CS698O Quiz 4 Solution

February 5, 2020

1 QUIZ 4

The sequence given is PLAY, STUDY, PLAY. So, $x_1 =$ PLAY, $x_2 =$ STUDY and $x_3 =$ PLAY.

1.1 $viterbi(1, c_k)$

$$\begin{aligned} viterbi(1, c_1) &= viterbi(1, SUNNY) \\ &= P(SUNNY|START) \times P(PLAY|SUNNY) \\ &= 0.8 \times 0.4 \\ &= 0.32 \end{aligned}$$

$$\begin{aligned} viterbi(1, c_2) &= viterbi(1, RAIN) \\ &= P(RAIN|START) \times P(PLAY|RAIN) \\ &= 0.2 \times 0.1 \\ &= 0.02 \end{aligned}$$

Refer to Figure 1 for the Trellis Diagram demonstrating $viterbi(1, c_k)$

1.2 $viterbi(2, c_k)$

$$\begin{aligned} viterbi(2, c_1) &= viterbi(2, SUNNY) \\ &= \max[\{P(SUNNY|SUNNY) \times viterbi(1, SUNNY)\}, \\ &\quad \{P(SUNNY|RAIN) \times viterbi(1, RAIN)\}] \times P(STUDY|SUNNY) \\ &= \max[(0.6 \times 0.32), (0.5 \times 0.02)] \times 0.2 \\ &= 0.0384 \end{aligned}$$

$$
\begin{aligned}
viterbi(2, c_2) &= viterbi(2, RAIN) \\
&= \max[\{P(RAIN|SUNNY) \times viterbi(1, SUNNY)\}, \\
&\quad \{P(RAIN|RAIN) \times viterbi(1, RAIN)\}] \times P(STUDY|RAIN) \\
&= \max[(0.4 \times 0.32), (0.5 \times 0.02)] \times 0.5 \\
&= 0.064
\end{aligned}
$$

Refer to Figure 2 for the Trellis Diagram demonstrating $viterbi(2, c_k)$

1.3 $viterbi(3, c_k)$

$$
\begin{aligned}
viterbi(3, c_1) &= viterbi(3, SUNNY) \\
&= \max[\{P(SUNNY|SUNNY) \times viterbi(2, SUNNY)\}, \\
&\quad \{P(SUNNY|RAIN) \times viterbi(2, RAIN)\}] \times P(PLAY|SUNNY) \\
&= [(0.6 \times 0.0384) + (0.5 \times 0.064)] \times 0.4 \\
&= 0.0128
\end{aligned}
$$

$$
\begin{aligned}
viterbi(3, c_2) &= viterbi(3, RAIN) \\
&= \max[\{P(RAIN|SUNNY) \times viterbi(2, SUNNY)\}, \\
&\quad \{P(RAIN|RAIN) \times viterbi(2, RAIN)\}] \times P(PLAY|RAIN) \\
&= [(0.4 \times 0.0384) + (0.5 \times 0.064)] \times 0.1 \\
&= 0.0032
\end{aligned}
$$

Refer to Figure 3 for the Trellis Diagram demonstrating $viterbi(3, c_k)$

1.4 Most Probable Sequence

$$
\begin{aligned}
\hat{y_3} &= \text{argmax}\{P(STOP|SUNNY) \times viterbi(3, SUNNY), \\
&\quad P(STOP|RAIN) \times viterbi(3, RAIN)\} \\
&= SUNNY
\end{aligned}
$$

$$
\begin{aligned}
\hat{y_2} &= backtrack(3, \hat{y_3}) \\
&= backtrack(3, SUNNY) \\
&= \text{argmax}[P(SUNNY|SUNNY) \times viterbi(2, SUNNY), \\
&\quad P(SUNNY|RAIN) \times viterbi(2, RAIN)] \\
&= RAIN
\end{aligned}
$$

$$
\begin{aligned}
\hat{y_1} &= backtrack(2, \hat{y_2}) \\
&= backtrack(2, RAIN) \\
&= \text{argmax}[P(RAIN|SUNNY) \times viterbi(1, SUNNY), \\
&\quad P(RAIN|RAIN) \times viterbi(1, RAIN)] \\
&= SUNNY
\end{aligned}
$$

So, the most probable sequence is SUNNY, RAIN, SUNNY. Refer to Figure 4 for the Trellis Diagram demonstrating the backtracking.

