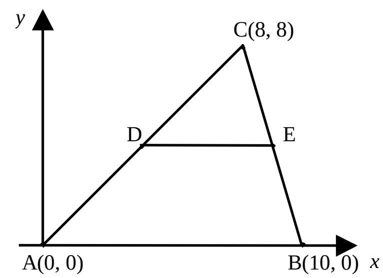
Analytic Geometry Practice Name: ____________________

1 Your are asked to prove the following proposition :

The segment joining the midpoints of two sides of a triangle is parallel to the third side and is half as long as the third side.

To prove : in ΔABC
A(0, 0) B(10, 0) C(8, 8)
D is the midpoint of $\overline{AC}$
E is the midpoint of $\overline{BC}$
Conclusion : $\overline{DE} \mathbin{/\!/} \overline{AB}$

$m\,\overline{DE} = \frac{1}{2}\,m\,\overline{AB}$



Statements	Justifications
1. The coordinates of point D are (4, 4) and those of point E are (9, 4)	1. ______?______
2. The slope of $\overline{DE}$ is : $\frac{4-4}{9-4} = \frac{0}{5} = 0$	2. Formula for slope
3. The slope of $\overline{AB}$ is : $\frac{0-0}{10-0} = \frac{0}{10} = 0$	3. Formula for slope
4. Therefore $\overline{DE} \mathbin{/\!/} \overline{AB}$	4. ______?______
5. $d(D, E) = \|9 - 4\| = 5$	5. Distance formula
6. $d(A, B) = \|10 - 0\| = 10$	6. Distance formula
7. Therefore $m\,\overline{DE} = \frac{1}{2}\,m\,\overline{AB}$	7. By substitution of the measures

What are the appropriate justifications to support statements 1 and 4?

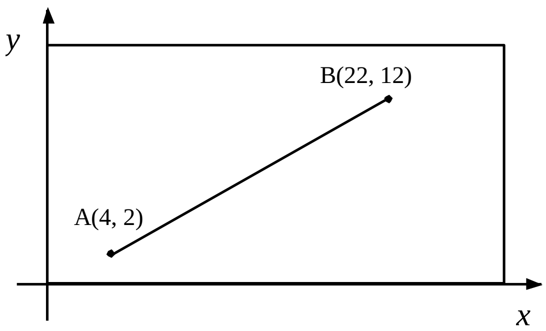
A) 1. Formula for the distance between a point and a line
4. Two lines are parallel if their slopes are equal.

B) 1. Formula for midpoint
4. Two lines are parallel if their slopes are equal.

C) 1. Formula for the distance between a point and a line
4. Two lines, not parallel to the vertical axis, are perpendicular if and only if their slopes are opposite of the reciprocals of each other.

D) 1. Formula for midpoint
4. Two lines, not parallel to the vertical axis, are perpendicular if and only if their slopes are opposite of the reciprocals of each other.

2 Vincent used a Cartesian plane to represent certain objects that make up a theatre set.



A third object must be placed on segment AB, at a point C located $\frac{3}{5}$ of the way along segment AB, starting from point A.

Identify the coordinates of point C.

A) $\left(\frac{74}{5}, 8\right)$

B) $\left(\frac{56}{5}, 6\right)$

C) $\left(\frac{54}{5}, 6\right)$

D) $\left(\frac{31}{4}, \frac{43}{4}\right)$

3 In a Cartesian plane, a line passes through the points A(-10, 6) and B(2, -3). C is a point on segment AB such that the measure of segment AC is equal to $\frac{2}{3}$ of the measure of segment AB $\left(\text{m }\overline{AC} = \frac{2}{3}\text{m }\overline{AB}\right)$.

What are the coordinates of point C?

A) (-6, 3)

B) (-4, 2)

C) (-2, 0)

D) $\left(-\frac{26}{5}, \frac{12}{5}\right)$

4 The coordinates of the vertices of a triangle are (1, 2), (5, 5) and (-2, 6).

In which interval does the area of this triangle fall?

