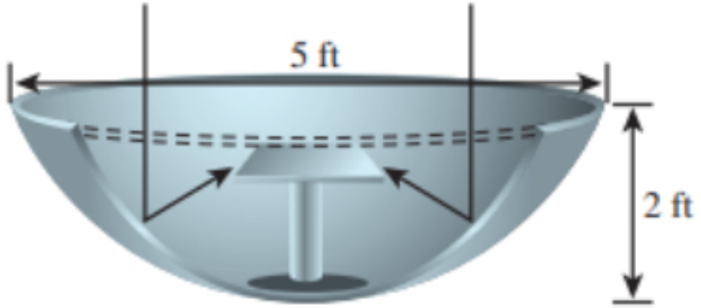
Practice (10 min)

1) Given the following information, write the equation of the parabola.

Vertex is (0,0) and Focus is at (0,2)

2) $y = x^2$. Find the directrix.

3) **Designing a Satellite Dish** The reflector of a television satellite dish is a paraboloid of revolution with diameter 5 ft and a depth of 2 ft. How far from the vertex should the receiving antenna be placed?



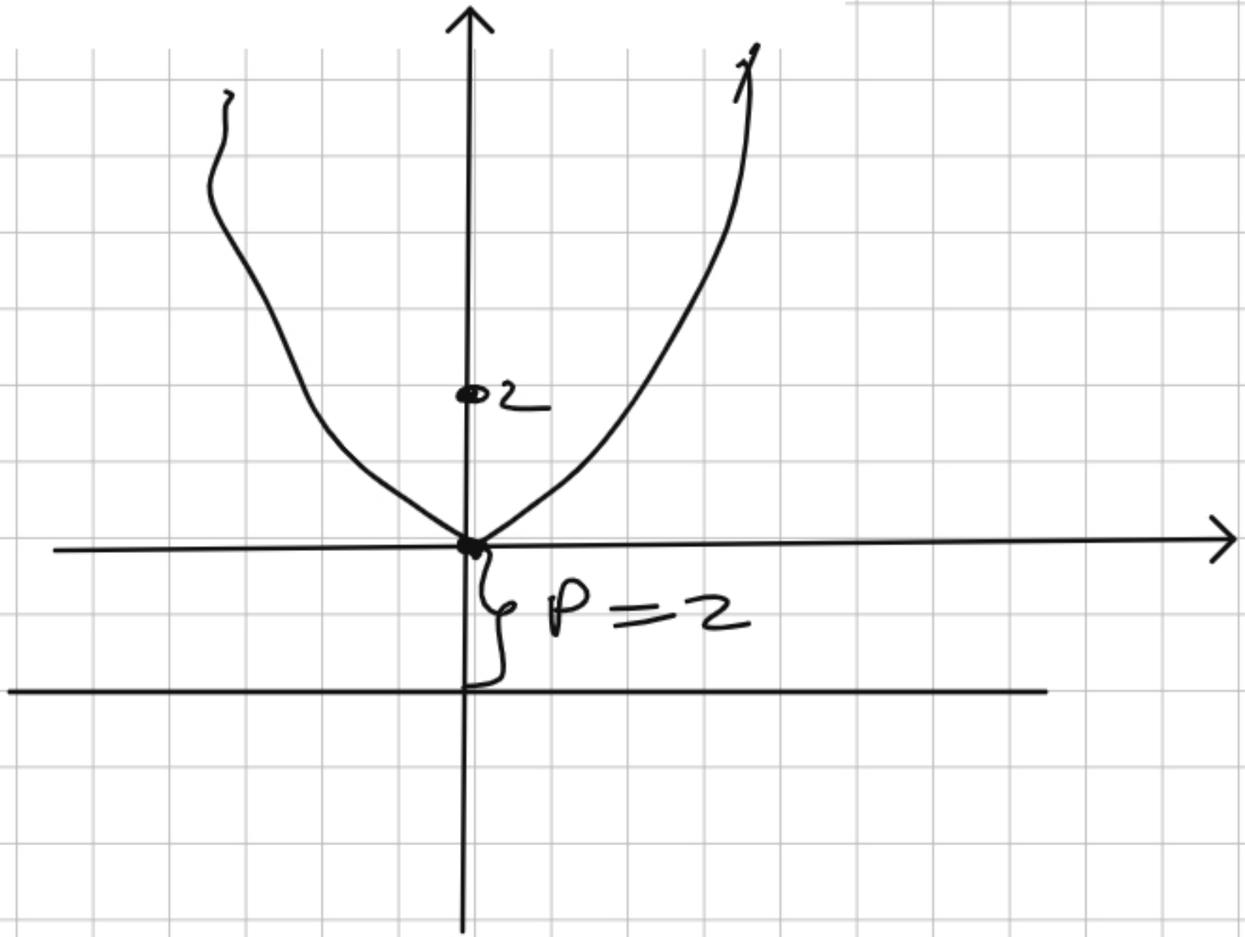
Given the following information, write the equation of the parabola.

