#4 To the fixed observer, unit vectors $\hat{i}, \hat{j}, \hat{k}$ also change with time. So,

$$\vec{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}$$

$$\Rightarrow \frac{d\vec{A}}{dt} = \frac{dA_1}{dt}\hat{i} + \frac{dA_2}{dt}\hat{j} + \frac{dA_3}{dt}\hat{k} + A_1 \frac{d\hat{i}}{dt} + A_2 \frac{d\hat{j}}{dt} + A_3 \frac{d\hat{k}}{dt}.$$

$$\Rightarrow \left.\frac{d\vec{A}}{dt}\right|_f = \left.\frac{d\vec{A}}{dt}\right|_m + A_1 \frac{d\hat{i}}{dt} + A_2 \frac{d\hat{j}}{dt} + A_3 \frac{d\hat{k}}{dt}.$$

$\frac{d\hat{i}}{dt}$ is perpendicular to $\hat{i}$ & lies in the plane of $\hat{j}$ & $\hat{k}$. So,

$$\frac{d\hat{i}}{dt} = \alpha_1 \hat{j} + \alpha_2 \hat{k}$$

$$\frac{d\hat{j}}{dt} = \alpha_3 \hat{k} + \alpha_4 \hat{i}$$

$$\frac{d\hat{k}}{dt} = \alpha_5 \hat{i} + \alpha_6 \hat{j}$$

$$\Rightarrow \hat{i}.\hat{j} = 0 \Rightarrow \hat{i}.\frac{d\hat{j}}{dt} + \frac{d\hat{i}}{dt}.\hat{j} = 0.$$

& $\hat{i}.\frac{d\hat{j}}{dt} = \alpha_4$ & $\hat{j}\frac{d\hat{i}}{dt} = \alpha_1 \Rightarrow \alpha_4 = -\alpha_1$

similarly, $\alpha_6 = -\alpha_3$

& $\alpha_5 = -\alpha_2$.

So, $$A_1 \frac{d\hat{i}}{dt} + A_2 \frac{d\hat{j}}{dt} + A_3 \frac{d\hat{k}}{dt} = (-\alpha_1 A_2 - \alpha_2 A_3)\hat{i} + (\alpha_1 A_1 - \alpha_3 A_3)\hat{j} + (\alpha_2 A_1 + \alpha_3 A_2)\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha_3 & -\alpha_2 & \alpha_1 \\ A_1 & A_2 & A_3 \end{vmatrix}.$$

If we choose, $\alpha_3 = \omega_1$, $-\alpha_2 = \omega_2$ & $\alpha_1 = \omega_2$, then the determinant is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_1 & \omega_2 & \omega_3 \\ A_1 & A_2 & A_3 \end{vmatrix} = \vec{\omega} \times \vec{A} \qquad \left[\vec{\omega} = \omega_1 \hat{i} + \omega_2 \hat{j} + \omega_3 \hat{k}\right]$$

$$\Rightarrow \left.\frac{d\vec{A}}{dt}\right|_f = \left.\frac{d\vec{A}}{dt}\right|_m + (\vec{\omega} \times \vec{A}).$$