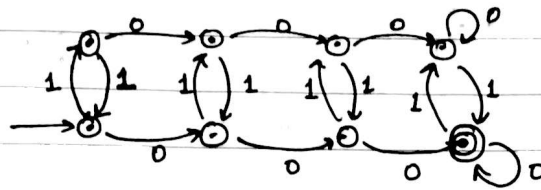


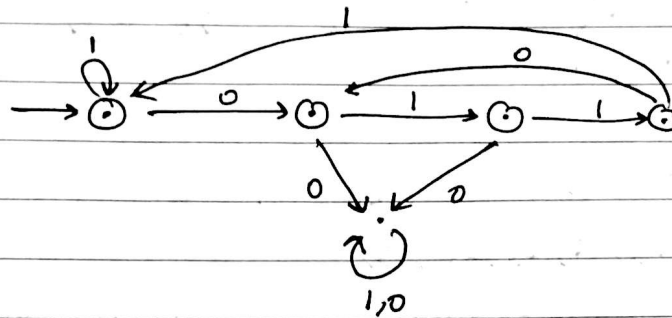
ToC : Problem set 1.

Q1

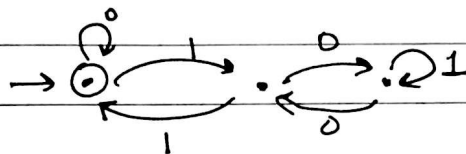
a)



b)



Q2



A' will be same as A . and because it is symmetric, so yes it will accept the set of binary whose reverse's decimal is divisible by 3, \therefore reverse EA.

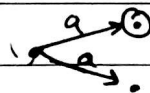
Q3

By definition of acceptance, x is accepted if $\delta(a, x) \in F$.

Since accepting and non-accepting states are interchanged

Implies all x that were accepted earlier, will not be accepted now and all that were not accepted will be accepted now, Hence language is complemented.

For NFA,



Due to this case, it is false

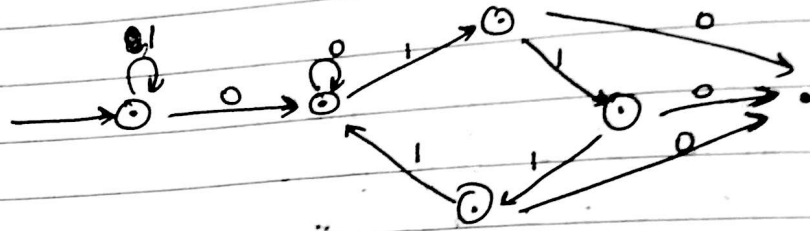
$\therefore a$ will be accepted in both cases

Q-4

Atleast one 1 and after first 1, every ~~00000~~ is followed immedi-
by atleast one 1.

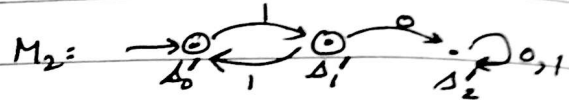
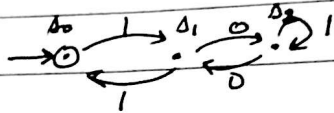
occurrence of ~~00000~~ ≥ 5 0's is followed
by atleast one 1.

Q-7.



Q-8.

$M_1:$
(L_1)



M_3 s.t. $L(M_3) = L_1 \cap L_2$
 $\hookrightarrow (Q_3, \Sigma, \delta_3, \Delta_3, F_3)$

$Q_3 = Q_1 \times Q_2$, $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$

$\Delta_3 = \{ \Delta_0, \Delta_0' \}$, $F_3 = F_1 \times F_2 = \{ (\Delta_0, \Delta_0'), (\Delta_0, \Delta_1') \}$

Q-5, 6, 9, 10

Problem Set 2

- Q-1. a) $(1^+ + 01^+)^*(00(1^+ + 01^+)^* + \epsilon)$
 or $1^*(011^*)^*(\epsilon + 00)(11^*0)^*1^*$
 b) $(0+1)^*(0(0+1)^*1 + 1(0+1)^*0)(0+1)^*$
 or $(0+1)^*01(0+1)^* + (0+1)^*10(0+1)^*$

- Q-2. a) ~~Set of all binary strings with at least one 11~~
~~repeating pattern after a given 11.~~
 b) ~~Every 1 is followed by at least one 0.~~ or null string.
 c) ~~At least 2 zeros~~ $3n+2$ zeros, $n \geq 0$.

- Q-3. a) $A = \{0^{n^2} \mid n \geq 0\}$
 $w = 0^{n^2} \Rightarrow |w| \geq n$ $xyz = w$, $|xy| \leq n$
 $|xy^2z| = |xyz| + |y| = n^2 + |y| \leq n^2 + n < (n+1)^2$
 $\Rightarrow xy^2z \notin A$

- b) $A = \{0^{n!} \mid n \geq 0\}$
 $w = 0^{n!} \Rightarrow |w| = n! \geq n$, $xyz = w$ $|xy| \leq n$
 $|xy^2z| = |xyz| + |y| \leq n! + n < (n+1)!$
 $\Rightarrow xy^2z \notin A$

- Q-8. a) ~~$A = \{0^n 1^m \mid m \leq n\}$~~ , ~~$B = \{0^k 1^k \mid k \geq 0\}$~~
 ~~$A \cap B = B \rightarrow$ not regular~~
 ~~$\Rightarrow A \& B$ are not regular (Closure property of regular sets under Intersection)~~
 ~~$\Rightarrow A$ is not regular~~