A) [10, 12[

B) [12, 14[

C) [16, 18[

D) [24, 26[

5 The equations of two parallel lines are as follows:

$$2x - 5y - 10 = 0$$
$$2x - 5y + 4 = 0$$

Rounded to the nearest tenth, what is the distance between these two lines?

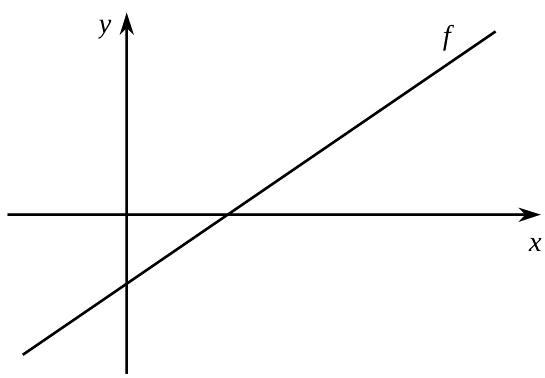
A) 2.5 units

B) 2.6 units

C) 2.7 units

D) 2.8 units

6 A function f is represented in the Cartesian plane below.



Which of the following statements is true?

A) The zero and the y-intercept of function f are positive.

B) The zero and the y-intercept of function f are negative.

C) The zero of function f is negative and the y-intercept of function f is positive.

D) The zero of function f is positive and the y-intercept of function f is negative.

7 Line *l* passes through point P (5, 8) in the Cartesian plane. Line *l* does not have a y-intercept.

What is the equation of line *l*?

A) $x = 5$

B) $x = 8$

C) $y = 5$

D) $y = 8$

8 The equation of line ℓ is $\frac{x}{r} + \frac{y}{t} = 1$, where $r > 0$, $t > 0$ and $r \neq t$.

Which of the following equations represents a line coincident with line ℓ?

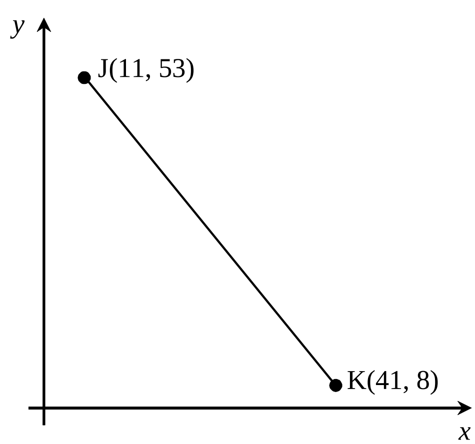
A) $y = -\frac{t}{r}x + r$

B) $y = -\frac{t}{r}x + t$

C) $y = -\frac{r}{t}x + r$

D) $y = -\frac{r}{t}x + t$

9 Point P(29, 26) is a point on line segment JK in the Cartesian plane on the right.



Which of the following statements describes the position of point P?

A) Point P is located $\frac{2}{3}$ of the way along segment JK, starting from point J.

B) Point P is located $\frac{2}{5}$ of the way along segment JK, starting from point J.

C) Point P divides segment JK in a ratio of 3:2, starting from point J.

D) Point P divides segment JK in a ratio of 5:3, starting from point J.

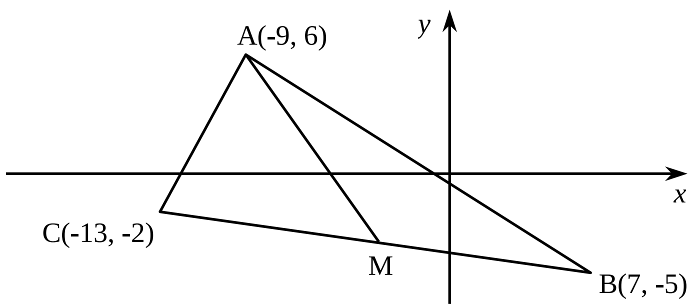
10 Points A, P and B are located on the same line. Point P is located between points A and B. The coordinates of points A and B are (2, -4) and (12, 1) respectively.

In addition, $\dfrac{m\ \overline{AP}}{m\ \overline{AB}} = \dfrac{3}{5}$.

