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268 RANKS IN TOP 10K

1 OUT OF 3 STUDENTS WHO CLEARED JEE MAINS SECURED A RANK IN JEE ADVANCED


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Dhruv Rambhia
AIR 43


Panchkula se
Madhav Goel
AIR 208


Gurugram se
Tanmay Gangwar
AIR 227

JEE Adv.

27 STUDENTS SCORED ABOVE 600

46 STUDENTS SCORED ABOVE 550


Patiala se
Shaurya Bhatia
682/720


Maharashtra se
Anjaney Pandey
670/720


Surat se
Tanishi Bhatt
670/720

NEET

157 Students
Scored above 90% in 12th


Lucknow se
Arya Verma
98.40%


Jharkhand se
Akshat Kumar
98.20%


Dubai se
Navya Ratnan
98.00%

12TH Boards

171 Students
Scored above 90% in 10th


Chennai se
Meera Ranjan
98.8%


Jhajjar se
Ayush Singh
98.2%


Patna se
Shambhavi Sinha
98.2%

10TH Boards


Malappuram se
Senin Ahammed
100%


Rajahmundry se
Reshma M
99.4%


Dahod se
Ummehani N
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Exercise 5.1

Question 1:

Express the given complex number in the form $a+ib:(5i)\left(-\frac{3}{5}i\right)$

Solution 1:

$(5i)\left(\frac{-3}{5}i\right)=-5\times\frac{3}{5}\times i\times i$

$=-3i^2$

$=-3(-1) \qquad [i^2=-1]$

$=3$

Question 2:

Express the given complex number in the form $a+ib:i^9+i^{19}$

Solution 2:

$i^9+i^{19}=i^{4\times2+1}+i^{4\times4+3}$

$=(i^4)^2\cdot i+(i^4)^4\cdot i^3$

$=1\times i+1\times(-i) \qquad [i^4=1,\ i^3=-i]$

$=i+(-i)$

$=0$

Question 3:

Express the given complex number in the form $a+ib:i^{-39}$

Solution 3:

$i^{-39}=i^{-4\times9-3}=(i^4)^{-9}\cdot i^{-3}$

$=(1)^{-9}\cdot i^{-3} \qquad [i^4=1]$

$=\frac{1}{i^3}=\frac{1}{-i} \qquad [i^3=-i]$

$=\frac{-1}{i}\times\frac{i}{i}$

$=\frac{-i}{i^2}=\frac{-i}{-1}=i \qquad [i^2=-1]$

Question 4:

Express the given complex number in the form a $a+ib$:

$3(7+i7)+i(7+i7)$

Solution 4:

$$3(7+i7)+i(7+i7)=21+21i+7i+7i^2$$
$$=21+28i+7\times(-1) \qquad [\because\ i^2=-1]$$
$$=14+28i$$

Question 5:

Express the given complex number in the form $a+ib$: $(1-i)-(-1+i6)$.

Solution 5:

$$(1-i)-(-1+i6)=1-i+1-6i$$
$$=2-7i$$

Question 6:

Express the given complex number in the form $a+ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Solution 6:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Question 7:

Express the given complex number in the form $a+ib$: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Solution 7:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$

$=\dfrac{17}{3}+i\dfrac{5}{3}$

Question 8:

Express the given complex number in the form $a+ib:(1-i)^4$

Solution 8:

$(1-i)^4=\left[(1-i)^2\right]^2$

$=\left[1^2+i^2-2i\right]^2$

$=[1-1-2i]^2$

$=(2i)^2$

$=(-2i)\times(-2i)$

$=4i^2=-4 \qquad \left[i^2=-1\right]$

Question 9:

Express the given complex number in the form $a+ib:\left(\dfrac{1}{3}+3i\right)^3$

Solution 9:

$\left(\dfrac{1}{3}+3i\right)^3=\left(\dfrac{1}{3}\right)^3+(3i)^3+3\left(\dfrac{1}{3}\right)(3i)\left(\dfrac{1}{3}+3i\right)$

$=\dfrac{1}{27}+27i^3+3i\left(\dfrac{1}{3}+3i\right)$

$=\dfrac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]$

$=\dfrac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]$

$=\left(\dfrac{1}{27}-9\right)+i(-27+1)$

$=\dfrac{-242}{27}-26i$

Question 10:

Express the given complex number in the form $a+ib:\left(-2-\dfrac{1}{3}i\right)^3$

Solution 10:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$

$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$

$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number $4-3i$.

