Problem Set 4

CS 340 - Theory of Computation
7 November 2018

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1. Are recursive sets and recursively enumerable sets closed under the operations: union, intersection and Kleene star ($*$)?
2. Are the following sets *"recursive"*, *"recursively enumerable but not recursive"* or *"not recursively enumerable"*:
   (a) $A = \{M\#q \mid M$ is a turing machine and q is state in M that is never reached on any input$\}$
   (b) $B = \{M \mid M$ is a turing machine that accepts at least 3 inputs$\}$
3. Is the following statement True?
   (a) If A is a recursive and $B \subseteq A$, then B is also recursive.
4. Prove or disprove the following statements:
   (a) For every recursive set L, there exists a deterministic total TM M with a single tape along with five states q_{start}, q_{aux1}, q_{aux2}, q_{accept} and q_{reject} such that $L(M) = L$.
   (b) For every recursive set, L there exists a total TM M which does not write on any cell (in other words, it does not change the content of the tape) such that $L(M) = L$.
5. Let L_1 and L_2 be two recursive sets. Let L_3 and L_4 be two recursively enumerable sets. Which of the following statements are False? Justify your answer.
   (a) $(L_1 \cup L_2) \setminus L_3$ is recursively enumerable.
   (b) $L_2 \setminus (L_3 \cup L_4)$ is recursively enumerable.
   (c) $L_3 \setminus L_1$ is recursive.
   (d) $L_3 \setminus L_2$ is recursive.
6. State whether the following sets are recursive.
   (a) $L_1 = \{M \mid M$ is a single tape TM that on any input x does not change the input content of the tape$\}$
   (b) $L_2 = \{M \mid M$ is a single tape TM that on any input x does not change any part of the tape$\}$