Problem 2.1: The integral $\mathbf{a} \equiv \int_S d\mathbf{a}$ is sometimes called the vector area of the surface *S*. If *S* happens to be *flat*, then $|\mathbf{a}|$ is the *ordinary* (scalar) area, obviously.

(a) Find the **vector area** of a hemispherical bowl of radius *R*.

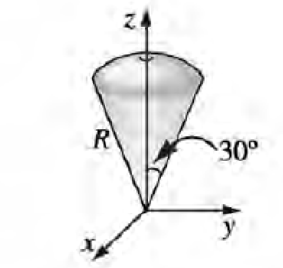
(b) Show that $\mathbf{a} = \mathbf{0}$ for any closed surface.

(c) Show that $\mathbf{a}$ is the same for all surfaces sharing the same boundary.

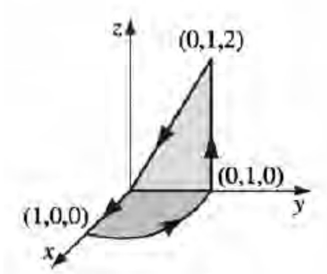
(d) Show that $= \frac{1}{2}\oint \mathbf{r} \times d\mathbf{l}$, where the integral is around the boundary line.

(e) Show that $\oint(\mathbf{c} \cdot \mathbf{r})\ d\mathbf{l} = \mathbf{a} \times \mathbf{c}$.

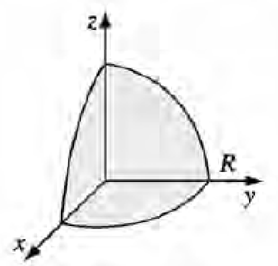
Problem 2.2: Check the divergence theorem for the function $\mathbf{v} = r^2 \sin\theta\,\hat{\mathbf{r}} + 4r^2 \cos\theta\,\hat{\boldsymbol{\theta}} + r^2 \tan\theta\,\hat{\boldsymbol{\phi}}$ using the volume of the "ice cream cone" shown in figure.



Problem 2.3: Compute the integral of $\mathbf{v} = (r cos^2\theta)\hat{\mathbf{r}} - (r\cos\theta\sin\theta)\hat{\boldsymbol{\theta}} + 3r\hat{\boldsymbol{\phi}}$ around the path shown in figure.



Problem 2.4: Compute the divergence theorem of the function $\mathbf{v} = r^2 \cos\theta\,\hat{\mathbf{r}} + r^2 \cos\phi\,\hat{\boldsymbol{\theta}} - r^2 \cos\theta \sin\phi\,\hat{\boldsymbol{\phi}}$ using as your volume one octant of the sphere radius *R*. Make sure you include the entire surface.

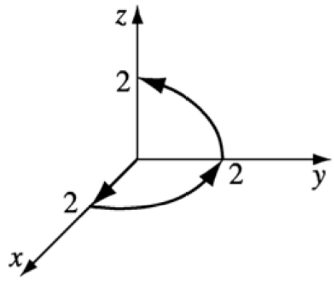


Problem 2.5: (a) Write the expression for the volume charge density $\rho(\mathbf{r})$ of a point charge *q* at $\mathbf{r}'$. Make sure that the volume integral of ρ equals *q*.

(b) What is volume charge density of an electric dipole, consisting of a point charge $-q$ at the origin and a point charge $+q$ at $\mathbf{a}$.

(c) What is the volume charge density (in spherical coordinates) of a uniform, infinitesimally thin spherical shell of radius R and total charge Q, centred at the origin.

Problem 2.6: Compute the gradient and Laplacian of the function $T = r(\cos\theta + \sin\theta\cos\phi)$ in spherical coordinates. Check the laplacian by converting T to Cartesian coordinates. Test the gradient theorem for this function from (0,0,0) to (0,0,2) along the path shown in figure.



Problem 2.7: Compute the divergence of the function

$\mathbf{v} = r\cos\theta\,\hat{\mathbf{r}} + r\sin\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\cos\phi\,\hat{\boldsymbol{\phi}}$

Check the divergence theorem for this function using volume as the inverted hemispherical bowl of radius R resting on the xy plane and centred at the origin.