# Test of Hypotheses

## 1 Basic terminologies and concept of hypotheses testing

*Hypotheses testing* is a process of trying to verify conjectures/claims about unknown quantities on the basis of evidence.

There are five important stages of hypotheses testing:

- Hypotheses – null hypothesis $H_0$, alternative hypothesis $H_a$.
- Test statistic (formula) – this is used to do the testing.
- Rejection region $R$ – the region to reject $H_0$.
- Value of test-statistic based on data.
- Conclusion – either reject $H_0$ or do not reject $H_0$.

A **statistical hypothesis** is a statement about the pdf $f(x;\theta)$, or the parameter $\theta$. We test the *null hypothesis* $H_0$ against the *alternative hypothesis* $H_a$, which are disjoint to each other.

The **rejection region** $R$ for hypotheses testing is a subset of the sample space that corresponds to rejection of null hypotheses $H_0$. It is also sometimes called *critical region*. The choice of rejection region is perhaps the most important part of test of hypotheses.

In testing the following four situations can happen:

| | **True situation** | |
|---|---|---|
| **Decision taken** | $H_0$ **is true** | $H_a$ **is true** |
| $H_0$ **is not rejected** | Correct decision | Type II error |
| $H_0$ **is rejected** | Type I error | Correct decision |

We try to find a rejection region such that both $P[\text{Type I error}]$ and $P[\text{Type II error}]$ are smallest. But except some ideal situations we are always forced to make both errors. What is even worse that in most cases, if we want to minimize Type I error, the probability of Type II error increases. So we build a critical region in such a way that the $P(\text{Type I error}) = \alpha$ (usually a small number) while $\beta := P(\text{Type II error})$ is minimum.

$\alpha := P(\text{Type I error})$ is called the **significance level** and is pre-determined. The popular choice of $\alpha = 0.01, 0.05$ or $0.10$, but one may use other values as well. So the permitted level of probability of Type I error, is usually small. This means that unless we have significant evidence against $H_0$, we do not reject $H_0$. Hence $H_0$ is protected to some extent. Under this restriction we have to find rejection region $R$ such that probability of Type II error is minimum. So if we reject $H_0$, that means there is significant evidence against $H_0$ in favor of $H_a$. So $H_a$ is preferred over $H_0$. However if we fail to reject $H_0$, that does not mean $H_0$ is true (or $H_0$ is accepted). It just says that there is not enough evidence against $H_0$. So we NEVER conclude: "$H_0$ is accepted", but we write: "$H_0$ is not rejected".

We define **power** of a test $:= 1 - \beta = 1 - P(\text{Type II error})$. So it means that our testing procedure should maximize the power. Suppose to test a certain pair of hypotheses, we come up with two different procedures with two different rejection regions with same significance level $\alpha$. In that case we should choose that rejection region which has higher power (because that means smaller Type II error). One of the goal is to find the most powerful tests (however that may not exist).

Most of the time rejection regions are expressed in terms of test statistic. A statistic $T = T(\boldsymbol{X})$ is called a **test statistic** if the distribution of $T$ under the null hypothesis $H_0$ does not depend on any unknown parameter.

**Example 1.** Let $X_1$ and $X_2$ be a random sample from $X$ with a pdf $f(x;\theta)$ for $\theta = 0, 1$, where

| | $x$ | |
|---|---|---|
| | 0 | 1 |
| $f(x;\theta=0)$ | 0.2 | 0.8 |
| $f(x;\theta=1)$ | 0.6 | 0.4 |

To test $H_0 : \theta = 0$ against $H_a : \theta = 1$, suppose we decide to reject $H_0$ if $X_1 + X_2 < 2$.

