MTH628 Weekly Recap 1

Harshit Bisht

January 13, 2019

1 Class 1

The course will largely concern with 3 topics:

1. Knot Theory
2. The Fundamental Group
3. Simplicial Homology

Those needing a background in topology are recommended to check out [1] and [2]. For those interested in reading further ahead, have a look at [3].

The class opened with definitions of two topological spaces being **homeomorphic**, followed by introducing the **Euler's invariant** and defining **topological invariants**. This was followed by a short proof of **the statement that there exist only 5 regular polyhedra**.

The concepts of knots was introduced along with defining the notion of a **knot invariant**, finishing with the **elementary knot moves** that can be used to manipulate them.

2 Class 2

The class started by defining a knot as a continuous embedding of the circle into 3-dimensional Euclidean space. The definition of two knot invariants in the **crossing number** $c(K)$ and **unknotting number** $u(K)$ followed, along with the definition and computation of Alexander and Jones polynomial of some simple knots. The class ended with a short result relating the Jones polynomial of a knot and its mirror image.

EXERCISE: Compute the Alexander polynomials for all knots with crossing numbers ≤ 6.

Those of you who are interested in the topic might like to browse ahead in [4],

or have a look at [5] (mentioned in class) that documents the first 1.7 million knots. You can find some interesting videos containing animations for elementary knot moves such as `https://www.youtube.com/watch?v=aqyyhhnGraw` and `https://www.youtube.com/watch?v=k9ub2mNyd9M` on YouTube. There are software tools available for rendering knot graphics, and you can have a look at `https://knotplot.com/` or similar others to explore.

3 Motivation/Further Reading

Knot theory and similar areas have a remarkable number of applications in many domains of science and engineering. Start exploring `https://en.wikipedia.org/wiki/Knot_theory#See_also` if you wish to find some, including some remarkable ones in `https://en.wikipedia.org/wiki/Molecular_knot` (chemical molecules that arrange themselves in the shape of knots) and Quantum Knots (outlined in [6]).

References

[1] Munkres, James R. Topology: a first course. Vol. 23. Englewood Cliffs, NJ: Prentice-Hall, 1975.

[2] Armstrong, Mark Anthony. Basic topology. Springer Science & Business Media, 2013.

[3] Munkres, James R. Elements of algebraic topology. CRC Press, 2018.

[4] Adams, Colin Conrad. The knot book: an elementary introduction to the mathematical theory of knots. American Mathematical Soc., 2004.

[5] Hoste, Jim, Morwen Thistlethwaite, and Jeff Weeks. "The first 1,701,936 knots." The Mathematical Intelligencer 20.4 (1998): 33-48.

[6] Hall, David S., et al. "Tying quantum knots." Nature Physics 12.5 (2016): nphys3624.