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1 NUMBER SYSTEM AND NUMBER SERIES

1.1 NUMBERS AND THEIR CLASSIFICATION

A number p may be,

(i) a **natural** number (N)
(ii) a **whole** number (W)
(iii) an **integer** (Z)
(iv) a **rational** number (Q)
(v) a **real** number (R)
(vi) an **irrational** number

For example,
set of **natural** numbers is $\{1, 2, 3, \ldots\}$
set of **whole** numbers is $\{0, 1, 2, 3, \ldots\}$
set of **integers** is $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
set of **rational** numbers is $\frac{1}{2}, \frac{3}{5}, -\frac{8}{5}, 0, +3, -150, \ldots$
set of **irrational** numbers is $\pi, \sqrt{2}, \sqrt{5}, \sqrt{7}, \ldots$

Besides the above cited number, we often come across numbers like $\sqrt{-8}, \sqrt{-7}, \sqrt{-5}, \sqrt{-1}, 3+\sqrt{-7}$ and so on. These are undefined numbers, called **complex** numbers.

A positive integer, except 1, is called a **prime** number, if its factors are 1 and the number itself 2, 3, 5, 7, ... are prime numbers.

1.2 TEST FOR DIVISIBILITY OF NUMBERS

There are certain tests for divisibility of numbers by any of the numbers 2, 3, 4, 5, 6, 8, 9, 10 and 11 such that by simply examining the digits in the given number, one can easily determine whether or not a given number is divisible by any of these numbers. Such tests are detailed as follows:

i. Divisibility by 2

If the last digit is an even number or it has zero (0) at the end.

Example: 74, 148, 1210 are all divisible by 2.

ii. Divisibility by 3

If the sum of the digits of the given number is divisible by 3.

Example: The sum of the digits of number 3705 is 3 + 7 + 0 + 5 = 15
Since 15 (the sum of digits) is divisible by 3, the number 3705 is also divisible by 3.

iii. Divisibility by 4

If the number formed by the last two digits of the given number is divisible by 4, or if the last two digits are '00'.

Example: 216560 is a number whose last two digits are 60. Since 60 is divisible by 4, the given number 216560 is also divisible by 4.

iv. Divisibility by 5

If the last digit of the given number is 0 or 5.

Example: 865, 1705, 4270 are all divisible by 5.

v. Divisibility by 6

If the given number is divisible by both 2 and 3.

Example: Let us consider the number 629130. It has 0 as the last digit, so it is divisible by 2.
Sum of the digits = 6 + 2 + 9 + 1 + 3 + 0 = 21
This sum 21 is divisible by 3, so the number is divisible by 3.
Since, 629130 is divisible by both 2 and 3, the number is also divisible by 6.

vi. Divisibility by 8

If the number formed by the last three digits of the given number is divisible by 8 or if the last three digits are '000'.

Example: The number 81976 has 976 as the last three digits. Since 976 is divisible by 8, 81976 is also divisible by 8. The number 6145000 ends with '000' and so, it is divisible by 8.

vii. Divisibility by 9

If the sum of the digits of the given number is divisible by 9.

Example: 870111 is a number the sum of whose digits = 8 + 7 + 0 + 1 + 1 + 1 = 18.
Since 18 (sum of digits) is divisible by 9, the number 870111 is also divisible by 9.

viii. Divisibility by 10

If the last digit of the number is zero (0).

Example: 730 has 0 at the end, so it is divisible by 10.

ix. Divisibility by 11

If the difference of the sum of its digits in odd places (i.e. first, third, fifth . . .) and the sum of its digits in even places (i.e. second, fourth, sixth . . .) is either zero (0) or a multiple of 11.

Example: Let us consider the number 647053.
Sum of digits at odd places = 6 + 7 + 5 = 18
Sum of digits at even places = 4 + 0 + 3 = 7
Difference of the sums = 18 – 7 = 11
Since the difference (= 11) is a multiple of 11, 647053 is also divisible by 11
Let us consider another number 9610260.
Sum of digits at odd places = 9 + 1 + 2 + 0 = 12
Sum of digits at even places = 6 + 0 + 6 = 12

Difference of the sums = 12 – 12 = 0
Since the difference is 0, 9610260 is divisible by 11.

1.3 GENERAL PROPERTIES OF DIVISIBILITY

There are some general properties of divisibility that help in determining the divisibility of a natural number by other natural numbers (other than detailed in 1.2)

Property 1

If a number x is divisible by another number y, then any number divisible by x, will also be divisible by y and by all the factors of y.

Example: The number 84 is divisible by 6. Thus any number that is divisible by 84, will also be divisible by 6 and also by the factors of 6, i.e. by 2 and by 3.

Property 2

If a number x is divisible by two or more than two co-prime numbers then x is also divisible by the product of those numbers.

Example: The number 2520 is divisible by 5, 4, 13 that are prime to each other (i.e. co-prime), so, 2520 will also be divisible by 20 (= 5 × 4), 65 (= 5 × 13), 52 (= 4 × 13).

Property 3

If two numbers x and y are divisible by a number 'p', then their sum $x + y$ is also divisible by p.

Example: The numbers 225 and 375 are both divisible by 5. Thus their sum = 225 + 375 = 600 will also be divisible by 5.

Note: It is also true for more than two numbers.

Property 4

If two numbers x and y are divisible by a number 'p', then their difference $x \sim y$ is also divisible by p.

Example: The numbers 126 and 507 are both divisible by 3. Thus their difference = 507 – 126 = 381 will also be divisible by 3.

1.4 TEST OF A PRIME NUMBER

A prime number is only divisible by 1 and by the number itself. The first prime number is 2. Every prime number other than 2 is odd, but every odd number is not necessarily a prime number. Again any even number (other than 2) cannot be a prime number. To test whether any given number (if odd) is a prime number or not, following steps are to be considered:

Step 1 Find an integer (x) which is greater than the approximate square root of the given number.

Step 2 Test the divisibility of the given number by every prime number less than x.

Step 3
- If the given number is divisible by any of them in Step 2, then the given number is NOT a prime number.
- If the given number is not divisible by any of them in Step 2, then the given number IS a prime number.

Example: Consider a number 203. Test if it is a prime number or not.

Step 1 The approximate square root of 203 is 14 plus. Take $x = 15$.

Step 2 Check the divisibility of 203 by the prime numbers less than 15 i.e. by 2, 3, 5, 7, 11, 13.

Step 3 203 is divisible by 7. Thus, it is not a prime number.

1.5 DIVISION AND REMAINDER

When a given number is not exactly divisible by any number, then there is a remainder number at the end of such division.

Suppose we divide 25 by 7 as,

$$\begin{array}{r l}
7) & 25 \;(3 \\
 & \underline{21} \\
 & 4
\end{array}$$

then, divisor = 7, dividend = 25

quotient = 3, and $\boxed{\text{remainder} = 4}$

So, we can represent it as

$$\begin{array}{l}
\text{divisor }) \; \underline{\text{dividend}} \; (\text{ quotient} \\
\qquad\qquad \text{remainder}
\end{array}$$

Thus $\boxed{\text{dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}}$ **(i)**

So, if a number x is divided by k, leaving remainder 'r' and giving quotient 'q' then the number can be found by using (i)

$$x = kq + r$$

Hence, if the number x is exactly divisible by k, then remainder $= r = 0$

$\therefore \quad x = kq$

and so $\quad \dfrac{x}{k} = q$, implying that x is divisible exactly by k and q is an integer.

1.5.1 Methods to Find a Number Completely Divisible by Another

Consider a given number x. When divided by d, it gives a quotient q and remainder 'r'.

It implies that the given number 'x' is not exactly divisible by 'd'

$$\begin{array}{l}
d \;)\; \underline{x} \;(\; q \\
\quad\;\; r
\end{array}$$

Now, to find a number exactly divisible by 'd', we can use either of the following two methods to reduce the remainder to zero. (If a number is exactly divisible, then remainder is zero).

Method 1

- By subtracting the remainder from the given number (dividend).

  $\therefore$ the required number that is exactly divisible by 'd' $= x - r$

  Hence 'remainder' is the **least number that can be subtracted** from any number to make it exactly divisible.

Method 2

- By adding the (divisor – remainder) to the given number.

  $\therefore$ the required number that is exactly divisible by $d = x +$ (divisor – remainder)

  Therefore, (divisor – remainder) is the **least number that can be added** to any given number to make it exactly divisible.

Example: Find the least number, that must be
(a) subtracted from
or (b) added to a given number 5029, to make it exactly divisible by 17.

Solution: On dividing 5029 by 17, we find that

$$
\begin{array}{r|l}
17) & 5029 \ (295 \\
 & \underline{34} \\
 & 162 \\
 & \underline{153} \\
 & 99 \\
 & \underline{85} \\
 & 14
\end{array}
$$

∴ remainder = 14.
(a) The least number to be subtracted to make it exactly divisible = remainder = 14. (By method 1)
(b) The least number to be added to make it exactly divisible = divisor – remainder = 17 – 14 = 3. (By method 2)

1.5.2 Greatest *n*-digit and Least *n*-digit Number Exactly Divisible by a Number

(a) To find out the **greatest *n*-digit** number exactly divisible by a divisor '*d*', we use Method 1 (1.5.1)
∴ the required number = Greatest *n*-digit number – remainder.
(b) To find out the **least *n*-digit** number exactly divisible by a divisor '*d*', we use Method 2 (1.5.1), because if we use method 1, then subtracting any number from the *n*-digit least number will reduce it to $(n - 1)$ digit number.
∴ the required number = Least *n*-digit number + (divisor – remainder)

Example: Find the (a) greatest 3-digit number divisible by 13.
(b) the least 3-digit number divisible by 13.

Solution: (a) 13) $\underline{999}$ (
11

∴ the required 3-digit greatest number
= 999 – 11 = 988

(b) 13) $\underline{100}$ (
9

∴ the required 3-digit least number
= 100 + (13 – 9)
= 104.

1.6 REMAINDER RULES

Rule 1

This rule is applied to find the **remainder for the smaller divison,** when the same number is divided by the two different divisors such that one divisor is a multiple of the other divisor and also the remainder for the greater divisor is known.

If the remainder for the greater divisor = r,
and the smaller divisor = d, then

Rule-1 states, that **when $r > d$**, the required remainder for the smaller divisor will be the remainder found out by dividing the 'r' by 'd'. [Case I]
and when $r < d$, then the required remainder is 'r' it self. [Case II]

Example: If a number is divided by 527, the remainder is 42.
What will be the remainder if it is divided by 17?

Solution: Here the same number is divided by two divisors: 527 and 17.

Now, $\frac{527}{17} = 31$, so, 527 is a multiple of 17
Hence Rule 1 can be applied.
Remainder for the greater divisor (i.e., for 527) = 42
Smaller divisor = 17.

$$\begin{array}{r|l} 17) & 42\ (\\ & \underline{34} \\ & 8 \end{array}$$

So, 8 = required remainder for smaller divisor (i.e. 17)
Hence, if 527 is divided by 17, the remainder will be 8.

Rule 2

If two different numbers a and b, on being divided by the same divisor leave remainders r_1 and r_2 respectively, then their sum $(a + b)$ if divided by same divisor will leave remainder R, given by

$$R = (r_1 + r_2) - \text{divisor}$$

$\Rightarrow$ The required remainder R = sum of remainders − divisor
(When sum is divided)

Note: If R becomes negative in the above equation, then the required remainder will be the sum of the remainders.
$\therefore$ the required remainder = sum of remainders

Example: Two different numbers, when divided by the same divisor, leave remainders 15 and 39 respectively, and when their sum was divided by the same divisor, the remainder was 7. What is the divisor?

Solution: Using the Rule 2

$$7 = (15 + 39) - \text{divisor}$$
$$\Rightarrow \quad \text{divisor} = 47$$

Example: Two different numbers, when divided by 47, leave remainders 13 and 23 respectively. If their sum is divided by the same number 47, what will be the remainder?

Solution: Using Rule 2,

The required remainder = $(13 + 23) - 47$
$= -11$

Since the remainder is (−) ve, so, the actual remainder will be 23 + 13 = 36 (refer to NOTE under Rule 2)

Rule 3

When two numbers, after being divided by the same divisor leave the same remainder, then the difference of those two numbers must be exactly divisible by the same divisor.

Example: Two numbers 147 and 225, after being divided by a 2-digit number, leave the same remainder. Find the divisor.

Solution: By Rule 3, the difference of 225 and 147 must be perfectly divisible by the divisor.

The difference = 225 − 147 = 78

Now, 78 = 13 × 2 × 3.

Thus, 1-digit divisors = 2, 3 and 2 × 3

2-digit divisors = 13 × 2, 13 × 3, 13, 13 × 2 × 3

∴ the possible divisors are 26, 39, 13, 78.

Rule 4

If a given number is divided successively by the different factors of the divisor leaving remainders r_1, r_2 and r_3 respectively, then the true remainder (i.e. remainder when the number is divided by the divisor) can be obtained by using the following formula:

True remainder = (first remainder) + (second remainder × first divisor) + (third remainder × first divisor × second divisor)

Example: A number, being successively divided by 5, 7 and 11 leaves 3, 1, 2 as remainders respectively. Find the remainder if the same number is divided by 385.

Solution: Here, the divisor is 385, whose factors are 5, 7 and 11.

∴ by Rule 3,

True remainder (i.e. remainder when divided by 385) = 3 + (1 × 5) + (2 × 5 × 7)

= 3 + 5 + 70

= 78

Rule 5

When $(x + 1)^n$ is divided by x, the remainder is always 1, where x and n are natural numbers.

Example: What will be the remainder when $(17)^{21}$ is divided by 16?

Solution: $(17)^{21} = (16 + 1)^{21}$,

∴ when $(16 + 1)^{21}$ is divided by 16, the remainder = 1.

Rule 6

When $(x - 1)^n$ is divided by x, then

the remainder = 1, when n is an even natural number

but the remainder= $x - 1$, when n is an odd natural number.

Example: What will be the remainder when $(29)^{75}$ is divided by 30?

Solution: $(29)^{75} = (30 - 1)^{75}$, here index = 75 (which is odd) so, when $(30 - 1)^{75}$ is divided by 30, the remainder will be $x - 1 = 30 - 1 = 29$

1.7 NUMBER SERIES

In the number series, some numbers are arranged in a particular sequence. All the numbers form a series and change in a certain order. Sometimes, one or more numbers are wrongly put in the number series. One is required to observe the trend in which the numbers change in the series and find out which number/numbers misfit into the series. That number/numbers is the ODD NUMBER of the series.

1.7.1 Important Number Series

Following are some of the important rules or order on which the number series can be made.

I. Pure Series

In this type of number series, the number itself obeys certain order so that the character of the series can be found out.

The number itself may be:

— perfect square
— perfect cube
— prime
— combination of above

II. Difference Series

Under this category, the **change in order** for the differences between each consecutive number of the series is found out as shown in Table 1.1.

III. Ratio Series

Under this category, the change in order for the ratios between each consecutive number of the series is found out as shown in Table 1.2.

IV. Mixed Series

Here, the numbers obeying various orders of two or more different type of series are arranged alternately in a single number series.

V. Geometric Series

Under this category, each successive number is obtained by multiplying (or dividing) the previous number with a fixed number. (See Table 1.2).

Example: 5, 35, 245, 1715, 12005, . . .
43923, 3993, 363, 33, 3, . . .

VI. Two-tier Arithmetic Series

Under this category, the differences of successive numbers form an arithmetic series. (See Table 1.1)

Example: 4, 5, 9, 16, 26, 39, 55, 74.

+1 +4 +7 +10 +13 +16 +19 → Arithmetic Series

3 3 3 3 3 3

VII. Three-tier Arithmetic Series

The differences of successive numbers form a two-tier arithmetic series. The successive term difference in this, in turn form an arithmetic series.

Table 1.1 Change in Order for the Difference Series

Series Code	*Nature/Order of the Number Series*	*Examples for the Series*	
D1	Difference between consecutive numbers is same.	2 7 12 17 +5 +5 +5	35 27 19 11 −8 −8 −8
D2	Differences between consecutive numbers are in arithmetic progression (A.P.)	15 26 42 63 +11 +16 +21 Here 11, 16, 21 are in A.P.	34 25 18 13 −9 −7 −5 Here −9, −7, −5 are in A.P.
D3	Difference between consecutive numbers is a perfect square number.	7 16 41 90 $+3^2$ $+5^2$ $+7^2$	70 54 45 41 -4^2 -3^2 -2^2
D4	Differences between consecutive numbers are multiples of a number.	11 23 47 95 +12 +24 +48 Here 12, 24 and 48 are multiples of 12.	204 160 127 105 −44 −33 −22 Here −44, −33 and −22 are multiples of 11.
D5	Differences between consecutive numbers are prime numbers.	2 3 5 8 13 +1 +2 +3 +5 Here, 1, 2, 3 and 5 are prime numbers.	29 22 15 4 −5 −7 −11 Here −5, −7 and −11 are all prime numbers.
D6	Difference between consecutive numbers is a perfect cube number.	7 15 42 167 $+2^3$ $+3^3$ $+4^3$	121 57 30 22 -4^3 -3^3 -2^3
D7	Difference between consecutive numbers are in geometric progression (G.P.).	1 2 5 14 41 124 +1 +3 +9 +27 +81 Here 1, 3, 9, 27 and 81 are in G.P. 33 17 9 5 3 2 −16 −8 −4 −2 −1 Here −16, −8, −4, −2 and −1 are in G.P.	

Table 1.2 Change In Order for the Ratio Series

Series Code	Nature or Order of the Number Series	Examples for the Series
R1	Ratio between each consecutive numbers is the same.	$32 \xrightarrow{\div 2} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4$ $3 \xrightarrow{\times 5} 15 \xrightarrow{\times 5} 75 \xrightarrow{\times 5} 375$
R2	Ratio between each consecutive numbers is in arithmetic progression (A.P.)	$2 \xrightarrow{\times 2} 4 \xrightarrow{\times 4} 16 \xrightarrow{\times 6} 96$ $420 \xrightarrow{\div 5} 84 \xrightarrow{\div 4} 21 \xrightarrow{\div 3} 7$
R3	Ratio between consecutive number is a perfect square number.	$3 \xrightarrow{\times 2^2} 12 \xrightarrow{\times 4^2} 192 \xrightarrow{\times 6^2} 6912$ $44100 \xrightarrow{\div 7^2} 900 \xrightarrow{\div 5^2} 36 \xrightarrow{\div 3^2} 4$
R4	Ratio between consecutive number is the multiple of a number.	$5 \xrightarrow{\times 2} 10 \xrightarrow{\times 4} 40 \xrightarrow{\times 8} 320$ Here 2, 4 and 8 are multiples of 2. $972 \xrightarrow{\div 9} 108 \xrightarrow{\div 6} 18 \xrightarrow{\div 3} 6$ Here 9, 6 and 3 are multiples of 3.
R5	Ratio between consecutive numbers is a prime number.	$5 \xrightarrow{\times 1} 15 \xrightarrow{\times 2} 30 \xrightarrow{\times 3} 90 \xrightarrow{\times 5} 450$ Here, 2, 3 and 5 are prime numbers. $770 \xrightarrow{\div 11} 70 \xrightarrow{\div 7} 10 \xrightarrow{\div 5} 2$ Here, 11, 7 and 5 are prime numbers.

(Contd.)

Table 1.2 (*Contd.*)

Series Code	*Nature or Order of the Number Series*	*Examples for the Series*
R6	Ratio between consecutive numbers is a perfect cube number.	2 2 54 6750 $\times 1^3$ $\times 3^3$ $\times 5^3$ 13824 216 8 1 $\div 4^3$ $\div 3^3$ $\div 2^3$
R7	Ratios between consecutive numbers are in geometric progression (G.P.).	1 1 2 8 64 1024 $\times$ 1 $\times$ 2 $\times$ 4 $\times$ 8 $\times$ 16 Here 1, 2, 4, 8 and 16 are in G.P. 729 27 3 1 $\times$ 27 $\times$ 9 $\times$ 3 Here 27, 9 and 3 are in G.P.

Example: 392, 245, 140, 70, 28, 7, 0 . . .

147 105 70 42 21 7

42 35 28 21 14 → Arithmetic Series

7 7 7 7

VII. Twin Series

Under this category, two series are alternatively placed in one.

Example: 1, 2, 3, –2, 5, –8, 7, –16, 9 . . . .

2 2 2 2

–4 –6 –8

–2 –2

(first, third, fifth, seventh terms are in A.P.)

(second, fourth, sixth, terms form two-tier Arithmetic Series).

1.8 THREE STEPS TO SOLVE A PROBLEM ON SERIES

Step 1 Determine whether the series is increasing, decreasing or alternating.

Step 2 If the series is increasing or decreasing, then check:

(a) if change is slow or gradual, then it is a difference series.

(b) if the change is equally sharp, throughout, then it is a ratio series.

(c) if the rise is very sharp initially, but slows down later, then the series may be formed by adding squared, or cubed numbers.

If the series is alternating or irregular, there may be either a mix of two series or two different kinds of operations going on alternately.

Step 3 Complete the series accordingly.

1.9 TWO-LINE NUMBER SERIES

A two-line numbe. series, as the name suggests, consists of number series in two lines. If one complete series is given in first line, with an incomplete series in second line, and it is given that the series in both the lines have the same definite rule, we need to work it out as follows:

Applying the very definite rule of the series in the first line, the series in second line can be completed. The pattern/type of series in the first line may be any of the types described in 1.7.

Example: 15 28 51 84 127 . . .

22 a b c d e . . .

In the first line, the differences of two successive terms of the series are 13, 23, 33, 43.

Hence following the pattern of first line series, the number series in second line are:

$a = 22 + 13 = 35$, $b = 35 + 23 = 58$, $c = 58 + 33 = 91$

$d = 91 + 43 = 134$, $e = 134 + 53 = 187$.

1.10 SUM RULES ON NATURAL NUMBERS

Rule 1

Sum of all the first n natural numbers (starting from 1) $= \dfrac{n(n+1)}{2}$

Example: Sum of 1 to 74 $= \dfrac{74 \times 75}{2} = 2775$

Rule 2

Sum of first n odd numbers (starting from 1) $= n^2$

Example: Sum of first 7 odd numbers $(1 + 3 + 5 + 7 + 9 + 11 + 13) = 72 = 49$

Rule 3

Sum of first n even numbers (starting from 1) $= n\ (n + 1)$

Example: Sum of first 9 even numbers $(2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18) = 9\ (9 + 1) = 90$.

Rule 4

Sum of squares of first n natural numbers (starting from 1) $= \dfrac{n(n+1)(2n+1)}{6}$

Example: Sum of squares of first 8 natural numbers

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = \frac{8(8+1)(2 \times 8+1)}{6}$$

$$= \frac{8 \times 9 \times 17}{6} = 204$$

Rule 5

Sum of cubes of first n natural numbers $= \left[\dfrac{n(n+1)}{2}\right]^2$

Example: Sum of cubes of first 6 natural numbers $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = \left[\dfrac{6(6+1)}{2}\right]^2 = 441$

Note: For applying Rule 2 and Rule 3, it is required to find how many odd numbers or even numbers are there in the given series.

In the first 'n' natural numbers,

if n is even, then there are $\dfrac{n}{2}$ odd numbers and $\dfrac{n}{2}$ even numbers

if n is odd, then there are $\dfrac{n+1}{2}$ odd numbers and $\dfrac{n-1}{2}$ even numbers

Example: From 1 to 30, as 30 is even, there are 15 odd numbers and 15 even numbers.

From 1 to 29, as 29 is odd, there are $\dfrac{29+1}{2} = 15$ odd numbers and $\dfrac{29-1}{2} = 14$ even numbers

1.11 BASE AND INDEX

If a number 'b' is multiplied by itself 'n' times, then the product is called the nth power of b, i.e.

$b \times b \times b$ . . . . up to n times = b^n

Here, b is called the base and n is called the index.

1.11.1 Last Digit (Digit at Unit's Place) in $(xyz)^n$

Here the given number is $\underbrace{(xyz)}_{\text{base}}^{n} \longrightarrow$ index

z is the last digit of the base.

To find out the last digit in $(xyz)^n$, following steps are to be followed.

Divide the index (n) by 4, then

Case I

if remainder = 0

then check if z is **odd** (except 5), then last digit = 1.

and if z is **even**, then last digit = 6.

Case II

if remainder = 1, then required last digit = last digit of base (i.e. z)

if remainder = **2**, then required last digit = last digit of $(z)^2$

if remainder = **3**, then required last digit = last digit of $(z)^3$

Note: If z is 5, then last digit in the product = 5

Example: Find the last digit in $(295073)^{130} \rightarrow n$

Solution: Dividing 130 by 4, the remainder = 2

∴ refering to Case II, the required last digit is the last digit of $(z)^2$, i.e. $(3)^2 = 9$, (because $z = 3$)

Example: Find the last digit in $(81678)^{199}$

Solution: Dividing 199 by 4, the remainder = 3

∴ the required last digit is the last digit of $(z)^3$, i.e. $(8)^3 = 512$ (as $z = 8$)

Hence the last digit is 2

1.11.2 Number of Zeroes at the End of a Product

On multiplying two or more given numbers, the zeroes are produced at the end of the resulting product due to the following reasons:

(a) If there is any zero at the end of any of the factors (or numbers being multiplied)

Example: 7 × 20 = 140

This zero is produced at the end of the product also

(b) If 5 or a multiple of 5 is multiplied by any even number.

Example: 45 × 12 = 540

$$\underset{\text{multiple of 5}}{45} \times \underset{\text{even}}{12} = 540$$

Combining the above two reasons, we may say that:

(i) Resolve all the given numbers into their factors.

(ii) Count the number of 2s and 5s.
i.e. $(5)^x \times (2)^y$, say.

(iii) No. of zeroes at the end of product = No. of 2s or no. of 5s, whichever is less

Example: Find the number of zeroes at the end of the product of :

$$15 \times 32 \times 25 \times 22 \times 40 \times 75 \times 98 \times 112 \times 125$$

Solution: We need not multiply these numbers to find the number of zeroes at the end of the resultant product.

All we need to do is to find the numbers of 5s and 2s in the given numbers by resolving the numbers into their factors:

$$(5 \times 3) \times (2^5 \times 5^2) \times (2 \times 11) \times (2^3 \times 5) \times (5^2 \times 3) \times (2 \times 7 \times 7) \times (2^4 \times 7) \times 5^3$$

Only no. of 2s and 5s are relevant here, so, we have:

(i) No. of 5s in $(5^{1+2+1+2+3})$ i.e. $5^9 = 9$

(ii) No. of 2s in $(2^{5+1+3+1+4})$ i.e. $2^{14} = 14$.
Since $9 < 14$.
the no. of zeroes = 9 at the end of the product.

1.12 BINARY NUMBER SYSTEM

This system has a base 2 and uses only 0 and 1; whereas the conventional decimal system having a base 10, uses 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

For any number system it is common that

— the position of each digit within a number affects the total value of the number. In fact, the position of each digit carries a specific weight according to the base of the system.

— each digit has a distinct value and it cannot equal or exceed the base of the system.

In the binary number system, the base = 2 and the number of digits used in the binary system is 2 (0 and 1). So the digits can have value, 0 or 1.

The positional weights of a binary number have been indicated in Table 1.3.

Table 1.3 Positional Weights of a Binary Number

Positions from Right to Left ←	*8th* 7	*7th* 6	*6th* 5	*5th* 4	*4th* 3	*3rd* 2	*2nd* 1	*1st* 0 ← *value*
Positional weight of a Binary number	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	128	64	32	16	8	4	2	1

Using Table 1.3, the decimal equivalents of any binary number can be found out.

$$(10101)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$
$$= 16 + 0 + 4 + 0 + 1 = (21)_{10}$$

Alternatively,

1	0	1	0	1
16	$\cancel{8}$	4	$\cancel{2}$	1

Hence, $(10101)_2 = 16 + 4 + 1 = (21)_{10}$

1.12.1 Conversion of a Decimal Number to a Binary Number

To convert a decimal number to a binary number, the following steps are to be considered.

Step 1 Divide the decimal number by 2.

Step 2 Go on dividing the quotients (obtained at each stage) by 2 till the quotient is 0.

Step 3 Write down the remainders on the right side after each of the above divisions.

Step 4 Arrange the remainders (as obtained in Step 3) in the reverse order to get the equivalent binary number.

For example, conversion of 25 to a binary number is given as:

			Remainder
2	25		1
2	12		0
2	6		0
2	3		1
2	1		1
	0	⇒ last quotient	Arrange the remainders

The equivalent binary number = $(11001)_2$

1.13 CALCULATION IN THE BINARY SYSTEM

Mathematical calculations (i.e. addition, subtraction and multiplication) in the binary system follow their own rules and are similar to those in the decimal system.

1.13.1 Binary Addition

It is easy to add two binary numbers. The rules for binary addition are as follows:

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 10$ (Put 0 in the same column and carry 1 to the **next left column**)

What is carry 1?

In the decimal system, when two numbers are added and the sum of two digits exceeds the highest digit (i.e. 9), then 1 is carried to the next higher digit position (left column).

Similarly, in binary system, when the sum of two digits (bits) exceeds the highest digit (i.e. 1), then 1 is carried to the next higher digit (bit) position (next left column).

In binary addition, the numbers are written one below the other with their rightmost digits aligned. In case of the numbers having fractional parts, the binary points are aligned. Adding is started from right to left, as done in the decimal system.

For example, 1011 + 111

carry	→	1	1	1	1	
(Augend) upper row	→		1	0	1	1
(Addend) lower row	→	+	0	1	1	1
		1 ←	0 ←	0 ←	1 ←	0

Carry 1 Carry 1 Carry 1 Carry 1

Explanation of Addition (column-wise)

Column 1 (rightmost column) 1 + 1 = 1 + $\boxed{0}$

carry result

Column 2 (from rightmost) 1 (carry) + 1 (upper row) = 1 0

0 + 1 (lower row) = $\boxed{1}$ result

carry

Column 3 (from rightmost) 1 (carry) + 0 (upper row) = 1

1 + 1 (lower row) = 1 $\boxed{0}$ result

← carry

Column 4 (from rightmost) 1 (carry) + 1 (upper row) = 1 0

0 + 0 (lower row) = $\boxed{0}$ result carry

and put in the left-most column

1.13.2 Binary Subtraction

It is easy to subtract a binary number from another binary number. The rules for binary subtraction are as follows:

0 − 0 = 0
1 − 1 = 0
1 − 0 = 1

But to find 0 − 1, we write 1 in the result and also we borrow 1 from the next left column.

What is Borrow 1?

In decimal system, when one number is subtracted from another and it happens that a greater digit is to be subtracted from a smaller one, then 10 is borrowed from the next digit position (next left column).

Similarly, in the binary system if it is required to subtract 1 from 0, then 1 is borrowed from the next digit position (next left column) and the result is 1.

For example, to find $(1000)_2 - (11)_2$, we proceed as follows.

Upper Row	1	0	0	0
Lower Row	0	0	1	1
Borrow	1	1	1	
		1	0	1

Explanation of Subtraction (Column-wise)

Column 1 (rightmost) — 0 – 1 yields a result $\boxed{1}$ and a borrow 1 which is placed below column 2.

Column 2 (from rightmost) — 0 – 1 yields a **temporary result 1** and a borrow 1 (placed below column 3).
From this temporary result 1, subtract borrow 1 which has already been placed below column 2. So, 1 – 1 yields a result $\boxed{0}$.

Column 3 (from rightmost) — 0 – 0 yields a **temporary result 0**. From this temporary result, **subtract borrow 1** (which has already been placed below column 3).
So, 0 – 1 yields a result $\boxed{1}$ and a borrow 1 (placed below column 4).

Column 4 (from rightmost) — 1 – 0 yields a temporary result 1. From this temporary result 1, subtract borrow 1 (which has already been placed below column 4).
So, 1 – 1 yields a result $\boxed{0}$.

1.13.3 Binary Multiplication

Binary multiplication is as simple as multiplication in decimal system. The four rules that are followed in multiplication of two binary numbers are summarised below.

0×0	$= 0$
0×1	$= 0$
1×0	$= 0$
1×1	$= 1$

For example, $(11011)_2 \times (101)_2$, we write as

			1	1	0	1	1
					1	0	1
			1	1	0	1	1
		0	0	0	0	0	
1	1	0	1	1			
1	0	0	0	0	1	1	1

carry (1) to the next higher digit position (next left column) and add, because

$1 + 1 = 1\ 0$

result

carry

1.13.4 Binary Division

In binary division, the method that is applied is similar to that in decimal system. The two rules which are followed here are,

$$\frac{0}{1} = 0 \quad \text{and} \quad \frac{1}{1} = 1$$

Here also, the value of $\frac{1}{0}$ is undefined.

For example, to divide 100111 by 111

```
111 ) 1 0 0 1 1 1 ( 1 0 1
      0 1 1 1
      ---------
          1 0 1 1
            1 1 1
          -------
            1 0 0
```

∴ Quotient = 101
Remainder = 100

Note: In the problems on binary system it has been found in the examinations that
0 is written as *
1 is written as •
Hence it is to be remembered that all the rules of binary system are applicable, but only type of coding may vary. Write * for 0 and • for 1.

Solved Examples

E-1 On dividing 15625 by 41, what is the quotient and the remainder?

S-1 Divisor = 41, dividend = 15625, quotient = 381 and remainder = **4.**

E-2 On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

S-2 Divisor $= \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \dfrac{397246 - 211}{865} = 459$ ∴ The divisor is **459.**

E-3 What is the number which on dividing $(x + ak)$ gives 'a' as the quotient and x as the remainder?

S-3 Divisor $= \dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \dfrac{x + ak - x}{a} = k$ ∴ the divisor is $\boldsymbol{k}$.

E-4 Find the least number, that must be subtracted from 87375, to get a number exactly divisible by 698.

S-4 On dividing 87375 by 698, the remainder is 125
By Method I (Refer 1.5.1)
The least number to be subtracted from the dividend is the remainder
∴ the least number to be subtracted = **125.**

E-5 What least number must be added to 49123 to get a number exactly divisible by 263

S-5 On dividing 49123 by 263, the remainder is 205.
By Method 2 (Refer 1.5.1)
the least number to be added to the dividend = divisor – remainder = 263 – 205 = 58
∴ the least number to be added = **58.**

E-6 Find the greatest number of 3 digits, which is exactly divisible by 35.

S-6 The greatest 3 digit number = 999.
On dividing 999 by 35, remainder = 19 Now by applying *Method* 2, we obtain
the required number = (dividend) – (remainder) = 999 – 19 = **980.**

E-7 Find the least number of 3 digits, which is exactly divisible by 14.

S-7 The least number of 3 digits = 100
On dividing 100 by 14, remainder = 2
To determine exactly divisible least number, we follow the method 2,
∴ The required number = Dividend + (Divisor – Remainder) = 100 + (14 – 2) = **112.**

E-8 A number when divided by 602 leaves a remainder 36. What remainder would be obtained by dividing the same number by 14?

S-8 Here divisor is a multiple of the other divisor, we find that one

i.e. $\frac{602}{14}$ = an integer. Rule 1 of Remainder Rules is to be used. (Refer 1.6) and so.

$$\begin{array}{r|l} 14) \; 36 & (2 \\ \underline{28} & \\ 8 & \end{array}$$

∴ the required remainder = 8

E-9 A number, when divided by 357, leaves a remainder 5. What remainder would be obtained by dividing the same number by 17?

S-9 Here, $\frac{357}{17}$ = an integer. i.e. one divisor is multiple of the other divisor.
∴ But the remainder by the greater divisor = 5, which is even less than the smaller divisor (=17), so using Case II of Rule-1, we find as
the required remainder by smaller divisor = the remainder by greater divisor = 5.

E-10 In each of the following number series, one number is wrong. Find out the odd number.
(i) 2, 8, 20, 44, 92, 184, 380
(ii) 60, 48, 38, 28, 24, 20, 18
(iii) 380, 188, 92, 48, 20, 8, 2
(iv) 3, 4.5, 9, 22.5, 67.5, 270, 945
(v) 7, 9, 17, 42, 91, 172, 293
(vi) 5, 15, 30, 135, 405, 1215, 3645
(vii) 2, 9, 28, 65, 126, 216, 344
(viii) 1, 2, 6, 21, 86, 445, 2676
(ix) 3, 5, 12, 38, 154, 914, 4634
(x) 696, 340, 168, 80, 36, 14, 3

(xi) 445, 221, 109, 46, 25, 11, 4

(xii) 1, 2, 4, 12, 36, 71, 144, 432

(xiii) 7, 10, 12, 14, 17, 19, 22, 24

(xiv) 1, 3, 10, 21, 64, 129, 356, 777

(xv) 200, 165, 148, 118, 104, 77, 68

(xvi) 2, 6, 24, 96, 285, 568, 567

(xvii) 6072, 1008, 200, 48, 14, 5, 3

(xviii) 3, 10, 36, 180, 1080, 7560

(xix) 318, 368, 345, 395, 372, 422, 400, 449

(xx) 54, 9, 15, 6, 24, 4, 16

(xxi) 444, 153, 156, 52, 60, 20, 28

(xxii) 2, 6, 12, 27, 58, 121, 248

(xxiii) 3, 9, 18, 54, 110, 324, 648

(xxiv) 5, 6, 15, 41, 89, 170, 291

(xxv) 8544, 1420, 280, 44, 18, 5

(xxvi) 1, 1, 4, 36, 586, 14400

(xxvii) 812, 398, 190, 90, 40, 16

(xxviii) 7, 8, 12, 24, 37, 62, 98

(xxix) 2, 2, 4, 12, 66, 420, 4620

(xxx) 7, 8, 10, 18, 17, 22, 28

(xxxi) 1, 3, 4. 8, 16, 36, 64

(xxxii) 12, 20, 19, 26, 24, 31, 30

(xxxiii) 4, 8, 11, 22, 18, 33, 25, 50

(xxxiv) –1, 2, 7, 14, 28, 34, 47

(xxxv) 100, 50, $33\frac{1}{3}$, 22, 20

(xxxvi) 1, 7, 16, 2, 8, 15, 3, 9, 18

(xxxvii) $\frac{1}{8}, \frac{1}{4}, \frac{3}{4}$, 4, 15, 90

(xxxviii) 6, 15, 19, 30, 32, 43, 45

(xxxix) 2348, 3437, 4346, 5436, 6344, 7433

(xxxx) 87, 86, 82, 75, 57, 32, –4

(xxxxi) 0, 3, 9, 15, 24, 35, 48

(xxxxii) $-\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, 2$

(xxxxiii) 2, 4, 6, 3, 9, 13, 4, 16, 20

(xxxxiv) 15, 1, 14, 2, 12, 4, 9, 11, 5

(xxxxv) 21, 28, 29, 36, 38, 46, 48, 55

(xxxxvi) 9, 7, 64, 6, 3, 18, 1, 8, 9

(xxxxvii) 27, 54, 58, 116, 232, 240, 244

(xxxxviii) –3, 9, 41, 113, 262, 577

S-10 (i)

2 8 20 44 92 $\boxed{184}$ 380

+6 +12 +24 +48 +96 +192

Wrong number = **184**, correct number = **188** (Refer D7, Table 1.1)

(ii)

60 48 38 $\boxed{28}$ 24 20 18

–12, –10, –8 –6 –4 –2

Wrong number = **28**, correct number = **30** (Refer D2, Table 1.1)

(iii)

380 188 92 $\boxed{48}$ 20 8 2

–192 –96 –48 –24 –12 –6

Wrong number = **48**, correct number = **44** (Refer D7, Table 1.1)

(iv)

3 4.5 9 22.5 67.5 $\boxed{270}$ 945

× 1.5 × 2 × 2.5 × 3 × 3.5 × 4

Wrong number = **270**, correct number = **236.25** (Refer R2, Table 1.2)

(v)

7 $\boxed{9}$ 17 42 91 172 293

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

(1) (9) (25) (49) (81) (121)

Wrong number = **9**, correct number = **8** (Refer D3, Table 1.1)

(vi)

5 15 $\boxed{30}$ 135 405 1215 3645

× 3 × 3 × 3 × 3 × 3 × 3

Wrong number = **30**, correct number = **45** (Refer R1, Table 2.1)

(vii)

2 9 28 65 126 $\boxed{216}$ 344

$1^3 + 1$ $2^3 + 1$ $3^3 + 1$ $4^3 + 1$ $5^3 + 1$ $6^3 + 1$ $7^3 + 1$

Wrong number = **216**, correct number = **217** (It is a pure number series).

(viii)

1 2 6 21 $\boxed{86}$ 445 2676

$\times 1 + 1$ $\times 2 + 2$ $\times 3 + 3$ $\times 4 + 4$ $\times 5 + 5$ $\times 6 + 6$

Wrong number = **86**, correct number = **88** (It is a mixed series).

(ix)

3 5 12 38 154 $\boxed{914}$ 4634

$\times 1 + 2$ $\times 2 + 2$ $\times 3 + 2$ $\times 4 + 2$ $\times 5 + 2$ $\times 6 + 2$

Wrong number = **914**, correct number = **772.**

(x)

696 $\boxed{340}$ 168 80 36 14 3

-352 -176 -88 -44 -22 -11

Wrong number = **340**, correct number = **344** (Refer D7, Table 1.1)

(xi)

445 221 109 $\boxed{46}$ 25 11 4

-224 -112 -56 -28 -14 -7

Wrong number = **46**, correct number = **53** (Refer D7, Table 1.2)

(xii)

1 2 4 12 36 $\boxed{71}$ 144 432

$\times 2$ $\times 2$ $\times 3$ $\times 3$ $\times 2$ $\times 2$ $\times 3$

Here, one series is multiple of 2 and other is multiple of 3 and both are repeated in a batch of two numbers.

Wrong number = **71**, correct number = **72**. It is a mixed series (Refer R4, Table 1.2)

(xiii)

7 10 12 14 17 $\boxed{19}$ 22 24

$+3$ $+2$ $+2$ $+3$ $+3$ $+2$ $+2$

Wrong number = **19**, correct number = **20**. It is a mixed series.

(xiv)

1 3 10 21 64 129 $\boxed{356}$ 777

$\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$ $\times 3 + 1$ $\times 2 + 1$

Wrong number = **356**, correct number = **388**. It is a mixed series.

(xv)

200 165 148 $\boxed{118}$ 104 77 68

$14^2 + 4$ $13^2 - 4$ $12^2 + 4$ $11^2 - 4$ $10^2 + 4$ $9^2 - 4$ $8^2 + 4$

Wrong number = **118**, correct number = **117**. It is a pure series (Refer 1.3.1 (I)).

(xvi)

2 6 $\boxed{24}$ 96 285 568 567

$\times 6 - 6$ $\times 5 - 5$ $\times 4 - 4$ $\times 3 - 3$ $\times 2 - 2$ $\times 1 - 1$

Wrong number = **24**, correct number = **25**.

(xvii)

6072 $\boxed{1008}$ 200 48 14 5 3

$-12 \div 6$ $-10 \div 5$ $-8 \div 4$ $-6 \div 3$ $-4 \div 2$ $-2 \div 1$

Wrong number = **1008**, correct number = **1010**.

(xviii)

3 $\boxed{10}$ 36 180 1080 7560

$\times 3$ $\times 4$ $\times 5$ $\times 6$ $\times 7$

Wrong number = **10**, correct number = **9** (Refer R2, Table 1.1).

(xix)

318 368 345 395 372 422 $\boxed{400}$ 449

+ 27 + 27 + 27

+ 27 + 27 + 27

Wrong number = **400**, correct number = **399**.

(xx)

54 9 $\boxed{15}$ 6 24 4 16

$\div 6$ $\div 6$ $\div 6$

Wrong number = **15**, correct number = **36**.

(xxi)

444 $\boxed{153}$ 156 52 60 20 28

$\div 3$ $\div 3$ $\div 3$

Wrong number = **153**, correct number = **148**.

(xxii)

2 $\boxed{6}$ 12 27 58 121 248

$\times 2 + 1$ $\times 2 + 2$ $\times 2 + 3$ $\times 2 + 4$ $\times 2 + 5$ $\times 2 + 6$

Wrong number = **6**, correct number = **5**.

(xxiii)

3 9 18 54 $\boxed{110}$ 324 648

$\times 3$ $\times 2$ $\times 3$ $\times 2$ $\times 3$ $\times 2$

Wrong number = **110**, correct number = **108**.

(xxiv)

5 6 15 $\boxed{41}$ 89 170 291

$+ 1^2$ $+ 3^2$ $+ 5^2$ $+ 7^2$ $+ 9^2$ $+ 11^2$

Wrong number = **41**, correct number = **40** (Refer D3, Table 1.1).

(xxv)

8544 1420 280 $\boxed{44}$ 18 5

$\div 6 - 4$ $\div 5 - 4$ $\div 4 - 4$ $\div 3 - 4$ $\div 2 - 4$

Wrong number = **44**, correct number = **66**.

(xxvi)

1 1 4 36 $\boxed{586}$ 14400

$\times 1^2$ $\times 2^2$ $\times 3^2$ $\times 4^2$ $\times 5^2$

Wrong number = **586**, correct number = **576** (Refer R3, Table 1.2).

(xxvii)

812   398 $\boxed{190}$ 90   40   16

$\div 2 - 8$ $\div 2 - 7$ $\div 2 - 6$ $\div 2 - 5$ $\div 2 - 4$

Wrong number = **190**, correct number = **192**.

(xxviii)

7   8   12 $\boxed{24}$ 37   62   98

$+1$ $+2^2$ $+3^2$ $+4^2$ $+5^2$ $+6^2$

Wrong number = **24**, correct number = **21** (Refer D3, Table 1.1).

(xxix)

2   2   4   12 $\boxed{66}$ 420   4620

$\times 1$ $\times 2$ $\times 3$ $\times 5$ $\times 7$ $\times 11$

Wrong number = **66**, correct number = **60** (Refer R5, Table 1.2).

(xxx)

7   8   10 $\boxed{18}$ 17   22   28

$+1$ $+2$ $+3$ $+4$ $+5$ $+6$

Wrong number = **18**, correct number = **13** (Refer D2, Table 1.1).

(xxxi)

1   3   4   8   16 $\boxed{36}$ 64

$4 \downarrow 1 + 3$ $8 \downarrow 1 + 3 + 4$ $16 \downarrow 1 + 3 + 4 + 8$ $36 \downarrow 1 + 3 + 4 + 8 + 16$ $64 \downarrow 1 + 3 + 4 + 8 + 16 + 32$

Wrong number = **36**, correct number = **32**.

(xxxii)

12   20   19   26 $\boxed{24}$ 31   30

$+8$ -1 $+7$ -1 $+6$ -1

Wrong number = **24**, correct number = **25**. It is a mixed series.

(xxxiii)

+ 7   + 7   + 7

4   8   11   22   18 $\boxed{33}$ 25   50

+ 14   + 14   + 14

Wrong number = **33**, correct number = **36**.

(xxxiv)

–1   2   7   14 $\boxed{28}$ 34   47

+ 3   + 5   + 7   + 9   + 11   + 13

Wrong number = **28**, correct number = **23**.

(xxxv)

100	50	$33\frac{1}{3}$	$\boxed{22}$	20
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$100 \div 1$	$100 \div 2$	$100 \div 3$	$100 \div 4$	$100 \div 5$

Wrong number = **22**, correct number = **25**.

(xxxvi)

1	7	16	2	8	$\boxed{15}$	3	9	18
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
x	y	z	x	y	z	x	y	z

There are three series x, y and z. mixed

x series contains 1, 2, 3

y series contains 7, 8, 9

z series contains 16, 15, 18. (Wrong number = **15**, correct number = **17**)

(xxxvii)

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{3}{4}$ $\boxed{4}$ 15   90

× 2   × 3   × 4   × 5   × 6

Wrong number = **4**, correct number = **3** (Refer R2, Table 1.2).

(xxxviii)

+ 13   + 13   + 13

6   15   19   30   32 $\boxed{43}$ 45

+ 15   + 15

Wrong number = **43**, correct number = **45**. It is a mixed series.

(xxxix)

2348 3437 4346 $\boxed{5436}$ 6344 7433

+ 1089 + 909 + 1089 + 909 + 1089

Here, 1089 and 909 are added alternately to continue the series.

Wrong number = **5436**, correct number = **5435**

(xxxx)

87 86 82 $\boxed{75}$ 57 32 -4

-1^2 -2^2 -3^2 -4^2 -5^2 -6^2

Wrong number = **75**, correct number = **73** (Refer D3, Table 1.1).