(a) **Find $P$(Type I error).**

$$P(\text{Type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P(X_1 + X_2 < 2 \mid \theta = 0).$$

Note that $X_1 + X_2 < 2$ if and only if

$$(X_1 = 0, X_2 = 0), \text{ or } (X_1 = 1, X_2 = 0), \text{ or } (X_1 = 0, X_2 = 1).$$

Also if $\theta = 0$, then $P(X = 0) = f(0;\theta = 0) = 0.2$, and $P(X = 1) = f(1;\theta = 0) = 0.8$.
Therefore if $\theta = 0$,

$$P(X_1 = 0, X_2 = 0) = P(X_1 = 0)\ P(X_2 = 0) = f(0;\theta = 0)f(0;\theta = 0) = 0.2 \times 0.2 = 0.04.$$

Similarly, $P(X_1 = 1, X_2 = 0) = 0.8 \times 0.2 = 0.16$, and $P(X_1 = 0, X_2 = 1) = 0.2 \times 0.8 = 0.16$.

$$\begin{aligned} P(\text{Type I error}) &= P(X_1 + X_2 < 2 \mid \theta = 0) \\ &= P((X_1 = 0, X_2 = 0), \text{ or } (X_1 = 1, X_2 = 0), \text{ or } (X_1 = 0, X_2 = 1) \mid \theta = 0) \\ &= P(X_1 = 0, X_2 = 0) + P(X_1 = 1, X_2 = 0) + P(X_1 = 0, X_2 = 1) \\ &= 0.04 + 0.16 + 0.16 = 0.36. \end{aligned}$$

(b) **Compute $P$(Type II error).**

$$P(\text{Type II error}) = P(\text{Do not reject } H_0 \mid H_a \text{ is true}) = P(X_1 + X_2 \geq 2 \mid \theta = 1).$$

Note that $X_1 + X_2 \geq 2$ if and only if $X_1 = 1, X_2 = 1$.
Now if $\theta = 1$, then $P(X = 1) = f(1;\theta = 1) = 0.4$. Thus

$$\begin{aligned} P(\text{Type II error}) &= P(X_1 + X_2 \geq 2 \mid \theta = 1) \\ &= P(X_1 = 0, X_2 = 1) = f(1;\theta = 1)f(1;\theta = 1) = 0.4 \times 0.4 = 0.16. \end{aligned}$$

(c) **Calculate power of this test.**

$$\text{Power} = 1 - P(\text{Type II error}) = 1 - 0.16 = 0.84.$$

# 2 Test of hypotheses in normal population

Let $X_1, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$.

## 2.1 Test of hypotheses for $\mu$

**Case 1: $\sigma$ is known.**

**Null hypothesis:** $H_0 : \mu = \mu_0$. **Test-statistic:** $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$.

| **Alternative hypothesis** | **Rejection region at $\alpha$ level of sig.** |
|---|---|
| $H_a : \mu > \mu_0$ | $Z \geq z_\alpha$ [upper-tailed test] |
| $H_a : \mu < \mu_0$ | $Z \leq -z_\alpha$ [lower-tailed test] |
| $H_a : \mu \neq \mu_0$ | $\|Z\| \geq z_{\alpha/2}$ [two-tailed test] |

Here the notation $z_\alpha$ implies $P(N(0,1)$ r.v. $> z_\alpha) = \alpha$.

**Case 2: $\sigma$ is unknown.**

**Null hypothesis:** $H_0 : \mu = \mu_0$. **Test-statistic:** $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$.

| **Alternative hypothesis** | **Rejection region at $\alpha$ level of sig.** |
|---|---|
| $H_a : \mu > \mu_0$ | $T \geq t_{\alpha;n-1}$ [upper-tailed test] |
| $H_a : \mu < \mu_0$ | $T \leq -t_{\alpha;n-1}$ [lower-tailed test] |
| $H_a : \mu \neq \mu_0$ | $\|T\| \geq t_{\alpha/2;n-1}$ [two-tailed test] |

**Warning:** In many books (and also in this note) the meaning of $t_{\alpha;df}$ is as depicted above. However, in our textbook of Rice, $t_{\alpha;df}$ implies $P(t_{df}$ r.v. $> t_{\alpha;df}) = 1 - \alpha$. So be careful.