What are the coordinates of point P?

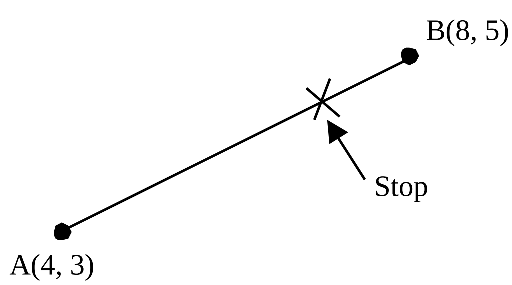
11 Find the coordinates of the mid-point M of segment AB whose end-points are : A(-1, -1) and B(4, 4).

12 In the Cartesian plane below, segment AM is a median of triangle ABC.



What are the coordinates of point M?

13 The following diagram shows a task that has to be done on an automobile driving test.

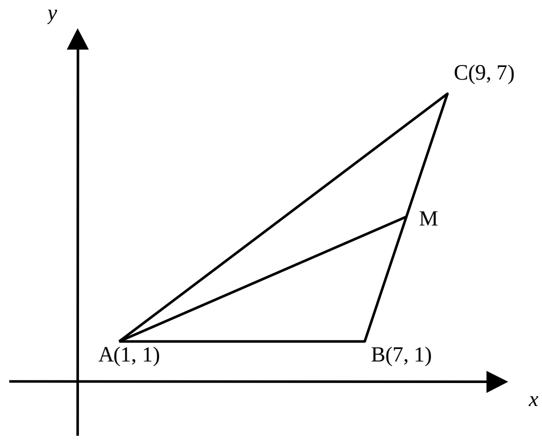


The instructor locates a stop sign $\dfrac{3}{4}$ of the way from A along segment AB.

What are the coordinates of the point that represents the location of the stop sign?

Show all the work needed to solve the problem.

14 In a Cartesian plane, the coordinates of the vertices of triangle ABC are A(1, 1), B(7, 1) and C(9, 7). The median AM is also drawn.

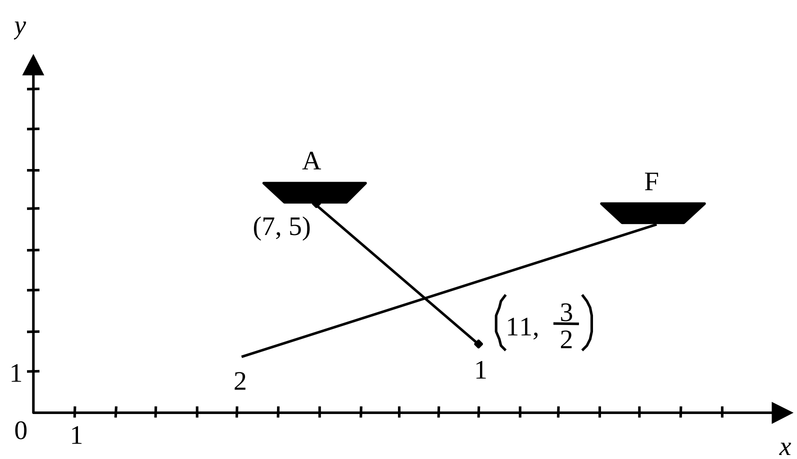


What is the perimeter of triangle AMB?

Show all the work needed to solve the problem.

15 Martin (M) was returning to wharf 1, having spent the day fishing. When he was $\frac{2}{3}$ of the way back, he met Jason (J) and they talked about their respective catches.

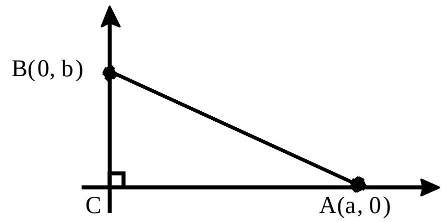
A graphic representation of the situation is shown below.



What are the coordinates of the meeting point of these two fishermen?

