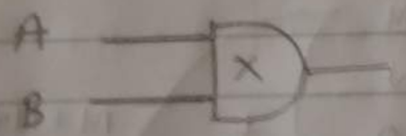
Ch-2 :- Minimizing Technique

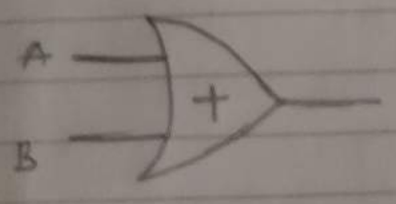
* Topics
① Boolean expression / postulate
② De-morgan Theorem
③ Duality (principle of duality)
④ min Term / max Term
(SOP) (POS)
↓ ↓
Sum of product | product of sum
⑤ Karnaugh map (K'map)
⑥ Don't care Condition.

—— x —— x ——

* Boolean Expression



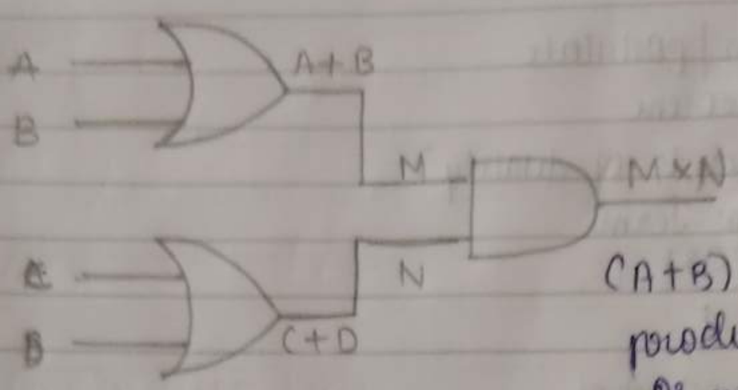
$y = AB$
↳ Boolean expression



$y = A+B$
↳ Sum of Boolean expression

Boolean expression is nothing but represent input & output in alphabetical manner.

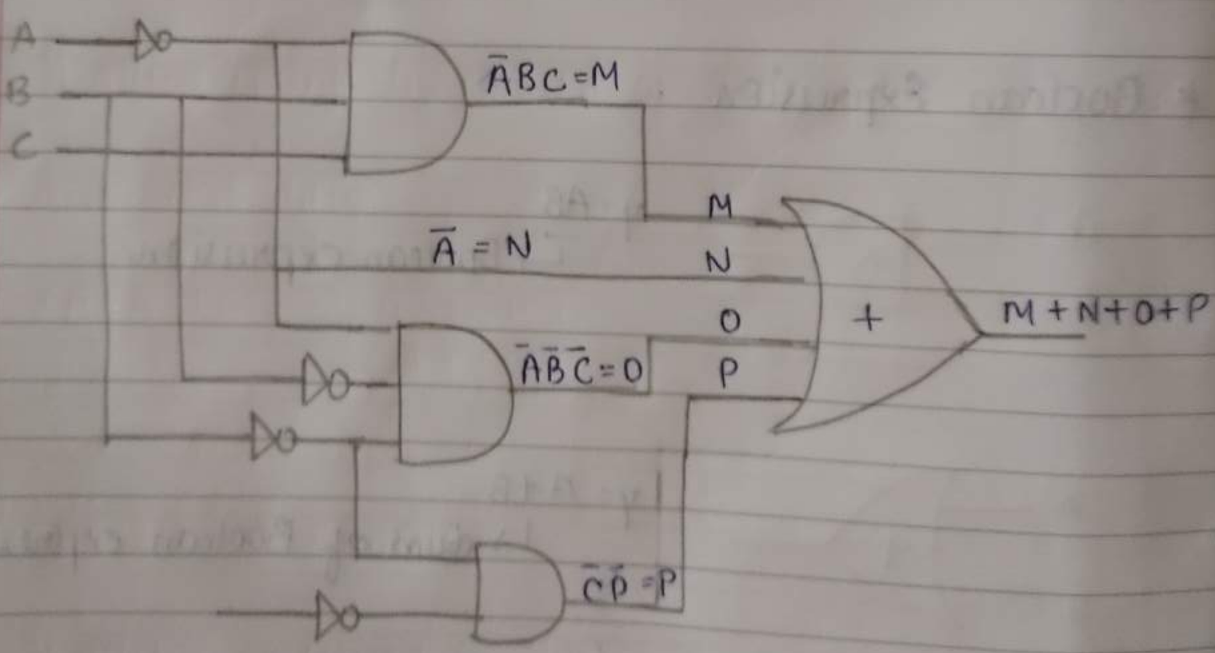
1)



