Special Topics in Natural Language Processing

CS698O

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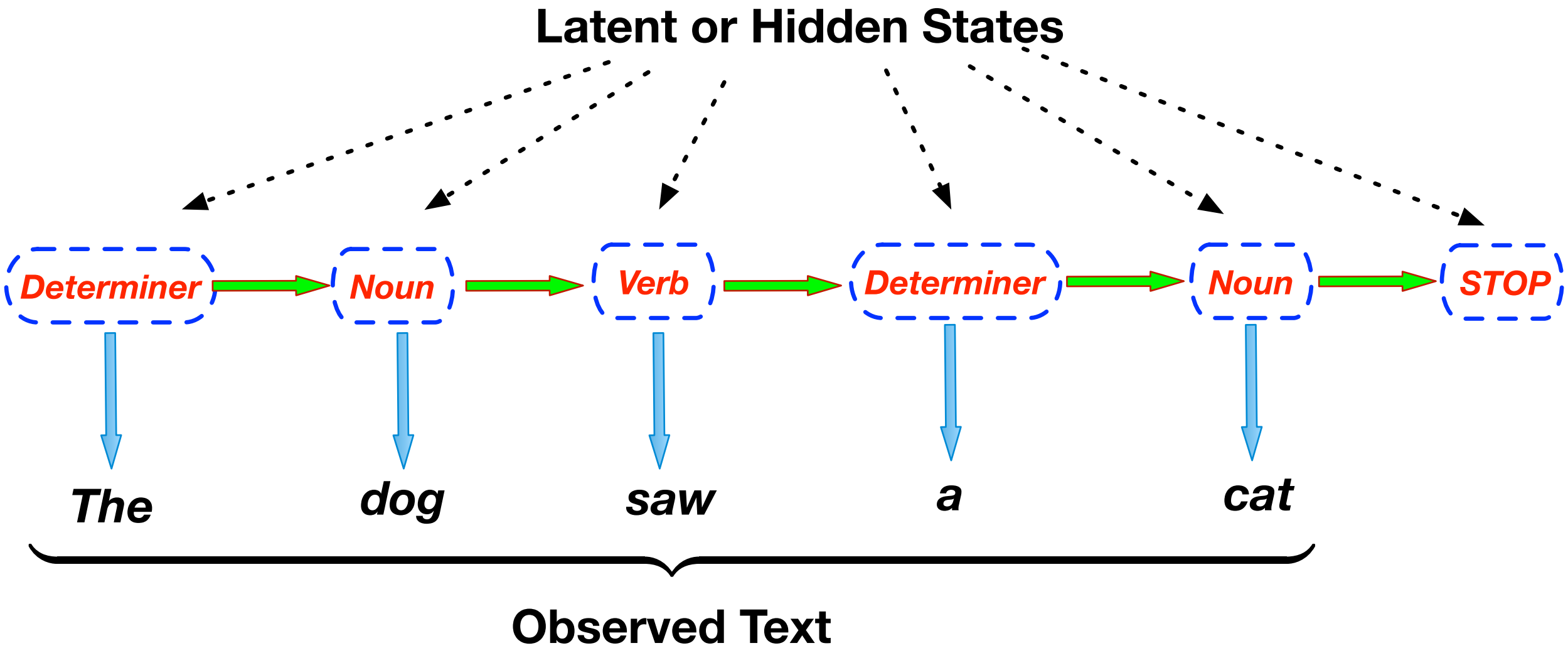
CSE Department, IIT Kanpur

Lecture 13: Sequence Prediction 4
Feb 3, 2020

Hidden Markov Models (HMM)



HMM Joint Distribution

- We have a generative model and are interested in the joint distribution

$$P(X, Y) = P\left(X_1 = x_1, \ldots, X_N = x_N, Y_1 = y_1, \ldots, Y_N = y_N\right)$$

$$P\left(X_1 = x_1, \ldots, X_N = x_N, Y_1 = y_1, \ldots, Y_N = y_N\right) =$$

$$P\left(y_1 | START\right) \times \left(\prod_{i=1}^{N-1} P\left(y_{i+1} | y_i\right)\right) \times \left(\prod_{i=1}^{N} P\left(x_i | y_i\right)\right) \times P\left(STOP | y_N\right)$$