Vertex is (0,0) and Focus is at (0,2)

~~$y^2 = 8x$~~

$x^2 = 4py$

$\boxed{x^2 = 8y}$

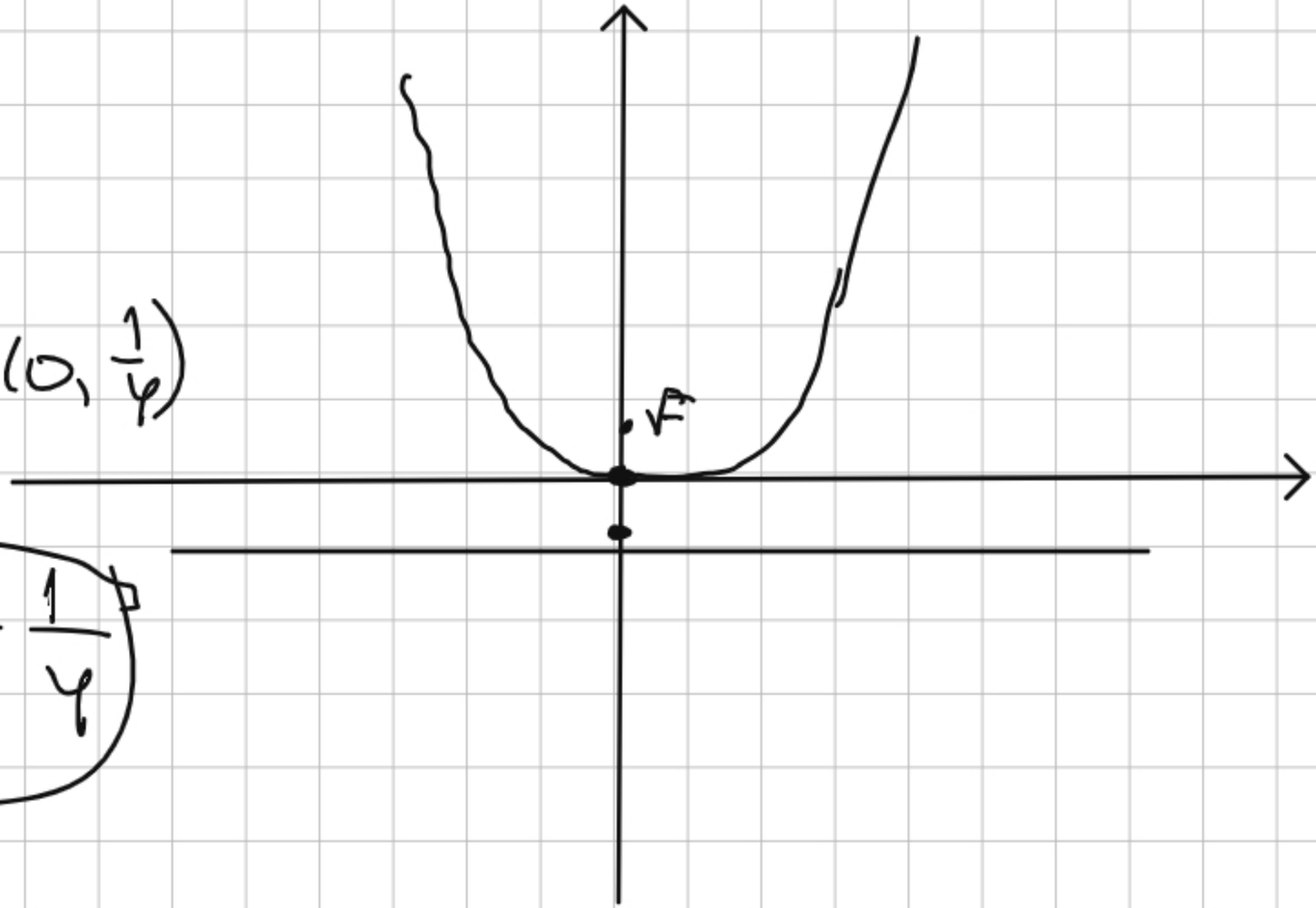


2) $y = x^2$ find the directrix

$x^2 = 4py$.