## 2.2 Test of hypotheses for $\sigma^2$

**Null hypothesis:** $H_0 : \sigma^2 = \sigma_0^2$. **Test-statistic:** $W = \dfrac{(n-1)S^2}{\sigma_0^2}$.

| **Alternative hypothesis** | **Rejection region at $\alpha$ level of sig.** |
|---|---|
| $H_a : \sigma^2 > \sigma_0^2$ | $W \geq \chi^2_{\alpha;n-1}$ [upper-tailed test] |
| $H_a : \sigma^2 < \sigma_0^2$ | $W \leq \chi^2_{1-\alpha;n-1}$ [lower-tailed test] |
| $H_a : \sigma^2 \neq \sigma_0^2$ | $W \leq \chi^2_{1-\alpha/2;n-1}$ or $W \geq \chi^2_{\alpha/2;n-1}$ [two-tailed test] |

Here the notation $\chi^2_{\alpha;df}$ implies $P(\chi^2_{df}$ r.v. $> \chi^2_{\alpha;df}) = \alpha$, where $df$ stands for degrees of freedom.

**Warning:** In our textbook of Rice, $\chi^2_{\alpha;df}$ implies $P(\chi^2_{df}$ r.v. $> \chi^2_{\alpha;df}) = 1 - \alpha$. So be careful.

# 3 Large sample approximate test of hypotheses

## 3.1 Large sample approximate test of hypotheses for $\mu$

Let $X_1, \ldots, X_n$ be a sample from a population with mean $\mu$ and standard deviation $\sigma^2$. Here the population may not be normally distributed. In that case if $n$ is large then to test:

**Null hypothesis:** $H_0 : \mu = \mu_0$.

**Test-statistic:**

$$Z = \begin{cases} \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}, & \text{if } \sigma \text{ is known;} \\ \\ \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}, & \text{if } \sigma \text{ is unknown.} \end{cases}$$

| **Alternative hypothesis** | **Rejection region at $\alpha$ level of sig.** |
|---|---|
| $H_a : \mu > \mu_0$ | $Z \geq z_\alpha$ [upper-tailed test] |
| $H_a : \mu < \mu_0$ | $Z \leq -z_\alpha$ [lower-tailed test] |
| $H_a : \mu \neq \mu_0$ | $\lvert Z\rvert \geq z_{\alpha/2}$ [two-tailed test] |

This test requires large sample, *i.e.*

- If $n < 20$, one needs normality assumption.
- If $20 \leq n \leq 40$, a little skewness in the distribution is allowed, but there should not be any outlier to apply large sample results.
- If $n > 40$, more skewed distribution, with a few outliers can be permitted to use large sample results.

## 3.2 Large sample approximate test of hypotheses for $p$

Suppose we are dealing with categorical variables, and a particular category has population proportion $p$, for which we want to construct a $100(1-\alpha)\%$ approximate C.I. For instance, suppose $p$ is the proportion of male in a certain population. We take a sample of size $n$ and find that $x$ of that sample is male. Then the sample proportion of male is $\hat{p} = \frac{x}{n}$. In that case,

**Null hypothesis:** $H_0 : p = p_0$. **Test-statistic:** $Z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$.

| **Alternative hypothesis** | **Rejection region at $\alpha$ level of sig.** |
|---|---|
| $H_a : p > p_0$ | $Z \geq z_\alpha$ [upper-tailed test] |
| $H_a : p < p_0$ | $Z \leq -z_\alpha$ [lower-tailed test] |
| $H_a : p \neq p_0$ | $\lvert Z\rvert \geq z_{\alpha/2}$ [two-tailed test] |

This test requires large sample, *i.e.* $np_0 > 9$ and $n(1-p_0) > 9$.

**Remark:** In some problems, the value of $\hat{p}$ will be given directly, and in some $x$ and $n$ will be given, where we have to compute $\hat{p} = x/n$.

