Exercises

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Calculus I

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Example 1: Find the domain and range of the following functions.

1. $f(x) = 1 + x^2$
2. $f(x) = 1 - \sqrt{x}$
3. $F(x) = \sqrt{5x + 10}$
4. $g(x) = \sqrt{x^2 - 3x}$
5. $f(t) = \dfrac{4}{3 - t}$
6. $G(t) = \dfrac{2}{t^2 - 16}$

Limit

Solution: Let D_f, R_f denote the domain and range of a function f, respectively.
1.
Domain: $D_f = \mathbb{R} = (-\infty, \infty)$.

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Range: $x^2 \geq 0 \Longrightarrow 1 + x^2 \geq 1$. So $R_f = [1, \infty)$.

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2.
Domain: Since $\sqrt{\cdot}$ is not defined for negative values, we have $x \geq 0$. So, $D_f = [0, \infty)$.

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Domain: Since $\sqrt{\cdot}$ is not defined for negative values, we have $x \geq 0$. So, $D_f = [0, \infty)$.

Range: $\sqrt{x} \geq 0$

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2.
Domain: Since $\sqrt{\cdot}$ is not defined for negative values, we have $x \geq 0$. So, $D_f = [0, \infty)$.

Range: $\sqrt{x} \geq 0 \implies -\sqrt{x} \leq 0$

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Domain: $D_f = \mathbb{R} = (-\infty, \infty)$. No point that makes the function undefined.

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Domain: Since $\sqrt{\cdot}$ is not defined for negative values, we have $x \geq 0$. So, $D_f = [0, \infty)$.

Range: $\sqrt{x} \geq 0 \Longrightarrow -\sqrt{x} \leq 0 \Longrightarrow 1 - \sqrt{x} \leq 1$.
Thus, $R_f = (-\infty, 1]$.
3.
Domain: $5x + 10 \geq 0 \Longrightarrow x \geq -\frac{10}{5} = -2 \Longrightarrow D_F = [-2, \infty)$.

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Range: $\sqrt{x} \geq 0 \Longrightarrow -\sqrt{x} \leq 0 \Longrightarrow 1 - \sqrt{x} \leq 1$.
Thus, $R_f = (-\infty, 1]$.
3.
Domain: $5x + 10 \geq 0 \Longrightarrow x \geq -\frac{10}{5} = -2 \Longrightarrow D_F = [-2, \infty)$.

Range: $y = \sqrt{5x + 10} \geq 0 \Longrightarrow F$ values can be any positive number. So, $R_F = [0, \infty)$.

4.
Domain: $x^2 - 3x = x(x-3) \geq 0 \Longrightarrow x \leq 0$ or $x \geq 3 \Longrightarrow$
$D_g = (-\infty, 0] \cup [3, \infty)$.

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Range: $\sqrt{x^2 - 3x} \geq 0$ So, $R_f = [0, \infty)$.

5.
Domain: $3 - t \neq 0 \Longrightarrow D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$.

Range: No $t \in D_f$ which makes f to be zero. Thus,
$R_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$.

Limit

6.
Domain: $t^2 - 16 \neq 0 \Longrightarrow x \neq 4$ and
$x \neq -4 \Longrightarrow D_G = \mathbb{R} \setminus \{-4, 4\} = (\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

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Domain: $t^2 - 16 \neq 0 \Longrightarrow x \neq 4$ and
$x \neq -4 \Longrightarrow D_G = \mathbb{R} \setminus \{-4, 4\} = (\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

Range:
i. $t \in (-\infty, -4) \Longrightarrow t < -4$

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Range:
i. $t \in (-\infty, -4) \implies t < -4 \implies t^2 > 16 \implies \frac{1}{t^2 - 16} > 0$.

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Range:
i. $t \in (-\infty, -4) \Longrightarrow t < -4 \Longrightarrow t^2 > 16 \Longrightarrow \frac{1}{t^2-16} > 0$.

ii. $-4 < t < 4 \Longrightarrow 0 \leq t^2 < 16$

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i. $t \in (-\infty, -4) \implies t < -4 \implies t^2 > 16 \implies \frac{1}{t^2 - 16} > 0$.

ii. $-4 < t < 4 \implies 0 \leq t^2 < 16 \implies -16 \leq t^2 - 16 < 0$

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Domain: $t^2 - 16 \neq 0 \Longrightarrow x \neq 4$ and
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Range:
i. $t \in (-\infty, -4) \Longrightarrow t < -4 \Longrightarrow t^2 > 16 \Longrightarrow \frac{1}{t^2 - 16} > 0$.

ii. $-4 < t < 4 \Longrightarrow 0 \leq t^2 < 16 \Longrightarrow -16 \leq t^2 - 16 < 0$
$\Longrightarrow \frac{1}{t^2-16} \leq -\frac{2}{16} = -\frac{1}{8}$.