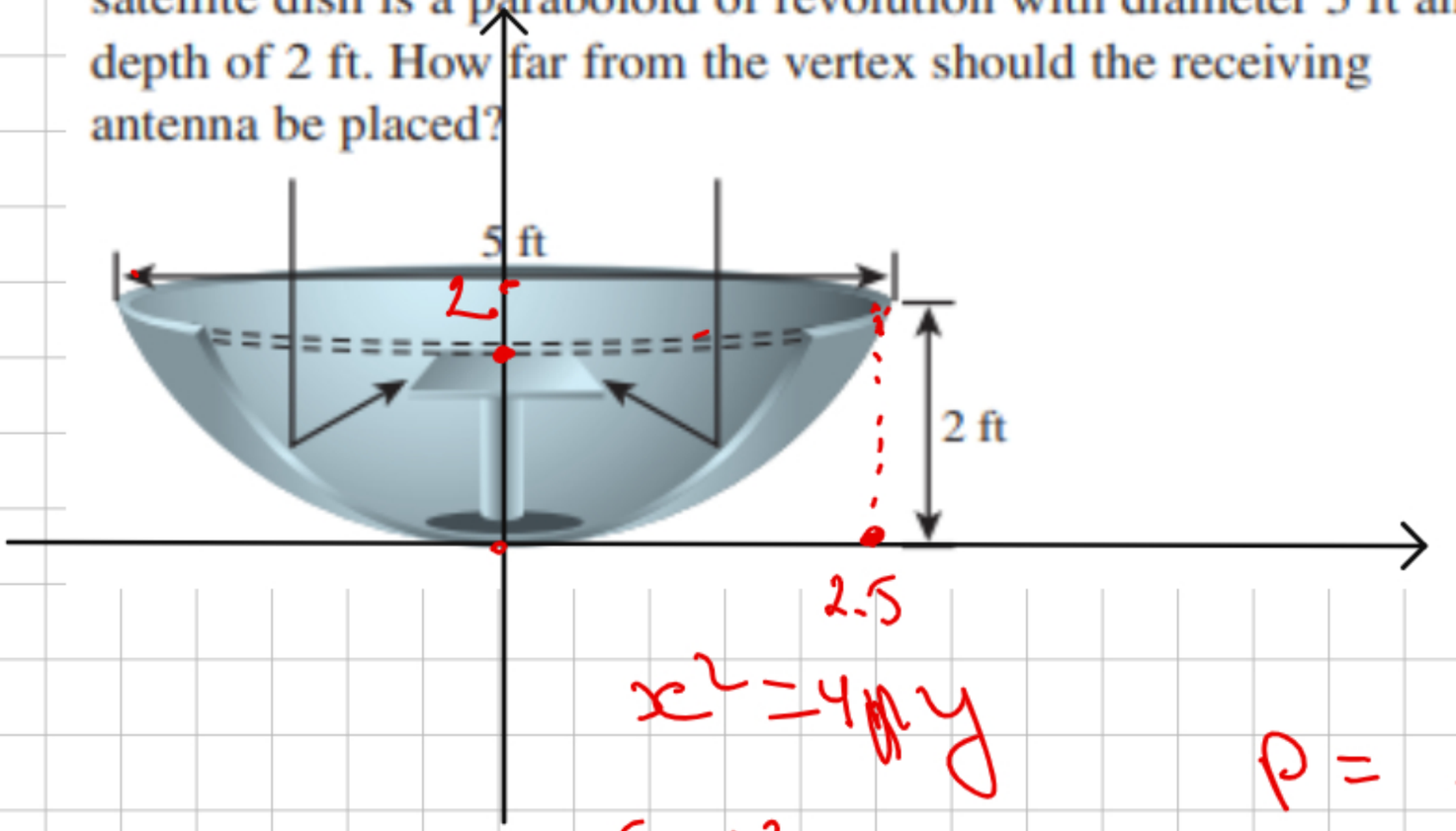
$4p = \frac{1}{4}$

$F(0, \frac{1}{4})$



directrix $y = -\frac{1}{4}$

. **Designing a Satellite Dish** The reflector of a television satellite dish is a paraboloid of revolution with diameter 5 ft and a depth of 2 ft. How far from the vertex should the receiving antenna be placed?



$x^2 = 4py$