Initial / Start Probability

Transition Probability

Emission Probability

End / Final Probability

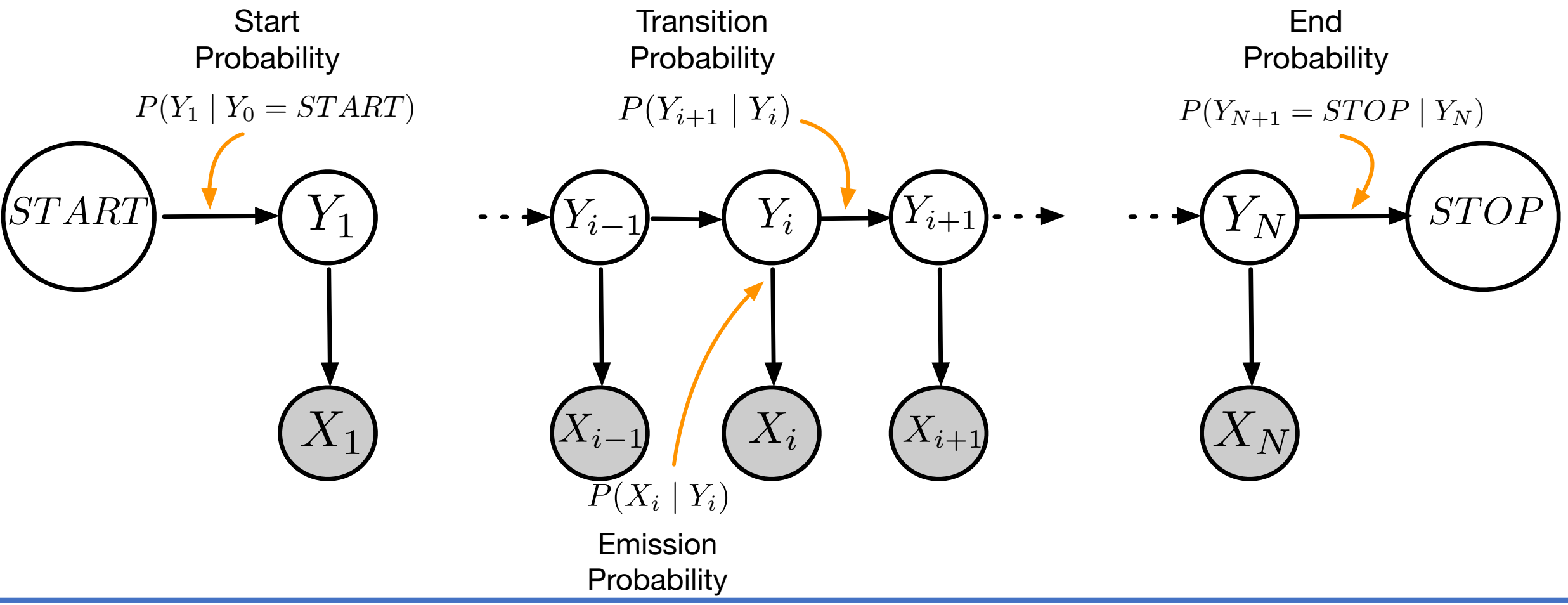
HMM Setting

Set of States (Λ)

$$\{c_0 = START, c_1, c_2, ..., c_K, c_{K+1} = STOP\}$$

Set of Observations

$$\{w_1, w_2, ...w_J\}$$



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$$P(Y_i = \text{NOUN} \mid Y_{i-1} = \text{VERB})$$ ✓

$$P(Y_i = \text{NOUN} \mid Y_{i-1} = \text{VERB},\ \text{CAPITAL}(X_i) = \text{True})$$ ✗

Maximum Entropy Markov Model (MEMM)

- Allow highly flexible representations, allowing features to be easily integrated into the model.

