CS638 Mid-Semester Examination

SAHIL DHULL

TOTAL POINTS

60 / 60

QUESTION 1

1 Question 1 10 / 10

✓ **- 0 pts** Correct

- 1 pts (i) incorrect

- 1 pts (ii) incorrect

- 1 pts (iii) incorrect

- 1 pts (iv) incorrect

- 2 pts (v) incorrect

- 2 pts (vi) incorrect

- 2 pts Rabin Automaton incorrect

-0.5: The transition from s0 to s1 should have label o1||o2

QUESTION 2

2 Question 2 10 / 10

✓ **- 0 pts** Correct

- 1 pts (a)(i) incorrect

- 1 pts (a)(ii) incorrect

- 1 pts (a)(iii) incorrect

- 1 pts (a)(iv) incorrect

- 1 pts (a)(v) incorrect

- 1 pts (a)(vi) incorrect

- 1 pts (b)(i) incorrect

- 1 pts (b)(ii) incorrect

- 1 pts (b)(iii) incorrect

- 1 pts (b)(iv) incorrect

QUESTION 3

3 Question 3 10 / 10

- 0 pts Correct

+ 1 Point adjustment

Fantastic. This has been very nicely done.

QUESTION 4

4 Question 4 10 / 10

✓ **- 0 pts** Correct

- 2 pts Wrong T'

- 4 pts Wrong T''

- 2 pts Wrong X'

- 2 pts Wrong X''

QUESTION 5

5 Question 5 10 / 10

✓ **- 0 pts** Correct

QUESTION 6

6 Question 6 10 / 10

✓ **- 0 pts** Correct

- 1 pts (a)(i) incorrect

- 1 pts (a)(ii) incorrect

- 1 pts (a)(iii) incorrect

- 1 pts (a)(iv) incorrect

- 1 pts (a)(v) incorrect

- 1 pts (a)(vi) incorrect

- 2 pts (b)(i) incorrect

- 2 pts (b)(ii) incorrect

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

Instructions: *Total:* **60 marks**

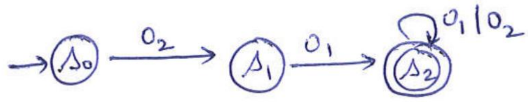
1. This question paper contains a total of 20 pages (20 sides of paper). Please verify.
2. Write your name, roll number, department on **every side of every sheet** of this booklet.
3. Write final answers **neatly with a pen** in the given boxes.

Problem 1. (10 points)
Draw the Buchi Automata for the following LTL formulas over atomic propositions o_1 and o_2.

(i) $\bigcirc o_1$
(ii) $\Diamond o_1$
(iii) $\Box o_1$
(iv) $o_1 \cup o_2$
(v) $\Box\Diamond o_1$
(vi) $\neg\Box\Diamond o_1$

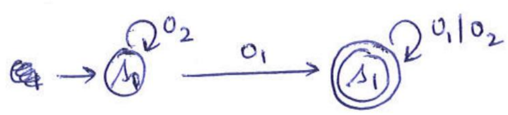
Indicate which B*ü*chi Automaton is deterministic and which one in non-deterministic. In case a B*ü*chi Automaton is non-deterministic, provide a deterministic Rabin Automaton for the LTL formula.

(i)



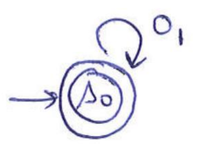
Deterministic

(ii)



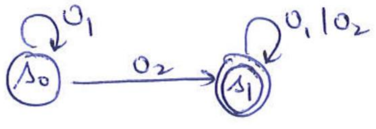
Deterministic

(iii)



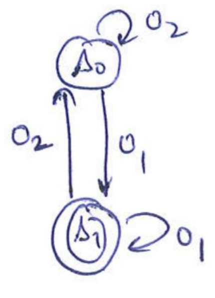
Deterministic

(iv) $o_1 \mathcal{U} o_2$



Deterministic

(v) $\Box\Diamond o_1$



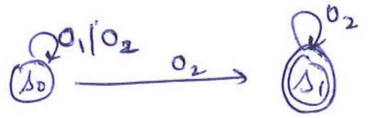
Deterministic

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

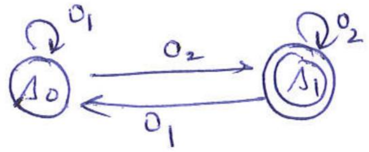
Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

(vi) $\neg \Box \Diamond o_1$



Non-deterministic

Deterministic Rabin Automata:



$F = \{ (s_1, s_0) \}$

ie $G_1 = \{s_1\}$, $B_1 = \{s_0\}$.

