Indian Institute of Technology, Kanpur

Department of Mathematics and Statistics

MTH-628A Topics in Topology

Mid-Semester Examination

Max Marks: 60

20. February, 2018.

18:00 p.m.-20: 00 p.m.

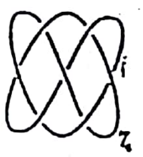
Each question carries 6 marks. DO ANY 10

1. Calculate the Jones polynomial for the figure eight knot 4_1 . Is the figure eight knot amphicheiral or not? Justify your answer.

2. Prove that if a knot K is amphicheiral then its Jones polynomial is palindromic

Converse is false Eg: 9_{42}

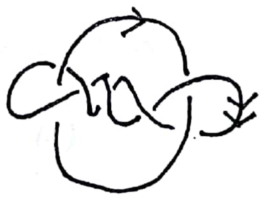
i.e. $V_K(t) = V_K(1/t)$



3. Is 7_4 amphicheiral or not ? Justify your answer.

4. Calculate the linking number of the following two-links, and determine if they are equivalent or not

L_1



L_2

5. Define when a knot K is said to be tricolourable. Show that if K_1 and K_2 are equivalent knots, then both are tricolourable or both are not triclourable.

6. Is the following knot tricolourable or not. Justify your answer.

7. Define the fundamental group of a topological space X and show that it is a topological invariant. However, show that the Converse is False by giving an example of topological spaces X and Y such that $\pi_1(X) = \pi_1(Y)$ but X is not homeomorphic to Y.

8. **Show that if X is path-connected, $\pi_1(X, X_0)$ is isomorphic to $\pi_1(X, X_1)$ for any two points X_0, X_1 in X.**

9. **Let S^1 be the unit circle in the plane. Suppose $f: S^1 \to S^1$ is a map which is not homotopic to the identity. Prove that $f(x) = -x$ for some point x of S^1.**

10. When is a topological space X said to be contractible ? Is a contractible space always simply connected ? Justify your answer.

11. Define a covering space E of B. Let $p: E \to B$ be a covering map; let B be connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$; then $p^{-1}(b)$ has k elements for every $b \in B$ (a k-fold cover).

12. Compute the fundamental group of the following spaces, giving brief reasons for your answer.

Cone on a circle ; $z^2 = x^2 + y^2$

(9.) Suppose $f(x) \neq -x \;\; \forall x$

Define $H: S^1 \times I \longrightarrow S^1$

$$H(x,t) = \frac{(1-t)\cdot f(x) + tx}{\|(1-t)f(x) + tx\|}$$

Since $f(x) \neq -x$

$\rightarrow$ denominator $\neq 0$

$H(x,0) = f(x)$

$H(x,1) = x$

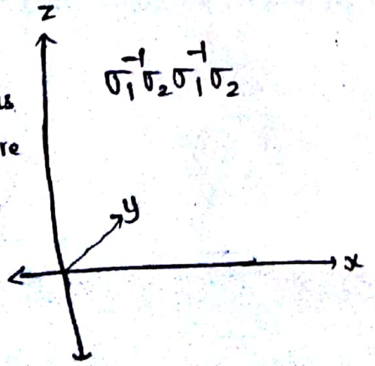
$\Rightarrow f$ is homotopic to x

$\rightarrow\leftarrow$ Contradiction

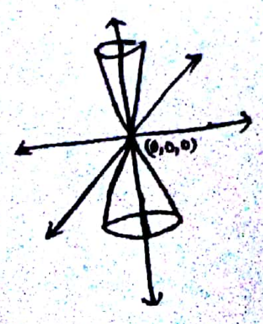
$\therefore f(x) = -x$ for some $x \in S^1$

Rotate 180° about Z-axis σ_1 and σ_2 are exchanged.

$\sigma_1^{-1}\sigma_2\sigma_1^{-1}\sigma_2$



(12.)



Let $\pi_1(C, (0,0,0))$

$= \{e\}$ star convex

INDIAN INSTITUTE OF TECHNOLOGY KANPUR
DEPARTMENT OF MATHEMTICS & STATISTICS

MTH-628A(TOPICS IN TOPOLOGY)
END-SEMESTER EXAMINATION

24 April 2018

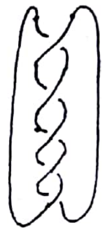
4-7 P.M. Max Marks: 80

$\frac{1}{v}P_{K^+} - vP_{K^-} = zP_{K_0}$

There are 22 questions. <u>Each question carries 5 marks. Do any 16 out of 22 questions</u>

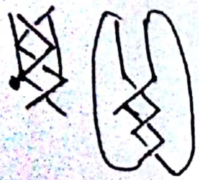
1. State the axioms defining the 2-variable HOMFLY polynomial $P_K(v,z)$. Show that the Jones polynomial and the Alexander polynomial can be obtained as special cases of the HOMFLY polynomial.