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$$\hat{Y}_{1:N} = \underset{y_{1:N} \in \Lambda}{\operatorname{argmax}} P(X_{1:N}, Y_{1:N} = y_{1:N})$$

$$= \underset{y_{1:N} \in \Lambda}{\operatorname{argmax}} \prod_{i=1}^{N+1} P(Y_i \mid Y_{i-1}) \times P(X_i \mid Y_i)$$

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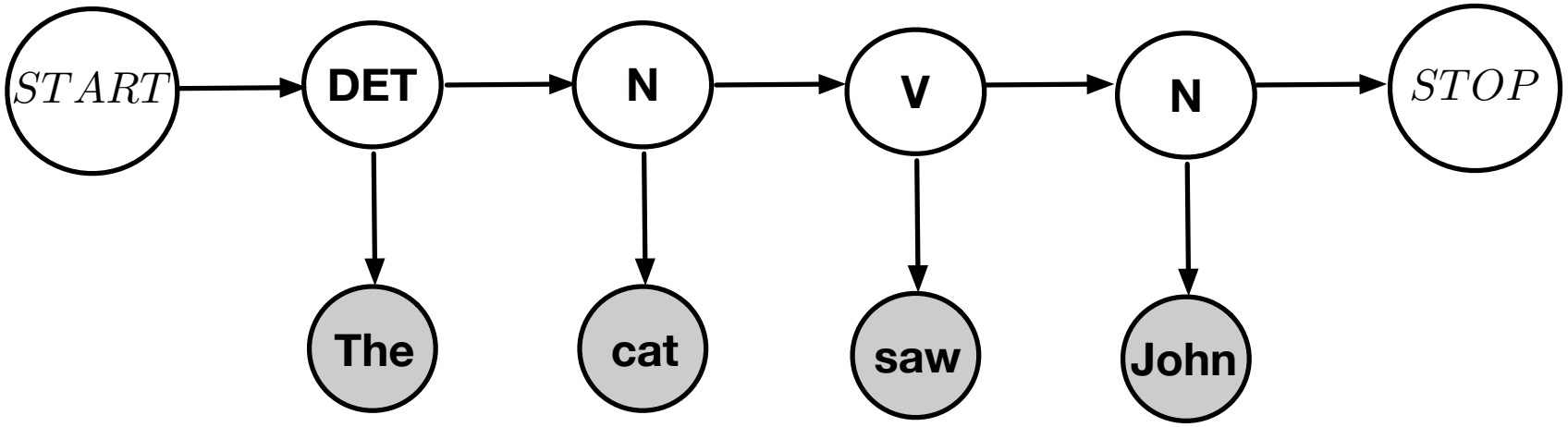
HMM vs MEMM