(xxxxi)

0 3 $\boxed{9}$ 15 24 35 48

+ 3 + 5 + 7 + 9 + 11 + 13

Wrong number = **9**, correct number = **8**.

(xxxxii)

$-\frac{1}{2}$ 0 $\frac{1}{2}$ $\boxed{\frac{1}{4}}$ $\frac{3}{2}$ 2

$+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{1}{2}$

Wrong number = $\mathbf{\frac{1}{4}}$, correct number = **1**.

(xxxxiii)

2 4 6 3 9 $\boxed{13}$ 4 16 20

2 + 4 3 + 9 4 + 16

Wrong number = **13**, correct number = **12**. It is a mixed series.

(xxxxiv)

+ 1 + 2 + 3

15 1 14 2 12 4 9 $\boxed{11}$ 5

-1 -2 -3 -4

There are two series arranged alternately.

Series 1 contains 1, 2, 4, 11

Seies 2 contains 15, 14, 12, 9, 5

Wrong number = **11**, correct number = **7** (Refer D2, Table 1.1).

(xxxxv)

21 28 29 36 38 $\boxed{46}$ 48 55

+ 7 + 1 + 7 + 2 + 7 + 3 + 7

Wrong number = **46**, correct number = **45**.

(xxxxvi)

9 7 64 6 3 $\boxed{18}$ 1 8 9

$9 \times 7 + 1$ $6 \times 3 + 1$ $1 \times 8 + 1$

Wrong number = **18**, correct number = **19**.

(xxxxvii)

27 54 58 116 $\boxed{232}$ 240 244

× 2 + 4 × 2 + 4 × 2 + 4

Wrong number = **232**, correct number = **120**.

(xxxxviii)

– 3 9 41 113 $\boxed{262}$ 577

+ 12 + 32 + 72 + 152 + 312

+ 20 + 40 + 80 + 160

Here, 20, 40, 80 and 160 are in G.P.

Wrong number = **262**, correct number = **265**.

E-11 7 13 78 83 415

3 *a* *b* *c* *d* *e*

What should replace *c*?

S-11 The pattern of the first line series is obtained as:

$7, 7 + 6, 13 \times 6, 78 + 5, 83 \times 5$

Therefore, the second line series can be completed on the basis of the same pattern, as:

$3, a = 3 + 6, b = 9 \times 6, c = 54 + 5, \ldots$

$\therefore$ The value of c is 59.

E-12 3 6 24 72 144 576

1 *a* *b* *c* *d* *e*

What should replace *d*?

S-12 The pattern of the first line series is obtained as:

$3, 3 \times 2, 6 \times 4, 24 \times 3, 72 \times 2, 144 \times 4$

Therefore, the second line series can be completed on the basis of the same pattern as:

$1, a = 1 \times 2, b = 2 \times 4, c = 8 \times 3, d = 24 \times 2$

$\therefore$ The value of d is 48

E-13 4 6 15 49 201 1011

15 *a* *b* *c* *d* *e*

What should replace *e*?

S-13 The pattern of the first line series is obtained as:

$4, 4 \times 1 + 2, 6 \times 2 + 3, 15 \times 3 + 4, 49 \times 4 + 5, 201 \times 5 + 6$

Therefore, the second line series can be completed on the basis of the same pattern as:
15, $a = 15 \times 1 + 2$, $b = 17 \times 2 + 3$, $c = 37 \times 3 + 4$, $d = 115 \times 4 + 5$, $e = 465 \times 5 + 6$
$\therefore$ The value of e is 2331

E-14 –1 0 10 65 345 1750
–2 a b c d e
What should come in place of d?

S-14 The pattern of the first line series is obtained as:
$-1, (-1 + 1) \times 5, (0 + 2) \times 5, (10 + 3) \times 5, (65 + 4) \times 5, (345 + 5) \times 5$
Therefore, the second line series can be completed on the basis of the same pattern as:
-2, $a = (-2 + 1) \times 5$, $b = (-5 + 2) \times 5$, $c = (-15 + 3) \times 5$, $d = (-60 + 4) \times 5$
$\therefore$ the value of d is –270

E-15 –1 0 –8 3 –52 –135
21 a b c d e
What should come in place of c?

S-15 The pattern of the first line series is obtained as:
$-1, (-1 \times 1) + 1^3, (0 \times 2) - 2^3, (-8 \times 3) + 3^3, (3 \times 4) - 4^3, -52 \times 5 - 5^3.$
Therefore, the second line series can be completed on the basis of the same pattern as :
21, $a = 21 \times 1 + 1^3$, $b = 22 \times 2 - 2^3$, $c = 36 \times 3 + 3^3$
$\therefore$ the value of c is 135.

E-16 3000 191 2216 847 1688 959
3435 a b c d e
What should come in place of b?

S-16 The pattern of the first line series is obtained as:
$3000, 3000 - 53^2, 191 + 45^2, 2216 - 37^2, 847 + 29^2, 1688 - 21^2$
Therefore, the second line series can be completed on the basis of the same pattern as:
3435, $a = 3435 - 53^2$, $b = 626 + 45^2$
$\therefore$ the value of b is 2651.

Directions (17–21): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer.

For example, if an interchange of number marked 1 and the number marked 4 is required to establish the series, your answer is 1. If it is not necessary to interchange the position of the numbers to establish the series, give 5 as your answer. Remember that when the series is established, the numbers change from left to right (i.e. from the unmarked number to the last marked number) in a specific order.

E-17 1200 40 1000 50 750 75
1) 2) 3) 4) 5)

S-17 The pattern of the series is:
$1200, 1200 \div 30, 40 \times 25, 1000 \div 20, 50 \times 15, 750 \div 10$
Here, it is not necessary to interchange the position of any number to establish the series.
Hence, 5) is the answer.

E-18 2 5 26545 177 4424 44
1) 2) 3) 4) 5)

S-18 The pattern of the series is:
$2, 2 \times 2 + 1, \underline{5 \times 3^2 - 1}, 44 \times 4 + 1, 177 \times 5^2 - 1, \underline{4424 \times 6 + 1}$
5) ——— to interchange ——— 2)

Here, the numbers marked 2) and marked 5) are to be interchanged in the position to establish the series.
Hence 2) is the answer.

E-19 1 1 1 2 8 4

1	1	1	2	8	4
	1)	2)	3)	4)	5)

S-19 The pattern of the series should go as

$$1,\quad 1^2,\quad 1^3,\quad 2,\quad \underbrace{2^2,}_{5)}\quad \underbrace{2^3}_{4)}$$

to interchange

Here, the numbers marked 4) and marked 5) are to be interchanged in their position to establish the series.
Hence 4) is the answer

E-20

48	16	13	12	17	25.25
	1)	2)	3)	4)	5).

S-20 The pattern of the series should go as

$$48,\quad 48 \times 0.25 + 4,\quad \underbrace{16 \times 0.5 + 4,}_{3)}\quad \underbrace{12 \times 0.75 + 4,}_{2)}\quad 13 \times 1 + 4,\quad 17 \times 1.25 + 4$$

to interchange

Here, the numbers marked 2) and marked 3) are to be interchanged in their position to establish the series.
Hence 2) in the answer.

E-21

82	83	165	9916	1983	496
	1)	2)	3)	4)	5).

S-21 The pattern of the series should go as

$$82,\quad 82 \times 1 + 1,\quad 83 \times 2 - 1,\quad \underbrace{165 \times 3 + 1,}_{5)}\quad 496 \times 4 - 1,\quad \underbrace{1983 \times 5 + 1}_{3)}$$

to interchange

Here, the numbers marked 5) and marked 3) are to be interchanged to establish the series.
Hence 3) is the answer.

E-22 Find the binary equivalents of
(i) 30; (ii) 27; (iii) 41

S-22 (i)

$$\begin{array}{r|l l l}
 & & & \text{Remainders} \\
2 & 30 & \longrightarrow & 0 \\ \hline
2 & 15 & \longrightarrow & 1 \uparrow \\ \hline
2 & 7 & \longrightarrow & 1 \\ \hline
2 & 3 & \longrightarrow & 1 \\ \hline
2 & 1 & \longrightarrow & 1 \\ \hline
 & 0 & &
\end{array}$$

So, the binary equivalent is $\mathbf{(11110)_2}$.

(ii) Remainders

$$\begin{array}{r|l l}
2 & 27 & \longrightarrow 1 \\ \hline
2 & 13 & \longrightarrow 1 \uparrow \\ \hline
2 & 6 & \longrightarrow 0 \\ \hline
2 & 3 & \longrightarrow 1 \\ \hline
2 & 1 & \longrightarrow 1 \\ \hline
 & 0 &
\end{array}$$

So, the binary equivalent is $\mathbf{(11011)_2}$.

(iii) Remainders

$$\begin{array}{r|l l}
2 & 41 & 1 \\ \hline
2 & 20 & 0 \uparrow \\ \hline
2 & 10 & 0 \\ \hline
2 & 5 & 1 \\ \hline
2 & 2 & 0 \\ \hline
2 & 1 & 1 \\ \hline
 & 0 &
\end{array}$$

So, the binary equivalent is $\mathbf{(101001)_2}$.

E-23 Simplify the following for question mark (?)

(i) $10110_2 - 1011_2 = (?)_2$
(ii) $1001_2 + 1010_2 = (?)_2$
(iii) $101_2 \times 100_2 = (?)_2$
(iv) $1100_2 \div 11_2 = (?)_2$
(v) $101_2 + 1100_2 \div 10_2 = (?)_2$

S-23 (i)

$$\begin{array}{ccccc l}
1 & 0 & 1 & 1 & 0 & \text{Upper row} \\
 & 1 & 0 & 1 & 1 & \text{Lower row} \\
1 & & 1 & 1 & & \text{Borrow} \\ \hline
0 & 1 & 0 & 1 & 1 &
\end{array}$$

Hence the required result is $\mathbf{(1011)_2}$.

(ii)

$$\begin{array}{ccccc}
 & 1 & 0 & 0 & 1 \\
+ & 1 & 0 & 1 & 0 \\ \hline
1 & 0 & 0 & 1 & 1
\end{array}$$

Hence the required result is $\mathbf{(10011)_2}$.

(iii)

$$\begin{array}{ccccc}
 & & 1 & 0 & 1 \\
 & \times & 1 & 0 & 0 \\
\hline
 & & 0 & 0 & 0 \\
 & 0 & 0 & 0 & \\
1 & 0 & 1 & & \\
\hline
1 & 0 & 1 & 0 & 0
\end{array}$$

Hence the required result is $\mathbf{(10100)_2}$.

(iv)

$$\begin{array}{l}
11\;)\;1100\;(\;100 \\
\quad\;\;\underline{11} \\
\quad\;\;000
\end{array}$$

Hence the required result is $\mathbf{(100)_2}$.

(v) $101_2 + 1100_2 \div 10_2$

Firstly,

$$\begin{array}{l}
10\;)\;1100\;(\;110 \\
\quad\;\;\underline{10} \\
\quad\quad\;10 \\
\quad\quad\;\underline{10} \\
\quad\quad\;\;00
\end{array}$$

Now $101_2 + 110_2$ is

$$\begin{array}{r}
101 \\
110 \\
\hline
1011
\end{array}$$

Hence the required result is $\mathbf{(1011)_2}$.

E-24 In a certain code, the symbol for 0 (zero) is * and for 1, it is ●. There are no other symbols for other numbers and all numbers greater than 1 are written using these two symbols only. The value of symbol 1 doubles itself everytime it shifts one place to the left.

Thus, 0 is written as *

1 is written as ●

2 is written as ●*

3 is written as ●●

On the above coding system, answer the following questions.

(i) What is the ratio of ●●* and ●**●?

(ii) Find the value of 10 ÷ 5 + 2 in the above code.

(iii) Evaluate: ●●* + ●*● – ●●●.

(iv) If 50% of ●*●* is added to ●*●, then what will be the result?

(v) Find the product of ●●● and ●*.

S-24 (i) ●●* = $(110)_2 = 4 + 2 = 6_{10}$

●**● = $(1001)_2 = 8 + 1 = 9_{10}$

$\therefore$ Required ratio = $\frac{6}{9} = \frac{2}{3}$.

(ii) $10 \div 5 + 2 = 4_{10}$

$$\begin{array}{r|l l} 2 & 4 & 0 \\ \hline 2 & 2 & 0 \\ \hline 2 & 1 & 1 \\ \hline & 0 & \end{array}$$

So, the required value is $(\mathbf{100})_2$ = ●**.

(iii) ●●* + ●*● – ●●●.

$= 110 + 101 - 111$

So, firstly

$$\begin{array}{cccc} & 1 & 1 & 0 \\ + & 1 & 0 & 1 \\ \hline 1 & 0 & 1 & 1 \\ - & 1 & 1 & 1 \\ \hline & 1 & 0 & 0 \end{array}$$

So, the required value is $(\mathbf{100})_2$, = ●**

(iv) ●*●* $= (1010)_2$

$= 8 + 2 = 10_{10}$

●*● $= (101)_2$

$= 4 + 1 = 5_{10}$.

So, 50% of ●*●* + ●*●

$= 50\%$ of $10_{10} + 5_{10}$

$= 5_{10} + 5_{10} = 10_{10}$

Now,

$$\begin{array}{r|l l} 2 & 10 & 0 \\ \hline 2 & 5 & 1 \\ \hline 2 & 2 & 0 \\ \hline 2 & 1 & 1 \\ \hline & 0 & \end{array}$$

So the required result is ●*●*.

(v) ●●● × ●*

$$\begin{array}{rrrr} & 1 & 1 & 1 \\ & & 1 & 0 \; \times \\ \hline & 0 & 0 & 0 \\ 1 & 1 & 1 & \\ \hline 1 & 1 & 1 & 0 \end{array}$$

i.e. ●●●*

REGULAR PROBLEMS

Section A: Number System

(1) In a division, find the divisor if dividend = 27541, quotient = 233 and remainder = 47
(a) 172 (b) 238 (c) 126 (d) 194 (e) 118

(2) The least number that must be subtracted from 104075 to make it exactly divisible by 437 is:
(a) 31 (b) 69 (c) 50 (d) 44 (e) 38

(3) The greatest 5-digit number that is exactly divisible by 100 is:
(a) 99899 (b) 99800 (c) 99900 (d) 99889 (e) 98990

(4) Which of the following numbers is a prime number?
(a) 541 (b) 323 (c) 217 (d) 551 (e) None

(5) Which is the least 7-digit number, that leaves a remainder of 3 when divided by 7?
(a) 1000003 (b) 1000010 (c) 1000005 (d) 1000002 (e) 1000007

(6) When a certain number is multiplied by 21, the product consists of only fours. The smallest such number is :
(a) 21164 (b) 4444 (c) 444444 (d) 444 (e) 3126

Hint: Assuming the product, you can find the other multiplicand

(7) A number when divided by 627 leaves a remainder 43. By dividing the same number by 19, the remainder will be
(a) 19 (b) 24 (c) 43 (d) 5 (e) 13

(8) What will be the remainder when $(29)^{36}$ is divided by 28?
(a) 0 (b) 1 (c) 29 (d) 5 (e) Cannot be determined

(9) The sum of all odd numbers from 1 to 41 is:
(a) 372 (b) 505 (c) 441 (d) 398 (e) 516

Hint: Refer 1.10

(10) What is the total number of numbers up to 9999?
(a) 98900 (b) 10000 (c) 9999 (d) 98100 (e) None of these

(11) The digit in the unit place in $(1038)^{67}$ is:
(a) 2 (b) 4 (c) 1 (d) 6 (e) 8

Hint: Refer 1.11.1

(12) The number of prime numbers in $(25)^{13} \times (10)^7 \times (27)^5$ is:
(a) 25 (b) 32 (c) 55 (b) 50 (e) 42

Hint: Resolve each number into its prime factors, then count the number of such factors by adding the index.

(13) The number of zeros at the end of the product **(SSC, '97)**
$16 \times 22 \times 15 \times 50 \times 65 \times 115 \times 18 \times 90$ is:
(a) 5 (b) 6 (c) 12 (d) 7 (10)

Hint: Refer 1.11.2 in the text

(14) What is the difference in intrinsic value and local value of 6 in 8631?
(a) 625 (b) 594 (c) 600 (d) 496 (e) 0

(15) How many such numbers are there between 1 and 100 such that each of which is not only divisible by 4, but also has one digit as 4 in the number?
(a) 5 (b) 12 (c) 6 (d) 15 (e) 7

(16) A number is greater than 3 but less than 8. Also, the number is greater than 6 but less than 10. What is the number?
(a) 5 (b) 4 (c) 9 (d) 6 (e) 7

(17) In a division, a student took 63 as divisor instead of 36. His answer was 24. The correct answer is:
(a) 42 (b) 32 (c) 48 (d) 28 (e) 38

(18) $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by:
(a) 3 (b) 11 (c) 13 (d) 17 (e) None of these

(19) How many numbers are there between 500 and 600 in which 9 occurs only once?
(a) 18 (b) 19 (c) 20 (d) 21 (e) 22

(20) Replace the * in the number 6* 106 by a suitable digit so that the number formed is exactly divisible by 11.
(a) 3 (b) 4 (c) 2 (d) 1 (e) 1

(21) The value of $101 + 102 + 103 + \ldots + 200$ is:
(a) 15050 (b) 20200 (c) 10909 (d) 16500 (e) None of these

(22) What are the values of '*a*' and '*b*', if 4266 *ab* is divisible by 45?
(a) 4 and 5 (b) 1 and 7 (c) 9 and 0 (d) 3 and 6 (e) either (a) or (c)

Hint: Refer 1.2

(23) The digit in the unit's place of the number
$17^{1999} + 11^{1999} - 7^{1999}$ is:
(a) 7 (b) 4 (c) 1 (d) 3 (c) None of these

Hint: Use of the concept (in ref 1.11.1) can be avoided here if we use a little common sense

(24) The number, one less than 7^{19} is divisible by:
(a) 49 (b) 7 (c) 16 (d) 18 (e) 6

Answers

1. (e)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)	**9.** (c)
10. (c)	**11.** (a)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (e)	**16.** (e)	**17.** (a)	**18.** (d)
19. (a)	**20.** (c)	**21.** (a)	**22.** (e)	**23.** (c)	**24.** (e)			

Section B: Number Series

(1) Which of the following does not fit in the series
(a) 3, 7, 12, 27, 51, 105, 204. **(RRB Kolkata '02)**

Hint: $3 \times 2 + 1, 7 \times 2 - 2, 12 \times 2 + 3 \ldots$

(b) 4443, 2433, 4322, 4511, 6221 **(RRB Kolkata '02)**

Hint: Digits add to 11

(c) 0, 6, 24, 90, 120, 210 **(RRB Kolkata '02)**
(d) 318, 368, 345, 395, 372, 422, 400, 449 **(AAO '98)**
(e) 196, 168, 143, 120, 99, 80 63
(f) 3, 4, 20, 38, 87, 168, 289 **(BSRB, Bang, '97)**
Hint: $+1^2, +3^2, +5^2 \ldots$
(g) 698, 343, 170, 82, 38, 16, 5 **(BSRB, Bang, '97)**
(h) 698, 554, 454, 390, 347, 338, 334 **(BSRB, Bang, '97)**
Hint: $-12^2, -10^2, -8^2 \ldots$
(i) 376, 188, 88, 40, 16, 4, –2 **(BSRB, Delhi, '93)**
Hint: $\div 2 - 4$
(j) 10, 15, 24, 35, 54, 75, 100 **(IBPS Bank PO, '02)**
Hint: $+ 5, + 9, + 13 \ldots.$
(k) 1, 3, 4, 7, 11, 18, 27, 47 **(IBPS Bank PO, '02)**
Hint: Middle number = difference of succeeding and proceeding number
(l) 2, 8, 32, 148, 765, 4626, 32431 **(IBPS, '02)**
Hint: $\times 2 + 2^2, \times 3 + 3^2, \times 4 + 4^2 \ldots$
(m) 2, 3, 11, 38, 102, 229, 443 **(IBPS, '02)**
Hint: Differences are $1^3, 2^3, 3^3 \ldots$
(n) 0, 6, 23, 56, 108, 184, 279 **(RRB, Bhubaneswar, 2000)**
Hint: $1^3 - 2^0, 2^3 - 2^1, 3^3 - 2^2, 4^3 - 2^3 \ldots$
(o) 49, 56, 64, 71, 81, 90, 100, 110 **(RRB, Bhubaneswar, 2000)**
(p) 13700, 1957, 326, 65, 16, 6, 2 **(Andhra Bank PO, '99)**
Hint: $-1 \div 7, \; -1 \div 6, \; -1 \div 5 \ldots$
(q) 5, 7, 13, 25, 45, 87, 117 **(Andhra Bank PO, '99)**

(2) What will come in place of the question mark (?) in the following number series? (Andhra Bank Specialist Officer's Exam, 2002)
(a) 2, 9, 30, 105, ?, 2195
(b) 3, 4, 12, 45, ?, 1005
(c) 1, 3, 9, 31, ?, 651
(d) 5, ?, 4, 7.5, 17, 45
(e) 15, 30, ?, 40, 8, 48

Direction (3–23): In each of the following questions, a number series is given. After the series, below it, a number is given followed by (A), (B), (C), (D) and (E). You have to complete the series starting with the number given, following the sequence for the given series on top. Then answer the question below it

(3) 616 496 397 317 254 **(SBI ASSC Bank PO, '97)**
838 A B C D E
What will come for E?

(4) 6 9 27 121.5 729 **(SBI ASS. Bank PO, '97)**
8 A B C D E
What will come for C?

(5) 6 14 35 111 149 **(SBI ASS. Bank PO, '97)**
3 A B C D E
What will come for E?

(6) 434 353 417 368 404 379 **(SBI ASS Bank PO, '97)**
108 A B C D E
What will come for D?

(7) 5 6 16 57 244 1245 **(RBI Grade-B, '02)**
2 A B C D E
What will come for D?

(8) 3 5 9 17 33 65 **(RBI Grade-B, '02)**
7 A B C D E
What will come for C?

(9) 7 4 5 9 20 52.5 **(RBI Grade-B, '02)**
3 A B C D E
What will come for C?

Hint: $\times 0.5 + 0.5$, $\times 1 + 1$, $\times 1.5 + 1.5$ . . .

(10) 3 10 32 111 460 2315 **(RBI Grade-B, '02)**
2 A B C D E
What will come for B?

Hint: $10 = 3 \times 1 + 7 \times 1$, $32 = 10 \times 2 + 6 \times 2$, $111 = 32 \times 3 + 5 \times 3$, $460 = 111 \times 4 + 4 \times 4$ . . . .

(11) 5 8 6 10 7 12 **(RBI Grade-B, '02)**
7 A B C D E
What will come for C?

Hint: $\times 2 - 2$, $\div 2 + 2$, $\times 2 - 2$, . . .

(12) – 4 2 10 96 6150 **(RBI Grade-B, '02)**
–10 A B C D E
What will come for D?

Hint: $\times 1^0 + 6$, $\times 2^1 + 6$, $\times 3^2 + 6$ . . .

(13) 4 6 15 79 704 8480 **(NABARD, '01)**
12 A B C D E
What will come for B?

(14) 200 184 193 157 182 118 167 **(NABARD '01)**
150 A B C D E
What will come for E?

Hint: -4^2, $+3^2$, -6^2, . . .

(15) 60 121 131 264 284 571 601 **(NABARD, '01)**
120 A B C D E
What will come for D?

(16) 0.25 1.25 –3 0 –64 **(NABARD, '01)**
45 A B C D E
What will come for C?

(17) 5 7 10 36 136 690 **(SBI PO, '01)**
2 A B C D E
What will come for D?

Hint: $\times 1 + 2$, $\times 2 - 4$, $\times 3 + 6$, . . .

(18)	8	9	13	12	8	9	
	12	A	B	C	D	E	

What will come for E?

(19)	3	20	118	587	2344		**(SBI PO, '01)**
	12	A	B	C	D	E	

What will come for C?

(20)	0	16	48	112	240		**(SBI PO, '01)**
	120	A	B	C	D	E	

What will come for D?

Hint: $(+ 2^1 \times 8)$, $(+ 2^2 \times 8)$ $(+ 2^3 \times 8)$ . . .

(21)	108	52	24	10	3		
	64	A	B	C	D	E	

What will come for D?

(22)	5	12	60	340			**(BSRB, Mumbai, '98)**
	7	A	B	C	D	E	

What will come for D?

Hint: $\times 8 - 28$, $\times 7 - 24$, $\times 6 - 20$ . . .

(23)	4	7	24	93			**(BSRB, Chennai, '98)**
	2	A	B	C	D	E	

What will come for D?

Answers

1. (a) 51, (b) 4443, (c) 90, (d) 400, (e) 196, (f) 20, (g) 343, (h) 347, (i) 188, (j) 35, (k) 27, (l) 32, (m) 229, (n) 108, (o) 71, (p) 6, (q) .87.

2. (a) 436, (b) 196, (c) 129, (d) 3, (e) 10.

3. 428 **4.** 162 **5.** 1889 **6.** 78 **7.** 172 **8.** 49 **9.** 6 **10.** 30 **11.** 14
12. –762 **13.** 23 **14.** 68 **15.** 524 **16.** 1611 **17.** 64 **18.** 13 **19.** 2477 **20.** 360
21. 0.25 **22.** 5044 **23.** 360

REAL PROBLEMS

Section A: Number System

(1) The greatest 5-digit number, that leaves a remainder of 19 if divided by 23 is:
(a) not possible (b) 99980 (c) 99982 (d) 99977 (e) 99962

Hint: Here the actual remainder is less than the desired remainder (= 19). So, it can not be found out by the conventional method of simply adding the desired remainder to the exactly divisible 5-digit greatest number. This is because conventional method is applicable only when actual remainder (= 18) is greater than the desired remainder.

(2) The four integers consecutively lower than 81, and the four consecutively higher than 81, are added together. This sum is divisible by:
(a) 7 (b) 9 (c) 11 (d) 13 (e) None of these

(3) In a question, divisor is $\frac{2}{3}$ of the dividend and twice the remainder. If the remainder is 5, then the dividend is **(SSC, '94)**
(a) 85 (b) 145 (c) 225 (d) 65 (e) None of these

(4) The number that is nearest to 2160 and exactly divisible by 52 is:
(a) 2132 (b) 2148 (c) 2184 (d) 2177 (e) None of these

Tips: Check if the remainder obtained is more or less than half the divisor, because nearest number is to be found out.

(5) What will be the remainder when $(16^{27} + 37)$ is divided by 17?
(a) 4 (b) 19 (c) 13 (d) 2 (e) 14

Hint: Refer the Remainder Rules

(6) What is the number of digits of the smallest number which when multiplied by 7 gives a result consisting entirely of nines?
(a) 3 (b) 6 (c) 5 (d) 4 (e) 7

(7) What will be the digit in the unit place in the product $(3807)^{194} \times (932)^{84}$?
(a) 9 (b) 1 (c) 2 (d) 4 (e) 7

Hint: Refer 1.11.1

(8) The numbers 1, 3, 5 . . . 25 are multiplied together. The number of zeroes at the right end of the product is:
(a) 22 (b) 8 (c) 13 (d) 6 (e) 0

(9) A number when divided by a divisor, leaves a remainder of 63. If the remainder is 55 when twice the number is divided by the same divisor, then the divisor is:
(a) 21 (b) 37 (c) 16 (d) 49 (e) None of these

Hint: Divisor must be always greater than the remainder. Use this concept. Do not go for calculation.

(10) The sum of two numbers is '*s*' and their quotient is $\frac{p}{q}$. The numbers are:

(a) $\frac{ps}{q}, \frac{qs}{p}$ (b) $\frac{s}{p}, \frac{s}{q}$ (c) $\frac{s-p}{q}, \frac{s-q}{p}$ (d) $\frac{ps}{p+q}, \frac{qs}{p+q}$ (e) None

(11) How many digits are required to number a book containing 200 pages?
(a) 200 (b) 600 (c) 492 (d) 372 (e) 250

(12) How many numbers between 101 and 300 are divisible by both 3 and 5?
(a) 107 (b) 20 (c) 127 (d) 14 (e) 34

(13) A number is multiplied by 5 and 25 is added to it. The result is divided by 5 and the original number is subtracted from the same. The remainder will be:
(a) 0 (b) 1 (c) 2 (d) 3 (e) 5

Tips: Do not waste your time by forming a linear equation. Rather use the concept of remainder

(14) $\left(\frac{1}{3} - \frac{1}{4}\right)$ is added to a number. From the sum so obtained, $\frac{1}{3}$ of $\frac{1}{4}$ is subtracted and the remainder is $\left(\frac{1}{3} + \frac{1}{4}\right)$. The number is:

(a) $\frac{1}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{3}{4}$

(15) A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?
(a) 9 (b) 3 (c) 0 (d) 4 (e) 1

(16) There is a number 8 * 20 which if multiplied by 6, the product is divisible by 8. The digit replacing * mark is:
(a) 4 (b) 1 (c) any digit in between 0 and 9 (d) 7 (e) 0

(17) If $B = 2 \times 4 \times 6 \ldots 98 \times 100$, then the number of zeroes at the end of B will be:
(a) 330 (b) 11 (c) 10 (d) 101 (e) 12

(18) In the product of $24 * \times 981 \times 79 \times 104$ if the digit in the unit place is 2, then what will come in place of the asterisk?
(a) 2 (b) 3 (c) 6 (d) 7 (e) either (a) or (d)

(19) A number consists of four digits having 8 in the unit's place. If the digit in the extreme left is shifted to the immediate right to the unit's place, keeping all other numbers as they are, the new number formed exceeds the original number by 1305. Find the original number. **(ASSL. Grade, '96)**
(a) 4358 (b) 2731 (c) 3478 (d) 3316 (e) 4387

(20) A 4-digit number divisible by 7 becomes divisible by 3 when 10 is added to it. The largest such number is:
(a) 9999 (b) 9996 (c) 9989 (d) 9987 (e) 9993

Answers

1. d)	**2.** (b)	**3.** (e)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (e)	**9.** (e)
10. (d)	**11.** (c)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (c)	**17.** (e)	**18.** (e)
19. (c)	**20.** (c)							

Section B: Number Series

(1) How many 7s are there in the following series that are not immediately followed by 3 but immediately preceeded by 8 **(ASM, '02)**
898762263269732872778737794
(a) 2 (b) 3 (c) 4 (d) 6 (e) 7

(2) If the given numbers are arranged in the descending order based on the sum of the digits of each number, which number will be in the middle?
842 641 961 479 715 216 523
(a) 961 (b) 216 (c) 479 (d) 715 (e) 523

(3) If by beginning with 1, consecutive numbers are continuously written to its right, then which digit will be written on thirty-first position?
(a) 1 (b) 2 (c) 3 (d) 0 (e) 4

(4) If the given numbers are arranged in such a way that each group of three assending numbers is followed by their LCM and the beginning number is 1, then 11th number is how many times of the fifth number?
1, 2, 3, 4, 5, 6, 5, 6, 6, 7, 60
(a) $\frac{3}{2}$ (b) 1.75 (c) 1.4 (d) $\frac{1}{3}$ (e) None of those

Directions (5–6): Study the following number series to answer these questions
2 6 7 5 4 3 7 4 8 9 4 3 2 5 4 7 9 8 6 8 7 1 2 5 3 7 6 8 9 3 6

(5) How many such numbers are there in the series which are not immediately followed by its multiple?
(a) 25 (b) 4 (c) 27 (d) 21 (e) 20

(6) If the order of last 15 numbers is reversed, which number will be eighth to the right of thirteenth number from left?
(a) 5 (b) 6 (c) 7 (d) 3 (e) 2

Directions (7–11): One number is wrong in the number series given in each of the following questions. You have to identify that number. Assuming that a new series starts with that number, and following the same logic as in the given series, which of the numbers given in (1), (2), (3), (4) and (5) given below the series will be the third number in the new series? **(SBI PO, '99)**

(7) 3 5 12 38 154 914 4634

(1) 1636 (2) 1222 (3) 1834 (4) 3312 (5) 1488

Hint: $\times 1 + 2, \times 2 + 2, \times 3 + 2 \ldots$ So, 914 is incorrect. The new series begins with 914 and third number in the new series will be 1834

(8) 3 4 10 34 136 685 1446

(1) 22 (2) 276 (3) 1374 (4) 72 (5) 12

(9) 214 18 162 62 143 90 106

(1) –34 (2) 110 (3) 38 (4) 10 (5) 91

Hint: $-(14)^2, +(12)^2, -(10)^2, +(8)^2, \ldots$ So, 143 is incorrect.

(10) 160 80 120 180 1050 4725 25987.5

(1) 60 (2) 135 (3) 3564 (4) 787.5 (5) 90

Hint: $\times\frac{1}{2}, \times\frac{3}{2}, \times\frac{5}{2}$ . . . Hence 180 is incorrect.

(11) 2 3 7 13 26 47 78

(1) 13 (2) 11 (3) 20 (4) 15 (5) 18

Directions (12–18): In each of the following questions, a number series is established if the positions of two out of the five marked numbers are interchanged. The position of the first unmarked number remains the same and it is the beginning of the series. The earlier of the two marked numbers whose positions are interchanged is the answer. For example, if an interchange of the number marked '1' and the number marked '4' is required to establish the series, your answer is 1. If it is not necessary to interchange the positions of the numbers to establish the series give '5' as your answer.

(12) 8	4	12	6	4	30
	(1)	(2)	(3)	(4)	(5)

Hint: $\times \frac{1}{2}, \times 1, \times 1\frac{1}{2}, \times 2 \ldots$ So, the numbers at (2) and (4) are to be interchanged

(13) 829	436	661	300	557	508
	(1)	(2)	(3)	(4)	(5)

Hint: $-23^2, +19^2, -15^2, +11^2 + \ldots$

(14) 6	56	1	19	11	529
	(1)	(2)	(3)	(4)	(5)

Hint: $\times 1^2 + 5, \times 1^2 - 10, \times 2^2 + 15 \ldots$

(15) 21	29	23	21	41	61
	(1)	(2)	(3)	(4)	(5)
(16) 4	0	–7	–45	–20	–94
	(1)	(2)	(3)	(4)	(5)
(17) 0	6	184	56	109	23
	(1)	(2)	(3)	(4)	(5)
(18) 375	363	356	344	336	324
	(1)	(2)	(3)	(4)	(5)

Directions (19–24): In each of the following questions a number series is given. A number in the series is replaced by a letter '*A*'. You have to find out the number that has been replaced by '*A*' and use this number to find out the value that should be in the place of the question mark in the equation following the series.

(19) 36 216 64.8 388.8 *A* 699.84 209.952 **(BSRB Mumbai PO, '99)**

$A \div 36 = ?$

(1) 61.39 (2) 0.324 (3) 3.24 (4) 6.139 (5) 32.4

(20) $\frac{3}{8}$ $\frac{3}{4}$ $\frac{9}{16}$ $\frac{9}{8}$ $\frac{27}{32}$ $\frac{27}{16}$ *A*. **(BSRB Mumbai PO, '99)**

$\sqrt{A} = ?$

(1) $\frac{3}{2}$ (2) $\frac{6}{8}$ (3) $\frac{6}{4}$ (4) $\frac{3}{4}$ (5) $\frac{9}{8}$

(21) 99 163 *A* 248 273 289

$\sqrt{2A+17} = ?$

(1) 20.5 (2) 21 (3) 20 (4) 20.7 (5) 19

(22) *A* 12 9 $7\frac{1}{5}$ 6 $5\frac{1}{7}$ **(BSRB Bangalore, 2000)**

18% of *A* + 24% of *A* = ?

(1) 7.56 (2) 8.20 (3) 9.42 (4) 6.38 (5) 10.64

(23) 125 *A* 1127 1176 9408 9472

$A^2 - 2A = ?$

(1) 23799 (2) 28063 (3) 25599 (4) 27850 (5) 18749

Hint: Series is $+ 6^2, \times 7, + 7^2, \times 8 \ldots$

(24) 14.8 17.2 *A* 22 2.8 41.2 **(BSRB Bangalore, 2000)**

25% of 25*A* = ?

(1) 77.5 (2) 73.5 (3) 172.5 (4) 86.5 (5) 92.8

Answers

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (3)	**8.** (4)	**9.** (5)
10. (2)	**11.** (2)	**12.** (2)	**13.** (1)	**14.** (1)	**15.** (1)	**16.** (3)	**17.** (2)	**18.** (5)
19. (3)	**20.** (5)	**21.** (2)	**22.** (1)	**23.** (3)	**24.** (1)			

2 HCF AND LCM OF NUMBERS

2.1 FACTORS

In a division, if a number f divides a number M completely (exactly) or in other words, if M is exactly divisible by f, then **'f' is the factor of M**.

Example: 5 divides 35 completely, so, 5 is a factor of 35.
Similarly, 2, 3, 4, 6 are all factors of 12, because each of the numbers 2, 3, 4, and 6 will divide 12 completely or, in other words 12 is divisible by 2, 3, 4 and 6.

2.2 MULTIPLES

From the above concept, if f is a factor of M, then M is a multiple of f.

Example: 63 is completely divisible by 7, 3, 9, 21. So, 63 is a multiple of 7 or 3 or 9 or 21.

2.3 PRINCIPLE OF PRIME FACTORISATION

Any natural number (>1) is either prime or non-prime (composite).

The principle of prime factorisation states:

Each non-prime (composite) number can be uniquely broken (reduced) into two or more prime numbers (prime factors). In other words, each non-prime number is divisible by any of the prime numbers.

With the use of this principle, a non-prime number is broken into its prime factor by dividing it with different prime numbers. This is known as division method of factorisation of a number. The same is explained in the following example.

Example: Resolve 20570 into its prime factors.

Division by prime number ↓			
	2	20570	
	5	10285	→ 1st Quotient
Prime	11	2057	→ 2nd Quotient
Factors	11	187	→ 3rd Quotient
	17	17	→ 4th Quotient
		1	

Thus, $20570 = 2 \times 5 \times 11 \times 11 \times 17$.
Hence, if the number is even, the division should start with 2; otherwise, rest of the prime numbers should be tried in succession.

2.4 HIGHEST COMMON FACTOR (HCF)

If two or more numbers are broken into their prime factors (as explained in 2.3), then the product of the maximum common prime factors in the given numbers is the H.C.F. of the numbers.

In other words, the HCF of two or more numbers is the greatest number (divisor) that divides all the given numbers exactly. So, HCF is also called the Greatest Common Divisor (GCD).

Example: Find the HCF of 72, 60, 96.
Here, we first find the prime factors of each given number.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

Here $\quad 72 = 2 \times 2 \times 2 \times 3 \times 3$

$\quad 60 = 2 \times 2 \times 3 \times 5$

$\quad 96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

and so $\quad$ HCF = product of maximum common prime factors $= 2 \times 2 \times 3 = 12$

Note: The common factors in the given numbers have been encircled.

2.5 LCM (LOWEST COMMON MULTIPLE)

The LCM of two or more than two numbers is the product of the highest powers of all the prime factors that occur in these numbers.

Example: Find the LCM of 36, 48, 64 and 72

$$\begin{array}{r|rrrr} 2 & 36, & 48, & 64, & 72 \\ \hline 2 & 18, & 24, & 32, & 36 \\ \hline 2 & 9, & 12, & 16, & 18 \\ \hline 2 & 9, & 6, & 8, & 9 \\ \hline 3 & 9, & 3, & 4, & 9 \\ \hline 3 & 3, & 1, & 4, & 3 \\ \hline & 1, & 1, & 4, & 1 \end{array}$$

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 576$.

2.6 PRODUCT OF TWO NUMBERS

HCF of numbers × LCM of numbers = Product of numbers.

i.e., if the numbers are A and B, then

$$\text{HCF of } A \text{ and } B \times \text{LCM of } A \text{ and } B = A \times B$$

2.7 DIFFERENCE BETWEEN HCF AND LCM

HCF of x, y and z	LCM of x, y and z
is the **Highest Divisor** which can exactly divide x, y and z.	is the **Least Dividend** which is exactly divisible by x, y and z.

Table 2.1 Rapid Information List

Ref. No.	*Type of Problem*	*Approach to Problem*
2.3	Find the *GREATEST NUMBER* that will *exactly* divide x, y and z.	Required number = *HCF* of x, y and z (greatest divisor)
2.4	Find the *GREATEST NUMBER* that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = *HCF* of $(x - a)$, $(y - b)$ and $(z - c)$
2.5	Find the *LEAST NUMBER* which is *exactly* divisible by x, y and z.	Required number = *LCM* of x, y and z (least dividend)
2.6	Find the *LEAST NUMBER* which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that $(x - a) = (y - b) = (z - c) = K$ (say). ∴ Required number = (*LCM* of x, y and z) − (K)
2.7	Find the *LEAST NUMBER* which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (*LCM* of x, y and z) + r.
2.8	Find the *GREATEST NUMBER* that will divide x, y and z leaving the same remainder in each case.	Required number = *HCF* of $(x - y)$, $(y - z)$ and $(z - x)$
2.9	Find the n-digit *GREATEST NUMBER* which when divided by x, y and z, (a) leaves no remainder (i.e., exactly divisible) (b) leaves remainder K in each case.	*LCM* of x, y and z = L (Step 1) L)*n-digit greatest number*((Step 2) *remainder* = R *By Rule I (Chapter I)*, (a) Required number = n-digit greatest number − R (b) Required number = [n-digit greatest number − R] + K

(*Contd.*)

(Contd.)

Ref. No.	*Type of Problem*	*Approach to Problem*
2.10	Find the n-digit *SMALLEST NUMBER* which when divided by x, y and z (*a*) leaves no remainder (i.e., exactly divisible) (*b*) leaves remainder K in each case.	*LCM* of x, y and $z = L$ (Step 1) L) n-digit greatest number ((Step 2) *remainder* $= R$ *By Rule II (Chapter I)* (a) Required number $= n$-digit smallest number $+ (L - R)$ (b) Required number $= n$-digit smallest number $+ (L - R) + K$
2.11	Find the *HCF* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*HCF* of fractions $= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$
2.12	Find the *LCM* of $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$	*LCM* of fractions $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$
2.13	Find the *HCF* of decimal numbers	**Step 1** Find the *HCF* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *HCF* of Step 1) from right to left according to the MAXIMUM decimal places among the given numbers.
2.14	Find the *LCM* of decimal numbers	**Step 1** Find the *LCM* of the given numbers **without** decimal. **Step 2** Put the decimal point (in the *LCM* of Step 1) from right to left at the place equal to the MINIMUM decimal places among the given numbers.

Solved Examples

E-1 Find the greatest number that will exactly divide 200 and 320.

S-1 Using the approach *2.3*,
required number = HCF of 200 and 320 = **40**.

E-2 Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.

S-2 Using the approach *2.4*,
required number = HCF of (148 − 4),
(246 − 6) and (623 − 11) i.e. HCF of 144, 240, 612 is **12**.

E-3 Find the least number which when divided by 27, 35, 45 and 49 leaves the remainder 6 in each case.

S-3 Using the approach *2.7*,
required number = (LCM of 27, 35, 45 and 49) + 6 i.e. 6615 + 6 = **6621.**

E-4 Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.

S-4 Using the approach *2.6*,

we get, $(36 - 25) = (48 - 37) = (64 - 53) = 11$

$\therefore$ required number = (LCM of 36, 48 and 64) − 11

= 576 − 11 = **565**.

E-5 Find the greatest possible length of a scale that can be used to measure exactly the following lengths of cloth; 3 m, 5 m, 10 cm and 12 m 90 cm.

S-5 The lengths of cloth to be measured are, 300 cm, 510 cm and 1290 cm.

$\therefore$ the required length of the scale is HCF of 300, 510 and 1290 i.e. 30

$\therefore$ the greatest possible length of the scale to be used = **30** cm.

E-6 Find the smallest number which when

(a) increased by 8 (or added by 8) ***(b)*** decreased by 8 (or subtracted by 8)

is exactly divisible by 15, 21, 30.

S-6 LCM of 15, 21, 30 = 210.

(a) the required number = LCM − (the number added) i.e. 210 − 8 = 202

(b) the required number = LCM + (the number subtracted) i.e. 210 + 8 = **218**.

E-7 Find

(a) the greatest number of 4 digits and ***(b)*** the smallest number of 4 digits

such that they are exactly divisible by 12, 15, 20 and 35.

S-7 ***(a)*** Using the approach *2.9(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 9999\ (\ 23 \\ \underline{9660} \\ 339 \end{array}$$

$\therefore$ required number = 9999 − 339 = **9663**.

(b) Using the approach *2.10(a)*,

Step 1 LCM of 12, 15, 20 and 35 = 420.

Step 2

$$\begin{array}{r} 420\ )\ 1000\ (\ 2 \\ \underline{840} \\ 160 \end{array}$$

$\therefore$ required number = 1000 + (420 − 160) = **1260**.

E-8 Four bells first begin to toll together and then at intervals of 6, 7, 8 and 9 seconds respectively. Find how many times the bells toll *together* in two hours and *at what interval they toll together* ?

S-8 LCM of 6, 7, 8 and 9 = 504.

$\therefore$ All the bells toll together after each interval of 504 seconds.

$\therefore$ in two hours, no. of times they toll together = $\dfrac{2 \times 60 \times 60}{504}$ times ≈ **14 times**.

E-9 Find the

(a) the greatest 3-digit number, and ***(b)*** the smallest 3-digit number

such that when they are divided by 12, 18, 21 and 28, it leaves a remainder 3 in each case.

S-9 ***(a)*** Using the approach *2.9(b)*,

Step 1 LCM of 12, 18, 21 and 28 = 252

Step 2 $\quad 252 \,)\, 9999 \,(\, 39$
$$\begin{array}{r} 9828 \\ \hline 171 \end{array}$$

$\therefore$ the required number = (9999 – 171) + 3 = **9931**.

(b) Using the approach *2.10(b)*,

Step 1 LCM of 12, 18, 21 and 28 = 252.

Step 2 $\quad 252 \,)\, 1000 \,(\, 3$
$$\begin{array}{r} 756 \\ \hline 244 \end{array}$$

$\therefore$ the required number = 1000 + (252 – 244) + 3 = **1011**.

E-10 Find the numbers between 200 and 300 such that when they are divided by 6, 8 or 9,

(a) it leaves no remainder, i.e. exactly divisible.

(b) it leaves in each case a remainder 5

S-10 Here, the number to be found out are the DIVIDENDS. Now, the LCM of 6, 8, 9 = 72.

(a) Multiples of 72 which lie between 200 and 300 are $72 \times 3 = 216$ and $72 \times 4 = 288$

(b) Here, the remainder is 5 in each case.

$\therefore$ Required numbers are (216 + 5) and (288 + 5) i.e. **221** and **293**.

E-11 There are two electrical wires, one is a 9 m 60 cm long aluminium wire and the other is a 5 m 12 cm long copper wire. Find the

(a) maximum length that can be equally cut from each wire in such a way that the total length of each wire is exactly divisible by it.

(b) how many such largest possible pieces are available in each kind of wire?

S-11 9 metre 60 cm = 960 cm and 5 metre 12 cm = 512 cm.

(a) The required largest piece = HCF of 960 and 512 cm, i.e. **64 cm.**

(b) $\therefore$ Number of such aluminium wire pieces = $\frac{960}{64}$ **nos.**

and number of such copper wire pieces = $\frac{512}{64}$ **nos.**

E-12 HCF and LCM of two numbers are 16 and 240 respectively. If one of the numbers is 48, find the other number.

S-12 We know that, HCF $\times$ LCM = Product of two numbers

$\therefore$ second number = $\frac{16 \times 240}{48}$ i.e. **80.**

E-13 Among how many students, 175 bananas and 105 oranges can be equally divided?

S-13 HCF of 175 and 105 = 35

$\therefore$ The required number of students is **35**, or factors of 35, namely 5 or 7.

E-14 Find out the HCF of 11, 0.121 and 0.1331.

S-14 *Step 1* HCF of 11, 121 and 1331 is **11**.

Step 2 Resultant HCF = **0.0011** (Since maximum decimal places = 4 in 0.1331).

E-15 Find out the LCM of 2.2, 540 and 1.08.

S-15 *Step 1* LCM of 22, 540 and 108 is **5940**.

Step 2 Here minimum decimal place = 1 (in 2.2)

So, resultant LCM = **594.**

E-16 Find out the HCF of 3^5 , 3^9 and 3^{14}.