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iii. $t > 4 \Longrightarrow t^2 > 16 \Longrightarrow t^2 - 16 > 0 \Longrightarrow \frac{2}{t^2-16} > 0$

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Domain: $t^2 - 16 \neq 0 \implies x \neq 4$ and
$x \neq -4 \implies D_G = \mathbb{R} \setminus \{-4, 4\} = (\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

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i. $t \in (-\infty, -4) \implies t < -4 \implies t^2 > 16 \implies \frac{1}{t^2-16} > 0$.

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iii. $t > 4 \implies t^2 > 16 \implies t^2 - 16 > 0 \implies \frac{2}{t^2-16} > 0$

Thus, $R_G = (-\infty, -\frac{1}{8}] \cup (0, \infty)$.

Example2: Specify the intervals over which the function is increasing and the intervals where it is decreasing.

1. $f(x) = -x^3$
2. $f(x) - \frac{1}{x^2}$
3. $f(x) = \frac{1}{|x|}$
4. $f(x) = \sqrt{|x|}$.
5. $f(x) = e^x$.
6. $f(x) = \ln(x)$.

Solution:

1. Let $x, y \in \mathbb{R}$ be any arbitrary two points in the domain of f with $x < y$. Then, $x^3 < y^3 \implies -x^3 > -y^3 \implies f(x) > f(y)$. So, f is decreasing everywhere.

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i. Suppose $x, y \in (-\infty, 0)$ with $x < y$. Then, $x^2 > y^2 \implies \frac{1}{x^2} < \frac{1}{y^2}$

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ii. Let $x, y \in (0, \infty)$ with $x < y$. Then, $x^2 < y^2$

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Recall:

$$|x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \\ 0, & x = 0 \end{cases}$$

i. Suppose $x, y \in (-\infty, 0)$ with $x < y$. Then, $-x > -y$.

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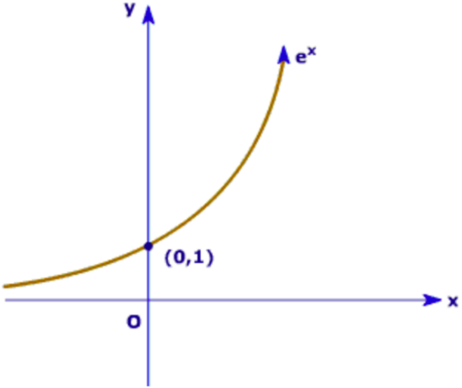
ii. Suppose $x, y \in (0, \infty)$ with $x < y$. Then, $\implies |x| < |y| \implies \sqrt{|x|} < \sqrt{|y|}$ $\implies f(x) < f(y)$. So, f is increasing on $(0, \infty)$.

5. Graph of exponential function $f(x) = e^x$



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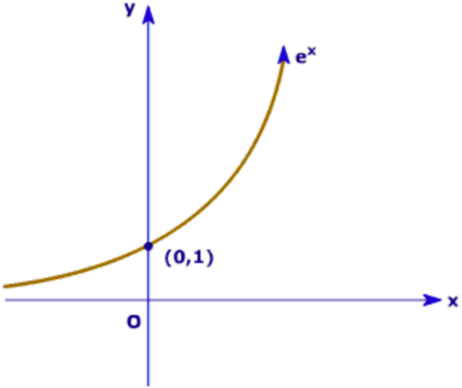
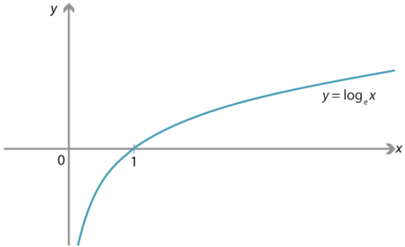


It increases everywhere.

