PHY305A
Exercise Set 7

1. Which of the following reactions are allowed by the conservation laws:
$$e^- + {}^1\mathrm{H} \to \gamma + \gamma\,,$$
$$n \to {}^1\mathrm{H} + e^- + \nu_e\,,$$
$${}^3\mathrm{He} + {}^3\mathrm{He} \to {}^4\mathrm{He} + {}^2\mathrm{H}\,.$$
If a reaction is not allowed, state which conservation law is violated. Note that here ^{1}H refers to hydrogen nucleus, which is simply a proton.

2. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of two particles of masses, m_1 and m_2, respectively. Show that, in terms of the relative velocity $\vec{v}$ and center of mass velocity $\vec{V}_c$, these can be expressed as
$$\vec{v}_1 = \vec{V}_c + \frac{m_2}{m_1 + m_2}\vec{v}\,,$$
$$\vec{v}_2 = \vec{V}_c - \frac{m_1}{m_1 + m_2}\vec{v}\,.$$

3. Determine the energy corresponding to the Gamow peak.

4. Make a Taylor expansion of ϵ_{PP1} around $T = 1.5 \times 10^7$K. Note that $T = 1.5 \times 10^7$K implies $T_6 = 15$. Keep only terms up to first order in $(T_6 - 15)$. Equate the zeroth and first-order terms of the two expansions and determine the parameters α and $\epsilon_{0,pp}$.

5. Verify the formula for ϵ_{PP1} derived in class. Proceed as follows: (a) Determine the formula for R_1 by explicitly substituting the expressions for $f(E_0)$ and $f''(E_0)$ in the equation for R_1. (b) Express the formula for $\rho\epsilon_{PP1}$, in units of ergs cm^{-3} sec^{-1}. (c) Obtain the expression for ϵ_{PP1} by using the results of (a), (b). (d) The value of $S(E_0)$ is given by, $S(E_0) \approx 3.78 \times 10^{-22}$ KeV·barns Convert this into cgs units. Note that 1 barn is equal to 10^{-24} cm^2. (e) Numerically compute the exponent and the coefficient.

6. (a) Make a rough estimate of the total number of neutrinos emitted by the Sun per second. Assume that only the PP1 chain contributes. By comparing the solar luminosity with the energy emitted per reaction, determine the total number of fusion reactions per second. Hence determine the neutrino emission rate. Compute also the solar neutrino flux at Earth. (b) The neutrino-proton cross section for typical solar neutrino energies is roughly 10^{-41} cm^2. Assume that an experiment to detect solar neutrinos uses a detector of mass 10,000 Kg. Make a rough estimate of the number of protons in this detector. Using the solar neutrino flux, determine the number of events, that is, neutrino-proton reactions, per second for this detector.

7. Check that the Jacobian of the transformation between $(\vec{v}_1, \vec{v}_2)$ and $(\vec{v}, \vec{V}_c)$ is unity. You only need to consider any particular component, x, y, or z of these velocity vectors and show that the corresponding Jacobian is unity.

8. Determine the production rate of ^{4}He in the PP2 chain assuming that all the intermediate nuclear products have reached steady state. Express the result in terms of the reaction rate of the first reaction.

9. Determine the rate of energy generation per unit mass for the PP2 chain assuming that all the intermediate products have reached their equilibrium concentration.

10. Consider the first reaction of the CNO cycle discussed in class. Proceeding as we did for the proton-proton fusion determine the formula for the reaction rate of this process, up to an overall factor $S(E)$ which we assume is a slowly varying function of E. Express the result as a power law in temperature in the vicinity of $T = 1.5 \times 10^7$ K in order to understand the temperature dependence of this rate.