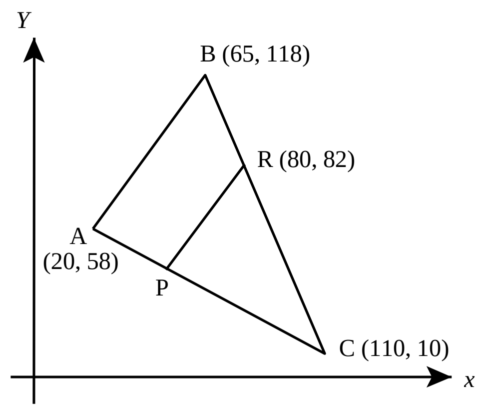
Show your work.

16 Given the right triangle shown on the right.



Prove that the midpoint of the hypotenuse of this right triangle is equidistant from the three vertices.

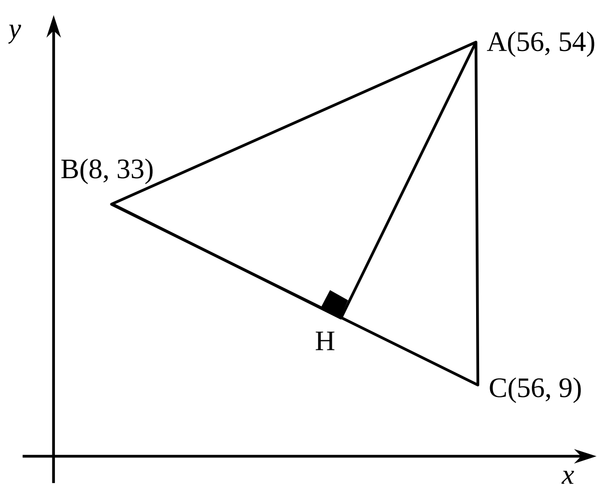
17 In the Cartesian plane on the right, a dilatation with centre C is applied to triangle ABC in order to produce triangle PRC.



What are the coordinates of point P?

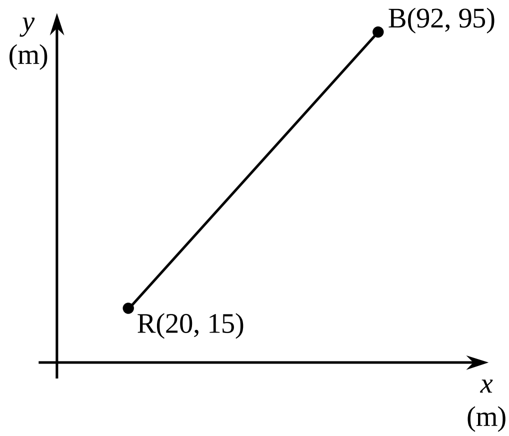
Show all your work.

18 Points A(56, 54), B(8, 33) and C(56, 9) are the vertices of a triangle. Segment AH is an altitude of this triangle.



What is the measure of altitude AH to the nearest tenth?
Show all your work.

19 In a cadet camp, two teams are assigned the task of finding an object hidden along a linear path. When the activity begins, the red team sets off from point R of the path, and the blue team sets off from point B. Path RB is represented in the following Cartesian plane. The scale of the graph is in metres.



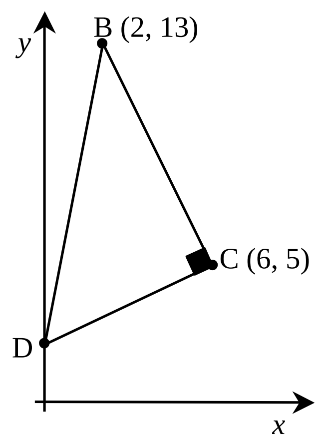
By the end of the activity, the red team's position divides path RB in the ratio 3:5. At the same time, the blue team is located one quarter of the distance from point B to point R.

Rounded to the nearest metre, what is the distance between the two teams by the end of this activity?

Show all your work.

20 In triangle BCD drawn in the Cartesian plane on the right, m∠BCD = 90°.

Point D is located on the *y*-axis.



What is the length of hypotenuse BD, to the nearest hundredth?