$(2.5)^2 = 4p \cdot 2$

$p = \frac{4 \cdot 2}{\frac{25}{4}} = \frac{32}{25}$

$F\left(0, \frac{32}{25}\right)$

6 min

* find the eq. of the parabola whose focus is at (1,1) and eq. directrix $y = -x-2$.

$(y-3)^2 = 16(x-1)$

$x+y+2=0$

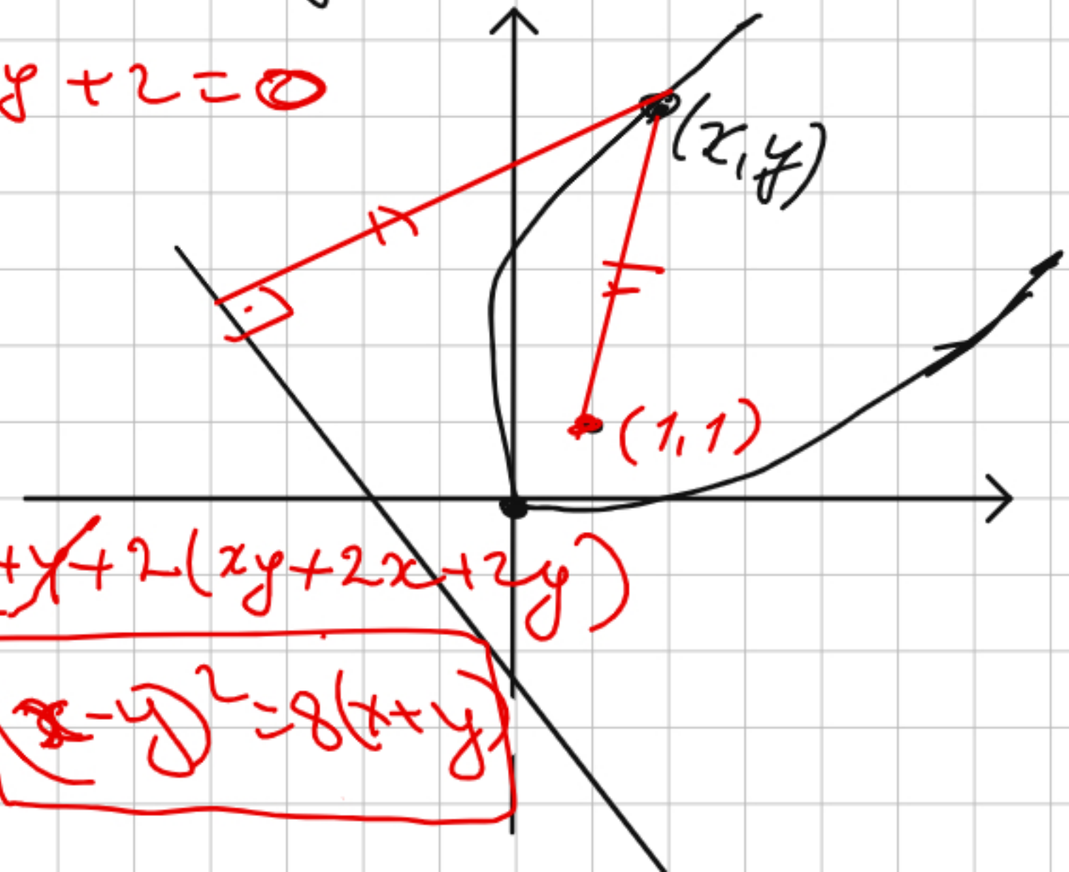
$\sqrt{(x-1)^2+(y-1)^2} = \dfrac{|x+y+2|}{\sqrt{1^2+1^2}}$

