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Employability Quantified

QUANTITATIVE ABILITY LEARNING LEVELS

Engineering Graduates

Annual Report 2011-12

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Introduction

"Mathematics is the *language* with which God has written the universe."
— *Galileo* Galilei, Italian physicist, 16th Century

The importance of mathematics has increased several-fold since Galileo first made this statement five centuries ago. While the famed Italian physicist was referring to the importance of mathematics in deciphering the natural laws of universe, the statement may be interpreted from multiple standpoints today. Given that the universe is written in mathematics, it naturally follows that all engineering systems, be it electrical, mechanical or civil, are built on the language of mathematics. Little wonder then that the word computer itself derives from the Latin word computare, meaning "to count, sum up, reckon together"! From another perspective, basic mathematics is required to perform activities like selling, buying, arranging, counting, etc. in day-to-day life.

Given the importance of mathematics, both from the standpoint of its application in daily life and its importance in various professions of engineering, we decided to analyze the mathematical competencies of engineering graduates in India. Aspiring Minds has regularly been publishing various editions of the National Employability Report[1], citing low employability figures for engineers in India, identifying quantitative ability to be one of the primary reasons. Besides the natural connection discussed above, our statistical studies with software engineers in IT services companies[2] and product companies[3] provide irrefutable evidence of the importance of quantitative ability for success in engineering jobs. For instance, quantitative ability is a strong ingredient for technical trainability, i.e., the ability of a person to learn computer programming and software. Our study shows candidates with high quantitative ability have a much higher success rate in IT software training, as compared to others. Interestingly, the hypothesis may be extended to

1 National Employability Report for Engineers 2011: *http://www.aspiringminds.in/docs/national_employability_report_engineers_2011.pdf* ; National Employability Report for Engineers 2009: *http://www.aspiringminds.in/docs/national_employability_study_IT_ aspiringminds.pdf*

2 Featured Profile: Software Engineer (Entry level): *http://www.aspiringminds.in/researchcell/featured_profiles/software-engineer-fresher-entry-level.html*

3 Software Engineers: Adaptive testing can improve interview convert rates phenomenally *http://www.aspiringminds.in/researchcell/articles/adaptive_testing_improve_interview_convert_rates.html*

the importance of quantitative ability in engineering domain trainability as well, be it electronics, mechanical or civil.

The employability report documents that around 46% engineers do not qualify for a job due to lack of quantitative skills (technical trainability). Therefore, the present report analyzes specific areas of quantitative ability in which engineers are most deficient and the implications arising from the same. It also highlights how engineers are found lacking in certain crucial mathematical skills taught at middle and high-school level, and goes on to suggest ways to bridge these gaps by interventions in the Indian Higher Education System to improve the quantitative ability of our students.

All findings in this report are based on objective test-attempt data of 55,000 engineering students based on AMCAT Quantitative Ability, a competency-based standardized assessment for quantitative aptitude. Each item (question) in the assessment is mapped to a competency, and the inferences for each competency are thus analyzed. This is by far the most comprehensive analysis of quantitative skills of engineers conducted in India.

We sincerely hope this report will be welcomed by educationists, policy-makers and industrial bodies in India. It provides inputs on skill gaps, areas where training should be focused and how to make pedagogic changes to the education system to impart better mathematical skills to our students. It is our view that the report will also interest mathematicians and education psychology personnel, as it shows relative trends in which mathematical skills are easy or hard to pick up[4].

We at Aspiring Minds are committed to present this analysis yearly to audit how our education system is faring in imparting better quantitative ability. For students, this is a means of empowerment to lead a better life, a higher probability of landing a good job and more importantly, a chance to innovate and understand the language of the universe.

Varun Aggarwal

Varun Aggarwal

Co-founder and Director
Aspiring Minds

4 Documented in Appendix

Methodology

This report is based on a sample of more than 55,000 engineering students from 250+ engineering colleges across multiple Indian states. All these candidates graduated in 2011[5]. The analysis and findings of this report are based on the results of these students on the Quantitative Ability module of AMCAT (Aspiring Minds Computer Adaptive Test), which is India's largest and only standardized employability test. AMCAT Quantitative Ability comprises basic mathematics, applied mathematics and engineering mathematics[6]. The module is adaptive, and its scoring scheme is based on item response theory, a globally accepted statistical technique to assess high-stake exams. The test was conducted under a proctored and credible environment ensured by Aspiring Minds.

While designing the AMCAT Quantitative Ability test, our instrument design team developed a competency framework to include majority of crucial quantitative skills required by engineers for entry-level technical jobs. Based on this competency framework, items (questions) for the test were designed. Special care was taken to ensure that items had good psychometric properties[7]. The competency allocation for each item was verified through a consensus by experts. After expert review on various parameters, the items were sampled on actual engineering test-takers. Questions with irrelevant statistical properties were weeded out and item response models for the rest of the questions were developed. These were then delivered adaptively using Aspiring Minds' proprietary item selection and delivery algorithms. The final assessment test showed reliability comparable to global standards.

In this report, our research team studied the performance of engineering graduates on each competency based on the response data of the items. For each competency, the percentage correct response rates for each tagged item were determined. The consensus response rate of items was considered as the actual performance of engineers on the given

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5 The sample was statistically balanced across various parameters to be representative of the true technical graduate population. A carefully chosen stratified sample was used for the study.

6 The detailed syllabus of the module is included in Appendix 1.

7 http://www.aspiringminds.in/researchcell/articles/how_to_create_test_blueprint.html

competencies. Items showing consensus as well as those acting as outliers were studied by experts to understand their respective behavior. The validity of some of the items was found to be influenced by biases such as elimination-strategy, rote learning, etc. Such items were not included in developing an estimate of the performance of engineers on each competency. Apart from testing performance in generalized competencies, various interesting trends that were found during the analysis are reported.

This report is organized as follows: for each area of mathematics, a list of competencies with their definition is provided, followed by the percentage of total engineers who exhibited possessing the competency. Sample questions for different competencies are also highlighted to provide insight into the construct of the competency. Finally, for each area, we share some key observations that will be of interest to people involved in academics, mathematics, education, policy making, etc.

Specific learning inferences from the study are summarized for interested readers in Appendix.

