Problem set 1

Ques-8

Hubble $\quad \theta = 1.22\dfrac{\lambda}{D} = \dfrac{1.22 \times 500\times10^{-9}}{2.4} \times \underbrace{206265}_{\text{rad to arc seconds.}}$

$= 0.052''$

VLA $\quad \theta = \dfrac{\lambda}{L} = \dfrac{2\times10^{-2}}{35\times10^{3}} \times 206265 = 0.12''$

VLBI $\quad \theta = \dfrac{\lambda}{L} = \dfrac{2\times10^{-2}}{10^{6}} \times 206265 = 4\times10^{-3}\,''$

Ques-7 $\quad E = h\nu$

For radio waves, $\quad E = \dfrac{6.626\times10^{-34}}{1.6\times10^{-19}} \times \dfrac{3\times10^{8}}{10\times10^{-2}}$ eV

$= 12.4\ \mu eV.$ $\qquad$ Rest is same

Ques-1

angular dia of Sirius : $\qquad 1'' = \dfrac{1 AU}{1 Pc}$

$\Rightarrow \theta = \dfrac{2\times1.71\times 6.96\times10^{5}\ km}{2.64\ Pc \times (1.496\times10^{8}\ km)} = \underline{6\times10^{-3}\ arc\,sec}$

Angle subtended by Jupiter

(i) $\theta_{max} = \dfrac{2\times69911\ km}{d_{min}\times(1.496\times10^{8} km)}$; $d_{min} = \dfrac{(7.78-1.496)\times10^{8} km}{\underbrace{(3.086\times10^{13}\ km)}_{1 Pc}}$

$= 45$ arc sec.

(ii) $\theta_{min} = \dfrac{2\times69911\ Km}{d_{max}\times(1.496\times10^{8}\ km)}$; $d_{max} = \dfrac{(7.78+1.496)\times10^{8}\ km}{3.086\times10^{13}\ km}$

$= 31$ arc sec.

Easier way: $\quad \theta = \dfrac{2\times69911\ km}{(7.78+1.496)\times10^{8}\ km} \times 206265$ arc sec.

Problem Set 3

Ques-1. $F = \frac{L}{4\pi r^2} = 1.36\times10^6\ erg/cm^2 s^2 = 1.36\ KW/m^2$

$m - M = 5\log\left(\frac{1\ AU}{10\times 206265\ AU}\right) = -31.6$

$\Rightarrow M = 4.8.$

Ques-2. $m = -2.5\log\frac{F}{F_0} \Rightarrow F_1 = F_0 10^{-2/5}$, $F_2 = F_0 10^{-4/5}$

$m_b = -2.5\log(10^{-2/5} + 10^{-4/5}) = \cancel{\ldots}$

Ques-3

$m - M = 5\log\left(\frac{r}{10 Pc}\right) = 10 \Rightarrow r = 10 Pc (10)^2 = \cancel{\ldots}$

Ques 4 $L_S = \cancel{4\pi R_S^2} \; 4\pi R_S^2 \sigma T_S^4$

~~Ill~~ Energy received on Earth surface Per second

$= \frac{L_S}{4\pi r^2} \times \pi R_E^2$

& Energy received = Energy emitted by BB

$\Rightarrow \frac{4\pi R_S^2 \sigma T_S^4}{4\pi r^2} \times \pi R_E^2 = 4\pi R_E^2 \sigma T_E^4 \Rightarrow T_E = \sqrt{\frac{R_S}{2r}}\, T_S$

Putting values, $T_E = 278.6$ K.

Ques-5 : Doubt. ✓

Ques-5 : Doubt

Ques-6: $B = \frac{\sigma T^4}{\pi}$, $B_\nu = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$, $x = \frac{h\nu}{kT}$

$B = \int_0^\infty B_\nu d\nu = \frac{2k^4T^4}{h^3c^2}\int_0^\infty \frac{x^3}{e^x - 1}dx = \frac{2k^4T^4\pi^4}{15h^3c^2} = \frac{\sigma T^4}{\pi}$

$\Rightarrow \sigma = \frac{2k^4\pi^5}{15h^3c^2}$

Ques 7. $\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2}d\lambda$, $B_\nu = \frac{2hc}{\lambda^3}\frac{1}{e^{hc/\lambda kT} - 1}$

$\Rightarrow B_\nu d\nu = -\left(\frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}\right)d\lambda = -B_\lambda d\lambda$

$\Rightarrow \int_0^\infty B_\nu d\nu = -\int_\infty^0 B_\lambda d\lambda = \int_0^\infty B_\lambda d\lambda$. Hence, $B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$

Ques-8 $\lambda . T = 0.290$ cm K.