Name:

Roll No:
e.g. 170001

Dept.:
e.g. CSE

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination

Date: February 23, 2018
Duration: 2 hours

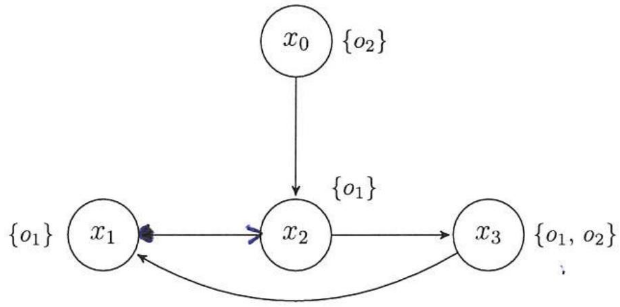
Name: Sahil Dhull

Roll No: 160607 (e.g. 170001)

Dept.: CSE (e.g. CSE)

Problem 2. (10 points)

(a) Consider the finite transition system shown in the figure below:



Indicate for each of the following LTL formulae the set of states for which the formula is fulfilled:

(i) $\bigcirc o_1$
(ii) $\bigcirc \bigcirc \bigcirc o_1$
(iii) $\square o_2$
(iv) $\square \Diamond o_1$
(v) $\square (o_2 \cup o_1)$
(vi) $\Diamond (o_1 \cup o_2)$

(b) Consider the following four algorithms to compute the largest satisfying region for an LTL formula ϕ in a transition system T:

(i) Apply model-checking algorithm at each state of T.
(ii) Apply model-checking algorithm to each state of the simulation quotient of T.
(iii) Apply model-checking algorithm to each state of the bisimulation quotient of T.
(iv) Apply formula guided analysis.

Compare the strength and weakness of the four algorithms

~~(a)~~
~~(i) $\phi = \bigcirc o_1$ $X_T^\phi = \{x_0, x_2, x_3\}$~~
~~(ii) $\phi = \bigcirc\bigcirc\bigcirc o_1$ $X_T^\phi = \{x_0\}$.~~
~~(iii) $\phi = \square o_2$ $X_T^\phi = \emptyset$~~
~~(iv) $\phi = \square \Diamond o_1$~~

(a) (i) $\phi = \bigcirc o_1$ $X_T^\phi = \{x_0, x_1, x_2, x_3\}$

(ii) $\phi = \bigcirc\bigcirc\bigcirc o_1$ $X_T^\phi = \{x_0, x_1, x_2, x_3\}$

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

(iii) $\phi = \Box o_2$ $\quad X_T^\phi = \phi$

(iv) $\phi = \Box \Diamond o_1$ $\quad X_T^\phi = \{x_0, x_1, x_2, x_3\}$

(v) $\phi = \Box(o_2 \mathcal{U} o_1)$ $\quad X_T^\phi = \{x_0, x_1, x_2, x_3\}$

(vi) $\phi = \Diamond(o_1 \mathcal{U} o_2)$ $\quad X_T^\phi = \{x_0, x_1, x_2, x_3\}$.

(b)

(i) Strength : Always gives the correct largest satisfying region.

Weakness : If no. of states is large, takes a lot of time and computation

(ii) Strength : Computes in less time even for large transition systems

Weakness : Gives an under-approximation of largest satisfying region.

(iii) Strength : Gives correct largest satisfying region. in less time

Weakness : Involves lots of redundant computations and some states are partitioned without need.

(iv) Strength : Gives the correct region, bypassing redundant calculation and partitions

Weakness : May take much time depending upon formula

Name:

Roll No:
e.g. 170001

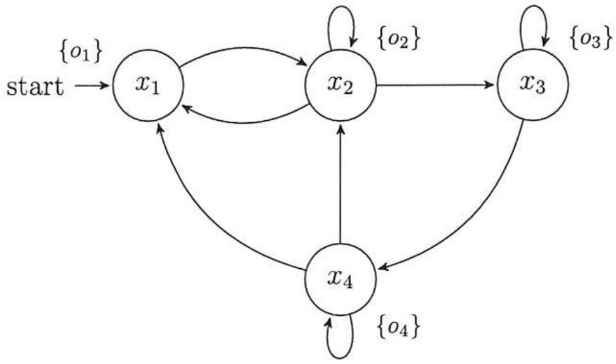
Dept.:
e.g. CSE

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

Problem 3. (10 points)
Consider the following finite transition system with no input :



: T

Apply the counterexample guided pruning methodology to generate all transition systems all states of which satisfy the LTL formula $\Diamond\Box o_3$.