S-16 Here the base of each number is same (= 3) but indices are different.

So, the required HCF = number with the minimum index, i.e. $\mathbf{3^5}$.

E-17 Find out the LCM of 4^5, 4^{-81}, 4^{12} and 4^7.

S-17 Here the base of each number is the same (= 4) but indices (or powers) are different.

So, the required LCM = number with the maximum index, i.e. $\mathbf{4^{12}}$.

REGULAR PROBLEMS

(1) What is the greatest possible length of scale to measure exactly the following lengths, 20 feet, 13 feet 9 inches, 17 feet 6 inches, 21 feet 3 inches?
(a) 1 feet 6 inches (b) 1 feet 3 inches (c) 9 inches
(e) 2 feet 4 inches (e) 6 inches

(2) The greatest number that will divide 410,751 and 1030 leaving a remainder 7 in each case is:
(a) 29 (b) 13 (c) 17 (d) 37 (e) 31

Hint: Since the number to be found out is a GREATEST DIVISOR, so HCF is to be found out

(3) The ratio of two numbers is 15 : 11. If their HCF is 13, then the numbers are:
(a) 75, 55 (b) 45, 22 (c) 104, 44 (d) 195, 143 (e) None

Hint: Since HCF is 13, so, the numbers will be $13 \times \underbrace{15 \text{ and } 13 \times 11}_{\text{terms of ratio}}$ (Delhi Metro Rail, 2002)

(4) The LCM of two numbers is 1296 and HCF is 96. If one of the numbers is 864, then the other is:
(a) 72 (b) 64 (c) 144 (d) 11664 (e) 36

(5) Three men start together to walk along a road at the same rate. The length of their strides are 68 cm, 51 cm and 85 cm respectively. How far will they go before they will be 'in step' again:
(a) 102 m (b) 1020 m (c) 10.2 m (d) 150 m (e) 17 cm

(6) How many times is the HCF of 48, 36, 72 and 24 contained in their LCM?
(a) 10 (b) 12 (c) 120 (d) 2 (e) 15

(7) The greatest 4-digit number exactly divisible by 88 is:
(a) 8888 (b) 9944 (c) 9988 (d) 9999 (e) 8899

(8) Find the least number of soldiers in a regiment, such that they stand in rows of 18, 15 and 25 and form a perfect square?
(a) 900 (b) 1600 (c) 2500 (d) 450 (e) 400

Hint: Find LCM and then multiply by the factors to make it a perfect square

(9) Three strings of a musical instrument vibrate 6, 8, and 12 times a second respectively. If all the three begin to vibrate simultaneously, find the shortest time interval before all three vibrate together again?
(a) 2 sec (b) 48 sec (c) $\frac{1}{2}$ sec (d) $\frac{1}{24}$ sec (e) 24 sec.

Hint: Time to vibrate once is $\frac{1}{6}$ sec, $\frac{1}{8}$ sec and $\frac{1}{12}$ sec.

(10) Which is the smallest number that can be subtracted from 1936 so that on being divided by 9, 10, 15 the remainder is 7 everytime? **(RRB Ajmer, '97)**
(a) 93 (b) 46 (c) 76 (d) 39 (e) 53

(11) The smallest number from which if 4000 subtracted, is exactly divisible by 7, 11 and 13, is
(a) 5001 (b) 2999 (c) 1000 (d) 6303 (e) 5101

Hint: Using the concept of LCM, we find, LCM of 7, 11, 13 = Required number – 4000

(12) Find the least number which when divided by 20, 25, 30, 36 and 48 leaves the remainders 15, 20, 25, 31 and 43 respectively.
(a) 2165 (b) 144 (c) 3595 (d) 3600 (e) 2875

(13) Find the H.C.F. of 2.4, 0.36 and 7.2 (RRB, Patna, 2002)
(a) 12 (b) 120 (c) 1.2 (d) 0.12 (e) 0.012

(14) Traffic lights at three different points are changing respectively at 24, 48 and 72 seconds. If all the three are changed together at 9 : 10 : 24 hours, then when will the next change take place together? **(RRB, Guwhati, '97)**
(a) 9 : 12 : 25 hrs (b) 9 : 10 : 48 hrs. (c) 9 : 12 : 48 hrs.
(d) 9 : 10 : 50 hrs (e) None

(15) The HCF of two numbers is 12 and their difference is also 12. The numbers are
(a) 12, 84 (b) 100, 112 (c) 40, 52 (d) 84, 96 (e) 120, 124

Hint: Do not try to calculate. Only check which of the given choices satisfy the given condition

(16) The sum of two numbers is 45 and their difference is $\frac{1}{9}$ of their sum. Their LCM is
(a) 200 (b) 100 (c) 90 (d) 180 (e) 250

(17) John has a camera that takes film that allows 24 exposures, whereas Nancy has a camera that takes film that allows 36 exposures. Both of them want to be able to take the same number of photographs and complete their rolls of film. How many rolls should each buy?
(a) 12 (b) 72 (c) 3 and 2 (d) 6 (e) 144

Answers

1. (b) **2.** (e) **3.** (d) **4.** (c) **5.** (c) **6.** (b) **7.** (b) **8.** (d) **9.** (c)
10. (d) **11.** (a) **12.** (c) **13.** (d) **14.** (c) **15.** (d) **16.** (b) **17.** (c)

REAL PROBLEMS

(1) The least number by which 825 must be multiplied in order to produce a multiple of 715 is **(Bank PO, '90)**
(a) 11 (b) 5 (c) 13 (d) 17 (e) 19

(2) The LCM of two number is 630 and their HCF is 9. If the sum of the numbers is 153, then the ratio of the two numbers is
(a) 70 (b) 9 (c) 0.7 (d) $\frac{10}{7}$ (e) (c) or (d)

(3) The HCF of two numbers each consisting of four digits is 103 their LCM is 19261, then the numbers are
(a) 1133, 1751 (b) 1621, 2031 (c) 3031, 3523 (d) 2979, 2277 (e) 1833, 1651

(4) The least number which is a multiple of 31 and when divided by 15, 24 and 32 leaves the remainders 2, 11 and 19 respectively is, **(RRB, Secunderabad, '01)**
(a) 2356 (b) 2387 (c) 2325 (d) 2418 (e) 2722

Hint: Here, the remainder in each case is less than the divisor by 13. The number can be 480 k – 13, where, the minimum value of k will make (480 k – 13) divisible by 31. Put k = 1, 2 . . . and check with the given choices.

(5) Three men start together to travel the same way around a circular track of 11 kms. Their speeds are 4, $5\frac{1}{2}$ and 8 kms per hour respectively. When will they meet at the starting point?

(a) 22 hrs. (b) 12 hrs. (c) 11 hrs. (d) 44 hrs. (e) 36 hrs.

(6) In a seminar, the number of participants in Physics, Chemistry and Mathematics are 96, 36 and 180 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being in the same subject.

(a) 12 (b) 21 (c) 36 (d) 26 (e) Not possible

(7) If the last divisor is 75 and the quotients are 3, 1, 1 and 3 respectively, in finding the HCF of two numbers by the method of division, then those two numbers are **(Mumbai Bank PO, '95)**

(a) 500, 1875 (b) 425, 1675 (c) 525, 1875 (d) 525, 1575 (e) 575, 1875

Hint: Assume two numbers as x and y. Follow the method of division to find HCF.

(8) Sum of two numbers is 56. If their LCM is 105, then the numbers are

(a) 7, 49 (b) 21, 34 (c) 24, 32 (d) 35, 21 (e) 27, 29

Tips: HCF of two numbers = HCF of (their sum and their LCM)

(9) The greatest number of three digits which when added to 45 is exactly divisible by 6, 8, 12 is **(RRB Kolkata Asst. Driver '01)**

(a) 963 (b) 987 (c) 984 (d) 980 (e) 1077

Hint: Find the greatest number which is less than 999 + 45 (= 1044) and divisible by the LCM of 6, 8 and 12 and then find the required number

(10) The sum of two numbers is pq and their difference is $\frac{1}{7}$ of their sum. Their HCF is

(a) $\frac{p+q}{pq}$ (b) $7\frac{(p-q)}{pq}$ (c) $\frac{12}{7}pq$ (d) $\frac{pq}{7}$ (e) $\frac{6}{7}pq$

(11) What is the smallest whole number that is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{21}$ and $3\frac{1}{7}$?

(a) 132 (b) 130 (c) 138 (d) 124 (e) 112

Hint: Required number = numerator of the LCM.

(12) A boy running up a stair case finds that when he goes up two steps at a time there is one step over; when he goes up three at a time there are two over and when he goes up four at a time, there are three over. Find the number of stairs, which is somewhere between 40 and 50. **(BSRB, '99)**

(a) 47 (b) 45 (c) 42 (d) 49 (e) None

Hint: If there were one more stair, the no. of stairs would have been exactly divisible by each of the numbers 2, 3 and 4 (i.e. no. of steps at a time). So, the remainder in each case is less than divisor by 1. The LCM of 2, 3 and 4 = 12

(13) A number when divided by 10 leaves a remainder 9, when divided by 9 leaves a remainder of 8, when divided by 8 leaves a remainder of 7 . . . . . . when divided by 2 leaves a remainder of 1. Determine the number. **(SSC, '96)**

(a) 31 (b) 1029 (c) 2519 (d) 1679 (e) 189

(14) When 1388, 3309 and 7151 are divided by a certain number of three digits, the remainders are the same. Find the remainder.

(a) 17 (b) 32 (c) 113 (d) 11 (e) 1921

(15) Four prime numbers are written in ascending order of their magnitudes. The product of first three is 715 and that of last three is 2431. What is the largest given prime number?

(a) 5 (b) 19 (c) 17 (d) 23 (e) 31

Hint: Let a, b, c and d be the prime numbers in ascending order.

$$a\boxed{bc} = 715 \quad \therefore \text{ HCF of 715 and 2431} = bc.$$

$$\boxed{bc}d = 2431 \quad \therefore d = \frac{bcd}{bc}$$

(16) The LCM of two numbers is 2900% more than their HCF and the sum of LCM and HCF of two numbers is 310. If one of the numbers is 20, find the other number. **(SSC, '93)**

(a) 290 (b) 150 (c) 75 (d) 300 (e) 58

Hint: Firstly find the LCM and HCF, by using two given conditions & then, solve.

(17) The LCM of three numbers is 4752 and HCF is 6. If the two numbers are 48 and 66, find the third least number

(a) 72 (b) 99 (c) 48 (d) 528 (e) 54

Hint: Let the third least number be $6x$, then

$$\begin{array}{r|lll} 6 & 48, & 66, & 6x \\ \hline & 8, & 11, & x \end{array}$$

$\Rightarrow$ LCM $= 6 \times 8 \times 11 \times x = 4752$ (given)

$\therefore$ $x = 9 \Rightarrow 6x = 6 \times 9 = 54$

(18) A man has certain number of small boxes to pack into parcels. If he packs 3, 4, 5 or 6 boxes in parcel, he is left with one over; if he packs 7 in a parcel, none is left over. What is the number of boxes he may have to pack?

(a) 301 (b) 400 (c) 309 (d) 405 (e) 105

Answers

1. (c)	**2.** (e)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (b)
10. (d)	**11.** (a)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (e)	**18.** (a)

3 FRACTION

3.1 DEFINITION

A number of the type $\frac{x}{y}$ which represents x number of parts out of y number of equal parts of a thing is called a **fraction**.

$\therefore$ Fraction $\frac{2}{7}$ represents 2 equal parts out of 7 equal parts of a thing. In the figure, the shaded part represents $\frac{2}{7}$

$\therefore$ FRACTION = $\frac{\text{Numerator}}{\text{Denominator}}$. Such a fraction is known as common fraction or vulgar fraction

- A fraction, whose denominator is 10 or 100 or 1000 etc. is called a **decimal fraction**.
- Fractions whose denominators are same, are called **like fractions**, e.g. $\frac{3}{8}, \frac{5}{8}$ are like fractions.
- Fractions whose denominators are different, are called **unlike fractions**, e.g. $\frac{3}{4}, \frac{9}{11}$ are unlike fractions.

3.1.1 Comparison of Fractions

Two or more different fractions can be compared with the help of the following rules:

Rule 1

When two fractions have the same denominator, the greater fraction is that which has the greater numerator.

Example: Thus, $\frac{5}{11}$ is greater than $\frac{3}{11}$

Rule 2

When two fractions have the same numerator, the greater fraction is that which has the smaller denominator.

Example: Thus, $\frac{7}{13}$ is greater than $\frac{7}{19}$

Rule 3

When two or more fractions with different denominators and different numerators are to be compared, then the following simple technique is to be used:

Step 1 Among all the given fractions,
let the maximum number of digits in the numerator = n
the maximum number of digits in the denominator = d

Step 2 Find $(d - n)$.

Step 3 If $(d - n) = 0$ or 1, multiply each fraction by 10.
If $(d - n) = 2, 3, 4 \ldots$ multiply each given fraction by $10^2, 10^3, 10^4 \ldots$ respectively.

Step 4 After multiplication, find only the integer value of the resultant fraction.

Step 5 If in step 4, any of the two fractions have the same integer value, then find the next decimal place and so on.

Step 6 Compare the integer/decimal values obtained in step 4 or step 5. The fraction having the maximum value is the greatest fraction.

Note: In order to write the given fraction in ascending order the smallest fraction is written first, then the next greater one and so on. In order to write the given fraction in descending order, the greatest fraction is written first, then the next smaller and so on.

Example: Arrange $\frac{7}{13}, \frac{493}{971}, \frac{87}{165}, \frac{123}{235}$ in descending order

Solution: Here, maximum no. of digits in the numerator $n = 3$ (in 493 or in 123) and maximum no. of digits in the denominator = $d = 3$ (in 971 or in 235)
Now, $d - n = 3 - 3 = 0$
So, multiply the given fractions by 10.

$\frac{7}{13} \times 10,\qquad \frac{493}{971} \times 10,\qquad \frac{87}{165} \times 10,\qquad \frac{123}{235} \times 10.$

5 5 5 5 (integer values)

Since the integer values are same, so, find the next decimal digit for these fractions,

$5.3 \qquad 5.0 \qquad \underbrace{\underline{5.2} \qquad \underline{5.2}}_{\text{same}}$

Since the value of two fractions are same, find the second decimal digit for these two fractions only, by dividing further

$5.3 \qquad 5.0 \qquad 5.27 \qquad 5.23.$

Now, $5.3 > 5.27 > 5.23 > 5.0$

$\Rightarrow \frac{7}{13} > \frac{87}{165} > \frac{123}{235} > \frac{493}{971}$ (in descending order)

3.2 FRACTIONAL PART OF A NUMBER

Fractional part of a number (or quantity) is simply the product of the related fraction and the given number.

Example: Consider a given number as 60, then

Two-thirds of 60 = $\frac{2}{3} \times 60 = 40$

$\therefore$ $\frac{2}{3}$rd of 60 is 40 (fractional part)

$$\boxed{\text{Fractional part of any number} = \text{number} \times \text{its related fraction.}} \quad \textbf{(1)}$$

3.2.1 Different Fractional Parts of the Same Number

Consider any number, say, 36

then $\frac{3}{4}$th of 36 = 27 (fractional part of 36)

$\frac{1}{9}$th of 36 = 4 (fractional part of 36)

$\frac{2}{3}$rd of 36 = 24 (fractional part of 36)

From this, we find that as the fraction changes, the fractional part of the same number also changes.

In our earlier examples, we find that

$$\frac{27}{3/4} = \frac{4}{1/9} = \frac{24}{2/3} \ldots = \underbrace{36}_{\text{Original number}} \quad \text{(Fixed)}$$

In such cases, equation (1) can be re-written as

$$\frac{\text{Any fractional number}}{\text{its related fraction}} = \text{Original } \underbrace{\text{number}}_{\text{it is fixed.}} \quad \textbf{(2)}$$

Example: A man travels $\frac{1}{4}$th part by scooter, $\frac{3}{8}$th by car and rest 48 km by bus. Find the total distance covered.

Solution: Here, total distance (i.e. original quantity) is to be found out.

$$\text{Fraction related to rest 48 km} = \underbrace{1}_{\text{related fraction to total distance}} - \left(\frac{1}{4} + \frac{3}{8}\right)$$

Using the relation (2)

$$\frac{\text{Any fractional number}}{\text{its related fraction}} = \text{Original number,}$$

Here, we find,

$$\frac{\text{Rest distance}}{\text{its related fraction}} = \text{total distance}$$

$$\Rightarrow \qquad \frac{48}{1-\left(\frac{1}{4}+\frac{3}{8}\right)} = \text{total distance}$$

$$\Rightarrow \qquad \text{total distance} = \frac{48}{3/8} = 128 \text{ km}$$

Example: If $\frac{3}{17}$th of a number is 18, then find its two-third.

Solution: Let $\frac{2}{3}$rd of number = x.

Since both the fractions $\frac{3}{17}$ and $\frac{2}{3}$ are to be found out for the same number, the relation (2) can be used as*

$$\frac{\text{Fractional number}}{\text{its related fraction}} = \frac{\text{Another fractional number}}{\text{its related fraction}} = \text{original number (always)}$$

$$\Rightarrow \qquad \frac{18}{3/17} = \frac{x}{2/3}$$

$$x = 68$$

$\therefore$ Two-third of the number is 68.

Note: This relation can be used to find another fractional part directly without finding the original number.

3.3 TO FIND THE FRACTION RELATED TO BALANCE (REST) AMOUNT

Conventionally, we have learnt that
Fraction related to balance (rest) part = 1 – (sum of all other fractions)
It is used when **all fractions are independent.** Following example will illustrate the fact.

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th part of his salary on education, $\frac{1}{4}$th part of his salary on clothing. He is now left with Rs. 550. Find his total salary.

Solution: Here, the spending on each item is independent, because each fraction has been indicated as out of total salary (original number).

$$\therefore \quad \text{fraction related to rest part} = 1 - \left(\frac{3}{8}+\frac{1}{12}+\frac{1}{4}\right)$$

$$= \frac{5}{24}$$

$$\therefore \qquad \text{total salary} = \frac{\text{Rest amount}}{\text{fraction related to rest part}} \qquad \textit{[Refer equation 2 of sec. 3.2.1]}$$

$$= \frac{550}{5/24} = \text{Rs } 2640.$$

Hence, for **independent fractions.**
Fraction for balance (rest) part = 1 – (sum of all independent fractions) **(3)**

Now, consider another example,

Example: A person spends $\frac{3}{8}$th part of his salary on food, $\frac{1}{12}$th of the rest part on education and $\frac{1}{4}$th of the remainder on clothing. He is now left with Rs 550. Find his total salary.

Solution: Here, spending on the second item (i.e. education) depends on the amount left after spending on the first item (i.e. food). Similarly, spending on the third item (i.e. clothing) depends on the amount left (remaining) after spending on the first item and the second item.

Here, spending on each item (except the first item) depends on the amount remaining, after spending on the previous item.

In such cases, all fractions (except the first one) are dependent on the previous fractions.

For **dependent fractions,**
Fraction for balance (rest) part = (1 – first fraction) × (1 – second fraction) **(4)**

So, in our example, using the relation 4

$$\text{fraction for balance (rest) part} = \left(1-\frac{3}{8}\right)\left(1-\frac{1}{12}\right)\left(1-\frac{1}{4}\right)$$

$$= \frac{5}{8} \times \frac{11}{12} \times \frac{3}{4}$$

$$= \frac{55}{128}$$

$$\therefore \quad \text{total salary} = \frac{\text{Rest amount}}{\text{Fraction related to rest part}}$$

$$= \frac{550}{55/128} = \text{Rs } 1280$$

Note: Observe the difference in the language of the two examples under 3.3

3.4 TO INSERT ANY NUMBER OF FRACTIONS IN BETWEEN TWO GIVEN FRACTIONS

Let two given fractions be $\frac{a}{b}$ and $\frac{x}{y}$. To insert a fraction lying between $\frac{a}{b}$ and $\frac{x}{y}$, the following steps are taken.

Step 1 The numerators of two given fractions are added to get the numerator of the **result** fraction, i.e. numerator of the result fraction = $a + x$

Step 2 The denominators are also added to get denominator of the **result** fraction. That is, denominator of the result fraction = $b + y$

Step 3 Result fraction = $\frac{a+x}{b+y}$

Hence, the result fraction so obtained has its magnitude (value) lying between the two given fractions. By this method, any number of fractions can be inserted between two given fractions.

Solved Examples

E-1 Arrange the following fractions in decreasing (descending) order:

(i) $\frac{5}{6}, \frac{3}{4}, \frac{5}{8}, \frac{6}{7}$ (ii) $\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{21}{50}$

(iii) $\frac{7}{12}, \frac{5}{16}, \frac{17}{36}, \frac{1}{3}$ (iv) $\frac{3}{5}, \frac{5}{7}, \frac{13}{16}, \frac{97}{104}$

(v) $\frac{2}{91}, \frac{5}{177}, \frac{22}{1091}, \frac{13}{558}$

S-1 Using the method 3.1.1

(i) Here $n = 1$ $d = 1$ $\therefore$ $d - n = 0$

So, multiply the given fractions by 10.

$\therefore$ $\frac{5}{6} \times 10 \approx 8$, $\frac{3}{4} \times 10 \approx 7$, $\frac{5}{8} \times 10 \approx 6$, $\frac{6}{7} \times 10 \approx 8$ (integer value)

Since two fractions have the same integer value (= 8), find the next decimal digit for these two fractions only namely

$\approx$ 8.3 7 6 8.5

Now, $8.5 > 8.3 > 7 > 6$ $\Rightarrow$ $\frac{6}{7} > \frac{5}{6} > \frac{3}{4} > \frac{5}{8}$ in descending order.

(ii) Here $n = 2$ $d = 2$ $\therefore d - n = 0.$

max^m no. of digits in numerator $= n = 2$ $\left(\text{in } \frac{21}{50}\right)$

max^m no. of digits in denominator $= d = 2$ $\left(\text{in } \frac{3}{10} \text{ or } \frac{21}{50}\right)$

So, multiply the given fraction by 10.

$\therefore$ $\frac{1}{2} \times 10 \approx 5$, $\frac{3}{5} \times 10 \approx 6$, $\frac{3}{10} \times 10 \approx 3$, $\frac{21}{50} \times 10 \approx 4$ (integer value)

Now, $6 > 5 > 4 > 3$ $\Rightarrow$ $\frac{3}{5} > \frac{1}{2} > \frac{21}{50} > \frac{3}{10}$

(iii) Here $n = 2$ $d = 2$ $\therefore$ $d - n = 0.$

So, multiply the given fraction by 10.

$\therefore$ $\frac{7}{12} \times 10 \approx 5$, $\frac{5}{16} \times 10 \approx 3$, $\frac{17}{36} \times 10 \approx 4$, $\frac{1}{3} \times 10 \approx 3$ (integer values)

Since the two fractions have the same integer value (= 3), find the next decimal place for these two fractions only.

i.e. $\approx$ 5 3.1 4 3.3

Now, $5 > 4 > 3.3 > 3.1$ $\Rightarrow$ $\frac{7}{12} > \frac{17}{36} > \frac{1}{3} > \frac{5}{16}$

(iv) Using the method 3.1.1, here, maximum number of digits in numerator = 2 $\left(\text{in } \frac{13}{16}\right)$.

Maximum number of digits in denominator = 3 $\left(\text{in } \frac{97}{104}\right)$.

So, $n = 2$ $d = 3$ $\therefore$ $d - n = 1$

So, multiply the numerator of the given fraction by 10.

$\therefore \frac{3}{5} \times 10 \approx 6, \frac{5}{7} \times 10 \approx 7, \frac{13}{16} \times 10 \approx 8, \frac{97}{104} \times 10 \approx 9$ (integer value)

Now, $9 > 8 > 7 > 6 \Rightarrow \frac{97}{104} > \frac{13}{16} > \frac{5}{7} > \frac{3}{5}$

(v) Here $n = 2$ (in 22 or in 13) $d = 4$ (in 1091) $\therefore$ $d - n = 2$

So, multiply the numerator of given fractions by 10^2

$\therefore \frac{2}{91} \times 100 \approx 2, \frac{5}{177} \times 100 \approx 2, \frac{22}{1091} \times 100 \approx 2, \frac{13}{558} \times 100 \approx 2$ (integer value)

All the fractions have the same integer value, so, find the next decimal place, i.e.

$\approx$ 2.1 2.8 2.0 2.3

Now, $2.8 > 2.3 > 2.1 > 2.0 \Rightarrow \frac{5}{177} > \frac{13}{558} > \frac{2}{91} > \frac{22}{1091}$

E-2 $\frac{5}{12}$ part of what amount will be equal to $3\frac{3}{4}$ part of Rs 100.

S-2 Let the amount be Rs x

$$\therefore \quad \frac{5}{12}x = 3\frac{3}{4} \times 100 \Rightarrow \frac{5}{12}x = \frac{15}{4} \times 100$$

$$\Rightarrow \quad x = \frac{12}{5} \times \frac{15}{4} \times 100 \Rightarrow x = \text{Rs } 900$$

$\therefore$ Required amount is Rs 900.

E-3 What fraction is 6 bananas in 5 dozens?

S-3 Required fraction = 6 out of 5 dozen

$$= \frac{6}{5 \times 12} = \frac{1}{10}.$$

E-4 There are 40 students in a class. One day only $\frac{7}{10}$ students were present. Find the number of absentees on that day.

S-4 In solving the problem on fraction, the whole quantity is *always* considered as 1.

$\therefore$ Number of absentees = Fraction of absentees $\times$ Total number

$$= \left(1 - \frac{7}{10}\right) \times 40 = 12 \text{ students.}$$

E-5 A man spent $\frac{2}{7}$ of his savings and still has Rs 1,000 left with him. What were his savings?

S-5 In this type of problem, if balance amount is given, then this amount is to be related to the balance part (fraction). Using relation 2, for savings.

$$\text{Savings} = \frac{\text{balance amount}}{\text{fraction related to balance part}}$$

$$\Rightarrow \quad \text{savings} = \frac{1000}{\left(1 - \frac{2}{7}\right)} = \text{Rs } 1400.$$

REGULAR PROBLEMS

(1) A badminton player, won 6 games and lost 4. The fraction of the games he won is:

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$ (e) $\frac{2}{5}$

(2) What fraction of 2 hours is 12 seconds?

(a) $\frac{1}{600}$ (b) $\frac{1}{12}$ (c) $\frac{1}{60}$ (d) $\frac{1}{5}$ (e) $\frac{3}{50}$

(3) A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m long can be cut from it?

(a) 16 (b) 21 (c) 13 (d) 11 (e) 17

(4) A lamp post has half of its length in mud, $\frac{1}{3}$ of its length in water and $3\frac{1}{3}$ m above the water. The total length of the post is:

(a) $4\frac{1}{6}$ m (b) $10\frac{1}{3}$ m (c) $16\frac{2}{3}$ m (d) 4 m (e) 20 m

(5) A man pays off $\frac{2}{5}$ of his debt and still has to pay Rs 240 to pay off the debt completely. The total amount of debt is:

(a) Rs 600 (b) Rs 400 (c) Rs 960 (d) Rs 480 (e) Rs 1200

(6) A drum of water is $\frac{3}{5}$ full. When 38 litres are drawn from it, it is just $\frac{1}{8}$ full. The half capacity of drum in litres is:

(a) 40 (b) 80 (c) 152 (d) 21.7 (e) 76

(7) The monthly salary of a man is Rs 480 and he spends $\frac{7}{8}$ of it. His income increases by $\frac{1}{6}$ of the present salary and his spending also increases by $\frac{2}{7}$ of the present expenditure. His savings will now

(a) increase by Rs 45 (b) decrease by Rs 40 (c) increase by Rs 40
(d) decrease by Rs 80 (e) decrease by Rs 60

(8) Which of the following fraction is the smallest?

(a) $\frac{7}{13}$ (b) $\frac{14}{33}$ (c) $\frac{11}{25}$ (d) $\frac{8}{15}$ (e) $\frac{9}{11}$

Hint: See 3.1.1

(9) Which of the following fraction is the greatest?

(a) $\frac{16}{21}$ (b) $\frac{11}{14}$ (c) $\frac{16}{19}$ (d) $\frac{16}{23}$ (e) $\frac{11}{17}$

(10) A man pays off $\frac{3}{20}$ of his debt every month. At the end of 6 months, his remaining debt is Rs A. How much amount has he cleared off in every month (in Rs)?

(a) $\frac{3A}{20}$ (b) $\frac{9A}{10}$ (c) $\frac{A}{10}$ (d) $\frac{3A}{10}$ (e) $\frac{3A}{2}$

(11) $\frac{3}{5}$ part of a kerosene tin is filled. If 6 bottles are taken out of it and 3 bottles are filled again, then half the tin is full. What is the capacity of the tin? (in bottles) **(RRB Guwahati, '97)**

(a) 20 (b) 30 (c) 45 (d) 50 (e) 40

(12) Reciprocal of sum of the reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ is:

(a) $\frac{1}{4}$ (b) $\frac{21}{44}$ (c) $\frac{4}{5}$ (d) $\frac{44}{21}$ (e) $\frac{15}{44}$

Hint: Start solving from backwards. First make reciprocals of $\frac{3}{5}$ and $\frac{7}{3}$ i.e. $\frac{5}{3}$ and $\frac{3}{7}$

Then sum it, as $\frac{5}{3}+\frac{3}{7}$, 8 then find reciprocal of the sum

(13) If the product of two numbers is 5 and one of the number is $\frac{3}{2}$, then what will be the sum of the numbers? **(RRB Trivendrum (Tech), '97)**

(a) $4\frac{1}{2}$ (b) $6\frac{1}{2}$ (c) $4\frac{5}{6}$ (d) 9 (e) $4\frac{2}{3}$

(14) In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By misake, he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. The given number is:

(a) 450 (b) 300 (c) 270 (d) 180 (e) 280

(15) One of the rational numbers between $\frac{2}{7}$ and $\frac{3}{14}$ is:

(a) $\frac{5}{14}$ (b) $\frac{3}{49}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (e) None

Hint: Refer

(16) Which of the following fractions is the greatest? **(RBI, '98)**

(a) $\frac{219}{337}$ (b) $\frac{221}{335}$ (c) $\frac{217}{339}$ (d) $\frac{215}{341}$ (e) $\frac{222}{339}$

Hint: Do not try to calculate. The greatest fraction can be found out by eliminating first the fractions with lower numerator and greater denominator, i.e, $\frac{215}{341}, \frac{217}{339}$, and $\frac{219}{337}$. Then compare $\frac{221}{335}$ and $\frac{222}{339}$

Answers

1. (c)	**2.** (a)	**3.** (e)	**4.** (e)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)
10. (e)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (e)	**15.** (c)	**16.** (b)		

REAL PROBLEMS

(1) One quarter of one-seventh of a land is sold for Rs 30000. What is the value of eight-thirty fifth of the land?
(a) Rs 192000 (b) Rs 212000 (c) Rs 27428 (d) Rs 36540 (e) Rs 150000

(2) It takes 40 days for a pond to get filled with rain water. If the level of water doubles each day, then how long would it take to fill $\frac{1}{4}$ of the pond?
(a) 10 days (b) 20 days (c) 30 days (d) 35 days (e) 38 days

(3) A post is divided into three parts, the first part is $\frac{1}{3}$ of the whole length, second $\frac{3}{8}$ of the first, and the third is 6 m 50 cm. The length of the post is:
(a) 15 m (b) 10 m (c) 12 m (d) $13\frac{1}{2}$ m (e) 18 m

(4) In a village, $\frac{5}{8}$ of the population are adults, $\frac{1}{2}$ of the adults are male, $\frac{4}{5}$ of adult females are illiterate. If 400 females are illiterate, then the population of the village is:
(a) 2000 (b) 1500 (c) 1800 (d) 1600 (e) 1200
Hint: Assume, population of village as x

(5) In a polling booth, total number of voters is 1575, of which 0.4 part are male voters. If a candidate gets 0.6 part of male voters and 0.4 part of female voters, then find how many votes did the candidate get?
(a) 189 (b) 756 (c) 378 (d) 630 (e) 945

(6) A man spends $\frac{1}{7}$ of his salary on food and $\frac{1}{2}$ of the remaining on clothing and $\frac{1}{3}$ of the remaining on entertainment. He is still left with Rs 600. How much does he spend on entertainment? **(BSRB, '95)**
(a) Rs 600 (b) Rs 450 (c) Rs 300 (d) Rs 700 (e) Rs 500
Hint: Refer E-6

(7) From a rope 30 metres long, a person cuts off as many pieces as possible, each $3\frac{1}{4}$ metres long. What fraction of the whole will be left?
(a) $\frac{1}{40}$ (b) $\frac{3}{4}$ (c) $\frac{8}{13}$ (d) $\frac{7}{13}$ (e) $\frac{13}{30}$

(8) A man left $\frac{1}{7}$ of his property to his daughter and the remaining to his sons to be equally divided among them. If the share of the each son be double of that of the daughter, find the number of sons. **(NABARD, '97)**

(a) 2 (b) 3 (c) 6 (d) 4 (e) 7

(9) A vessel, full of water, weighs 16.5 kg. When the vessel is $\frac{1}{4}$ full, it weighs 5.25 kg. The weight of the empty vessel (in kg) is:

(a) 1.125 (b) 4.5 (c) 1.5 (d) 3 (e) 2.5

(10) A scooter before overhauling requires $\frac{2}{3}$ hour service time every 45 days, while after overhauling it requires $\frac{2}{3}$ hour service time every 60 days. What fraction of pre-overhauling service time is saved in the latter case? **(MBA, '81)**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$ (e) $\frac{4}{9}$

(11) Sundari, Kusu and Jyoti took two tests each. Sundari secured $\frac{24}{60}$ marks in the first test and $\frac{32}{40}$ marks in the second test. Kusu secured $\frac{35}{70}$ marks in the first test and $\frac{54}{60}$ marks in the second test. Jyoti secured $\frac{27}{90}$ marks in the first test and $\frac{45}{50}$ marks in the second test. Who among them did register maximum progress? **(BSRB Bangalore, 2000)**

(a) Only Sundari (b) Only Kusu (c) Only Jyoti
(d) Both Sundari and Kusu (e) Both Kusu and Jyoti

Hint: Tabulate the score in each test with **common denominator** so that the progress for each person in second test over the first test can be found & compared.

$$\begin{array}{llcccccl} & & \text{I} & & \text{II} & & & \\ \text{Sundari} & \rightarrow & \frac{4}{10} & \rightarrow & \frac{8}{10} & \rightarrow & \text{2 times} & \because \frac{8}{4} = 2 \\ \text{Kusu} & \rightarrow & \frac{5}{10} & \rightarrow & \frac{9}{10} & \rightarrow & \text{Less than 2 times} & \because \frac{9}{5} < 2 \\ \text{Jyoti} & \rightarrow & \multicolumn{3}{c}{\boxed{\frac{3}{10} \rightarrow \frac{9}{10}}} & \rightarrow & \multicolumn{2}{l}{\text{Maximum progress, as in II test, score is 3 times the I test in terms of fraction}} \end{array}$$

(12) A boy on being asked $\frac{13}{14}$ of a certain fraction had made the mistake of dividing the fraction by $\frac{13}{14}$ and so got an answer that exceeded the correct answer by $\frac{3}{65}$. The correct is:

(a) $\frac{14}{45}$ (b) $\frac{12}{65}$ (c) $\frac{13}{45}$ (d) $\frac{2}{7}$ (e) $\frac{196}{585}$

(13) A has twice as much money as B. They play together, and at the end of the first game, B wins one third of A's money from A; what fraction of the sum that B now has, must A win back in the second game so that they may have exactly equal money?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{10}$ (e) $\frac{5}{18}$

Hint: Assume, before the start of first game, B has Rs 1 and A has Rs 2

(14) I bought a number of mangoes at 35 for 2. I divided the whole into two equal parts, one part of which I sold at 17, and the other at 18 mangoes per Rs 1. I spent and received an integral number of rupees, but bought the least possible number of mangoes. How many did I buy?

(a) 21420 (b) 24120 (c) 22014 (d) 1225 (e) 612

Hint: Assume that I buy 35 mangoes. Then on selling, I get $\left(\underbrace{\frac{35}{2}}_{\text{No. of mangoes}} \times \underbrace{\frac{1}{17}}_{\text{Unit price}} + \frac{35}{2} \times \underbrace{\frac{1}{18}}_{\text{Unit price}}\right) = \text{Rs } \frac{1225}{612}$

But the number of rupees is an integer.

so, I must receive 612 times $\frac{1225}{612}$

Hence no. of mangoes I buy is 612 times 35

(15) Find out that minimum fraction which when added to $\frac{29}{12} + \frac{15}{16}$ will give a complete number.

(a) $\frac{21}{38}$ (b) $\frac{31}{38}$ (c) $\frac{31}{48}$ (d) $\frac{17}{48}$ (e) $\frac{23}{38}$

(16) In a class, 18 boys are there whose height is more than 160 cm. If they are three-fourth of the total number of boys and the total number of boys is two-third of the total number of students, then how many girls are there in the class?

(a) 18 (b) 6 (c) 12 (d) 24 (e) 8

(17) The fuel indicator in a car shows $\frac{1}{5}$th of the fuel tank as full. When 22 more litres of fuel are poured into the tank, the indicator rests at the three-fourth of the full mark. The capacity of the fuel tank (in litres) is:

(a) 30 (b) 40 (c) 36 (d) 28 (e) 45

Answers

1. (a) **2.** (e) **3.** (c) **4.** (d) **5.** (b) **6.** (c) **7.** (a) **8.** (b) **9.** (c)
10. (d) **11.** (c) **12.** (c) **13.** (d) **14.** (a) **15.** (c) **16.** (c) **17.** (b)

4 SIMPLIFICATION AND APPROXIMATION

4.1 OPERATION ORDER SEQUENCE

For simplifying an expression containing various types of fractions, the order of various operations involved should be strictly maintained. A simple technique for arranging the expression in the proper sequence, is by placing them in the order of the first letter appearing in **VBODMAS** where.

1. **V** Stands for vinculum or bar as ($\overline{\quad}$)
2. **B** stands for bracket and operation of brackets in the order (), {} and then []
3. **O** stands for 'of'
4. **D** stands for division (÷)
5. **M** stands for multiplication (×)
6. **A** stands for addition (+)
7. **S** stands for subtraction (–)

4.2 APPLICATION FOR ALGEBRAIC FORMULA

Some algebraic formulae are used in solving the problems on simplification. Following important formula are to be memorised:

1. $(a + b)^2 = a^2 + b^2 + 2ab.$
2. $(a - b)^2 = a^2 + b^2 - 2ab.$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2).$
4. $(a + b)^2 - (a - b)^2 = 4ab.$
5. $(a + b) \times (a - b) = a^2 - b^2.$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   $= a^3 + b^3 + 3ab\,(a + b).$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   $= a^3 - b^3 - 3ab\,(a - b).$
8. $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
10. $a^m \times a^n = a^{m+n}$
11. $a^m \div a^n = a^{m-n}$
12. $a^3 + b^3 + c^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$, then the above identity reduces to $a^3 + b^3 + c^3 = 3abc$

4.2.1 Square Root and Square

When a number is multiplied by itself, the product obtained is called the square of the number since $6 \times 6 = 36$.

$\therefore$ 36 is the square of 6 or $6^2 = 36$

Also, $-3 \times -3 = 9 \Rightarrow (-3)^2 = 9$

$\therefore$ 9 is the square of -3

The square root of a given number is equal to the value whose square is the given number and the sign for square root is '$\sqrt{}$'

Since $6^2 = 36$ then

$$\sqrt{36} = 6, \text{ i.e. square root of 36 is 6}$$

$$\sqrt{25} = 5, \text{ i.e. square root of 25 is 5}$$

$$\sqrt{-25} = \text{an imaginary quantity}$$

$\therefore$ Square root of a negative number is an imaginary quantity

1. $\sqrt{?} = y$

   Then the required number = y^2
2. $\sqrt{a^2 \times b^2} = ab$
3. $\sqrt{a^3 \times b^3} = ab\sqrt{ab}$
4. $\sqrt{a^4 \times b^4 \times c^4} = a^2b^2c^2$
5. $\sqrt{a^n \times b^m} = a^{n/2} \times b^{m/2}$

4.2.2 Division Method for Finding the Square Root

$$\sqrt{64009} = ?$$

Step 1 Pairing the digits from right to left, we get

$$6\ \overline{40}\ \overline{09}$$

Step 2 Then take the first pair, here it is only '6' and find the largest whole number whose square is equal to 6 or less than 6. Such a whole number is 2.

Step 3 Hence, write 2 in the quotient and also in the divisor. (see next page)

Step 4 Subtract $2 \times 2 = 4$ from 6. The remainder is then 2.

Step 5 Bring down the second pair of digits (i.e. 40) double the quotient (i.e. $2 \times 2 = 4$) and write the result on the left of 240 and then repeat Step-2 till the remainder is zero. The whole process can be enumerated step-by-step as shown in the following table.

2	6 $\overline{40}$ $\overline{09}$	253 = Quatient
+ 2	4	
45	2 $\overline{40}$	
+ 5	2 25	(since 45 × 5 = 225)
503	15 $\overline{09}$	(since 503 × 3 = 1509)
	15 09	
	0	

$\therefore \quad \sqrt{64009} = 253$ (i.e. Quotient)

4.2.3 Properties of a Perfect Square Number

A number whose exact square root (which must be a whole number can be obtained, is called a *perfect square*:

(a) A number ending with 2, 3, 7 or 8 cannot be a perfect square.
(b) The last digit of a perfect square must be 0, 1, 4, 5, 6 or 9.
(c) A number ending with odd number of zeroes cannot be a perfect square, e.g. 9000, 25000, 16000, etc. are not perfect squares.
(d) A perfect square number is either exactly divisible by 3 or leaves a remainder of 1, when divided by 3.
e.g. 64 if divided by 3, will leave a remainder of 1
36 is exactly divisible by 3.
(e) A perfect square number is either exactly divisible by 4 or leaves a remainder of 1, when divided by 4
e.g. 81 if divided by 4, will leave a remainder of 1.
100 is exactly divisible by 4.

Note: Above properties are very useful to check if a given number is a perfect square or not.

4.2.4 Square Root of Vulgar Fraction

$$\sqrt{\frac{3}{7}} = ?$$

Step 1 Multiply the numerator and the denominator by the donominator.

Step 2 Find the square root of the new numerator and divide it by the new denominator.

$$\therefore \quad \sqrt{\frac{3}{7}} = \sqrt{\frac{3}{7} \times \frac{7}{7}} = \frac{\sqrt{21}}{7} = \frac{4.582}{7} = 0.654$$

4.2.5 Square Root of a Rational Decimal Fraction

$\boldsymbol{\sqrt{387.09126} = ?}$

Step 1 Pair the integer part first

$\overline{3}\,\overline{87}$

Step 2 Check the number of decimal places.
If it is odd, then affix a zero on the extreme right of decimal part to make the even number of decimal places.
Here, no. of decimal places = 5, so, after placing a zero, it becomes .091260

Step 3 Pair the decimal part accordingly

$$.\overline{09}\,\overline{12}\,\overline{60}$$

Step 4 Start finding the square root by the division method as explained in 4.2.2 and put the decimal point in the square root as soon as the integer part is exhausted.

1	$\overline{3}\,\overline{87}\cdot\overline{09}\,\overline{12}\,\overline{60}$	19.674
+ 1	1	
29	2 87	
+ 9	261 (integer part is over)	since 29 × 9 = 261
386	26 09	
6	2316	since 386 × 6 = 2316
3927	293 12	
7	27489	since 3927 × 7 = 27489
39344	18236 60	
	157376	since 39344 × 4 = 157376

The square root of 387. 09126 = $\sqrt{387.09126}$
= 19.674

4.3 SIMPLIFICATION OF DECIMAL FRACTION

The number of digits which are present on the RIGHT OF A DECIMAL POINT is called the number of decimal places.

That is, 32.0075 has four digits on the right of the decimal point. Therefore, the number is expressed to four decimal places.

A WHOLE NUMBER can also be written as a decimal fraction by putting a decimal after its LAST DIGIT and adding as many zeros as are required.
e.g. 12 = 12.0 = 12.000 and so on.

4.3.1 Addition

For addition of a decimal number with another decimal number or with another whole number write the given number in such a way that the number of decimal places are equal for all the numbers.

e.g. 1 + 0.59 + 0.008

Here maximum number of decimal places = 3 (three) in 0.008.
Convert all the number so that they have 3 decimal places.
∴ 1 + 0.59 + 0.008 = 1.000 + 0.590 + 0.008 = 1.598

4.3.2 Subtraction

In subtraction also, the given numbers are to be wirtten in such a way that the number of decimal places become equal for all the numbers (empty places are filled up with zeroes).

e.g. 2 − 0.283
In 0.283, number of decimal places = 3
In 2, number of decimal places = 0
So, make 2 as having 3 decimal places, i.e. 2.000
$\therefore$ 2 − 0.283 = 2.000 − 0.283 = 1.717.

4.3.3 Multiplication

0.005 × 0.08 × 0.4= ?

Step 1 Multiply the number only, i.e. 5 × 8 × 4 = 160

Step 2 Add the total number of decimal places in the given number, i.e. 3 + 2 + 1 = 6

Step 3 Write the result of Step 1 and convert it to a number with decimal places as obtained in Step 2 by shifting the decimal point to the left.
i.e. by six decimal places, we then get
0.005 × 0.08 × 0.4 = 0.000160 = 0.00016
Similarly $\underbrace{0.03 \times 0.7 \times 2}_{\text{Total of 3 decimal places}} = 0.042$

4.3.4 Division of Decimals

(a) When the Divisor (or Denominator) is a Whole Number

e.g. $\frac{3.0056}{7}$

Step 1 Simply divide the number without considering the decimal points given i.e. 7) 30056 (4293.7

Step 2 Count the no. of decimal places in the given number. Here it has 4 decimal places in 3.0056.

Step 3 Shift the decimal point in the quotient obtined to the same no. of decimal places as in Step-2

Hence the result becomes. $\overset{\text{Shift}}{42937}$ = 0.42937

(b) When the Divisor (Denominator) is also a Decimal Number

e.g. $\frac{12.598}{27.08 \times 1.417}$

Step 1 Shift the decimal point to the right of the numerator and of the denominator such that
— total decimal point shift in numerator = total decimal point shift in denominator.
— there is no decimal place left after the shift.
Here, no. of decimal place in numerator (in 12.598) = 3
no. of decimal place in denominator (in 27.08 and 1.417) = 2 + 3 = 5
since 5 > 3, so, total shift in decimal point to be made (in numerator and denominator) = 5
Now, 5 decimal point shifts are made,

$$\frac{12\overbrace{59800}^{5\text{ shift}}}{27\underbrace{08}_{2\text{shift}} \times 1\underbrace{417}_{3\text{shift}}} = \frac{1259800}{2708 \times 1417}$$

Step 2 Division process is continued with the resulting fraction obtained in step 1.

4.4 SIMPLIFICATION OF A MIXED FRACTION

A mixed fraction consists of two parts, the integer part and the fractional part.

e.g. $2\frac{7}{18}$ has 2 as an integer and $\frac{7}{18}$ as a fraction.

In fact $2\frac{7}{18} = 2 + \frac{7}{18}$

4.4.1 Addition

$12\frac{5}{8} + 13\frac{7}{11} = ?$

Step 1 Add the integer part only i.e. $12 + 13 = 25$

Step 2 Add the fractional part only i.e. $\frac{5}{8} + \frac{7}{11} = 1\frac{23}{88}$

Step 3 Add the results obtained in Step 1 and Step 2

$$\therefore \quad 12\frac{5}{8} + 13\frac{7}{11} = 25 + 1\frac{23}{88} = 26\frac{23}{88}$$

4.4.2 Subtration

$10\frac{3}{7} - 18\frac{1}{6} = ?$

Step 1 Subtract the integer part only, i.e. $10 - 18 = -8$

Step 2 Subtract the fraction part only, i.e. $\frac{3}{7} - \frac{1}{6} = \frac{11}{42}$

Step 3 Add the result obtained in Step 1 and Step 2

Hence, $$10\frac{3}{7} - 18\frac{1}{6} = -8 + \frac{11}{42} = -8 + 1 - 1 + \frac{11}{42} = -(8-1) - \left(\frac{42-11}{42}\right) = -7 - \frac{31}{42}$$

$$= -\left(7 + \frac{31}{42}\right)$$ *[Please refer to Section 4.4.3]*

$$= -7\frac{31}{42}$$

Similarly $$12\frac{9}{11} - 15\frac{5}{8} = (12 - 15) + \left(\frac{9}{11} - \frac{5}{8}\right)$$

$$= -3 + \frac{17}{88} = -(3-1)\ \frac{88-17^{*}}{88} = -2\frac{71}{88}$$

*Explanation of in between steps of adding 1 & subtracting 1 have been explained in the previous problem, however, this step can be directly obtained after little practice.

