Problem Set 1

CS 340 - Theory of Computation

16 August 2018

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1. Construct DFAs accepting the following languages over $\Sigma = \{0, 1\}$:

   (a) $L_1 = \{w \mid w$ contains at least three 0s and even number of 1s$\}$

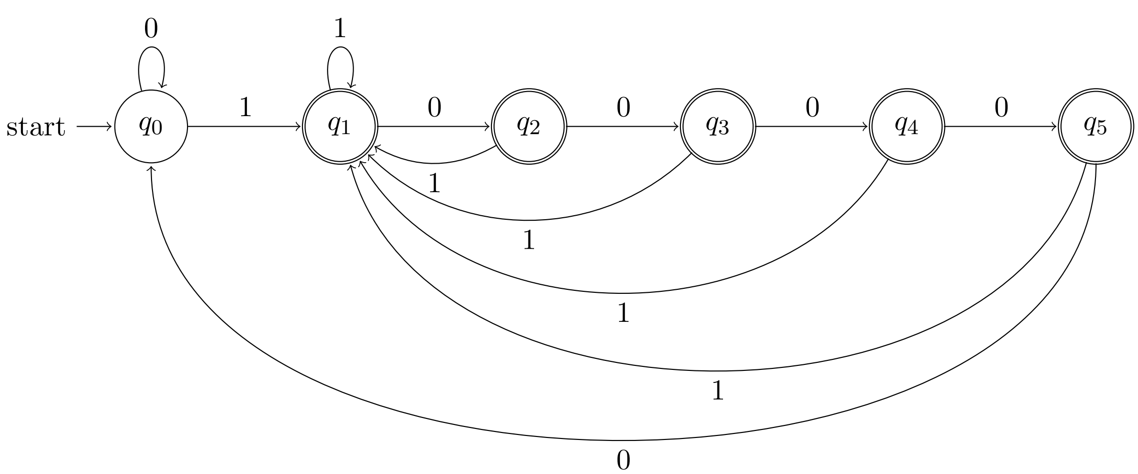
   (b) $L_2 = \{w \mid$ every consecutive block of three letters in w contains at most one 0$\}$

   For the second part, assume that all strings of length less than three are in L_2 except 00.

2. Construct a DFA A of not more than three states for the set of binary strings whose decimal representation is divisible by 3. Let A' be the FA formed by reversing the direction of the edges of A and swapping the start and accepting state. Is it true that A' is a DFA that accepts the set of binary strings whose reverse's decimal representation is divisible by 3?

3. Prove that if we swap the accepting and non-accepting states of a DFA, then the resulting DFA will accept the complement of the language of the original DFA. Can we say the same for an NFA?

4. What is the language accepted by the following DFA?



5. Prove that for every NFA there exist an equivalent NFA which has only a single accepting state. Can we say the same for a DFA?

6. Given a DFA M is it possible to determine if $L(M)$ is finite?

7. Construct DFAs for the following languages over alphabet $\{0, 1\}$:

   (a) $\{x \mid x$ has two 0's separated by a number of positions that is a multiple of 4.$\}$

8. Consider the following languages:

- $L_1 = \{x \mid \ x \text{ is binary representation of multiple of } 3\} \cup \{\epsilon\}$
- $L_2 = \{x \mid x \text{ is a binary string and decimal of any prefix of } x \text{ is not of form } 3m+2, \text{ where } m \geq 0\} \cup \{\epsilon\}$

Construct a DFA for $L_1 \cap L_2$.

9. Let A and B be regular sets over the alphabet Σ. Let *shuffle*(A, B) be the set defined as follows:

   $\{w \in \Sigma^* \mid w = a_1 b_1 \cdots a_k b_k, \text{ where } a_1 \cdots a_k \in A \text{ and } b_1 \cdots b_k \in B \text{ and each } a_i, b_i \in \Sigma\}$

   Prove that regular sets are closed under shuffle.

10. Which of the followings are True or False. Justify you answer.

    (a) For every NFA N with k states, there exists a DFA M with at most 2^k states such that $L(M) = L(N)$.

    (b) For every DFA M with 2^k states, there exists an NFA N with at most k states such that $L(M) = L(N)$.