$\phi = \Diamond \Box o_3$

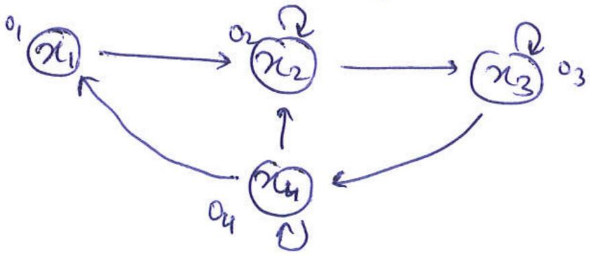
Let the initial TS be T.

(i) A counter example is $(o_1 o_2)^\omega$.

2 transitions can be removed (a) $x_1 \to x_2$ or (b) $x_2 \to x_1$

Removing a results in blocking TS. so we remove (b) to form T_1

T_1 :

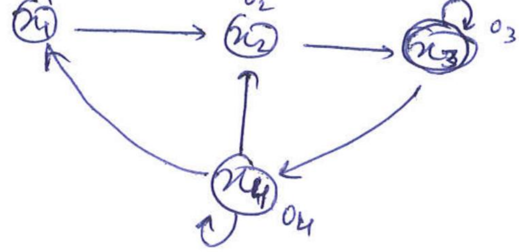


(ii) Counter example: $o_1\, o_2^\omega$.

Transitions that can be removed $\underbrace{x_1 \to x_2}_{\text{Blocking}}$, ~~and~~ or $x_2 \to x_2$

So, it becomes T_2 with no self loop on x_2.

T_2 :



Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

(iii) $o_1 (o_2\, o_3\, o_4)^\omega$

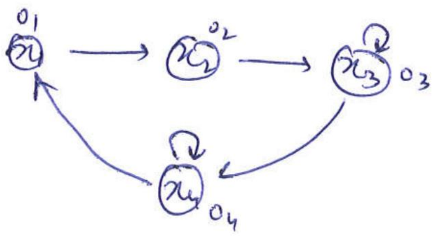
3 transitions can be removed:

a) $x_2 \rightarrow x_3$ × Blocking TS

b) $x_3 \rightarrow x_4$ ✓ Results in x_4 being out $\longrightarrow T_3$

c) $x_4 \rightarrow x_2$ ✓ $\longrightarrow T_4$

T_3:

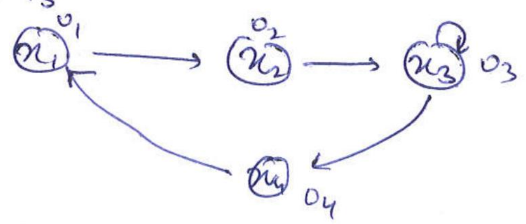

T_4:


$T_3 \models \phi$.

(iv) For T_4, a counterexample is $o_1\, o_2\, o_3\, o_4^\omega$

which can be removed by removing $x_4 \rightarrow x_4$

T_4 becomes T_5

T_5 :


(v) For T_5, counter example: $(o_1\, o_2\, o_3\, o_4)^\omega$

4 ways :

a) $x_1 \rightarrow x_2$ × Blocking TS

b) $x_2 \rightarrow x_3$ × Blocking TS

c) $x_3 \rightarrow x_4$ $\longrightarrow T_3$

d) $x_4 \rightarrow x_1$ × Blocking TS.

So, only $T_3 \models \phi$ and no other TS formed from pruning satisfies the formula $\phi = \Diamond \Box o_3$

Name: Sahil Dhull

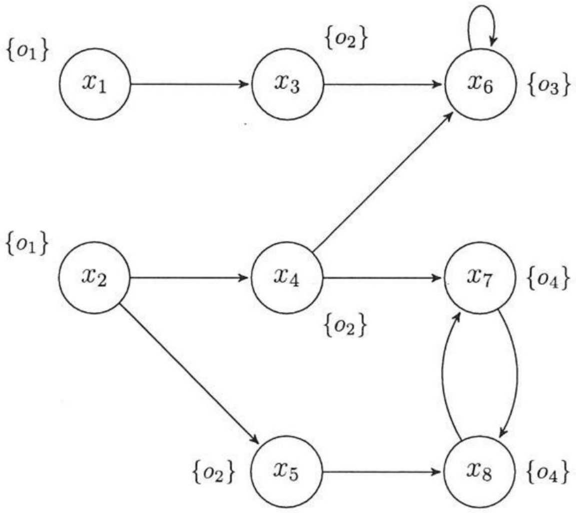
Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