Executive Summary

This study looked into the quantitative skills of engineers across India, which is one of the basic constitutive skills for pursuing engineering education and engineering as a profession. The key findings are as follows.

Over 30% engineers do not have basic quantitative skills as required in day-to-day life and entry-level engineering jobs.

Over 30% engineers do not possess mathematical skills needed in day-to-day life for doing simple transactions, counting, arranging, etc. The competencies required for these operations involve having a basic understanding of decimals, powers, simple mathematical operations, ratio, fractions and the ability to apply these concepts to real-world problems. These skills are required in all engineering and analytics jobs in the knowledge-based industry. For instance, an engineer who cannot multiply/divide decimal numbers (the total being an alarming 42%) would face difficulty in doing basic engineering calculations. Lack of these skills incapacitates the candidate to be suitable for any engineering job, rendering him unemployable. Furthermore, these candidates may face challenges in their day-to-day life in doing tasks involving simple mathematical operations.

Suggested Intervention

These skills should be learned by students in middle and high-school before they start engineering education[8]. Whereas schools need to ensure that such skills are successfully imparted to students, engineering colleges must take cognizance of the fact that these skills are prerequisites for the four-year technical syllabus of engineering. To enable all inducted students to learn their courses well, the gaps in basic mathematical understanding of each inducted student must be identified and bridged in the first semester itself. Methodologies to assess, identify, train and track the quantitative ability of students from the first year need to be institutionalized. By bridging these gaps in the first semester, institutions would be able to produce

8 Our findings are in line with Pratham's ASER report, which observes that even after the initial four years in school, two-thirds of children were not up to the level prescribed by the Std 1 textbook.

engineers with higher employability following proper training in technical skills.

Less than 40% engineers can perform 'problem solving' by cross-application of concepts.

There is a sharp fall in the percentage of engineers who can 'cross apply' mathematical constructs to solve practical problems, as opposed to straightforward, theoretical application (formulae-based) of standalone concepts. For each area of mathematics identified in the report, the percentage of engineers who can solve questions based on direct formulae of the concept is 60% and above, whereas any cross-application of concept makes the percentage fall to under 40%.

Engineering design is all about cross-application of multiple concepts to solve a given real-world problem; theoretical knowledge without the understanding or skill of application is a big handicap. Lack of such a skill in a majority of the engineers is a strong impediment in system and product design in India.

Suggested Intervention

It is essential to encourage more hands-on engineering exercises, besides individualized courses in multiple disciplines. There are a few different ways of doing this. Firstly, students should be encouraged to build real-world engineering projects. These projects naturally contain quantitative problem solving, in addition to cross-domain engineering problem-solving. Such projects would be interesting to students and would challenge them to apply the concepts rather than learn them by rote. At least 25% of the courses at engineering colleges should have a project component apart from regular quizzes and exams. From a longer term perspective, multidisciplinary and engineering design courses may be introduced in colleges to encourage cross-application of not just quantitative ability concepts but multiple engineering disciplines.

Fewer than 30% engineers can apply engineering mathematics concepts.

Increasing engineering innovation is an important task for India. The prime minister recently articulated how research has declined in India in recent years[9].

9 http://timesofindia.indiatimes.com/home/science/China-ahead-of-India-in-scientific-research-says-PM-Manmohan-Singh/articleshow/11349667.cms

Since mathematics is the language of engineering, high competence in engineering mathematics is important for furthering the frontiers of research and innovation. We found that a very small percentage of engineers show competence in applying engineering mathematics to solve problems. For instance, only 24% engineers can apply probability and permutation-combination to solve complex problems. These areas are important for understanding the science behind engineering: algorithm design, communication theory, building complex models for physical and mechanical systems, etc. Lack of these skills not only impedes advanced product design, but also holds back high quality research and innovation.

Suggested Intervention

Excellence in any system is driven by inspiration. Without inspiration, no level of curriculum and teaching can lead to excellence. While good teaching is indispensable, colleges need to find ways to institutionalize a culture of inspiration and love for mathematics. This, unfortunately, is a problem for which no standard solutions exist.

I. Decimal Fractions

Competency categorization of decimals and fractional numbers

A. Understanding of central tenets of Decimals and Fractions.

Ability to do simple operations with decimals and fractional numbers, including:

- Performing simple mathematical operations on decimal numbers and fractions, such as addition, subtraction, multiplication and division.
- Conversion of fractions into decimals and vice versa.
- Proper and mixed fractions.
- Comparison of fractions, including negative fractions.

70% have competency*

This competency does not measure accuracy or speed of calculation. All calculation involved are simple, focusing only on the conceptual understanding of decimals and fractional numbers.

Exposure : Introduced in grade VII, and continues on to grade VIII in CBSE school education. In ICSE, it is introduced in Class VI continues on to grade VII.

Relevance : This is required in day-to-day arithmetic for any engineering discipline, since most calculations comprise decimal numbers.

B. Ability to intuit about decimal/fractional and perform advanced operations.

Ability to think about decimal numbers intuitively, solve problems by combining other mathematical concepts not immediately apparent, and being able to quickly approximate numbers. This includes but is not limited to:

- Terminating and non-terminating recurring decimals.
- Calculating roots and powers of decimal numbers.
- Solving problems involving decimal numbers through intuitive understanding.

51% have competency*

Such understanding shows a highly evolved intuition of numbers, beyond basic arithmetic and requires conceptual understanding to be creative with numbers.

Exposure : Such competency is developed only through practice, passion and innate ability, in addition to regular classroom study. Some of these concepts are covered in Standard IX in both CBSE and ICSE curriculum.

Relevance : Possessing such a competency helps an engineer tremendously in designing and debugging systems, which naturally requires approximations, back-of-envelope calculations and creative intuition. It is also a must for those in analytical roles in the Knowledge Process Outsourcing sector and consultancies.