Solution 11:

Let $z=4-3i$

Then,

$\bar{z}=4+3i$ and $\left|z\right|=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of $4-3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Solution 12:

Let $z=\sqrt{5}+3i$

Then, $\bar{z}=\sqrt{5}-3i$ and $|z|^2=\left(\sqrt{5}\right)^2+3^2=5+9=14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number $-i$

Solution 13:

Let $z=-i$

Then, $\bar{z}=i$ and $|z|^2=1^2=1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

Question 14:

Express the following expression in the form of $a+ib$.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Solution 14:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Exercise 5.2

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Solution 1:

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})^2$

$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+3$

$\Rightarrow r^2=4 \qquad [\cos^2\theta+\sin^2\theta=1]$

$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$

$\therefore$ Modulus = 2

$\therefore 2\cos\theta=-1$ and $2\sin\theta=-\sqrt{3}$

$\Rightarrow \cos\theta=\dfrac{-1}{2}$ and $\sin\theta=\dfrac{-\sqrt{3}}{2}$

Since both the values of $\sin\theta$ and $\cos\theta$ negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant,

Argument $=-\left(\pi-\dfrac{\pi}{3}\right)=\dfrac{-2\pi}{3}$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $-\dfrac{2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

Solution 2:

$z=-\sqrt{3}+i$

Let $r\cos\theta=-\sqrt{3}$ and $r\sin\theta=1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-\sqrt{3})^2+1^2$

$\Rightarrow r^2=3+1=4 \qquad [\cos^2\theta+\sin^2\theta=1]$

$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$

$\therefore$ Modulus = 2

$\therefore 2\cos\theta=-\sqrt{3}$ and $2\sin\theta=1$

$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ [As θ lies in the II quadrant]

Thus, the modulus an argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: $1-i$

Solution 3:

$1-i$

Let $r\cos\theta = 1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ $[\text{Conventionally, } r > 0]$

$\therefore \sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\frac{\pi}{4}$ $[\text{As } \theta \text{ lies in the IV quadrant}]$

$\therefore 1-i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$

This is the required polar form.

Question 4:

Convert the given complex number in polar form: $-1+i$

Solution 4:

$-1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ [Conventionally, r > 0]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$ and $\sqrt{2}\sin\theta = 1$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in the II quadrant]

It can be written,

$\therefore\ -1+i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: $-1-i$

Solution 5:

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$ [As θ lies in the III quadrant]

$\therefore -1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$

$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$

This is the required polar form.

Question 6:

Convert the given complex number in polar form: -3

Solution 6:

-3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 9$

$\Rightarrow r^2 = 9$

$\Rightarrow r = \sqrt{9} = 3$ [Conventionally, r > 0]

$\therefore\ 3\cos\theta = -3$ and $3\sin\theta = 0$

$\Rightarrow \cos\theta = -1$ and $\sin = 0$

$\therefore \theta = \pi$

$\therefore\ -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3}+i$

Solution 7:

$\sqrt{3}+i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3+1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \sqrt{4} = 2$ $[\text{Conventionally, } r > 0]$

$\therefore 2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$ and $\sin\theta = \dfrac{1}{2}$

$\therefore \theta = \dfrac{\pi}{6}$ [As θ lies in the I quadrant]

$\therefore\ \sqrt{3}+i = r\cos\theta + ir\sin\theta = 2\cos\dfrac{\pi}{6} + i2\sin\dfrac{\pi}{6} = 2\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: i

Solution 8:

i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

$\Rightarrow r^2 = 1$

$\Rightarrow r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \dfrac{\pi}{2}$

$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}$

This is the required polar form.