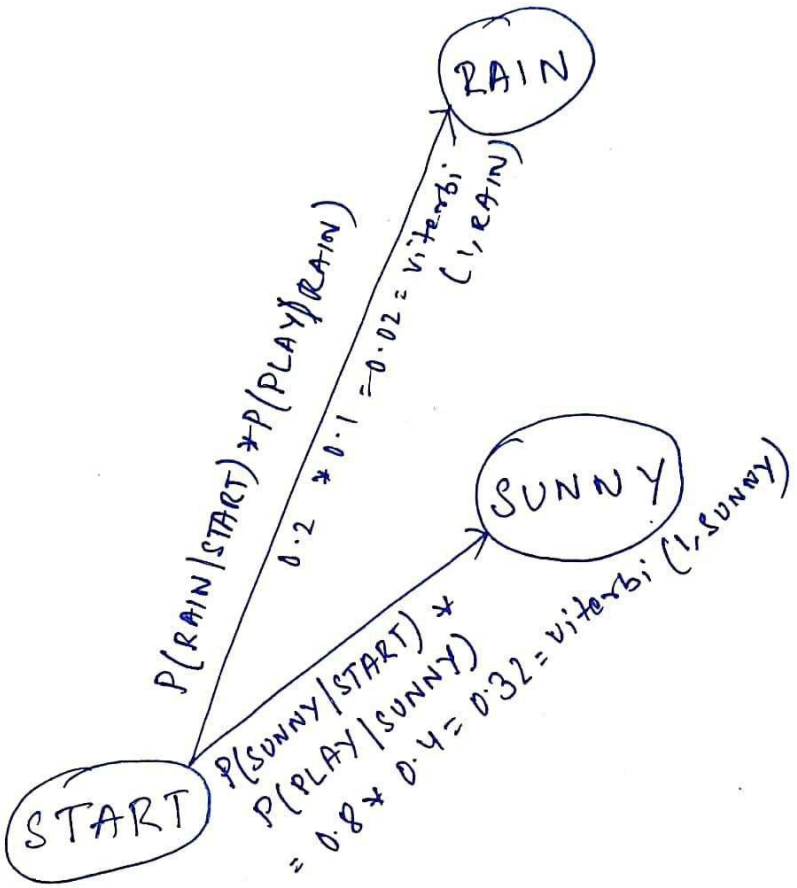

Figure 1: Trellis Diagram showing $viterbi(1, c_k)$ where $k \in \{1, 2\}$

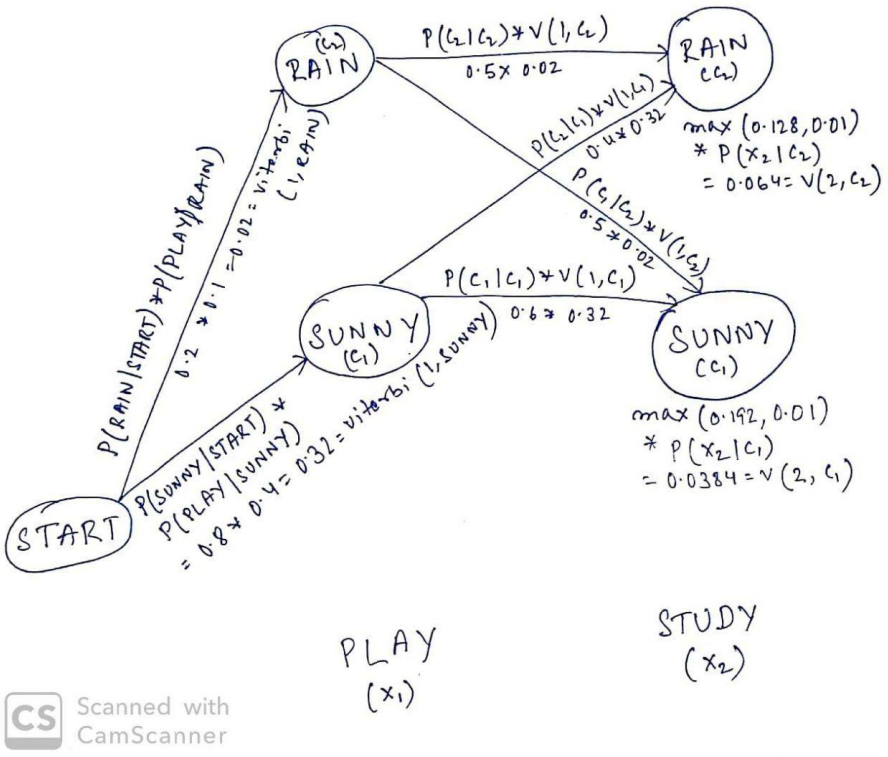

Figure 2: Trellis Diagram showing $viterbi(2, c_k)$ where $k \in \{1, 2\}$

