CS315: Database Systems
Query Optimization

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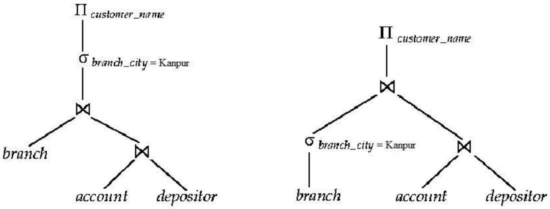
Mon 12:00-13:15, Tue 9:00-10:15

Evaluation Plan

- Equivalent expressions provide alternate ways of executing a query



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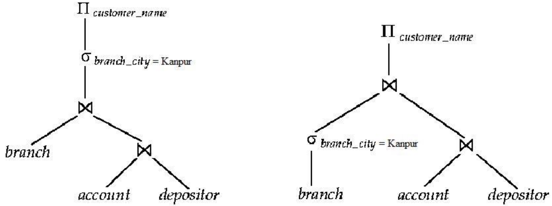
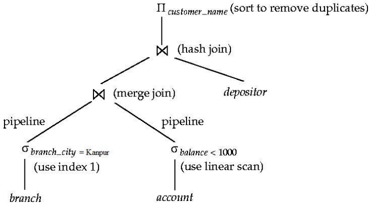


- An evaluation plan also specifies the algorithms



- Cost-based query optimization

Equivalent Expressions

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  - Order of tuples does not matter
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- Equivalence rule: specifies which expressions are equivalent
- Equivalent expressions are systematically generated by repeatedly applying equivalence rules and replacing one form by another
- Evaluation plans must account for all algorithms used
  - Merge join may be costlier than hash join, but since it provides a sorted output, a higher level aggregation will be faster

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18. $\Pi_L(E_1 \cup E_2) = \Pi_L(E_1) \cup \Pi_L(E_2)$

Example Schema

- branch (bname, bcity, assets)
- customer (cname, cstreet, ccity)
- account (ano, bname, bal)
- loan (lno, bname, amt)
- depositor (cname, ano)
- borrower (cname, lno)

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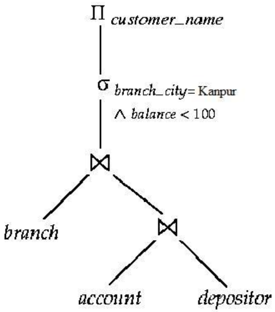
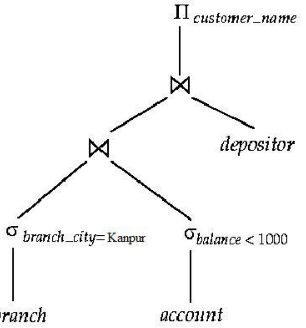


(a) Initial expression tree



(b) Tree after multiple transformations

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- Find the number of join order for $r_1 \bowtie r_2 \bowtie \cdots \bowtie r_n$
- If order of relations cannot change, it is the possible ways of bracketization or the number of full binary trees
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$$J_{n-1} = \frac{1}{n}\binom{2n-2}{n-1} = \frac{(2n-2)!}{n!(n-1)!}$$

  - When $n = 3$, $J_2 = 2$: $(r_1 \bowtie r_2) \bowtie r_3$ and $r_1 \bowtie (r_2 \bowtie r_3)$
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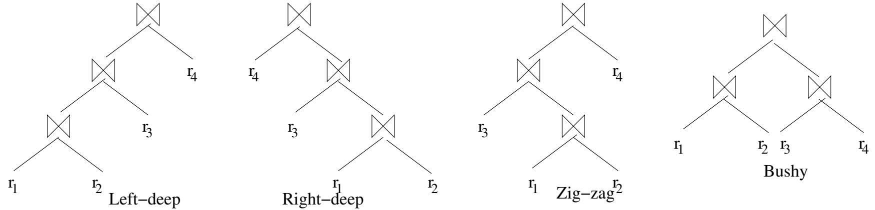
- Too many
- Use left-deep join tree
  - Right side of each join is a single relation, and not an intermediate result of join of two or more relations
- Similarly, right-deep join trees can be defined
- A tree where at least one child of an internal node is a single relation is called a zig-zag tree
- A general tree is also called a bushy tree



Number of Trees

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  - Bushy trees: $(n-1)^{\text{th}}$ Catalan number $\frac{(2n-2)!}{n!(n-1)!}$
    - $C(1) = 1; C(n) = \sum_{i=1}^{n-1} C(i).C(n-i)$

Algorithm for Join Order

- Consider set S as the join of n relations
- S can be represented as $S_1 \bowtie (S - S_1)$ for any non-empty proper subset $S_1 \subset S$
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- Time complexity is exponential in n
- Interesting sort order: Particular order of records that are useful later
  - Example: Merge join produces tuples in sorted order which makes later merge joins faster
  - Example: Sorted order makes later grouping and aggregation faster
- Algorithm should find the best subset for *each* interesting sort order

Heuristics in Query Optimization

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  - Reduces size of relations

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  - Hybrid hash joins can be used
- Perform semantic optimizations
  - Find all employees earning more than their manager
  - May use domain knowledge to return empty result directly

Statistics

- For each relation r
  - Number of tuples n_r
  - Number of blocks b_r
  - Blocking factor, i.e., number of tuples that fit in a block $f_r = \lfloor n_r / b_r \rfloor$
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- If $R \cap S$ is a foreign key in S referencing R, then it is exactly n_s
- Every tuple of r can join with $n_s/v_s(A)$ when joining attribute is A, thereby prducing $n_r.n_s/v_s(A)$ joined tuples
- Reversing r and s, estimate becomes $n_s.n_r/v_r(A)$
- Lower size is the better estimate
- Histograms can improve the estimates

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