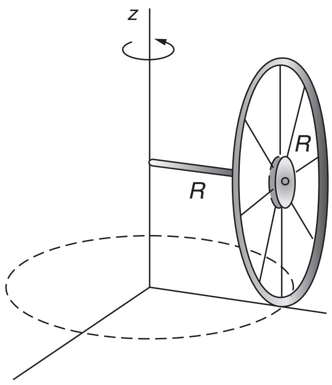
Problems

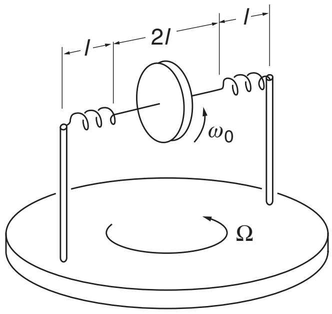
*For problems marked *, refer to page 523 for a hint, clue, or answer.*



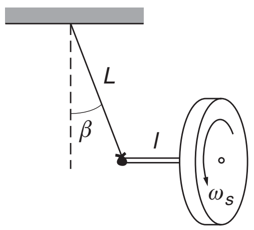
8.1 *Rolling hoop*
A thin hoop of mass M and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center, as shown. The hoop circles around the z axis with angular speed Ω.

(a) What is the instantaneous angular velocity $\boldsymbol{\omega}$ of the hoop?

(b) What is the angular momentum $\mathbf{L}$ of the hoop? Is $\mathbf{L}$ parallel to ω? (The moment of inertia of a hoop for an axis along its diameter is $\frac{1}{2}MR^2$.)

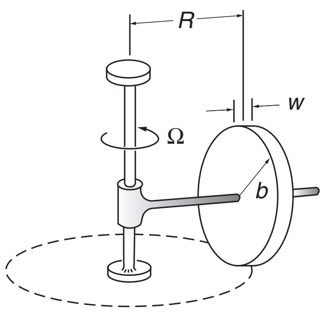


8.2 *Flywheel on rotating table*
A flywheel of moment of inertia I_0 rotates with angular velocity ω_0 at the middle of an axle of length $2l$. Each end of the axle is attached to a support by a spring which is stretched to length l and provides tension T. You may assume that T remains constant for small displacements of the axle. The supports are fixed to a table that rotates at constant angular velocity Ω, where $\Omega \ll \omega_0$. The center of mass of the flywheel is directly over the center of rotation of the table. Neglect gravity and assume that the motion is completely uniform so that nutational effects are absent. The problem is to find the direction of the axle with respect to a straight line between the supports.



8.3 *Suspended gyroscope*
A gyroscope wheel is at one end of an axle of length A. The other end of the axle is suspended from a string of length B. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_0. Its spin angular velocity is ω_s. Neglect the masses of the shaft and string.

Find the angle β that the string makes with the vertical. Assume that β is so small that approximations like $\sin\beta \approx \beta$ are justified.



8.4 *Grain mill**
In an old-fashioned rolling mill, grain is ground by a disk-shaped millstone that rolls in a circle on a flat surface, driven by a vertical shaft. Because of the stone's angular momentum, the contact force with the surface is greater than the weight of the wheel.

Assume that the millstone is a uniform disk of mass M, radius b, and width w, and that it rolls without slipping in a circle of radius R with angular velocity Ω. Find the ratio of the contact force with respect to the surface to the weight of the stone.

8.5 *Automobile on a curve*

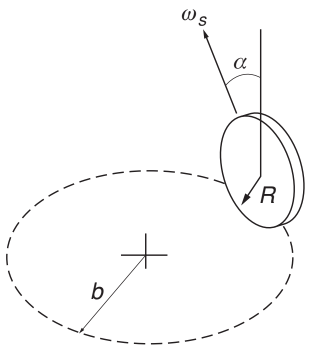
When an automobile rounds a curve at high speed, the loading (weight distribution) on the wheels is markedly changed. For sufficiently high speeds the loading on the inside wheels goes to zero, at which point the car starts to roll over. This tendency can be avoided by mounting a large spinning flywheel on the car.

(*a*) In what direction should the flywheel be mounted, and what should be the sense of rotation, to help equalize the loading? (Be sure that your method works for the car turning in either direction.)

(*b*) Show that for a disk-shaped flywheel of mass m and radius R, the requirement for equal loading is that the angular velocity ω of the flywheel is related to the velocity of the car V by

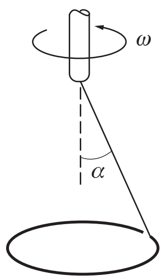
$$\omega = 2V\frac{Mb}{mR^2},$$

where M is the total mass of the car and flywheel, and b is the height of the center of mass of the car (including the flywheel) above the road. Assume that the road is unbanked.



8.6 *Rolling coin**

A coin of radius b and mass M rolls on a horizontal surface at speed V. If the plane of the coin is vertical the coin rolls in a straight line. If the plane is tilted, the path of the coin is a circle of radius R. Find an expression for the tilt angle of the coin α in terms of the given quantities. (Because of the tilt of the coin the circle traced by its center of mass is slightly smaller than R but you can ignore the difference.)



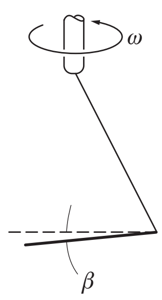
8.7 *Suspended hoop*

A thin hoop of mass M and radius R is suspended from a string through a point on the rim of the hoop. If the support is turned with high angular velocity ω, the hoop will spin as shown, with its plane nearly horizontal and its center nearly on the axis of the support. The string makes angle α with the vertical.

(*a*) Find, approximately, the small angle β between the plane of the hoop and the horizontal. Assume that the center of mass is at rest.

(*b*) Find, approximately, the radius of the small circle traced out by the center of mass about the vertical axis.

(*c*) Find a criterion for the validity of the assumption that motion of the center of mass can be neglected. (With skill you can demonstrate this motion with a rope. It is a favorite cowboy lariat trick.)



8.8 *Deflected hoop*

A child's hoop of mass M and radius b rolls in a straight line with velocity V. The top of the hoop is given a light tap with a stick at