Exercise 5.3

Question 1:

Solve the equation $x^2 + 3 = 0$

Solution 1:

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1$, $b = 0$, and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 0^2 - 4\times1\times3 = -12$

Therefore, the required solutions are

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times1} = \frac{\pm\sqrt{12}i}{2} \qquad [\sqrt{1} = i]$$

$$= \frac{\pm2\sqrt{3}i}{2} = \pm\sqrt{3}i$$

Question 2:

Solve the equation $2x^2 + x + 1 = 0$

Solution 2:

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 2$, $b = 1$ and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4\times2\times1 = 1 - 8 = -7$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2\times2} = \frac{-1 \pm \sqrt{7}i}{4} \qquad [\sqrt{-1} = i]$$

Question 3:
Solve the equation $x^2+3x+9=0$

Solution 3:
The given quadratic equation is $x^2+3x+9=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=3$, and $c=9$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=3^2-4\times1\times9=9-36=-27$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-27}}{2(1)}=\frac{-3\pm3\sqrt{-3}}{2}=\frac{-3\pm3\sqrt{3}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

Question 4:
Solve the equation $-x^2+x-2=0$

Solution 4:
The given quadratic equation is $-x^2+x-2=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=-1$, $b=1$, and $c=-2$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=1^2-4\times(-1)\times(-2)=1-8=-7$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2(-1)}=\frac{-1\pm\sqrt{7}i}{-2} \qquad \left[\sqrt{-1}=i\right]$$

Question 5:
Solve the equation $x^2+3x+5=0$

Solution 5:
The given quadratic equation is $x^2+3x+5=0$
On comparing the given equation with $ax^2+bx+c=0$,
We obtain $a=1$, $b=3$, and $c=5$
Therefore, the discriminant of the given equation is
$D=b^2-4ac=3^2-4\times1\times5=9-20=-11$
Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-11}}{2\times1}=\frac{-3\pm\sqrt{11}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

Question 6:
Solve the equation $x^2-x+2=0$

Solution 6:

The given quadratic equation is $x^2 - x + 2 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = 1,\ b = -1,$ and $c = 2$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-1)^2 - 4\times1\times2 = 1-8 = -7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-1)\pm\sqrt{-7}}{2\times1} = \frac{1\pm\sqrt{7}i}{2} \qquad \left[\sqrt{-1}=i\right]$$

Question 7:

Solve the equation $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Solution 7:

The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = \sqrt{2},\ b = 1,$ and $c = \sqrt{2}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = 1^2 - 4\times\sqrt{2}\times\sqrt{2} = 1-8 = -7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-1\pm\sqrt{-7}}{2\times\sqrt{2}} = \frac{-1\pm\sqrt{7}i}{2\sqrt{2}} \qquad \left[\sqrt{-1}=i\right]$$

Question 8:

Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Solution 8:

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$,

We obtain $a = \sqrt{3},\ b = -\sqrt{2},$ and $c = 3\sqrt{3}$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = \left(-\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(3\sqrt{3}\right) = 2-36 = -34$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right)\pm\sqrt{-34}}{2\times\sqrt{3}} = \frac{\sqrt{2}\pm\sqrt{34}\,i}{2\sqrt{3}} \qquad \left[\sqrt{-1}=i\right]$$

Question 9:

Solve the equation $x^2+x+\frac{1}{\sqrt{2}}=0$

Solution 9:

The given quadratic equation is $x^2+x+\frac{1}{\sqrt{2}}=0$

This equation can also be written as $\sqrt{2}x^2+\sqrt{2}x+1=0$

On comparing the given equation with $ax^2+bx+c=0$, we obtain $a=\sqrt{2}$, $b=\sqrt{2}$, and $c=1$

$\therefore$ Discriminant $(D)=b^2-4ac=\left(\sqrt{2}\right)^2-4\times\left(\sqrt{2}\right)\times 1=2-4\sqrt{2}$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$=\left(\frac{-\sqrt{2}\pm\sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right)\qquad\left[\sqrt{-1}=i\right]$$

$$=\frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

Question 10:

Solve the equation $x^2+\frac{x}{\sqrt{2}}+1=0$

Solution 10:

The given quadratic equation is $x^2+\frac{x}{\sqrt{2}}+1=0$

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing the given equation with $ax^2+bx+c=0$,

We obtain $a=\sqrt{2}$, $b=1$, and $c=\sqrt{2}$

$\therefore$ Discriminant $(D)=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}\qquad\left[\sqrt{-1}=i\right]$$

Miscellaneous Exercise

Question 1:

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Solution 1:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$
$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$
$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$
$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$
$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$
$$=\left[-1+\frac{i}{i^2}\right]^3$$
$$=\left[-1-i\right]^3$$
$$=(-1)^3\left[1+i\right]^3$$
$$=-\left[1^3+i^3+3\cdot1\cdot i(1+i)\right]$$
$$=-\left[1+i^3+3i+3i^2\right]$$
$$=-\left[1-i+3i-3\right]$$
$$=-\left[-2+2i\right]$$
$$=2-2i$$

Question 2:

For any two complex numbers z_1 and z_2, prove that

$\operatorname{Re}\left(z_1z_2\right)=\operatorname{Re}z_1\operatorname{Re}z_2-\operatorname{Im}z_1\operatorname{Im}z_2$

Solution 2:

Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$

$$\therefore\ z_1z_2=\left(x_1+iy_1\right)\left(x_2+iy_2\right)$$
$$=x_1\left(x_2+iy_2\right)+iy_1\left(x_2+iy_2\right)$$
$$=x_1x_2+ix_1y_2+iy_1x_2+i^2y_1y_2$$

$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$

$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$

$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$

$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$

Hence, proved.

Question 3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form

Solution 3:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)}$$ [On multiplying numerator and denominator by $(14+5i)$]

$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$

$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

Question 4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Solution 4:

$$x - iy = \sqrt{\frac{a-ib}{c-id}}$$

$$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}}$$ [On multiplying numerator and denominator by $(c+id)$]

$$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

$$\therefore (x-iy)^2 = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

$$\Rightarrow x^2-y^2-2ixy = \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2-y^2 = \frac{ac+bd}{c^2+d^2}, -2xy = \frac{ad-bc}{c^2+d^2} \quad ......(1)$$

$$(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{ad-bc}{c^2+d^2}\right) \qquad \text{[Using (1)]}$$

$$= \frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{(c^2+d^2)^2}$$

$$= \frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{(c^2+d^2)^2}$$

$$= \frac{a^2(c^2+d^2)+b^2(c^2+d^2)}{(c^2+d^2)^2}$$

$$= \frac{(c^2+d^2)(a^2+b^2)}{(c^2+d^2)^2}$$

$$= \frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

Question 5:

Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$, (ii) $\frac{1+3i}{1-2i}$

Solution 5:

(i) Here, $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let $r\cos\theta=-1$ and $r\sin\theta=1$

On squaring and adding, we obtain $r^2\left(\cos^2\theta+\sin^2\theta\right)=1$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=2$

$\Rightarrow r^2=2$ $\qquad\left[\cos^2\theta+\sin^2\theta=1\right]$

$\Rightarrow r=\sqrt{2}$ $\qquad$[Conventionally, $r>0$]

$\therefore\ \sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow\cos\theta=\dfrac{-1}{\sqrt{2}}$ and $\sin\theta=\dfrac{1}{\sqrt{2}}$

$\therefore\ \theta=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ $\qquad$[As θ lies in II quadrant]

$\therefore\ z=r\cos\theta+ir\sin\theta$

$=\sqrt{2}\cos\dfrac{3\pi}{4}+i\sqrt{2}\sin\dfrac{3\pi}{4}=\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

This is the required polar form.

(ii) Here, $z=\dfrac{1+3i}{1-2i}$

$=\dfrac{1+3i}{1-2i}\times\dfrac{1+2i}{1+2i}$

$=\dfrac{1+2i+3i-6}{1+4}$

$=\dfrac{-5+5i}{5}=-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta$

$=1$ on squaring and adding, we obtain $r^2\left(\cos^2\theta+\sin^2\theta\right)$

$=1+1$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=2$

$\Rightarrow r^2=2$ $\qquad\left[\cos^2\theta+\sin^2\theta=1\right]$

$\Rightarrow r=\sqrt{2}$ $\qquad\left[\text{Conventionally, } r>0\right]$

$\therefore\sqrt{2}\cos\theta=-1$ and $\sqrt{2}\sin\theta=1$

$\Rightarrow\cos\theta=\dfrac{-1}{\sqrt{2}}$ and $\sin\theta=\dfrac{1}{\sqrt{2}}$

$\therefore\ \theta=\pi-\dfrac{\pi}{4}=\dfrac{3\pi}{4}$ $\qquad$[As θ lies in II quadrant]

$\therefore\ z=r\cos\theta+ir\sin\theta$

$=\sqrt{2}\cos\dfrac{3\pi}{4}+i\sqrt{2}\sin\dfrac{3\pi}{4}=\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

This is the required polar form.