$(x-1)^2+(y-1)^2 = \dfrac{(x+y+2)^2}{2}$

$2(x^2-2x+1+y^2-2y+1) = x^2+y^2+1+2(xy+2x+2y)$

$x^2+y^2 = 8x+8y+2xy$

$(x-y)^2 = 8(x+y)$



⊙ $x^2+y^2=0$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

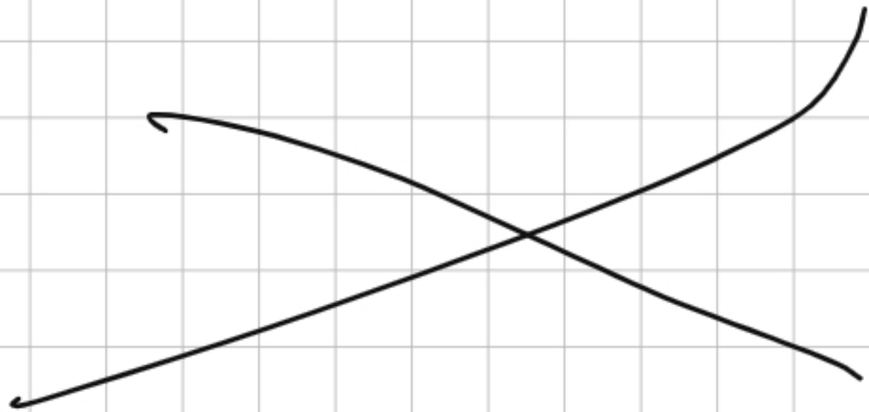
2

$Ax^2+Bxy+Cy^2=0$

$A\frac{x^2}{y^2}+B\frac{x}{y}+C=0$ $\quad \frac{x}{y}=t$

$At^2+Bt+C=0$

$D>0$ two roots



$\frac{x}{y}=t_1$

$y=t_1x$

$y=t_2x$

$B^2-4AC>0$

t_1, t_2

$Ax^2 + Bxy + Cy^2 = 0$ $\quad y \neq 0 \ (\div y^2)$

$A\left(\frac{x}{y}\right)^2 + B\left(\frac{x}{y}\right) + C = 0$ $\quad (0,0)$

$At^2 + Bt + C = 0$

$D < 0$ $\qquad B^2 - 4AC < 0$

no real roots

$$At^2 + Bt + C = 0$$

$$D = 0 \quad t_1 = t_2 = t$$

$$\frac{x}{y} = t \quad \Rightarrow \quad y = xt$$

$y = \frac{1}{x}$ ~ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$y = \frac{1}{x}$

$yx = 1$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

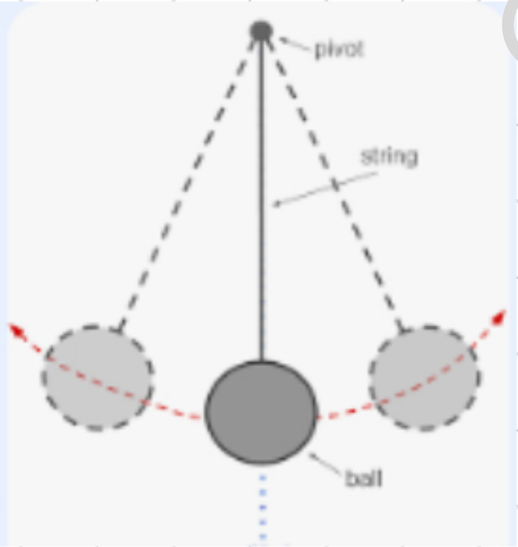
$D = B^2 - 4AC$

$0 \cdot x^2 + 1 \cdot xy + 0 \cdot y^2 + 0 \cdot x + 0 \cdot y - 1 = 0$

$D = 1^2 - 4 \cdot 0 \cdot 0 = 1 > 0$

Pendulum Velocity vs. Position A pendulum that swings toward and away from a motion detector has a distance (in feet) from the detector of $x(t) = 5 + 3\sin(\pi t + \pi/2)$ and a velocity (in ft/sec) of $y(t) = 3\pi\cos(\pi t + \pi/2)$, where t represents time (in seconds).