(iii) $12\frac{5}{7} - 10\frac{2}{3} = (12 - 10) + \left(\frac{5}{7} - \frac{2}{3}\right) = 2 + \frac{1}{21} = 2\frac{1}{21}$

(iv) $5\frac{2}{3} - 2\frac{1}{7} + 6\frac{3}{8} = (5 - 2 + 6) + \left(\frac{2}{3} - \frac{1}{7} + \frac{3}{8}\right)$

$$= 9 + \left(\frac{112 - 24 + 63}{168}\right) = 9\frac{151}{168}$$

4.4.3 Subtraction of a Whole Number and Fraction

$5 - \frac{31}{48} = ?$

Step 1 Subtract 1 from the whold number i.e. + (5 – 1) = (4)

Step 2 Subtract the numerator from denominator and write in the numerator i.e. $\frac{31}{48} = \frac{48 - 31}{48} = \frac{17}{48}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $5 - \frac{31}{48} = 4\frac{17}{48}$

$\therefore \quad 12 - \frac{11}{52} = (12 - 1)\ \frac{52 - 11}{52} = 11\frac{41}{52}$

and $\quad -6 + \frac{23}{36} = -(6 - 1)\ \frac{36 - 23}{36} = -5\frac{13}{36}$

4.4.4 Easy Method For Simplification

5542 + ? + 1369 = 4200

Step 1 First always put 'x' for (?)

Step 2 Proceed and follow the rules to find the value for 'x' (or finding the value of?)

4.4.5 Multiplication of a Whole Number and a Fraction

$4 \times 16\frac{2}{3} = ?$

Step 1 Multiply the integer part by the whole number
i.e. $4 \times 16 = 64$

Step 2 Multiply the fraction part by the whole number i.e. $4 \times \frac{2}{3} = 2\frac{2}{3}$

Step 3 Add the results obtained in Step 1 and Step 2

i.e. $64 + 2\frac{2}{3} = 66\frac{2}{3}$

$\therefore \quad 4 \times 16\frac{2}{3} = 66\frac{2}{3}$

4.4.6 Division of Mixed Fraction by a Whole Number

$16\frac{2}{3} \div 4 = ?$

Step 1 Divide the integer part by the whole number,

i.e. $\frac{16}{4} = 4$

Step 2 Divide the fractional part by the whole number,

i.e. $\frac{2}{3} \div 4 = \frac{1}{6}$

Step 3 Add the results obtained Step 1 and Step 2,

i.e. $4 + \frac{1}{6} = 4\frac{1}{6}$

$\therefore \quad 16\frac{2}{3} \div 4 = 4\frac{1}{6}$

4.5 CONTINUED FRACTIONS AND ITS SIMPLIFICATION

Fractions of the form

(a) $7 + \cfrac{1}{4 + \cfrac{1}{5 + \cfrac{1}{3}}}$ or (b) $\cfrac{1}{2 - \cfrac{3}{8 + \cfrac{1}{4 - \cfrac{1}{5}}}}$

are called continued fractions

Simplification Rule

To simplify a continued fraction begin at the bottom and work upwards.

Example: Simplify $\cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{6}}}}$

$$= \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{\frac{7}{6}}}} = \cfrac{1}{3 + \cfrac{1}{5 + \frac{6}{7}}}$$

$$= \cfrac{1}{3 + \cfrac{1}{\frac{41}{7}}} = \cfrac{1}{3 + \frac{7}{41}}$$

$$= \frac{1}{\frac{130}{41}}$$

$$= \frac{41}{131}$$

4.6 RECURRING DECIMALS

A decimal fraction in which a digit or set of digits is repeated continually is called a Recurring or Periodic decimal.

e.g. $\frac{1}{3} = 0.333...$

Here, on performing the division, it is found that the remainder is always 1 and in the quotient, the digit 3 is continually repeated. Hence it is written as $0.\dot{3}$, where the *dot* over 3 indicates that the 3 has to be continually repeated.

Similarly, $\frac{1}{7}$ is

7) 1.000000 (0.142857 142857...

So, if we continue the division, we shall get the same set of figures 142857 again and again and in the same order.

Therefore $\frac{1}{7} = 0.\dot{1}4285\dot{7}$ or $0.\overline{142857}$

The repeated digits or repeated set of digits is called the period of the recurring decimal.
There are two types of Recurring Decimals,

(a) *Pure recurring decimal:* Such a decimal in which all the decimal digits recur, e.g. $0.\dot{1}4285\dot{7}$

(b) *Mixed recurring decimal:* Such a decimal in which all the decimal digits do not recur, e.g. $0.71\dot{6}\dot{7}$

4.6.1 Conversion of a Pure Recurring Decimal to the form $\frac{p}{q}$

Steps (a) Write the decimal part without the decimal point as the numerator.
(b) Write as many 9s as there are different repeating digits for the denominator.

e.g. $0.\overline{587} = \frac{587}{999}$, $\underset{\downarrow}{3}.\overline{17} = 3 + \frac{17}{99}$

(remains unchanged as it is integer part)

4.6.2 Conversion of a Mixed Recurring Decimal to the form $\frac{p}{q}$

Step (1) First, write the decimal part without the decimal point and subtract the non-repeating part from it and write the result in the numerator

Step (2) Write a number in the denominator with as many 9s as there are repeating digits in the decimal part and followed by as many zeroes as there are non-repeating digits in the decimal part.

Example: Express $7.00\dot{8}\dot{1}$ as vulgar fraction $\left(\frac{p}{q}\text{ form}\right)$

Solution $7.04\dot{8}\dot{1}$

$$= 7\frac{481-04}{9900}$$

$$= 7\frac{477}{9900}$$

4.7 IMPORTANT DERIVATIONS

(i) $\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{+m/n}$

(ii) $\left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{+m/n}$

(iii) If $a \div b$, then $a = c \times b$ and $a \div c = b$

(iv) $N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2$

4.8 APPROXIMATE VALUE

In this type of questions, candidates do not have to find out the exact value, but all they have to do is

Step 1 To round off the numbers given in the question

Step 2 To simplify

Step 3 To round off the result obtained in Step-2

Very Important: In some of the questions, the choices given are very close to each other. In such case, Step-1 is to be avoided, and we should go directly to Step-2.

4.8.1 Rounding Off Numbers

On some occasions for ease in simplification, we require only a rough estimation and not the exact value. In such cases we round off the values to the nearest tens, or hundreds or thousands.

Rounding off a number to the nearest ten, hundred or thousand means finding the multiple of 10, 100 or 1000 which is closest to (or approximate) the original number. It can be done by the following procedure.

(a) rounding off to the nearest 10: Replace the digit at unit's place by 0. If the replaced digit is 5 or more, then add 1 to the digit at tens place, otherwise digit at tens place remains unchanged.

e.g. $47 \xrightarrow[\text{off}]{\text{rounded}} 50$

↓

>5, so 1 is added to digit at tens place i.e. 1 + 4 = 5 (digit at tens place after rounding off)

But, $92 \longrightarrow 90$

↓

less than 5, so, 9 (digit at tens place) remains unchanged.

$75 \longrightarrow 80$

↓

equal to 5, so 1 is added to 7 i.e. 1 + 7 = 8 becomes the ten's place digit

$295 \longrightarrow 300$

(b) rounding off to the nearest 100: Replace the digit at unit and tens places by 00. If the replaced digit at **tens place** is 5 or more, then add 1 to the digit at hundreds place, otherwise the digit at hundreds place remains unchanged.

e.g. $264 \xrightarrow{+1} 300$

↓

>5, so 1 is added to digit at hundreds place i.e. 1 + 2 = 3

$5660 \longrightarrow 5700$, because 660 is rounded off as 700.

$841 \longrightarrow 800$

(c) rounding off to the nearest 1000: Replace the ones, tens and hundreds digits by 000. If the replaced digit at hundreds place is 5 or more, then add 1 to the digit at thousands place, otherwise not.

$1973 \longrightarrow 2000.$

NB: Whether a given number is to be rounded off to the nearest 10, or 100 or 1000, it depends on the other numbers involved in the simplification. It will be explained in the examples provided subsequently.

4.8.2 Rounding off a Number to a Decimal Place

To round off a number to the rth decimal place; following steps are to be checked.

Step 1 Check the digit immediately, next right to the rth place.

Step 2 If the next right digit is 5 or more, then add 1 to the digit in the rth place, otherwise the digit remains unchanged.

Step 3 Delete all the digits in places to the right of the rth place.

e.g. $5.792 \xrightarrow[\text{to 2nd place}]{\text{rounded off}} 5.79$

$5.795 \longrightarrow 5.8$

Example: What approximate value should come in place of (?) in the following equation

$9876 \div 24.96 + 215.005 - ? = 309.85$

Solution: $9876 \div 24.96 + 215.005 - ? = 309.85$

Put x in place of ?, then approximating the terms to the nearest values,

$9900 \div 25 + 215 - x = 310$

$$\Rightarrow \quad x = \frac{9900}{25} + 215 - 310$$

$\Rightarrow \quad x = 396 + 215 - 310$
$= 301 \approx 300$

Therefore the approximate value in place of (?) is 300.

Example: What approximate value should come in place of (?) in the following equation.

96895 + 589 + 22497 = ?

(a) 120000 (b) 125000 (c) 122000 (d) 99000 (e) 130000.

Solution: Here, out of the 5 choices, the values in three choices are very close to each other. Now, put (x) in place of (?) and after approximating, we get

$$x = 96895 + 589 + 22497$$
$$\approx 96900 + 600 + 22500$$
$$= 120000$$

Hence answer is (a)

Solved Examples

E-1 Simplify

$$10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6-4})\}\right]$$

S-1
$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - 2)\}\right] \qquad \overline{6-4} = 2\ (V)$$

$$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - 5\}\right] \qquad (7 - 2) = 5\ (B)$$

$$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right] \qquad \{6 - 5\} = 1\ (B)$$

$$= 10\frac{1}{2} - 9\frac{1}{2} = \mathbf{1}. \qquad \left[8\frac{1}{2} + 1\right] = 9\frac{1}{2}\ (B)$$

E-2 $0.75 \times 0.75 + 0.25 \times 0.75 \times 2 + 0.25 \times 0.25$

S-2 Let $0.75 = a$ and $0.25 = b$

By 4.2 (1), we have $a^2 + 2ab + b^2 = (a + b)^2$

$= (0.75 + 0.25)^2 = \mathbf{1}.$

E-3 Simplify $\dfrac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - (0.87 \times 0.13)}$

S-3 Let $0.87 = a$ and $0.13 = b$

$$\therefore \quad \frac{a^3 + b^3}{a^2 + b^2 - ab} = \frac{(a+b)(a^2 - ab + b^2)}{(a^2 + b^2 - ab)} = a + b \qquad \textit{[Refer 4.2]}$$

$$= 0.87 + 0.13 = \mathbf{1}.$$

E-4 Find the missing number.

$$\frac{9840}{\sqrt{?}} = 410$$

S-4 $\therefore$ Required Number $= \left(\dfrac{9840}{410}\right)^2$ *[Refer 4.2.1]*

$= (24)^2 =$ **576.**

E-5 Simplify $(40^2 - 30^2) = 10 \times ?$

S-5 Let $40 = a$, $30 = b$, required number $= x$

$a^2 - b^2 = (a + b)(a - b)$

$\therefore$ Required Number $(x) = \dfrac{40^2 - 30^2}{10} = \dfrac{(40+30)(40-30)}{10} =$ **70.**

E-6 $3^4 \times 3^6 \div 3^9 = ?$

S-6 here $a = 3$ and the base is the same. Now, on multiplication, the exponents are added, and on division the difference of the exponents are taken. Now applying 4.2 (10) and (11), we get the required number $(x) = 3^{(4+6-9)} = 3$

E-7 $2 + \sqrt{2} + \dfrac{1}{2+\sqrt{2}} + \dfrac{1}{\sqrt{2}-2} = ?$ **(AGE '93)**

S-7 $2 + \sqrt{2} + \left[\dfrac{\sqrt{2}-2+2+\sqrt{2}}{(2+\sqrt{2})(\sqrt{2}-2)}\right]$ [Since $a = \sqrt{2}$ $b = 2$

$= 2 + \sqrt{2} + \dfrac{2\sqrt{2}}{2-4} \Rightarrow 2 + \sqrt{2} + \dfrac{2\sqrt{2}}{-2} =$ **2.** $\therefore (a+b)(a-b) = a^2 - b^2$]

E-8 If $x * y = (x+2)^2 (y-2)$ then $7 * 5 = ?$

S-8 Substituting $x = 7$ and $y = 2$, we get,

$7 * 5 = (7+2)^2 (5-2)$

$= (9)^2 \times 3 =$ **243.**

E-9 If m and n are whole numbers such that $m^n = 121$, then $(m-1)^{n+1} = ?$

S-9 Given that $m^n = 121 \Rightarrow m^n = (11)^2$

Hence $m = 11$ and $n = 2$, and substituting these values,

$(m-1)^{n+1} = (11-1)^{2+1} = 10^3 =$ **1000.**

E-10 If $\dfrac{x}{y} = \dfrac{3}{4}$, then $\dfrac{6}{7} + \dfrac{(y-x)}{(y+x)} = ?$

S-10 Substruting $x = 3k$

$y = 4k$, we get,

$\dfrac{6}{7} + \dfrac{(y-x)}{(y+x)} = \dfrac{6}{7} + \dfrac{4k-3k}{4k+3k}$

$= \dfrac{6}{7} + \dfrac{1}{7} =$ **1.**

E-11 If $\sqrt{x} - \sqrt{y} = 1$ and $\sqrt{x} + \sqrt{y} = 17$, then $\sqrt{xy} = ?$

S-11 $\sqrt{x} + \sqrt{y} = 17$ **(i)**

and, $\sqrt{x} - \sqrt{y} = 1$ **(ii)**

Adding equations (i) and (ii) $\sqrt{x} = 9$

Subtracting equation (ii) from (i), $\sqrt{y} = 8$

Substituting these values,

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y} = 9 \times 8 = \mathbf{72}.$$

E-12 $\dfrac{? \div 12}{0.2 \times 3.6} = 2$

S-12 Putting x in place of ?, we get

$$\frac{x \div 12}{0.2 \times 3.6} = 2$$

$$\Rightarrow \quad x \div 12 = 2 \times 0.2 \times 3.6 \Rightarrow \frac{x}{12} = 2 \times 0.2 \times 3.6$$

$$\Rightarrow \quad x = 12 \times 2 \times 0.2 \times 3.6 = \mathbf{17.28}.$$

E-13 $\sqrt{? \times 7} \times 18 = 84$

S-13 Subsituting x for ?, we get,

$$\sqrt{x \times 7} \times 18 = 84$$

$$\Rightarrow \quad \sqrt{x \times 7} = \frac{84}{18}$$

$$\Rightarrow \quad \left(\sqrt{x \times 7}\right)^2 = \left(\frac{84}{18}\right)^2$$ (squaring both sides)

$$\Rightarrow \quad x \times 7 = \left(\frac{84}{18}\right)^2 \Rightarrow x = \frac{84 \times 84}{18 \times 18 \times 7} = \mathbf{3.11}.$$

E-14 $\left(2\dfrac{3}{x}\right) \times \left(y\dfrac{1}{2}\right) = 7\dfrac{3}{4}$ **(MBA, '82)**

S-14 Taking the quotients 2, y and 7, we get

$2y = 7$, which gives the quotient as 3 $\left(\text{Since } y = \dfrac{7}{2} = 3\dfrac{1}{2}\right)$

$\therefore$ $y = 3$. Substituting the value of y, we get,

$$2\frac{3}{x} \times 3\frac{1}{2} = 7\frac{3}{4}$$

Now, $\dfrac{7\frac{3}{4}}{3\frac{1}{2}} = 2\frac{3}{x} \Rightarrow 2\frac{3}{14} = 2\frac{3}{x}$

$\therefore \quad x = \mathbf{14}$ and $y = \mathbf{3}$

E-15 $2^{2^?} = 256$ **(IA, '87)**

S-15 Putting x for ?, we get

$2^{2^x} = 256 = 2^8$

$\Rightarrow \quad 2^{2^x} = 2^{2^3}$

$\therefore \quad x = \mathbf{3}.$

E-16 $2^{x+13} = 4^{x+2}$ then $x = ?$

S-16 $2^{x+13} = (2^2)^{x+2}$

$\Rightarrow \quad 2^{x+13} = 2^{2x+4}$

$\Rightarrow \quad x + 13 = 2x + 4 \quad \Rightarrow \quad x = \mathbf{9}.$

E-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12} = ?$

S-17 $3\sqrt{27} - \sqrt{75} + \sqrt{12}$

$= 3\sqrt{3\times3\times3} - \sqrt{5\times5\times3} + \sqrt{2\times2\times3}$

$= 3 \times 3\sqrt{3} - 5\sqrt{3} + 2\sqrt{3}$

$= (9 - 5 + 2)\ \sqrt{3} = \mathbf{6\sqrt{3}}$

E-18 If $5\frac{3}{x} \times y\frac{1}{2} = 19$, then the value of $(x, y) = ?$

S-18 Equating the quotients of both sides,

$5y = 19$, we get the quotient as 3 $\left(\text{Since } \frac{19}{5} = 3\frac{4}{5}\right)$

$\therefore y = 3$

Hence, $5\frac{3}{x} \times 3\frac{1}{2} = 19$

$\Rightarrow \quad 5\frac{3}{x} = \dfrac{19}{\frac{7}{2}} \Rightarrow 5\frac{3}{x} = 5\frac{3}{7}$

$\therefore \quad x = 7$ so $x = \mathbf{7}$ and $y = \mathbf{3}.$

E-19 $8\frac{1}{4} + 8\frac{1}{2} + ? = 20\frac{1}{8}$

S-19 Substituting x for ?, we get,

$8\frac{1}{4} + 8\frac{1}{2} + x = 20\frac{1}{8}$

$\Rightarrow\ x = 20\frac{1}{8} - 8\frac{1}{4} - 8\frac{1}{2}$

$= (20 - 8 - 8) + \left(\frac{1}{8} - \frac{1}{4} - \frac{1}{2}\right)$ *[Refer 4.4.2]*

$= 4 + \left(\frac{1-2-4}{8}\right)$

$= 4 + \frac{-5}{8} = (4 - 1)\ \frac{8-5}{8}$ *[Refer 4.4.3]*

$= \mathbf{3\frac{3}{8}}$.

E-20 $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$

S-20 Putting x for (?), we get *[Refer 4.4.4]*

$\sqrt{1296} \times 2.25 = x^2$

$36 \times 2.25 = x^2 \Rightarrow x = \sqrt{36 \times 2.25} \Rightarrow x = 6 \times 1.5$ (Since $\sqrt{1296} = 36$)

$\therefore\ x = \mathbf{9}$.

E-21 65% of ? = 124.90 – 63.15

S-21 Putting x for (?),

$\frac{65}{100}$ of $x = 61.75$

$\Rightarrow x = \frac{61.75}{65} \times 100 \Rightarrow x = \mathbf{95}$.

E-22 If $\frac{a}{a+b} = \frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to? **(IA, '79)**

S-22 Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e. if $a = 17$, then $a + b = 23$

or $b = 6$

$\therefore\ a - b = 17 - 6 = 11$

hence $\frac{a+b}{a-b} = \mathbf{\frac{23}{11}}$.

E-23 $\frac{\sqrt{24} + \sqrt{216}}{\sqrt{96}} = ?$

S-23 Putting x for (?) and solving for x, we get

$x = \frac{2\sqrt{6} + 6\sqrt{6}}{4\sqrt{6}} \Rightarrow x = \frac{8\sqrt{6}}{4\sqrt{6}}$

$\therefore\ x = \mathbf{2}$.

E-24 Simplify $(x + y - z)^2 - (x - y + z)^2$

S-24 Using the formula *[Refer 4.2 v]*

$a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow \quad (x + y - z)^2 - (x - y + z)^2$

$= (x + y - z + x - y + z)(x + y - z - x + y - z) = 2x(2y - 2z)$

$= \mathbf{4xy - 4xz}$.

E-25 If $x = 5$ and $y = -2$, then what is the value of $(x - 2y)^{1/y}$?

S-25 $(x - 2y)^{1/y}$

$$= (5 + 4)^{1/-2} = (9)^{1/-2} = \frac{1}{(9)^{1/2}}$$

$$= \frac{1}{\sqrt{9}} = \frac{\mathbf{1}}{\mathbf{3}}.$$

E-26 $2\frac{2}{3} \div \frac{4}{5}$ of ? $= \frac{1}{6}$

S-26 Putting x for (?) and solving for x, we get,

$$\Rightarrow \quad \frac{8}{3} \div \frac{4}{5}x = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{3} = \frac{1}{6} \times \frac{4}{5}x \quad \Rightarrow \quad x = \frac{8 \times 6 \times 5}{3 \times 4}$$

$\Rightarrow \quad x = \mathbf{20}$.

E-27 Find the value of

(i) $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees. **(ii)** $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

S-27 **(i)** $\frac{5}{6}$ of $\frac{1}{20}$ of 24 rupees.

$= \frac{5}{6} \times \frac{1}{20} \times 24$ rupees $=$ **1 rupee**.

(ii) $\frac{4}{7}$ of 14 times of $2\frac{1}{4}$ kg.

$= \frac{4}{7} \times 14 \times \frac{9}{4}$ kg. $=$ **1.8 kg**.

E-28 $\left(11 \div 2\frac{1}{5}\right) \div \frac{11}{5}$ of $2\frac{1}{2} - 2$

S-28 Applying **VBODMAS** Rules, we get *[Refer 4.1]*

$$= \left(11 \times \frac{1}{2\frac{1}{5}}\right) \div \frac{11}{5} \text{ of } 2\frac{1}{2} - 2 \qquad [B]$$

$$= 5 \div \frac{11}{5} \times \frac{5}{2} - 2 \qquad [O]$$

$$= 5 \times \frac{5}{11} \times \frac{5}{2} - 2 = \frac{125}{22} - 2$$ **[D][M]**

$$= \mathbf{3\frac{15}{22}}.$$ **[S]**

E-29 $\dfrac{\frac{2}{3}+\frac{1}{5}-\frac{1}{10}}{\frac{4}{5}\times\frac{1}{8}+\frac{1}{2}}$

S-29 $= \dfrac{\frac{20+6-3}{30}}{\frac{1}{10}+\frac{1}{2}} = \dfrac{\frac{23}{30}}{\frac{1+5}{10}}$

$$= \frac{23}{30}\times\frac{10}{6} = \mathbf{1\frac{5}{18}}$$

E-30 $\left(\frac{?}{18}\right) \times \left(\frac{?}{162}\right) = 1$ **(BSRB, '92)**

S-30 Putting x for (?) and solving it for x, we get

$$\frac{x}{18}\times\frac{x}{162} = 1$$ *[Refer 4.4.2.]*

$\Rightarrow \quad x^2 = 18 \times 162$

$\Rightarrow \quad x^2 = 18 \times 18 \times 9 \Rightarrow x = 18 \times 3$

$\therefore \quad x = \mathbf{54}.$

E-31 $\dfrac{(0.55)^2+(0.07)^2+(0.027)^2}{(0.055)^2+(0.007)^2+(0.0027)^2} = ?$

S-31 Let $0.55 = a$, $0.07 = b$ and $0.027 = c$

Then, the given expression becomes

$$\frac{a^2+b^2+c^2}{(0.1\times a)^2+(0.1\times b)^2+(0.1\times c)^2} = \frac{[a^2+b^2+c^2]}{0.01[a^2+b^2+c^2]}$$

$$\frac{1}{0.01} = \mathbf{100}.$$

E-32 $\dfrac{137\times137\times137+133\times133\times133}{137\times137-137\times133+133\times133} = ?$

S-32 Let $137 = a$ and $133 = b$

Then, the given expression becomes

$$\frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a + b$$ *[Refer 4.2 (8)]*

Putting the value of a and b, we get

$= 137 + 133 =$ **270.**

E-33 $\dfrac{20.25 \times 2.80}{28.35}$

S-33 $\dfrac{20\frac{1}{4} \times 2.80}{28.35}$ $\qquad\left(\text{Since} \quad 20.25 = 20\frac{1}{4}\right)$

$$= \frac{\frac{81}{4} \times 2.8}{28.35} = \frac{81 \times 0.7}{28.35}$$

$$= \frac{56.70}{28.35} = \mathbf{2.}$$

E-34 $54 \div 66 \div 33 = ?$

S-34 $\dfrac{54}{66} \div 33$

$$= \frac{9}{11} \div 33 = \frac{9}{11} \times \frac{1}{33} = \mathbf{\frac{3}{121}}$$

E-35 $\dfrac{2?9}{4} = \dfrac{916}{16}$

S-35 Putting x for (?) we get

$$\frac{2x9}{4} = \frac{916}{16} \quad \Rightarrow \quad \frac{2x9}{4} = \frac{229}{4}$$

$\therefore \quad x = \mathbf{2}$

E-36 $\sqrt{\dfrac{?}{10}} = 0.011$

S-36 Putting x for (?) we get

$$\sqrt{\frac{x}{10}} = 0.011 \qquad \text{(Since } x \text{ is under square root)}$$

$$\frac{x}{10} = (0.011)^2 \qquad \text{(squaring both sides)}$$

$\Rightarrow \quad x = 10 \times 0.000121$

$\therefore \quad x = \mathbf{0.00121}.$

E-37 $\sqrt{\dfrac{67.6}{?}} = 0.26$ **(SBI, '80)**

S-37 Putting x for (?) we get

$$\sqrt{\frac{67.6}{x}} = 0.26$$

$\Rightarrow \quad \frac{67.6}{x} = (0.26)^2$ (squaring both sides)

$\Rightarrow \quad x = \frac{67.6}{0.0676} \Rightarrow x = \mathbf{1000}.$

E-38 $\sqrt{\frac{0.324}{10}} = ?$

S-38 Putting x for (?), we get

$$\sqrt{\frac{0.324}{10}} = x$$

Here x is not under square root, hence squaring is not done.

$\Rightarrow \quad \sqrt{0.0324} = x$

$\Rightarrow \quad x = \sqrt{\frac{324}{10,000}} \Rightarrow x = \sqrt{\frac{18 \times 18}{100 \times 100}}$

$\Rightarrow \quad x = \frac{18}{10} \Rightarrow x = \mathbf{0.18}.$

E-39 $\frac{(63+36)^2 + (63-36)^2}{63^2 + 36^2} = ?$ **(ITI, '83)**

S-39 Putting 63 = a and 36 = b in the given expression, we get

$\Rightarrow \quad x = \frac{(a+b)^2 + (a-b)^2}{a^2 + b^2}$

$\Rightarrow \quad x = \frac{2(a^2 + b^2)}{a^2 + b^2}$ (Since $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$)

$x = \mathbf{2}.$ *[Refer 4.2 (iii)]*

E-40 44.60 × 2.50 = ?

S-40 Putting x for (?), we get

$x = 44.60 \times 2.50 \Rightarrow x = 44.60 \times \frac{10}{4}$ $\left(\text{Since } 2.50 = \frac{10}{4}\right)$

$\Rightarrow \quad x = 11.15 \times 10$

$\Rightarrow \quad x = \mathbf{111.5}.$

E-41 $\frac{14 \times 14 - 46}{11 \times 6 - (4)^2} = ?$

S-41 Putting x for (?) and applying **VBODMAS** Rule,
We get,

$$x = \frac{196-46}{66-16} \Rightarrow x = \frac{150}{50} = \mathbf{3}.$$

E-42 2002 – 2002 ÷ 10.10 = ? **(IA, '80)**

S-42 Putting x for (?) and applying **VBODMAS** Rule, we get

$$x = 2002 - \frac{2002}{10.10}$$

$$\Rightarrow x = 2002 - 200 \Rightarrow x = \mathbf{1802}.$$

E-43 $\dfrac{18-3\times 4+2}{6\times 5-3\times 8} = ?$

S-43 Putting x for (?) and applying **VBODMAS** Rule, we get

$$\Rightarrow x = \frac{18-12+2}{30-24} \Rightarrow x = \frac{18+2-12}{30-24} \Rightarrow x = \frac{20-12}{30-24} \quad \text{[M][A]}$$

$$\Rightarrow x = \frac{8}{6} \quad \text{[S]}$$

$$\therefore x = \frac{\mathbf{4}}{\mathbf{3}}.$$

E-44 Express 5005 into its prime factors. **(SSC, '86)**

S-44 5005 = 5 × 1001
5 × 7 × 143 = **5 × 7 × 11 × 13.**

E-45 $(4^3)^4 \div (4^2)^3 \times (4^5)^0 = ?$ **(ITI, '84)**

S-45 Put x for (?) , Since all base are equal to 4, hence, put a = 4.

$$\Rightarrow x = (a^3)^4 \div (a^2)^3 \times (a^5)^0$$

$$\Rightarrow x = a^{12} \div a^6 \times 1 \Rightarrow x = a^{12-6} \Rightarrow x = a^6 \quad (\text{Since } (a^5)^0 = 1)$$

(Since $a^m \div a^n = a^{m-n}$) *[Refer 4.2 (9)]*

$x = \mathbf{4^6}$.

E-46 Find the value of x in the equation. **(SSC, '86)**

$$\sqrt{1+\frac{25}{144}} = 1+\frac{x}{12}$$

S-46 $\sqrt{\dfrac{144+25}{144}} = 1+\dfrac{x}{12}$

$$\Rightarrow \sqrt{\frac{169}{144}} = 1+\frac{x}{12} \Rightarrow \frac{13}{12} = 1+\frac{x}{12}$$

$$\Rightarrow x = \mathbf{1}.$$

E-47 If $\frac{b}{a}=\frac{1}{2}$, what is the value of the expression $\frac{a-b}{a+b}+\frac{2}{3}$

S-47 Given that $\frac{b}{a}=\frac{1}{2}$

i.e. if $b = 1$, then $a = 2$ (assume it, for convenience)

So, $\frac{a-b}{a+b}+\frac{2}{3} = \frac{2-1}{2+1}+\frac{2}{3} = \mathbf{1}.$

E-48 If $\sqrt{18\times 14\times x} = 84$, then x is equal to ? **(Auditors, '86)**

S-48 $\sqrt{18\times 14\times x} = 84$

Since x is under square root, so, squaring both sides we get

$18 \times 14 \times x = 84 \times 84 \Rightarrow x = \frac{84\times 84}{18\times 14}$

$\therefore \quad x = \mathbf{28}.$

E-49 $(243)^{0.8} \div (243)^{0.4} = ?$ **(BSRB, '88)**

S-49 Putting x for (?) and $243 = a$, we get, *[Refer 4.2 (ix)]*

$x = (a)^{0.8} \div (a)^{0.4} = (a)^{0.8-0.4}$

$\Rightarrow \quad x = a^{0.4}$

$x = (243)^{0.4}$

$x = (243)^{4/10} \Rightarrow x = (3^5)^{2/5} \Rightarrow x = 3^2$

$\Rightarrow \quad x = \mathbf{9}.$

E-50 $1.2 \times 1.2 + 0.8 \times 0.8 + 2.4 \times 0.8 = ?$

S-50 Putting x for (?) and $1.2 = a$, $0.8 = b$,
we get

$x = a^2 + b^2 + 2ab \Rightarrow x = (a + b)^2$

$\Rightarrow \quad x = (1.2 + 0.8)^2 \Rightarrow x = \mathbf{4}.$

E-51 $\dfrac{\frac{64}{121}-\frac{9}{64}}{\frac{8}{11}+\frac{3}{8}} = ?$ **(SBI, '86)**

S-51 $x = \frac{(64^2-9\times 121)}{121\times 64}\times\frac{8\times 11}{(8\times 8+3\times 11)}$

$\Rightarrow \quad x = \frac{(64^2-3\times 3\times 11\times 11)}{(11\times 11\times 8\times 8)}\times\frac{8\times 11}{(64+33)}$

$$\Rightarrow \quad x = \frac{(64+33)(64-33)}{11 \times 8} \times \frac{1}{(64+33)}$$

$$\Rightarrow \quad x = \mathbf{\frac{31}{88}}.$$

E-52 $\dfrac{0.1+0.75}{2.5+0.05} \div \left(0.125 + \dfrac{1}{4.8}\right) = ?$

S-52 Putting x for (?)

$$\Rightarrow \quad x = \frac{0.85}{2.55} \div \left(\frac{1}{8} + \frac{10}{48}\right) \quad \Rightarrow \quad x = \frac{1}{3} \div \left(\frac{16}{48}\right)$$

$$\Rightarrow \quad x = \mathbf{1}.$$

E-53 $\left(\dfrac{216}{1}\right)^{-2/3} \div \left(\dfrac{27}{1}\right)^{-4/3} = ?$ **(Bank PO, '79)**

S-53 Putting x for (?), we get

$$\Rightarrow \quad \left(\frac{216}{1}\right)^{-2/3} \div \left(\frac{27}{1}\right)^{-4/3} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad (6^3)^{2/3} \div (3^3)^{4/3} = x \qquad \Rightarrow \quad x = \frac{6^2}{3^4} \quad \Rightarrow \quad x = \mathbf{\frac{4}{9}}$$

E-54 $8^{5/3} \div (125)^{-2/3} = ?$

S-54 Putting x for (?)

$$(2^3)^{5/3} \times (125)^{2/3} = x \qquad (\text{Since } a^m \div b^{-n} = a^m \times b^n)$$

$$\Rightarrow \quad x = 2^5 \times (5^3)^{2/3} \quad \Rightarrow \quad x = 32 \times 25$$

$$\therefore \quad x = \mathbf{800}.$$

E-55 $\left(\dfrac{1}{6^{-2}}\right) \times (81)^{-3/4} = ?$

S-55 Putting x for (?) and solving

$$\frac{6^2}{1} \times \left(\frac{1}{81}\right)^{3/4} = x \qquad \left(\text{Since } \left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}\right)$$

$$\Rightarrow \quad x = \frac{6^2}{(3^4)^{3/4}} \quad \Rightarrow \quad x = \frac{6 \times 6}{3 \times 3 \times 3}$$

$$x = \mathbf{\frac{4}{3}}.$$

E-56 $\sqrt{147}+\sqrt{27} = ? \times \sqrt{3}$

S-56 Putting x for (?) and solving it for x,

$$\sqrt{3\times7\times7}+\sqrt{3\times3\times3} = x\times\sqrt{3}$$

$$\Rightarrow \quad x\times\sqrt{3} = 7\sqrt{3}+3\sqrt{3} \quad \Rightarrow \quad x\sqrt{3} = 10\sqrt{3}$$

$\therefore \quad x = \mathbf{10}.$

E-57 $\sqrt{98}-\sqrt{50} = ? \times \sqrt{2}$ **(Bank PO, '80)**

S-57 Putting x for (?) and solving it for x,

$$\sqrt{7\times7\times2}-\sqrt{5\times5\times2} = x\times\sqrt{2}$$

$$\Rightarrow \quad 7\sqrt{2}-5\sqrt{2} = x\times\sqrt{2}$$

$\therefore \quad x = \mathbf{2}.$

E-58 $\sqrt{\dfrac{9}{25}}+\sqrt{3\dfrac{1}{16}} = ?$

S-58 $\dfrac{3}{5}+\sqrt{\dfrac{49}{16}} = x \quad \Rightarrow \quad x = \dfrac{3}{5}+\dfrac{7}{4} \quad \Rightarrow \quad x = \mathbf{2\dfrac{7}{20}}$

E-59 $\sqrt{0.01+\sqrt{0.0064}} = ?$

S-59 Putting x for (?) and solving it for x,

$$\sqrt{0.01+0.08} = x \quad \Rightarrow \quad \sqrt{0.09} = x$$

$\therefore \quad x = \mathbf{0.3}$

E-60 $\sqrt{\dfrac{25.6}{36.1}} \div \sqrt{\dfrac{12.1}{81\times0.1}} = ?$

S-60 Putting x for (?), we get

$$\sqrt{\frac{256}{361}} \div \sqrt{\frac{121}{81}} = x \quad \Rightarrow \quad x = \frac{16}{19} \div \frac{11}{9}$$

$$\therefore \quad x = \frac{16\times9}{11\times19} = \mathbf{\frac{144}{209}}$$

E-61 Express the number 51 as the difference of squares of two numbers. **(Bank PO, '82)**

S-61 Using the formula

$$N = \left[\frac{N+1}{2}\right]^2 - \left[\frac{N-1}{2}\right]^2, \text{ where } N = \text{ original number}$$

Put $N = 51$

$$\Rightarrow \quad 51 = \left[\frac{51+1}{2}\right]^2 - \left[\frac{51-1}{2}\right]^2$$

$$\Rightarrow \quad \mathbf{51 = (26)^2 - (25)^2}.$$

E-62 Find the number whose seventh part multiplied by its eleventh part gives 1,232.

S-62 Let x be the number such that

$$\frac{x}{7} \times \frac{x}{11} = 1232$$

$\Rightarrow \quad x^2 = 7 \times 11 \times 1232 \quad \Rightarrow \quad x^2 = 7 \times 11 \times 7 \times 11 \times 4 \times 4$

$\Rightarrow \quad x = 7 \times 11 \times 4 \quad \Rightarrow \quad 308$

$\therefore$ the number is **308**.

E-63 Find the square root of $\dfrac{\left(3\frac{1}{4}\right)^4 - \left(4\frac{1}{3}\right)^4}{\left(3\frac{1}{4}\right)^2 - \left(4\frac{1}{3}\right)^2}$

S-63 Let $\left(3\frac{1}{4}\right)^2 = a$ and $\left(4\frac{1}{3}\right)^2 = b$, then

$$\text{Given expression} = \frac{a^2 - b^2}{a - b} = a + b$$

$$= \left(3\frac{1}{4}\right)^2 + \left(4\frac{1}{3}\right)^2 = \left(\frac{13}{4}\right)^2 + \left(\frac{13}{3}\right)^2$$

$$= \frac{169}{16} + \frac{169}{9} = 169\left(\frac{1}{16} + \frac{1}{9}\right) = 169 \times \frac{25}{144}$$

$$\text{Required square root} = \sqrt{169 \times \frac{25}{144}} = \sqrt{13^2 \times \frac{5^2}{12^2}}$$

$$= \frac{65}{12} = \mathbf{5\frac{5}{12}}$$

E-64 The highest score in an innings was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder. These scores differed by 8 runs. What was the total score in the innings? **(NDA, '88)**

S-64 Let the total score be x runs, such that

$$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8 \quad \Rightarrow \quad \frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$$

$$\Rightarrow \quad \frac{2}{9}x \times \frac{2}{9} = 8 \quad \Rightarrow \quad x = 162$$

$\therefore$ The total score in the innings was **162**

E-65 Simplify

$$\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$$ **(SSC, '94)**

S-65 $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5}$

$= 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}$

$= 1 + \frac{1}{8} + [(-1^2)^{2/5} \times (2^5)^{4/5}] = 1 + \frac{1}{8} + [2^4] = \mathbf{17\frac{1}{8}}.$

E-66 Simplify

$(243)^{0.12} \times (243)^{0.08}$ **(SSC, '94)**

S-66 $(243)^{0.12} \times (243)^{0.08}$

$= (243)^{0.12 + 0.08} = (243)^{0.2}$ (Since $a^m \times a^n = a^{m+n}$)

$= (3^5)^{1/5} = \mathbf{3}.$

E-67 If $\sqrt{2^n} = 64$, then find the value of n **(AGE, '90)**

S-67 $\sqrt{2^n} = 64$

$\Rightarrow \quad (2^n)^{1/2} = 2^6 \;\Rightarrow\; \frac{n}{2} = 6$ (Since bases are same)

$\therefore \quad n = \mathbf{12}.$

E-68 $\left(\frac{-1}{216}\right)^{-2/3} = ?$ **(SSC, '94)**

S-68 Putting x for (?), we get

$$x = \left(\frac{-1}{216}\right)^{-2/3}$$

$\Rightarrow \quad x = (-216)^{2/3} \;\Rightarrow\; x = (-6^3)^{2/3}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \mathbf{36}.$

E-69 $(-2)^{-(2)^{(-2)}} = ?$ **(SSC, '94)**

S-69 Putting x for (?)

$$x = (-2)^{-(2)^{(-2)}}$$

$\Rightarrow \quad x = \left(-\frac{1}{2}\right)^{(2)^{(-2)}}$ $\left(\text{Since } \left(-\frac{a}{b}\right)^{-m/n} = \left(-\frac{b}{a}\right)^{m/n}\right)$

$\Rightarrow \quad x = \left[\frac{1}{4}\right]^{-2}$ $\left(\text{Since } \left(\frac{-1}{2}\right)^2 = \frac{1}{4}\right)$

$\Rightarrow \quad x = (4)^2 \;\Rightarrow\; x = \mathbf{16}.$

E-70 $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$ **(NDA, '83)**

S-70 Putting x for (?), we get

$$11\frac{1}{3} \times 4\frac{8}{10} \div x = 22\frac{2}{3}$$

$$\Rightarrow \quad 11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3} \times x$$ (If $a \div b = c$ then $a = b \times c$)

$$\Rightarrow \quad x = \frac{1}{2} \times 4\frac{8}{10}$$ $\left(\text{Since } \dfrac{11\frac{1}{3}}{22\frac{2}{3}} = \dfrac{1}{2}\right)$

$$\Rightarrow \quad x = \mathbf{2.4}.$$

E-71 $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$ **(CDS, '80)**

S-71 Putting x for (?) and solving for it

$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$

Assume, $1.06 = a$ and $0.04 = b$

$\therefore \quad (a + b)^2 - x = 4ab$

$\therefore \quad x = (a - b)^2 = (1.06 - 0.04)^2 = \mathbf{1.0404}.$ $[(a + b)^2 - (a - b)^2 = 4ab]$

E-72 If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a} + \frac{1}{b}$ **(MBA, '87)**

S-72 $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

$$= \frac{\sqrt{a^2 + b^2 + 2ab}}{ab}$$ (Since $a + b = \sqrt{(a+b)^2}$)

$$= \frac{\sqrt{45 + 2 \times 18}}{18} = \frac{\pm 9}{18} = \pm\frac{\mathbf{1}}{\mathbf{2}}$$

E-73 If $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only. **(MBA, '87)**

S-73 $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd} \quad \Rightarrow \quad \frac{a^2 + b^2}{c^2 + d^2} = \frac{2ab}{2cd}$

$$\Rightarrow \quad \frac{a^2 + b^2 + 2ab}{c^2 + d^2 + 2cd} = \frac{a^2 + b^2 - 2ab}{c^2 + d^2 - 2cd}$$ (By componendo and dividendo)

$$\Rightarrow \quad \left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$$

$$\therefore \quad \frac{\mathbf{a+b}}{\mathbf{a-b}} = \pm\frac{\mathbf{c+d}}{\mathbf{c-d}}$$

E-74 Simplify

$$\frac{a^{1/2}+a^{-1/2}}{1-a}+\frac{1-a^{-1/2}}{1+\sqrt{a}}$$ **(MBA, '87)**

S-74 $\dfrac{a^{1/2}+a^{-1/2}}{1-a}+\dfrac{1-a^{-1/2}}{1+\sqrt{a}}$

$$=\frac{a^{1/2}+a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})}+\frac{1-a^{1/2}}{1+a^{1/2}}$$ [Since $1-a=(1)^2-(a^{1/2})^2=(1+a^{1/2})(1-a^{1/2})$]

$$=\frac{a^{1/2}+a^{-1/2}+(1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})}$$

$$=\frac{a^{1/2}+a^{-1/2}+1-a^{-1/2}-a^{1/2}+1}{(1-a)}$$

$$=\mathbf{\frac{2}{1-a}}.$$

E-75 Solve $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$ **(MBA, '87)**

S-75 $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$

The given equation is of the form

$5^2+12^2=13^2$ (By the Pythogoras theorem of numbers)

Comparing the two equations, we find

$$\sqrt{x}=2$$

$\Rightarrow$ $x=\mathbf{4}$.

E-76 ***Directions (i)-(iv):*** What approximate value should come in place of the question mark (?) in the following questions:

(i) 139% of 459 + $5\frac{1}{2}$ of 384 = ? **(BSRB Bombay PO, '97)**

(ii) $\sqrt{2000}\times 0.7=(?)^2$ **(BSRB Bangalore PO, '97)**

(iii) ?% of 8999 + $\dfrac{599}{3}$ = 26300 **(BSRB Bangalore PO, '97)**

(iv) 3.9% 99 + $\dfrac{4}{9}$ of 700 = 40% of ? **(BSRB Bangalore PO, '97)**

S-76 (i) Assuming x for ? and approximating the terms to closest values, we get

$$x=140\%\text{ of }460+\frac{11}{2}\times 384$$

$\Rightarrow \quad x = \dfrac{140 \times 460}{100} + \dfrac{11}{2} \times 384$

$\Rightarrow \quad x = 644 + 2112 = 2756 \approx 2800$

$\therefore$ the required value is **2800**.

(ii) Assuming x for ? and approximating the terms to its closest values, we get

$$(x)^2 = \sqrt{2000} \times 0.7$$
$$= 20\sqrt{5} \times 0.7$$
$$= 20 \times 2.23 \times 0.7$$
$$= 31.22 \approx 31$$

$\therefore \quad x = \sqrt{31} = 5.56 \approx$ **5.6**.

(iii) Assuming x for ? and approximating the terms to its nearest values, we get

$$x\% \text{ of } 9000 = 26300 - \frac{600}{3}$$

$$\Rightarrow \quad x = \frac{26300 - 200}{9000} \times 100$$
$$= \mathbf{290}.$$

(iv) Assuming x for ? and approximating the terms to its nearest values, we get

$$40\% \text{ of } x = 4\% \text{ of } 100 + \frac{4}{9} \times 700$$

$$\Rightarrow \quad \frac{2}{5} \times x = 4 + 311$$

$$\Rightarrow \quad x = \frac{315 \times 5}{2} = \frac{1575}{2} = 787 \approx \mathbf{790}.$$

REGULAR PROBLEMS

(1) $\dfrac{2 \div (2 \times 2)}{(2 \div 2) \times 2} = ?$ **(UTI, '90)**

(a) 2 (b) 1 (c) 4 (d) $\frac{1}{4}$ (e) None of these

(2) Find the missing number: **(RRB Mumbai, '98)**

$\dfrac{8}{7} \times \dfrac{7}{12} \div ? = \dfrac{4}{9}$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{9}$ (d) $\frac{9}{2}$ (e) $\frac{1}{3}$

(3) Simplify: $2\frac{1}{2}$ of $\frac{3}{4}\times\frac{1}{2}\div\frac{3}{2}+\frac{1}{2}\div\frac{3}{2}\left(\frac{2}{3}-\frac{1}{2}\text{ of }\frac{2}{3}\right)$ to get

(a) 1 (b) $3\frac{3}{8}$ (c) $2\frac{7}{3}$ (d) $1\frac{5}{8}$ (e) $2\frac{3}{4}$

(4) If $\sqrt{13.69} = 3.7$ then **(ASM Exam, '01)**

$\sqrt{1369}+\sqrt{0.1369}+\sqrt{0.001369} = ?$

(a) 37.407 (b) 34.307 (c) 37.470 (d) 34.707 (e) 37.737

(5) If $46^2 - 44^2 = 45p$, then the value of 'p' will be:

(a) 10 (b) 15 (c) 4 (d) 12 (e) 6

(6) $\frac{196}{?} = \frac{?}{36}$ **(IA, '82)**

(a) 28 (b) 84 (c) 56 (d) 16.3 (e) 24

(7) If $a * b = a + b + \sqrt{ab}$, then $6 * 24$ is equal to **(RRB Bhopal, '98)**

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

(8) Find the value of $45^3 - 65^3 + 20^3$ **(RRB Ajmer, '98)**

(a) –175500 (b) 165500 (c) 0 (d) –174500 (e) –140055

Hint: Here, $a + b + c = 45 - 65 + 20 = 0$. Then use the identity no. 12 of 4.2

(9) $\frac{0.538\times0.538-0.462\times0.462}{1-0.924} = ?$ **(Bank PO, '83)**

(a) 2 (b) 1.08 (c) 0.076 (d) 0.987 (e) 1

(10) What should come in place of the (?) mark? **(SBI PO, '99)**

$\frac{?}{24} = \frac{72}{\sqrt{?}}$

(a) 12 (b) 16 (c) 114 (d) 144 (e) None of these

(11) Find the value of $(512)^{-2/9}$

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

(12) $8 \times 12\frac{1}{2} - 75 = 1\frac{2}{3} \times ?$

(a) 15 (b) 16 (c) 25 (d) $\frac{3}{5}$ (e) $2\frac{1}{3}$

Hint: $8 \times 12\frac{1}{2} = 100$

(13) $\sqrt{12\frac{3}{4}+13\frac{1}{4}+?} = 6$

(a) 10 (b) 16 (c) 12 (d) 36 (e) 18

(14) $7 - [? - \{4 - 7\} - \{5 - (4 - 5) + 2\}] = 16$

(a) 2 (b) 6 (c) 4 (d) 3 (e) – 4

(15) If $\sqrt{x}-\sqrt{y}=1$ and $\sqrt{x}+\sqrt{y}=17$ then $\sqrt{xy}=?$ **(BSRB, '93)**

(a) 51 (b) 16 (c) $\sqrt{72}$ (d) 72 (e) $\sqrt{17}$

(16) If $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{?}}=2$, then (?) is

(a) $2\sqrt{3}$ (b) 24 (c) 12 (d) 144 (e) $3\sqrt{2}$

(17) A man eats 'x' bananas in a week. In how many days will he eat 84 bananas?

