CS345: Assignment 4

Q1 Given two sorted integer arrays A and B of size n each. Let C denote the $Merge(A, B)$. Devise an $O(\log n)$ algorithm to report the median of C.

Q2 A machine converts a note of denomination k into notes of denominations $\lfloor k/2 \rfloor, \lfloor k/3 \rfloor, \lfloor k/4 \rfloor$ and $\lfloor k/5 \rfloor$. One can use the machine any number of times to maximize the value of their currency notes. Let $max(k)$ denote the largest value a currency note of face value k can be converted to.

For example a note of face value 12 can be converted to notes of values $6, 4, 3, 2$. Then 6 value note can be convertex to the notes of value $3, 2, 1, 1$. Thus the total value becomes 16. Verify that no further conversion helps increase the value. So $max(12) = 16$.

Devise a divide and conquer algorithm to compute $max(k)$ having time complexity $O(k)$. Hint: Count the number of nodes in the sub-instance tree.

Q3 (a) Determine the number of binary sequences of length n in which no two 1s are adjacent. For example for $n = 3$, the sequences are $000, 100, 010, 001, 101$. What is the time complexity of your algorithm. (b) Modify the program to compute the number of such binary sequences which use exactly k 1s. If this count is denoted by $T(n, k)$, then from the above example we find that $T(3, 1) = 3$.

Q4 Let $a_1, a_2, \ldots, a_n$ be a sequence of integers. Find a longest subsequence $a_{j_1}, a_{j_2}, \ldots a_{j_k}$ such that $a_{j_i} \leq a_{j_{i+1}}$ for all i.

Q5 Let $a_1, a_2, \ldots, a_n$ be a sorted sequence of integers. Design an efficient algorithm to find if there exists j such that $a_j = j$. Hint: Use divide and conquer.

Q6 Extend the longest common sequence algorithm to compute the longest sequence which is a subsequence of each of the three given sequences.

Q7 Given a sequence of integers and an integer r. Design an algorithm to find a subsequence (not substring) which adds up to k. Hint: Use divide and conquer technique.

Q8 Given a sequence of integers $a_1, a_2, \ldots, a_n$, we want to find three integers, a_i, a_j, a_k such that $a_i + a_j = a_k$. Design an $O(n^2. \log\ n)$ complexity algorithm for this problem. Can you design an $O(n^2)$ algorithm?

Q9 Given a sequence $b_1, \ldots, b_n$ of non-negative integers. Determine a sequence $a_1, \ldots a_n$ of non-negative integers such that $\sum_i |a_i - a_{i-1}|$ is maximum subject to the condition that $a_i \leq b_i$ for all i.

Q10 Let $a_1, a_2, \ldots, a_n$ be a sequence of integers. Then a pair of indices, $(i, i+j)$ is said to be an inversion if $a_i > a_{i+j}$. Design an efficient algorithm to compute the total number of inversion in a sequence. Hint: Suitable modify Mergesort algorithm.

Q11 Given a text sequence $T : a_1, a_2, \ldots, a_n$ and a pattern sequence $P : b_1, b_2, \ldots, b_m$. Design an algorithm to compute the number of ways P is a subsequences (not substrings) of T. For example, if T = "rabbbit" , P = "rabbit", then the answer is 3.