So, all transition systems satisfying LTL formula $\phi = \Diamond \Box o_3$ are

Name: Sahil Dhull

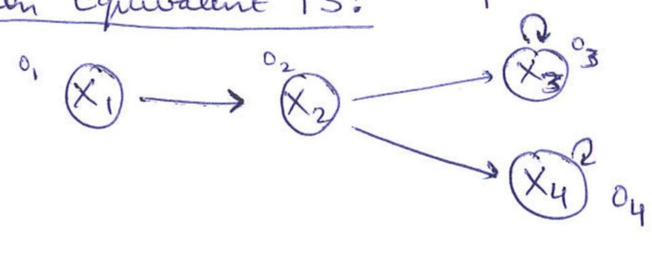
Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

Problem 4. (10 points)
Consider the following finite transition system T with no input :



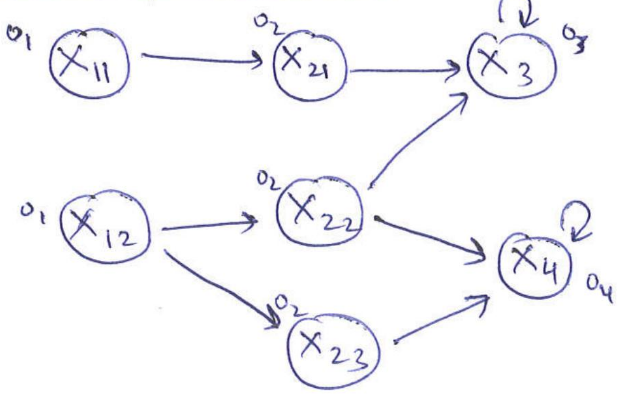
Construct a Simulation equivalent transition system T' and a bisimulation equivalent transition system T''. Consider the temporal logic formula $(o_1 \vee o_2) \cup o_3$. Analyze T' and determine which states in T, denoted by X', will satisfy the temporal logic formula. Perform the same analysis on T'' and determine the set of states X'' that will satisfy the temporal logic formula.

Simulation Equivalent TS: T'



$con(X_1) = \{x_1, x_2\}$
$con(X_2) = \{x_3, x_4, x_5\}$
$con(X_3) = \{x_6\}$
$con(X_4) = \{x_7, x_8\}$

Bisimulation Equivalent TS: T''



$con(X_{11}) = \{x_1\}$
$con(X_{12}) = \{x_2\}$
$con(X_{21}) = \{x_3\}$
$con(X_{22}) = \{x_4\}$
$con(X_{23}) = \{x_5\}$
$con(X_3) = \{x_6\}$
$con(X_4) = \{x_7, x_8\}$

 Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001)   Dept.: CSE (e.g. CSE)

$\phi = (o_1 \vee o_2) \,\mathcal{U}\, o_3$

In simulation equivalent TS:

$S_T = \{X_3\}$, $S_\perp = \{X_4\}$

$\Rightarrow$ $X_2 \in S_?$ $\Rightarrow$ X_1 also belongs in $S_?$

So, set of states that satisfy ϕ are $con(X_3)$

$X' = \{x_6\}$.

In Bisimulation equivalent TS:

$S_T = \{X_{11}, X_{21}, X_3\}$

$S_\perp = \{X_{23}, X_4\}$

and $S_? = \{X_{12}, X_{22}\}$

So, set of states that satisfy ϕ are $con(X_{11}) \cup con(X_{21}) \cup con(X_3)$

$\Rightarrow$ $X'' = \{x_1, x_3, x_6\}$.

Name:

Roll No:
e.g. 170001

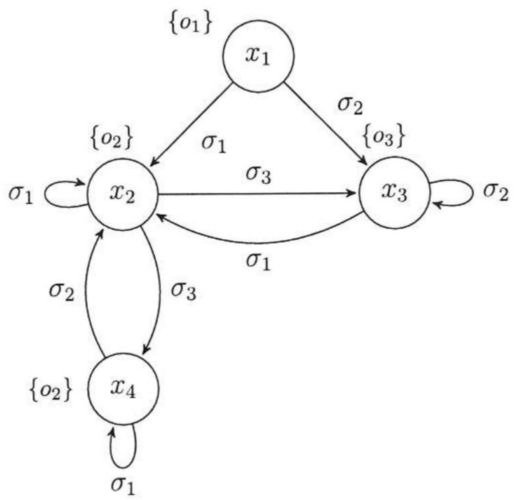
Dept.:
e.g. CSE

 Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

Problem 5. (10 points)
Consider the following finite transition system T with the set of inputs $\Sigma = \{\sigma_1, \sigma_2, \sigma_3\}$ and the set of outputs $O = \{o_1, o_2, o_3\}$:

