Unit 2

Minimization Technique.

Important Topics :-

→ Boolean postulate/properties.
→ De-morgan Theorem.
→ Duality property.
→ Minterm and MaxTerm (SOP, POS)
→ k'map.
→ k'map with Don't care Condition.
→ Quine mc Cluskey.
→ Variable Entered Map (VEM).

Boolean Properties.

Commutative Law	Associative Law	Distributive Law.
⇒ When position of input is changed then also output will be same.	$A+(B+C)=(A+B)+C$ $A\cdot(B\cdot C)=(A\cdot B)\cdot C$.	$A+(BC)$ = It can also be written as :-
$A+B=B+A$. Both are same.	We can write in both ways. The output will be same.	$A+(BC)=(A+B)(A+C)$
$A\cdot B=B\cdot A$. Both are same.		$A\cdot(B+C)=AB+AC$.

Other Boolean properties are :-

1. $A + A = A$.
2. $A + 1 = 1$
3. $A + 0 = A$
4. $A + \overline{A} = \cancel{A}\ 1$

5. $A \cdot A = A$
6. $A \cdot 1 = A$
7. $A \cdot 0 = 0$
8. $A \cdot \overline{A} = 0$.

9. $\overline{\overline{A}} = A$.

10. Redundancy law :- $\cancel{A * B/C}$. $A + AB$

$$= A(1 + B)$$

$$1 + B = 1.$$

$$\therefore \text{Ans} = A.$$

:- $A \cdot (A + B)$

$$= A \cdot A + A \cdot B.$$

$$= A + AB.$$

$$= A\underbrace{(1 + B)}_{1}$$

$$= \boxed{A}$$

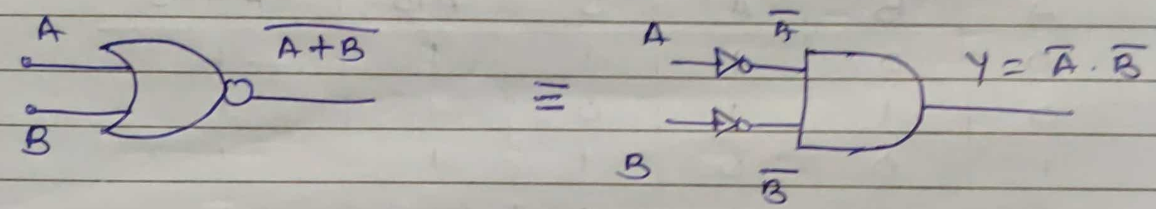
De-Morgan Theorem.

⇒ Compliment of Union of two sets is the Intersection of their Compliments and the Compliments of intersection of two sets is the Union of their Compliments.

1st Theorem :- $\overline{A+B} = \overline{A} \cdot \overline{B}$

↳ The Compliment of OR gate is equal to individual Compliment of AND gate.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$



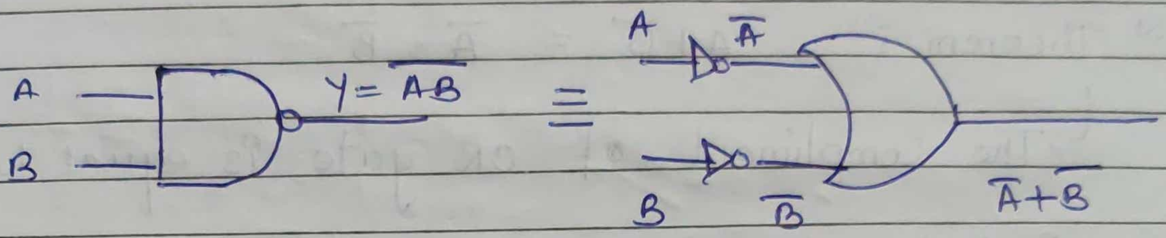
Truth Table

A	B	$\overline{A}$	$\overline{B}$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

⇒ more than two input can also be Implemented in same way.

2nd Theorem :- $\overline{A \cdot B} = \bar{A} + \bar{B}$

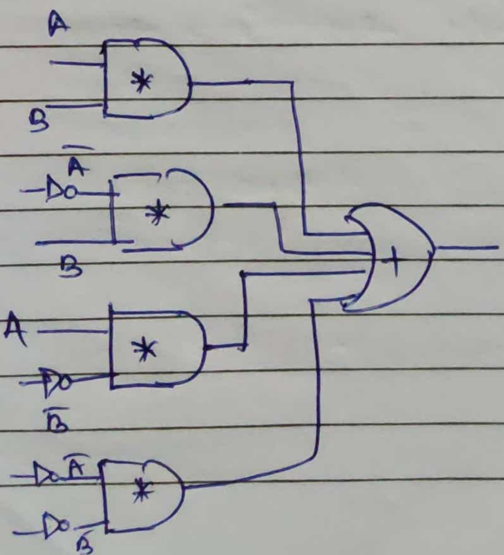
↳ The Compliment of AND (NAND) is equal to individual compliment of OR gate.

