CS345: Assignment 6

Q1 Given a positive integer n and a set of positive integers $f_1, \ldots, f_n$. Design a rooted binary tree with n leaf nodes such that $\sum_{i=1}^{n} f_i.depth(l_i)$ is minimum, where l_i is the i-th leaf node and $depth(l_i)$ is the distance of l_i from the root.

Q2 (a) Given set of pairs of positive numbers $(V_1, p_1), (V_2, p_2), \ldots, (V_n, p_n)$ and a positive number V such that $\sum_i V_i \geq V$. Design a greedy algorithm to compute V_i' for all i such that $\sum_i p_i.V_i'$ is maximum subject to the conditions that (i) $0 \leq V_i' \leq V_i$ for all i, and (ii) $\sum_i V_i' \leq V$.

(b) Prove that your algorithm does not always compute an optimum solution if the first condition is replaced by $V_i' \in \{0, V_i\}$ for all i.

Q3 Let M be a graphic matroid for a graph $G = (V, E)$ and let $\mathcal{J}$ be those subsets of E which induce a tree (not arbitrary forest). Show that $(E, \mathcal{J})$ is a connected sub-matroid.

Q4 Give an example of a matrix matroid and a dependent set in it which is not critically dependent. Give an example of a circuit in it.