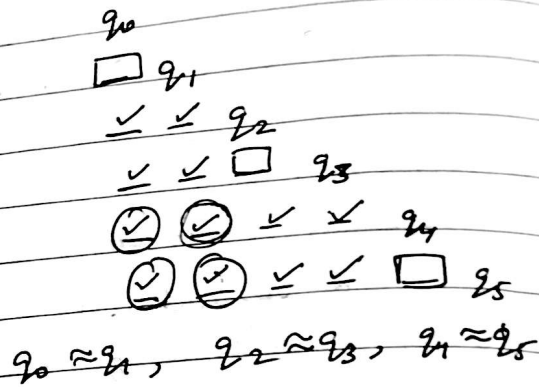
- b) $A = \{0^n 1^m \mid m \neq n\}$, $B = \{0^k 1^k \mid k \geq 0\}$
 ~~$A \cap B = \emptyset$~~ $\bar{A} \cap 0^* 1^* = B \rightarrow$ not regular $\Rightarrow \bar{A}$ is not regular
 \downarrow
 regular \Downarrow
 A is not regular

Q7.

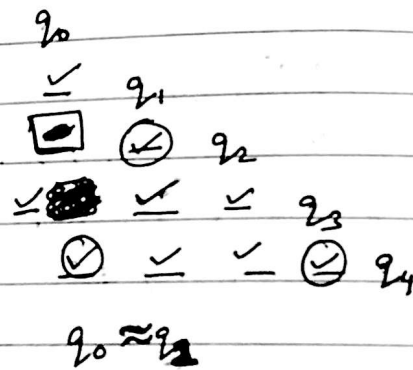
$$1^* 0 (11^* 0)^* 0 (0+1)^*$$

Q6.

	0	1
q ₀	q ₁	q ₃
q ₁	q ₀	q ₂
F q ₂	q ₄	q ₅
F q ₃	q ₄	q ₅
q ₄	q ₅	q ₄
q ₅	q ₅	q ₅



	0	1
q ₀	q ₁	q ₂
F q ₁	q ₁	q ₄
q ₂	q ₁	q ₂
F q ₃	q ₁	q ₂
q ₄	q ₁	q ₃



Q8. (a)

$$A = \{0^n 1^m \mid m \geq n \geq 0\}$$

Assumption:

Assume it to be regular

Construct its DFA^(M) and reverse all edges. Let it be ~~A~~

let it be M'. Then since M is DFA, M' is also DFA.

Let A' be ~~regular~~ self associated with M'.

$$A' = \{1^m 0^n \mid m \geq n \geq 0\}$$

Construct a homomorphism with h(0)=1 & h(1)=0.

$$A'' = h(A) = \{0^m 1^n \mid m \geq n \geq 0\}$$

Since A and A' are regular $\Rightarrow A \cap A''$ is also regular (Closure Property).

$$A \cap A' = \{0^n 1^n \mid n \geq 0\} = B$$

But B is not regular \Rightarrow Assumption is false $\Rightarrow A$ is not regular

Q5 b)

Co

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To show:

Q5 b) Let DFA M be associated with L . s.t. $L(M) = L$

Construct DFA M' s.t. for every edge $a_i a_j$ in M ,
insert a node in between and make it $a_i \Delta a_j$
and change final node to Δ

Every edge a_i in M is replaced by $a_i \Delta$ (by inserting a state in between). It can be easily argued that M' is also a DFA. $\Rightarrow L_1$ is and $L_1 = L(M') \Rightarrow L_1$ is regular.

a) Using Pumping's Lemma:

$$L_1 = \{w \mid w = a_1 \Delta a_2 \Delta a_3 \Delta \dots \Delta a_n \Delta^n \text{ where } a_1, a_2, \dots, a_n \in L\}$$

$$\text{Take } w = a_1 \Delta a_2 \Delta^2 a_3 \Delta^3 \dots \Delta^n \Delta^n$$

$$|w| \geq n. (\exists n)$$

To show $\forall n; y, z$ with $xyz = w$. $\exists k$ s.t. $xy^kz \notin L_1$

$$\text{Let } y = a_i \text{ then } xy^2z \notin L_1$$

$$y = \Delta, \quad xy^2z \notin L_1$$

$$y = a_i \Delta, \quad xy^2z \notin L_1$$

$$y = a_i \Delta^k, \quad xy^2z \notin L_1$$

So, $\nexists \forall y$ $xy^kz \notin L_1$ for some k

Q-4

Problem set 3

Q1. (ii) $L = \{w \in \{0,1\}^* \mid |w| \text{ is odd and middle symbol is } 0\}$.

CFG, $G = (N, \Sigma, P, S)$

$N = \{S\}, \Sigma = \{0,1\}$.

P include $S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$

(i) $S \rightarrow 0S \mid 0S1 \mid 1S0 \mid \epsilon$. $\rightarrow \#_0(w) \geq \#_1(w)$
 ~~$S \rightarrow 0S \mid 00S1 \mid 01S0 \mid 0S10 \mid 1S00 \mid \epsilon$~~ $\rightarrow \#_0(w) > \#_1(w)$. X.

$S \rightarrow 0A \mid A0 \mid 0$

$A \rightarrow 0A \mid 0A1 \mid 1A0 \mid \epsilon$. $\rightarrow \#_0(A) \geq \#_1(A)$

Q2 (i) $S \rightarrow AC' \mid A'C \mid \epsilon$

$A \rightarrow aAb \mid \epsilon$

$C \rightarrow bCc \mid \epsilon$

$C' \rightarrow cC' \mid \epsilon$

$A' \rightarrow aA' \mid \epsilon$.

(ii) $S \rightarrow aaqA \mid \epsilon$

$A \rightarrow aAb \mid Ab \mid \epsilon$.

(iii) $S \rightarrow ase \mid bBc \mid \epsilon$

$B \rightarrow bBc \mid \epsilon$.

Q-3

$L(G) = \{w \mid \#_a(w) = \#_b(w)\}$.