Prove that the plot of velocity versus position (distance) is an ellipse.



6 min

Pendulum Velocity vs. Position A pendulum that swings toward and away from a motion detector has a distance (in feet) from the detector of $x(t) = 5 + 3\sin(\pi t + \pi/2)$ and a velocity (in ft/sec) of $y(t) = 3\pi\cos(\pi t + \pi/2)$, where t represents time (in seconds).

Prove that the plot of velocity versus position (distance) is an ellipse.

$$\begin{cases} \sin(\pi t + \pi/2) = \dfrac{x-5}{3} \\ \cos(\pi t + \pi/2) = \dfrac{y}{3\pi} \end{cases}$$

$$\frac{(x-5)^2}{3^2} + \frac{y^2}{(3\pi)^2} = 1$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

[5 min]

Find an equation in standard form for the parabola that satisfies the given conditions.

Focus (0, 5), directrix $y = -5$

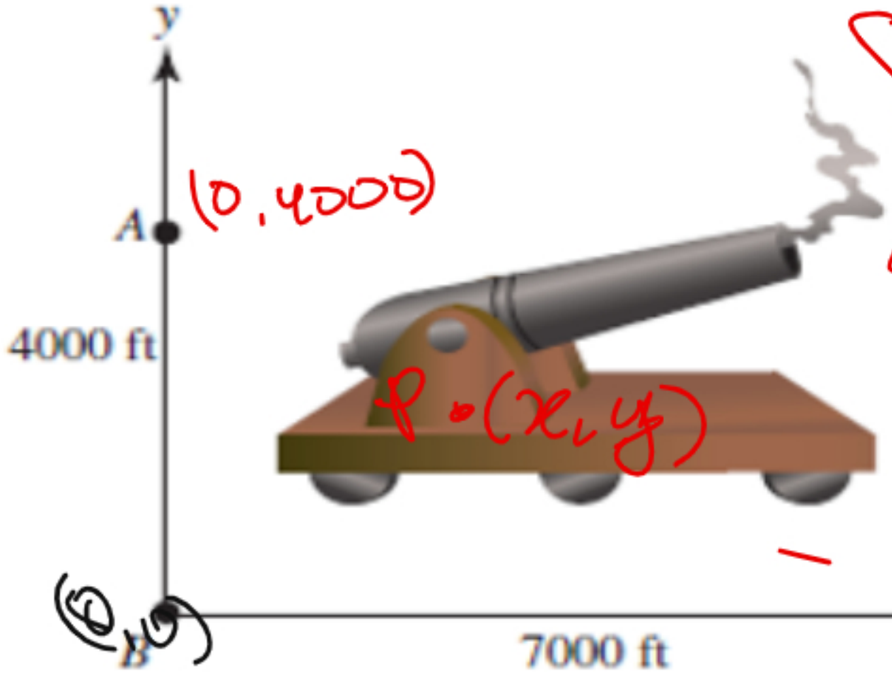
$20y = x^2$ $\quad x^2 = 20y$

find an equation in standard form for the ellipse that satisfies the given conditions.

. Major axis endpoints $(\pm 5, 0)$, minor axis length 4

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Gun Location Observers are located at positions A, B, and C with A due north of B. A cannon is located somewhere in the first quadrant as illustrated in the figure. A hears the sound of the cannon 2 sec before B, and C hears the sound 4 sec before B. Determine the bearing and distance of the cannon from point B. (Assume that sound travels at 1100 ft/sec.)