6. Graph of exponential function $f(x) = \ln(x)$



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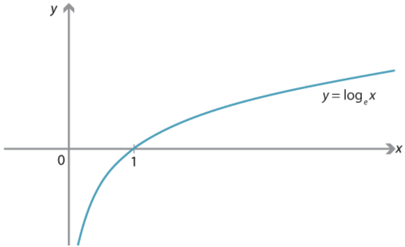

It increases everywhere.

Limit

Example3: Say whether the function is even, odd, or neither.

1. $f(x) = x^2$.
2. $f(x) = x^3$.
3. $f(x) = \cos(x)$
4. $f(x) = \sin(x)$
5. $f(x) = \frac{1}{x^2+x+1}$
6. $f(x) = \frac{x^2+1}{x-1}$

Limit

Solution:

1. $f(-x) = (-x)^2 = x^2 = f(x) \implies f$ is even.

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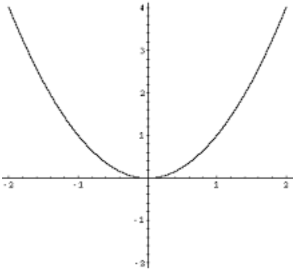

$y = x^2$

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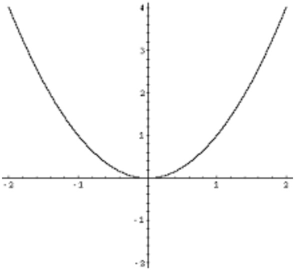
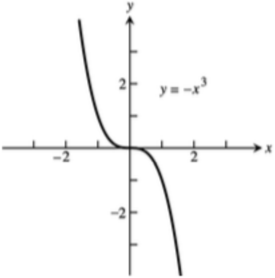


$y = x^2$

Symmetric about the $y-$axis.

Limit

2. $f(-x) = (-x)^3 = -x^3 = -f(x) \Longrightarrow f$ is odd.



Limit

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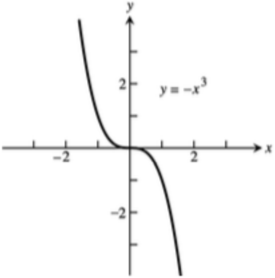

Symmetric about the origin

Limit

3. $f(-x) = \cos(-x) = \cos = (x) \Longrightarrow f$ is even.

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Neither even nor odd.

Calculate the following limit

$$\lim_{x\to 0} \frac{\sqrt{3x+4}-2}{x}.$$

$$F(x) = \begin{cases} \frac{x^3-64}{x^2-16}, & x \neq 4 \\ 4, & x = 4 \end{cases}$$

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Limit

Find the following limits:

1. $\lim\limits_{x \to -5} (2x + 7)$,
2. $\lim\limits_{x \to 4} (-x^2 + 2x + 3)$,
3. $\lim\limits_{x \to \frac{1}{2}} 2x^2(x + 8)$,
4. $\lim\limits_{x \to 3} \dfrac{3x + 4}{3x^2 + 2x + 1}$,
5. $\lim\limits_{x \to 2} (3x^2 + 4)^{\frac{3}{4}}$.

Limit

Solution:

1.

Using Sum and Constant multiple Rules gives

$$\lim_{x\to -5}(2x+7) = 2\lim_{x\to -5} x + \lim_{x\to -5} 7 = 2(-5)+7 = -3.$$

2. Using Sum, Constant multiple and Power Rules gives

$$\lim_{x\to 4}(-x^2+2x+3) = -(\lim_{x\to 4} x)^2 + 2\lim_{x\to 4} x + \lim_{x\to 4} 3 = -4^2 + 2\cdot 4 + 3 = -5.$$

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3. Using Sum, Product and Power Rules:

$$\lim_{x\to \frac{1}{2}} 2x^2(x+8) = 2\cdot\left(\frac{1}{2}\right)^2\cdot(\frac{1}{2}+8) = 2\cdot\frac{1}{4}\cdot\frac{17}{2} = \frac{17}{4}$$

.

Limit

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.

4. Using Sum, Constant Multiple, Power and Quotient Rules:

$$\lim_{x\to 3}\frac{3x+4}{3x^2+2x+1} = \frac{3\cdot 3+4}{3\cdot 3^2+2\cdot 3+1} = \frac{13}{34}$$

.