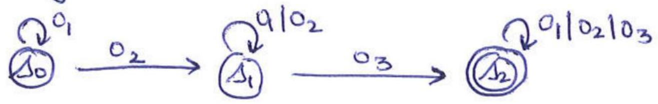


For the LTL formula $(o_1 \cup o_2) \wedge \Diamond o_3$, construct the control automaton. Show all the steps involved in this construction clearly.

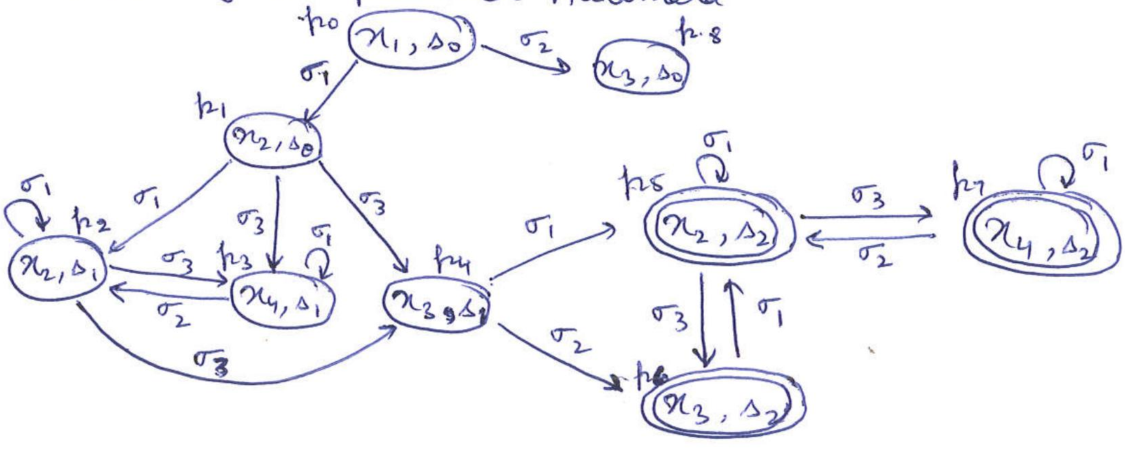
$\phi = (o_1 \mathcal{U} o_2) \wedge \Diamond o_3.$

Steps:

1. Constructing the Buchi Automata (B_ϕ):



2. Forming the product Automata



$F_P = \{p_5, p_6, p_7\}$

Name: Sahil Dhull
Roll No: 160607 (e.g. 170001)
Dept.: CSE (e.g. CSE)

3) Computing $A^+(F_P)$

$A^+(F_P) = \{p_4, p_5, p_6, p_7\}$

$\Rightarrow W_P = \{p_4, p_5, p_6, p_7\}$

4) $W_{P_0} = W_P \cap S_{P_0}$ where S_{P_0} = ~~Po~~ starting state.

$= \phi$

$\Rightarrow$ No initial state can ~~guara~~ gaurantee a visit to final states

$\Rightarrow X_f^\phi = \phi$.

Hence no control automata.

which is clear from the fact that on σ_3, x_2 can go to x_4 or x_3.

Hence inputs can't control the Transition System

Hence No control automata T_C is possible for the given formula $\phi = (o_1 \mathcal{U} o_2) \wedge \Diamond o_3$.

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination

Name:

Roll No: e.g. 170001

Dept.: e.g. CSE

Date: February 23, 2018
Duration: 2 hours

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001) Dept.: CSE (e.g. CSE)

Problem 6. (10 points)
(a) Give an example of the following discrete-time systems, with the state vector $x(k) \in \mathbb{R}^2$ and control input $u \in \mathbb{R}$.

(i) An uncertain-parameter piecewise affine control system.
(ii) A fixed parameter piecewise affine control system.
(iii) An uncertain-parameter piecewise affine autonomous system.
(iv) An autonomous additive uncertainty piecewise affine system.
(v) A fixed parameter linear autonomous system.
(vi) A switched linear system with two modes.

