CS345: Assignment 8

Q1. (a) Suppose we have n identical processors. Then show that the dot-product of two vectors, $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$, can be computed in $O(\log\ n)$ time. Hint: All $p_i = a_i b_i$ can be computed in $O(1)$ time. Then show that $\sum_i p_i$ can be computed in $O(\log\ n)$ time using the processors.

(b) Show that the product of two $n \times n$ matrices can be computed in $O(\log\ n)$ using n^3 identical processors.

(c) Show that inverse of two lower triangular matrices can be computed in $O(\log^2\ n)$ time using n^3 identical processors.

Q2. Prove or disprove.

(a) A system of linear equations can have no solution,

(b) A system of linear equations can have exactly one solution.

(c) A system of linear equations can have exactly two solutions.

(d) A system of linear equations can have countably infinite solutions but not uncountable infinite solutions.

Q3. A square matrix with integral entries is called *unimodular* if its determinant is 1 or -1. Show that its inverse exists and it is also a unimodular matrix.

Q4. Let S be a subspace of $\mathbb{R}^n$. Let $S^\perp$ denote its perpendicular subspace. Let B_1 and B_2 be some bases of the two subspaces respectively. Then show that $B_1 \cup B_2$ is a basis of the whole space.

Q5. Following matrix is called Vandermonde matrix.

$$A = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ a_1 & a_1^2 & a_1^3 & \ldots & a_1^n \\ a_2 & a_2^2 & a_2^3 & \ldots & a_2^n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_n & a_n^2 & a_n^3 & \ldots & a_n^n \end{bmatrix}$$

Let $det(A) = f_n(x_1, x_2, \ldots, x_n)|_{x_1=a_1,\ldots,x_n=a_n}$.

Using the fact that $det(A) = det(A^T)$, deduce a recurrence relation for f_n.

Q6. Let M be a full rank matrix of size $m \times n$ where $m \leq n$.

(a) Show that $\{M \cdot x | x \in \mathbb{R}^n\}$ is a vector space. What is the dimension of this space?

(b) Show that $\{x \in mathbbR^n | M \cdot x = 0\}$ is a vector space. What is its dimension?

Q7. Consider any 4×5 matrix A and perform divide and conquer based LUP decomposition showing each step.