Show all your work.

Introduction to Vectors

1. Let $A(7,1,-1)$, $B(4,-2,-1)$, $C(3,0,-5)$, and $D(-6,-9,-5)$ be points in 3-space.
   (a) Find the vector $\overrightarrow{AB}$.
   (b) Find a vector equation for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (c) Is the point D on the line L?

2. Let $A(5,-2,1)$, $B(0,-3,4)$, $C(1,-1,2)$, and $D(7,2,1)$ be points in 3-space.
   (a) Find the magnitude of the vector $\overrightarrow{AB}$.
   (b) Find the unit vector in the same direction as $\overrightarrow{AB}$.
   (c) Find *parametric equations* for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (d) Is the point D on the line L? Justify your answer.

3. Let $A(3,0,-2)$, $B(5,1,-3)$, $C(-1,-2,0)$, and $D(-2,3,-9)$ be points.
   (a) Find *parametric equations* for the line L that passes through A and B.
   (b) Determine if C is on the line containing A and B. Justify your answer.
   (c) Show that triangle ABD is a right triangle.

4. Let $A(-3,6,1)$, $B(4,1,-1)$, and $C(7,-2,-3)$.
   (a) Find $\|\overrightarrow{BC}\|$.
   (b) Find the unit vector in the opposite direction of $\overrightarrow{BC}$.
   (c) Find *parametric equations* for the line L that passes through A and is parallel to $\overrightarrow{BC}$.
   (d) Find a point on the line L other that A.
   (e) Find a *vector equation* for the plane containing the three points A, B, and C.

5. Let L be the line with vector equation
   $(x,y,z) = (1,2,3) + t(4,5,6)$.
   For each equation below, determine if it is also a vector equation for the line L.
   (a) $(x,y,z) = (2,4,6) + t(8,10,12)$
   (b) $(x,y,z) = (-3,-3,-3) + t(8,10,12)$
   (c) $(x,y,z) = (-3,-3,-3) + t(4,5,6)$
   (d) $(x,y,z) = (1,2,3) + t(3,3,3)$

6. Find an equation of the line passing through $(1,-4)$ and $(3,7)$:
   (a) in standard form $Ax + By = C$;
   (b) in vector form;
   (c) in parametric form.

7. Find an equation of the line through $(1,-5)$ with slope$= -\frac{2}{3}$:
   (a) in standard form $Ax + By = C$;
   (b) in vector form;
   (c) in parametric form.

8. Find a vector equation of the line L which:
   (a) is parallel to $(2,-1,0)$ and passes through $P(1,-1,3)$.
   (b) passes through $P(3,-1,4)$ and $Q(1,0,-1)$.
   (c) is parallel to $(1,2,-7)$ and passes through $O(0,0,0)$.
   (d) passes through $P(1,0,-3)$ and parallel to the line $\begin{cases} x = -1 + 2t \\ y = 2 - t \\ z = 3 + 3t \end{cases}$
   (e) passes through $P(2,-1,1)$ and parallel to the line $(x,y,z) = (2,1,0) + t(-1,0,1)$

9. For each set of planes determine if the intersection is a plane, a line, a point, or the empty set $\emptyset$.
   (a) $\begin{cases} 20x + 30y - 30z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$
   (b) $\begin{cases} 20x + 30y - 30z = 4 \\ -16x - 24y + 24z = -5 \end{cases}$
   (c) $\begin{cases} 20x + 30y - 20z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$
   (d) $\begin{cases} x + y = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$
   (e) $\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$
   (f) $\begin{cases} x + y + z = 0 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$

10. Find a standard equation $(ax + by = c)$ for the line in $\mathbb{R}^2$ that fits each description:
    (a) Through $(1,3)$ and perpendicular to $\langle 7,-2\rangle$.
    (b) Through $(1,3)$ and parallel to $\langle 7,-2\rangle$.
    (c) Through $(2,0)$ and $(0,9)$.