(b) Consider an autonomous additive uncertainty piecewise affine system $\mathcal{S} : x(k+1) = A_l x(k) + c_l$ where $l \in L$, L being a finite index set, and $c_l \in P_l^c$, P_l^c being a polytopic set. Given a polytope X_l with the set of vertices $V = \{v_1, \ldots, v_n\}$, provide a mathematical procedure to compute $Post(X_l)$ where the dynamics in the region l is given by $\mathcal{S}$.

Also, provide a mechanism to decide whether there exists a transition between two polytopes X_l and $X_{l'}$ when the dynamics in the polytopic region X_l is $\mathcal{S}$.

(b) $x(k+1) = A_l x(k) + c_l \quad , l \in L.$

$V = \{v_1, \ldots v_n\}$. → vertices of X_l.

Procedure:

1) Compute

$v_i' = A_l v_i + c_l \quad \forall\, c_l \in P_l^c.$

So, the new set of vertices

$V' = \{v_1', v_2' \;--- \; v_n'\}$

will be formed for some new polytopic region X_l'

2) We have the region X_l'.

for all $l'' \in L$,

if $X_l' \cap X_{l''} \neq \phi$

$Post(X_l) = Post(X_l) \cup \{l''\}.$

end if

end for

This procedure will compute the set $Post(X_l)$.

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001)

Dept.: CSE (e.g. CSE)

Method to check if there exists a transition from X_ℓ to $X_{\ell'}$.

1 ~~for~~ given X_ℓ, $X_{\ell'}$

Compute Post(X_ℓ).

2. If $\ell' \in$ Post(X_ℓ)

then $\ell' \in \delta(\ell)$

ie if ℓ' is in post(X_ℓ), then there exists a transition from ℓ ~~to~~ ℓ'.

(a) (i) $x(k+1) = A_\ell x(k) + B_\ell u(k) + C_\ell$

$B_\ell = \begin{bmatrix} \ell \\ 1 \end{bmatrix}$, $C_\ell = \begin{bmatrix} 1 \\ a\ell \end{bmatrix}$, $a \in [0,1]$

$A_\ell = \begin{bmatrix} a\ell & 1 \\ 1 & a\ell \end{bmatrix}$ $\quad x \in R^2$, $u \in R$.

$x \in X_\ell$ if $\ell - 1 <$ y-coordinate $(x) < \ell$.

(ii) $x(k+1) = A_\ell x(k) + B_\ell u(k) + C_\ell$.

$A_\ell = \begin{bmatrix} \ell & 1 \\ 1 & \ell \end{bmatrix}$ $B_\ell = \begin{bmatrix} \ell \\ 1 \end{bmatrix}$, $C_\ell = \begin{bmatrix} 1 \\ \ell \end{bmatrix}$

And regions defined are same as above.

(iii) $x(k+1) = A_\ell x(k) + C_\ell$

$A_\ell = \begin{bmatrix} a\ell & 1 \\ 1 & a\ell \end{bmatrix}$, $C_\ell = \begin{bmatrix} 1 \\ a\ell \end{bmatrix}$ $\quad a \in [0,1]$

· Regions defined are same as in (i)

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name: Sahil Dhull

Roll No: 160607 (e.g. 170001)   Dept.: CSE (e.g. CSE)

(iv) $$x(k+1) = A_l x(k) + C_l$$

$$A_l = \begin{bmatrix} l & 1 \\ -1 & l \end{bmatrix}, \quad C_l = \begin{bmatrix} 1 \\ a l \end{bmatrix} \qquad a \in [0,1]$$

(v) $$x(k+1) = A_l x(k)$$

$$A_l = \begin{bmatrix} l & 1 \\ 1 & l \end{bmatrix}$$

(vi) ~~Let~~ $x(k+1) = A_\gamma x(k)$.

~~Let~~ There are 2 modes let γ_1, γ_2.

if y-coordinate < 10, γ_1 mode

else γ_2 mode.

$$A_{\gamma_1} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad A_{\gamma_2} = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

Indian Institute of Technology Kanpur
CS638 Formal Methods for Robotics & Automation
Mid-Semester Examination
Date: February 23, 2018
Duration: 2 hours

Name:

Roll No: e.g. 170001 **Dept.:** e.g. CSE

BLANK SPACE: Any answers written here will be left ungraded.
No exceptions.
You may use this space for rough work.

BLANK SPACE: Any answers written here will be left ungraded.
No exceptions.
You may use this space for rough work.

Name:

Roll No:
e.g. 170001

Dept.:
e.g. CSE