# 4 Likelihood ratio test: Neyman-Pearson lemma

Suppose $L_{\boldsymbol{X}}(\theta)$ is the likelihood function based on the sample $\boldsymbol{X} = (X_1, \ldots, X_n)$. Define

$$\lambda(\boldsymbol{x}; \theta_0, \theta_1) = \frac{L_{\boldsymbol{X}}(\theta_0)}{L_{\boldsymbol{X}}(\theta_1)}, \qquad \text{and} \qquad R^* = \{\lambda(\boldsymbol{x}; \theta_0, \theta_1) \le k\},$$

where $k$ is a constant such that

$$P(R^* \mid \theta_0) = \alpha, \quad i.e. \quad P(\lambda(\boldsymbol{x}; \theta_0, \theta_1) \le k \mid \theta_0) = \alpha.$$

Then $R^*$ is the critical region of a most powerful test for $H_0 : \theta = \theta_0$ against $H_a : \theta = \theta_1$ with significance level $\alpha$.

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**Example 2.** Suppose $X_1, \ldots, X_n$ is a random sample from exponential distribution with mean $\theta$, *i.e.* from the pdf, $f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}$, if $x > 0$, and zero otherwise. We want to build the rejection region for the most powerful test for $H_0 : \theta = \theta_0$, against $H_a : \theta = \theta_1$.

The likelihood function is

$$L_{\boldsymbol{X}}(\theta) = \prod_{i=1}^{n} f(X_i; \theta) = \prod_{i=1}^{n} \frac{1}{\theta} e^{-X_i/\theta} = \theta^{-n} e^{-\sum_{i=1}^{n} X_i/\theta} = \theta^{-n} e^{-n\bar{X}/\theta},$$

and the likelihood ratio is

$$\lambda(\boldsymbol{X}) = \frac{L_{\boldsymbol{X}}(\theta_0)}{L_{\boldsymbol{X}}(\theta_1)} = \frac{\theta_0^{-n} e^{-n\bar{X}/\theta_0}}{\theta_1^{-n} e^{-n\bar{X}/\theta_1}} = \left(\frac{\theta_0}{\theta_1}\right)^{-n} e^{-n\bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)}.$$

So by Neyman-Pearson, the rejection region of most powerful test is of the form $R^* = \{\lambda(\boldsymbol{X}) \le k\}$. Now

$$\begin{aligned}
& \lambda(\boldsymbol{X}) = \left(\frac{\theta_0}{\theta_1}\right)^{-n} e^{-n\bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)} \le k \\
\Rightarrow \quad & e^{-n\bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)} \le k \left(\frac{\theta_0}{\theta_1}\right)^{n} =: k_0 \\
\Rightarrow \quad & -n\bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right) \le \ln k_0 =: k_1 \\
\Rightarrow \quad & \bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right) \ge -\frac{k_1}{n} =: k_2.
\end{aligned}$$

Notice if $\theta_0 < \theta_1$, then $\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right) > 0$, and so

$$\bar{X}\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right) \ge k_2 \quad \Rightarrow \quad \bar{X} \ge k_2 \left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)^{-1} =: k_3.$$

Similarly we obtain $R^*$ for the case $\theta_0 > \theta_1$.

Therefore, the rejection region of most powerful test is of the form:

- $R^* = \{\bar{X} \ge k_3\}$, if $\theta_0 < \theta_1$.
- $R^* = \{\bar{X} \le k_3\}$, if $\theta_0 > \theta_1$.

The above result also shows that a suitable test statistic is $\bar{X}$.

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**Example 3.** Suppose $X_1, \ldots, X_n$ is a random sample from pdf, $f(x;\theta) = \theta x^{\theta-1}$, if $0 < x < 1$, and zero otherwise. We want to build the rejection region for the most powerful test for $H_0 : \theta = \theta_0$, against $H_a : \theta = \theta_1$.