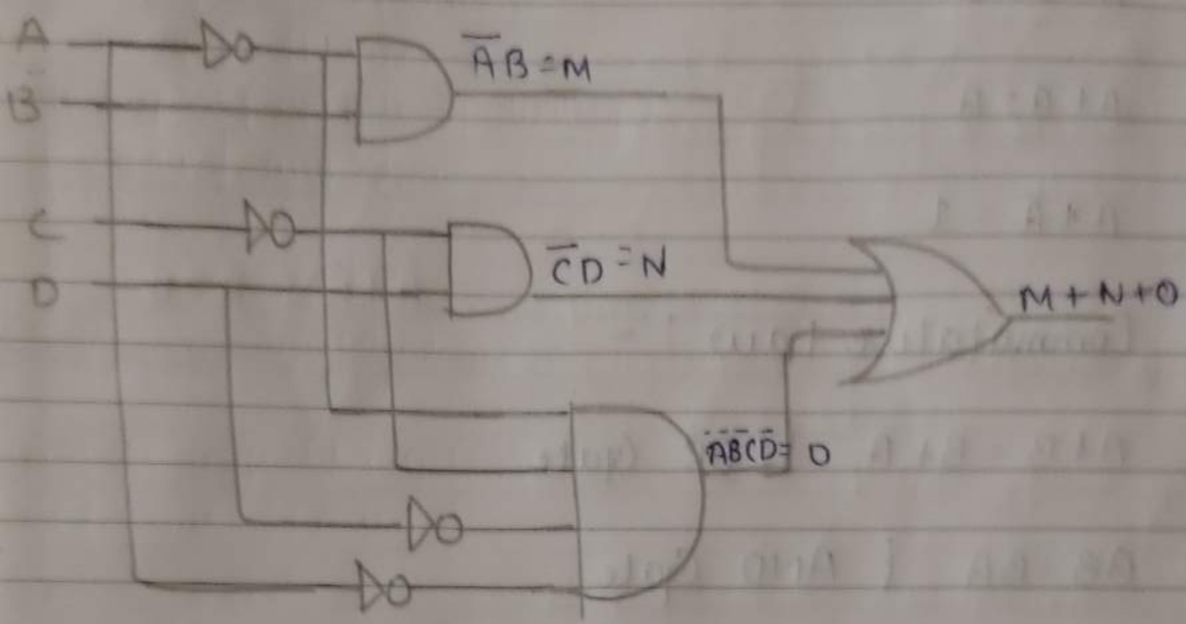
$(A+B).(C+D)$

product of sum (POS) or max term.

2) $\bar{A}BC + \bar{A}\bar{B}\bar{C} + \bar{A} + \bar{C}\bar{D}$



3) $\bar{A}B + \bar{C}D + \bar{A}B\bar{C}\bar{D}$



* Boolean postulates / Law

① $A \cdot 1 = A$ (Identity Law)

② $A \cdot 0 = 0$ (Null law)

③ $A \cdot A = A$
$AAAA = A$

4. $A \cdot \bar{A} = 0$
$\left\{ \begin{array}{l} A = 0 \\ \bar{A} = 1 \end{array} \right\} = 0 \cdot 1 = 0$

multiplication terms.

(5) $A+0=A$

(6) $A+1=1$

(7) $A+A=A$

(8) $A+\bar{A}=1$

(9) Commutative Law :-

- $A+B=B+A$ } OR Gate
- $AB=BA$ } AND Gate

It allow change in position of AND & OR Variable.