* Highlights the % of engineering students who possess the competency

Sample Questions

Competency Level (i)	Competency Level (ii)
Identify the smallest decimal number from the following: (a) 0.5 (b) 1/0.5 (c) 0.555555 (d) $(0.5)^2$	0.5 + 0.5 x 0.33 + 0.33 x 0.25 + … Given the series above, what would be the sum of the first 10 terms: (a) 9/10 (b) 10/11 (c) 11/10 (d) 9/13
(Requires: Simple understanding and comparison of decimal numbers.)	*(Requires: Identifying pattern by converting decimal numbers into fractions.)*
72% answered correctly	**26% answered correctly**

Observations

The results lead us to make the following observations:

- An average of 70% of engineering graduates understand concepts related to central tenets of decimals and fractions correctly and can do associated basic mathematical operations.
- Among these, a lower percentage (58%), have the ability to divide and multiply decimal numbers, as compared to those who can perform addition and subtraction (79%).
- A much lower percentage of candidates are able to handle negative fractional numbers.
- Only half of the engineers demonstrated intuitive understanding of decimals commonly required for approximations, back-of-envelope calculations and analytics.

Whereas 59% of examinees were able to arrange 2 or more fractions in ascending order, this number fell to 43% if one or more of the fractions involved were negative.

II. Divisibility

Competency categorization of number divisibility

A. Ability to factorize and knowledge of basic divisibility rules.

Awareness of divisibility rules limited to digits from 2 to 11. The numbers in these cases were exactly divisible by the divisor, and ability to work with remainders was not assessed.

Since any large number degenerates into smaller, easily manageable numbers (for instance, to check whether 98234 is divisible by 9, one only needs to check if 9+8+2+3+4 is divisible by 9), this competency does not require a very high degree of mathematical skill.

77% have competency

Exposure : Is introduced in grade VI, and is reintroduced at a slightly more abstract level in grade VIII in CBSE school education, while in ICSE curriculum it is taught through classes VI to VIII.

Relevance : This competency is needed for simple operations such as breaking amounts into denomination of currency notes, performing simple storage calculations given quantities of objects, box dimensions, etc. It also enhances speed of calculations and the ability to do mental math. It is a must for any quantitative job such as that of engineering design, software design, analytics, etc.

B. Ability to solve problems by creatively combining divisibility rules with basic number.

This competency not only requires knowledge of number divisibility, but also intuitive understanding and comfort with numbers in general. It calls for cross-application of knowledge of other mathematical concepts, arriving at solutions based on intuitive understanding and making calculated guesses. This does require a moderate level of attention to detail and accuracy. This competency includes:

42% have competency

- Identifying recurring patterns in exponential powers to perform divisibility checks.
- Deducing divisibility test rules for two or three-digit numbers as divisors.
- Forming algebraic equations to identify a number, given division details including remainder, quotient, etc.

Exposure : These concepts are covered in Standard VIII in CBSE and Standard IX in ICSE; however, mastery of this competency requires practice, passion and high intuitive ability.

Relevance : The competency is required for all analytical jobs and those in advanced engineering, product or software design. The absence of this competence could be a big handicap in basic engineering and software design jobs.

Sample Questions

Competency Level (i)	Competency Level (ii)
What least value must be assigned to * so that the number 3626*8 is divisible by 8?	In how many ways can the number 76176 be written as a product of two different factors ?
(a) 0 (b) 4 (c) 5 (d) 8	(a) 23 (b) 22 (c) 19 (d) 18
(Requires: Checking divisibility rule for 8.)	*(Requires: Factorization and counting.)*
75% answered correctly	**35% answered correctly**

Observations

The foregoing analysis points to the following observations:

- On an average, 77% of engineering graduates possess knowledge of basic divisibility rules and can identify whole multiples of single digit numbers.
- While divisibility by 2, 3 and multiples of 2 is commonly understood by 86% of the engineering graduates, this percentage falls to 70% when divisors like 6 and 9 are considered.
- Only 42% of engineering graduates develop higher-level intuitive understanding of concepts not directly taught in school or college. The rest are unable to intuit ways of problem solving that do not have a defined solution process. An example of the same is shared above (Competency level (ii)), which was solved by only 35% people.

While divisibility by 2, 3 and multiples of 2 is commonly understood by 86% of the engineering graduates, this percentage falls to 70% when divisors like 6 and 9 are considered.

- We find that engineering graduates are more comfortable with factorizing and solving algebraic equations, but are unable to intuit ways to solve puzzle-like problems involving divisibility. For instance, the problem "If a, b, c are roots of the equation $1x^3 - 4x^2 + 6.5x + 3.5 = 0$, then what is the value of $a^2 + b^2 + c^2$?" was successfully solved by 66% of engineering graduates, whereas only 43% were able to solve "Find the remainder when (888 . . . repeat 63 times) is divided by 9". Even though factorizing algebraic equations (symbolic factorization) is considered more difficult than numerical divisibility by experts, empirical trends show otherwise. This is because the former is taught in schools by means of set solving mechanisms and formulae, whereas the latter requires intuitive understanding and application of various concepts, and not necessarily a formula-based approach.

III. HCF and LCM

Competency categorization of HCF, LCM and factorization

A. Understanding basics of HCF and LCM, including calculation.

This competency highlights an elementary understanding of the difference between HCF and LCM, what they signify and the ability to calculate them.

Exposure : HCF, LCM and prime factorization is first introduced in Standard VI in CBSE & ICSE boards.

62% have competency

Relevance : Understanding of HCF and LCM is a basic skill required in day-to-day calculations and all engineering-related professions.

B. Ability to solve word problems based on concepts of LCM and HCF:

The competencies tested include the following, amongst others:

- Properties of HCF and LCM (for instance, HCF X LCM = Product of numbers).
- Application of concepts of HCF and LCM along with other mathematical concepts such as ratios, square and square roots.

45% have competency

Exposure : Basic-level word problems for HCF and LCM are introduced in Class IX in CBSE and ICSE syllabus.

Relevance : Application of HCF and LCM is required in day-to-day calculations and is a must for any analytical/engineering profession.

C. Understanding properties of HCF and LCM, and seamlessly combining different mathematical concepts in solving word problems.

This requires a deeper understanding of HCF and LCM, factorization and their properties. It also tests the ability to decipher real-world problems as checking for HCF and LCM.

Exposure : The concepts are taught in Standard VI but are not used often and their application in other mathematical areas is also low. However, the word problems are an important aspect, covered in standards VIII and IX.