$T = 2.73$ K $\Rightarrow \lambda = 0.106$ cm. Hence $\nu = 2.82 \times 10^{11}$ Hz.

~~Ques-9 T = 28000 K, r = 5.16 x 10^11 cm. (Doubt)~~

Ques-10 (a) ν is very small $\Rightarrow \frac{h\nu}{kT} << 1 \Rightarrow e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$

$$\Rightarrow B_\nu = \frac{2h\nu^3/c^2}{1+\frac{h\nu}{kT}-1} = 2kT\frac{\nu^2}{c^2}$$

(b) $$F_\nu = \int d\Omega\, B_\nu \cos\theta = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta\, 2kT_0 \cos\theta \frac{\nu^2}{c^2} \cdot \cos\theta$$

$$= \frac{8}{3}\pi k T_0 \frac{\nu^2}{c^2}$$

(c) Similar to z- coordinate in any transformation.

Ques-9 $\rightarrow$ Data in this ques doesn't match actual data, so answers won't match

a) $L_D = \sigma T^4 S = 1.16 \times 10^{38}$ erg/s. $= 1.16 \times 10^{31}$ W

g) $\lambda . T. = 0.290$ cm-K $\Rightarrow \lambda = 10^{-5}$ cm $= 10^{-7}$ m $= 0.1\,\mu$m

b) $L_{sun} = 3.9 \times 10^{26}$ W, $L_{vega} = 40 L_{sun}$, $r_{vega} = 7.7$ Pc

$\Rightarrow$ ~~$M = -2.5 \log \frac{L_D}{L_{vega}}$~~ $\Rightarrow M = -2.5 \log\left(\frac{L_D}{(10)^2} \times \frac{(7.7)^2}{L_{vega}}\right)$

$\Rightarrow M = -6.6$

c) $$m = -2.5 \log\left(\frac{L_D}{(180)^2} \times \frac{(7.7)^2}{L_{vega}}\right) = -0.33$$

d) $\mu = m - M = 6.27$

e) $F_S = \sigma T^4 = 3.5 \times 10^{10}$ W/m²

f) $$F_E = \frac{L_D}{4\pi r^2} = \frac{1.16 \times 10^{31}}{4\pi (180\,Pc)^2} = 3 \times 10^{-8}\ \text{W/m}^2$$

Ques-10 (c) Any coordinate system whose z' axis makes an angle of ϕ with current z-axis has temperature dependance as

$T = T_0 \cos(\theta' + \phi)$. [Think in terms of Euler transformation & how rotation about z-axes doesn't make any difference]

Ques-5 (1st)

$$B = -2.5 \log \frac{F_B}{F_{B0}}, \quad V = -2.5 \log \frac{F_V}{F_{V0}}$$

$$B - V = 1 = -2.5 \log \frac{F_B}{F_{B0}} \times \frac{F_{V0}}{F_V} \Rightarrow \frac{F_B}{F_V} = \frac{F_{B0}}{F_{V0}} 10^{-0.4}$$

Ques-5 (2nd) - Data not given, ques seems easy

Problem Set 4

Ques-1 For H-atom, $\frac{N_2}{N_1} = 4\, e^{-\frac{10.4}{kT}}$ $\Rightarrow$ $\frac{N_2}{N_1+N_2} = \frac{4e^{-10.4/kT}}{1+4e^{-10.4/kT}}$

$k = 8.6 \times 10^{-5}\ eV/K$.

at 8000 K, $\frac{N_2}{N_1+N_2} \sim \frac{N_2}{N_1} = 1.49 \times 10^{-6}$

at 11000 K, $\frac{N_2}{N_1+N_2} \sim \frac{N_2}{N_1} = \cancel{8.72\times10^{-5}}\ 8\times10^{-5}$

$\frac{N_{II}}{N_I} = 1.74\, a \times 10^{-5}\, e^{-\frac{158}{a}}$ where $a = 8/11$ for 8000/11000 K.