2. Calculate the HOMFLY polynomial for the knot 5_1.

5_1  

3. Using question 2, or otherwise calculate the Jones polynomial for the knot 5_1.

4. Draw the rational knot $K_1 = [5,1,4]$ and determine the rational number Draw the rational knot $K_2 = [2,-2,2,-2,2,4]$ and determine whether K_2 is equivalent to K_1



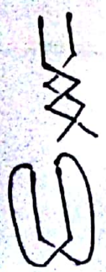
5. Is 5_1 amphicheiral or not ? Justify your answer.

6. Use the Alexander's Theorem to find a braid representation for the knot 6_2.

6_2 

7. Show that the two words w_1 *and* w_2 represent the same five string braid up to equivalence

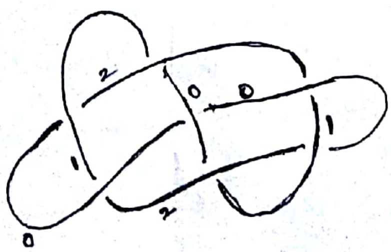
$$w_1 = \sigma_2\sigma_1\sigma_2\sigma_1\sigma_2^{-1}\sigma_4$$
$$\text{and } w_2 = \sigma_2^2\sigma_3\sigma_1\sigma_3^{-1}\sigma_4$$

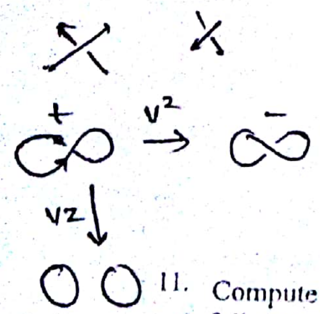


8. State the Markov Theorem for closed n −braids.

9. Construct a spatial graph for the complete graph on 6 vertices K_6 , and show that it contains a pair of linked triangles (i.e. the Hopf link).

10. Determine whether the following knot 7_4 is tricolourable or not.





7_4 

11. Compute the fundamental group of the circle S^1 giving complete reasons for your answer and full statements of theorems used.

12. Determine the Euler Number of a connected sum of two surfaces of dimension2. Hence determine the Euler Number of a connected sum of n-tori T_n , and the Euler Number of a connected sum of h projective planes U_h .

$P: 1 = v^2(1) + vzP_{a_2}$

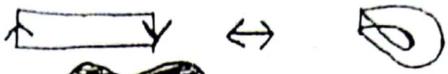
$\frac{1-v^2}{vz} = P_{a_2}$

What surface is the connected sum of a torus and a projective plane.

13. State and prove the BROUWER'S. Fixed Point Theorem for the disc D.

14. Prove that there are exactly five regular polyhedron in nature, and describe them.

15. Compute the fundamental group of the following spaces, giving brief reasons

[1] Mobius Strip 

[2] Solid Double Torus 

[redacted]

[4] Torus with a disc attached in the centre

[5] R^3 with the non-negative x,y and z axes deleted

16. A compact topological space X is said to have the fixed point property if every continuous map $f: X \to X$ has at least one fixed point. Determine whether the following topological spaces have the fixed point property or not. Justify your answer in each case, with reasons

(i) Sphere S^2

(ii) Torus T

(iii) Finite cone $z^2 = x^2 + y^2,\ 0 \le z \le 1$

17. 18. For the following spaces –(i) construct a triangulation (ii) write down the simplicial homology groups with brief reasons (iii) calculate the Euler Characteristic and (iv) show that

$$\chi(K) = \sum_{i=0}^{n} (-1)^i \beta_i .$$

[1] Four circles meeting at a point.

[2] [redacted] TORUS $S^1 \times S^1$

[3] Double Torus

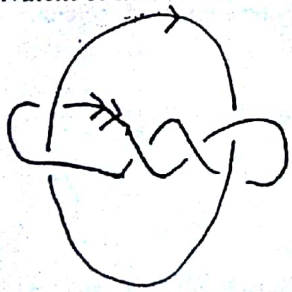
[4] One point union of two spheres

[5] One point union of a sphere S^2 and a circle S^1

19. Prove that R^n is homeomorphic to R^m if and only if $m = n$.

20. 21. State and prove the Euler-Poincare Theorem. [10]

22. Determine the linking numbers of the following 2-component links, and determine whether they are equivalent or not.



L_1

L_2

[5]

$\frac{1}{4} + 1$

$\frac{1}{3} + \frac{1}{5} - \frac{1}{2}$

$\frac{10+6-15}{30}$

$\frac{1}{3} + \frac{1}{3} - \frac{1}{2}$

$\frac{2}{3} - \frac{1}{2}$

$\frac{1}{3} + \frac{1}{4} - \frac{1}{2}$

$\frac{1}{3} - \frac{1}{4}$

$\frac{1}{3} + \frac{1}{6} - \frac{1}{2}$

$\frac{2\times4}{3}$ $\frac{2\times16}{4}$