)\,\beta}{n-1} < \frac{2+(n-2)\,\sigma}{n} \\ &\Leftrightarrow \left(n^2-5n+2\right)\sigma > n\,(n-5)\,\beta+2, \\ \overline{co}^r < \overline{co}^s &\Leftrightarrow \frac{2+2\sigma}{n-1} < \frac{2+(n-2)\,\sigma}{n} \\ &\Leftrightarrow \left(n^2-5n+2\right)\sigma > 2.\end{aligned}$$

both of which hold if n is large enough and σ is valued considerably higher than β. □

Proposition 4.

Proof. From Proposition 1, we know

$$L_{SJL} \leq \mathscr{L} = \begin{cases} \frac{\alpha Y^H}{(1+\epsilon)} \left(\frac{\alpha\delta}{1-\delta Q+\alpha\delta} \right) \equiv \hat{L} & \text{if} \quad \delta \leq \frac{1}{Q} \\ \frac{\alpha Y^H}{(1+\epsilon)} \equiv \tilde{L} & \text{if} \quad \delta \geq \frac{1}{Q} \end{cases},$$

$$V_{SJL}^R = \frac{n\alpha Y^H - nL_{SJL}(1+\epsilon)}{1-\delta Q}.$$

1. Below, we show that the loan ceiling $\mathscr{L}$, and consequently the borrower welfare V^R, can be increasing in n for some δ.

$$\frac{\partial \mathscr{L}}{\partial n} = \begin{cases} \frac{\partial \hat{L}}{\partial n} = \left(\frac{\alpha^2 \delta Y^H}{1+\epsilon} \right) \frac{\delta \frac{\partial Q}{\partial n}}{(1-\delta Q+\alpha\delta)^2} & \text{if} \quad \delta \leq \frac{1}{Q} \\ \frac{\partial \tilde{L}}{\partial n} = 0 & \text{if} \quad \delta \geq \frac{1}{Q} \end{cases}.$$

   Clearly, $\hat{L}$ can be increasing in n if and only if $\frac{\partial Q}{\partial n} \geq 0$. Therefore, $\mathscr{L}$ can be increasing in n if and only if $\frac{\partial Q}{\partial n} \geq 0$ and $\delta \leq \frac{1}{Q}$.

2. Is the borrower welfare V_{SJL}^R increasing in n and α? We should recall that V^R is increasing in L because we assumed that a larger loan increases the project outcome more than the repayment (step 1 of the model); that is, $\frac{\partial V^R}{\partial L} > 0$. Thus, $\frac{\partial V^R}{\partial n} = \left(\frac{\partial V^R}{\partial L}\right)\left(\frac{\partial L}{\partial n}\right)$ is positive where $\frac{\partial L}{\partial n} \geq 0$; and $\frac{\partial V^R}{\partial \alpha} = \left(\frac{\partial V^R}{\partial L}\right)\left(\frac{\partial L}{\partial \alpha}\right)$ is positive where $\frac{\partial L}{\partial \alpha} \geq 0$. In other words, V^R is increasing in n and α whereever L is increasing in n and α.

□

Lemma 1.

Proof. For a complete network of size n,

$$\begin{aligned} Q &= \sum_{i=0}^{n-1} \binom{n}{i} \alpha^{n-i} (1-\alpha)^i \\ &= [1-(1-\alpha)^n], \\ \frac{\partial Q}{\partial n} &= -(1-\alpha)^n \ln(1-\alpha). \end{aligned}$$

Clearly, for these networks $\frac{\partial Q}{\partial n} \geq 0$. Proposition 5.

Assume borrower $j = m$ is the borrower with the highest expected repayment in the network with $v_m^R = \frac{\alpha Y^H - \alpha k_m R}{1-\delta Q}$, the lender's optimization problem can be written as

$$\begin{aligned} \underset{L_{FJL}, R_{FJL}}{maximize} \quad V_{FJL}^R \quad &= \frac{n\alpha Y^H - QnR_{FJL}}{1-\delta Q} \\ s.t. \qquad\qquad &QR_{FJL} \geq L_{FJL}(1+\epsilon) \\ &nR_{FJL} \leq Y^H \\ &nR_{FJL} < \frac{\delta\left(1-\delta^T\right)\left(\alpha Y^H - \alpha k_m R_{FJL}\right)}{1-\delta Q}, \end{aligned}$$

Since V_{FJL}^R is decreasing in R_{FJL}, the lender sets the lowest R_{FJL} possible; that is, $R_{FJL} = \frac{L_{FJL}(1+\epsilon)}{Q}$. Replacing R_{FJL} into the objective function and constraints, we will have

$$\begin{aligned} \underset{L_{FJL}, R_{FJL}}{maximize} \quad V_{FJL}^R \quad &= \frac{n\alpha Y^H - QnR_{FJL}}{1-\delta Q} \\ s.t. \qquad\qquad &L_{FJL} \leq \frac{QY^H}{n(1+\epsilon)} \equiv \tilde{L}_{FJL} \\ &L_{FJL} < \frac{QY^H}{(1+\epsilon)}\left(\frac{\delta\left(1-\delta^T\right)\alpha}{n - n\delta Q + \delta\left(1-\delta^T\right)\alpha k_m}\right) \equiv \hat{L}_{FJL}. \end{aligned}$$

From the above constraints, a feasible loan L must satisfy $L \leq \min\left\{\tilde{L}_{FJL}, \hat{L}_{FJL}\right\}$. It is not difficult to verify that $\hat{L}_{FJL} \leq \tilde{L}_{FJL}$ if and only if

$$\frac{\delta\left(1-\delta^T\right)\alpha}{n - n\delta Q + \delta\left(1-\delta^T\right)\alpha k_m} \leq \frac{1}{n},$$

or

$$\delta \leq \frac{n}{n\alpha\left(1-\delta^T\right) + nQ - \alpha k_m\left(1-\delta^T\right)} \equiv \tilde{\delta}_{FJL}.$$

Thus, $L \leq \hat{L}_{FJL}$ if $\delta \leq \tilde{\delta}_{FJL}$ and $L \leq \tilde{L}_{FJL}$ if $\delta \geq \tilde{\delta}_{FJL}$. □

Lemma 2.

Proof. Consider $\hat{L}_{FJL}$ and $\tilde{\delta}_{FJL}$defined in Proposition 5. It can be verified that $\hat{L}_{FJL}$ increases and $\tilde{\delta}_{FJL}$ decreases when T grows larger.

$$\frac{\partial \hat{L}_{FJL}}{\partial T} = \left(\frac{QY^H}{1+\epsilon}\right)\left[\frac{n\alpha\delta^{T+1}\left(\ln\delta\right)\left(\delta Q-1\right)}{\left(n-n\delta Q+\delta\left(1-\delta^T\right)\alpha k_m\right)^2}\right] \geq 0$$
$$\frac{\partial \tilde{\delta}_{FJL}}{\partial T} = \frac{n\alpha\delta^{T}\left(\ln\delta\right)\left(n-k_m\right)}{\left[n\alpha\left(1-\delta^T\right)+nQ-\alpha k_m\left(1-\delta^T\right)\right]^2} \leq 0.$$

□