The likelihood function is

$$L_{\boldsymbol{X}}(\theta) = \prod_{i=1}^{n} f(X_i;\theta) = \prod_{i=1}^{n} \theta X_i^{\theta-1} = \theta^n \left(\prod_{i=1}^{n} X_i\right)^{\theta-1},$$

and the likelihood ratio is

$$\lambda(\boldsymbol{X}) = \frac{L_{\boldsymbol{X}}(\theta_0)}{L_{\boldsymbol{X}}(\theta_1)} = \frac{\theta_0^n \left(\prod_{i=1}^n X_i\right)^{\theta_0-1}}{\theta_1^n \left(\prod_{i=1}^n X_i\right)^{\theta_1-1}} = \left(\frac{\theta_0}{\theta_1}\right)^n \left(\prod_{i=1}^{n} X_i\right)^{\theta_0-\theta_1}.$$

So by Neyman-Pearson, the rejection region of most powerful test is of the form $R^* = \{\lambda(\boldsymbol{X}) \le k\}$. Now

$$\begin{aligned}
\lambda(\boldsymbol{X}) = \left(\frac{\theta_0}{\theta_1}\right)^n \left(\prod_{i=1}^{n} X_i\right)^{\theta_0-\theta_1} \le k \quad &\Rightarrow \quad \left(\prod_{i=1}^{n} X_i\right)^{\theta_0-\theta_1} \le k \left(\frac{\theta_1}{\theta_0}\right)^n =: k_0 \\
&\Rightarrow \quad (\theta_0 - \theta_1) \ln \left(\prod_{i=1}^{n} X_i\right) \le \ln k_0 =: k_1 \\
&\Rightarrow \quad (\theta_0 - \theta_1) \left(\sum_{i=1}^{n} \ln X_i\right) \le k_1.
\end{aligned}$$

Notice if $\theta_0 < \theta_1$, then $(\theta_0 - \theta_1) < 0$, and so

$$(\theta_0 - \theta_1) \left(\sum_{i=1}^{n} \ln X_i\right) \le k_1 \quad \Rightarrow \quad \sum_{i=1}^{n} \ln X_i \ge \frac{k_1}{(\theta_0 - \theta_1)} =: k_2.$$

Similarly we obtain $R^*$ for the case $\theta_0 > \theta_1$.

Therefore, the rejection region of most powerful test is of the form:

- $R^* = \left\{\sum_{i=1}^{n} \ln X_i \ge k_2\right\}$, if $\theta_0 < \theta_1$.
- $R^* = \left\{\sum_{i=1}^{n} \ln X_i \le k_2\right\}$, if $\theta_0 > \theta_1$.

The above result also shows that one can consider $\sum_{i=1}^{n} \ln X_i$ as a suitable test statistic.

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**Example 4.** Read Example A of Chapter 9.2 about testing $\mu$ in $N(\mu, \sigma^2)$ from the textbook.

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**Example 5.** Let $p$ be the probability of head of a particular coin. We want to test

$$H_0 : p = p_0, \qquad \text{against} \qquad H_a : p = p_1,$$

based on $n$ tosses of the coin.

Let us define the random variable:

$$X_i = \begin{cases} 1, & \text{if the } i^{th} \text{ toss results in head;} \\ 0, & \text{if the } i^{th} \text{ toss results in tail.} \end{cases}$$

Hence for any $i = 1, \ldots, n$, we have $P(X_i = 1) = P(H) = p$, and $P(X_i = 0) = P(T) = 1 - p$; *i.e.*

$$f(x; p) = P(X = x) = p^x(1-p)^{1-x}, \qquad x = 0, 1.$$

Thus we can consider $(X_1, \ldots, X_n)$ to be a sample from the above mentioned pdf $f(x; p)$. Hence the likelihood function is

$$L_{\boldsymbol{X}}(p) = \prod_{i=1}^{n} f(X_i; p) = \prod_{i=1}^{n} \left[p^{X_i}(1-p)^{1-X_i}\right] = p^{\sum_{i=1}^n X_i}(1-p)^{\sum_{i=1}^n (1-X_i)} = p^{n\bar{X}}(1-p)^{n(1-\bar{X})}.$$