(a) $\dfrac{84}{x}$ (b) $12x$ (c) $\dfrac{588}{x}$ (d) $\dfrac{12}{x}$ (e) $84x$

(18) 10 raised to the fifth power may be expressed as:

(a) 10×5 (b) 5^{10} (c) $\sqrt[5]{10}$
(d) $10 \times 10 \times 10 \times 10 \times 10$ (e) $(5)^{10/5}$

(19) Which of the following is the same as $50 \div 12$?
(a) $10\,(5 \div 3)$ (b) $(50 \div 6) + (50 \div 6)$ (c) $25 \div 12 \times 2$
(d) $50 \div 4 \times 3$ (e) $50 \div 4 \div 3$

(20) If $\left(\sqrt{3}\right)^x = 1$, then the value of x is

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 0 (e) 2

(21) $(16^{0.16} \times 2^{0.36})$ is equal to

(a) 64 (b) 16 (c) 2 (d) $\dfrac{1}{2}$ (e) 1

(22) If $0.5 \times A = 0.0003$, then the value of A will be
(a) 0.6 (b) 0.06 (c) 0.0006 (d) 0.006 (e) 0.175

Hint: Put $0.5 = \dfrac{1}{2}$, for easy calculation

(23) Reciprocal of $\sqrt[5]{12\dfrac{209}{243}}$ is equal to

(a) $\dfrac{5}{3}$ (b) $\dfrac{3}{5}$ (c) $\dfrac{2}{5}$ (d) $\dfrac{5}{4}$ (e) $\dfrac{1}{3}$

(24) A decimal fraction is multiplied by itself. If the product is 477.4225, then the fraction is
(a) 19.325 (b) 23.715 (c) 22.75 (d) 21.85 (e) 18.65

(25) Simplify: $2.5 - \dfrac{1}{3.25 - \dfrac{2.5}{0.75+0.50}}$ **(Goods Guard Ex, '99)**

(a) 0.50 (b) 1.70 (c) 1.25 (d) 0.80 (e) 1.18

(26) Simplify: $\dfrac{0.\dot{3} \times 1.\dot{0}\dot{6}}{0.\dot{5} \times 0.\dot{4}}$

(a) $1.43\dot{1}\dot{8}$ (b) $0.2\dot{8}$ (c) $1.0\dot{3}\dot{2}$ (d) $1.6\dot{4}$ (e) $1.\dot{8}\dot{8}$

(27) If $\sqrt{\sqrt[3]{0.000001 \times x}} = 0.4$, then the value of x is

(a) $2^{1/6}$ (b) 4096 (c) $4^{1/6}$ (d) 64 (e) $4^{1/3}$

(28) $3\frac{1}{x} \times 3\frac{3}{4} = 12\frac{1}{2}$, then the value of '$x$' is

(a) 1 (b) $\frac{1}{3}$ (c) 2 (d) 3 (e) $\frac{1}{2}$

(29) If $3.2 \div 64 * 10 = 2.45 - 1.95$, which of the following should replace the asterisk ($*$)? **(TC Clerk, '97)**

(a) + (b) − (c) × (d) ÷ (e) insufficient data

(30) The value of $(9.9)^3$ is

(a) 970.299 (b) 981.009 (c) 981.999 (d) 998.99 (e) 990.989

Hint: Write $9.9 = 10 - 0.1$ and the use the identity $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

(31) Square of 21 of 64 is equal to

(a) square of 168 (b) square of 165 (c) square of 179
(d) square of 167 (e) square of 1344

(32) What is the value of x in $\frac{2}{x} - \frac{5}{3x} = \frac{1}{3}$ $(x \neq 0)$?

(a) cannot be determined (b) −2 (c) 3
(d) −1 (e) 1

(33) The value of $(16^{0.16} \times 2^{0.36})$ is equal to

(a) 0 (b) 1 (c) 2 (d) −1 (e) 4

(34) If '+' means '×', '−' means ÷, ÷ means + and '×' means '−', then what will be the value of $20 \div 40 - 4 \times 5 + 6 = ?$

(a) 0 (b) 13.5 (c) 60 (d) −15 (e) None of these

(Elec. Appr. Exam, '01)

(35) If the given interchanges are made in signs and numbers, which one of the four equations would be correct? **(RRB Secunderabad, '01)**

Given interchanges: signs × and ÷, numbers 4 and 9

(a) $94 \times 7 \div 47 = 329$ (b) $47 \times 9 \div 94 = 18$ (c) $49 \times 7 \div 49 = 7$
(d) $94 \times 7 \div 97 = 324$ (e) None

(36) Which of the following equations are equivalent? **(Mumbai PO, '99)**

(1) $\left(\frac{1}{2}M + \frac{2}{3}N\right)^2$ (2) $\frac{4}{9}N^2 + \frac{1}{4}M^2 + \frac{2}{3}MN$

(3) $\left(\frac{M}{2} + \frac{2}{3}N\right)\left(\frac{1}{2}M - \frac{2}{3}N\right)$ (4) $\frac{1}{4}\left(M + \frac{4}{3}N\right)^2$

(a) only (2) and (3) (b) only (1) and (4) (c) only (1) and (2)
(d) only (1) and (3) (e) only (1), (2) and (4)

Directions (37-41): What approximate value will come in place of the question mark (?) in the following equation?

(37) $\sqrt{625.04} \times 16.96 + 136.001 \div 17 = ?$ **(RBI, '02)**

(a) 4.18 (b) 4.41 (c) 425 (d) 433 (e) None

(38) $\left(115\frac{1}{24} + 234.92\right) \times 5\frac{3}{37} = ?$ **(Bank PO, '97)**

(a) 1400 (b) 1750 (c) 1350 (d) 1200 (e) 1650

(39) ? % of 6147 = $2\frac{1}{2} \times 245.76$ **(BSRB Mumbai Hindi Officer's, '97)**

(a) 16 (b) 10 (c) 18 (d) 20 (e) 15

(40) $5.6 \times 2569 + 2058 = 157\% \times 6529 + ?$

(a) 5800 (b) 6300 (c) 6200 (d) 6500 (e) 6000

(41) $0.\overline{91} + \sqrt{999} + \sqrt{111} + 0.\overline{11} = ?$

(a) 43 (b) 40 (c) 39 (d) 42 (e) 4.23

Answers:

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (a)	**9.** (e)
10. (d)	**11.** (b)	**12.** (a)	**13.** (a)	**14.** (e)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)
19. (e)	**20.** (d)	**21.** (c)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (b)	**26.** (a)	**27.** (b)
28. (d)	**29.** (c)	**30.** (a)	**31.** (a)	**32.** (e)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (e)
37. (d)	**38.** (b)	**39.** (b)	**40.** (c)	**41.** (a).				

REAL PROBLEMS

(1) If $x = 0.5$ and $y = 0.2$, then $\sqrt{0.6} \times (3y)^x$ is equal to:

(a) 1.0 (b) 0.5 (c) 0.6 (d) 1.1 (e) $\sqrt{1.8}$

(2) $\sqrt[3]{1+\sqrt{2}} \cdot \sqrt[6]{3-2\sqrt{2}}$ is equal to:

(a) $2 - \sqrt{2}$ (b) $\sqrt{2} - 1$ (c) 1 (d) $3 - 2\sqrt{2}$ (e) $2 + \sqrt{2}$

(3) The value of the expression $\frac{x-1}{x^{3/4} + x^{1/2}} \cdot \frac{x^{1/2} + x^{1/4}}{x^{1/2} + 1} \cdot x^{1/4}$

when $x = 16$, is:

(a) 4 (b) 3 (c) 2 (d) 1 (e) 16

(4) $999\frac{998}{999} \times 13 = ?$

(a) $13888\frac{4}{999}$ (b) $12999\frac{986}{999}$ (c) $11988\frac{994}{999}$ (d) $12990\frac{1}{999}$ (e) $12907\frac{904}{999}$

Hint: $999\frac{998}{999} = 999 + \frac{998}{999} = 1000 - \frac{1}{999}$

(5) Arrange the following surds in ascending order of magnitude:

(i) $\sqrt[4]{3}$, (ii) $\sqrt[6]{7}$, (iii) $\sqrt[12]{48}$.

(a) (i), (ii), (iii) (b) (ii), (iii), (i) (c) (i), (iii), (ii) (d) (iii), (ii), (i) (e) (iii), (i), (ii)

Hint: The LCM of the orders of given surds, i.e. 4, 6, 12 are 12. Convert each one of the given surds into a surd of order 12

(6) Simplify: $\dfrac{2^{2^{2^2}} \div \left[\left(2^2\right)^3\right]^4}{\left(4^{4^4}\right) \div (4^4)^4}$

(a) 256 (b) 1024 (c) 64 (d) $\dfrac{1}{2^{488}}$ (e) 2^{4^2}

Hint: $a^{2^3} = a^{2\times2\times2} = a^8$, but $(a^2)^3 = a^{2\times3} = a^6$

(7) $38\frac{2}{3}$ divided by $\frac{2}{8}$ can be expressed as:

(a) $38\frac{2}{3}\times\frac{1}{4}$ (b) $\frac{2}{8}\times\frac{116}{3}$ (c) $\left(\frac{1}{4}\times38\right)+\left(\frac{1}{4}\times\frac{2}{3}\right)$ (d) $\frac{116}{3}\times4$

(e) $(38\times4)+\left(\frac{1}{4}\times\frac{2}{3}\right)$

(8) If $2^n = \dfrac{4^5+4^5+4^5}{3^5+3^5}\times\dfrac{6^5+6^5+6^5+6^5}{2^5+2^5+2^5}$ and $n > 0$, then the value of n^2 is:

(a) 11 (b) 121 (c) 169 (d) 7 (e) 81

(9) If $4^b - 4^{b-1} = 24$, then $(2b)^b$ equals:

(a) 25 (b) $25\sqrt{5}$ (c) 125 (d) $5\sqrt{5}$ (e) $\sqrt{5}$

(10) If $a^3 + b^3 = 0$, then:

(a) $a + b = \sqrt{2ab}$ (b) $a + b = a^2 + b^2$ (c) $a + b = a^2 - b^2 + ab$

(d) $a + b = \sqrt{ab}$ (e) $a + b = \sqrt{3ab}$

(11) Replace the (*) mark in

$$1+\cfrac{1}{1+\cfrac{1}{1+(*)}} = \frac{8}{5}.$$

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) $\frac{1}{3}$ (e) $\frac{1}{4}$

(12) If $m\sqrt{m}\times m^3 \div m^{-3/2} = m^{(a+2)}$, then the value of $a^{(a+2)}$ is:

(a) 1876 (b) 2304 (c) 16 (d) 256 (e) 4096

(13) Evaluate: $1 + \frac{1}{1\times 2} + \frac{1}{1\times 2\times 4} + \frac{1}{1\times 2\times 4\times 8} + \frac{1}{1\times 2\times 4\times 8\times 16}$ up to four places of decimals.

(a) 1.6096 (b) 1.7062 (c) 1.3214 (d) 1.6416 (e) None of these

Hint: Do not waste your time by calculating individual term. Adopt any other convenient method

(14) Find the value of '*b*', if *b* is a natural number and

$$\sqrt{b\sqrt[3]{b}} - \sqrt[3]{b\sqrt{b}} = \sqrt{b}$$

(a) 4 (b) 16 (c) 256 (d) 64 (e) 128

(15) Evaluate: $\frac{(0.15)^2 + (0.28)^2}{(0.45)^2 + (0.84)^2} - \frac{(0.28)^3 + (0.47)^3 - (0.75)^3}{3(0.28)(0.47)(0.75)}$.

(a) 1 (b) 0.8 (c) $1\frac{1}{9}$ (d) 2.5 (e) $-\frac{8}{9}$

(16) If $\sqrt{\sqrt[3]{0.000001x}} = 0.2$, then $\frac{\sqrt{x}}{0.1}$ equals to:

(a) 14.14 (b) 640 (c) 80 (d) 20 (e) None of these

Hint: Remove the radicals one by one

(17) If $\left(\frac{p^2}{q^2}\right)^{5x+7} = \left(\frac{q^3}{p^3}\right)^{x-8}$, then the value of $(5x + 7)$ is:

(a) 12 (b) $10\frac{11}{13}$ (c) 17 (d) $7\frac{2}{9}$ (e) $-20\frac{1}{7}$

Hint: Do not try to find the value of $5x + 7$ directly from the given relation

(18) What should come in place of the question mark (?) in the following equation?

(BSRB Patna PO, '01)

$$75^{7.5} \div 75^{3/2} \times 75^{-3} = \left(\sqrt{75}\right)^?$$

(a) 6 (b) $5\sqrt{3}$ (c) $\sqrt{3}$ (d) 3 (e) None of these

(19) What will replace the question mark (?) in the following equation?

$$27^{\sqrt{?-7}} = \frac{6^{19} \times 153}{2^{16} \times 136}$$

(a) 8 (b) 16 (c) 56 (d) 23 (e) 32

(20) If $5^x = 6^y = 30^7$, then the value of $\frac{xy}{x+y}$ is:

(a) $\frac{1}{7}$ (b) 3 (c) 6 (d) 7 (e) 1

(21) If $7x + 10 = x^2 - \frac{1}{N} = 13x - 2$, what is the value of N?

(a) $-\frac{1}{20}$ (b) 20 (c) 18 (d) $\frac{1}{18}$ (e) 22

(22) If $\frac{x(x+2)}{(x+4)4} = \frac{0.\dot{3} \times 1.\dot{0}\dot{6}}{0.\dot{5} \times 0.\dot{4}}$, then the value of x is:

(a) 9 (b) 7 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$ (e) None of these

Hint: Do not waste your time by solving it like a quadratic equation

Directions (23-27): Following (A) to (H) are combinations of an operation and an operand:

(A) means $\div 3$ (B) means $\times 3$ (C) means $- 3$ (D) means $+ 3$
(E) means $\div 2$ (F) means $\times 2$ (G) means $- 2$ (H) means $+ 2$

You have been given one or more of these as answer choices for the following questions. Select the appropriate choice to replace the question mark in the equations. **(SBI PO, '99)**

(23) $42 \times 21 - 12? = 880$

(a) A (b) F (c) G (d) D (e) None of these

Hint: $42 \times 21 = 882 \quad \therefore \quad 12? = 882 - 880 = 2$ Checking the given combination in place of ? By trial, (A) $\rightarrow 12 \div 3 = 4 \neq 2$, then
for combination (F) $\rightarrow 12 \times 2 = 24 \neq 2$,
(G) $\rightarrow 12 - 2 = 10 \neq 2$ and so on

(24) $36 + 12\ ? = 48$

(a) A followed by D (b) B followed by G (c) A followed by B
(d) F followed by H (e) None of these

Hint: Here the number on right hand side i.e. 48 is comparatively smaller than previous one (i.e. 880). So, we can directly test the combination of operation as per choices i.e. (a) $\rightarrow 36 + 12 \div 3 + 3 = 43 \neq 48$

(25) $48\ ? + 12 \times 4 = 80$

(a) E followed by B (b) D followed by A (c) B followed by F
(d) F followed by A (e) None of these

(26) $18 \times 3 \div 2 + 3 < 27\ ?$

(a) D followed by G (b) A followed by G (c) D followed by H
(d) D followed by A (e) None of these

(27) $(48 + 9) \div 19 \times 2 = 12\ ?$

(a) B followed by E (b) A followed by H (c) A followed by D
(d) C followed by A (e) None of these

(28) What would be the maximum value of Q in the following equation?

2P4 + 7R9 + 4Q7 = 1380

(a) cannot be determined (b) 9 (c) 6 (d) 7 (e) 8

Directions (29-36): What approximate value should replace the question mark (?) in the following equations:

(29) 208.78 of $7\frac{3}{5}\% + 423.547$ of $24\frac{39}{50}\% = ?$

(a) 120 (b) 117 (c) 123 (d) 114 (e) 130

Tips: Both the parts have % term, so take the % common outside a bracket. Hence calculation of % at each step should be avoided.

(30) 159% of 6531.8 + 5.5 × 1015.2 = ? + 5964.9 **(Mumbai PO, '98)**

(a) 11,000 (b) 11,500 (c) 10,000 (d) 10,800 (e) 12,000

(31) $152\sqrt{?} + 795 = 8226 - 3486$ **(Baroda PO, '99)**

(a) 675 (b) 550 (c) 860 (d) 925 (e) 500

(32) $\frac{5}{7}$ of 1596 + 3015 = ? – 2150

(a) 5500 (b) 6300 (c) 49000 (d) 7400 (e) 68000

(33) $16\sqrt{524} + 1492 - 250.0521 = ?$

(a) 1500 (b) 1350 (c) 2200 (d) 1800 (e) 1600

(34) 857 of 14% – 5.6 × 12.128 = ? **(Chennai PO, 2000)**

(a) 45 (b) 52 (c) 65 (d) 60 (e) 40

(35) 6.39 × 15.266 + 115.8 of $\frac{2}{5}$ = ?

(a) 160 (b) 150 (c) 145 (d) 170 (e) 130

(36) $33\frac{1}{3}$ % of 768.9 + 25% of 161.2 – 68.12 = ?

(a) 220 (b) 245 (c) 235 (d) 250 (e) 230

Hint: Before simplification, put the fractional equivalent of the percentage values as $33\frac{1}{3}\% = \frac{1}{3}$ and $25\% = \frac{1}{4}$

Directions (37-40): Find out the approximate value that should come in place of the question mark in the following questions. (You are not expected to find the exact value).

(37) $\sqrt{45689} = ?$

(a) 170 (b) 280 (c) 320 (d) 210 (e) 430

Hint: Do not try to find the square root by conventional method, but, in reverse way, check the square of which alternative gives closest value to 45689. This checking can be done mentally as square of 2, i.e. $(2)^2$ can give the first digit as 4. So either (b) or (d) can be the answer. Next you check $(28)^2 > 25^2$ (625) it is not the starting digit of the given number. So (d) 210 is the choice.

(38) $\frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$

(a) 30000 (b) 900000 (c) 300000 (d) 3000 (e) 60000

(39) $\frac{2}{5} + \frac{7}{8} \times \frac{17}{19} \div \frac{6}{5} = ?$

(a) $3\frac{1}{2}$ (b) $1\frac{1}{2}$ (c) 2 (d) 1 (e) $\frac{8}{11}$

(40) 399.89 + 206 × 11.009 = ?

(a) 2700 (b) 3100 (c) 6566 (d) 4336 (e) 2400

Directions (41-44): Four of the five parts numbered as (a), (b), (c), (d) and (e) in the following equations are exactly equal. You have to find out the part that is not equal to the other parts.

(41) $\underset{(a)}{\sqrt[3]{729}+\sqrt[3]{625}} = \underset{(b)}{\sqrt{324}+\sqrt{256}} = \underset{(c)}{\sqrt[3]{216}\times\sqrt[3]{81} - \frac{1}{2} \text{ of } 40}$

$= \underset{(d)}{\sqrt{441}+\sqrt[3]{2197}} = \underset{(e)}{\sqrt[3]{5832}+\sqrt[3]{2744}}$

(42) $\underset{(a)}{8362.64 + 768.3 - 190.57} = \underset{(b)}{593.38 + 604.7 + 7742.29}$

$= \underset{(c)}{2235.925 \times 4} = \underset{(d)}{9931.04 - 990.67} = \underset{(e)}{17880.74 \div 2}$

Hint: You have to do actual calculation here. Never try to round off

(43) $\underset{(a)}{10.36 + 69.802 + 24.938} = \underset{(b)}{2207.1 \div 21} = \underset{(c)}{16\frac{2}{3}\% \text{ of } 630.6}$

$\underset{(d)}{32.84375 \times 3.2} = \underset{(e)}{\frac{1}{5} \text{ of } \frac{1}{9} \text{ of } 4729.4}$

(44) $\underset{(a)}{x(x + 5) + 2(3x + 2)} = \underset{(b)}{(x + 2)^2 + 7x} = \underset{(c)}{(x + 3)(x - 2) + 10(x + 1)}$

$= \underset{(d)}{(x - 2)^3 - (x - 3)^3 - 2(x^2 - 13x + 8)} = \underset{(e)}{(x + 7)(x + 2) + 2(x - 5)}$

(45) If $\sqrt[3]{91125} = 45$, then the value of

$\sqrt[3]{91.125}+\sqrt[3]{0.091125}+\sqrt[3]{0.000091125}$

(a) 49.95 (b) 5.495 (c) 4.995 (d) 5.405 (e) 4.545

Tips: On finding cube roots $\left(\sqrt[3]{\ }\right)$ the number of decimal places get reduced to $\frac{1}{3}$, as on finding square roots $\left(\sqrt[2]{\ }\right)$, the number of decimal places reduces to $\frac{1}{2}$

(46) If $x = \frac{1}{11} \div \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5}}}$, then find the value of $\left(x + \frac{5}{6}\right)$.

(a) 3 (b) $\frac{1}{5}$ (c) $5\frac{5}{6}$ (d) $\frac{1}{6}$ (e) 1

(47) If $(x + p + m) = p^3 + m^3 + x^3 - 3xpm$ and the sum of any two numbers is 12, then what is the ratio of that sum to the third number?

(a) -4 (b) -1 (c) 1 (d) -12 (e) 4

(48) If $8700 \div x = 300$ and $4590 \div y = 170$, then $(x - y) \times (x + y) = ?$

(a) 29 (b) 56 (c) 112 (d) 27 (e) 81

Hint: Refer to 4.7

(49) If $\frac{8}{a} = \frac{3}{a} - 10$, then $\frac{a}{8} - \frac{3}{a} = ?$

(a) -10 (b) $\frac{95}{16}$ (c) $\frac{16}{95}$ (d) $\frac{24}{a^2 - 8}$ (e) $\frac{a^2 - 24}{8a}$

Answers

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (b)
10. (e)	**11.** (a)	**12.** (e)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)
19. (c)	**20.** d)	**21.** (a)	**22.** (b)	**23.** (e)	**24.** (c)	**25.** (d)	**26.** (c)	**27.** (b)
28. (c)	**29.** (a)	**30.** (c)	**31.** (a)	**32.** (b)	**33.** (e)	**34.** (b)	**35.** (c)	**36.** (e)
37. (d)	**38.** (c)	**39.** (d)	**40.** (a)	**41.** (e)	**42.** (c)	**43.** (e)	**44.** (d)	**45.** (c)
46. (e)	**47.** (b)	**48.** (c)	**49.** (b)					

5 PERCENTAGE

5.1 INTRODUCTION

The word 'per cent' means per hundred. Thus, 19 per cent means, 19 parts out of 100 parts. This can also be written as $\frac{19}{100}$.

Therefore, per cent is a fraction whose denominator is 100, and the numerator of this fraction is called the **Rate percent**. So, $\frac{19}{100}$ = 19 per cent. Here, 19 per cent is the rate. The sign for per cent is '%'.

Table 5.1 Fractional Equivalents of Important Percentages

$1\% = \frac{1}{100}$	$2\% = \frac{1}{50}$	$4\% = \frac{1}{25}$	$8\% = \frac{2}{25}$	$16\% = \frac{4}{25}$	$64\% = \frac{16}{25}$	$96\% = \frac{24}{25}$
$5\% = \frac{1}{20}$	$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$40\% = \frac{2}{5}$	$60\% = \frac{3}{5}$	$80\% = \frac{4}{5}$	$120\% = \frac{6}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$25\% = \frac{1}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$	$50\% = \frac{1}{2}$	$87\frac{1}{2}\% = \frac{7}{8}$	$100\% = 1$
$8\frac{1}{3}\% = \frac{1}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$	$83\frac{1}{3}\% = \frac{5}{6}$	$133\frac{1}{3}\% = \frac{4}{3}$	

Note: Similarity along the horizontal rows are to be observed for memorizing table 5.1.

5.2 FRACTION TO RATE PER CENT

To convert (or express) any fraction $\frac{a}{b}$ to rate per cent, multiply it by 100 and put $a(\%)$ sign.

$$\Rightarrow \underbrace{\boxed{\frac{a}{b}}}_{\text{a fraction}} = \underbrace{\boxed{\frac{a}{b} \times 100\%}}_{\text{rate per cent}}$$

Example: Express $\frac{3}{4}$ in rate per cent

Required rate per cent = $\frac{3}{4} \times 100\% = 75\%$

5.3 RATE PER CENT TO FRACTION

To convert a rate per cent to a fraction, divide it by 100 and delete the % sign.

Example: 8% can be converted to a fraction as $\frac{8}{100}$

5.4 RATE PER CENT OF A NUMBER

Rate per cent of a number is the product of equivalent fraction (of rate per cent) and the number.

$$\Rightarrow \quad p\% \text{ of } A = \left(\frac{p}{100}\right) \times A$$

Example: To find out 25% of 500

Solution: Required value = 25% of 500

$$= \underbrace{\left(\frac{25}{100}\right)}_{\text{equivalent fraction for 25\%}} \times 500.$$

$$= 125$$

5.4.1 Relation Among Rate Per cent, Number and Value

Let us consider a number, N.

Then N is considered as the base over which value of different rate per cents are found out,

$$10\% \text{ of } N = \frac{10}{100} \times N = \frac{N}{10} \text{ (value)}$$

$$25\% \text{ of } N = \frac{25}{100} \times N = \frac{N}{4} \text{ (value)}$$

and so on.

Therefore, it is found that as the rate per cent changes, its related value for the same number will also change.

Conversely, different values stand for different rate per cents of the same number. As in the above example, $\frac{N}{10}$ stands for 10% of N; $\frac{N}{4}$ stands for 25% of N and so on.

In the above context, a very useful relation is derived as:

$$\boxed{\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (base)}} \qquad \textbf{(1)}$$

Example: 9% of what number is 36?

Solution: Using the relation 1,

the required number (base number) = $\frac{36}{9\%}$

$$= \frac{36}{9} \times 100$$
$$= 400$$

Note: Here, 36 is the value and its rate % of base number = 9%

Example: If 30% of a number is 48, then what is 70% of the number?

Solution: Here, unitary method can be used to save the time.

$$30\% \rightarrow 48$$

$$\Rightarrow \qquad 1\% \rightarrow \frac{48}{30}$$

$$\Rightarrow \qquad 70\% \rightarrow \frac{48}{30} \times 70 = 112$$

Hence, the required value is 112

Example: If 40% of the number exceeds the 25% of it by 54. Find the number.

Solution: Using the formula (1)

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (i.e. base number)}$$

Here, 54 stands for the difference of (40% and 25% of number)

$$\Rightarrow \qquad \frac{54}{(40-25)\%} = \text{number}$$

$$\Rightarrow \qquad \text{required number} = \frac{54}{40-25} \times 100 = 360$$

5.5 EXPRESSING A GIVEN QUANTITY AS A PERCENTAGE OF ANOTHER GIVEN QUANTITY

Let one given quantity be x and another given quantity be y. It is often asked to find what percentage of y is x. Here both quantities (x and y) should be in same units. If not, they should be converted into the same unit.

Concept

The question requires us to express one given quantity 'x' as a percentage of another given quantity 'y'.

Since y is the basis of comparison, so, y will be in the denominator. But x is to be converted as percentage of y, hence x will be in the numerator of the fraction. Now to convert the fraction to percentage, we will multiply it by 100. So, we get

$$\text{the required percentage} = \frac{x}{y} \times 100\%.$$

Example: To find '30 is what per cent of 150' or 'what percentage of 150 is 30 ?

Solution: Using the earlier concept, we find here that 150 is the basis of comparison and hence 150 will be in the denominator.

$$\text{The required percentage} = \frac{30}{150} \times 100\% = 20\%.$$

Example: ?% of 320 = 86.4

Solution: Here, 320 is the basis of comparison and it will be in the denominator.

$$\therefore \quad \text{required percentage} = \frac{86.4}{320} \times 100\% = 27\%$$

5.6 CONVERTING A PERCENTAGE INTO DECIMALS

Case I

Let the percentage be a positive integer, then
place a decimal point after two places from the extreme right of the integer to convert it into a decimal. If the percentage is a single digit number, add one zero to the left of it and then place the decimal point for its conversion. % Sign is removed after conversion.

Example: 67% may be converted into decimals as 0.67, because 67% = $\frac{67}{100}$ = 0.67

8% may be written as 0.08 → {Zero added to its left to make it a two digit number so that decimal point can be placed.}

253% is equivalent to 2.53

Case II

Let the percentage be a decimal fraction
The percentage being a decimal fraction, shift decimal by two places to the left. Add zero to the left of the fraction, if needed.

Two place left — Shifted

Example: 3.5% may be written as 0.035 (zero is added to its left so that decimal point can be shifted by two places to the left)

0.7% may be written as 0.007

Case III

Let the percentage be a fraction

If the percentage is a fraction of the form $\frac{a}{b}$, then convert it into a decimal fraction and then follow the rule detailed in case II

Example: $\frac{1}{4}$ % is equivalent to 0.25% which may be converted into decimals as 0.0025

5.7 CONVERTING A DECIMAL INTO A PERCENTAGE

In this case, the method of 5.6 is reversed, i.e. shift the decimal point two places to the right. Add zero to the extreme right if required. Then add % sign.

Shift

Example: 0.45 may be expressed as 45%

0.032 is equivalent to 3.2%

1.7 is equivalent to 170%

zero is added so that decimal point can be shifted by two places.

5.8 EFFECT OF PERCENTAGE CHANGE ON ANY QUANTITY (NUMBER)

If any number (quantity) is increased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100 + x}{100}\right)$$

or

$$= \text{original number} \times (1 + \text{decimal equivalent of } x\%).$$

Similarly, if any number (quantity) is decreased by $x\%$, then

$$\text{new number (quantity)} = \text{original number} \times \left(\frac{100 - x}{100}\right)$$

or

$$= \text{original number} \times (1 - \text{decimal equivalent of } x\%).$$

Note: In case of percentage decrease, a (–)ve sign is put before x, otherwise the formula is same.

Example: The present salary of A is Rs 3000. This will be increased by 15% in the next year. What will be the increased salary of A?

Solution: Here, the salary is to be increased by 15%.

15% is equivalent to 0.15

$$\therefore \text{ the increased salary} = 3000\,(1 + 0.15) \text{ or } 3000 \times \frac{100 + 15}{100}$$

$$= 3000 \times 1.15$$

$$= \text{Rs } 3450.$$

5.9 TWO STEP CHANGE OF PERCENTAGE FOR A NUMBER

In the first step, a number is changed (increased or decreased) by $x\%$, and in the second step, this changed number is again changed (increased or decreased) by $y\%$, then net percentage change on the original number can be conveniently found out by using the following formula,

$$\boxed{\underset{(+\text{ or }-)}{\text{net \% change}} = x + y + \frac{xy}{100}} \qquad (2)$$

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign remains.

Example: If a number is increased by 12% and then decreased by 18%, then find the net percentage change in the number.

Solution: Using the formula (2)

$$\text{net \% change} = x + y + \frac{xy}{100}$$

where $x = 12 \quad y = -18$

$$\Rightarrow \quad \text{net \% change} = 12 - 18 + \frac{(12)\times(-18)}{100}$$
$$= -6 - 2.16$$
$$= -8.16$$

(–) sign signifies that there is percentage decrease in the result. Therefore –8.16 indicates net 8.16% decrease of the given number as a result of 12% increase and 18% decrease.
It also implies that 12% increase and 18% decrease are equivalent to 8.16% decrease.

5.10 PERCENTAGE CHANGE AND ITS EFFECT ON PRODUCT

$$\text{Let} \quad \underset{\text{variable}}{\underset{\downarrow}{A}} \quad \times \quad \underset{\text{variable}}{\underset{\downarrow}{B}} \quad = \text{result (product)}$$

If A is changed (increased or decreased) by $x\%$, and also B is changed (increased or decreased) by $y\%$, then the net percentage change (increase/decrease) of the product of A and B can be found out easily by the formula

$$\boxed{\underset{(+\text{ or }-)}{\text{net \% change in product}} = x + y + \frac{xy}{100}} \qquad \textbf{(2A)}$$

If x or y indicates decrease in percentage, then put a (–)ve sign before x or y, otherwise positive sign remains.
This formula (2A) is same as formula (2)

Above formula can be used to find out the *net percentage change*, if it involves the *product of any two variable quantities which have also the % change.*

Application of the Formula (2A)

The formula (2A) can be used to find out

(a) % effect on expenditure, when rate and consumption are changed, since rate × consumption = expenditure [$A \times B$ = product]

(b) % effect on area of rectangle/square/triangle/circle, when its sides/radius are changed, since $\text{Side}_1 \times \text{Side}_2$ = area, or radius × radius = area [$A \times B$ = product]

(c) % effect on distance covered, when time and speed are changed, since time × speed = distance. [$A \times B$ = product]

Example: If the length of rectangle increases by 30% and the breadth decreases by 12%, then find the % change in the area of the rectangle.

Solution: Since, length × breadth = area, and both the length and breadth are changed, so, using the formula (2A), we get

$$\text{net \% change in product} = x + y + \frac{xy}{100}$$

where $x = 30,\ y = -12$

$$\Rightarrow \qquad \text{net \% change in area} = 30 - 12 + \frac{30 \times -12}{100}$$
$$= 18 - 3.6$$
$$= +14.4$$

It implies that there is 14.4% increase in the area of the rectangle.

5.10.1 To keep the product of two variable quantity as fixed

As we have seen in 5.10, $A \times B$ = product, where A and B are two quantities which are changing and product is also changing.

Now, we want to **keep the product fixed**, even if A and B are changed (increased/decreased). Then, if one quantity increases, the other quantity will decrease and vice-versa so that product remains unchanged.

Hence the net percentage effect on product is zero in the formula (2A).
Put net % change in product = 0 in formula (2A),

$$x + y + \frac{xy}{100} = 0$$

$$\Rightarrow \qquad y = -\frac{x}{100 + x} \times 100, \text{ (–) ve sign shows decrease}$$

From the above derivation, we thus find that

if one quantity A increases by $x\%$, then other quantity B decreases by $\boxed{\frac{x}{100 + x} \times 100\ \%}$

and if one quantity A decreases by $x\%$, then putting (–) x, in place of x,

we find that the other quantity B increases by $\boxed{\frac{x}{100 - x} \times 100\ \%.}$

These formula are used to find out:

(a) % change in either rate or consumption, when expenditure is to be kept fixed, because, rate × consumption = expenditure [$A \times B$ = product = fixed]

(b) % change in either length or breadth, when area of rectangle is to be kept fixed, because, length × breadth = area [$A \times B$ = product = fixed]

(c) % change in either time or speed, when distance is to be kept fixed because, time × speed = distance [$A \times B$ = product = fixed]

Example: If the price of coffee is increased by 10%, then by how much percentage must a house wife reduce her consumption, to have no extra expenditure?

Solution: Since price × consumption = expenditure and expenditure has to be kept fixed (or unchanged), so, when the price increases by 10%, the % reduction in consumption $= \dfrac{10}{100+10} \times 100\%$

$= 9\dfrac{1}{11}\%.$

5.11 RATE CHANGE AND CHANGE IN QUANTITY AVAILABLE FOR FIXED EXPENDITURE

Let the original rate of an item = Rs x per unit quantity.
Expenditure is fixed.

$$\text{Quantity of the item available} = \frac{\text{Expenditure}}{x} \qquad \textbf{(a)}$$

Now, the original rate (x) changes (increase/decrease) to a new rate. Since the amount of expenditure is fixed, so, with the *change in rate*, it is obvious that the quantity of the item available in equation (a) will also change (decrease/increase) accordingly.
Hence, due to rate change,

$$\text{New quantity of the item available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$\Rightarrow$ Original quantity available ± change in quantity available $= \dfrac{\text{Expenditure}}{\text{New rate}}$, Using equation (a), we get

$$\Rightarrow \boxed{\frac{\text{Expenditure}}{x} \pm \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}} \qquad \textbf{(3)}$$

Put (–)ve when quantity available decreases due to rate change, (x = original rate per unit quantity.)

Example: A reduction of 25% in the price of sugar enables the person to get 10 kg more on a purchase for Rs 600. Find the reduced rate of sugar.

Solution: Let the original rate = Rs x per kg.
Since, there is a rate reduction of 25%, so,
New rate (or reduced rate) $= (1 - 0.25)\ x$

$= 0.75\ x = \dfrac{3}{4}\ x$

Expenditure = Rs 600.

Using the formula (3)

$$\frac{\text{Expenditure}}{x} + \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$$\Rightarrow \qquad \frac{600}{x} + 10 = \frac{600}{\frac{3}{4}x}$$

(+ 10, for quantity available increases after rate change)

$$\Rightarrow \qquad \frac{600}{x}\left(\frac{4}{3} - 1\right) = 10 \quad \Rightarrow \quad x = 20$$

Therefore, reduced rate = $\frac{3}{4}x = \frac{3}{4} \times 20$ = Rs 15/kg.

5.12 % EXCESS OR % SHORTNESS

When a number A exceeds the another number B by x %, then % shortness of $B = \frac{x}{100 + x} \times 100$

It implies that B is less than A by $\frac{x}{100 + x} \times 100\%$.

Similarly, if a number A is short of (or less than) B by x%, then % excess of $B = \frac{x}{100 - x} \times 100$

i.e. B is more than A by $\frac{x}{100 - x} \times 100\%$

Example: If the income of Ram is more than that of Shyam by 25%, then by how much percentage Shyam's income is less than that of Ram?

Solution: Required % shortness (less) income of Shyam $= \frac{25}{100 + 25} \times 100\%$

$= 20\%$.

Therefore, income of Shyam is 20% less than that of Ram.

Solved Examples

E-1 Express the following in terms of percentage:

(a) 0.4 *(b)* 1.0 *(c)* $\frac{5}{3}$ *(d)* $\frac{7x}{y}$ *(e)* 1.23

S-1 *(a)* Refer 5.2, multiply the decimal fraction by 100.

$\therefore$ 0.4 = (0.4 × 100) % = **40 %**.

(b) 1.0 = (1.0 × 100) % = **100 %**.

(c) $\frac{5}{3} = \left(\frac{5}{3} \times 100\right) \% = \mathbf{166\frac{2}{3}}$ %.

(d) $\frac{7x}{y} = \left(\frac{7x}{y} \times 100\right) \% = \frac{\mathbf{700x}}{\mathbf{y}}$ %.

(e) 1.23 = (1.23 × 100) % = **123 %**.

E-2 Express the following in terms of fractions:

(a) $22\frac{1}{2}$ % *(b)* 35% *(c)* $\frac{a^2}{b}$ % *(d)* 0.3% *(e)* $\frac{7}{2}$ %

S-2 ***(a)*** Divide the given percentage by 100 to convert it into a fraction. *[Refer 5.3]*

$$\therefore\quad 22\frac{1}{2}\ \% = \frac{22\frac{1}{2}}{100} = \frac{45}{2\times 100} = \mathbf{\frac{9}{40}}.$$

(b) $35\ \% = \frac{35}{100} = \mathbf{\frac{7}{20}}.$

(c) $\frac{a^2}{b}\ \% = \frac{a^2}{b\times 100} = \mathbf{\frac{a^2}{100b}}.$

(d) $0.3\ \% = \frac{0.3}{100} = \mathbf{\frac{3}{1000}}.$

(e) $\frac{2}{7}\ \% = \frac{2}{7\times 100} = \mathbf{\frac{1}{350}}.$

E-3 Find

(a) 9% of 27

(b) 0.02% of 6500

(c) $\frac{7}{2}$ % of 80

(d) 125% of 64

(e) 10% of 5% of 320

S-3 ***(a)*** Refer 5.4, Multiply the number by $\frac{p}{100}$, if p% of the number is to be calculated.

$$\therefore\quad 9\ \%\text{ of } 27 = \frac{9}{100}\times 27 = \mathbf{\frac{243}{100}}.$$

(b) $0.02\ \%\text{ of } 650 = \frac{0.02}{100}\times 6500 = \mathbf{\frac{13}{10}}.$

(c) $\frac{7}{2}\ \%\ 80 = \frac{7}{2\times 100}\times 80 = \mathbf{\frac{14}{5}}.$

(d) $125\%\text{ of } 64 = \frac{125}{100}\times 64 = \mathbf{80}.$

(e) $10\%\text{ of } 5\%\text{ of } 320 = \frac{10}{100}\times\frac{5}{100}\times 320 = 1.6 = \mathbf{\frac{8}{5}}.$

E-4 Find the following:

(a) 36 is what % of 144.

(b) $\frac{7}{8}$ is what % of $\frac{3}{4}$.

(c) What % of 80 is 16.

(d) 0.625 is equal to what % of $1\frac{7}{28}$.

(e) 36 × 14 is what % of 1400.
(f) R is what % of N.

S-4 Refer 5.5, the required percentage = $\frac{x}{y} \times 100$ %, where y is the base.

(a) Here, the base is 144. So, the denominator will be 144.

$$\therefore \text{ required percentage} = \frac{36}{144} \times 100\% = 25\%.$$

(b) Here, the base is $\frac{3}{4}$.

$$\therefore \text{ required percentage} = \frac{7/8}{3/4} \times 100\ \% = 116\ \frac{2}{3}\ \%.$$

(c) Here, the base is 80.

$$\therefore \text{ required percentage} = \frac{16}{80} \times 100\% = 20\%.$$

(d) Here, the base is $1\frac{7}{28}$

$$\therefore \text{ required percentage} = \frac{0.625}{1\frac{7}{28}} \times 100 = 50\%.$$

(e) Here, the base is 1400

$$\therefore \text{ required percentage} = \frac{36 \times 14}{1400} \times 100 = 36\%.$$

(f) Here, the base is N

$$\therefore \text{ required percentage} = \frac{R}{N} \times 100 = \frac{100R}{N}\ \%.$$

E-5 Find the following:
(a) 36 is 6 % of what?
(b) 2.5 is 5 % of what?
(c) 12 is 25 % of 20 % of what?
(d) $\frac{4}{7}$ is 24 % of what?

S-5 Refer 5.4.1, using equation (1),

$$\frac{\text{any value}}{\text{its rate \% of number}} = \text{number (original)}$$

(a) required number = $\frac{36}{6\%} = \frac{36}{6} \times 100 = 600.$

(b) required number = $\frac{2.5}{5\%} = \frac{2.5}{5} \times 100 = 50.$

(c) required number = $\dfrac{12}{25\% \times 20\%} = \dfrac{12}{25 \times 20} \times 100 \times 100 = 240$

(d) required number = $\dfrac{4/7}{24\%} = \dfrac{4}{7 \times 24\%} = \dfrac{4}{7 \times 24} \times 100 = \dfrac{50}{21}$

E-6 25 % of a number is 20, what is 40 % of that number? Also find the number.

S-6 25 % $\rightarrow$ 20

$\Rightarrow$ 40 % $\rightarrow \dfrac{20}{25} \times 40 = 32$

Refer 5.4.1, using equation (1), the number = $\dfrac{\text{any value}}{\text{its rate }\%} = \dfrac{20}{25\%}$

$\Rightarrow$ required number = $\dfrac{20}{25} \times 100 = 80$

E-7 p_1 % of number N_1 is equal to p_2 % of number N_2. Find what per cent of N_1 is N_2?

S-7 It is required to find what per cent of N_1 is N_2, i.e. the base is N_1

$\therefore$ required percentage = $\dfrac{N_2}{N_1} \times 100\%$.

It is given that $\dfrac{p_1}{100} \times N_1 = \dfrac{p_2}{100} \times N_2 \quad \therefore \quad \dfrac{N_2}{N_1} = \dfrac{p_1}{p_2}$

Putting the value of $\dfrac{N_2}{N_1}$, we find the required percentage = $\left(\dfrac{p_1}{p_2} \times 100\right)\%$

$\therefore$ N_2 is equal to $\left(\dfrac{p_1}{p_2} \times 100\right)\%$ of N_1

E-8 A number A exceeds B by 25%. By what per cent is B short of A?

S-8 Refer 5.12

$$\% \text{ Short} = \frac{\% \text{ excess}}{(100 + \% \text{ excess})} \times 100$$

$$= \frac{25}{(100 + 25)} \times 100 = \mathbf{20}$$

E-9 A number X is short of Y by 40%. By what per cent is Y in excess of X?

S-9 Refer 5.12. Using the formula,

$$\% \text{ Excess} = \frac{\% \text{ short}}{100 - \% \text{ short}} \times 100$$

$$= \frac{40}{100 - 40} \times 100 = \mathbf{66\frac{2}{3}}\ \%.$$

E-10 The ratio of salary of a worker in July to that in June was $2\frac{1}{2} : 2\frac{1}{4}$. By what % was the salary of July more than salary of June? Also find by what %, salary of June was less than of July.

S-10 Let Salary of July $= \frac{5}{2}x$

and Salary of June $= \frac{9}{4}x$

Refer 5.5, here the basis of comparison is either the salary of June or the salary of July.
Salary of July more than that of June by per cent

$$= \frac{\text{Difference}}{\text{Salary of June}} \times 100$$

$$= \frac{\left(\frac{5}{2} - \frac{9}{4}\right)x}{\frac{9}{4}x} \times 100 = \mathbf{11\frac{1}{9}\ \%}$$

Salary of June less than that of July by per cent

$$= \frac{\text{Difference}}{\text{Salary of July}} \times 100 \qquad = \frac{\left(\frac{5}{2} - \frac{9}{4}\right)x}{\frac{5}{2}x} \times 100 = \mathbf{10\%}.$$

E-11 In 1987, the enrolment in a school was 1,500. Next year it increased by 10 %. What was the enrolment in 1988?

S-11 Enrolment in 1988 $= 1500 \times \frac{110}{100} = \mathbf{1,650.}$

E-12 The side of a square increases by $p\%$, then find by what % does its area increase?

S-12 Suppose, side of a square $= b$
Original area of the square $= b^2$, i.e. result $= A \times B$
Here, both sides are increased by $p\%$,
So, for finding out the % change in area, Refer formula (2A).

Net % change in area $= x + y + \frac{xy}{100}$, where, $x = y = +\ p$

$\Rightarrow$ Net % change in area $= p + p + \frac{p^2}{100}$

$$= 2p + \frac{p^2}{100}$$

Hence area increases by $\left(2\boldsymbol{p} + \frac{\boldsymbol{p}^2}{\mathbf{100}}\right)$ %.

Note: This formula is also applicable when the radius of a circle is increased by $p\%$.

Then its area increases by $\left[2p + \left(\frac{p}{10}\right)^2\right]\%$.

E-13 The daily wage is increased by 15 %, and a person now gets Rs 23 per day. What was his daily wage before the increase?