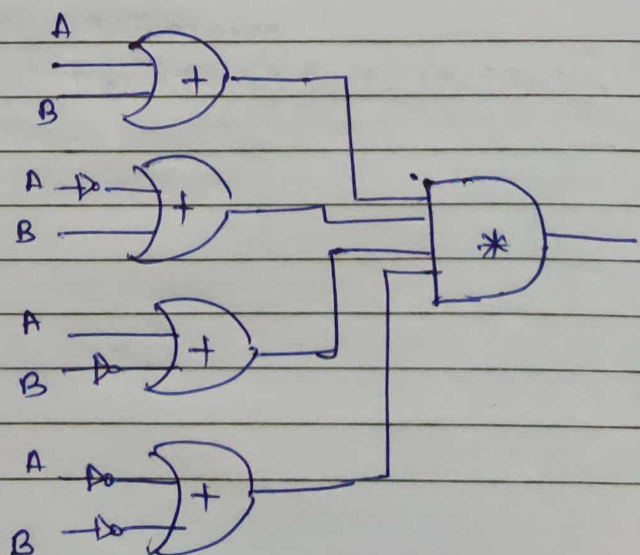


Truth Table

A	B	$\bar{A}$	$\bar{B}$	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0.

SOP (minterm)	POS (maxterm)
→ Sum of product. (+) (*)	→ product of sum. (*) (+)
→ It is also called a minterm.	→ It is also called a Maxterm.
→ It is Summation of all product terms.	→ It is product of all Summation terms.
→ It is Denoted as Σm.	→ It is denoted as ΠM.
→ If we take two Input A & B, then it can be written as	→ If we take two Input A & B, then it can be written as
$A \cdot B + \bar{A}B + A\bar{B} + \bar{A}\bar{B}$	$(A+B) \cdot (\bar{A}+B) \cdot (A+\bar{B}) \cdot (\bar{A}+\bar{B})$
→ Compliment = 0's.	→ Compliment = 1's.
→ In k'map, group of 1's is calculated.	→ In k'map, group of 0's is calculated.





Example :- simplify Boolean function.

$$A+B\left(AC+(B+\overline{C})D\right)$$

$$= A+B\left[AC+(B+\overline{C})D\right]$$

$$= A+B\left[AC+BD+\overline{C}D\right].$$

$$= A+ABC+\underbrace{B.B}_{B}.D+B\overline{C}D$$

$$= A+ABC+BD+B\overline{C}D.$$

$$= A\underbrace{\left[1+BC\right]}_{1}+BD\underbrace{\left[1+\overline{C}\right]}_{1}.$$

$$= \boxed{A+BD} \leftarrow \text{Ans}.$$

Example :- Simplify Boolean function using k'map.

$F = \overline{A}\,\overline{B}\,\overline{D} + \overline{A}CD + \overline{A}BC$.

$D = \overline{A}B\overline{C}D + ACD + A\overline{B}\,\overline{D}$ (Don't care condition).

Solution :-

$F = \overline{A}\,\overline{B}\,\overline{D} + \overline{A}CD + \overline{A}BC$.

Total Variable = A, B, C, D (04).

Find missing in Each term.

$= \overline{A}\,\overline{B}\,\overline{D}\,(C+\overline{C}) + \overline{A}CD\,(B+\overline{B}) + \overline{A}BC\,(D+\overline{D})$

$= \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \underline{\overline{A}BCD} + \overline{A}\,\overline{B}CD + \cancel{\overline{A}BCD} + \overline{A}BC\overline{D}$.

= Remove Repeated Term.

$= \underbrace{\overline{A}\,\overline{B}C\overline{D}}_{0010 \to 2} + \underbrace{\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}}_{0000 \to 0} + \underbrace{\overline{A}BCD}_{0111 \to 7} + \underbrace{\overline{A}\,\overline{B}CD}_{0011 \to 3} + \underbrace{\overline{A}BC\overline{D}}_{0110 \to 6}$.

$D = \overline{A}B\overline{C}D + ACD + A\overline{B}\,\overline{D}$

$= \overline{A}B\overline{C}D + ABCD + \underline{A\overline{B}CD} + \cancel{A\overline{B}CD} + A\overline{B}\,\overline{C}\,\overline{D}$

$= \underbrace{\overline{A}B\overline{C}D}_{0101 \to 5} + \underbrace{ABCD}_{1111 \to 15} + \underbrace{A\overline{B}CD}_{1011 \to 11} + \underbrace{A\overline{B}\,\overline{C}\,\overline{D}}_{1000 \to 8}$.

put 1's at Number = 0, 2, 3, 6, 7.

put X at Numbers = 5, 8, 11, 15.

put 0's at Number = Remaining i.e 1, 4, 5, 9, 10, 12, 13, 14.