11. Find a standard equation $(ax + by + cz = d)$ of the plane in $\mathbb{R}^3$ that fits each description:
    (a) Through $(4,0,-1)$ and parallel to both $\langle 5,1,-1\rangle$ and $\langle -2,3,0\rangle$.
    (b) Through $(6,6,-2)$, $(1,6,1)$, and $(2,9,1)$
    (c) Through $(1,1,4)$, $(2,-3,1)$, and $(-1,5,2)$
    (d) Through $(2,1,1)$, $(3,2,3)$, and $(-2,-1,3)$

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–4, find the vertex, focus, directrix, and focal width of the parabola, and sketch the graph.

1. $y^2 = 12x$ **2.** $x^2 = -8y$

3. $(x + 2)^2 = -4(y - 1)$ **4.** $(y + 2)^2 = 16x$

In Exercises 5–12, identify the type of conic. Find the center, vertices, and foci of the conic, and sketch its graph.

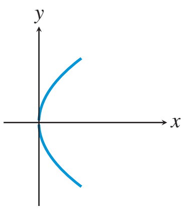
5. $\dfrac{y^2}{8} + \dfrac{x^2}{5} = 1$ **6.** $\dfrac{y^2}{16} - \dfrac{x^2}{49} = 1$

7. $\dfrac{x^2}{25} - \dfrac{y^2}{36} = 1$ **8.** $\dfrac{x^2}{49} - \dfrac{y^2}{9} = 1$

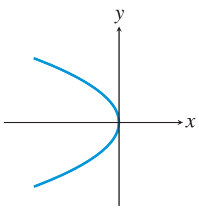
9. $\dfrac{(x+3)^2}{18} - \dfrac{(y-5)^2}{28} = 1$ **10.** $\dfrac{(y-3)^2}{9} - \dfrac{(x-7)^2}{12} = 1$

11. $\dfrac{(x-2)^2}{16} + \dfrac{(y+1)^2}{7} = 1$ **12.** $\dfrac{y^2}{36} + \dfrac{(x+6)^2}{20} = 1$

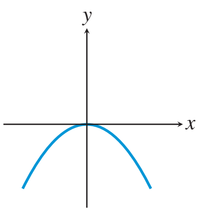
In Exercises 13–20, match the equation with its graph.



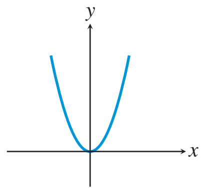
(a)



(b)



(c)



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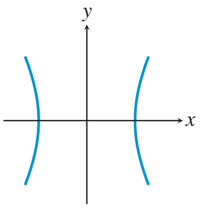
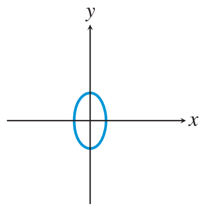


(e)



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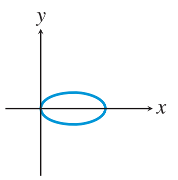
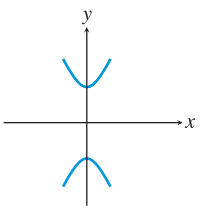


(g)



(h)

13. $y^2 = -3x$ (b) **14.** $\dfrac{(x-2)^2}{4} + y^2 = 1$ (g)

15. $\dfrac{y^2}{5} - x^2 = 1$ (h) **16.** $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$ (e)

17. $\dfrac{y^2}{3} + x^2 = 1$ (f) **18.** $x^2 = y$ (d)

19. $x^2 = -4y$ (c) **20.** $y^2 = 6x$ (a)

In Exercises 21–28, identify the conic. Then complete the square to write the conic in standard form, and sketch the graph.

21. $x^2 - 6x - y - 3 = 0$

22. $x^2 + 4x + 3y^2 - 5 = 0$

23. $x^2 - y^2 - 2x + 4y - 6 = 0$

24. $x^2 + 2x + 4y - 7 = 0$

25. $y^2 - 6x - 4y - 13 = 0$

26. $3x^2 - 6x - 4y - 9 = 0$

27. $2x^2 - 3y^2 - 12x - 24y + 60 = 0$

28. $12x^2 - 4y^2 - 72x - 16y + 44 = 0$

29. Prove that the parabola with focus $(0, p)$ and directrix $y = -p$ has the equation $x^2 = 4py$.

30. Prove that the equation $y^2 = 4px$ represents a parabola with focus $(p, 0)$ and directrix $x = -p$.

In Exercises 31–36, identify the conic. Solve the equation for y and graph it.