S-13 Original daily wage $= \dfrac{\text{Increased daily wage}}{100 + \%\text{ increase}} \times 100$ ***[Refer 5.8]***

$$= \frac{23}{115} \times 100 = \mathbf{20}$$

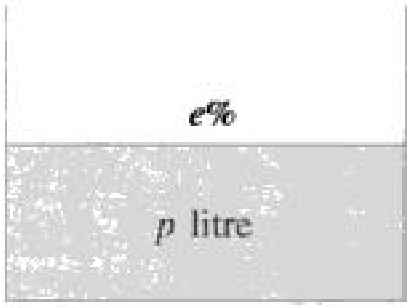
NB: $\rightarrow$ *In case of decrease use* (–) *ve sign,* before % value.

E-14 'p' litres of oil was poured into a tank and it was still e % empty. How much oil must be poured into the tank in order to fill it to the brim. Find the capacity of the tank.

S-14 Suppose, 'x' litres are to be poured to fill the tank completely, then, $x = e$ % of capacity

Now, $p + e$ % of capacity = full capacity $\Rightarrow$ $p + \dfrac{e}{100} \times$ capacity = capacity

$$\Rightarrow \quad \text{capacity} = \frac{p \times 100}{100 - e}$$



$$\therefore \quad x = e\% \text{ of capacity} = \frac{e}{100} \times \frac{p \times 100}{100 - e} \text{ litres}$$

$x = \dfrac{pe}{100 - e}$ litres = amount of oil to be poured to fill the tank completely.

E-15 A student X passes his examination with 515 marks, having scored 3% above the minimum. If Y had obtained 710 marks, what % would he have been above the minimum?

S-15 $\dfrac{\text{Marks of Y}}{\text{Marks of X}} = \dfrac{100 + \%\text{ above minimum of Y}}{100 + \%\text{ above minimum of X}}$

$$\frac{710}{515} = \frac{100 + \text{Y}}{100 + 3} \Rightarrow \text{Y} = +\ 42\ \%$$

Hence Y gets 42% above minimum.

Note: Similarly, if the % marks is below minimum, formula would have been,

$$\frac{\text{Marks of Y}}{\text{Marks of X}} = \frac{100 - \%\text{ below minimum of Y}}{100 - \%\text{ below minimum of X}}$$

Remember,

$$\frac{\text{Marks of Y}}{\text{Marks of X}} = \frac{100 \pm \%\text{ above/below minimum of Y}}{100 \pm \%\text{ above/below minimum of X}}$$

Here, (+) ve sign indicates above minimum and (–)ve sign indicates below minimum.

E-16 If a is x more then b and b is $y\%$ less than a, find the relation between x and y.

S-16 *[Refer 5.12]*

$$y\% = \frac{x}{100+x} \times 100\%$$

$$\Rightarrow \quad y = \frac{x \times 100}{100+x} \quad \Rightarrow \quad \frac{100+x}{100 \times x} = \frac{1}{y}$$

$$\Rightarrow \quad \frac{1}{y} - \frac{1}{x} = \frac{\mathbf{1}}{\mathbf{100}}.$$

NB: *y and x represent only the numerical value, i.e. if a is 3% more than b put only x = 3.*

E-17 The ratio of number of boys and girls in a school is 3 : 2 if 20% of the boys and 25% of the girls are holding scholarship, find the % of school students who

(a) hold scholarship

(b) do not hold scholarship

S-17 Percentage of scholarship holders

= (Boys × % boys who are scholarship holders) + (Girls × % girls who are scholarship holders)

$$= \left(\frac{3}{2+3} \times 20\right) + \left(\frac{2}{3+2} \times 25\right) = 22.$$

Similarly, percentage of non scholarship holders = $\left(\frac{3}{2+3} \times 80\right) + \left(\frac{2}{2+3} \times 75\right) = 78$

(Since 100 – 20 = 80, 100 – 25 = 75)

E-18 Groundnut oil is now being sold at Rs 27 per kg. During last month its cost was Rs 24 per kg. Find by how much % a family should reduce its consumption, so as to keep the expenditure the same.

S-18 New rate = Rs 27/kg

Original rate = Rs 24/kg.

∴ Change in rate = 27 – 24 = Rs 3

Here, the % change in consumption can be found out directly without finding the % change in rate by the following short-out method.

Short-cut: Since, % change in rate = $\frac{\text{rate change}}{\text{old rate}} \times 100$

So, % change in consumption = $\frac{-\text{ rate change}}{\text{New rate}} \times 100$ (observe the difference in the denominator of these two formula)

$$= \frac{-3}{27} \times 100\ \% = -11\frac{1}{9}\ \%$$

Hence, family has to reduce its consumption by $\mathbf{11\frac{1}{9}}$ %.

E-19 A reduction of Rs 2 per kg enables a man to purchase 4 kg more sugar for Rs 16. Find the original price of sugar.

S-19 Here expenditure if fixed (= Rs 16), but as the rate **reduces** (by Rs 2/kg), so, the quantity of sugar available **increases** (by 4 kg). Let original price be Rs x/kg, using the formula 3. *[Refer 5.11]*

$$\frac{\text{Expenditure}}{x} + \text{change in quantity available} = \frac{\text{Expenditure}}{\text{New rate}}$$

$$\Rightarrow \quad \frac{16}{x} + 4 = \frac{16}{x-2} \quad \Rightarrow \quad x^2 - 2x - 8 = 0 \quad \Rightarrow \quad (x = 4)\ (x - 2) = 0 \quad \Rightarrow x = 4 \text{ or } -2$$

Considering the +ve value, original price = **Rs 4 per kg**.

E-20 If the price of coffee is increased by 3%, by how much % must a housewife reduce her consumption of coffee, so as to have no extra expenditure?

S-20 Refer 5.10.1

$$\text{For fixed expenditure, \% change in consumption} = \frac{\text{\% change in rate}}{100 + \text{\% change in rate}} \times 100$$

$$\therefore \quad \text{decrease in consumption} = \frac{100 \times 3}{100 + 3}\,\% = \frac{300}{103}\,\%.$$

$$\therefore \quad \text{\% decrease in consumption} = 2\,\frac{94}{103}\,\%$$

The housewife must reduce her consumption of coffee by $\mathbf{2\,\frac{94}{103}\,\%}$.

E-21 If from a man's salary of Rs 'S'', p% is deducted on house rent, k% of the rest on education and q% of the rest on food, and still there is balance Rs 'b' left, find the value of 'S'.

S-21 Here we know that this is a case of **dependent activity**, also refer the chapter on Fraction.

$\therefore \quad (1 - p\,\%) \times (1 - k\,\%) \times (1 - q\,\%)$ of $S = b$.

$$\Rightarrow \quad S = \frac{b}{(1 - p\,\%) \times (1 - k\,\%) \times (1 - q\,\%)}.$$

E-22 From a man's salary, 10% is deducted on tax, 20% of the rest is spent on education, and 25% of the rest is spent on food. After all these expenditures, he is left with Rs 2,700. Find his salary.

S-22 Using the relation of E-21, we get,

$$\frac{(100 - 10)}{100} \times \frac{(100 - 20)}{100} \times \frac{100 - 25}{100} \times \text{salary} = 2700$$

$$\frac{90}{100} \times \frac{80}{100} \times \frac{75}{100} \times \text{salary} = 2700$$

$$\therefore \quad \text{Salary} = \frac{2700}{90 \times 80 \times 75} \times 100 \times 100 \times 100 = \mathbf{5000.}$$

E-23 When the price of sugar was increased by 32%, a family reduced its consumption in such a way that the expenditure on sugar was only 10% more than before. If 30 kg were consumed per month before, find the new monthly consumption.

S-23 We know that, expenditure = price × consumption.

So, new expenditure becomes 110% of 30

$\Rightarrow \quad \frac{110}{100} \times 30 = \frac{132}{100} \times$ new consumption [Expenditure increases by 10%]

$\therefore$ New consumption = **25kg.**

E-24 A man's income is increased by Rs 1, 200 and at the same time, the rate of tax to be paid is reduced from 12% to 10%. He now pays the same amount of tax as before. What is his increased income if 20% of his income is exempted from tax in both cases?

S-24 Since, the same percentage of his income is exempted from tax in both cases, this data is not to be considered.

Now, let x be the increased income.

Amount of Tax_1 = Amount of Tax_2

$(x - 1200) \times 12\% = x \times 10\%.$

x = **Rs 7,200 = income after increase.**

E-25 When N is reduced by 4, it becomes 80% of itself. What is the value of N?

(Bank P.O. '91)

S-25 The problem implies that (100 – 80) % of $N = 4$

$$20\% \text{ of } N = 4 \quad N = \frac{4}{20} \times 100 \qquad \text{(Since } 100 - 80 = 20)$$

$\therefore \qquad N =$ **20.**

E-26 When N is increased by 6, it becomes 102% of itself. What is the value of N? **(IA '84)**

S-26 The problem implies that

$(102 - 100)\ \%$ of $N = 6$

$$\Rightarrow \qquad 2\% \text{ of } N = 6 \Rightarrow N = \frac{6 \times 100}{2}$$

$\therefore \qquad N =$ **300.**

E-27 Increase

(i) 200 by 60 % (ii) 11 by 100 %

(iii) 35 by 200 % (iv) 48 by $12\frac{1}{2}$ %

(v) 1000 by 3.5 %

S-27 If a number (N) is to be increased by $p\%$ then multiply N by $\frac{100 + p}{100}$ *[Refer 5.8]*

(i) So, 200 becomes $200 \times \frac{(100 + 60)}{100}$, i.e. **320.**

(ii) 11 increases to $11 \times \frac{100 + 100}{100}$, i.e. **22.**

(iii) Increased number $= N \times \frac{100 + p}{100}$

$$= 35 \times \frac{100 + 200}{100} = \mathbf{105.}$$

(iv) Increased number $= N \times \dfrac{100 + p}{100}$

$$= 48 \times \frac{100 + 12\frac{1}{2}}{100}$$

$$= 48 \times \frac{225}{200}$$

$$= 48 \times \frac{9}{8} = \mathbf{54}.$$

(v) Increased number $= N \times \dfrac{100 + p}{100}$

$$= 1000 \times \frac{100 + 3.5}{100}$$

$$= 10 \times 103.5 = \mathbf{1{,}035}.$$

E-28 Decrease

(i) 200 by 40 % (ii) 16 by 25 %

(iii) 216 by $37\frac{1}{2}$ % (iv) 300 by 2.5 %

(v) 1000 by $\frac{1}{5}$ % (vi) 50 by $12\frac{1}{2}$ %

S-28 In order to decrease a number N by p%, multiply N by $\dfrac{100 - p}{100}$. *[Refer 5.11]*

(i) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 200 \times \frac{100 - 40}{100} \quad \mathbf{120}.$$

(ii) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 16 \times \frac{100 - 25}{100} = \mathbf{12}.$$

(iii) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 216 \times \frac{100 - 37\frac{1}{2}}{100}$$

$$= 216 \times \frac{62\frac{1}{2}}{100}$$

$$= 216 \times \frac{125}{200}$$

$$= 216 \times \frac{5}{8} = \mathbf{135}.$$

(iv) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 300 \times \frac{100 - 2.5}{100}$$

$$= 3 \times 97.5 = \mathbf{292.5}.$$

(v) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 1000 \times \frac{100\frac{1}{5}}{100}$$

$$= 10 \times \frac{499}{5} = \mathbf{998}.$$

(vi) Decreased number $= N \times \dfrac{100 - p}{100}$

$$= 50 \times \frac{100 - 12\frac{1}{2}}{100}$$

$$= 50 \times \frac{175}{200}$$

$$= 50 \times \frac{7}{8} = \mathbf{43.75}.$$

E-29 If 10% of an electricity bill is deducted, Rs 45 is still to be paid. How much was the bill?

S-29 Here Rs 45 refers to (100 – 10) % of the bill.

Since 90% of bill = 45

$\Rightarrow$ 100% of bill $= \dfrac{45}{90} \times 100 = 50$

Hence the bill was 50.

E-30 The weight of a sand bag is 40 kg. In a hurry, it was weighed as 40.8 kg. Find the error percentage. **(Bank PO '89)**

S-30 $\% \text{ Error} = \dfrac{\text{False weight} - \text{Actual weight}}{\text{Actual weight}} \times 100$

$$= \frac{40.8 - 40}{40} \times 100 = 2\%$$

$\therefore$ The error is **2%**.

E-31 If the price of one kg of cornflakes is increased by 25%, the increase is Rs 10. Find the new price of cornflakes per kg. **(C.D.S '86)**

S-31 Using the formula (1)

$$\text{Original price} = \frac{\text{Difference in price}}{\text{Difference in per cent}} \times 100$$

$$= \frac{10}{25} \times 100 = 40$$

$$\therefore \text{ New price} = 40 \times \frac{125}{100} = 50$$

$\therefore$ New price of cornflakes per kg is **Rs 50.**

E-32 The price of a book is reduced by 25%, what is the ratio of

(i) change in price to the old price?

(ii) old price to the new price?

Also find

(iii) the factor by which the old price should be multiplied to give the new price.

S-32 Consider the old price = Rs 100.

(i) $\dfrac{\text{Change in price}}{\text{Old price}} = \dfrac{25}{100} = \mathbf{\dfrac{1}{4}}$

(ii) $\dfrac{\text{Old price}}{\text{New price}} = \dfrac{100}{75} = \mathbf{\dfrac{4}{3}}$

(iii) Refer 5.8

$$\text{Decreased price (new price)} = \text{old price} \times \frac{100 - p}{100}, \text{ where } p = 25$$

$$\text{New price} = \text{old price} \times \left(\frac{75}{100}\right)$$

So the required factor is $\mathbf{\dfrac{3}{4}}$.

E-33 If A is more than B by 10%, then find

(i) $B = A \times ?$

(ii) $A = B \times ?$

(iii) $\dfrac{A}{B} = ?$

S-33 Since A is more than B by 10%,

(i) So $A = B \times \frac{100+10}{100}$

$\Rightarrow \quad \boldsymbol{B = A \times \frac{10}{11}}$.

(ii) $\boldsymbol{A = B \times \frac{11}{10}}$.

(iii) $\frac{A}{B} = \frac{100+10}{100}$

$\therefore \quad \frac{A}{B} = \boldsymbol{\frac{11}{10}}$.

E-34 If X is 20% less than Y, then find

(i) $\frac{X}{Y} = ?$

(ii) $\frac{Y-X}{Y} = ?$

(iii) $\frac{X}{Y-X} = ?$

S-34 Since X is 20% less than Y, therefore

(i) $\frac{X}{Y} = \frac{100-20}{100}$

$\frac{X}{Y} = \boldsymbol{\frac{4}{5}}$.

(ii) X is 20% less than Y, therefore, if Y is 100 then X is 80

$\Rightarrow \quad \frac{Y-X}{Y} = \frac{100-80}{100}$

$\therefore \quad \frac{Y-X}{Y} = \boldsymbol{\frac{1}{5}}$.

(iii) $\frac{X}{Y-X} = \frac{80}{100-80}$

$\frac{X}{Y-X} = \boldsymbol{\frac{4}{1}}$.

E-35 If $2\frac{1}{2}$ % of the weight of a table is 0.2 kg, then what will be 120% of it? **(C.D.S '83)**

S-35 $2\frac{1}{2}$ % of weight = 0.2 kg (use the unitary method)

$$\Rightarrow \quad 120\% = \frac{0.2}{2\frac{1}{2}} \times 120 = \frac{0.2 \times 120}{5} \times 2 = 9.6$$

$\therefore$ 120% of the weight of the table is 9.6 kg.

REGULAR PROBLEMS

(1) The number 0.05 is what percentage of 20?
(a) 1.5 (b) 0.025 (c) 0.25 (d) 2.5 (e) 25

(2) 12 is 0.2% of?
(a) 2400 (b) 600 (c) 240 (d) 6000 (e) 24

(3) In an election, one of the two candidates got 30% of the total votes polled, but he lost by 210 votes. What was the total number of votes polled?
(a) 525 (b) 700 (c) 610 (d) 300 (e) 520

(4) There are 850 students in a class. Out of these, 44% are Muslims, 28% Hindus, 10% Sikhs and remaining students belong to the other communities. How many students are there of other communities? **(RRB Secunderabad, '01)**
(a) 173 (b) 143 (c) 153 (d) 163 (e) 133

(5) The price of an article is cut by 10%, to restore to its original value, the new price must be increased by:
(a) 10% (b) $9\frac{1}{11}$% (c) 11% (d) $11\frac{1}{9}$% (e) 90%

Hint: Use the concept: if A is x% less than B, then B exceeds A by $\frac{x}{100-x}$%

$A \downarrow$ new price, $B \downarrow$ original price

(6) The difference between one-fifth of 1000 and one-fifth per cent of 1000 is:
(a) 800 (b) 80 (c) 198 (d) 998 (e) 0

(7) A man spends 80% of his income and saves the rest. What percentage of his expenditure does he save?
(a) 20 (b) 25 (c) 30 (d) 40 (e) Data insufficient

(8) 15% of x subtracted from x is equal to multiplying x by which number?
(a) 0.15 (b) $\frac{23}{20}$ (c) 115 (d) $\frac{17}{20}$ (e) $\frac{0.85}{100}$

(9) A pudding is made of 400 gm sugar, 200 gm of eggs, 800 gm of flour and 100 gm of dry fruit. The percentage of sugar present in the pudding is :
(a) $11\frac{1}{9}$% (b) 40% (c) 20% (d) $26\frac{2}{3}$% (e) $12\frac{1}{2}$%

(10) If 16% of 40% of a number is 8 then the number is
(a) 200 (b) 225 (c) 125 (d) 325 (e) 512

(11) The radius of a circle is so increased that its circumference increases by 5%. Then the area of the circle will increase by:
(a) 10% (b) 25% (c) 10.25% (d) 12.5% (e) 5%

Hint: Refer 5.10

(12) 40% of the population of a town are men and 35% are women. If the number of children are 20000, then the number of men will be:
(a) 3200 (b) 80000 (c) 32000 (d) 3,20,000. (e) 2,00,000.

(13) A medical student has to secure 40% marks to pass. He gets 80 and fails by 60 marks. Find the maximum marks.
(a) 150 (b) 250 (c) 350 (d) 450 (e) 500

Hint: Refer 5.4.1. Here, maximum marks is the (base) number, because % pass marks is related as 40% of maximum marks

(14) 40% of the students of a college are from West Bengal and out of this, 40% are from Kolkata. What % of the students are not from Kolkata?
(a) 60 (b) 16 (c) 40 (d) 84 (e) 20

(15) A school has a student population of 560. The number of girls is $14\frac{2}{7}$ % of the number of boys. How many girls are in the school?
(a) 100 (b) 70 (c) 80 (d) 140 (e) 240.

Hint: $14\frac{2}{7}\% = \frac{1}{7}$

(16) A man saves $3\frac{1}{3}\%$ from his salary of Rs 750 every month. In how many months will he be able to save an amount equal to his monthly salary?
(a) 40 months (b) 50 months (c) 30 months (d) 45 months (e) None of these

(17) In an examination, 'P' scored 130 points, which are 10 points above 40%, and Q scored 75%. The points scored by Q are
(a) 225 (b) 250 (c) 200 (d) 275 (e) 300

(18) When 75% of a number is added to 75, the result is the number again. The number is
(a) 150 (b) 300 (c) 100 (d) 450 (e) 350

(19) The percentage of total quantity represented by a 12° sector in a circle graph (pie diagram) is,
(a) 12% (b) 24% (c) $33\frac{1}{3}\%$ (d) $3\frac{1}{3}\%$ (e) 36%

Hint: Total quantity in a circle graph (pie diagram) = 360°

(20) If the price of sugar is increased by 7%, then by how much per cent should a housewife reduce her consumption of sugar, to have no extra expenditure
(a) 7 over 107% (b) 107 over 100% (c) 100 over 107%
(d) 7% (e) 93 over 100%.

Hint: Refer 5.10.1

(21) What is the total number of customers in a shop on a particular day, when 31% of them purchases on credit, and the number of those who do cash purchase exceeds the number of credit purchases by 247?
(a) 605 (b) 560 (c) 650 (d) 1650 (e) 620

(22) 5 out of 2250 parts of the earth is sulphur. The percentage of sulphur in the earth is
(a) 11 over 50% (b) 2 over 9 (c) 1 over 45 (d) 2 over 45 (e) None of these

(23) In Mathematics examination, a student scored 30% in the first paper of 180 marks. How much % marks should he score in the second paper of 150 marks if he is to get an overall percentage of at least 50%
(a) 20 (b) 74 (c) 30 (d) 65 (e) 70

Tips: Do not try to calculate the marks obtained in each paper, because here the % marks are used in the problem and also % marks is to be foundout. Total marks = 180 + 150 = 330, As per question, 50% (330) = 30% of 180 + x% of 150.

(24) Nagamani had a car to sell. Loknayak offered him a sum of money for the car that he refused as it was 13%, below its value. Loknayak then offerred Rs 450 more and the second offer was 5% more than the estimated value. What was the value of the car?
(a) Rs 3000 (b) Rs 2500 (c) Rs 3800 (d) Rs 2800 (e) None

Answers

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (d)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (d)
10. (c)	**11.** (c)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (a)	**18.** (b)
19. (d)	**20.** (a)	**21.** (c)	**22.** (b)	**23.** (b)	**24.** (b)			

REAL PROBLEMS

(1) Which number is 40% less than 360?
(a) 90 (b) 144 (c) 216 (d) 270 (e) 288

(2) p is 6 times that q. By what per cent is q less than p?
(a) 500 (b) $83\frac{1}{3}$ (c) 5 (d) 20 $16\frac{2}{3}$

(3) If 90% of A = 30% of B and $B = x\%$ of A, then the value of x is
(a) 600 (b) 800 (c) 900 (d) 300 (e) None

(4) If $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of a number is 12 then 30 per cent of the number will be **(BSRB, Chennai, 2000)**
(a) 48 (b) 64 (c) 54 (d) 42 (e) None of these

(5) A piece of cotton cloth 20 m long, shrinks by 0.5% after washing. The length of the cloth after washing is
(a) 19.5 m (b) 19.9 m (c) 19.95 m (d) 19.92 m (e) 19.05 m.

(6) The percentage by which 120 is to be diminished to get 90 is: **(RRB Kolkata, '01)**
(a) 30 (b) 25 (c) 20 (d) $33\frac{1}{3}$ (e) None of these

(7) Two numbers are 20% and 25% less than the third number. By how much per cent is the second number to be enhanced to make it equal to the first number?
(a) $6\frac{2}{3}$ (b) $6\frac{1}{4}$ (c) 25 (d) $33\frac{1}{3}$ (e) 20

Tips: When any number 'x' is related to two or more than two numbers in terms of percentage, then it is convenient to assume $x = 100$

(8) 2 over 3 is? per cent of 5 over 7. The value of (?) is
(a) 90 (b) 93.33 (c) 94 (d) 9 (e) None of these

(9) There is an increase of 30% in the production of milk chocolates in Amul Dairy in one month. If now it is 9100 milk chocolates per month, what was it one month ago?
(a) 13000 (b) 10300 (c) 8400 (d) 7000 (e) 11700

(10) A positive number is by mistake divided by 6 instead of being multiplied by 6. What is the % error on the basis of correct answer?
(a) 3 (b) 97 (c) 17 (d) 83 (e) 100

(11) When 30 per cent of a number is added to another number the second number increases to its 140 per cent. The second number = $x\%$ of the first number. The value of x is **(Bank of Baroda PO, '99)**

(a) 130 (b) 75 (c) $133\frac{1}{3}$ (d) $33\frac{1}{3}$ (e) 110

(12) If the length of a rectangle is increased by 20% and the breadth is reduced by 20%, what will be the effect on its area? **(Guwahati PO, '99)**

(a) 4% increase (b) 6% increase (c) 4% decrease (d) No change (e) None of these

(13) If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area? **(SBI PO, '99)**

(a) No change (b) 16% increase (c) 8% decrease (d) 16% decrease (e) None of these

(14) If two numbers are respectively 20% and 50% of the third number, then what % is the first number of the second?

(a) 30 (b) 70 (c) 40 (d) 30 (e) 50.

(15) In measuring the side of a square, an error of 5% in excess is made. The error percentage in the calculated area is

(a) $10\frac{1}{4}$ (b) $10\frac{3}{4}$ (c) $1\frac{3}{4}$ (d) 25 (e) 5

(16) A number exceeds by 40 when added by 20% of itself. The number is

(a) 200 (b) 60 (c) 80 (d) 320 (e) 120

(17) A rainy day occurs once in every 25 days. Half of the rainy days produce rainbows. The percentage of days having no rainbows is :

(a) 2 (b) $12\frac{1}{2}$ (c) 98 (d) $87\frac{1}{2}$ (e) 50

(18) In a class of 52 students, 25% are rich and others are poor. There are 20 females in the class, of whom 55% are poor. How many rich males are there in the class? **(NABARD, '96)**

(a) 13 (b) 4 (c) 39 (d) 2 (e) 28

(19) When any number is divided by 12, then dividend becomes $\frac{1}{4}$ of the other number. By how much per cent is first number greater than the second number? **(BSRB, Chennai, 2000)**

(a) 200 (b) 150 (c) 300 (d) data inadequate (e) None

(20) Naresh's monthly income is 30% more than that of Raghu. Raghu's monthly income is 20% less than that of Vishal. If the difference between the monthly incomes of Naresh and Vishal is Rs 800, what is the monthly income of Raghu? **(Baroda PO, '99)**

(a) Rs 16000 (b) Rs 20000 (c) Rs 12000 (d) Data inadequate (e) None

Hint: Since the difference of income has been given in respect of Naresh and Vishal. So, at first, find the relation of income for Naresh with Vishal and equate the difference.

(21) The rate for admission to an exhibition was Rs 5 and was later reduced by 20%. As a result, the sale proceeds increased by 44%.

The percentage increase in attendance was:

(a) 80 (b) 24 (c) 64 (d) 20 (e) None of these

Hint: Rate $\times$ attendance = sale proceeds

6 AVERAGE

6.1 INTRODUCTION

The idea of **average** is not new to us. We all are familiar with the following types of statements.

(i) The average runs scored by Sachin Tendulkar in a series is 72.

(ii) The average marks secured by Kana is 78%.

If a man earns Rs 40, Rs 50, Rs 56, Rs 46 and Rs 48 on five consecutive days of a week, then he earns a total of Rs (40 + 50 + 56 + 46 + 48) = Rs 240.

To find his average earning per day, his total earning is divided by the number of days, i.e.,

$$\text{Average} = \frac{240}{5} = \text{Rs } 48$$

Average earning does not mean that he earned Rs 48 everyday . But had he earned Rs 48 everyday, then his total earnings would have also been Rs 240 in 5 days.

Hence, to find the average of given quantities:

Step 1 The given quantities are added to get a **Sum**

Step 2 The **Sum** is divided by the **Number of items** to get the **Average.**

$$\therefore \quad \frac{\text{Sum of all the items}}{\text{Number of items}} = \text{Average} \qquad \textbf{(1)}$$

Note: The average is also called the **Mean.**

The quantities, whose average is to be determined, *should be in the same unit.*

Hence, $$\boxed{\text{Sum of all the items} = \text{Average} \times \text{no. of items}} \qquad \textbf{(2)}$$

6.2 AVERAGE OF DIFFERENT GROUPS

Sometimes, the average of two different groups are known and the average of a third group (made by combining these two groups) is to be found out.

Let,

	Group 1	+	Group 2	makes	Combined Group (1 + 2)
No. of items =	m		n		$m + n$
Average =	a		b		A
Sum of all items =	ma		nb		$ma + nb$

$$\text{Therefore, average of combined group} = \frac{\text{Sum of all items}}{\text{No. of items}}$$

$$\boxed{A^* = \frac{ma + nb}{m + n}} \qquad \textbf{(3)}$$

*This formula is also applicable for more than two groups forming the combined group.

Example: The average weight of 17 girls is 20 kg and that of 23 boys is 22 kg. Find the average weight of the class.

Solution:

	Girls	Boys
No. in the class =	17	23
Average =	20	22

$$\therefore \quad \text{average weight of the class} = \frac{17 \times 20 + 23 \times 22}{17 + 23}$$

$$= 21.15 \text{ kg.}$$

6.3 ADDITION OR REMOVAL OF ITEMS AND CHANGE IN AVERAGE

Since, $\text{Average} = \dfrac{\text{Sum of the items}}{\text{Number of items}}$

So, the original average may change (increase/decrease), if number of items change. The number of items may change in the following two cases,

Case I

When one or more than one NEW items are added

Let the average of N items = A

Now, 'n' New items are **added** and the average increases or decreases by x, then

$$\text{Average of New items added} = A \pm \left(1 + \frac{N}{n}\right)x \qquad \textbf{(4)}$$

Use (–), when average decreases
(+), when average increases.

When only one New item is added, put $n = 1$, then

Value* of the New item added = $A \pm (N + 1)x$

Case II

When one or more than one items are removed

In this case, items are removed, so on placing $-\dfrac{N}{n}$ in place of $+\dfrac{N}{n}$ in formula (4), it becomes,

* Here, it deals with ONE item only, so, average has got no meaning and thus, 'average' is replaced by 'value' of that ONE item.

Example: When a man weighing 80 kg is replaced by another man in a group of five persons, the average weight decreases by 3 kg. What is the weight of new man?

Solution: Using formula (6),

Weight of new man – Weight of removed man = $-Nx$ (–ve, average decrease)

$\Rightarrow$ Weight of new man – 80 = -5×3

$\Rightarrow$ Weight of new man = 80 – 15

= 65 kg.

6.5 SOME PROBLEM-SPECIFIC FORMULAE

(i) Before 't' years, the average age of 'N' members of a family was 'T' years. If the average remains same even after one more member joins the family, then present age of new member = $T - Nt$.

Example: Four years ago, the average age of six members of a family was 26 years. On the birth of a child in the family, the average remains the same. Find the present age of the child.

Solution: Present age of the child = $26 - 6 \times 4$

= 2 years.

(ii) Out of the given numbers, if the average of first 'n' numbers is 'x' and that of last 'n' numbers is 'y', then

First number – last number = $n(x - y)$.

Example: The average temperature of June, July and August was 31°C. The average temperature of July, August and September was 30 °C. If the temperature of June was 29 °C, find the temperature of September.

Solution: In the given problem, four months have been indicated, i.e.

June,	July,	August,	September
1 (first)	2	3	4 (last)

Out of these, the average temperature of first three (n = 3) months = 31°C = x

and the average temperature of last three (n = 3) months = 30 °C = y

Then, by the formula in (ii)

Temperature of first month – temperature of last month = $n(x - y)$

$\Rightarrow$ temperature of June – temperature of September = 3(31 – 30).

$\Rightarrow$ 29 – temp. of September = 3

$\therefore$ Temperature of September = 26 °C

Solved Examples

E-1 The average age of students in section A of 40 students is 10 years and the average age of students in section B of 30 students is 12 years. Find the average age of students in both sections taken together.

S-1 Here, average of 40 + 30 students is to be found out. Refer 6.2

$$\text{Average} = \frac{ma + nb}{m + n} = \frac{40 \times 10 + 30 \times 12}{(40 + 30)} = \frac{760}{70} = 10.855 \text{ years}$$

$\therefore$ Average age of all the students = 10.86 years.

E-2 The average of 5 quantities is 6. The average of three of them is 4. What is the average of remaining two quantities?

S-2 Let the average of two quantities be x,
then as per question,

$$6 = \frac{3 \times 4 + 2 \times x}{5}$$

$\Rightarrow \quad x = 9$

$\therefore$ required average = 9.

E-3 30 pens and 75 pencils were purchased for Rs 510. If the average price of a pencil was Rs 2.00, find the average price of a pen.

S-3 [Refer 6.1], using the formula (2),
Average of quantities × Number of quantities = Sum of quantities
Here quantity is the cost of pen.
Let average price of pen be Rs y

$\therefore \quad 30 \times y + 75 \times 2 = 510$ = Sum of the cost.

$\Rightarrow \quad y = 12,$

$\therefore$ Average price of pen = **Rs 12.**

E-4 The average age of A and B is 20 years. If C were to replace A, the average would be 19 and if C were to replace B, the average would be 21. What are the ages of A, B and C ? **(MBA '82)**

S-4 Say, a, b, c are the ages of A, B, and C
Since the average age of A and B is 20 years, so,

$$\frac{a+b}{2} = 20$$

$\Rightarrow \quad a + b = 40$ **(i)**

As per question, when C replaces A, average drops by $19 - 20 = -1$,
So, using formula (6), we get, (refer 6.4)

$c - a = -2 \times 1$ **(ii)**

Similarly, when C replaces B, average increases by $21 - 20 = +1$,

$c - b = 2 \times 1$ **(iii)**

Adding (i), (iii) and (iii), we get

$c = 20,$

Then from (ii), $a = 22$ and from (iii), $b = 18$

$\therefore$ Age of A = 22 yrs, age of B = 18 yrs, age of C = 20 yrs.

E-5 The average monthly expenditure of a family was Rs 2,200 during first 3 months, Rs 2,550 during next 4 months and Rs 3,120 during last 5 months of the year. If the total saving during the year was Rs 1,260, find average monthly income.

S-5 Total yearly income = yearly expenditure + yearly saving

$= [2200 \times 3 + 2550 \times 4 + 3120 \times 5] + 1260$ = Rs 33,660

Average monthly income = $\frac{33660}{12}$ = **Rs 2,805.**

E-6 The average temperature on Tuesday, Wednesday and Thursday was 37 °C. The average temperature on Wednesday, Thursday and Friday was 38 °C. If the temperature on Friday was 39 °C, find the temperature on Tuesday.

S-6

	Average Temperature
Tue + (Wed + Thurs)	37 °C
Fri + (Wed + Thurs)	38 °C

It is same.

Here, Tuesday is replaced by Friday. So, using the relation 6 for *replacement of* one quantity only.
Replacing quantity – replaced quantity = Change in average × Number of quantity

$\therefore$ Temperature of Friday – Temperature of Tuesday = (+) 1 × 3

$\Rightarrow$ 39 – Temperature of Tuesday = + 3

$\Rightarrow$ Temperature of Tuesday = 39 – 3 = **36 °C**.

E-7 The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.

S-7 Here, weight of the teacher added and final average of the group increases by 0.5 kg. Since, here only one item (i.e. weight of one teacher) has been added, so using the formula *[Refer 6.3]*

$\therefore$ weight of teacher $= A + (N + 1)\ x$

$= 48 + (29 + 1) \times 0.5 = 63$ kg

E-8 There are 50 boys in a class. Their average weight is 45 kg. When one boy leaves the class, the average reduces by 100 g. Find the weight of the boy who left the class.

S-8 Since here only one item (i.e. weight of boy who leaves the class) has been removed, so, using formula (5),

Value of one item removed $= A - (1 - N)\ x$ (here, average drops)

$\Rightarrow \therefore$ weight of boy who left $= 45 - (1 - 50) \times 0.1$ [100 gm = 0.1 kg]

$= 45 + 4.9$

$= 49.9$ kg.

$\therefore$ weight of boy who left the class is **49.9 kg.**

E-9 The average of 11 results is 50. If the average of first six results is 49 and that of last six results is 52, find the sixth result.

S-9 Average of 11 results

1	2	3	4	5	6	7	8	9	10	11

Average of last 6 results = 52

Average of first 6 results = 49

From the above diagram, it is quite obvious that the 'sixth result' is included twice, once in the first six results and second in the last six results.

$\therefore$ Value of the sixth result = (Sum of first six results) + (Sum of last six results) – (Sum of 11 results)
$= 6 \times 49 + 6 \times 52 - 11 \times 50 =$ **56.**

E-10 A batsman's runs just before the last match of the season, adds up to 750. In his last two innings, he scores only 6 runs, and his average drops by 2. Find his final average of the season.

S-10 For a batsman, average $= \dfrac{\text{Total runs scored}}{\text{Total number of innings playing}}$

Suppose,
Just before the last match, the batsman played "N" number of innings.

$\therefore$ Original average $A = \dfrac{750}{N}$

Using formula (4),

average of two innings added $= A - \left(1 + \dfrac{N}{n}\right)x$ [(–)ve, average drops]

(3) Out of four numbers, the average of the first three is 15 and that of the last three is 16. If the last number is 19, then first number is:
(a) 15 (b) 16 (c) 18 (d) 19 (e) 22

Tips: 1 2 3 4
↓ ↓
15 16

(4) A cricket player makes 200 runs in the 15th over. In doing so, his average at the end of the 14th over increases by 10. What was his average at the end of 15th over? **(RRB Guwahati, '97)**
(a) 60 (b) 50 (c) 40 (d) 45 (e) 35

(5) The average of Suresh's marks in English and History is 55. His average marks in English and Science is 65. What is the difference between the marks that he obtained in History and Science? **(Baroda Bank PO, '97)**
(a) 40 (b) 60 (c) 20 (d) data inadequate (e) None

(6) A body weighs 121 grams and 125.44 grams on two different pans of a faulty balance. Its true weight will be:
(a) 4.44 gms (b) 120 gms (c) 123.22 gms (d) 130 gms (e) 122 gms

(7) The average of three consecutive odd numbers is 14 more than one-third of the first of these numbers. What is the last of these numbers?
(a) 17 (b) 19 (c) 15
(d) data inadequate (e) None of these

(8) Ten years ago, the average age of a family of four members was 24 years. Three children having been born, the average age of the family is same today. What are the present ages of children, if two children are identical twins and differ by two years from the younger one?
(a) 12, 12, 10 (b) 8, 8, 6 (c) 13, 13, 11 (d) 14, 14, 12 (e) None

(9) The captain of a cricket team of 11 players is 25 years and the wicket keeper is 3 years older. If the age of these two are excluded, the average age of the remaining players is 1 year less than the average age of the whole team. The average age of the whole team is
(a) 24 years (b) 21 years (c) 26 years (d) 22 years (e) 25 years

(10) What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the two fractions?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$ (e) None of these

(11) In a coconut groove, $(x + 2)$ trees yield 60 nuts per year, x trees yield 120 nuts per year and $(x - 2)$ trees yield 180 nuts per year. If the average yield per year is per tree be 100. Then the value of x is
(a) 8 (b) 4 (c) 12 (d) 10 (e) 14

(12) If the average weight of boys of Ram's age and height is 105 kg, and if Ram weighs 110% of the average, then the weight of Ram is
(a) 105 kg (b) data insufficient (c) 115.5 kg
(d) 110 kg (e) 107.5 kg

(13) After a certain number of matches, a bowler has had 200 runs knocked off him. In the next match, he takes 3 wickets for 52 runs and his average goes up by 1. The new average of the bowler is
(a) 9 (b) $8\frac{1}{3}$ (c) $9\frac{1}{3}$ (d) 8 (e) either (a) or (c)

Hint: Bowler's average = $\dfrac{\text{runs knocked off}}{\text{no. of wickets taken}}$, Refer formula no. 4

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$$= \frac{36}{210} \times 6{,}300$$
$$= \text{Rs } 1{,}080$$

Solved Examples

E-1 If $0.75 : x :: 5 : 8$ then find x. **(Bank P.O. '86)**

S-1 See 7.5, product of extremes = product of middle terms

$$0.75 \times 8 = x \times 5$$
$$\Rightarrow \quad x = \mathbf{1.2}.$$

E-2 The mean proportion of 0.32 and 0.02 is what? **(RB1 '83)**

S-2 If x be the required mean proportional then, *[Refer 7.6]*

$$\Rightarrow \quad x^2 = 0.32 \times 0.02$$
$$x = \mathbf{0.08}.$$

E-3 Find the third proportional to 16 and 24

S-3 Here 24 is the middle number *[Refer 7.6]*

$$\therefore \quad \text{third proportional} = \frac{(24)^2}{16} = 36.$$

E-4 Find the fourth proportional to 16, 4 and 4. **(L1C '83)**

S-4 $16 : 4 :: 4 : x$ *[Refer 7.6]*

$$\Rightarrow \quad x = \frac{4 \times 4}{16} = 1.$$

E-5 $\frac{3}{48}$ is what part of $\frac{1}{12}$? **(Indian Air Lines '88)**

S-5 Required part is the ratio of $\frac{3}{48}$ and $\frac{1}{12}$

$$\therefore \quad \text{part} = \frac{\frac{3}{48}}{\frac{1}{12}} = \frac{3}{48} \times \frac{12}{1} = \mathbf{\frac{3}{4}} \textbf{ part.}$$

E-6 7 is what part of 8?

S-6 Required part = $\mathbf{\frac{7}{8}}$.

E-7 The ratio of number of boys and girls in a school is 4 : 3. If there are 480 boys in the school, find the number of girls in the school.

S-7 $4 : 3 = 480 : x$

$$\Rightarrow \quad x = \frac{3 \times 480}{4} = \textbf{360 girls.}$$

E-8 Divide Rs 72 in the ratio 4 : 5

S-8 First part = $\dfrac{\text{First ratio term}}{\text{Sum of ratio terms}} \times \text{Total amount}$

$$= \frac{4}{(4+5)} \times 72 = \text{Rs } 32$$

S-24 L.C.M. of 3, 4, 5 and 6 is 60

So, the pens are distributed among A, B, C and D in the ratio

$$\frac{1}{3} \times 60 : \frac{1}{4} \times 60 : \frac{1}{5} \times 60 : \frac{1}{6} \times 60$$

i.e. 20 : 15 : 12 : 10.

$\therefore$ Total number of pens = $20x + 15x + 12x + 10x = 57x$

For minimum number of pens, $x = 1$

$\therefore$ The person should have at least **57 pens**.

REGULAR PROBLEMS

(1) The ratio of number of males to number of females in a club is 7 : 4. If there are 84 males in the club, the total number of members in the club are: **(BSRB, '78)**

(a) 126 (b) 132 (c) 136 (d) 148 (e) 168

(2) The fourth proportional to 5, 6 and 150 is:

(a) 30 (b) 125 (c) 180 (d) 120 (e) 135

(3) Two numbers are in the ratio of 3:5. If 9 be subtracted from each, they are in the ratio of 12 : 23. The first number is:

(a) 27 (b) 33 (c) 55 (d) 49 (e) None of these

Hint: See E-13 **(RRB Secunderabad, '01)**

(4) A map is drawn on the scale of 4 mm for each 16 km. Two places are shown on the map at the distance of 7.2 mm. How far away they are from each other?

(a) 22.3 km (b) 9.21 km (c) 28.8 km (d) 21.8 km (e) 25.4 km

Hint: Use the rule of proportion

(5) If a 13 m-long iron rod weighs 23.4 kg, then what will be the weight of 6 m-long iron rod? **(RRB Bhopal, '98)**

(a) 7.2 kg (b) 12.4 kg (c) 10.8 kg (d) 18 kg (e) None

(6) The ratio of two number is 3 : 8 and their difference is 115. The greater number is:

(a) 69 (b) 115 (c) 184 (d) 230 (e) 240

Hint: Assume the numbers as $3x$ and $8x$, $\left(\text{do not try to write as } \dfrac{\text{first number}}{\text{second number}} = \dfrac{3}{8}\right)$

(7) The sum of three numbers A, B and C is 98. If $A : B = \dfrac{2}{3}$ and $B : C = \dfrac{5}{8}$, then B is:

(a) 15 (b) 20 (c) 30 (d) 32 (e) 45

(8) If $3x = 8y$ and $5y = 9z$, then $\dfrac{x}{z} = ?$

(a) $\dfrac{72}{15}$ (b) $\dfrac{83}{15}$ (c) $\dfrac{9}{8}$ (d) $\dfrac{11}{83}$ (e) None

Hint: Do not try to calculate by finding $x : y : z$, but only put the value of x and z in terms of y in $\dfrac{x}{z}$.

8 PARTNERSHIP AND SHARE

8.1 INTRODUCTION

When more than one person invests in the same business jointly, the "profit or loss is shared in the ratio of the investments of the partners." The persons who invest money are called **Partners**.

8.1.1 Investment and Share

(a) If there is profit in the business run by A and B then,

$$\frac{\text{Amount of investment of } A \times \text{No. of months invested by } A}{\text{Amount of } B\text{'s investment} \times \text{No. of months invested by } B} = \frac{\text{Profit of } A}{\text{Profit of } B}$$

$$\Rightarrow \quad \frac{A\text{'s Monthly Equivalent of Investment (MEI)}}{B\text{'s Monthly Equivalent of Investment (MEI)}} = \frac{\text{Profit of } A}{\text{Profit of } B}$$

(b) If more than two persons invest money in a business
then, MEI of A : MEI of B : MEI of C = Profit for A : Profit for B : Profit for C

(c) If there is loss in business,
MEI of A : MEI of B : MEI of C = Loss of A : loss of B : loss of C.

(d) If the number of months invested or period of investment is the same for each partner, then,
A's investment : B's investment : C's investment = A's profit : B's profit : C's profit

Solved Examples

E-1 A and B together invested Rs 12,000 in a business. At the end of the year, out of a total profit of Rs 1,800, A's share was Rs 750. What was the investment of A?

S-1 Since profits are shared in the ratio of their investments

$$\therefore \quad \frac{A\text{'s investment}}{B\text{'s investment}} = \frac{\text{profit share of } A}{\text{profit share of } B} \qquad \text{(Money invested by } A \text{ and } B \text{ for the same period)}$$

$$= \frac{750}{1800 - 750} = \frac{750}{1050} = \frac{5}{7}$$

$$\therefore \quad \text{Investment of } A = \frac{5}{5+7} \times 12000 = \textbf{Rs 5,000}.$$

Let the total time be 12 months

$\therefore$ A's profit : B's profit : C's profit = MEI of A : MEI of B : MEI of C

$$= \frac{1}{6}\times\left(\frac{1}{6}\times 12\right):\frac{1}{3}\times\left(\frac{1}{3}\times 12\right):\frac{1}{2}\times 12$$

$$= \frac{1}{3}:\frac{4}{3}:6$$

$$= 1 : 4 : 18$$

$\therefore$ Share of $A = \dfrac{1}{1+4+18} \times 2300 =$ **Rs 100.**

REGULAR PROBLEMS

(1) The annual income of A is 10% less than that of B whose income is 20% more than that of C. If the monthly income of C is Rs 200, find the total annual income of A, B, and C together.
(a) Rs 7,046 (b) Rs 7,772 (c) Rs 6,872 (d) Rs 7,872 (e) None of these

(2) A sum of money is to be divided among A, B and C in the ratio 2 : 3 : 7. If the total share of A and B together is Rs 1,500 less than C, what is A's share in it? **(NIC, '80)**
(a) Rs 1,000 (b) Rs 1,500 (c) Rs 2,000
(d) Data insufficient (e) None of these

Hint: $7x - (2x + 3x) = 1,500 \quad \therefore x = 750 \quad \therefore A = 2x = 1,500.$

(3) Nirmal and Kapil started a business investing Rs 9,000 and Rs 12,000 respectively. After 6 months, Kapil withdrew half of his investment. If after a year, the total profit was Rs 4,600, what was Kapil's share in it?
(a) Rs 2,000 (b) Rs 2,600 (c) Rs 1900 (d) Rs 2,300 (e) None of these

(4) The ratio between the ages of Amar and Kabir at present is 2 : 3. After three years, the ratio of their ages will be 3 : 4. Find the present age of Amar in years.
(a) 6 (b) 9 (c) 15
(d) Data insufficient (e) None of these

Hint: $\dfrac{\text{Amar's age}}{\text{Kabir's age}} = \dfrac{2x-3}{3x-3} = \dfrac{3}{4}.$

(5) Rs 750 is distributed among A, B and C such that A's share : B's share = 2 : 3 and B's share : C's share = 6 : 5. The share of A is,
(a) Rs 150 (b) Rs 175 (c) Rs 200 (d) Rs 250 (e) None of these

(6) The cost of a black and white TV and a colour TV are in 3 : 8 and total price of both is Rs 12,100. The difference in their prices is
(a) Rs 6,600 (b) Rs 6,050 (c) Rs 5,500 (d) Rs 5,100 (e) Rs 5,000

Hint: $\dfrac{8x+3x}{8x-3x} = \dfrac{12,100}{?} \Rightarrow ? = \dfrac{5}{11}\times 12,100 = 5,500$

(7) A profit of Rs 450 is divided between two partners, one of whom has contributed Rs 1,200 for 5 months and the other Rs 750 for 4 months. How much amount the second partner received?
(a) Rs 300 (b) Rs 425 (c) Rs 150 (d) Rs 175 (e) None of these

9 MIXTURES

9.1 DEFINITION

Mixtures are generally of two types. When two different **ingredients** are mixed together, it is known as **simple mixture**, e.g. a mixture of water and milk; water and pure spirit.

