Problem Set 2

CS 340 - Theory of Computation

1 September 2018

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1. Let $\Sigma = \{0, 1\}$. Give a regular expression α such that $L(\alpha)$ corresponds to the following sets:
   (a) Set of all binary strings with at most one occurrence of 00.
   (b) Set of all binary strings with at least one 0 and at least one 1.

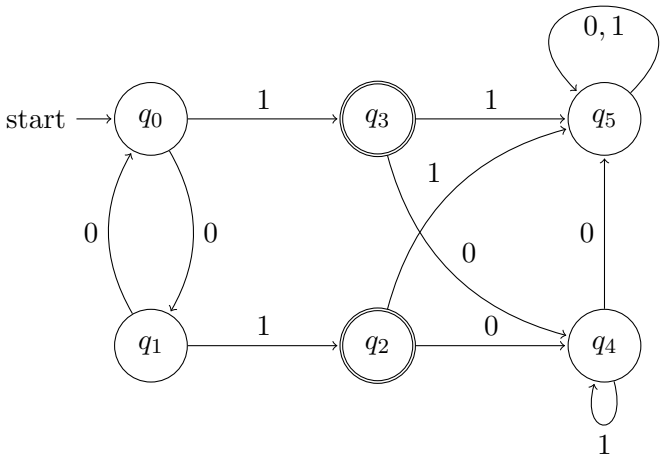
2. Let $\Sigma = \{0, 1\}$. Describe the regular set corresponding to the following regular expressions in words:
   (a) $(0 + 1)^*11(0^*1^*)^*$
   (b) $(0 + 10)^*$
   (c) $(1^*01^*01^*)(1^*01^*01^*01^*)^*$

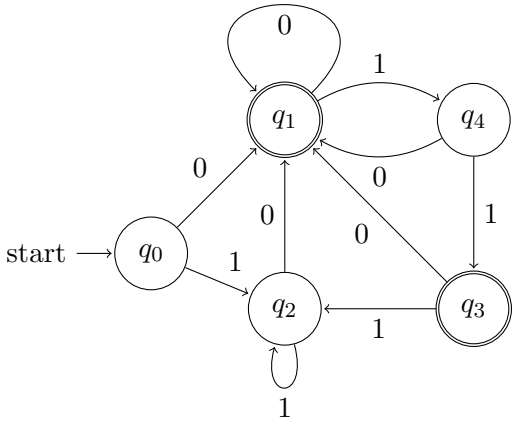
3. Prove that the following sets are not regular:
   (a) $\{0^{n^2} \mid n \geq 0\}$
   (b) $\{0^{n!} \mid n \geq 0\}$

4. Let L be a regular set. Prove that the following sets are regular.
   (a) $L_1 = \{x \mid \text{there exist } y \text{ such that } xy \in L \text{ and } |x| = 2|y|\}$
   (b) $L_2 = \{x \mid \text{there exist } y \text{ such that } yx \in L \text{ and } |x| = 2|y|\}$

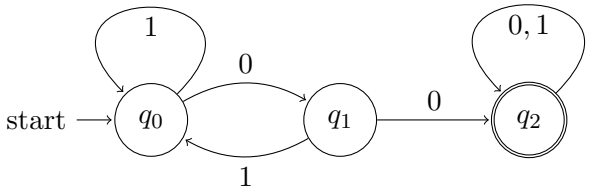
5. Let L be a regular set over Σ. Are the following sets are regular? Clearly justify your answer.
   (a) $L_1 = \{w \mid w = a_1 s a_2 s s a_3 s s s \ldots a_n s^n, \text{where } s, a_1, \ldots, a_n \in \Sigma \text{ and } a_1 a_2 \ldots a_n \in L\}$
   (b) $L_2 = \{w \mid w = a_1 s a_2 s a_3 s \ldots a_n s, \text{where } s, a_1, \ldots, a_n \in \Sigma \text{ and } a_1 a_2 \ldots a_n \in L\}$

6. Minimize the following DFAs:





7. Give a regular expression α such that $L(\alpha) = L(M)$ for the following DFA M:



8. Recall that $\{0^n1^n \mid n \geq 0\}$ is not regular. Use the closure properties of regular sets to argue that the following sets are not regular.

   (a) $\{0^n1^m \mid m \geq n \geq 0\}$

   (b) $\{0^n1^m \mid m \neq n \geq 0\}$