26% have competency

Relevance : Possessing such competency indicates a degree of comfort with numbers and an ability to work with abstract concepts, which has a direct bearing on system design, algorithm analysis, etc.

Sample Questions

Competency Level (i)

What will be obtained if 5 is subtracted from the HCF of 138, 144 and 84?

(a) 1

(b) 6

(c) 8

(d) 7

(Requires: Simple HCF calculation

67% answered correctly

Competency Level (ii)

The LCM and HCF of two positive numbers are 1000 and 50, respectively. Which of the following could be the pair of such numbers?

(a) 250 & 200

(b) 20 & 2500

(c) 400 & 125

(d) 40 & 1250

(Requires: Use of product property of HCF and LCM.)

42% answered correctly

Competency Level (iii)

Four timers beep after every 30 minutes, 1 hour, 3/2 hour and 1 hour 45 minutes, respectively. All the devices beeped together at 12 noon. They will again beep together at:

(a) 6pm

(b) 12 midnight

(c) 3am

(d) 9pm

(Requires: Understanding that one requires to find out LCM of the numbers to solve the problem.)

27% answered correctly

Observations

The following observations can be made:

- 62% of engineering graduates understand constructs of HCF and LCM, and this number stays consistent across different measures of the same competency.
- While basic tenets of HCF and LCM are lucid, competence on properties and conditional word problems drops by 20% percentage points.
- From the perspective of deciphering the application of LCM and HCF in real-world problems, the understanding is very weak and an average of only 30% engineering graduates are able to do this successfully. This can be assessed through word problems such as "A light blinks after every 3 seconds, another light blinks after every 5 seconds and the third one blinks after every 16 seconds. How many times do they blink together in half an hour?"

On average only 30% engineering graduates are able to understand the application of LCM and HCF in real-world problems

Competency Level (i)

Q. What will be obtained if 5 is subtracted from the HCF of 138, 144 and 84?

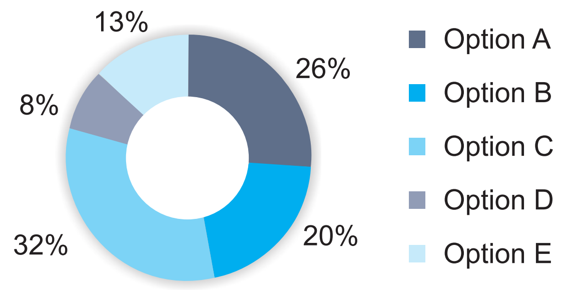
(a) 1 (b) 6

(c) 8 (d) 9

(e) 3

Correct answer : (a)

Break-up of options selected



IV. Powers

Competency categorization of powers (exponents)

A. Understanding of exponential properties.

This competency highlights familiarity and comfort with basic properties of exponential functions involving both positive and negative exponents.

80% have competency

Exposure : These form part of the Standard VIII curriculum in CBSE, whereas in ICSE exponents, squares, cubes are introduced in Grade VI.

Relevance : Exponents often result in equations describing the physical world, which form an integral part of electronics, electrical and mechanical engineering. They are also an important construct used in counting, which is foundational to the study of computer algorithms.

B. Ability to solve algebraic equations involving exponents and ability to work with decimal fraction base and exponents.

This competency is reflective of an understanding of various aspects of exponents which can be employed for problem solving. It represents an intuitive knowledge of being able to adjudge where these properties can be utilized in order to solve problems in the shortest and quickest possible way. The learning levels that are considered for this competency are:

68% have competency

- Roots and powers of whole numbers (4-digits or more) and decimals.
- Solving linear and quadratic equations based on exponential properties.

Exposure : The concepts are covered in Standards VIII and IX and are employed frequently in problem solving.

Relevance : Similar as above

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Sample Questions

Competency Level (i)

If $2^{2n-2} = 4^{2n+2}$ then the value of n is:

(a) -1
(b) -2
(c) 2
(d) 1

(Requires: Basic exponential properties.)

78% answered correctly

Competency Level (ii)

Simplify: (a-2 - 1)/(a + 1)

(a) $a^{-2}-a^{-1}$
(b) $a^{-1}-a^{2}$
(c) $a^{2}-a^{-3}$
(d) $a-a^{-2}$

(Requires: Algebraic simplification of exponents.)

64% answered correctly

Observations

This analysis enables us to make the following observations:

- Additive, multiplicative and divisive properties of powers (both positive and negative) are among the most well-understood mathematical concepts, with 80% of engineering graduates displaying a comfort of working with these.
- Even with fractional, negative or algebraic powers, this level of comprehension remains unaffected at 80%
- A lower number of engineering graduates are able to accurately calculate square roots of whole numbers and decimals.

V. Word problems:
averages, ratios and fractions

Competency categorization of word problems dealing with averages, ratios and fractions

A. Ability to decipher word problems on ratios and fractions involving a single variable.

This competency measures the ability to convert simple word problems into mathematical equations. Two concepts being assessed here are averages and ratios and fractions. The learning levels have been assigned based on the number of variables involved in a single equation, the basic level comprising one variable.

61% have competency

Exposure : Such problems are commonly studied in upper-primary school, from classes VI to VIII.

Relevance : This is a skill of much importance, constituting a part of the basic arithmetic required for any engineering discipline.

B. Acumen to comprehend word problems relating to averages, ratios and fractions involving change and more than one variable.

This competency is correlated with an ability to be able to formulate and solve equations involving more than one variable. Where ratios are concerned, a typical construct being measured is the ability to logically understand and quantitatively determine the implication of a change in the ratio and proportions.

39% have competency

Exposure : While the topics are covered in upper-primary school, Standard IX mathematics has a comprehensive coverage of word problems with more than one variables.