Question 6:

Solve the equation $3x^2-4x+\frac{20}{3}=0$

Solution 6:

The given quadratic equation is $3x^2-4x+\frac{20}{3}=0$

This equation can also be written as $9x^2-12x+20=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=9, b=-12$ and $c=20$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=(-12)^2-4\times9\times20=144-720=-576$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(12)\pm\sqrt{-576}}{2\times9}=\frac{12\pm\sqrt{576}\,i}{18} \qquad \left[\sqrt{-1}=i\right]$$

$$=\frac{12\pm24i}{18}=\frac{6(2\pm4i)}{18}=\frac{2\pm4i}{3}=\frac{2}{3}\pm\frac{4}{3}i$$

Question 7:

Solve the equation $x^2-2x+\frac{3}{2}=0$

Solution 7:

The given quadratic equation is $x^2-2x+\frac{3}{2}=0$

This equation can also be written as $2x^2-4x+3=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=2, b=-4$ and $c=3$

Therefore, the discriminant of the given equation is

$D=b^2-4ac=(-4)^2-4\times2\times3=16-24=-8$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-4)\pm\sqrt{8}}{2\times2}=\frac{4\pm2\sqrt{2}\,i}{4} \qquad \left[\sqrt{-1}=i\right]$$

$$=\frac{2\pm\sqrt{2}i}{2}=1\pm\frac{\sqrt{2}}{2}i$$

Question 8:

Solve the equation $27x^2-10x+1=0$

Solution 8:

The given quadratic equation is $27x^2-10x+1=0$

On comparing this equation with $ax^2+bx+c=0$, we obtain $a=27, b=-10$ and $c=1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4\times 27\times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-10)\pm\sqrt{-8}}{2\times 27} = \frac{10\pm 2\sqrt{2}\,i}{54} \qquad \left[\sqrt{-1}=i\right]$$

$$= \frac{5\pm\sqrt{2}i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i$$

Question 9:

Solve the equation $21x^2 - 28x + 10 = 0$

Solution 9:

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain $a = 21, b = -28$ and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4\times 21\times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{-(-28)\pm\sqrt{-56}}{2\times 21} = \frac{28\pm\sqrt{56}\,i}{42} \qquad \left[\sqrt{-1}=i\right]$$

$$= \frac{28\pm 2\sqrt{14}\,i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i$$

Question 10:

If $z_1 = 2 - i,\ z_2 = 1 + i,$ find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$

Solution 10:

$z_1 = 2 - i,\ z_2 = 1 + i$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i}\times\frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{(1^2 - i^2)}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$=|1+i|=\sqrt{1^2+1^2}=\sqrt{2}$

Thus, the value of $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$ is $\sqrt{2}$.

Question 11:

If $a+ib=\dfrac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2=\dfrac{(x^2+1)^2}{(2x^2+1)^2}$

Solution 11:

$a+ib=\dfrac{(x+i)^2}{2x^2+1}$

$=\dfrac{x^2+i^2+2xi}{2x^2+1}$

$=\dfrac{x^2-1+i2x}{2x^2+1}$

$=\dfrac{x^2-1}{2x^2+1}+i\left(\dfrac{2x}{2x^2+1}\right)$

On comparing real and imaginary parts, we obtain

$a=\dfrac{x^2-1}{2x^2+1}$ **and** $b=\dfrac{2x}{2x^2+1}$

$\therefore a^2+b^2=\left(\dfrac{x^2-1}{2x^2+1}\right)^2+\left(\dfrac{2x}{2x^2+1}\right)^2$

$=\dfrac{x^4+1-2x^2+4x^2}{(2x+1)^2}$

$=\dfrac{x^4+1+2x^2}{(2x^2+1)^2}$

$=\dfrac{(x^2+1)^2}{(2x^2+1)^2}$

$\therefore a^2+b^2=\dfrac{(x^2+1)^2}{(2x^2+1)^2}$

Hence, proved.