Thus the likelihood ratio

$$\lambda(\boldsymbol{X}) = \frac{L_{\boldsymbol{X}}(p_0)}{L_{\boldsymbol{X}}(p_1)} = \frac{p_0^{n\bar{X}}(1-p_0)^{n(1-\bar{X})}}{p_1^{n\bar{X}}(1-p_1)^{n(1-\bar{X})}} = \left[\frac{p_0(1-p_1)}{p_1(1-p_0)}\right]^{n\bar{X}} \left(\frac{1-p_0}{1-p_1}\right)^n.$$

By Neyman-Pearson lemma, the rejection region of the most powerful test of significance level $\alpha$ is of the form $R^* = \{\lambda(\boldsymbol{X}) \le k\}$, for some constant $k$ so that $P(R^* \mid \theta_0) = \alpha$. Now

$$\begin{aligned}
\lambda(\boldsymbol{X}) = \left[\frac{p_0(1-p_1)}{p_1(1-p_0)}\right]^{n\bar{X}} \left(\frac{1-p_0}{1-p_1}\right)^n &\le k, \\
\Rightarrow \left[\frac{p_0(1-p_1)}{p_1(1-p_0)}\right]^{n\bar{X}} \le k \left(\frac{1-p_1}{1-p_0}\right)^n &=: k_0, \\
\Rightarrow (n\bar{X}) \ln \left[\frac{p_0(1-p_1)}{p_1(1-p_0)}\right] &\le \ln k_0, \\
\Rightarrow \bar{X} C(p_0, p_1) \le \frac{\ln k_0}{n} &=: k_1,
\end{aligned}$$

where

$$C(p_0, p_1) = \ln \left[\frac{p_0(1-p_1)}{p_1(1-p_0)}\right].$$

**Case $p_0 < p_1$:** In this case

$$p_0 < p_1 \;\Rightarrow\; p_0 - p_0 p_1 < p_1 - p_0 p_1 \;\Rightarrow\; p_0(1-p_1) < p_1(1-p_0) \;\Rightarrow\; \frac{p_0(1-p_1)}{p_1(1-p_0)} < 1 \;\Rightarrow\; C(p_0, p_1) < 0.$$

Hence from (1), we get

$$\bar{X} C(p_0, p_1) \le k_1 \;\Rightarrow\; \bar{X} \ge \frac{k_1}{C(p_0, p_1)} =: k_2.$$

So to test $H_0 : p = p_0$, against $H_a : p = p_1$, where $p_0 < p_1$, the most powerful rejection region will have the form $R^* = \{\bar{X} \ge k_2\}$, for some constant $k_2$ such that $P(R^* | p_0) = \alpha$.

**Case $p_0 > p_1$:** In this case

$$p_0 > p_1 \;\Rightarrow\; p_0 - p_0 p_1 > p_1 - p_0 p_1 \;\Rightarrow\; p_0(1-p_1) > p_1(1-p_0) \;\Rightarrow\; \frac{p_0(1-p_1)}{p_1(1-p_0)} > 1 \;\Rightarrow\; C(p_0, p_1) > 0.$$

Hence from (1), we get

$$\bar{X} C(p_0, p_1) \le k_1 \;\Rightarrow\; \bar{X} \le \frac{k_1}{C(p_0, p_1)} =: k_2.$$

So to test $H_0 : p = p_0$, against $H_a : p = p_1$, where $p_0 > p_1$, the most powerful rejection region will have the form $R^* = \{\bar{X} \le k_2\}$, for some constant $k_2$ such that $P(R^* | p_0) = \alpha$.

Notice one important fact, that since each of $X_i$'s is a binary variable taking two values 0 and 1 (depending on whether you get tail or head respectively), hence

$$\begin{aligned}
&\sum_{i=1}^{n} X_i = \text{ number of heads obtained in } n \text{ tosses}, \qquad \text{and,} \\
&\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{\text{number of heads obtained in } n \text{ tosses}}{n} = \hat{p}, \text{the sample proportion of head.}
\end{aligned}$$

Thus to test $H_0 : p = p_0$, against $H_a : p = p_1$, the most powerful rejection region will have the form:

- $R^* = \{\hat{p} \geq k_2\}$, if $p_0 < p_1$,
- $R^* = \{\hat{p} \leq k_2\}$, if $p_0 > p_1$,

for some constant $k_2$ such that $P(R^* | p_0) = \alpha$.