Relevance : Similar as above

Sample Questions

Competency Level (i)	Competency Level (ii)
If Ramesh scored 68 out of hundred then how much would he out of 75? (a) 54 (b) 51 (c) 48 (d) 45	I read 4/9th of a book on one day, and 3/5 of the remainder on another day. If now there were 60 pages unread, the book contains: (a) 260 (b) 270 (c) 240 (d) 210
(Requires: Simple ratio calculation.)	*(Requires: Algebraic formulation based on ratios.)*
68% answered correctly	**30% answered correctly**

Observations

The analysis of the above-mentioned competency in word problems says that:

- An equal percentage of engineering graduates are comfortable with uni-variable word problems involving averages and ratios & fractions. In both cases, 62% of engineering graduates display an aptitude for these concepts.
- Simple problems such as "If candies were distributed amongst A, B and C in the ratio 4:6:14 and B's share was 66 candies, then how many candies were distributed in all?" were solved by only 60% of the sampled graduates.
- Understanding and ability to calculate how a given ratio changes as a result of variations in the scenario is low, with 34% engineering graduates ascertaining these accurately. Such problems offer a peek into real-life, practical scenarios which an engineering graduate ought to know.
- Basic-level problems such as "If Ramesh scored 68 out of 100, what would his score be if the exam was marked out of a total of 75 instead?" and "Which is a better investment: 15% stock at 190 or 6% stock at 320?" were correctly answered by 68% and 53% of graduates, respectively, which does not present an encouraging picture.

Which is a better investment: 15% stock at 190 or 6% stock at 320?

VI. Word problems: Inverse relationships

Competency categorization of word problems with inverse proportion

A. Understanding of elementary-level word problems on concept of work and time.

This competency assesses the ability to be able to solve simple work-time problems involving two variables. These problems have been separated from other word problems since they necessitate an understanding of the concept of inverse relationship; i.e., when number of men increases, time required decreases, and hence this category measures a different construct than other word problems.

64% have competency

Exposure : Time & work and inverse variation problems are introduced in Standard VIII.

Relevance : This is an extremely crucial skill, something useful for a non-specialist, and forms a part of the basic arithmetic required for any engineering discipline.

B. Ability to work with inverse relationship concepts involving three variables and more than one mathematical concept.

This higher-level competency reflects the comfort in working with inverse relationships involving three variables. The constructs measured also include combining inverse relationship problems with other mathematical concepts such as ratios, linear equations, proportions, etc.

38% have competency

Exposure : Competitive examinations and preparation for higher studies.

Relevance : Similar as above.

Sample Questions

Competency Level (i)

20 men can complete a piece of work in 10 days. In how many days will 25 men complete the same work?

(a) 6 (b) 8

(c) 9 (d) 12

(Requires: Understanding Simple inverse relationship)

70% answered correctly

Competency Level (ii)

Ronald and Elan are working on an assignment. Ronald takes 8 hours to type 36 pages on a while Elan takes 6 hours to type 39 pages. How long will they take working together to type an assignment of 275 pages?

(a) 24 (b) 26

(c) 25 (d) 28

(Requires: Working with multiple inverse relationships combined together)

30% answered correctly

Sample Questions

Competency Level (i)	Competency Level (ii)
20 men can complete a piece of work in 10 days. In how many days will 25 men complete the same work? (a) 6 (b) 8 (c) 9 (d) 12 *(Requires: Understanding Simple inverse relationship)* **70% answered correctly**	Ronald and Elan are working on an assignment. Ronald takes 8 hours to type 36 pages on a while Elan takes 6 hours to type 39 pages. How long will they take working together to type an assignment of 275 pages? (a) 24 (b) 26 (c) 25 (d) 28 *(Requires: Working with multiple inverse relationships combined together)* **30% answered correctly**

Observations

Following observations can be made:

- The cognitive skills required to deal with inverse relationships are typically assessed through two concepts – work & time and pipes. While the construct measured remains the same, it is observed that 70% of engineering graduates are comfortable when inverse relationship between two variables is to be determined ("If 3 engineers can write a piece of code in 9 hours, how many engineers are required to write it in 6 hours?"); by contrast, this percentage drops to 60% when the concept of combined time or 'together' is introduced ("A heater can heat the hall in 20 minutes while another takes 35 minutes. If both heaters are switched on at the same time, how long will it take to heat the room?").
- 62% of engineering graduates are unable to deduce problems involving inverse relationships if three variables need to be simultaneously balanced. This indicates that engineering graduates find it more difficult to work with three variables than constructs where the concepts of proportions and inverse relationships are being simultaneously measured. 56% engineers are unable to comprehend constructs where more tedious calculations are required as a result of changing proportions.
- As the number of conditions is increased, the percentage falls further, and only 28% of engineering graduates are able to extrapolate the above concepts. Since the fundamental constructs remain unchanged, it is inferred that intuitive understanding is lacking.

VII. Word problems – Profit and loss

Competency categorization of Profit, Loss and Discounts

A. Ability to perform simple mathematical operations related to profit, loss & discount.

This competency tests elementary understanding of profit and loss, The students' familiarity with terms, such as profit, loss, discount, mark-up, selling price, marked price, etc. are evidenced from this competency. Following is a representation of the constructs being measured:

66% have competency

- Solving for the missing variable, given the relationship with another variable. The relationship could be expressed in ratio, fractions, decimals or require back calculation or any combination of these.

Exposure : Simple concepts of profit and loss are introduced in Standard VII in CBSE and Standard IX in ICSE.

Relevance : This is a skill of much importance, being useful for a non-specialist, and is utilized in efficiency calculations required in engineering disciplines.

B. Ability to comprehend successive changes in cost and sale price:

This competency ascertains ability to calculate sales price, given cost and successive discounts, mark-downs, followed by markups, etc. It also involves calculations to find out cost price for the given sale price, and intervening changes to the cost price. The competency is a measure of understanding that successive discounts of a% and b% are not equal to a single discount of (a+b)%.

46% have competency

Exposure : This concept is introduced in Standard IX in CBSE and ICSE boards.

Relevance : Similar as above.

C. Ability to work with three variables and strong intuitive understanding of concepts of profit and loss.

This competency highlights the ability to work with vagueness and abstraction when one variable is expressed not in absolute terms, but in relation to another. It also measures the ability to balance 3 variables and understanding the implication of successive markup/discount in terms of real-life situations. All these aspects signify a strong clarity of basic concepts, irrespective of the form they are presented in.

35% have competency

Exposure : These concepts need to be practiced, and form part of competitive exams and other higher-education entrance exams.

Relevance : Similar as above.