(acc. to formula)

$\therefore \frac{N_{II}}{N_I+N_{II}} \approx \frac{N_{II}}{N_I}$ | $\frac{N_{II}}{N_I} = \begin{cases} 0.038, & T = 8000\,K \\ 16.8, & T = 11000\,K \end{cases}$

for 8000 K, $\frac{N_{II}}{N_I+N_{II}} \approx$ 0.038

11000 K, $\frac{N_{II}}{N_I+N_{II}} = \cancel{1.1\times10^{-10}}$ 0.94

$\frac{N_2}{N_I+N_{II}} = \frac{N_2/N_I}{1+N_{II}/N_I} \approx \frac{N_2/N_1}{1+N_{II}/N_I}$

Hence for 8000 K, $\frac{N_2}{N_I+N_{II}} = 1.48 \times 10^{-6}$

for 11000 K, $\frac{N_2}{N_I+N_{II}} = \cancel{6.72\times10^{-5}}\ 4.5\times10^{-6}$

$m_e = 9.1\times10^{-31}\,kg$, $P_e = 20\ N/m^2$

$k = 1.4\times10^{-23}\,kg\,m^2/s^2\,K$

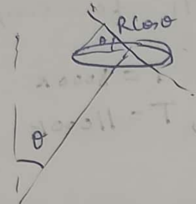
Problem Set 2

Q1. Adv:
- No chromatic aberration
- Larger aperture possible
- No irregularities as compared to lens

Q2 $0.545'' = \frac{1\,AU}{r\,(\text{in Pc})} \Rightarrow r = \left(\frac{1\,AU}{0.545''}\right) Pc$

Transverse speed $= \frac{10.3''}{\text{year}} \times \left(\frac{1\,AU}{0.545''}\right) = \left(\frac{10.3}{0.548}\right)$ AU/year

Q-3



Semi_Major axis $= R$
Semi-minor axis $= R\cos\theta$
$\Rightarrow$ eccentricity $= \sin\theta$

Q-~~7~~ ~~Simple~~ Use the equations:

$\cos\alpha \cos\delta = \cos\lambda \cos\beta$ — ①

$\sin\delta = \sin\theta \cos\beta \sin\lambda + \cos\theta \sin\beta$ — ②

& $\sin\beta = -\sin\theta \cos\delta \sin\alpha + \cos\theta \sin\delta$ — ③

Diff ② & substitute ① $\Rightarrow \dot{\delta} = \sin\theta \cos\alpha \dot{\lambda}$

Diff ③ & substitute $\dot{\delta}$, $\Rightarrow \dot{\alpha} = (\cos\theta + \sin\theta \sin\alpha \tan\delta)\dot{\lambda}$

Ques 15

1 sidereal day extra in 365 solar days

$\Rightarrow 360°$ in 365 solarday

$\Rightarrow 2°$ in $\frac{365}{360} \times 2 = \frac{365}{180}$ days ≈ 2 days.

It would move west

Problem set 6

Ques-5

$$V_g = -4\pi G \int_0^R dr\, M(r)\, \rho r \qquad M(r) = \frac{4}{3}\pi r^3 \rho$$

$$= -4\pi G \int_0^R \frac{4\pi}{3}\rho^2 \, r^4 dr$$

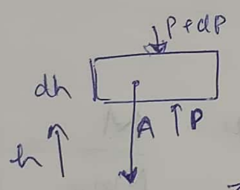
$$V_g = \frac{-16\pi^2}{3} G \rho^2 \frac{R^5}{5}$$

$$= -\frac{16}{15}\pi^2 G \rho^2 R^5$$

Ques-3

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \qquad e^2 = \alpha \hbar c$$

Ques-1



$$dP\, A = -(\rho A\, dh) g \Rightarrow \frac{dP}{dh} = -\rho g$$

$$\int \frac{dz}{T(z')} = \int_0^z \frac{dz}{(T_0 - \beta z)} = \frac{\left[\log(T_0 - \beta z)\right]_0^z}{-\beta}$$

$$= -\frac{1}{\beta}\log\left(\frac{T_0 - \beta z}{T_0}\right) = \frac{1}{\beta}\log\left(\frac{T_0}{T_0 - \beta z}\right)$$

