Department of Mathematics & Statistics

MTH-102A Ordinary Differential Equations

Assignment II

1. $\star$ Solve the following separable equations
(i) $y' = \frac{1+y^2}{1+x^2}$ (ii) $\sqrt{1-x^2}\, y' + \sqrt{1-y^2} = 0$.

2. $\star$ Solve the following non-linear equations by converting them in to a separable equation.
(i) $xy' - y = \sqrt{x^2+y^2}$ (ii) $(x - \sqrt{xy})y' = y$ (iii) $y' + \frac{2}{x}y = \frac{3x^2y^2+6xy+2}{x^2(2xy+3)}$.

3. $\star$ Solve the following initial value problems and find the maximal interval on which the solution is defined.
(i) $x^2y' = y^2 + xy - x^2$ with $y(1) = 2$ (ii) $y' = -2x(y^2 - 3y + 2)$ with $y(0) = 3$.

4. $\star$ Show that the equations
(i) $(4x^3y^3+3x^2)+(3x^4y^2+6y^2)y' = 0$ and (ii) $(ye^{xy}\tan x + e^{xy}\sec^2 x)+(xe^{xy}\tan x)y' = 0$.
are exact and solve them.

5. $\star$Find an integrating factor of
   (a) $(2xy^3 - 2x^3y^3 - 4xy^2 + 2x) + (3x^2y^2 + 4y)y' = 0$,
   (b) $2xy^3 + (3x^2y^2 + x^2y^3 + 1)y' = 0$ and
   (c) $\left(3xy + 6y^2\right) + \left(2x^2 + 9xy\right) y' = 0$.

   and solve them.

6. $\star$ Solve the initial value problem $y' + 2xy = -e^{-x^2}\left(\frac{3x+2ye^{x^2}}{2x+3ye^{x^2}}\right)$ with $y(0) = -1$.

7. $\star$ Find the Picard iterates of $y' = y$ with $y(0) = 1$.

8. $\star$ Find the first three Picard iterates of $y' = 1 + y^3$ with $y(1) = 1$.

9. Solve the equation $y' = \frac{ax+by+h}{cx+dy+k}$ where a, b, c, d, h and k are constants.

10. Show that the separable equation $-y + (x + x^6)y' = 0$ can be converted to an exact equation by multiplying with an integrating factor.

11. Let $a,\ b,\ c,\ d \in \mathbb{R}$ be such that $ad - bc \neq 0$ and $m,\ n \in \mathbb{R}$. Show that the equation $(ax^my + by^{n+1}) + (cx^{m+1} + dxy^n)y' = 0$ has an integrating factor of the form $x^\alpha y^\beta$.

12. Construct the first two Picard iterates of $y' = (x^2 + y^2)$ with $y(0) = 1$.

13. Construct the Picard iterates of $y' = 2t(y + 1)$ with $y(0) = 0$ and show that $y(t) \to e^{t^2} - 1$.

14. Show that the solution y of $y' = x^2 + e^{-y^2}$ with $y(0) = 0$ exists for $0 \leq x \leq \frac{1}{2}$ and $|y(x)| \leq 1$ for $0 \leq x \leq \frac{1}{2}$.

15. Show that $W := \{y : \mathbb{R} \to \mathbb{R} : y \text{ is a solution of } y' + py = 0\}$ is a vector space; here $p : \mathbb{R} \to \mathbb{R}$ is a continuous function. What is the dimension of W?

16. Solve the given Bernoulli equations.
(i) $7xy^6y' - 2y^7 = -x^2$ (ii) $x^2y' + 2y = 2e^{\frac{1}{x}}y^{\frac{1}{2}}$.

17. Miscellaneous Problems. Solve the following equations.
(i) $y' + \frac{t}{1+x^2}y = 1 - \frac{x^3}{1+x^4}y$ (ii) $y' = k(a-y)(b-y)$ where $a,\ b > 0$
(iii) $y' = -y\sqrt{x}\sin x$.