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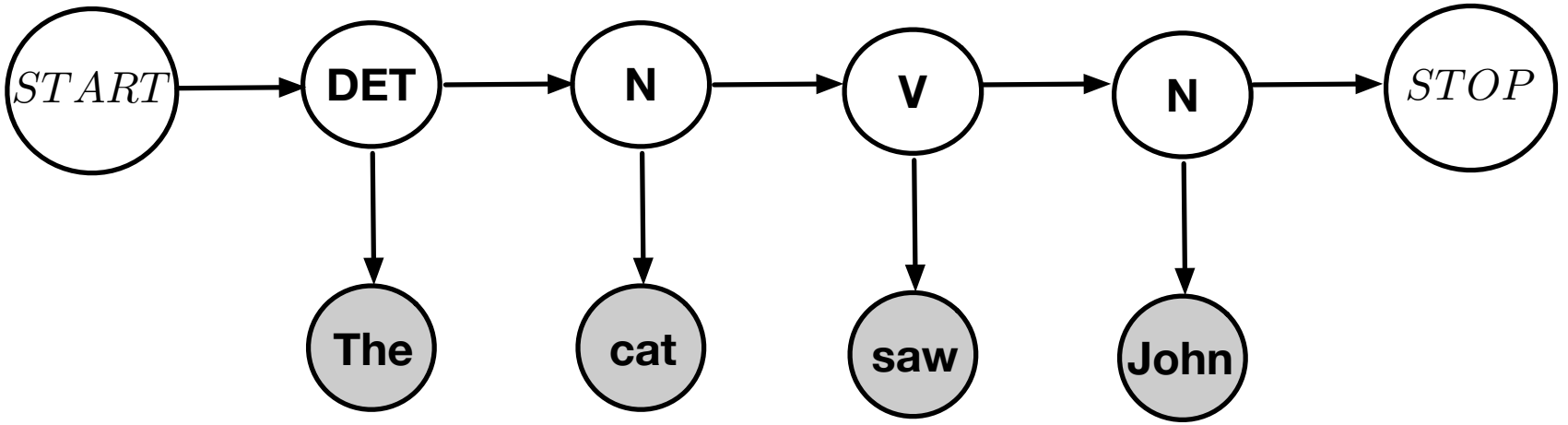
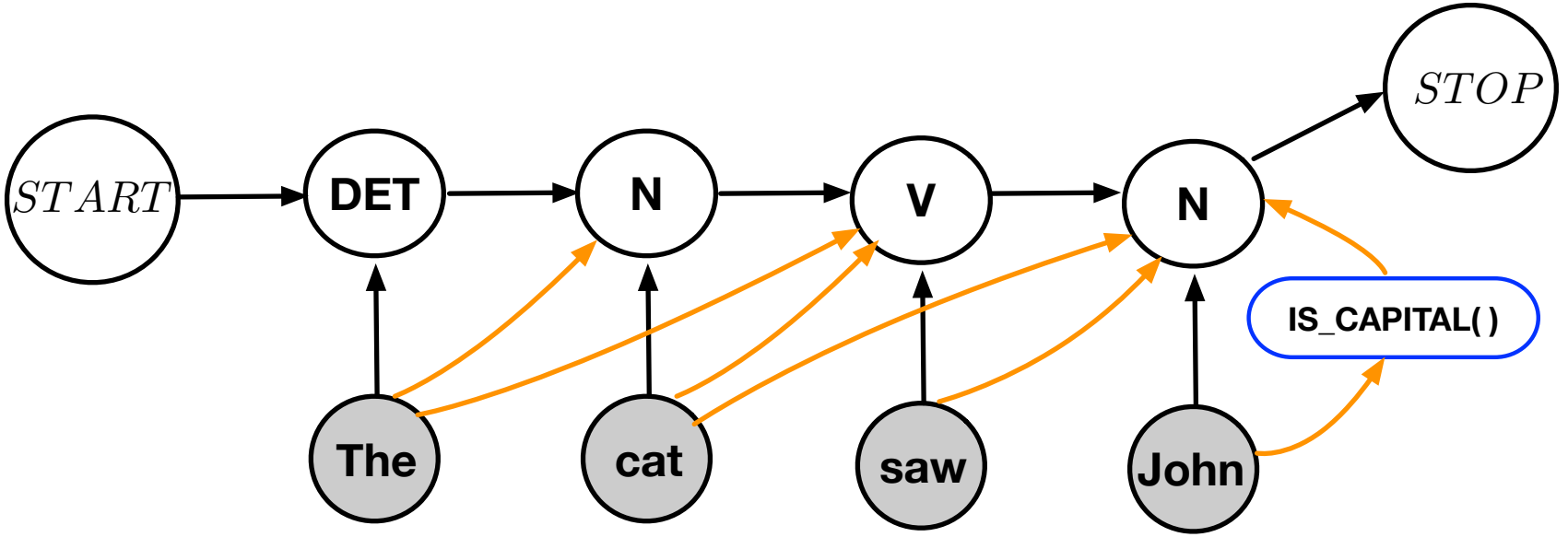

START
DET
N
V
N
STOP
The
cat
saw
John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Why Discriminative Models?

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  - If we are interested only in output (hidden states), does it make sense to model joint distribution?
  - Model the conditional distribution directly *P(Y | X)*

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Three Questions:

1. ***Feature Vectors***: How do we define feature vector?
2. ***Learning Problem***: How do we learn parameters?
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$$f(h_i, y_i) \in \mathbb{R}^d$$

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- Each feature can capture any information in history *h* in conjunction with tag *y*

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- Each feature can capture any information in history *h* in conjunction with tag *y*
- A much richer set of features can be employed for the tagging task

MEMM: Feature Vectors

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$$f(h_i, y_i) \in \mathbb{R}^d \qquad h_i =< y_{i-1}, x_1 \ldots x_N, i >$$

WORD/TAG FEATURES

$$f_{100}(h, y) = \begin{cases} 1 & \text{If } x_i = \texttt{base} \text{ and } y = \text{VB} \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

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SUFFIX FEATURES

$$f_{101}(h, y) = \begin{cases} 1 & \text{If } x_i \text{ ends in } \texttt{ing} \text{ and } y = \text{VBG} \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

MEMM: Feature Vectors

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PREFIX FEATURES

$$f_{102}(h, y) = \begin{cases} 1 & \text{If } x_i \text{ starts with } \texttt{pre} \text{ and } y = \text{NN} \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