Sample Questions

Competency Level (i)

A man bought 400 meters of cloth for Rs. 40,000 and sold it at a rate of Rs. 200 per one and a half meter. What was his percentage profit or loss?

(a) 36% loss
(b) 25% profit
(c) 33% profit
(d) 27% loss

(Requires: Elementary understanding of profit and loss.)

54% answered correctly

Competency Level (ii)

Shreya sold 20 necklaces for Rs. 600 and suffered a loss of 20%. At what price should she sell 24 necklaces in order to earn 20%?

(a) 1050
(b) 1080
(c) 950
(d) 1250

(Requires: Understanding of successive changes in profit and loss.)

35% answered correctly

Competency Level (iii)

A shopkeeper buys 200 kg rice at Rs. 7.25 per kg and 400 kg at Rs. 5.75/kg. He mixes and sells 1/3rd of the mixture at Rs. 6/kg. At what price/kg should he sell the remaining mixture so that he may earn a profit of 20% on the whole?

(a) Rs. 8
(b) Rs. 8.25
(c) Rs. 6
(d) Rs. 9.25

(Requires: Ability to work with multiple variables and intuitive understanding of profit and loss constructs.)

24% answered correctly

Observations

The crucial observations from this analysis are as follows:

- While 70% of engineering graduates are comfortable with simple profit and loss problems, the number reduces to 66% primarily because of problems involving back calculation and abstraction when absolute figures are not mentioned.
- Abstract concepts can be classified into two types -first, where the variables are not presented as absolute figures and need to be represented as algebraic variables; and second, where the relationship of the variables is expressed in terms of one other or a third independent variable. While the abstraction does not pose a problem (compared to overall averages), the number of variables has a marked impact on comprehension: abstract constructs with 2 variables are understood by 66% of engineering graduates whereas this percentage drops to 31% when 3 variables are introduced.
- Sample question involving abstraction and three variables: The fundamental concept of successive changes to the Sale Price of an item is comprehended by an average of 42% engineering graduates. This percentage bumps up by 7 percentage points if the abstraction is removed.

Neha sold a book at a profit of 20%. Had she bought the book for 10% less and sold it for Rs. 10 more, she would have still earned a profit of 20%. What is the cost price of the book?

VIII. Probability

Competency categorization of Probability

A. Ability to comprehend the concept of probability and understanding the implication of 'or'.

This indicates a familiarity with the concept of probability and a basic aptitude of being able to decipher problems related to chance. Awareness of the concept of event, success and failure, and differentiation of single measure of success vs. one of the two possibilities, along with understanding of independent events are indicated. This competency includes:

65% have competency

- Basic probability with single measure of success in a single draw.
- Probability when any one of two outcomes is regarded as success in a single draw.
- Probability of independent events.

Exposure : Introduced in Standard IX in CBSE board and Standard X in ICSE board.

Relevance : Probability theory forms the basis of and has applications in almost all engineering disciplines, be it understanding of algorithms, compilers, microprocessors, communication, signal processing, mechanical design, mathematical modeling, etc. It is quite fundamental to any higher-level understanding of engineering and a must for post-graduate studies in engineering.

B. Comprehension of tenets of probability in more than one draw or in more than one event.

Here the measure of conditional probability is adjudged. The competency checks understanding of higher-level concepts of probability where the outcomes are dependent on more than one draw or more than one event.

36% have competency

- Conditional probability where success is dependent on more than one draw of both dependent and independent events.
- Conditional probability where more than one event is used to determine success.

Exposure : These concepts are taught in schools at various levels, along with topics covered in previous competency, but require practice to be mastered.

Relevance : Similar to above.

C. Strong incisive knowledge of combinatorics and probability, and the ability to work with them together.

This competency reflects and ability to be able to combine concepts of combinations (basic-to medium-level) and probability, in order to arrive at the probability of one or more events. Concept of circular probability is also measured here.

Exposure : Both permutation, combination and probability are introduced in Standard XII. This competency requires a lot of practice, intuition and understanding of both permutation-combination and probability.

28% have competency

Relevance : Similar to above.

Sample Questions

Competency Level (i)	Competency Level (ii)	Competency Level (iii)
In a single throw of dice, what is the probability to get a number greater than 3? (a) 1/2 (b) 2/3 (c) 1/3 (d) 1/6	There are two boxes A and B. Box A has two red and three blue balls. Box B has five red and two blue balls. A ball is drawn from each bag randomly. What is the probability that both balls are red? (a) 5/7 (b) 2/7 (c) 4/5 (d) 1/5	A bag contains 3 white, 4 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are blue is: (a) 8/153 (b) 10/153 (c) 10/143 (d) 9/143
(Requires: Simple counting to ascertain probability.) **56% answered correctly**	*(Requires: Joint probability of events.)* **41% answered correctly**	*(Requires: Probability and combinatorics applied together.)* **29% answered correctly**

Observations

The analysis of probability competency enables us to make the following observations:

- Only 66% of engineering graduates are able to understand fundamental probability problems which only require knowledge of the basic formula:

  $$\textit{Probability of event} = \frac{\textit{Number of favourable outcomes}}{\textit{Total number of outcomes}}$$

  Sample question: In a year, what is the probability of the month to be January?

- An average of 64% of engineering graduates understand the importance of 'or' in probability, which requires them to sum up the probabilities of more than one outcome. Representative questions are of the type: “In a single throw of dice, what is the probability of getting a number greater than or equal to 3?”

- Only 32% of engineering graduates are comfortable with set theory in probability. While this is a fundamental concept of probability theory, it is observed that terminologies like mutually exclusive events are not understood by candidates. Difference between independent events and mutually exclusive events is not clear.

Only 66% of engineering graduates are able to understand fundamental probability problems which only require knowledge of the basic formula

- A marginally higher, 38% of engineering graduates display acumen for probability concepts where the outcome is dependent on more than one draw. Difference between probability with replacement and probability without replacement is poorly understood, with only 26% being able to understand the difference.

- Given certain choices, ascertaining probability when a combination of two or more outcomes is defined as success, is comprehended by 30% of the sample population. These problems require an amalgamation of combination formulas and probability to arrive at the solution, for example: “Kunti is determining where to travel; she has 3 brochures for Asia, 4 for Europe and 7 for Africa and wants to take the decision based on random selection. She closes her eyes and picks 3 brochures at random. What is the probability that all of these are places in Africa?”