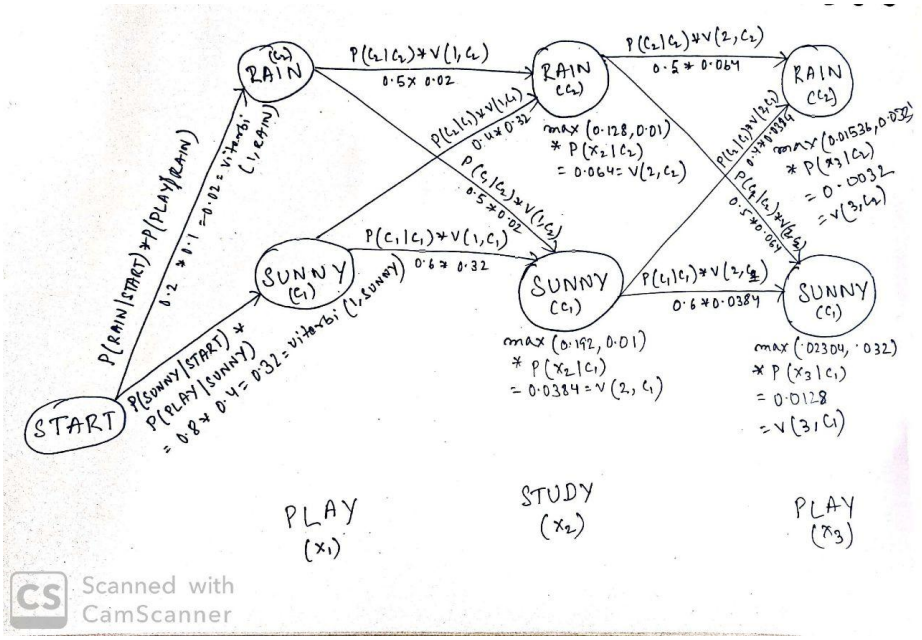

Figure 3: Trellis Diagram showing $viterbi(3, c_k)$ where $k \in \{1, 2\}$

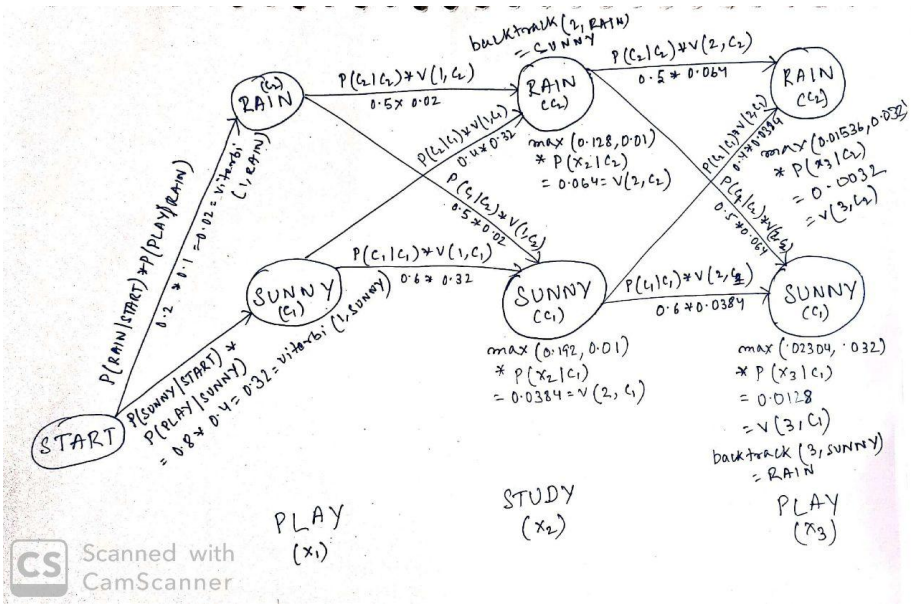

Figure 4: Trellis Diagram showing backtracking.