Rate of Change

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BHOS

Calculus

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When $D = 10$ m, the area is changing with respect to the diameter at the rate of $(\pi/2)10 = 5\pi$ m^2/m ≈ 15.71 m^2/m.

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$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$