MTH628 Weekly Recap 2

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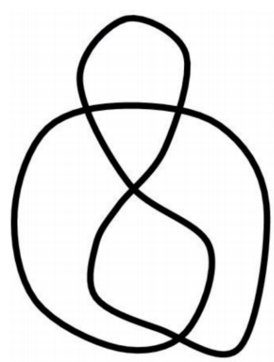
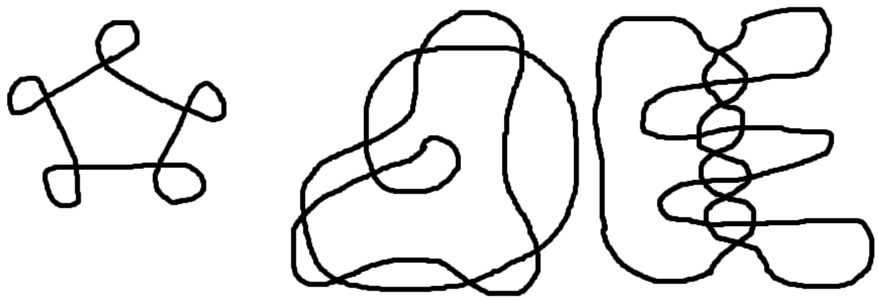
A game of knots



Consider the knot projection as shown above. Find a friend and play the following game with them: similar to tic-tac-toe, choose an intersecting crossing point and make one of the strands go over or under the other in turns. Make one person's goal to make the final diagram be the unknot (the unknotter), and the other person's goal be to finish at any other knot/link (the knotter). Play twice, taking the first turn once each.

1. Who won the game? Do you think it had anything to do with who started first? If yes, can you formalise the argument that outlines the connection?

2. Try playing the game for any other arbitrary squiggles you can draw on paper. Are you able to generalize the patterns you observed to some properties of the diagrams? If you are unable to decide on what squiggles to play with, use the following:



3. Can you isolate any families of diagrams where the unknotter or the knotter can always win provided they can choose who goes first?

(This activity has been adapted from [3]. Please go through it for a different approach to knot theory!)

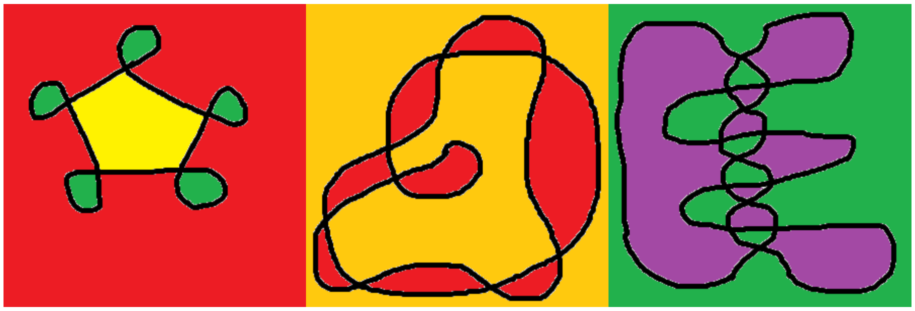
Class 3

Class 3 introduced another knot invariant in the HOMFLY polynomial (see [4] for the original paper). The class also discussed linking number and tricolorability as knot invariants, after introducing Reidemeister moves for working with knot diagrams rather than knots themselves.

Class 4

Class 4 talked about Reidemeister moves (For a proof of them preserving knot equivalence, please see [2]). After formally introducing tricolorability and p-colorability, the class looked at a proof for them being knot invariants via the Reidemeister moves. The class also introduced braid groups and the intuition behind their connection with knots.

EXERCISE: It is always a good idea to look for new ideas for knot invariants. Consider a knot diagram and try coloring it so each neighbouring component has a different color. Is the minimum number of colors you need over all possible knot diagrams a knot invariant? Consider the following colourings for intuition (Please not that these figures are not knot diagrams. You need to make the crossings go over or under for that):



For exploring more about knots, feel free to browse any of the recommended books or going through The Knot Atlas! (clickable link in [5])

References

[1] Adams, Colin Conrad. The knot book: an elementary introduction to the mathematical theory of knots. American Mathematical Soc., 2004.

[2] Murasugi, Kunio. Knot theory and its applications. Springer Science & Business Media, 2007.

[3] Johnson, Inga, and Allison Henrich. An Interactive Introduction to Knot Theory. Courier Dover Publications, 2017.

[4] Freyd, Peter, et al. "A new polynomial invariant of knots and links." Bulletin of the American Mathematical Society 12.2 (1985): 239-246.

[5] The Knot Atlas.