Substitute - -

Ques-2

$$P(z) \approx P_0 e^{-z/z_0}$$

~~$T^\gamma P^{1-\gamma} = C$~~

$$\frac{dT}{dr} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} \qquad T = K P^{1-\frac{1}{\gamma}}$$

$$\cancel{= K\left(1 - \frac{1}{\gamma}\right) P^{-1/\gamma} \frac{dP}{dr}}$$

Assuming $z = 0$
i.e. at earth's surface

$$\frac{dT}{dr} = \frac{2}{7}\frac{T_0}{P_0 e^{-z/z_0}}\frac{(-1)P_0 e^{-z/z_0}}{(z_0)} = -\frac{2}{7}\frac{T_0}{z_0}$$

$$\left|\frac{dT}{dr}\right| = \frac{2}{7}\frac{300\,K}{7.3\,Km} = 11.7\, K/km. < 15\, K/Km$$

So, unstable.

Ques 4

$$\sigma_\lambda n = k_\lambda \rho \quad , \quad \sigma = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2$$

(i) assuming only Hydrogen,

$$\Rightarrow k_\lambda = \frac{\sigma n}{\rho}$$

$$n = \frac{N}{V} = \frac{2N_H}{V}$$

$$\frac{n}{\rho} = \left(\frac{2N_H}{V}\right) \times \left(\frac{V}{M}\right) = \frac{2N_H}{M} = \frac{2}{m_H}$$

$$\Rightarrow k_\lambda = \frac{2}{m_H}\sigma = \frac{16\pi}{3m_H}\left(\frac{e^2}{m_e c^2}\right)^2$$

(ii) $X = 0.7$, $Y = 0.3$.

$$\frac{n}{\rho} = \frac{N}{M} = \frac{1}{\mu m_H}$$

$$\mu = \frac{M}{N m_H} \Rightarrow \frac{N}{M} = \frac{1}{\mu m_H}$$

$$\Rightarrow k_\lambda = \frac{\sigma}{\mu m_H}$$

$$\mu = \frac{1}{2X + \frac{3Y}{4}} = \frac{1}{1.625}$$

$$= \frac{1.625}{m_H}\sigma .$$

Ques 6 Let there be k elements with $Z > 2$.

~~$N = 2m$~~

$$N = 2N_H + 3N_{He} + \sum_{i=1}^{k} (Z_i + 1) N_{Z_i}$$

$$\frac{\text{mass fraction of } Z_i}{M_{Z_i}} = \frac{N_{Z_i}(A_{Z_i} m_H)}{M}$$

$$\Rightarrow N = 2N_H + 3N_{He} + \sum_{i=1}^{k} \frac{M_{Z_i} M}{A_{Z_i} m_H}(Z_i + 1)$$

$$= 2N_H + 3N_{He} + \sum_{i=1}^{k} \frac{1}{2}\frac{M}{m_H} M_{Z_i}$$

$$\Rightarrow N = 2N_H + 3N_{He} + \frac{M}{2m_H} \underbrace{\sum_{i=1}^{k} M_{z_i}}$$

$$N = 2N_H + 3N_{He} + \frac{M}{2m_H}(Z')$$

$$\mu = \frac{M}{\cancel{p} N m_H} = \frac{M}{\left(2\frac{MX}{m_H} + \frac{3MY}{4m_H} + \frac{MZ'}{2m_H}\right) m_H}$$

$$= \frac{1}{2X + 3Y/4 + Z'/2}$$

Problem set 7

Ques-1

$$e^- + {}^1H \longrightarrow \gamma + \gamma$$

Not possible, $L=1$ for LHS, $L=0$ for RHS

$$n \longrightarrow {}^1H + e^- + \nu_e$$

Not possible, $L=0$ for LHS, $L=2$ for RHS

$$^3He + {}^3He \longrightarrow {}^4He + {}^2H$$

Not sure. But some particles are definitely missing

$n \rightarrow p$ is happening.

Ques-2 Easy

Ques-3

$$E^{3/2} = \frac{bkT}{2}$$

For solar interior, $T = 1.5 \times 10^7 K$, $b = \frac{2^{3/2} Z_1 Z_2 e^2 \pi^2 \sqrt{\mu}}{h}$

Ques-4