10) Associative law

- $(A+B)+C \Rightarrow A+(B+C)$
  $= (A+C)+B$

- $(ABC)D \Rightarrow (ABD)C$

It allows Grouping of Variable

11) Distributive Law :-
If allow factoring or multiplying out of expression

$A(B+C) = AB + AC$

$A+BC = (A+B)(A+C)$

* De-morgan Theorem:

(1) $\overline{\overline{A}\cdot\overline{B}} = A+B$

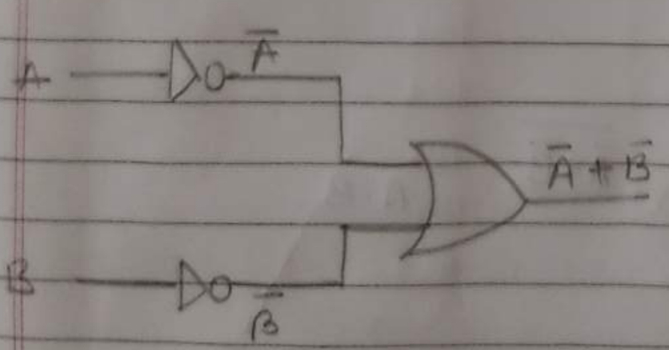
(2) $\overline{\overline{A}+\overline{B}} = A\cdot B$

1) NAND

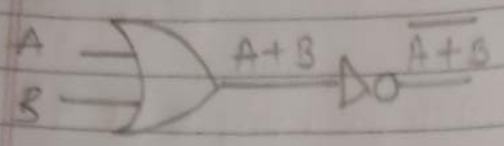
Break the Compliment (Bar)
then $* = +$
$+ = *$

$\overline{A}$ & $\overline{B}$

$\overline{A\cdot B} = \overline{A}+\overline{B}$

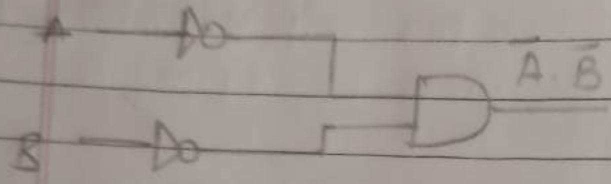


2) NOR

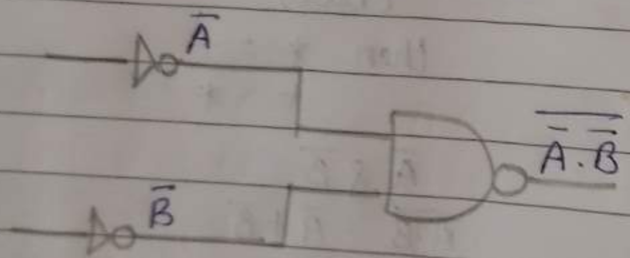


$\equiv$

$\overline{A+B} = \overline{A \cdot B}$

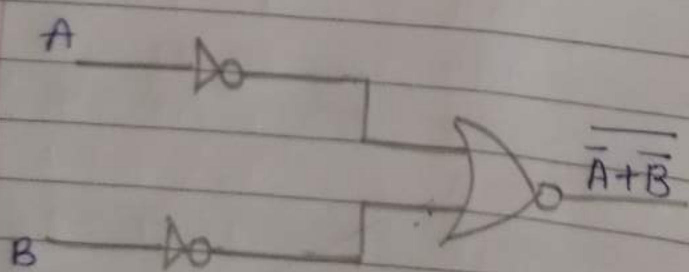


3)



$\overline{\overline{A} \cdot \overline{B}} = A + B$

4)



$\cdot$ A.B.

5) $\overline{A \cdot B \cdot C \cdot D} = \bar{A} + \bar{B} + \bar{C} + \bar{D}$

$\overline{A \cdot \bar{B} \cdot C \cdot \bar{D}} = \bar{A} + B + \bar{C} + D$

· Truth Table.

A	B	$\bar{A}$	$\bar{B}$	AB	$\overline{\bar{A}+\bar{B}}$	A+B

* Min term & max term.

