Problem 1.1: The plot of the following function looks like a hill on the *xy* plane:

$$h(x, y) = \exp\left[\frac{2xy - 3x^2 - 4y^2 - 18x + 28y - 5}{60}\right]$$

(a) Where is the top of the hill located?

(b) How high is the hill?

(c) In what direction is the slope steepest at the point (1,1)?

(d) How steep is the slope of $h(x,y)$ at the point (1,1) in the direction $\mathbf{n} = (\hat{x}x + \hat{y}y)$?

Problem 1.2: Let **r** be the separation vector from a fixed point (x', y', z') to the point (x, y, z) and let r be its length. Show that:

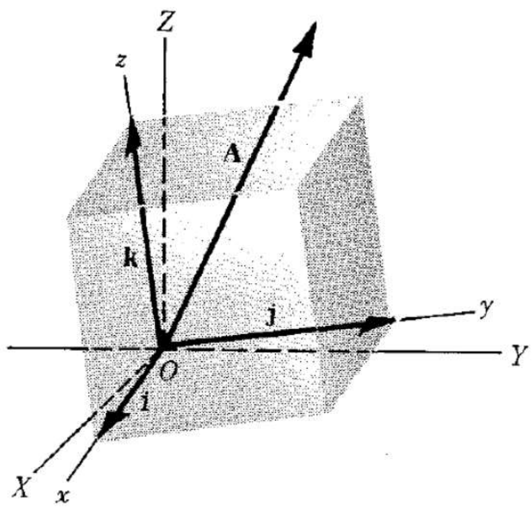
(a) $\boldsymbol{\nabla}(r^2) = 2\mathbf{r}$

(b) $\boldsymbol{\nabla}(1/r) = -\hat{\mathbf{r}}/r^2$

(c) what is the general formula for $\boldsymbol{\nabla}(r^n)$?

Problem 1.3: Suppose that f is a function of two variables (y and z) only. Show that the gradient $\boldsymbol{\nabla} f = (\partial f/\partial y)\hat{\mathbf{y}} + (\partial f/\partial z)\hat{\mathbf{z}}$ transforms as a vector under rotations.

Problem 1.4: An observer stationed at a point which is fixed relative to an *xyz* coordinate system with origin O (see figure) observes a vector $\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$. Later he finds out that he and his coordinate system are actually rotating with respect to an *XYZ* coordinate system, taken as fixed in space and having origin also at O. If $\left.\frac{d\mathbf{A}}{dt}\right|_f$ and $\left.\frac{d\mathbf{A}}{dt}\right|_m$ denote respectively the time derivatives of **A** with respect to the fixed and moving coordinate system, show that there exists a vector quantity $\boldsymbol{\omega}$ such that



$$\left.\frac{d\mathbf{A}}{dt}\right|_f = \left.\frac{d\mathbf{A}}{dt}\right|_m + (\boldsymbol{\omega} \times \mathbf{A})$$

Problem 1.5: A vector **V** is called irrotational if curl **V** = 0.

(a) Find constants a, b and c so that following vector is irrotational.

$$\mathbf{V} = (-4x - 3y + az)\mathbf{i} + (bx + 3y + 5z)\mathbf{j} + (4x + cy + 3z)\mathbf{k}$$

(b) Show that **V** can be expressed as the gradient of a scalar function.

Problem 1.6: Sketch the vector function $\mathbf{v} = \frac{\hat{\mathbf{r}}}{r^2}$, and compute it's divergence and explain the result.

Problem 1.7: Let P_1, P_2 and P_3 be points fixed relative to an origin O and let $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ be position vectors from O to each point. Suppose the vector equation $a_1\mathbf{r}_1 + a_2\mathbf{r}_2 + a_3\mathbf{r}_3 = 0$ holds with respect to origin O. show that it will hold with respect to any other origin O' if and only if $a_1 + a_2 + a_3 = 0$.

Problem 1.8: Find the equation of a straight line that passes through two given points A and B having position vectors **a** and **b** with respect to the origin.

Problem 1.9: Find a unit vector **u** parallel to the resultant **R** of vectors $\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$ and $\mathbf{r}_2 = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.