31. $3x^2 - 8xy + 6y^2 - 5x - 5y + 20 = 0$

32. $10x^2 - 8xy + 6y^2 + 8x - 5y - 30 = 0$

33. $3x^2 - 2xy - 5x + 6y - 10 = 0$

34. $5xy - 6y^2 + 10x - 17y + 20 = 0$

35. $-3x^2 + 7xy - 2y^2 - x + 20y - 15 = 0$

36. $-3x^2 + 7xy - 2y^2 - 2x + 3y - 10 = 0$

In Exercises 37–48, find the equation for the conic in standard form.

37. Parabola: vertex $(0, 0)$, focus $(2, 0)$ $y^2 = 8x$

38. Parabola: vertex $(0, 0)$, opens downward, focal width $= 12$

39. Parabola: vertex $(-3, 3)$, directrix $y = 0$

40. Parabola: vertex $(1, -2)$, opens to the left, focal length $= 2$

41. Ellipse: center $(0, 0)$, foci $(\pm 12, 0)$, vertices $(\pm 13, 0)$

42. Ellipse: center $(0, 0)$, foci $(0, \pm 2)$, vertices $(0, \pm 6)$

43. Ellipse: center $(0, 2)$, semimajor axis $= 3$, one focus is $(2, 2)$. $x^2/9 + (y - 2)^2/5 = 1$

44. Ellipse: center $(-3, -4)$, semimajor axis $= 4$, one focus is $(0, -4)$. $(x + 3)^2/16 + (y + 4)^2/7 = 1$

45. Hyperbola: center $(0, 0)$, foci $(0, \pm 6)$, vertices $(0, \pm 5)$

46. Hyperbola: center $(0, 0)$, vertices $(\pm 2, 0)$, asymptotes $y = \pm 2x$ $x^2/4 - y^2/16 = 1$

47. Hyperbola: center $(2, 1)$, vertices $(2 \pm 3, 1)$, one asymptote is $y = (4/3)(x - 2) + 1$

48. Hyperbola: center $(-5, 0)$, one focus is $(-5, 3)$, one vertex is $(-5, 2)$. $y^2/4 - (x + 5)^2/5 = 1$

In Exercises 49–54, find the equation for the conic in standard form.

49. $x = 5 \cos t$, $y = 2 \sin t$, $0 \le t \le 2\pi$ $x^2/25 + y^2/4 = 1$

50. $x = 4 \sin t$, $y = 6 \cos t$, $0 \le t \le 4\pi$ $y^2/36 + x^2/16 = 1$

51. $x = -2 + \cos t$, $y = 4 + \sin t$, $2\pi \le t \le 4\pi$

52. $x = 5 + 3 \cos t$, $y = -3 + 3 \sin t$, $-2\pi \le t \le 0$

53. $x = 3 \sec t$, $y = 5 \tan t$, $0 \le t \le 2\pi$ $x^2/9 - y^2/25 = 1$

54. $x = 4 \sec t$, $y = 3 \tan t$, $0 \le t \le 2\pi$ $x^2/16 - y^2/9 = 1$

In Exercises 55–62, identify and graph the conic, and rewrite the equation in Cartesian coordinates.

55. $r = \dfrac{4}{1 + \cos \theta}$ **56.** $r = \dfrac{5}{1 - \sin \theta}$

57. $r = \dfrac{4}{3 - \cos \theta}$ **58.** $r = \dfrac{3}{4 + \sin \theta}$

59. $r = \dfrac{35}{2 - 7 \sin \theta}$ **60.** $r = \dfrac{15}{2 + 5 \cos \theta}$

61. $r = \dfrac{2}{1 + \cos \theta}$ **62.** $r = \dfrac{4}{4 - 4 \cos \theta}$

In Exercises 63–74, use the points $P(-1, 0, 3)$ and $Q(3, -2, -4)$ and the vectors $\mathbf{v} = \langle -3, 1, -2 \rangle$ and $\mathbf{w} = \langle 3, -4, 0 \rangle$.