When two or more simple mixtures (made of same ingredients of same or different proportions) are mixed together to form another mixture, it is known as a **compound mixture**.

9.2 ALLIGATION RULE

Alligation literally means "linking". The **alligation rule** states that,

'When different quantities of same or different ingredients, of different cost (value) are mixed together to produce a mixture of a mean cost (value), the ratio of their quantities are inversely proportional to the differences in their cost from the mean cost (value)'

$$\frac{\text{Quantity of smaller cost ingredient}}{\text{Quantity of larger cost ingredient}} = \frac{\text{Larger cost} - \text{Mean cost}}{\text{Mean cost} - \text{Smaller cost}}$$

Let C_1 = cost price of 1st ingredient

C_2 = cost price of 2nd ingredient

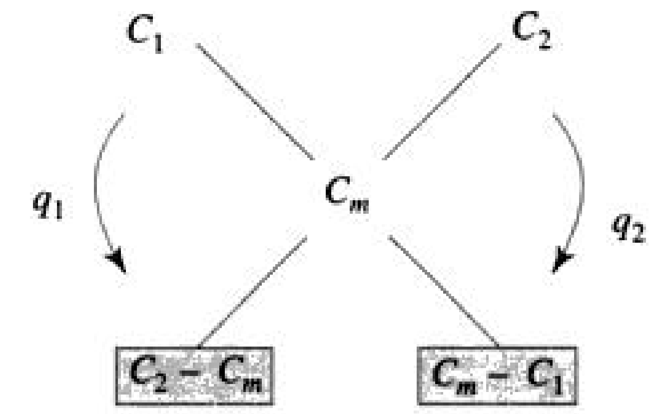
Now, these two ingredients are mixed to produce a mixture such that

C_m = cost price of the mixture (mean price)

then, the ratio of the quantity of two ingredients needed to produce this mixture is given by:

$$\frac{\text{quantity of 1}^{\text{st}}\text{ ingredient}}{\text{quantity of 2}^{\text{nd}}\text{ ingredient}} = \frac{C_2 - C_m}{C_m - C_1}$$

Diagram Representation



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9.4 SIX GOLDEN RULES TO SOLVE PROBLEMS ON MIXTURE

Assumptions

(a) *In Ratio form:* There is one given mixture of P kg (say) having two ingredients A and B in the ratio $a : b$

OR

(b) *In Percentage form:* There is one given mixture of P kg (say) containing two ingredients A and B. Ingredient A is $m\%$ of the mixture.

Rule 1

Quantity of each ingredient is to be found out by using the Ratio–Quantity Rule *[Refer 9.2.1]*

e.g. Quantity of $A = \frac{a}{a+b} \times P$, as per the assumption in (a)

Quantity of $B = \frac{b}{a+b} \times P$

Rule 2

This rule is applicable when the quantity of any ingredient is expressed as percentage of the mixture [as per the assumptions (b)]

e.g. quantity of $A = m\%$ of P, as per assumption (b) and quantity of $B = (100 - m)\%$ of P.

Rule 3

If x kg (say) of any ingredient is added to the mixture, then in the final, that particular ingredient increases by x kg and the amount of mixture also increases by x kg, but the quantity of other ingredient remains unchanged.

e.g. (a) **In Ratio form:** Let x kg of ingredient A be added to the mixture, then

$$\text{new ratio of } A \text{ and } B = \frac{\text{Quantity of } A}{\text{Quantity of } B} = \frac{\left(P \times \frac{a}{a+b}\right) + x}{P \times \frac{b}{a+b}}$$ *[Refer Rule-1]*

(b) **In Percentage form:** Let x kg of ingredient A be added to the mixture, then

$$\text{new \% of } A \text{ in the mixture} = \frac{\text{New quantity of } A}{\text{Amount of mixture}} \times 100\%$$

$$\Rightarrow \quad \text{\% of } A \text{ in the mixture} = \frac{m\% \text{ of } P + x}{P + x} \times 100\%$$ [See Assumption (b)]

Similarly,

$$\text{new \% of } B \text{ in the mixture} = \frac{\text{Quantity of } B}{\text{Amount of the mixture}} \times 100\%$$

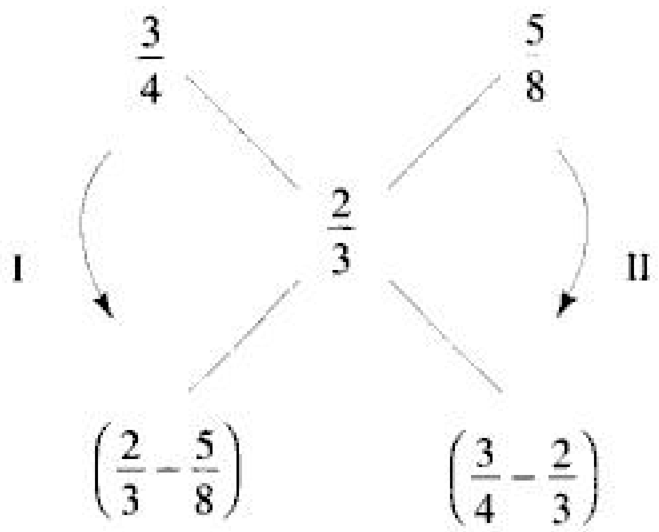
$$\text{new \% of } B \text{ in the mixture} = \frac{(100 - m)\% \text{ of } P}{P + x} \times 100\%$$

By applying Alligation Rule,

$$\frac{\text{Quantity of Mixture I in the new mixture}}{\text{Quantity of Mixture II in the new mixture}}$$

$$= \frac{\frac{2}{3} - \frac{5}{8}}{\frac{3}{4} - \frac{2}{3}}$$

$$= \frac{1}{2}$$



The required ratio of two mixtures is 1 : 2

Rule 6

Two mixtures of the same ingredients are mixed.

If x kg (say) of any ingredient A is added to the above mixture, then the value of x can be found out by the following equation:

x + quantity of A in Mix I + quantity of A in Mix II = quantity of A in new mixture.

Example: In two mixtures, spirit and water are related in the ratios of 3 : 5 and 7 : 4. 24 gallons of mixture I, 44 gallons of mixture II and 25 gallons of spirit are mixed together. What is the final ratio of spirit and water?

Solution: Here, 25 gallons of spirit is added to the mixture.

∴ quantity of spirit in both the mixtures and in the final mixture is to be considered.

Let the ratio of spirit and water in the final mixture be $m : n$.

Now, as per question,

quantity of spirit in Mix I + spirit in Mix II + pure spirit = quantity of spirit in final mixture

(By rule-6)

$$\Rightarrow \quad \left(\frac{3}{3+5} \times 24\right) + \left(\frac{7}{7+4} \times 44\right) + 25 = \frac{m}{m+n}\,(24 + 44 + 25)$$

$$\Rightarrow \quad \frac{9 + 28 + 25}{24 + 44 + 25} = \frac{m}{m+n}$$

$$\Rightarrow \quad \frac{m}{m+n} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{2}{1}$$

Therefore, the final ratio of spirit and water is 2 : 1

9.5 REMOVAL AND REPLACEMENT BY EQUAL AMOUNT

Case I

Removal of an amount from a mixture

Let the mixture contain A and B

Amount of mixture = M

$\therefore$ Cost price of mixture = $C_m = 40 \times \dfrac{100}{(100+25)}$ (Since 25% gain)

= 32 paise

$\therefore \quad \dfrac{q_{42}}{q_{24}} = \dfrac{32-24}{42-32} = \dfrac{8}{10} = \dfrac{4}{5}$

$\therefore \quad q_{42} = \dfrac{4}{5} \times 25 = \mathbf{20\ kg.}$ (Since q_{24} = 25 kg given)

E-6 A mixture of 40 litres of milk and water contains 10% water. How much water must be added to make 20% of water in the new mixture? **(FT, '92)**

S-6 $\dfrac{q_{\text{milk}}}{q_{\text{water}}} = \dfrac{90\% \text{ of } 40 \text{ litres}}{10\% \text{ of } 40 \text{ litres}}$

Let x litres of water be added, then,

$\dfrac{q_{\text{milk}}}{q_{\text{water}}} = \dfrac{(0.9 \times 40)}{(0.1 \times 40) + x} = \dfrac{80}{20}$ (Since 20% water in new mixure)

$\Rightarrow$ x = **5**, hence, 5 litres of water is added.

E-7 Five litres of water is added to a certain quantity of pure milk costing Rs 3 per litre. If by selling the mixture at the same price as before, a profit of 20% is made, what is the amount of pure milk in the mixture? **(SSC, '80)**

S-7 Sale price of mixture = Cost price of pure milk

$\Rightarrow \quad \dfrac{120}{100} \times$ CP of mixture = 3.0 (Since a profit of 20% is made)

$\Rightarrow$ CP of mixture = $\dfrac{3 \times 100}{120}$ = Rs 2.5 per litre

$\therefore$ Using formula (a), after assuming x = amount of pure milk

mean CP of mixture = 2.5 = $\dfrac{x \times 3 + 5 \times 0}{x+5}$ (Since CP of water = 0)

$\Rightarrow \quad x = 25$

$\therefore$ amount of pure milk = **25 litres**

NB: When water is mixed with any other ingredient to form a mixture such that
Sale price of mixture = Cost price of ingredient,
Then,

$\dfrac{\text{Quantity of water}}{\text{Quantity of ingredient}}$ = % profit made due to mixing of water

E-8 To 5 litres, of 20% sulphuric acid, 5 litres of 100% pure sulphuric acid is added. What is the strength of the acid in the mixture now?

S-8 % strength of acid in mixture = $\dfrac{5 \times 20 + 5 \times 100}{5+5}$ (here, quantity is in %) *[Refer 9.3]*

= **60%**.

E-17 A bottle is full of dettol. One-third of it is taken out and then an equal amount of water is poured into the bottle to fill it. This operation is done four times. Find the final ratio of dettol and water in the bottle.

S-17 The bottle originally contains dettol only
Let the bottle contain 1 litre of dettol originally,
So, using the formula [Refer 9.5 (Case II)]

$$\frac{\text{Amount of } A \text{ (dettol) left}}{\text{Amount of } B \text{ (water) left}} = \frac{\left(1-\frac{x}{a}\right)^n}{1-\left(1-\frac{x}{a}\right)^n}$$

$$\Rightarrow \qquad \frac{\text{Dettol}}{\text{Water}} = \frac{\left(1-\frac{\frac{1}{3}}{1}\right)^4}{1-\left(1-\frac{\frac{1}{3}}{1}\right)^4}$$

$$= \frac{\left(\frac{2}{3}\right)^4}{1-\left(\frac{2}{3}\right)^4}$$

$$= \frac{16}{65}$$

Finally, the bottle contains dettol and water in the ratio **16 : 65**.

REGULAR PROBLEMS

(1) How much chicory at Rs 4 a kg should be added to 15 kg of tea at Rs 10 a kg so that the mixture be worth Rs 6.50 per kg?
(a) 15 kg (b) 18 kg (c) 12 kg (d) 21 kg (e) 10 kg

Hint: Do not waste time by applying alligation rule to find the ratio of quantity of chicory and quantity of tea. It is better to use formula (4) of mean cost price directly. Assume quantity of chicory = x kg

(2) Six litres of a 20% solution of alcohol in water are mixed with four litres of 60% solution of alcohol in water. The alcoholic strength of the mixture is:
(a) 40% (b) 36% (c) 26% (d) 30% (e) 32%

Hint: Use formula (4) directly to find mean strength of the mixture.

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(16) An alloy contains copper and manganeese in the ratio of 8 : 5 and another alloy contains copper and tungsten in the ratio of 5 : 3. If equal weights of the two are melted together to form the third alloy, then the weight of manganeese per kg in the new alloy is:

(a) $\frac{79}{208}$ kg (b) $\frac{13}{5}$ kg (c) $\frac{5}{13}$ kg (d) $\frac{5}{26}$ kg (e) $\frac{8}{13}$ kg

(17) A cup of milk contains 3 parts of pure milk and 1 part of water. How much mixture must be withdrawn and water substituted ~~in~~ order that the resulting mixture may be half milk and half water?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$ (e) $\frac{1}{2}$

Answers

1. (b)	**2.** (d)	**3.** (e)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (a)	**9.** (b)
10. (e)	**11.** (c)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (a)	

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S-9 Total job assigned = $10 \times 40 = 400$ men days.

Say, after x days of the commencement of work, 4 men leave.

$\therefore$ 4 men work for x days. $\therefore$ $M_1D_1 = M_2D_2 + M_3 \times D_3$.

$10 \times 40 = 4 \times x + 6 \times 50 \Rightarrow x = \mathbf{25}$. (Since 6 men work throughout 50 days)

They left 25 days after the commencement of work.

E-10 Four lorries carrying 4 tons each move 128 tons in 8 days. In how many days will 6 lorries carrying 3 tons each move 540 tons?

S-10 $$\frac{N_1 \times D_1 \times R_1}{W_1} = \frac{N_2 \times D_2 \times R_2}{W_2}$$ *[Refer 10.2.1]*

$$\frac{4 \times 8 \times 4}{128} = \frac{6 \times D_2 \times 3}{540} \Rightarrow D_2 = \mathbf{30\ days}.$$

E-11 A hostel has provisions for 250 students for 35 days. After 5 days, a fresh batch of 25 students were admitted to the hostel. Again after 10 days, a batch of 25 students left the hostel. How long will the remaining provisions survive?

S-11

	$D_1 = 10$	$D_2 = x$	
After 5 days,	after 10 days,	remaining x days	
25 students added,	B 25 students left	C	D
	$N_1 = 250 + 25 = 275$	$N_2 = 275 - 25 = 250$	

Remaining food provisions at B (after 5 days) = $ND = 250 \times (35 - 5)$ student days

$\therefore$ $ND = N_1D_1 + N_2D_2 \Rightarrow (250 \times 30) = 275 \times 10 + 250 \times x$

$\Rightarrow x = \mathbf{19}$. $\therefore$ The remaining provisions would last for 19 days.

E-12 A garrison of 3000 men has provisions for 25 days, when given at the rate of 900 g per head. At the end of 11 days, a reinforcement arrives and it was found that now the provision will last 10 days more, when given at the rate of 840 g per head. What is the strength of reinforcement?

(MBA, '82)

S-12 Let strength of reinforcement be x

Remaining food provisions after 11 days = $3000 \times (25 - 11) \times 900$

Total men after 11 days = $(3000 + x)$

$\therefore$ $3000 \times 14 \times 900 = (3000 + x) \times 10 \times 840$ *[Refer 10.1]*

$\Rightarrow x = \mathbf{1{,}500}$. $\therefore$ The reinforcement had 1,500 men.

E-13 Six diesel engines consume 900 litres of diesel, when each one is running for 5 h a day. How much diesel will be required by 9 engines, each running 8 h a day when 5 diesel engines of former type consume as much diesel as 8 diesel engines of the latter type.

S-13 Using the formula,

$$N_1 \times D_1 \times \frac{R_1}{W_1} = N_2 \times D_2 \times \frac{R_2}{W_2}$$

Since 5 diesel engines of I type = 8 diesel engines of II type

$$\therefore \quad R_1 = \frac{1}{5} \quad \text{and} \quad R_2 = \frac{1}{8}$$

$W_1 = 900$ litres, $W_2 = ?$ (*Since Amount of work $\equiv$ Diesel consumption*)

11 TIME, WORK AND WAGES

In most of the problems on time and work, one of the following basic parameters is to be calculated:

(a) Time: Time needed by more than one person to complete a job or time for which a person(s) actually worked on the assigned job.

(b) Alone time: Time needed by single person to complete a job.

(c) Work: The amount of total work (assigned) or the part of total assigned work actually done.

11.1 BASIC CONCEPTS

Concept 1

Total amount of a complete job (or assigned job) = 1, always, unless otherwise specified.

Concept 2

If any person 'M' completes a job **alone** in t days, then **alone time** for 'M' = t

Concept 3

1 day's work by any person = $\left(\dfrac{1}{\text{alone time}}\right)^{\text{th}}$ part of total work

Example: Ram can polish the floor of a building in 16 days. Find the work done by Ram in one day.

Solution: Here, alone time for Ram = 16 days, so 1 day's work by Ram = $\frac{1}{16}$ th part of total work.

Concept 4

The reciprocal of 1 day's work gives the alone time i.e., alone time (or time to complete a job by a single person) = $\dfrac{1}{\text{1 day's work}}$

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(i) in proportion to the work done by each person, or
(ii) in proportion to the 1 day's work of each person, or
(iii) in inverse proportion to the alone time of each person.

Solved Examples

E-1 Tuktuki and Rasmani can do a job alone in 20 days and 30 days respectively. In how many days the job will be finished if they work together?

S-1 Here, we can use a direct formula, if $a = 20$, $b = 30$, then

$$\text{Combined required time} = \frac{ab}{a+b} \qquad \text{[formula derived by using concept (6)]}$$

$$= \frac{20 \times 30}{20 + 30} = 12 \text{ days.}$$

E-2 Mohan and Sohan can do a job in 12 days. Sohan alone can finish it in 28 days. In how many days can Mohan alone finish the work?

S-2 Using concept (5),
(Mohan + Sohan)'s 1 day work = Mohan's 1 day work + Sohan's 1 day work

$$\Rightarrow \frac{1}{12} = \text{Mohan's 1 day work} + \frac{1}{28}$$

$$\Rightarrow \text{Mohan's 1 day work} = \frac{1}{12} - \frac{1}{28} = \left(\frac{1}{21}\right)\text{th of work}$$

$$\therefore \text{Mohan's alone time} = \frac{1}{1/21} \qquad \text{[concept (4)]}$$

$$= 21 \text{ days}$$

Short-Cut if $T = 12$, $a = 28$, then

$$\text{Required time} = \frac{Ta}{a-T} = \frac{28 \times 12}{28 - 12} = 21 \text{ days.}$$

E-3 Mary and Maurice can do a piece of work in 10 days and 15 days respectively. They work together for 3 days and then Maurice leaves. Mary finishes the remaining work alone. In how many days is the total work finished?

S-3 Let the total work be finished in 'T' days.
Now, using the concept (8), and (7)

$$\frac{\text{no. of days Mary worked}}{\text{alone time}} + \frac{\text{no. of days Maurice worked}}{\text{alone time}} = 1$$

$$\Rightarrow \frac{T}{10} + \frac{3}{15} = 1$$

$$\Rightarrow \frac{T}{10} = 1 - \frac{1}{15}$$

$$\Rightarrow T = 10 \times \frac{4}{5} = 8$$

$\therefore$ Total work is finished in 8 days.

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(14) P and Q can weave a mat in 12 days, Q and R together do it in 15 days. If P is twice as good a workman as R, find in what time Q will do it alone?

(a) 30 days (b) 22 days (c) 18 days (d) 24 days (e) 20 days

Hint: $P = R/2$

(15) If 3 men and 2 boys together earn Rs 306 in 9 days while 7 men and 3 boys earn Rs 639 in the same time. In how many days will 8 men and 6 boys together earn Rs 376?

(a) 7 days (b) 5 days (c) 3 days (d) 6 days (e) 4 days

Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (e)	**7.** (c)	**8.** (b)	**9.** (e)
10. (b)	**11.** (e)	**12.** (c)	**13.** (d)	**14.** (e)	**15.** (e)			

Example: Two pipes A and B can separately fill a cistern in $7\frac{1}{2}$ and 5 minutes respectively and a waste pipe C can carry off 14 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 1 hour. How many litres does it hold?

Solution: After opening all three pipes, the full cistern gets emptied in 1 hour = 60 minutes.

$\Rightarrow$ Net work done by all three pipes in 1 minute = $-\frac{1}{60}$ (–)ve, for cisterm in emptied.

(or part or cistern emptied/filled in 1 minute)
As per concept (B-3),

$$(\text{work done by inlets in 1 minute}) - (\text{work done by outlets in 1 minute}) = -\frac{1}{60}$$

Here, the outlet is the waste pipe C.

$$\Rightarrow \left(\frac{1}{7\frac{1}{2}} + \frac{1}{5}\right) - (\text{work done by outlets in 1 minute}) = -\frac{1}{60}$$

$$\Rightarrow \text{work done by waste pipe in 1 minute} = \frac{2}{15} + \frac{1}{5} + \frac{1}{60}$$

$$= \frac{7}{20}$$

Using concept (C),

$$\text{Capacity of the cistern} = \frac{\text{flow rate in 1 min by waste pipe}}{\text{work done by waste pipe in 1 min}}$$

$$= \frac{14}{\frac{7}{20}} = \textbf{40 litres.}$$

Solved Examples

E-1 A fill pipe can fill $\frac{3}{4}$ of cistern in 12 minutes. In how many minutes can it fill $\frac{1}{2}$ of cistern?

S-1 Alone time = $\dfrac{\text{time of open}}{\text{work done}}$ (Concept A-1)

For the same pipe, alone time will be fixed, so,

$$\frac{\text{time of open}_1}{\text{work done}_1} = \frac{\text{time of open}_2}{\text{work done}_2}$$

$$\Rightarrow \quad \frac{12}{\frac{3}{4}} = \frac{x}{\frac{1}{2}}$$

$$\Rightarrow \quad x = \textbf{8 minutes.}$$

$$\Rightarrow \qquad \frac{50}{6\frac{1}{4}} = \frac{\text{Cost price}}{100}$$

$$\Rightarrow \qquad \text{Cost price} = \textbf{Rs 800}.$$

Example: If 30% loss on selling an article makes the trader to suffer a loss of Rs 45, then find the selling price of the article.

Solution: In the conventional method we have to find the cost price first and only then can we find the sale price, although the problem has not asked to find the cost price.

In such case, relation 13.3 is useful to find out the sale price directly.

$$\frac{\text{loss}}{\%\text{ loss}} = \frac{\text{Sale price}}{100 - \%\text{loss}}$$

$$\Rightarrow \qquad \frac{45}{30} = \frac{\text{Sale price}}{100 - 30}$$

$$\Rightarrow \qquad \text{Sale price} = \textbf{Rs 105}$$

13.4 AN ARTICLE SOLD AT TWO DIFFERENT SELLING PRICE

Since, the same article is sold at two different prices, so, cost price remains the same in both the cases. Using the relation (1), of 13.3,

$$\frac{\text{Cost price}}{100} = \frac{\text{Sale price}}{100 + \%\text{ gain}}$$

We can combine both the cases as

$$\frac{\text{Cost price}}{100} = \boxed{\frac{\text{Sale price}_1}{100 + \%\text{ gain}_1} = \frac{\text{Sale price}_2}{100 + \%\text{ gain}_2}} \quad \text{it can be used directly.}$$

Example: By selling an article for Rs 450, a man loses 25%. At what price will he sell in order to gain 50%?

Solution: Using the relation 13.4, we can directly find the second sale price, without finding the cost price of the article.

$$\frac{\text{Sale price}_1}{100 + \%\text{ gain}_1} = \frac{\text{Sale price}_2}{100 + \%\text{ gain}_2}$$

$$\frac{450}{\underset{\downarrow}{100 - 25}} = \frac{\text{Sale price}_2}{100 + 50}$$

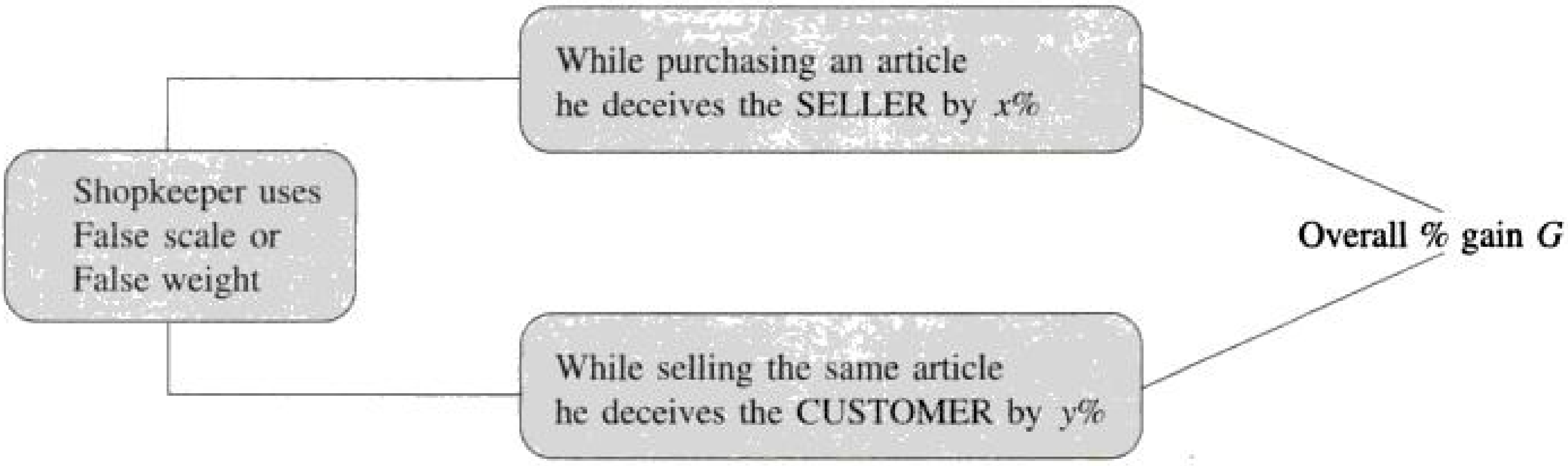
[(–)ve for % loss]

$$\Rightarrow \qquad \text{Sale price}_2 = \textbf{Rs 900}.$$

$$\Rightarrow \qquad G = \frac{100}{19} = 5\frac{5}{19}\%$$

$\therefore$ The gain percentage of the shopkeeper is $5\frac{5}{19}\%$

13.8 USE OF FALSE SCALE FOR PURCHASING AND SELLING AN ARTICLE



$$G = \left[\frac{(100+x)(100+y)}{100} - 100\right]\%$$

13.9 % GAIN OR % LOSS ON SELLING PRICE

Sometimes % gain or % loss is indicated on the basis of selling price, which is not actual. So, it needs to be recalculated on cost price which is known as real % gain or % loss, because % gain or % loss is always on cost price.

$$\text{Real \% gain or \% loss (on cost price)} = \frac{\text{\% gain or \% loss on sale price}}{100 - \text{\% gain or \% loss on sale price}}$$

Put (–)ve, when it is % loss.

13.10 % GAIN OR % LOSS ON WHOLE PROPERTY

Assume,

Percentage or part of the whole property being sold (*A*)	*% Gain or % Loss incurred by selling* (*B*)	*Product* $A \times B$
xth part or $x\%$ of whole or quantity x	% gain = $g_1\%$ (say)	$\Rightarrow x \times g_1$
yth part or $y\%$ of whole or quantity y	% loss = $l_1\%$	$\Rightarrow -y \times l_1^*$
Remaining zth part or $z\%$ of whole or rest quantity of whole (= z)	% gain = $g_2\%$	$\Rightarrow z \times g_2$

and so on, then.

[*(–)ve sign because it is loss]

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E-6 By selling a horse for Rs 455, a man loses 9%. If he sells it for Rs 555, what would be his gain or loss per cent?

S-6 [Refer 13.4], using the formula,

$$\frac{S_1}{100+x_1} = \frac{S_2}{100+x_2}$$

$$\Rightarrow \quad \frac{455}{100+(-9)} = \frac{555}{100+x_2} \quad \Rightarrow \quad \frac{455}{100-9} = \frac{555}{100+x_2}$$

$\therefore$ $x_2 = +\ 11\%$. [(+)ve sign indicates it is gain.]

$\therefore$ The man has a **gain of 11%**.

E-7 If a merchant estimates his profit as 20% of the selling price, what is his real profit per cent?

S-7 [Refer 13.9], using the formula,

$$\text{Real profit \% (\% profit on C.P.)} = \frac{\text{\% profit on S.P.}}{100 - \text{\% profit on S.P.}} \times 100$$

$$= \frac{20}{100-20} \times 100 = \mathbf{25\%}.$$

NB: Real % profit is always more than the % profit on S.P.

E-8 How much per cent above the cost price should a shopkeeper mark his goods so as to earn a profit of 26% after allowing a discount of 10% on the marked price?

S-8 [Refer 13.6], using the formula, Marked price (1 – % discount) = Cost price (1 + % gain)

[M = Marked price, C = Cost price]

$$M\ (100 - d) = C\ (100 + g) \quad \Rightarrow \quad M\ (100 - 10) = C\ (100 + 26)$$

$$\Rightarrow \quad M = \frac{126}{90}\ C = 1.4\ C = (1 + 0.4)\ C$$

i.e. M is + 0.4 or 40% above C

Marked price is **40%** above the cost price.

E-9 A vendor sells 10 oranges for a rupee gaining thereby 40%. How many oranges did he buy for a rupee?

S-9 Always the unit price is to be put.

i.e. Sale price for 1 orange = Rs $\frac{1}{10}$ [Refer 13.1], Using the formula, $\%\ x = \left(\frac{\text{SP}}{\text{CP}} - 1\right) \times 100.$

$$\Rightarrow \quad 40 = \left[\frac{1}{10 \times C} - 1\right] \times 100 \quad \Rightarrow \quad \frac{1}{10 \times C} = \frac{40}{100} + 1 = \frac{14}{10}$$

$\Rightarrow$ $C = \frac{1}{14}$. So, he bought **14 oranges** per rupee.

E-10 A cloth merchant says that due to slump in the market, he sells the cloth at 10% loss, but he uses a false metre-scale and actually gains 15%. Find the actual length of the scale. **(AAO '82)**

S-10 Here cost price is not equal to selling price because he sells the cloth at 10% loss.

[Refer 13.7], using the formula,

$$\frac{100+G}{100+x} = \frac{\text{True Scale}}{\text{False Scale}}$$

Here, overall gain G = 15% and loss x = – 10% [(–)ve sign for loss]

(15) A shopkeeper increased the price of a product by 50% and later on he reduce the price by 50%. Shopkeeper's loss was:
(a) 0% (b) 2.5% (c) 25% (d) 0.25% (e) 100%

(16) A trader bears a loss of 25% by selling 40 needles for a rupee. How many needles should he sell for a rupee in order to earn a profit of 20%?
(a) 23 (b) 20 (c) 25 (d) 27 (e) 30

Tips: Always consider the unit sale price in such problem, Unit sale price means sale price of one needle. So, $\dfrac{\frac{1}{40}}{100-25} = \dfrac{\frac{1}{x}}{100+20}$

(17) A shopkeeper purchased an electric heater marked at Rs 200 at successive discounts of 10% and 15% respectively. He spent Rs 7 on packaging and sold the electric heater for Rs 200. Find his gain percent.
(a) No loss or gain (b) 25 (c) 30
(d) 40 (e) 50

(18) A CD-music system when sold at a certain price gives a gain of 20%. If sold for thrice that price, the gain percent will be:
(a) 60 (b) 160 (c) 360 (d) 200 (e) 260

(19) What is the equivalent discount of the 20%, 10%, 5% discount series?
(a) 35 (b) $17\frac{1}{2}$ (c) 28 (d) 31.6 (e) 33

Hint: Refer text

(20) A shopkeeper bought 150 calculators at the rate of Rs 250 per calculator. He spent Rs 2500 on transportation and packing. If the marked price of calculator is Rs 320 per calculator and the shopkeeper gives a discount of 5% on the marked price, what will be the percentage profit gained by the shopkeeper?
(a) 20 (b) 14 (c) 15 (d) 16 (e) None of these
(BSRB Hyderabad PO, '99)

21) The cost price of 20 pencils is equal to the selling price of 25 pencils. The loss percent in the transaction is: **(ITI, '93)**
(a) 5 (b) 20 (c) 25 (d) 30 (e) 40

22) Anindita sells her car for Rs 5 lakhs and loses something. Had she sold it for Rs 5.60 lakh, her gain would have been double the former loss. Find the cost price of the car.
(a) Rs 5.50 lakhs (b) Rs 6.20 lakhs (c) Rs 5.20 lakhs (d) Rs 5.40 lakhs (e) None

Hint: Follow the concept of cost price and sale price

23) A dishonest milkman buys milk at Rs 6 per litre and adds one third of water to it and sells the mixture at Rs 7.20 per litre. The gain is:
(a) 40% (b) $\frac{80}{3}$% (c) 60% (d) 25% (e) 20%

Tips: Water added is also sold at Rs 7.20 per litre

24) A trader had 6 quintals of rice. He sold a part of it at 7% profit and the rest at 17% profit, so that he made a total profit of 11%. How much rice did he sell at 17% profit?
(a) 200 kg (b) 220 kg (c) 240 kg (d) 260 kg (e) None of these

Hint: Refer text 13.10

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so, if the value of P, R or T changes, then the value of simple interest (SI) will also change.
It implies that

— all three parameters P, R and T may change
— any two of these may change and one parameter remains fixed
— any one parameter may change and remaining two parameters are fixed

In all these cases, simple interest (SI) will change
In such cases,

$$\text{Change in simple interest} = \frac{\text{Product of fixed parameters}}{100} \times \underbrace{(\text{difference of product of variable parameters})}_{\text{Changing}}$$

Example: At 5% simple interest, a sum of Rs 500 fetches Rs 36 more than the amount of interest accrued on Rs 380 for the same period and rate. Find the time.

Solution: Here, only the principal is variable. The time and rate are fixed. Using the relation (in 14.2).

$$\text{Change in simple interest} = \frac{\text{product of fixed parameters}}{100} \times (\text{difference of product of variable parameters})$$

$$36 = \frac{T \times 5}{100} \times (\underbrace{500 - 380}) \longrightarrow \text{it is only difference because there is one variable parameter}$$

$\Rightarrow$ $T = 6$ years

Example: The simple interest on a certain sum for 4 years at 7.5% per annum is Rs 95 less than the simple interest on the same sum for 3.5 years at 10% per annum. Find the sum.

Solution: Here the principal is fixed. Let the principal (sum) be Rs P
Time and rate are variable (changing)
Using the relation (in 14.2),

$$\text{Change in simple interest } (SI) = \frac{\text{product of fixed parameters}}{100} \times (\text{difference of product of variable parameters})$$

$$95 = \frac{P}{100} \times (4 \times 7.5 - 3.5 \times 10) \qquad \text{(here, } P \text{ is only fixed parameter)}$$

$\Rightarrow$ $P = 95 \times 20 = 1900$

Hence, the required sum = Rs 1900

14.3 AMOUNT

PRINCIPAL (SUM)		SIMPLE INTEREST		AMOUNT
→ actually borrowed money	+	→ interest accrued on the sum borrowed	=	→ money to be returned by the borrower

A = Amount = Principal + Simple Interest = $P + SI$

In terms of principal, $A = P + SI = P + \dfrac{PRT}{100} = P\left(1 + \dfrac{RT}{100}\right)$

In terms of simple interest, $A = P + SI = \dfrac{100 \times SI}{RT} + SI = SI\left(1 + \dfrac{100}{RT}\right)$

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$$R = 2\frac{1}{2}\% = \frac{5}{2}\%,\ T = 2 \text{ yrs.}$$

$$630 = P\left[1 + \frac{2\times 5}{100\times 2}\right] = 630 = P \times \frac{105}{100}$$

$\Rightarrow$ $P = 600$. $\therefore$ The sum is **Rs 600.**

E-6 A sum of money lent out at simple interest amounts to Rs 720 after 2 years and to Rs 1,020 after a further period of 5 years. Find the sum and the rate %.

S-6 Using relation (1), $A - P = \dfrac{PRT}{100}$, we find that R and P do not change here

so, $\dfrac{A_1 - P}{A_2 - P} = \dfrac{PRT_1}{PRT_2} \Rightarrow \dfrac{720 - P}{1{,}020 - P} = \dfrac{2}{5}$

$\Rightarrow$ $P = 600$

$\therefore$ sum = Rs 600

Putting $P = 600$ in above relation, we get

$$720 - 600 = \frac{600 \times R \times 2}{100}$$

$\Rightarrow$ $R = 10$

$\therefore$ Rate = **10% p.a.**

E-7 Simple Interest on a certain sum is 16 over 25 of the sum. Find the rate per cent and time, if both are equal. **(UTI '90)**

S-7 $S.I. = \dfrac{P\times R\times T}{100} \Rightarrow \dfrac{16}{25}\times P = \dfrac{P\times R\times R}{100}$ (Since $R = T$ given) *[Refer 14.6]*

$\Rightarrow R^2 = \dfrac{1{,}600}{25} = 64 \Rightarrow R = 8$, Hence, $R = 8\%$, $T =$ **8 years.**

E-8 A milk man borrowed Rs 2,500 from two money lenders. For one loan, he paid 5% p.a. and for the other, he paid 7% p.a. The total interest paid for two years was Rs 265. How much did he borrow at each rate? **(MBA '86)**

S-8 A simple interest for the total amount = Rs 265

Rate of Interest on total amount, R_m = ?

$$R_m = \frac{100\times S.I.}{T\times P} = \frac{100\times 265}{2{,}500\times 2} = \frac{11}{2}\%$$

By Rule of Alligation, $\dfrac{\text{Sum borrowed at 5\% p.a.}}{\text{Sum borrowed at 7\% p.a.}} = \dfrac{7 - \frac{11}{2}}{\frac{11}{2} - 5} = \dfrac{3}{1}$

$\therefore$ Sum at 5% p.a.

$= \dfrac{3}{1+3} \times 2{,}500 =$ **Rs 1,875**

(18) The simple interest on a sum of money is $\frac{1}{9}$ of the principal and the number of years is equal to the rate % p.a. The rate % p.a. is

(a) 3% (b) $3\frac{1}{3}$% (c) 10% (d) $\frac{1}{3}$% (e) 9%.

(19) The simple interest on a certain sum of money at 4% p.a. for 4 years is Rs 19 more than interest on the same sum for 3 years at 5% p.a. Find the sum of money

(a) 6,700 (b) 7,500 (c) 8,000 (d) 8,750 (e) 9,500

(20) The simple interest on Rs 800 at 6% per month for 9 months is

(a) Rs 51.60 (b) Rs 36 (c) Rs 432 (d) Rs 360 (e) None of these

(21) If Rs 450 amount to Rs 540 in 4 years, what will it amount to in 6 years at the same rate %?

(a) Rs 600 (b) Rs 585 (c) Rs 700 (d) Rs 640 (e) None of these

Tips: Since Principal and Rate are same in both cases; So, simple interest per year will be the same. This concept is used here.

(22) The interest on a sum of money at the end of $2\frac{1}{2}$ years is $\frac{4}{5}$ of the sum. The rate per cent per year is

(a) 10% (b) 20% (c) 16% (d) 32% (e) 40%

(23) A man lent Rs 600 for 2 years and Rs 200 for 3 years, at the same rate at simple interest and received only Rs 90 as interest. What was the rate % p.a.?

(a) 12% (b) 5% (c) 6% (d) $8\frac{1}{3}$% (e) 4%

Hint: Rate is same $\therefore$ $\frac{R[P_1T_1 + P_2T_2]}{100} = SI$

(24) A certain sum of money amounts to $\frac{5}{4}$ of itself in 5 years. The rate per cent p.a. is

(a) 10% (b) 5% (c) 7% (d) 11% (e) $12\frac{1}{2}$%

(25) In what time will a sum of money double itself at 3% per annum at simple interest?

(a) $33\frac{1}{3}$ yrs. (b) 30 yrs. (c) $13\frac{1}{3}$ yrs. (d) 6 yrs. (e) $23\frac{1}{3}$ yrs.

(26) Namrata deposited Rs 8,000 which amounted to Rs 9,200 after 3 years at simple interest. Had the interest been 2% more, she would get how much?

(a) Rs 1,680 (b) Rs 9,860 (c) Rs 9,980 (d) Rs 9,680 (e) Rs 10,620

(27) The rate of interest on a sum of money is 3% p.a. for the first four years, 5% p.a. for the next four years and 6% p.a. for the period beyond 8 years. If the simple interest accrued on the sum for a total period of 9 years is Rs 817, the sum is

(a) Rs 21.50 (b) Rs 18.50 (c) Rs 29.00 (d) Rs 215 (e) None

(28) What quarterly payment will discharge a debt of Rs 2,280 due in two years at 16% p.a. simple interest?

(a) Rs 1,000 (b) Rs 1,600 (c) Rs 250 (d) Rs 360 (e) Rs 450

(Bank PO, '89)

Answers

1. (a)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (b)
10. (b)	**11.** (c)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (b)
19. (c)	**20.** (c)	**21.** (b)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (e)
28. (c)								

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rate % p.a. for 3 years = 10% = R_1 (say)

then t_1 = 3 years

rate % p.a. for next 2 years = 20% = R_2 (say)

then t_2 = 2 years

Then, amount after $(t_1 + t_2)$ years is given by

$$A = P\left(1+\frac{R_1}{100}\right)^{t_1} \times \left(1+\frac{R_2}{100}\right)^{t_2}$$

$\Rightarrow$ amount after 5 years $= 4{,}000\left(1+\frac{10}{100}\right)^{3} \times \left(1+\frac{20}{100}\right)^{2}$

$$= 4{,}000 \times \left(\frac{11}{10}\right)^{3} \times \left(\frac{6}{5}\right)^{2}$$

$$= 7{,}667$$

$\therefore$ amount after 5 years = **Rs 7,667**.

Case III

When interest is compounded yearly but **time is a fraction**

Consider, time $= 5\frac{3}{4}$ years

In this case,

$$\text{Amount } A = P\underbrace{\left(1+\frac{R}{100}\right)^{5}}_{\text{whole part of time}} \times \underbrace{\left(1+\frac{\frac{3}{4}\times R}{100}\right)}_{\text{fraction part of time}}$$

Similarly,

$$\text{amount for } 2\frac{1}{6} \text{ years} = P\left(1+\frac{R}{100}\right)^{2} \times \left(1+\frac{\frac{1}{6}\times R}{100}\right)$$

Example: Find the compound interest on Rs 2,400 at 20% per annum for $3\frac{3}{4}$ years.

Solution: Amount $= 2{,}400\times\left(1+\frac{20}{100}\right)^{3} \times \left(1+\frac{\frac{3}{4}\times 20}{100}\right)$

$$= 2{,}400\times\left(1+\frac{1}{5}\right)^{3}\left(1+\frac{3}{20}\right)$$

$$= 4{,}769.28$$

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amount of each instalment = Rs a
and time = t years

Then, $$a\left[\left(\frac{100}{100+R}\right)+\left(\frac{100}{100+R}\right)^2+\ldots+\left(\frac{100}{100+R}\right)^t\right] = \text{Borrowed amount } B$$

15.8 GROWTH

In our day-to-day life we observe that there are some entities such as population of a city, the value of property, the height of a tree, weight and height of a child, the number of bacteria, etc, which increase in magnitude over a period of time. This relative increase in quantity is called growth.
Growth per unit of time is called the rate of growth.
In this section, we shall illustrate how the formulae for computing amount and compound interest are directly used to calculate the growth of population, height of a tree, number of bacteria and all such entities.

15.8.1 Population Growth

Let P be the population at the beginning of a certain year.

(i) If the constant rate of growth be R% per annum (i.e. R persons per 100 persons per year), then

$$\text{Population after '}t\text{' years} = P\left(1+\frac{R}{100}\right)^t$$

which is exactly similar to finding the amount after 't' years *[Refer 15.3]*

$$\text{Net increase in population during '}t\text{' years} = P\left[\left(1+\frac{R}{100}\right)^t-1\right] \qquad (\text{Since, } CI = A - P)$$

which is exactly similar to finding the compound interest after 't' years *[Refer 15.3]*

(ii) If the rate of growth be R_1% p.a. during first 't_1' years and R_2% p.a. during next t_2 years, then

$$\text{population after } (t_1 + t_2) \text{ years} = P\left(1+\frac{R_1}{100}\right)^{t_1}\times\left(1+\frac{R_2}{100}\right)^{t_2}$$

which is exactly similar case to finding the amount after $(t_1 + t_2)$ years when rate % is not same for every year *[Refer Case II, 15.4]*

(iii) If the constant rate of decrease be R% per annum, then

$$\text{population after } t \text{ years} = P\left(1-\frac{R}{100}\right)^t$$

which is obtained by simply putting $-R$ in place of R in the basic formula. because there is decrease in rate per cent.

15.8.2 Growth of Bacteria, Height of Tree, Production of a Factory

To find the bacteria count, the height of a tree, or production of a certain commodity and such other entities, we use similar formulae as detailed in 15.8.1

Example: The present population of a town is 25,000. If it increases at the rate of 5% per annum, what will be its population after 2 years?

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MEMORY TABLES

Table 16.1 Formulae on Stock

Part	I	II	III	IV	V
	No. of Stock =	$\frac{\text{Total Stock}}{100}$ =	$\frac{\text{Purchase Cost}^*}{\text{M.V.} + \text{brokerage}}$ =	$\frac{\text{Sale Realisation}}{\text{M.V.} - \text{brokerage}}$ =	$\frac{\text{Annual Income}}{\text{Rate \%}}$ =

*Purchase Cost = Total Investment

NB: Equate any of the TWO PARTS to find out the unknown as per the data available.

Table 16.2 Formulae on Shares and Debentures

Part	I	II	III	IV
	No. of Shares =	$\frac{\text{Investment (or Purchase Cost)}}{\text{M.V.}(1 + \text{\%brokerage})}$ =	$\frac{\text{Sale Realisation}}{\text{M.V.}(1 - \text{\% brokerage})}$ =	$\frac{\text{Annual Income} \times 100}{\text{Dividend \%} \times \text{Face Value}}$ =

NB: Equate any of the TWO PARTS as per the requirements of the given problem.
M.V. stands for Market Value of one share/debenture/stock.

$$= \frac{100}{1 - \%5 + \frac{1}{4}\%} \times 7 = \frac{100}{1 - 0.05 + 0.0025} \times 7 = 735$$

Rate % obtained = $\frac{735}{100}$ = **7.35%.**

E-8 Find the market value of a 6% stock in which an income of Rs 244 is derived by investing Rs 1,220, brokerage being $\frac{1}{4}$%.

S-8 $\frac{\text{Annual Income}}{\text{Rate \%}} = \frac{\text{Investment}}{(\text{Market Value + brokerage})}$ *[Refer Parts III and V in Table 1]*

$$\Rightarrow \quad \frac{244}{6} = \frac{1,220}{\text{Market Value} + \frac{1}{4}}$$

$\therefore$Market Value + $\frac{1}{4}$ = 30 $\Rightarrow$ Market Value = **29.75.**

E-9 How much should one invest in $6\frac{2}{3}$% stock at 110 to secure an annual income of Rs 300.

S-9 $\frac{\text{Annual Income}}{\text{Rate \%}} = \frac{\text{Investment}}{\text{Market Value + \% brokerage}}$ *[Refer Parts III and V in Table 1]*

$$\frac{300}{I} = \frac{20}{3 \times 110}$$

$\Rightarrow$ $I = \frac{30 \times 110 \times 300}{20}$ $\Rightarrow$ I = Rs 4,500 $\therefore$ The investment is **Rs 4,500.**

E-10 Find what a purchaser would have to pay for 250 shares of Rs 20 each quoted at Rs 74. What would be the gain to the share holder, if he had purchased the shares at par?

S-10 Face Value of 1 share = Rs 20
Market Value of 1 share = Rs 74
The amount paid by the buyer (purchaser) = 250 × 74 = Rs 18,500
The Purchase Cost by share holder = 250 × 20 = 5,000
$\therefore$ Gain by the share holder = 18,500 – 5,000 = **Rs 13,500.**

E-11 A man invests Rs 4,220 in $6\frac{1}{2}$% stock at 105. On selling the invested stock, how much will he realise? $\left(\text{brokerage} = \frac{1}{2}\%\right)$.