IX. Permutation and combination

Competency categorization of permutation and combination

A. Understanding of fundamentals of counting, and principles of permutation and combination:

This competency, more than anything, represents a basic-level comfort with permutations and combinations. It checks whether the candidate can differentiate between permutation and combination and use their canonic formulae to solve a given problem. The skills being assessed are:

59% have competency

- Distinguishing between permutation and combination.
- Fundamental principles of counting and basic permutation-combination formulae.
- Permutation with no repeats, including selection from more than one set.
- Elementary combination problems including selection from more than one set.

Exposure : This is a higher-level concept which is introduced only in high school, i.e. Standard X.

Relevance : Permutation and combination forms the basis of understanding computer science algorithms, optimization, discrete number sets and analytical studies.

B. Understanding of permutation and combination with conditions such as repetition and multiple subsets.

This represents a comfort with permutation and combination where the candidate needs to make some interpretation and choice. He/she needs to identify that the given problem deals with certain exceptions and requires some intermediate formulation beyond the canonic formulae. The knowledge of advance formulae may also be required.
This includes:

39% have competency

- Conditional counting with no repeats
- Permutation with repeats
- Indentifying combinations from more than one subset with ordering.

Exposure : These are constructs taught in high-school but represent higher learning levels than basic constructs.

Relevance : Such exceptions often occur in computer science algorithms, discrete mathematics, digital circuits and optimization formulations. An understanding of these is a necessary prerequisite for advanced engineering studies.

C. Strong grasp over permutation and combination measured through cross-application of more than one concept.

This competency represents absolute comfort with all aspects of permutation and combination. This skill is acquired through continuous practice. Learning levels include:

- Constructs involving simultaneous use of permutations and combinations.
- Understanding of the concept of 'at least'.
- Problems requiring simultaneous use of multiple concepts of permutation and combination.

24% have competency

Exposure : The constructs are taught in high school, but this skill being at a higher learning

Relevance : Similar as above.

Sample Questions

Competency Level (i)	Competency Level (ii)	Competency Level (iii)
Mayank is going for a holiday. He wants to take 3 t-shirts from 5 t-shirts he has. In how many ways can he make his choice? (a) 15 (b) 10 (c) 8 (d) 20	In how many different ways can the letters of the word 'OCTAVE' be arranged so that the vowels always come together? (a) 156 (b) 180 (c) 144 (d) 204	8 questions are to be answered, choosing at least 3 from each of part A and B. If there are 5 questions in part A and 6 in part B, in how many ways can 8 questions be answered? (a) 145 (b) 155 (c) 165 (d) 175
(Requires: Simple combination application.)	*(Requires: Conditional permutation-combination.)*	*(Requires: Combining multiple permutation-combination concepts.)*
46% answered correctly	**37% answered correctly**	**28% answered correctly**

Observations

The following observations follow from this analysis:

Only 24% of all engineering graduates are conversant with higher-level concepts of combinatorics.

- Multiplication principle of counting[10], where the total number of combinations is to be determined, given multiple choices for each constitutive element, is most easily understood out of all aspects of combinatorics. A typical problem of this kind is: “Sushil needs to go for a meeting. He has 3 formal shirts and 8 trousers. In how many ways can he make a choice about what to wear?” 69% of engineering graduates display an understanding of this concept as compared to an average of 59% when other elementary permutation combination problems are also taken into consideration.
- Whereas 59% engineers are able to solve direct permutation-combination questions, in case a choice between permutation or combination formulae has to be made for solving a particular word problem, the percentage falls to 46%. This shows that a large proportion of engineers do not have an understanding of difference between permutation and combination, or at least its applications.
- Multiplicative principles of counting, given certain conditions and permutations with repeats are the next most-easily understood constructs and are understood- by 42% engineers. Permutations under different conditions comes a distant third at 29%.
- Only 24% of all engineering graduates are conversant with higher-level concepts of combinatorics. The understanding for combinations is lower than that of permutations, and the concept of 'at least' in counting is not well-grasped.

10 http://www.cs.odu.edu/~toida/nerzic/content/counting/basic_principles.html

Appendix

AMCAT Quantitative Ability(technical): Structure and Design

The Aspiring Minds Quantitative Ability test is designed to assess the quantitative skills of an engineering college graduate, and covers topics that have been taught in school as part of the national curriculum. The test assesses an examinee's ability across various levels – from checking understanding of basic mathematical properties governing each topic, to more application-oriented problems which necessitate an understanding of the concepts as well as use of intuitive and logical ability in a two-step process to arrive at solutions. The test also consists of word problems depicting real-life situations that assess the examinee's ability to identify the construct a mathematical problem is presenting, as well as the ability to integrate different mathematical concepts in order to arrive at a solution.

Quantitative Ability

Basic Numbers

Decimals and fractions

Basic algebra

Factorization, divisibility, HCF, LCM

Odd, even, prime, rational numbers

Word problems

Appilication of algebra to real world
Direct and inverse Proportion

Common Applications
Speed-time-distance
Profit, Loss
Percentage, age relations, Mixtures
Miscellaneous

