CS345: Assignment 5

Q1 (a) A regular graph is a graph in which all vertices have the same degree. In a k-regular graph all vertices have degree k.

Prove that every regular bipartite graph has a perfect matching (a matching is perfect if all vertices are matched in it.) Hint: First show that this graph must have equal number of vertices in both the vertex sets. Use Hall's theorem to prove the existence of a perfect matching.

(b) In a k-regular bipartite graph show that there exist k perfect matchings $M_1, M_2, \ldots, M_k$ such that $M_i \cap M_j = \emptyset$ for all $i \neq j$.

Q2 Let M be a matching in a graph G. Let v be an unmatched vertex in (G, M). If there does not exist an augmenting path starting from v in (G, M), then show that there exists a maximum matching M^* in G in which v is unmatched, i.e., $v \in S_1$ in Gallai Edmonds decomposition.

Q3 Let S_1, S_2, S_3 be the Gallai Edmonds decomposition of a graph G.

(a) Let M_1 and M_2 be two maximum matchings of G and U_1 and U_2 be the respective sets of unmatched vertices. Let x belong to $U_1 \setminus U_2$ and let P be the maximal M_2-M_1 alternating path starting from x. Show that P does not visit any vertex of S_3.

(b) Let M be a maximum matching in G and U denote the set of unmatched vertices. Recall that for every $y \in S_1$ there exists an $x \in U$ such that there exists an even length alternating path from x to y. Prove the converse here. Show that if P is an even length alternating path from some $x \in U$ to some vertex y, then y belongs to S_1.

(c) Given M and U as in part (b) let P be any even length alternating path from some $x \in U$. Show that P does not visit any vertex of S_3.