S-11 Here, amount of stock is not known.
Purchase cost of stock = Investment = Rs 4,220

$$\therefore \quad \frac{\text{Sale Realisation}}{\text{Market Value – brokerage}} = \frac{\text{Purchase Cost}}{\text{Market Value + brokerage}}$$

[Refer Parts III and IV in Table 1]

$$\therefore \quad \frac{\text{Sale Realisation}}{105 - \frac{1}{2}} = \frac{4,220}{105 + \frac{1}{2}}$$

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(13) A man performs $\frac{2}{25}$ of his total journey by bus, $\frac{21}{50}$ by car and the remaining 2 km on foot. Find the total journey. **(Bank PO, '90) {2.5 km}**

Hint: Let total journey = x km.

$$\therefore \quad \frac{2}{25}x + \frac{21}{50}x + 2 = x \quad \Rightarrow \quad x = 2.5.$$

(14) Normally it takes 3 hours for a train to run from A to B. One day, due to a minor trouble, the train had to reduce the speed by 12 km/h and so it took $\frac{3}{4}$ of an hour more than usual. What is the distance from A to B. **{180 km}**

Hint: Let d be the distance from A to B.

$$\frac{d}{t_1} - \frac{d}{t_2} = V_1 - V_2$$ *[Refer 17.4]*

$$\Rightarrow \quad \frac{d}{3} - \frac{d}{3\frac{3}{4}} = 12 \quad \Rightarrow \quad d = 180 \text{ km}.$$

REAL PROBLEMS

(1) A car takes 3 hours to cover a distance of 180 km. If the distance is to be covered in $2\frac{1}{2}$ hours, what should be the speed of the car?
(a) 90 (b) 60 (c) 36 (d) 72 (e) None of these

Hint: Takes 3 hours is useless data.

(2) A train runs at 45 km/h. How far does it go in 6 seconds?
(a) 72 metres (b) 60 metres (c) 75 metres (d) 70 metres (e) 150 metres

Hint: 1 km/h = $\frac{5}{18}$ m/s.

(3) Joseph walked 1 km/h slower than usual and he could return home in $\frac{9}{8}$ of his usual time. His normal walking rate is
(a) 8 km/h (b) 9 km/h (c) 10 km/h (d) 11 km/h (e) None of these

Hint: $\left(1 - \frac{8}{9}\right)$ usual speed = 1 km/h.

(4) A car travels a certain distance at 60 km/h and comes back at 50 km/h. Find the average speed for total journey.
(a) 50 km/h (b) 45 km/h (c) 48 km/h (d) 55 km/h (e) None of these

(5) A motor cyclist travels for 10 hours, the first half at 21 km/h and the other half at 24 km/h. Find the distance travelled.
(a) 225 km (b) 224 km (c) 200 km (d) 324 km (e) 350 km

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$$\frac{\text{True time interval}}{5} = \frac{1}{1+\frac{1}{60}}$$

$$\text{True time interval} = \frac{5 \times 60}{61} \text{ hours} = 4\frac{56}{61} \text{ hours}$$

$$\text{True time} = 1 \text{ p.m.} + 4\frac{56}{61} \text{ hours}$$

$$= 5 \text{ p.m. and } \frac{56}{61} \times 60 \text{ minutes} = \mathbf{55\frac{5}{61}} \textbf{ minutes past 5 p.m.}$$

E-7 At what time between 2 o' clock and 3'o clock will the two hands be at right angles to each other?

S-7 [Refer 21.3 (II)

Here $H \times 30 < A°$ since $H = 2$ and $A = 90°$

$$\text{Required time } T = \frac{2}{11}(H \times 30 + A) \text{ minutes past } H$$

$$\text{and } \frac{2}{11}(360 + H \times 30 - A) \text{ minutes past } H$$

$$= \frac{300}{11} \text{ minutes and 60 minutes past 2}$$

$$= \mathbf{27\frac{3}{11}} \textbf{ minutes past 2} \text{ and } \textbf{at 3 o' clock.}$$

E-8 The minute hand of a clock overtakes (or coincides) the hour hand at intervals of 65 minutes of correct time. How much does the clock gain or lose in 12 hours?

S-8 [Refer 21.2 (K)]

In a correct clock, both the hands coincide (or overtake) at an interval of $65\frac{5}{11}$ minutes.

But, here it coincides at interval of 65 minutes

Since $65\frac{5}{11} > 65$, so, the clock is gaining time

Using the formula, [Refer 21.4], we get

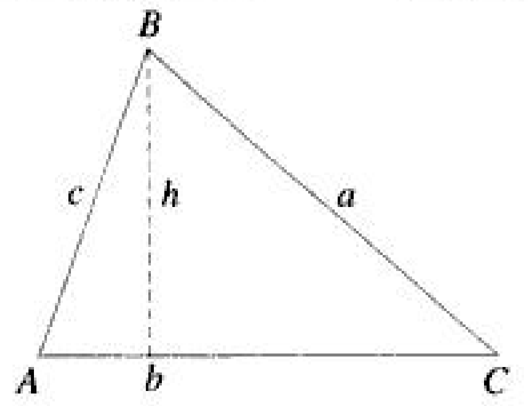
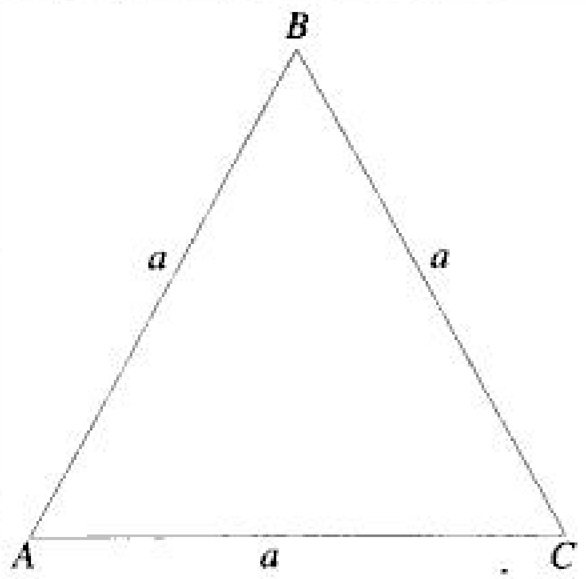
$$\text{total time gained in } T \text{ hours} = (T \times 60)\frac{\left(65\frac{5}{11} - x\right)}{x} \text{ minutes}$$

where $T = 12$ hours, $x = 65$

$$\text{Total time gained in 12 hours} = (12 \times 60)\frac{\left(65\frac{5}{11} - 65\right)}{65} \text{ minutes}$$

$$= 12 \times 60 \times \frac{5}{11} \times \frac{1}{65} \text{ minutes} = \mathbf{5\frac{5}{143}} \textbf{ minutes.}$$

RAPID INFORMATION LIST
Area of Plane Figures

Plane Figure/ Type	Figure	Side/Angles	Diagonal	Perimeter (P)	Area (A)
1. Triangle (a) Any	 a, b and c are three sides of the triangle; h is the altitude and AC is the base	$a \neq b \neq c$ $\angle A \neq \angle B \neq \angle C$ $\angle A + \angle B + \angle C = 180°$	—	$P = a + b + c = 2s$	$A = \frac{1}{2} \times$ base $\times$ altitude $= \frac{1}{2} \times$ any side $\times$ length of $\perp r$ dropped on that side $= \sqrt{s(s-a)(s-b)(s-c)}$
(b) Equilateral	 a is the length of each side	$AB = BC = CA = a$ $\angle A = \angle B = \angle C = 60°$	—	$P = 3a$	$A = \frac{\sqrt{3}}{4} a^2$

(Contd.)

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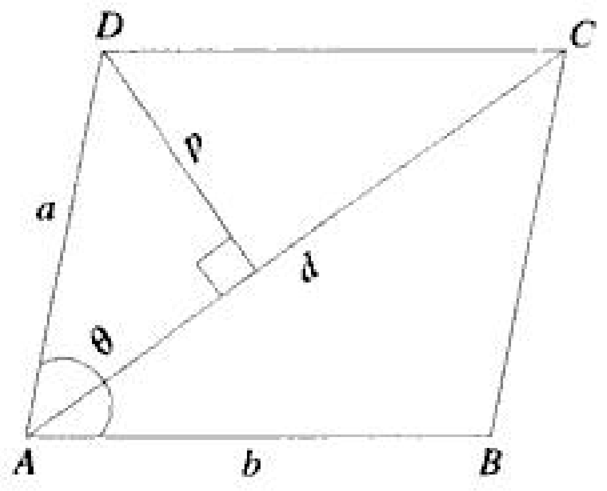
Plane Figure/Type	Figure	Side/Angles	Diagonal	Perimeter (p)	Area (A)
(g) Parallelogram	 a, b are two sides d is the diagonal p is the perpendicular dropped on that diagonal θ is the angle between a and b	$AB = DC = b$ $AD = BC = a$ $AB \parallel DC$ $AD \parallel BC$ $\angle A = \angle C = \theta$ $\angle B = \angle D$ $= 180 - \theta$	Bisect each other If $d_1 = BD$ $d_2 = AC$ then, $d_1^2 + d_2^2$ $= 2(a^2 + b^2)$	$p = 2(a + b)$	$A = p \times d$ $= ab \sin \theta$ $= 2 \times$ area of ΔABC $= 2\sqrt{\dfrac{s(s-a)(s-b)}{(s-d)}}$ where $s = \dfrac{a+b+d}{2}$

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(7) The length of two equal sides in an isosceles triangle are 12.5 metre and the third side is 20 metre. Find the area of triangle and the perimeter.

Hint: For isosceles Δ, Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

(8) In an isosceles right-angled triangle, the length of one leg is 10 metre. Find its area and its perimeter. **(MBA '86)**

Hint: For isosceles right Δ, Area $A = \frac{1}{2}$ (Leg)^2; Perimeter $P = 3.414 \times$ Leg.

(9) In an isosceles right angled triangle, the perimeter is 20 metre. Find its area.

Hint: For isosceles right angled Δ, $A = 0.043\ p^2$.

(10) In a right angled-triangle, the length of two legs are 24 and 10 metres respectively. Find the length of hypotenuse and the area of triangle.

(11) In a right-angled triangle, the area is 30 m^2 and one of its legs is 12 metres. Find its other leg and hypotenuse.

(12) The length and breadth of a rectangle are 12 and 8 metres respectively. Find its area and perimeter.

(13) The length and breadth of a rectangle are in the ratio 9 : 5. If its area is 720 m^2, find its perimeter.

Hint: Let the length = $9x$; breadth = $5x$; $9x \times 5x = 720 \quad \therefore \quad x = 4$.

(14) The length and breadth of a rectangle plot are 30 and 12 metres respectively. Find the cost for

(i) Cultivating it, if the rate is Rs 2.5 per square metre,

(ii) Fencing it, if the rate is Rs 1.30 per metre.

(15) A rectangular grass field is 112 m $\times$ 78 m. It has a gravel path 2.5 metre wide all round it on the inside. Find the area of gravel path and the cost of constructing it at the rate of 80 paise per square metre.

Hint: Here pathway is inside of rectangular grass field

$\therefore$ Area of gravel path $= 2w\ (l + b - 2w)$

$= 2 \times 2.5\ (112 + 78 - 2 \times 2.5) = 925$ m^2.

(16) A rectangular plot 15 m $\times$ 10 m, has a path of grass outside it. If the area of grassy pathway is 54 m^2, find the width of the path,

Hint: Area of path way $= 2W\ (l + b + 2W)$.

(17) One side and the diagonal of a rectangle are 40 metres and 50 metres respectively. Find its area and perimeter.

Hint: Area of rectangle $= l \times \sqrt{d^2 - l^2} = 40\ \sqrt{50^2 - 40^2} = 1{,}200$ m^2

Perimeter of rectangle $= 2\ \left(l + \sqrt{d^2 - l^2}\right) = 2\ \left(40 + \sqrt{50^2 - 40^2}\right) = 140$ metres

(18) A rectangular lawn 70 m $\times$ 30 m has two roads each 5 metres wide, running in the middle of it, one parallel to the length and the other parallel to the breadth. Find the cost of gravelling the road at the rate of Rs 4 per square metre.

Hint: Area of two parallel paths $= w\ (l + b - w) = 5\ (70 + 30 - 5) = 475$ m^2 *[Refer 22.3.1]*

Cost of gravelling $= 4 \times 475 =$ Rs 1,900.

(19) The length and the diagonal of a rectangle are 60 and 75 cms respectively. Find its breadth.

(20) The perimeter and the diagonal of a rectangle are 18 and 5 metres respectively. Find its area.

Hint: In a rectangle, $\left(\frac{P}{2}\right)^2 = d^2 + 2A$

(28) If the ratio of the area of two squares is 2 : 1, then the ratio of the perimeters of the square is

(a) 1 : 2 (b) 1 : $\sqrt{2}$ (c) $\sqrt{2}$: 1 (d) 2 : 1 (e) $\sqrt{2}$ – 1 : 1

(29) The base of parallelogram is 18 metres and its height is 10 metres. Its area is

(a) 180 m^2 (b) 240 m^2 (c) 90 m^2 (d) 140 m^2 (e) 360 m^2

(30) If the side of square is increased by 200%, the area of the square is increased by

(a) 200% (b) 400% (c) 800% (d) 300% (e) 250%

(31) When radius of a circle is decreased by 100%, the area of the circle

(a) decreases by 50% (b) vanishes
(c) increases by 100% (d) decreases by 200%
(e) decreases by 400%

Hint: If the radius of a circle or side of a square increases by p% then, the area increases by $\left[2p+\left(\frac{p}{10}\right)_2^2\right]$%, if the radius of a circle or side of square decreases by p% the area decreases by $\left[2p-\left(\frac{p}{10}\right)^2\right]$%.

(32) If the area of a square increases by 96%, then the side of a square increases by

(a) 96% (b) 485 (c) 14% (d) 40% (e) 24%

Hint: If the area of a square or a circle increases by p%, then the side of a square or radius of circle increases by $\left(10\sqrt{100+p}-100\%\right)$.

(33) In the area of a circle decreases by 36%, then the radius of a circle decreases by

(a) 20% (b) 18% (c) 36% (d) 64% (e) 6%

Hint: If the area of circle or a square decreases by p% then the radius (or diameter) of a circle or side of a square decreases by $\left(100-10\sqrt{100-p}\right)$%.

(34) The side of a rhombus is 26 metres and length of one of its diagonals is 20 metres. The area of the rhombus is

(a) 520 m^2 (b) 240 m^2 (c) 260 m^2 (d) 300 m^2 (e) 280 m^2

(35) How many plants will be there in a circular bed whose outer edge measure 30 cms, allowing 4 cm^2 for each plant?

(a) 18 (b) 750 (c) 24 (d) 120 (e) 36

Hint: Circumference of circular bed C = 30 cm (given)

Area of circular bed $A = \dfrac{C^2}{4\pi}$ $\quad\therefore$ number of plants = $\dfrac{A}{4}$

space for each plant = 4 cm^2.

(36) The area of a rectangular field is 144 m^2. If the length had been 6 metres more, the area would have been 54 m^2 more. The original length of the field is

(a) 22 metres (b) 18 metres (c) 16 metres (d) 24 metres (e) 12 metres

(37) When the circumference and area of a circle are numerically equal, then the diameter is numerically equal to

(a) area (b) circumference (c) $\dfrac{\pi}{2}$
(d) 2π (e) 4

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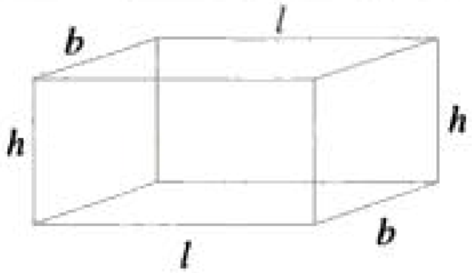
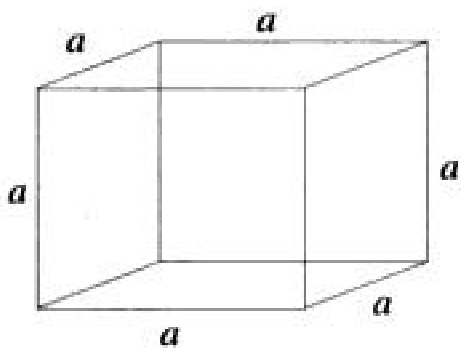
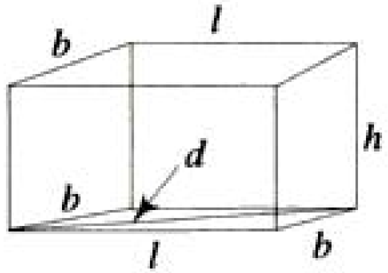
TABLE 23.1 RAPID INFORMATION LIST

Volume and Area of Solid Figures

Name of Solid Figure	Figure	Shape of Curved Surface/Base	Surface Area			Volume (V)
			Curved (C)	Base (B)	Total (C + 2B)	
Cuboid (Rectangular Solid)	 l is the length b is the breadth h is the height Total number of faces = 6	**Surface** Rectangular side face. = 4 s **Base** Rectangular Base = 1 Top = 1	$2\ (bh + lh)$	lb	$2\ (bh + lh + lb)$	$l \times b \times h$ $= \sqrt{A_1\ A_2\ A_3}$ where A_1, A_2 and A_3 are areas of base, side and end face respectively
Cube	 Each side (edge) = a It has 6 equal faces	**Surface** Square side faces = 4 **Base** Square Base = 1 Top = 1	$4a^2$	a^2	$6a^2$	a^3
Right Parallelopiped	 It is also a type of cuboid	**Surface** Rectangular **Base** Parallelogram	$2\ (lh + bh)$	$2\sqrt{s\,(s-a)\,(s-b) \times (s-d)}$		Base area × Height

(Contd.)

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$\therefore$ Weight of wire in kilograms $= \dfrac{\pi \times 300 \times 1000}{4 \times 81 \times 100 \times 100} \times \dfrac{270}{0.027} =$ **2910 kg**.

E-12 A reservoir is supplied water by a pipe 6 cms in diameter. How many pipes of 3 cms diameter would discharge the same quantity, supposing the velocity of water is same?

S-12 Number of discharge pipes $= \dfrac{\text{Volume of water in supply pipe}}{\text{Volume of water in each discharge pipe}}$

Since velocity is same, take unit length of each pipe for calculating the volume of water content in supply pipe and each discharge pipe.

$\therefore$ No. of discharge pipes $= \dfrac{\pi \times (3)^2 \times 1}{\pi \times \left(\dfrac{3}{2}\right)^2 \times 1} =$ **4**.

E-13 Find the weight of iron in a pipe whose interior and exterior diameters measures 10 and 12 cms respectively, and length 4 m; one cubic metre of iron weighs 7,800 kg.

S-13 Internal diameter $d_i = 10$ cms $= \dfrac{10}{100}$ m $= 0.1$ m

External diameter $d_e = 12$ cms $= \dfrac{12}{100}$ m $= 0.12$ m

length $l = 4$ m.

Pipe is a hollow cylinder.

$\therefore$ Volume of iron = External Volume – Internal Volume

$$= \frac{\pi}{4}(d_e^2 - d_i^2) \times l = \frac{\pi}{4}\ (0.12^2 - 0.10^2) \times 4 = \frac{22}{7} \times 0.22 \times 0.02 = 0.0138 \text{ m}^3$$

$\therefore$ Weight of iron $= 0.0138 \times 7800 =$ **108 kg**.

E-14 A right pyramid 10 m high has a square base for which the diagonal is 10 m. Find its volume and lateral surface.

S-14 Volume of pyramid $= \dfrac{1}{3} \times$ Base area $\times$ Height

Base is a square, with diagonal $(d) = 10$ m.

$\therefore$ Base area $= \dfrac{d^2}{2} = \dfrac{10^2}{2}$ Volume of pyramid $= \dfrac{1}{3} \times \dfrac{10^2}{2} \times 10 = 166\dfrac{2}{3}$ m^3

Lateral (Curved) surface of pyramid $= \dfrac{1}{2} \times$ base perimeter $\times$ slant height (since base is square)

Side of square base $(a) = \dfrac{d}{\sqrt{2}} = \dfrac{10}{\sqrt{2}}$ m and base perimeter $= 4a$

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For example, say, the ratios obtained in analysis of data are, $\frac{13}{447}, \frac{45}{508}, \frac{28}{583}, \frac{18}{531}$, to find the highest and the lowest among them, following method is adopted for **mental calculation.**

Apply Reverse operation i.e. straight way divide the denominator of the ratio by the numerator to find how many times the denominator is of the numerator.

$$\Rightarrow \qquad \frac{447}{13}, \frac{508}{45}, \frac{583}{28}, \frac{531}{18}$$

No. of times >30, 10 20 <30 (closer value is not required for comparison)

↓ ↓

maximum value minimum valueof no. of times

↓ ↓

minimum ratio maximum ratio

Since, reverse operation has been done, so, maximum no. of times will indicate the minimum ratio and minimum no. of times correspond to maximum ratio.

R(4) To find the highest and the lowest among the ratios (<1) when numerator < denominator.

Step 1 Approximate the given ratio (if the no. of digits in numerator/denominator is more than 2)

Step 2 Multiply the numerator by 10 and get the result fraction

Step 3 Find only the integer value of the result fraction

Step 4 If any of the integer value of the result fraction is same then find the next decimal place and so on.

Step 5 Compare the value of the result fractions. The maximum ratio will have maximum value.

For example, the ratios are $\frac{471}{525}, \frac{324}{411}, \frac{648}{749}$

Step 1 Approximated as $\frac{47}{52}, \frac{32}{41}, \frac{65}{75} \left(\approx \frac{13}{15}\right)$

Step 2 Multiply by 10, $\frac{470}{52}, \frac{320}{41}, \frac{130}{15}$

Step 3 Find the integer value, >9, <8, <9

Further calculation is not required because one fraction >9 and other <8.

Hence $\frac{471}{525}$ is the highest ratio and $\frac{324}{411}$ is the lowest ratio.

R(5) To find which of the given ratio is more or less than a specified limit.

Consider, $\frac{31}{83}, \frac{15}{46}, \frac{22}{65}$, then it is required to find which of the among ratio is less than $\frac{1}{3}$

It is convenient to check which denominator is $\frac{3}{1} = 3$ times the numerator. So, we find that

$$\frac{83}{31} < 3, \quad \boxed{\frac{46}{15} > 3,} \quad \frac{65}{22} < 3$$

↓

required ratio

% increase (or change) in no. of scooters during 1994-98 = $\frac{55-20}{20} \times 100 = 175\%$

Average % increase (or change) during 1994-98 = $\frac{\text{\% increase during 1994-98}}{\text{No. of entires during 1994-98}}$

$= \frac{175}{5}\% = 35\%$

A(3) *To find the value of which year (or the entry in a table) is closer to the average value of a given period*

Step 1 Find the average value for the given period

Step 2 Find the difference = any value (or entry) – average value. Minimum the difference, closer the value to average.

Step 3 If the **difference** is **same** for any two different entry (or values) then find the

% variation over the average = $\frac{\text{difference}}{\text{average}} \times 100$

Since difference is same, so,
More the average, less the % variation, closer the value to the average.

Example: Production figures of five types of car by a company is given as below

Type of cars

Year	*A*	*B*	*C*	*D*	*E*	*Total*
1995	20	10	15	8	20	73
1996	12	12	18	12	12	66
1997	24	14	17	14	15	84
1998	26	10	16	15	9	76
1999	22	16	14	16	11	79

Example (1): In which of the given years the production of type D car was close to its average production over the years?

Example (2): In which of the given years the production of type D car was close to the average production of all the cars in the year?

Solution (1): The average production of type *D* car over the years = $\frac{8+12+14+15+16}{5} = \frac{65}{5} = 13.$

Difference with average is same (=1) for 1996 and 1997, and the average is same (=13). So, the production of type D car in both 1996 and 1997 are close to its average production over the years.

Solution (2): It has striking difference (dissimilarity) from the above question because the value of average varies for each year (i.e. horizontally)

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S-4 *(e)*

Pairs of years	Total production of E-type scooters
1993 & 1995	48 + 115 $\neq$ 200 thousands
1995 & 1997	115 + 185 $\neq$ 200 thousands
1994 & 1997	85 + 185 $\neq$ 200 thousands
1992 & 1994	76 + 84 $\neq$ 200 thousands

Note: 2 lakhs = 200 thousands, so, it is better to check in which pair of years, the total production = 200 thousands for *E*-type scooters.

E-5 In which of the given years the total production of all types of scooters was minimum?
(a) 1994 (b) 1996 (c) 1997 (d) 1995 (e) None

S-5 *(a)*

Year	Total Production of all type scooters
1992	180 + 60 + 84 + 100 + 76 = 500
1993	210 + 90 + 32 + 80 + 48 = 460
1994	135 + 30 + 44 + 95 + 85 = 389 → Minimum
1995	190 + 85 + 69 + 125 + 115 = 444
1996	260 + 95 + 120 + 80 + 120 = 675
1997	240 + 140 + 161 + 90 + 185 = 816

Directions (6-10): Study the following table and answer the questions given below it.

Students Going Abroad For Study From Different States/UTs

State/U.T.	*Year*				
	1995	*1996*	*1997*	*1998*	*1999*
Maharashtra	723	840	900	920	925
Kerala	1035	940	1200	1400	1500
Karnataka	750	600	830	575	900
West Bengal	500	550	450	600	525
Delhi	1500	1625	1700	1475	1800
Andhra Pradesh	800	840	875	925	785
Total	5308	5395	5955	5895	6435

E-6 What was the per-cent increase in the number of students from West Bengal from 1997 to 1998?

(a) $66\frac{2}{3}$ (b) $33\frac{1}{3}$ (c) 150 (d) 50 (e) $16\frac{2}{3}$

S-6 *(b)*

$$\%\text{ increase from 1997 to 1998 for West Bengal} = \frac{\text{Value (1998)} - \text{Value (1997)}}{\text{Value (1997)}} \times 100$$

$$= \frac{600 - 450}{450} \times 100 = 33\frac{1}{3}\,\%$$

E-7 In the case of which state/U.T. was there a continuous increase in the number of students over the given years?
(a) Kerala (b) Delhi (c) Andhra Pradesh (d) Maharashtra
(e) None

S-7 *(d)*

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E-36 In which district the $\frac{SMT}{MT}$ is maximum?

(a) A (b) B (c) C (d) D (e) E

S-36 *(b)* Since $\frac{SMT}{MT}$ <<1, applying reverse operation, we can find $\frac{MT}{SMT}$ as

		A	B	C	D	E
$\frac{MT}{SMT}$	$\rightarrow$	>25	<25 (Minimum)	25	>25	25

Hence $\frac{SMT}{MT}$ is maximum in district B.

E-37 In which district the MTs are best performers in making Trained volunteer teachers?

(a) A (b) B (c) C (d) D (e) E

S-37 *(b)* As per question, the district with maximum $\frac{TVT}{MT}$ is to be identified.

	A	B	C	D	E	
$\frac{TVT}{MT}$	$\frac{180}{13}$	$\frac{210}{14}$	$\frac{220}{20}$	$\frac{240}{22}$	$\frac{270}{22}$	(approximation done)
	<15	$=\boxed{15}$ maximum				

$\leftarrow$ further division not required as numerator is not >300, so in these districts, $\frac{TVT}{MT}$ cannot be 15.

E-38 In district C, how many MTs are to be included to reach 54000 mark in ILLs?

(a) 240 (b) 40 (c) 80 (d) 60 (e) 140

S-38 *(b)*

Let the no. of MTs = x, for ILLs = 54000 in district C

$\Rightarrow \frac{45000}{200} = \frac{54000}{x} \Rightarrow x = 240$, so 40 nos. of MTs are to be included

E-39 The literacy programme is adjudged commendable if the $\frac{SMT}{ILL}$ bears the ratio of 1 : 7000, then how many districts are adjudged commendable?

(a) One (b) Two (c) None (d) Three (e) All

S-39 *(b)*

Here, the district with $\frac{SMT}{ILL} = \frac{1}{7000}$, is to be identified

Tips: Do not calculate $\frac{SMT}{ILL}$, because difference of no. of digits of the SMT and ILL are same for all districts. So, approximate ILLs in '000 and divide by SMT to check which one is >7

A	B	C	D	E	
$\frac{38}{5}$	$\frac{43}{6}$	$\frac{45}{8}$	$\frac{52}{8}$	$\frac{57}{9}$	(Approximating numerator X
>7	>7	<6	<7	<7	dividing by 1000)

commendable.

Hence, districts A & B i.e. two are adjudged commendable

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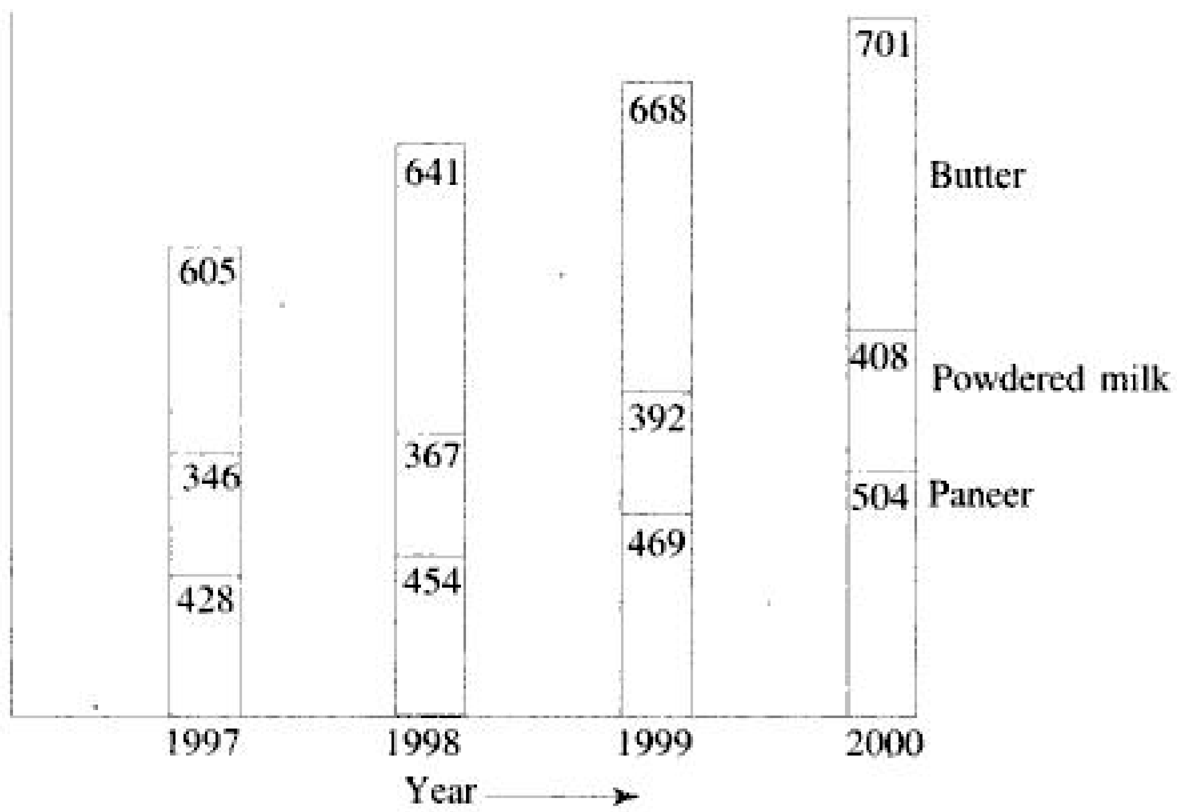
(8) In which year is the ratio of earnings from polished diamond and the earnings from gold jewellery, the highest?
(a) 1990 (b) 1991 (c) 1992 (d) 1993 (e) 1994

(9) In which year is the percentage increase in the earnings from the export of polished diamond over the preceding year, the maximum?
(a) 1990 (b) 1991 (c) 1992 (d) 1993 (e) 1994

Directions (10-11): Study the bar chart and answer the questions given below.

A Milk Dairy—Production ***(in '00 kg)***



(10) If the average earning of one kg of butter, powdered milk and paneer are 13 : 15 : 10, then the percentage growth in the earning from the dairy products produced in 1999 over 1998 was (assuming no change in relative price)
(a) $13\frac{1}{3}$ (b) 11 (c) 16 (d) 5 (e) Data inadequate

Hint: Without knowing the absolute value of earning per kg for each item, the percentage growth cannot be found out.

(11) In the year 2000, it was found that the requirement of milk for one kg of butter, powdered milk and paneer is 7.5 kgs, 6 kgs and 5.5 kgs respectively. Approximately, how many tonnes of milk are required in 2000?
(a) 104 (b) 743 (c) 1708 (d) 1047 (e) 1074

Directions (12-13): Study the following chart and answer the questions given below.

(12) What per-cent of roast coffee is domestically consumed?
(a) 55.46 (b) 44.53 (c) 65.5 (d) 35 (e) $33\frac{1}{3}$

Hint: Domestic Consumption = Production – Export, so, in terms of percentage, the production = 100,

$$\text{then required percentage} = \left(1 - \frac{104}{\frac{108^\circ}{360^\circ} \times 625}\right) \times 100 \qquad (360^\circ = 100\%)$$

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Type 3: **Problems based on Ages**

E-9 The ages (in years) of Mohan and Sohan are in the ratio 5 : 7. If Mohan were 9 years older and Sohan 9 years younger, the age of Mohan would have been twice the age of Sohan. Find their ages.

S-9 Let the ages of Mohan and Sohan be $5x$ years and 7 years respectively, then as per question

$$(5x + 9) = 2\ (7x - 9)$$
$$\Rightarrow \quad 5x + 9 = 14x - 18$$
$$\Rightarrow \quad 9x = 27$$
$$\Rightarrow \quad x = 3$$

Hence the age of Mohan = $5 \times 3 = 15$ years

Age of Sohan = $7 \times 3 = 21$ years.

E-10 The sum of the ages of the father and his son is 54 years. After 18 years, father will be twice as old as his son. Find their present ages.

S-10 Let the age of the father be x years,

the age of the son = $54 - x$, [Since their sum of ages = 54]

then, as per question,

$$(18 + x) = 2(54 - x + 18)$$
$$\Rightarrow \quad 18 + x = 2(72 - x)$$
$$\Rightarrow \quad 18 + x = 144 - 2x$$
$$\Rightarrow \quad 3x = 126$$
$$\Rightarrow \quad x = 42$$

Hence, the age of the father = 42 years.

age of the son = $54 - 42 = 12$ years.

E-11 Ten years ago, a father was four times as old as his son. After 20 years, he will be twice as old as his son. Determine their present ages.

S-11 Although the given problem can be solved with the help of conventional linear equations, it may be time consuming. So, a short-cut approach has been developed to tackle this type of problem.

Past	Present	Future
10 years (ago)	—	20 years (after)
4 times	—	2 times

$$\text{Present age of the younger person (here, it is of son)} = \frac{10(4-1)+20(2-1)}{\underbrace{(4-2)}_{\text{difference of times}}}$$

$$\therefore \text{ present age of the son} = \frac{10 \times 3 + 20}{2}$$
$$= 25 \text{ years.}$$

Now, as per question,

$$(\text{present age of father} + 20) = 2(25 + 20)$$
$$\text{present age of father} = 70 \text{ years.}$$

Hence, present ages of father and son are 70 years and 25 years respectively.

E-12 The age of woman is three times that of her daughter. Six years ago, the woman was five times as old as her daughter at that time. Find the present age of the woman.

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24. 137% of 12345 = ?
(1) 17000 (2) 15000
(3) 1500 (4) 14300
(5) 900

25. 3739 + 164 × 27 = ?
(1) 105400 (2) 4000
(3) 8200 (4) 690
(5) 6300

Directions (26-30): Study the following table to answer the given questions:

Per centage of Marks obtained by seven students in six subjects

Subjects → *(Max. Marks)* *↓ Students*	*Eng* *(60)*	*His* *(40)*	*Comp* *(130)*	*Maths* *(150)*	*Science* *(120)*	*Econ* *(80)*
Meera	100	80	50	90	90	60
Subodh	80	70	80	100	80	40
Kunal	90	70	60	90	70	70
Soni	60	60	65	80	80	80
Richu	50	90	62	80	85	95
Irene	40	60	64	70	65	85
Vijay	80	80	35	65	50	75

26. What is the total marks obtained by Meera in all the subjects?
(1) 448 (2) 580
(3) 470 (4) 74.67
(5) None of these

27. What is the average marks obtained by these seven students in History? (rounded off to two digits)
(1) 72.86 (2) 27.32
(3) 24.86 (4) 29.14
(5) None of these

28. How many students have got 60% or more marks in all the subjects?
(1) One (2) Two
(3) Three (4) None
(5) None of these

29. What is the overall per centage of Kunal?
(1) 64 (2) 65
(3) 75 (4) 64.24
(5) None of these

30. In which subject is the overall per centage the best?
(1) Maths (2) Economics
(3) History (4) Science
(5) None of these

Direction (31-35): Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the questions. Read the questions and both the statements and

Give answer (1) if the data in **statement I alone are sufficient** to answer the question, while the data in **statement II alone are not sufficient** to answer the question.

Give answer (2) if the data in **statement II alone are sufficient** to answer the question, while the data in **statement I alone are not sufficient** to answer the question.

Give answer (3) if the data **either in statement I alone or in statement II alone are sufficient** to answer the question.

Give answer (4) if the data even in both the **statements I and II together are not sufficient** to answer the question.

Give answer (5) if the data in both the **statements I and II together are necessary** to answer the question.

31. Train '*A*' running at a certain speed crosses another train '*B*' running at a certain speed in the opposite direction in 12 seconds. What is the length of train '*B*'?
I. The length of both the trains together is 450 metres.
II. Train '*A*' is slower than train '*B*'.

32. The area of a rectangle is equal to the area of a right-angled triangle. What is the length of the rectangle?
I. The base of the triangle is 40 cm.
II. The height of the triangle is 50 cm.

33. What was the total compound interest on a sum after three years?
I. The interest after one year was Rs 100 and the sum was Rs 1000.

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73. A certain distance is covered by a cyclist at a certain speed. If a jogger covers half the distance in double the time, the ratio of the speed of the jogger to that of the cyclist is
(1) 1 : 2 (2) 2 : 1
(3) 1 : 4 (4) 4 : 1

74. A and B are two stations 500 km apart. A train starts from A and moves towards B at 20 km/hour. Another train starts from B at the same time and moves towards A at 30 km/hour. The distance of crossing point of the two trains from A (in km) is
(1) 100 (2) 200
(3) 250 (4) 300

75. A car completes a certain journey in 8 hours. It covers half the distance at 40 km/hour and the rest at 60 km/hours. The length of the journey (in km) is
(1) 420 (2) 400
(3) 384 (4) 350

76. *A* and *B* can do a piece of work in 18 days, *B* and *C* in 24 days, *A* and *C* in 36 days. In what time can they do it all working together?
(1) 12 days (2) 13 days
(3) 16 days (4) 26 days

77. A takes twice as much time as *B* and thrice as much time as *C* to finish a piece of work. Working together, they can finish the work in 2 days. The number of days required by *B* to do the work alone is
(1) 4 (2) 6
(3) 8 (4) 12

78. A pipe can fill a tank in 8 minutes and another pipe can empty 6 kiloliters of water in 1 minute. If both the pipes are opened together the empty tank can be filled up in 20 minutes. The capacity of the tank (in kilometers) is
(1) 50 (2) 60
(3) 75 (4) 80

79. 3 men and 2 women can do a piece of work in 15 days. 2 men and 3 women can do the same work in 18 days. The number of days to be taken by 1 man and 1 woman to do the work is
(1) $20\frac{10}{11}$ (2) $30\frac{10}{11}$
(3) $40\frac{10}{11}$ (4) $45\frac{10}{11}$

80. A contractor engaged 100 labourers for a construction work to be completed in 96 days. But only $\frac{1}{7}$ of the work was done in $\frac{1}{6}$th of the scheduled time. The additional number of labourers that will be required to complete the work in time is
(1) 20 (2) 40
(3) 30 (4) 15

81. A train crosses a telegraph pole in 6 seconds and a 220 metre long bridge in 16 seconds. The speed (in m/sec) of the train is
(1) 44 (2) 22
(3) 30 (4) 60

82. By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than usual. The time taken at his usual speed is
(1) 30 minutes (2) 60 minutes
(3) 75 minutes (4) 90 minutes

83. The average of first four of five numbers is 26 and that of the last four is 25. The difference between the last number and the first number is
(1) 1 (2) 4
(3) 5 (4) 16

84. *A* can run one kilometer in 4 minutes 54 seconds and *B* can run this distance in 5 minutes. At the start of one kilometer race, *B* should stand how much ahead of *A*, so that both of them reach the finish point together?
(1) 20 metre (2) 16 metre
(3) 18 metre (4) 14.5 metre

85. *A* and *B* walk on a circular path whose circumference is 35 km. They start walking from the same place in the same direction at the same time. Speed of *A* is 4 km/hr and speed of *B* is 5 km/hr. They will meet earliest again after
(1) 15 hours (2) 21 hours
(3) 35 hours (4) 42 hours

86. $\sqrt[3]{1-\frac{91}{216}}$ is equal to

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Percentage of Marks Obtained by Six Students in Different Subjects

Student	Subject					
	History	Geography	Mathematics	Science	Economics	English
	Maximum Marks					
	50	50	100	75	100	120
A	86	82	77	72	69	66
B	80	74	73	66	76	84
C	68	92	89	78	75	72
D	76	84	83	81	59	60
E	72	66	82	87	62	78
F	84	64	93	63	81	54

46. What are the average marks obtained by all students in Geography?
    (1) 41.5 (2) 42.5
    (3) 38.5 (4) 39.5
    (5) None of these
47. Marks obtained by *E* in Economics are **approximately** what percent of the total marks obtained by him in all subjects together?
    (1) 12 (2) 8
    (3) 10 (4) 22
    (5) 17
48. What is the overall percentage of marks obtained by all the students in English?
    (1) 83 (2) 69
    (3) 71 (4) 81
    (5) None of these
49. What are total marks obtained by *D* in History, Mathematics, Science and English together?
    (1) 256.25 (2) 275.55
    (3) 235.75 (4) 253.75
    (5) None of these
50. What are the total marks obtained by *B* in all subjects together?
    (1) 376.3 (2) 373.6
    (3) 376.6 (4) 373.3
    (5) None of these

Answers

1. (1)	**2.** (5)	**3.** (2)	**4.** (4)
5. (3)	**6.** (5)	**7.** (1)	**8.** (4)
9. (2)	**10.** (3)	**11.** (1)	**12.** (4)
13. (4)	**14.** (1)	**15.** (5)	**16.** (2)
17. (5)	**18.** (2)	**19.** (3)	**20.** (4)
21. (4)	**22.** (3)	**23.** (5)	**24.** (3)
25. (1)	**26.** (3)	**27.** (5)	**28.** (1)
29. (4)	**30.** (5)	**31.** (2)	**32.** (3)
33. (1)	**34.** (4)	**35.** (1)	**36.** (5)
37. (3)	**38.** (1)	**39.** (2)	**40.** (4)
41. (1)	**42.** (3)	**43.** (4)	**44.** (4)
45. (2)	**46.** (3)	**47.** (5)	**48.** (2)
49. (4)	**50.** (1)		

SET 21

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Directions (1–10): What should come in place of the question mark (?) in the following questions?

1. $\sqrt{571536} \div 42 \times ? = 5850$
   (1) 420 (2) 240
   (3) 315 (4) 325
   (5) None of these
2. $(34)^{56} \times (34)^{-53} = ?$
   (1) 39304 (2) 1156
   (3) 170504 (4) 102
   (5) None of these
3. $378.35 + 478 \div 12.5 = ?$
   (1) 508.268 (2) 416.59
   (3) 425.28 (4) 68.508
   (5) None of these
4. $(550\% \text{ of } 250) \div 275 = ?$
   (1) 15 (2) 1.5
   (3) 0.5 (4) 25
   (5) None of these
5. $334.41 + 47.26 + 1.25 + 5 + 0.66 = ?$
   (1) 411.24 (2) 396.15
   (3) 388.58 (4) 376.85
   (5) None of these
6. $74844 \div ? = 54 \times 63$
   (1) 34 (2) 42
   (3) 22 (4) 54
   (5) None of these
7. $(21.35)^2 + (12.25)^2 = ?$
   (1) 171.4125 (2) 605.885
   (3) 604.085 (4) 463.8125
   (5) None of these
8. $124 + 56 \times 1.5 - 12 = ?$
   (1) –1890 (2) 252
   (3) 230 (4) 196
   (5) None of these
9. $\sqrt[3]{1092727} = ?$
   (1) 108 (2) 99
   (3) 97 (4) 107
   (5) None of these
10. $(46351 - 36418 - 4505) \div ? = 1357$
    (1) 4 (2) 6
    (3) 3 (4) 2
    (5) None of these

Directions (11–15): What should come in place of the question mark (?) in the following number series?

11. 124 228 436 ? 1684 3348
    (1) 844 (2) 852
    (3) 872 (4) 834
    (5) None of these
12. 1108 1117 1142 1191 ? 1481
    (1) 1312 (2) 1300
    (3) 1272 (4) 1204
    (5) None of these
13. 25 30 70 260 1280 ?
    (1) 6400 (2) 7680
    (3) 6380 (4) 7660
    (5) None of these
14. 8484 4248 2112 1074 513 ?
    (1) 201 (2) 280.5
    (3) 256.5 (4) 171
    (5) None of these
15. 154 162 226 ? 954 1954
    (1) 242 (2) 554
    (3) 442 (4) 642
    (5) None of these
16. The number obtained by interchanging the two digits of a two-digit number is less than the original number by 18. The sum of the two digits of the number is 16. What is the original number?
    (1) 97 (2) 87
    (3) 79
    (4) cannot be determined
    (5) None of these
17. By how much is $\frac{4}{5}$ of 1150 less than $\frac{5}{6}$ of 1248?
    (1) 140 (2) 115
    (3) 125 (4) 120
    (5) None of these
18. In how many different ways can the letters of the word 'OPERATE' be arranged?
    (1) 5040 (2) 720

(1) 482 (2) 346
(3) 556 (4) 384
(5) None of these

42. If $(74)^2$ is subtracted from the square of a number, the answer so obtained is 5340. What is the number?
(1) 98 (2) 102
(3) 104 (4) 110
(5) None of these

43. In a sale, a perfume is available at a discount of 15% on the selling price. If the perfume's discounted selling price is Rs 3675.40, what was the original selling price of the perfume?
(1) Rs 4,324 (2) Rs 4,386
(3) Rs 4,400 (4) Rs 4,294
(5) None of these

44. The product of two consecutive even numbers is 582168. Which is the smaller number?
(1) 760 (2) 762
(3) 764 (4) 766
(5) 756

45. The population of a town was 48600. It increased by 25% in the first year and decreased by 8% in the second year. What will be the population of the town at the end of two years?
(1) 65610 (2) 55580
(3) 60750 (4) 64850
(5) None of these

Directions (46–50): What approximate value should come in place of the question mark (?) in the following questions? (You are not expected to calculate the exact value.)

46. $54.35 \times 39.87 \div 13.35 = ?$
(1) 174 (2) 156
(3) 162 (4) 168
(5) 152

47. $\sqrt{3219} \times \sqrt{4178} = ?$
(1) 3953 (2) 3528
(3) 3498 (4) 3667
(5) 3591

48. $(749 - 325 - 124) \div (1254 - 1100) = ?$
(1) 2 (2) 4
(3) 6 (4) 8
(5) 12

49. $(47)^2 + 3.25 \times 2.5 = ?$
(1) 1624 (2) 1535
(3) 1687 (4) 1593
(5) 1699

50. 115% of $624 + \frac{2}{7}$ of $419 = ?$
(1) 887 (2) 833
(3) 765 (4) 756
(5) 787

Answers

1. (4)	**2.** (1)	**3.** (2)	**4.** (5)
5. (3)	**6.** (3)	**7.** (2)	**8.** (4)
9. (5)	**10.** (1)	**11.** (2)	**12.** (1)
13. (4)	**14.** (5)	**15.** (3)	**16.** (1)
17. (4)	**18.** (3)	**19.** (2)	**20.** (5)
21. (3)	**22.** (1)	**23.** (5)	**24.** (4)
25. (2)	**26.** (4)	**27.** (3)	**28.** (2)
29. (5)	**30.** (1)	**31.** (3)	**32.** (5)
33. (2)	**34.** (3)	**35.** (4)	**36.** (1)
37. (4)	**38.** (5)	**39.** (2)	**40.** (3)
41. (4)	**42.** (3)	**43.** (1)	**44.** (2)
45. (5)	**46.** (3)	**47.** (4)	**48.** (1)
49. (5)	**50.** (2)		

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