5 min

$t_A = \frac{|PA|}{v_0}$

$t_B = \frac{|PB|}{v_0}$

$t_C = \frac{|PC|}{v_0}$

$\begin{cases} t_B - t_A = 2 \\ t_B - t_C = 4 \end{cases}$

(0, 4000)

$P = (x, y)$

(7000, 0)

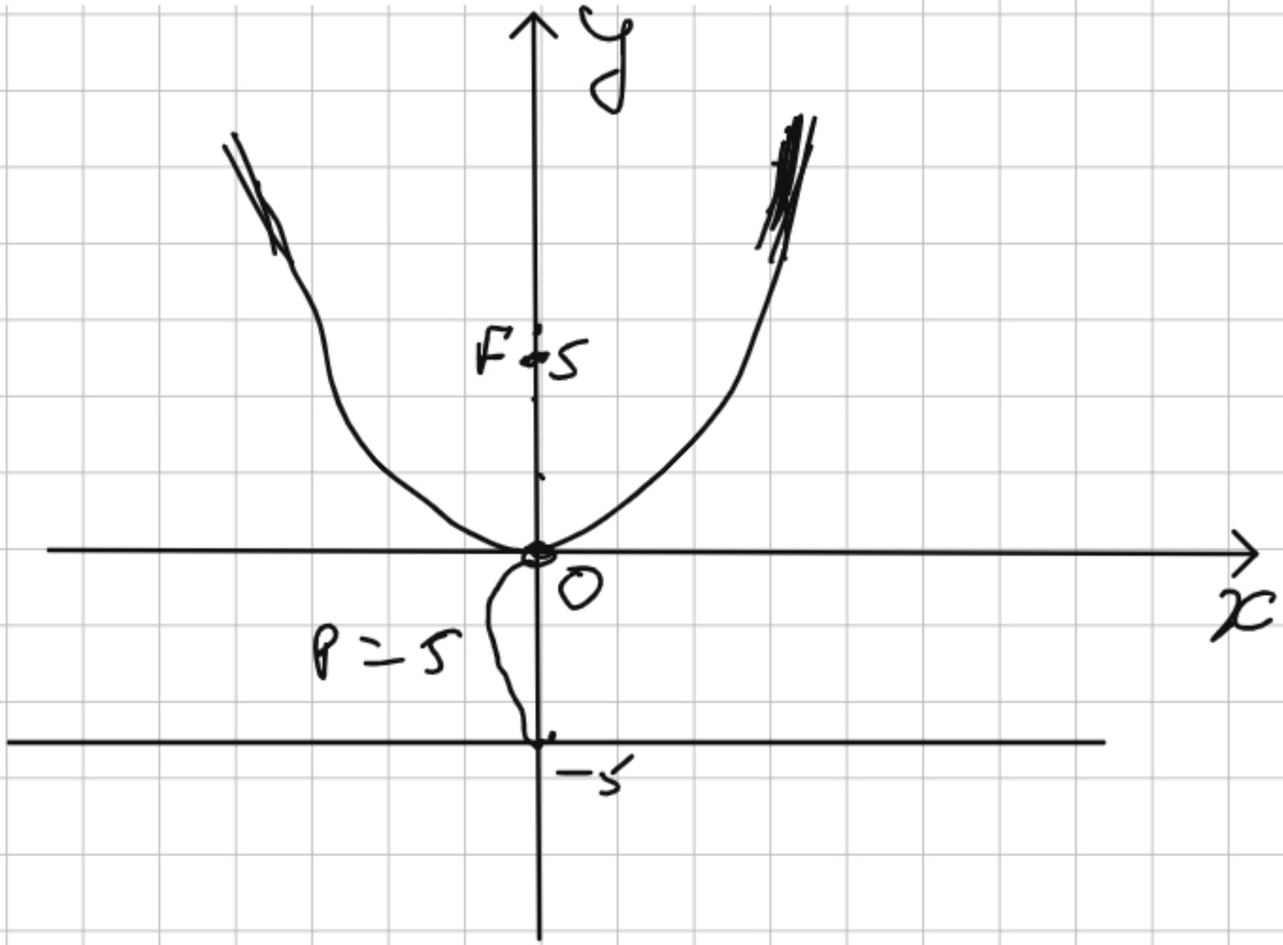
$$\begin{cases} \frac{\sqrt{x^2+y^2}}{v_0} - \frac{\sqrt{x^2+(y-4000)^2}}{v_0} = 2 \\ \frac{\sqrt{x^2+y^2}}{v_0} - \frac{\sqrt{(x-7000)^2+y^2}}{v_0} = 4 \end{cases}$$

Find an equation in standard form for the parabola that satisfies the given conditions.