① $-\Sigma_m$	(Π_m)
② SOP	$-(POS)$
③ $-(AB+BD+AC)$	$(A+B)(B+D)(A+C)$

* Difference b/w min term & max term

min term	max term
① Sum of product	① product of sum.
② In SOP the boolean logic AND operation is done for variable & expression & then OR Gate is used.	

(3) The minterm define the minimum no of Combination of input

(4) It is defined by AND Gate

(5) When input is zero of any Variable then that Variable is Compliment term.

* Min Gate $\longrightarrow$ AND Gate

Eg - 3 input - A, B, C

	A	B	C	A B C	minterm
0	0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
1	0	0	1	$\bar{A}\bar{B}C$	m_1
2	0	1	0	$\bar{A}B\bar{C}$	m_2
3	0	1	1	$\bar{A}BC$	m_3
4	1	0	0	$A\bar{B}\bar{C}$	m_4
5	1	0	1	$A\bar{B}C$	m_5
6	1	1	0	$AB\bar{C}$	m_6
7	1	1	1	ABC	m_7
8					
				non-Canonical	Canonical

* 3 different form of SOP

1) Canonical form
2) non-Canonical form
3) minimal

* Canonical (SOP) → Group of minimal

* non-Canonical

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + ABC + A\bar{B}C$$

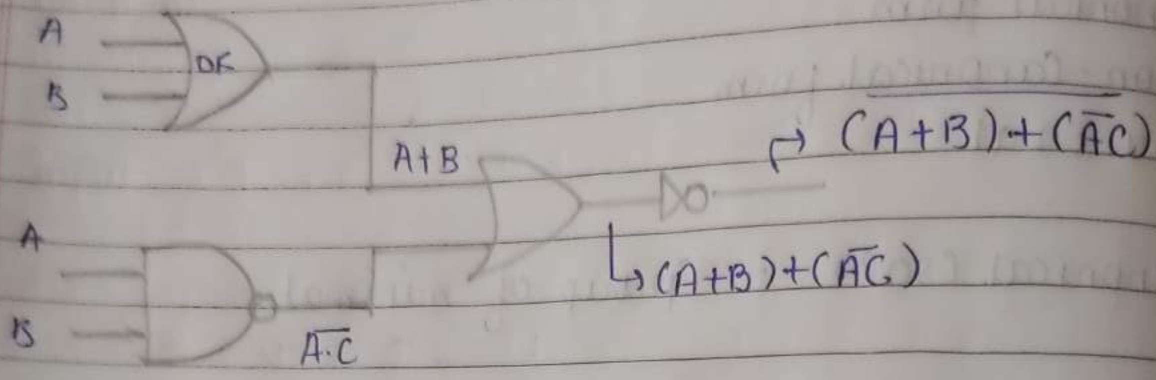
$$\bar{A}\bar{C}\underbrace{(\bar{B}+B)}_{1} + AC\underbrace{(B+\bar{B})}_{1}$$