Total Input = 4.

$2^4 = 16$ (Block) k map.

$CD \backslash AB$	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1 (0)	(4)	(12)	X (8)
$\bar{C}D$ 01	(1)	X (5)	(13)	(9)
CD 11	1 (3)	1 (7)	X (15)	X (11)
$C\bar{D}$ 10	1 (2)	1 (6)	(14)	(10)

$\Rightarrow \bar{B}\bar{C}\bar{D} + CD + \bar{A}C\bar{D}$.

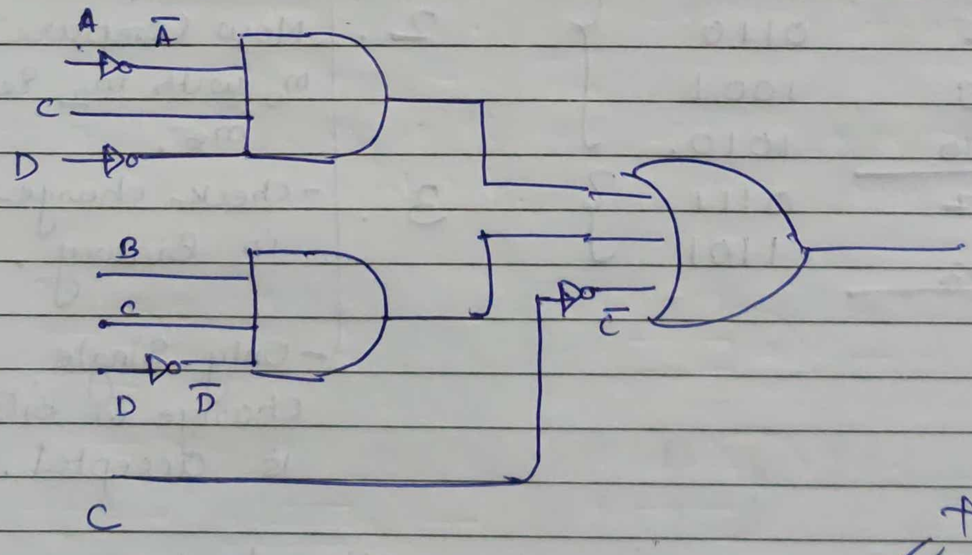
Example:- Use k'map & draw logic diagram. (3)

$F(w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$

Solution:- 16 Block k'map is used

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1 (0)	1 (4)	1 (12)	1 (8)
$\bar{C}D$ 01	1 (1)	1 (5)	1 (13)	1 (9)
CD 11	(3)	(7)	(15)	(11)
$C\bar{D}$ 10	1 (2)	1 (6)	1 (14)	(10)

$\Rightarrow \overline{C} + \overline{A}C\overline{D} + BC\overline{D}$



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Quine Mc cluskey Method. (Tabular Method).

Ex: $f = \Sigma_m (0, 2, 3, 6, 7, 8, 9, 10, 13)$.

1st Selection of Prime Number.

	Minterm	Binary
0	m_0	0000
2	m_2	0010
3	m_3	0011
6	m_6	0110
7	m_7	0111
8	m_8	1000
9	m_9	1001
10	m_{10}	1010
13	m_{13}	1101

Next Step:- Arrange all Binary Values according to number of 1's.

2nd

minterm	Binary	No. of 1's
m_0	0000	~~0000~~ 0
m_2	0010	1
m_8	1000	
m_3	0011	2
m_6	0110	
m_9	1001	
m_{10}	1010	
m_7	0111	3
m_{13}	1101	

Next step:- Now Compare m_0 with m_2 & m_8.

- Check change in Binary.
- Only Single change in Bits is accepted.

→ If there is change in two Bits, dont consider.

→ when there is change in Single Bit, — is placed at that bit.

Notes - Dont consider (dont write) if two bits are changing.

minterm	Binary.	prime Implicit.
$m_0 - m_2$	0000 0010	00_0
$m_0 - m_8$	0000 1000	_000.
$m_2 - m_3$	0010 0011	001_
$m_2 - m_6$	0010 0110	0_10.
$m_2 - m_9$	0010 1001	There are ~~two~~ three Bits change. (Not accepted)
$m_2 - m_{10}$	0010 1010	_ϕ0_.
$m_8 - m_9$	1000 1001	100_
$m_8 - m_{10}$	1000 1010	10_0
$m_3 - m_7$	0011 0111	0_11
$m_6 - m_7$	0ϕ10 0111	011_
$m_9 - m_{13}$	ϕ00ϕ 1101	1_01

Next step :- Compare the Prime Implicit & mark — when there is single change.

minterm	Binary.	prime Implicit.
$m_0 - m_2$	0000	
$m_0 - m_8$.	0010	-0
	0000	
	1000.	