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BI-GRAM FEATURES

$$f_{103}(h, y) = \begin{cases} 1 & \text{If } < y_{i-1}, y_i > = < \text{DT,NN} > \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

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UNI-GRAM FEATURES

$$f_{104}(h, y) = \begin{cases} 1 & \text{If } < y > = < \text{NN} > \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

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CONTEXTUAL FEATURES

$$f_{105}(h, y) = \begin{cases} 1 & \text{If previous word } x_{i-1} = \texttt{the} \text{ and } y = \text{VB} \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

MEMM: Feature Vectors

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$$f(h_i, y_i) \in \mathbb{R}^d \qquad h_i =< y_{i-1}, x_1 \ldots x_N, i >$$

CONTEXTUAL FEATURES

$$f_{106}(h, y) = \begin{cases} 1 & \text{If next word } x_{i+1} = \texttt{the} \text{ and } y = \text{VB} \\ 0 & \text{otherwise} \end{cases}$$

Ratnaparkhi, 96

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OTHER FEATURES

- Spelling features which consider weather a word being tagged contains number, contains a hyphen, or contains an upper-case letter
- Contextual features that consider the word at x_{i-2} and x_{i+2} in conjunction with the current tag *y*

Ratnaparkhi, 96

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Parameter Estimation in MEMM

TRAINING DATA $(x^{(k)}, y^{(k)}) \qquad k = 1, \ldots, m$

$$x^{(k)} = \{x_1^{(k)}, \ldots, x_{N_k}^{(k)}\}$$

$$y^{(k)} = \{y_1^{(k)}, \ldots, y_{N_k}^{(k)}\}$$

$$p(y_i^{(k)} \mid h_i^{(k)}; \theta) = \frac{exp(\theta^T f(h_i^{(k)}, y_i^{(k)}))}{\sum_{y' \in K} exp(\theta^T f(h_i^{(k)}, y'))}$$

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How do we estimate parameters?

Parameter Estimation in MEMM

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$$\mathcal{L}(\theta) = \sum_{k=1}^{m} \sum_{i=1}^{N_k} log\ p(y_i^{(k)} \mid h_i^{(k)}; \theta) - \frac{\lambda}{2} \sum_{j=1}^{d} \theta_j^2$$

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$$\theta^* = \underset{\theta \in \mathbb{R}^d}{\operatorname{argmax}}\, \mathcal{L}(\theta)$$

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DECODING

$$\underset{y_1,\ldots,y_N}{\text{argmax}} \; P(Y_1 = y_1, \cdots, Y_N = y_N \mid X_1 = x_1, \cdots, X_N = x_N)$$

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DECODING IN HMM

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DECODING IN MEMM: VITERBI

$$\underset{y_1,\ldots,y_N}{\operatorname{argmax}} \prod_{i=1}^{N} p(y_i \mid h_i; \theta)$$

$$\text{viterbi}(k, c_k) = \max_{y_1,\ldots,y_k} \prod_{i=1}^{k} p(y_i \mid h_i; \theta)$$

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HMM:

DECODING IN MEMM: VITERBI

$$\underset{y_1,\ldots,y_N}{\operatorname{argmax}} \prod_{i=1}^{N} p(y_i \mid h_i; \theta)$$

$$\text{viterbi}(k, c_k) = \max_{y_1,\ldots,y_k} \prod_{i=1}^{k} p(y_i \mid h_i; \theta)$$

$$\text{viterbi}(k, c_k) = \max_{c_l} \text{viterbi}(k-1, c_l) \times p(Y_k = c_k \mid h_k; \theta)$$

HMM:

$$\text{viterbi}(i, c_k) = \left(\max_{c_l} \text{viterbi}(i-1, c_l) \times P\left(Y_i = c_k \mid Y_{i-1} = c_l\right) \right) \times P\left(X_i \mid Y_i = c_k\right)$$

VITERBI ALGORITHM

Initialization: $\text{viterbi}(0, START) = 1$

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Summary

1. It is difficult to consider features in HMM models.
2. MEMMs make it possible to include rich set of features in sequence prediction
3. MEMMs are discriminative models
4. Features can be included in MEMMs model via indicator functions
5. Parameters can be estimated using MLE
6. Decoding in MEMMs can be done via Viterbi Algorithm.

References

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