Furthermore, it indicates that $\hat{p}$ can be a suitable test statistic for this test.

# 5 Minitab to perform one sample tests

## 5.1 $z$-test for $\mu$:

STEP 1: **Stat** → **Basic Statistics** → **1-Sample Z...**.

STEP 2: A new dialog box opens.

- If the data are stored in a column in the worksheet, select that column in the box SAMPLES IN COLUMNS:.
- If only the summary statistics are available, then select the option SUMMARIZED DATA, and type the values of $n$ in the box SAMPLE SIZE and $\bar{X}$ in the box MEAN.

STEP 3: Type in the value of $\sigma$ in the box STANDARD DEVIATION:. [If you are using (**??**), then type $S$ in this box.]

STEP 4: Check the box PERFORM HYPOTHESIS TEST and type the value of $\mu_0$ in the box HYPOTHESIZED VALUE.

STEP 5: The default $H_a$ is $\mu \neq \mu_0$. If you want to change the $H_a$, click on the button **Options** and choose the appropriate $H_a$ from the drop down menu ALTERNATIVE:. Press the button **OK**.

STEP 6: Press the button **OK**. The output will be available on the session page.

## 5.2 $t$-test for $\mu$:

STEP 1: **Stat** → **Basic Statistics** → **1-Sample t...**.

STEP 2: A new dialog box opens.

- If the data are stored in a column in the worksheet, select that column in the box SAMPLES IN COLUMNS:.
- If only the summary statistics are available, then select the option SUMMARIZED DATA, and type the values of $n$ in the box SAMPLE SIZE, $\bar{X}$ in the box MEAN and $S$ in the box STANDARD DEVIATION:.

STEP 3: Check the box PERFORM HYPOTHESIS TEST and type the value of $\mu_0$ in the box HYPOTHESIZED VALUE.

STEP 4: The default $H_a$ is $\mu \neq \mu_0$. If you want to change the $H_a$, click on the button **Options** and choose the appropriate $H_a$ from the drop down menu ALTERNATIVE:. Press the button **OK**.

STEP 5: Press the button **OK**. The output will be available on the session page.

### 5.3 $\chi^2$-test for $\sigma^2$:

Step 1: **Stat** $\rightarrow$ **Basic Statistics** $\rightarrow$ **1 Variance...**.

Step 2: A new dialog box opens.

- If the data are stored in a column in the worksheet, select that column in the box Columns:.
- If only the summary statistics are available, then select the option Sample standard deviation from the drop down menu under Data, and type the values of $n$ in the box Sample size and $S$ in the box Sample standard deviation:.

Step 3: Check the box Perform hypothesis test, choose from the drop down menu Hypothesized variance and type the value of $\sigma_0^2$ in the box Value:.

Step 4: The default $H_a$ is $\sigma^2 \neq \sigma_0^2$. If you want to change the $H_a$, click on the button **Options** and choose the appropriate $H_a$ from the drop down menu Alternative:. Press the button **OK**.

Step 5: Press the button **OK**. The output will be available on the session page.

### 5.4 $z$-test for $p$:

Step 1: **Stat** $\rightarrow$ **Basic Statistics** $\rightarrow$ **1 Proportion...**.

Step 2: A new dialog box opens.

- If the data are stored in a column in the worksheet, select that column in the box Samples in columns:.
- If only the summary statistics are available, then select the option Summarized data, and type the values of $x$ in the box Number of events: and $n$ in the box Number of trials:.

Step 3: Check the box Perform hypothesis test and type the value of $p_0$ in the box Hypothesized value.

Step 4: The default $H_a$ is $p \neq p_0$. If you want to change the $H_a$, click on the button **Options** and choose the appropriate $H_a$ from the drop down menu Alternative:. Press the button **OK**.

Step 5: Press the button **OK**. The output will be available on the session page.