General Maths

Permutation-Combination Basic Probability

Factorial Notation, Principles of counting

P&C with repetition, restriction and counting

Probability, Addition and Multiplication Law

Events: Dependent. Independent, exclusive events

Power and Logarithms

Factorial and negative powers

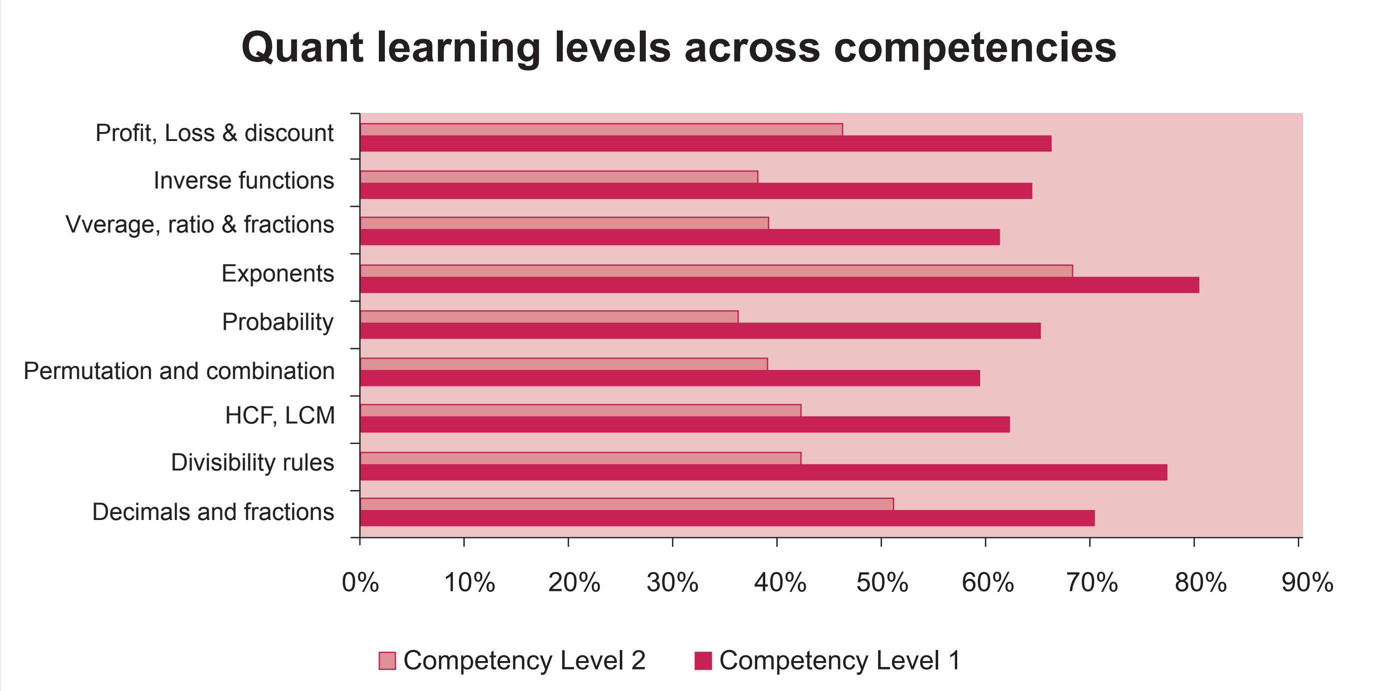
Graphical, intutive understanding

Logarithm/antilog definition and properties
Word problems

Mixed with algebra

Maths for IT/Analytical/ Engineering Purposes

Summary of key mathematical findings:



- Engineering graduates find it much harder to divide/multiply decimal numbers and to handle negative fractions. These numbers are a dismal 58% and 43%, respectively. Candidates seem to have a stronger grasp over high-school math topics such as probability and combinatorics.
- Despite the methodology remaining the same, divisibility by 6 and 9 (70%) is understood by lesser people than divisibility by 2 and 3 (86%).
- It is interesting to note that problems involving inverse equations are comprehended by higher number of people (64%) than problems that require direct calculation of proportions (61%).The same is also true for elementary-level probability, which contrary to assumption, is comprehended by more engineers (65%) than basic calculations involving HCF and LCM (62%). Our hypothesis is that this is due to trainability and greater focus on topics such as inverse proportion problems and probability in competitive examinations.
- HCF and LCM finds lesser acceptance among graduates, likely due to lower application of this topic in other mathematical problems. However, these have pragmatic value and form the basis of optimization analysis. With only 30% engineering graduates showing an aptitude for this competency, it is clear that applied learning is largely being missed for more 'assessment oriented' learning.
- In accordance with the complexities involved, students display highest proclivity towards multiplicative principles of counting, followed by elementary permutation-combination problems, followed by conditional permutation-combination and permutations with repeats. Combinations with repetition turn out to be the least comprehended.

- While 65% graduates being comfortable with basic-level probability seems to be a heartening number, it is also alarming to note that 30% of engineering graduates do not comprehend probability at all. Even extremely elementary questions like "A bag is full of 20 bananas and no other fruit. Rajeev draws a fruit from the bag. What is the probability that he will draw a banana?" were not understood by about one-third of engineering graduates.
- Averages and ratio problems render themselves more difficult to shortcuts and back-calculation, thereby reducing trainability for these concepts. This phenomenon reflects in the fact that 40-50% of engineering graduates cannot even solve basic-level problems involving very simple ratio balancing. Strangely, this number is greater than the number of engineers who are unable to work with inverse ratios (30%).
- One-third of engineering graduates display inaptitude for profit-and-loss concepts, which essentially are applications of percentages and ratios. Even if one is unaware of the formula to calculate successive discounts, intuitive understanding would be sufficient to decipher these problems, however the number there is dismal at 42%

About Aspiring Minds

Aspiring Minds is India's leading employability solutions company, headquartered in Gurgaon. Aspiring Minds offers scientific assessments with an innovative large-scale sourcing model analogous to a GRE-for-job concept. The state-of-the-art assessment tools developed by Aspiring Minds have been used across industry verticals to help recruit the right people, develop profile-wise employability benchmarks and assess workforce health.

Aspiring Minds' intelligent adaptive assessments span across Language, Cognitive skills, Domain knowledge and Personality. A strong in-house research and development team with alumni from IITs and MIT form the development back bone of the patent pending assessment tools.

AMCAT® - the flagship product is India's Largest Employability Test. Conducted across the country throughout the year, AMCAT has been taken by over 750,000 candidates in 1300+ campuses, spread across 23 states. Tens of thousands of candidates secure their dream jobs every year through AMCAT.

Powered by a highly dedicated management team drawn from the IITs and IIMs, over 180 full-time employees, and a pan-India operational presence, Aspiring Minds has helped leading brands across verticals to improve their recruitment process efficiency and the quality of talent they hire.Aspiring Minds products and solutions have been adopted by leading corporates including HCL, Genpact, Accenture, L&T Finance, Keane, Mphasis, Infosys, Ericsson, Sapient, John Deere, Tavant, Tally, among others.

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