Question 12:

Let $z_1=2-i,\ z_2=-2+i$. Find (i) $\operatorname{Re}\left(\dfrac{z_1z_2}{\bar{z}_1}\right)$, (ii) $\operatorname{Im}\left(\dfrac{1}{z_1\bar{z}_1}\right)$

Solution 12:

$z_1 = 2-i,\ z_2 = -2+i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4+2i+2i-i^2 = -4+4i-(-1) = -3+4i$

$\overline{z_1} = 2+i$

$\therefore \frac{z_1 z_2}{\overline{z_1}} = \frac{-3+4i}{2+i}$

On multiplying numerator and denominator by $(2-i)$, we obtain

$$\frac{z_1 z_2}{\overline{z_1}} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$

$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1 \overline{z_1}} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$$\text{Im}\left(\frac{1}{z_1 \overline{z_1}}\right) = 0$$

Question 13:

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

Solution 13:

Let $z = \frac{1+3i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$

$$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$$

Let $z = r\cos\theta + ir\sin\theta$

i.e., $r\cos\theta = \frac{-1}{2}$ and $r\sin\theta = \frac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{2}} \qquad [\text{Conventionally, } r > 0]$$

$$\therefore \frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad \text{[As } \theta \text{ lies in the II quadrant]}$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

Question 14:

Find the real numbers x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

Solution 14:

Let $z = (x-iy)(3+5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x+5y) + i(5x-3y)$$

$$\therefore \bar{z} = (3x+5y) - i(5x-3y)$$

It is given that, $\bar{z} = -6-24i$

$$\therefore (3x+5y) - i(5x-3y) = -6-24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \quad ....(i)$$

$$5x - 3y = 24 \quad ....(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{aligned} 9x + 15y &= -18 \\ 25x - 15y &= 120 \\ \hline 34x &= 102 \end{aligned}$$

$$\therefore x = \frac{102}{34} = 3$$

Putting the value of x in equation (i), we obtain

$$3(3) + 5y = -6$$

$$\Rightarrow 5y = -6 - 9 = -15$$

$$\Rightarrow y = -3$$

Thus, the values of x and y are 3 and -3 respectively.

Question 15:

Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

Solution 15:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$
$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$
$$=\frac{4i}{2}=2i$$
$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Question 16:

If $(x+iy)^3=u+iv$, then show that: $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

Solution 16:

$$(x+iy)^3=u+iv$$
$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$
$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$
$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$
$$\Rightarrow (x^3-3xy^2)+i(3x^2y-y^3)=u+iv$$

On equating real and imaginary parts, we obtain

$$u=x^3-3xy^2,\ v=3x^2y-y^3$$
$$\therefore\ \frac{u}{x}+\frac{v}{y}=\frac{x^3-3xy^2}{x}+\frac{3x^2y-y^3}{y}$$
$$=\frac{x(x^2-3y^2)}{x}+\frac{y(3x^2-y^2)}{y}$$
$$=x^2-3y^2+3x^2-y^2$$
$$=4x^2-4y^2$$
$$=4(x^2-y^2)$$
$$\therefore\ \frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$$

Hence, proved.

Question 17:

If α and β are different complex numbers with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Solution 17:

Let $\alpha=a+ib$ and $\beta=x+iy$

It is given that, $|\beta| = 1$

$\therefore \sqrt{x^2 + y^2} = 1$

$\Rightarrow x^2 + y^2 = 1 \;\ldots\ldots(i)$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right| \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using}(1)]$$

$$\therefore \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$$

Question 18:

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$

Solution 18:

$|1-i|^x = 2^x$

$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$

$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$

$\Rightarrow 2^{x/2} = 2^x$

$\Rightarrow \frac{x}{2} = x$

$\Rightarrow x = 2x$

$\Rightarrow 2x - x = 0$

$\Rightarrow x = 0$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of nonzero integral solutions of the given equation is 0.

Question 19:

If $(a+ib)(c+id)(e+if)(g+ih) = A+iB,$, then show that:

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2.$

Solution 19:

$(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$|(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad \because [|z_1 z_2| = |z_1||z_2|]$

$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

On squaring both sides, we obtain

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$. Hence proved.

Question 20:

If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least positive integral value of m.

Solution 20:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$$\Rightarrow i^m = i^{4k}$$

$\therefore m = 4k,$ where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is $4(=4\times1)$.