Focus $(0, 5)$, directrix $y = -5$

$x^2 = 4py$

$\boxed{x^2 = 20y}$



find an equation in standard form for the ellipse that satisfies the given conditions.

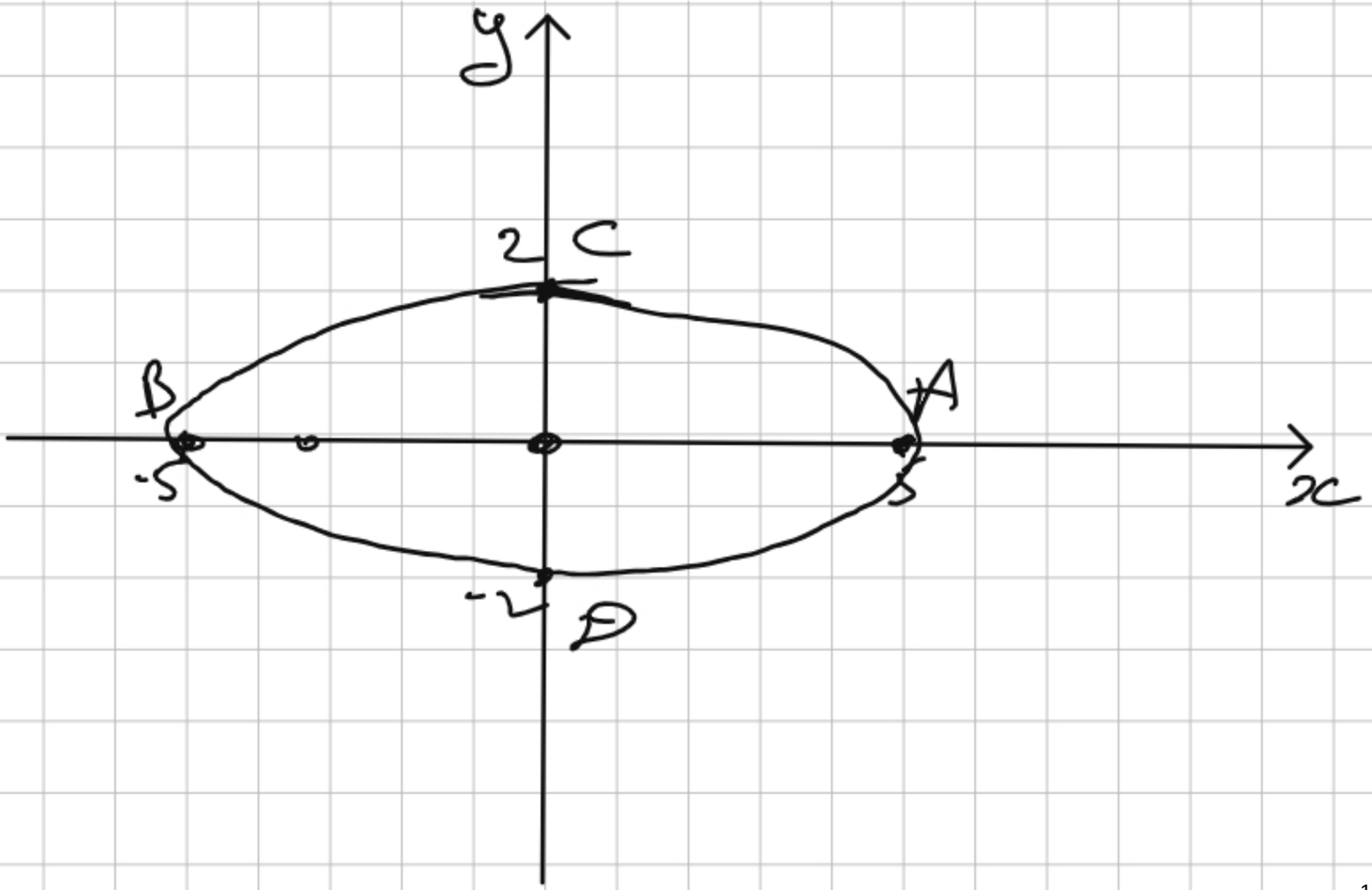
. Major axis endpoints $(\pm 5, 0)$, minor axis length 4

$2B = 4$

$B = 2$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$



Hyperbolic navigation.