$$\bar{A}\bar{C} + AC$$

$$\underbrace{(\bar{A}+A)}_{1}(\bar{A}+C)(\bar{C}+A)$$

$$(\bar{A}+C)(A+\bar{C})$$

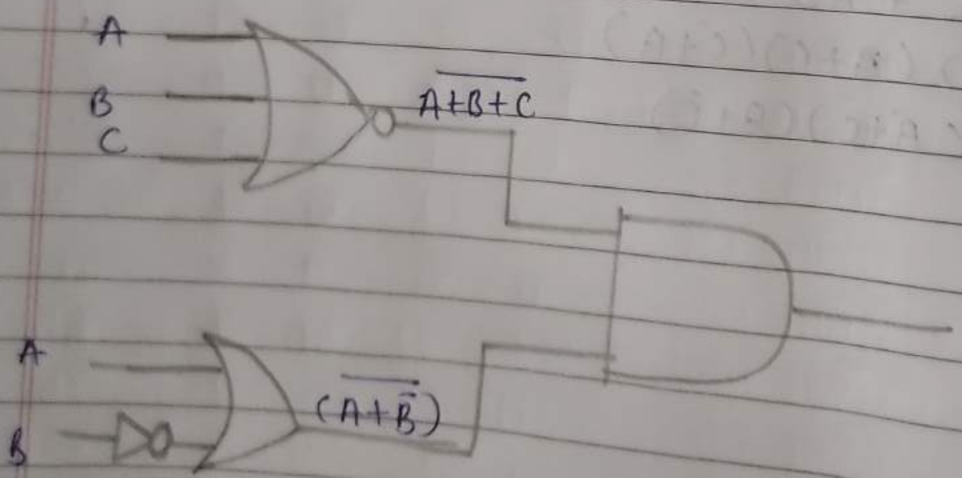
* Boolean expression



2) $(\overline{A+B+C})\,(A+\bar{B})$

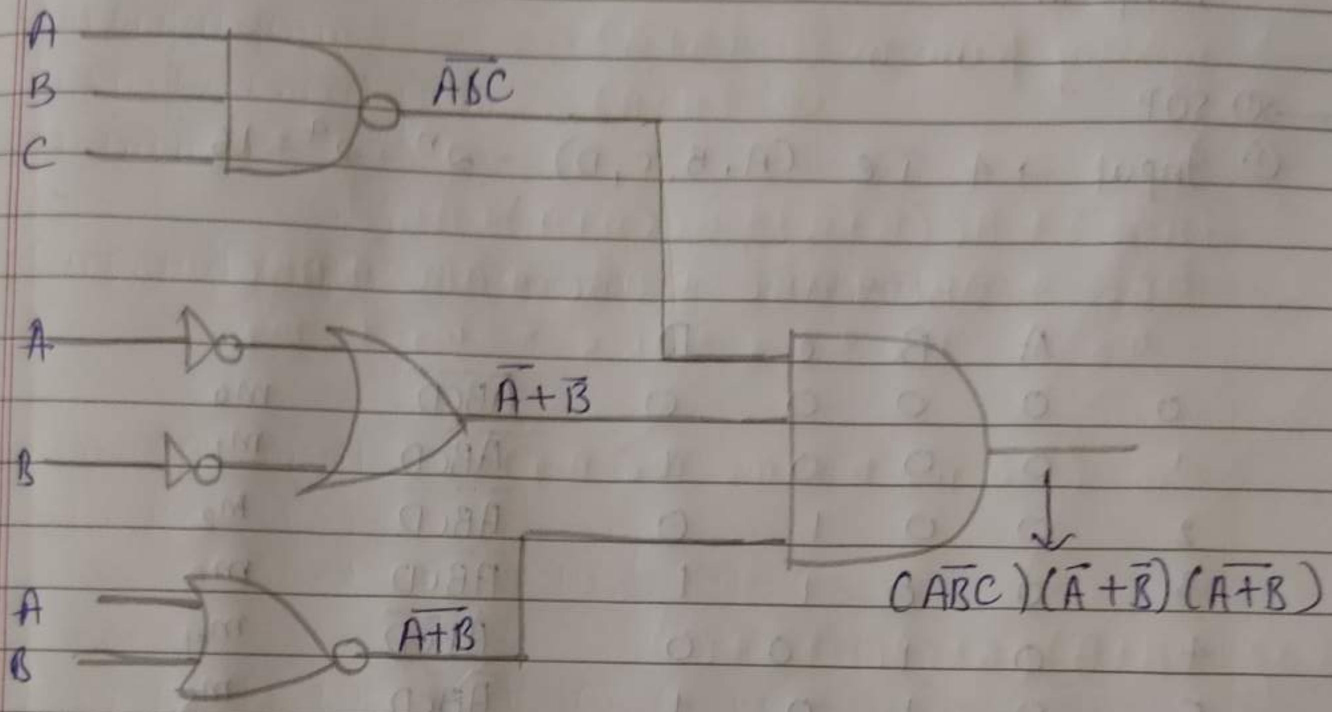
→ OR Gate $A+B+C$ AND Gate

NOR Gate $\overline{A+B+C}$



3) $(\overline{ABC})(\overline{A}+\overline{B})(\overline{A+B})$

→



* Difference b/w SOP & POS

	SOP	POS
①	Sum of product	product of sum
②	min Term	max term
③	m	M
④	Σm	Πm
⑤	$(AB+AC+BC)$	$(A+B)\cdot(A+C)\cdot(B+C)$
⑥	Bar = 0	Bar = 1