63. Compute the distance from P to Q. $\sqrt{69}$

64. Find the midpoint of segment PQ. $(1, -1, -1/2)$

65. Compute $\mathbf{v} + \mathbf{w}$. $\langle 0, -3, -2 \rangle$

66. Compute $\mathbf{v} - \mathbf{w}$. $\langle -6, 5, -2 \rangle$

67. Compute $\mathbf{v} \cdot \mathbf{w}$. -13

68. Compute the magnitude of $\mathbf{v}$. $\sqrt{14}$

69. Write the unit vector in the direction of $\mathbf{w}$. $\langle 3/5, -4/5, 0 \rangle$

70. Compute $(\mathbf{v} \cdot \mathbf{w})(\mathbf{v} + \mathbf{w})$. $\langle 0, 39, 26 \rangle$

71. Write an equation for the sphere centered at P with radius 4.

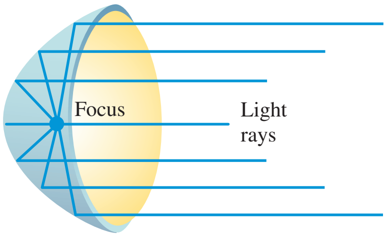
72. Write parametric equations for the line through P and Q.

73. Write a vector equation for the line through P in the direction of $\mathbf{v}$. $\mathbf{r} = \langle -1, 0, 3 \rangle + t\langle -3, 1, -2 \rangle$

74. Write parametric equations for the line in the direction of $\mathbf{w}$ through the midpoint of PQ.

75. Parabolic Microphones B-Ball Network uses a parabolic microphone to capture all the sounds from the basketball players and coaches during each regular season game. If one of its microphones has a parabolic surface generated by the parabola $18y = x^2$, locate the focus (the electronic receiver) of the parabola. $(0, 4.5)$

76. Parabolic Headlights Specific Electric makes parabolic headlights for a variety of automobiles. If one of its headlights has a parabolic surface generated by the parabola $y^2 = 15x$ (see figure), where should its lightbulb be placed? $(3.75, 0)$



77. Writing to Learn Elliptical Billiard Table Elliptical billiard tables have been constructed with spots marking the foci. Suppose such a table has a major axis of 6 ft and minor axis of 4 ft.

(a) Explain the strategy that a "pool shark" who knows conic geometry would use to hit a blocked spot on this table.

(b) If the table surface is coordinatized so that $(0, 0)$ represents the center of the table and the x-axis is along the focal axis of the ellipse, at which point(s) should the ball be aimed?

78. Weather Satellite The Nimbus weather satellite travels in a north-south circular orbit 500 meters above Earth. Find the following. (Assume Earth's radius is 6380 km.)

(a) The velocity of the satellite using the formula for velocity v given for Exercises 43 and 44 in Section 8.5 with $k = 1$ 7.908 km/sec

(b) The time required for Nimbus to circle Earth once 1 hr 25 min

79. Elliptical Orbits The velocity of a body in an elliptical Earth orbit at a distance r (in meters) from the focus (the center of Earth) is

$$v = \sqrt{3.99 \times 10^{14}\left(\frac{2}{r} - \frac{1}{a}\right)} \text{ m/sec},$$

where a is the semimajor axis of the ellipse. An Earth satellite has a maximum altitude (at *apogee*) of 18,000 km and has a minimum altitude (at *perigee*) of 170 km. Assuming Earth's radius is 6380 km, find the velocity of the satellite at its apogee and perigee.

80. Icarus The asteroid Icarus is about 1 mi wide. It revolves around the Sun once every 409 Earth days and has an orbital eccentricity of 0.83. Use Kepler's first and third laws to determine Icarus's semimajor axis, perihelion distance, and aphelion distance.