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Exercise 12.1

Question 1:

Find the coordinates of the point which divides the line segment joining the points (-2, 3, -2, 3)

- 5) and (1, -4, 6) in the ratio
- (i) 2:3 internally, (ii) 2:3 externally.

Solution 1:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (-2, 3, 5) and (1, -4, 6) internally in the ratio 2:3

$$x = \frac{2(1) + 3(-2)}{2+3}$$
, $y = \frac{2(-4) + 3(3)}{2+3}$, and $z = \frac{2(6) + 3(5)}{2+3}$

i.e.,
$$x = \frac{-4}{5}$$
, $y = \frac{1}{5}$, and $z = \frac{27}{5}$

Thus, the coordinates of the required point are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$

(ii) The coordinates of point R that divides the line segment joining points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ externally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m - n}, \frac{my_2 + ny_1}{m - n}, \frac{mz_2 + nz_1}{m - n}\right)$$

Let R (x, y, z) be the point that divides the line segment joining points (-2, 3, 5) and (1, -4, 6) externally in the ratio 2:3

$$x = \frac{2(1) - 3(-2)}{2 - 3}$$
, $y = \frac{2(-4) - 3(3)}{2 - 3}$, and $z = \frac{2(6) - 3(5)}{2 - 3}$

i.e.,
$$x = -8$$
, $y = 17$, and $z = 3$

Thus, the coordinates of the required point are (-8, 17, 3).

Ouestion 2:

Given that P (3, 2, -4), Q (5, 4, -6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Solution 2:

Let point Q (5, 4, -6) divide the line segment joining points P (3, 2, -4) and R (9, 8, -10) in the ratio k:1.

Therefore, by section formula,

$$(5, 4, -6) = \left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow$$
 9k + 3 = 5k + 5

$$\Rightarrow 4k = 2$$



$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (-2, 4, 7) and (3, -5, 8).

Solution 3:

Let the YZ plane divide the line segment joining points (-2, 4, 7) and (3, -5, 8) in the ratio k:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1}, \frac{k(-5)+4}{k+1}, \frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$$\Rightarrow \frac{3k-2}{k+1} = 0$$

$$\Rightarrow 3k-2=0$$

$$\Rightarrow k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4:

Using section formula, show that the points A (2, -3, 4), B (-1, 2, 1) and C $\left(0, \frac{1}{3}, 2\right)$ are collinear.

Solution 4:

The given points are A (2, -3, 4), B (-1, 2, 1), and C $\left(0, \frac{1}{3}, 2\right)$

Let P be a point that divides AB in the ratio k:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking
$$\frac{-k+2}{k+1} = 0$$
, we obtain $k = 2$.

For k = 2, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$

i.e., $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.



Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

Solution 5:

Let A and B be the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6)

Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-4)}{1+2}\right) = (6, -4, -2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1}, \frac{2(-16)+1(2)}{2+1}, \frac{2(6)+1(-4)}{2+1}\right) = (8,-10,2)$$

Thus, (6, -4, -2) and (8, -10, 2) are the points that trisect the line segment joining points P (4, 2, -6) and Q (10, -16, 6).

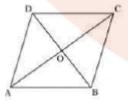
Miscellaneous Exercise

Question 1:

Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) and C (-1, 1, 2). Find the coordinates of the fourth vertex.

Solution 1:

The three vertices of a parallelogram ABCD are given as A (3, -1, 2), B (1, 2, -4), and C (-1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

: Mid-point of AC = Mid-point of BD



$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow (10,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

$$\Rightarrow x = 1, y = 2 \text{ and } z = 8$$

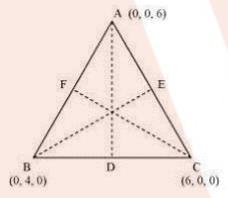
Thus, the coordinates of the fourth vertex are (1, -2, 8).

Question 2:

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Solution 2:

Let AD, BE, and CF be the medians of the given triangle ABC



Since AD is the median, D is the mid-point of BC

Coordinates of point D =
$$\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3,2,0)$$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$$

Since BE is the median, E is the mid-point of AC.

Coordinate of point E =
$$\left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{0+0}{2}\right) = (3,0,3)$$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = \sqrt{9+16+9} = \sqrt{34}$$

Since CF is the median, F is the mid-point of AB

Coordinates of point
$$F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0,2,3)$$

Length of CF =
$$\sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

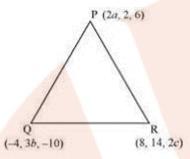
Thus, the lengths of the medians of $\triangle ABC$ are 7, $\sqrt{34}$, and 7.



Question 3:

If the origin is the centroid of the triangle PQR with vertices P (2a, 2, 6), Q (-4, 3b, -10) and R (8, 14, 2c), then find the values of a, b and c.

Solution 3:



It is known that the coordinates of the centroid of the triangle, whose vertices are

$$(x_1, y_1, z_1)(x_2, y_2, z_2)$$
 and (x_3, y_3, z_3) , are

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}$$

Therefore, coordinates of the centroid of

$$\Delta PQR = \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right) = \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of $\triangle PQR$

$$\Rightarrow \frac{2a+3}{3} = 0, \frac{3b+16}{3} = 0$$
 and $\frac{2c-4}{3} = 0$

$$\Rightarrow a = -2, b = \frac{16}{3}$$
 and $c = 2$

Thus, the respective values of a, b and c are -2, $-\frac{16}{3}$ and 2

Question 4:

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, -2, 5).

Solution 4:

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, -2, 5). Accordingly, AP = $5\sqrt{2}$



$$\Rightarrow AP^2 = 50$$

$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$

$$\Rightarrow$$
 9 + 4 + b^2 + 4 b + 25 = 50

$$\Rightarrow b^2 + 4b - 12 = 0$$

$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$

$$\Rightarrow$$
 $(b+6)(b-2)=0$

$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinate of the required points are (0, 2, 0) and (0, -6, 5)

Question 5:

A point R with x-coordinate 4 lies on the line segment joining the points P(2, -3, 4) and Q(8, 0, 10). Find the coordinates of the point R.

[Hint: suppose R divides PQ in the ratio k: 1.] The coordinates of the point R are given by

Solution 5:

The coordinates of points P and Q are given as P (2, -3, 4) and Q (8, 0, 10). Let R divide line segment PQ in the ratio k:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the x-coordinate of point R is 4

$$\frac{8k+2}{k+1} = 4$$

$$8k + 2 = 4k + 4$$

$$4k = 2$$

$$k = \frac{1}{2}$$

Therefore, the coordinates of points R are

$$\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$$

Question 6:

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Solution 6:



The coordinates of points A and B are given as (3, 4, 5) and (-1, 3, -7) respectively. Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$= x^{2} + 9 - 6x + y^{2} + 16 - 8y + z^{2} + 25 - 10z$$

$$= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$$

$$PB^2 = (x-1)^2 + (y-3)^2 + (z-7)^2$$

$$= x^2 - 2x + y^2 - 6y + z^2 + 14z + 59$$

Now, if
$$PA^2 + PB^2 = k^2$$
, then

$$(x^2 -6x + y^2 -8y + z^2 -10z +50) + (x^2 +2x + y^2 -6y + z^2 +14z +59) = k^2$$

$$\Rightarrow$$
 2x² + 2y² + 2z² - 4x - 14y + 4z + 109 = k^2

$$\Rightarrow 2(x^2+y^2+z^2-2x-7y+2z)=k^2-109$$

$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is b $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$.

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