Ex:-

Q $A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$ - Canonical form & Find it's minterm

→ ① SOP

② Input → 4 i.e (A, B, C, D) $= 2^n = 2^4 = 16$

	A	B	C	D		
0	0	0	0	0	$\overline{ABCD}$	m_0
1	0	0	0	1	$\bar{A}\bar{B}\bar{C}D$	m_1
2	0	0	1	0	$\bar{A}\bar{B}C\bar{D}$	m_2
3	0	0	1	1	$\bar{A}\bar{B}CD$	m_3
4	0	1	0	0	$\bar{A}B\bar{C}\bar{D}$	m_4
5	0	1	0	1	$\bar{A}B\bar{C}D$	m_5
6	0	1	1	0	$\bar{A}BC\bar{D}$	m_6
7	0	1	1	1	$\bar{A}BCD$	m_7
8	1	0	0	0	$A\bar{B}\bar{C}\bar{D}$	m_8
9	1	0	0	1	$A\bar{B}\bar{C}D$	m_9
10	1	0	1	0	$A\bar{B}C\bar{D}$	m_{10}
11	1	0	1	1	$A\bar{B}CD$	m_{11}
12	1	1	0	0	$AB\bar{C}\bar{D}$	m_{12}
13	1	1	0	1	$AB\bar{C}D$	m_{13}
14	1	1	1	0	$ABC\bar{D}$	m_{14}
15	1	1	1	1	$ABCD$	m_{15}

Q. $\underset{①}{A\bar{B}C} + \underset{②}{\bar{A}\bar{B}} + \underset{③}{AB\bar{C}D}$

→ ① In each term find out missing variable.

missing term is D in ① (C, D) in ②

② In SOP multiply the missing variable in the form $(D+\bar{D})$

③ Simplify the eq^n.

$A\bar{B}C(D+\bar{D}) + \bar{A}\bar{B}(C+\bar{C})(D+\bar{D}) + AB\bar{C}D$

$ABCD + ABCD + ABC \quad + AB$

$A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

$m_{11} + m_{10} + m_3 + m_2 + m_1 + m_0 + m_{13}$

$\Sigma_m (m_0, m_1, m_2, m_3, m_{10}, m_{11}, m_{13})$

Q 2) $\bar{A}\bar{B}\bar{C} + A\bar{C} + \bar{B}C + ABC$

① ② ③ ④

→ missing term in ② $(B+\bar{B})$

③ $(A+\bar{A})$

$(\bar{A}\bar{B}\bar{C}) + A\bar{C}(B+\bar{B}) + \bar{B}C(A+\bar{A}) + ABC$

$\bar{A}\bar{B}\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + ABC$

$m_0 + m_6 + m_4 + m_5 + m_1 + m_7$

$\Sigma m\,(m_0\ m_1\ m_4\ m_5\ m_6\ m_7)$.

Q 3) $(A+\bar{B})(B+C)(\bar{A}+\bar{B}+\bar{C})$

→ ① POS

② POS | Input = 3 (A, B, C) = $2^3 = 8$

	A	B	C	Boolean eqⁿ
0	0	0	0	$A+B+C$
1	0	0	1	$A+B+\bar{C}$
2	0	1	0	$A+\bar{B}+C$
3	0	1	1	$A+\bar{B}+\bar{C}$
4	1	0	0	$\bar{A}+B+C$
5	1	0	1	$\bar{A}+B+\bar{C}$
6	1	1	0	$\bar{A}+\bar{B}+C$
7	1	1	1	$\bar{A}+\bar{B}+\bar{C}$

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missing term in ① $C.\bar{C}$
② $A.\bar{A}$

$(A+\bar{B}+(C.\bar{C}))(B+C+(A.\bar{A}))(\bar{A}+\bar{B}+\bar{C})$

$(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

$\quad m_2 \qquad m_3 \qquad m_0 \qquad m_4 \qquad m_7$

$\Pi_m(m_0, m_2, m_3, m_4, m_7)$.