5. Using appropriate rules:

$$\lim_{x\to 2}(3x^2+4)^{\frac{3}{4}} = (3\cdot 2^2+4)^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 8.$$

Limit

Find the limits:

1. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$,
2. $\lim\limits_{x \to 0} \dfrac{\sqrt{5x + 4} - 2}{x}$,
3. $\lim\limits_{x \to 0} \dfrac{\frac{1}{x-1} + \frac{1}{x+1}}{x}$,
4. $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$,
5. $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$,
6. $\lim\limits_{x \to 1} \dfrac{x - 1}{\sqrt{x + 3} - 2}$,
7. $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2 + 8} - 3}{x + 1}$.

Limit

Solution:

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Canceling the $(x - 4)$'s gives a simpler fraction with the same values as the original for $x \neq 4$:

$$\frac{x^2 - 16}{x - 4} = \frac{(x + 4)(x - 4)}{x - 4} = (x + 4) \quad \textit{if} \quad x \neq 4.$$

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We find the limit of these values as $x \to 4$ by substitution:

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x+4)(x-4)}{x - 4} = \lim_{x \to 4} (x + 4) = 4 + 4 = 8.$$

Limit

2. First we multiply the numerator and the denominator by the Conjugate of the numerator. Then,

$$\lim_{x\to 0}\frac{\sqrt{5x+4}-2}{x}=\lim_{x\to 0}\frac{(\sqrt{5x+4}-2)(\sqrt{5x+4}+2)}{x(\sqrt{5x+4}+2)}=\lim_{x\to 0}\frac{5x}{x(\sqrt{5x+4}+2)}$$
$$=\lim_{x\to 0}\frac{5}{(\sqrt{5x+4}+2)}=\frac{5}{2+2}=\frac{5}{4}.$$

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$$=\lim_{x\to 0}\frac{5}{(\sqrt{5x+4}+2)}=\frac{5}{2+2}=\frac{5}{4}.$$

3. $\lim\limits_{x\to 0}\dfrac{\frac{1}{x-1}+\frac{1}{x+1}}{x}=\lim\limits_{x\to 0}\dfrac{\frac{x+1+x-1}{x^2-1}}{x}=\lim\limits_{x\to 0}\dfrac{2x}{x(x^2-1)}=\lim\limits_{x\to 0}\dfrac{2}{(x^2-1)}=$
$\dfrac{2}{0^2-1}=-2.$

Limit

4. $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2-1} = \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x+2}{x+1} = \dfrac{1+2}{1+1} = \dfrac{3}{2}.$

Limit

4. $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x^2-1} = \lim\limits_{x\to 1} \dfrac{(x-1)(x+2)}{(x+1)(x-1)} = \lim\limits_{x\to 1} \dfrac{x+2}{x+1} = \dfrac{1+2}{1+1} = \dfrac{3}{2}.$

5. $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9} = \lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim\limits_{x\to 9} \dfrac{1}{\sqrt{x}+3} = \dfrac{1}{\sqrt{9}+3} = \dfrac{1}{6}.$

Limit

4. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x+2)}{(x+1)(x-1)}=\lim\limits_{x\to 1}\dfrac{x+2}{x+1}=\dfrac{1+2}{1+1}=\dfrac{3}{2}.$

5. $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}=\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}=\lim\limits_{x\to 9}\dfrac{1}{\sqrt{x}+3}=\dfrac{1}{\sqrt{9}+3}=\dfrac{1}{6}.$

6.

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}=\lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{x-1}=\lim_{x\to 1}(\sqrt{x+3}+2)=\sqrt{4}+2=4.$$

Limit

4. $\lim_{x\to 1}\frac{x^2+x-2}{x^2-1}=\lim_{x\to 1}\frac{(x-1)(x+2)}{(x+1)(x-1)}=\lim_{x\to 1}\frac{x+2}{x+1}=\frac{1+2}{1+1}=\frac{3}{2}.$

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$$\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}=\lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x+3}+2)}{x-1}=\lim_{x\to 1}(\sqrt{x+3}+2)=\sqrt{4}+2=4.$$

7.

$$\lim_{x\to -1}\frac{\sqrt{x^2+8}-3}{x+1}=\lim_{x\to -1}\frac{(\sqrt{x^2+8}-3)(\sqrt{x^2+8}+3)}{(\sqrt{x^2+8}+3)(x+1)}$$

$$=\lim_{x\to -1}\frac{x^2-1}{(\sqrt{x^2+8}+3)(x+1)}=\lim_{x\to -1}\frac{x-1}{\sqrt{x^2+8}+3}=\frac{-1-1}{(-1)^2+3}=-\frac{2}{4}=-\frac{1}{2}.$$