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Section 52: Problem 5 Solution

> Working problems is a crucial part of learning mathematics. No one can learn topology merely by poring over the definitions, theorems, and examples that are worked out in the text. One must work part of it out for oneself. To provide that opportunity is the purpose of the exercises.
>
> — *James R. Munkres*

Let A be a subspace of $\mathbb{R}^n$; let $h : (A, a_0) \to (Y, y_0)$. Show that if h is extendable to a continuous map of $\mathbb{R}^n$ into Y , then h_* is the trivial homomorphism (the homomorphism that maps everything to the identity element).

First note, that h is continuous. Then, h_* is the trivial homomorphism iff for every loop f in A based at a_0 , $h_*([f]) = [h \circ f] = [e_{y_0}]$ iff for every loop f in A based at a_0 , $h \circ f$ is nulhomotopic. Consider $F : I \times I \to Y$ defined by $F(s,t) = \overline{h}((1-t)f(s) + ta_0)$ where $\overline{h}$ is a continuous extension of h . Then, $F(s,0) = \overline{h}(f(s)) = (h \circ f)(s)$ since $\overline{h} \equiv h$ on A and f is a path in A . Moreover, $F(s,1) = \overline{h}(a_0) = h(a_0) = y_0$.