CS698O Quiz 3 Solution

February 5, 2020

1 QUIZ 3

The sequence given is PLAY, STUDY, PLAY. So, $x_1 =$ PLAY, $x_2 =$ STUDY and $x_3 =$ PLAY.

1.1 $forward(1, c_k)$

$$\begin{aligned} forward(1, c_1) &= forward(1, SUNNY) \\ &= P(SUNNY|START) \times P(PLAY|SUNNY) \\ &= 0.8 \times 0.4 \\ &= 0.32 \end{aligned}$$

$$\begin{aligned} forward(1, c_2) &= forward(1, RAIN) \\ &= P(RAIN|START) \times P(PLAY|RAIN) \\ &= 0.2 \times 0.1 \\ &= 0.02 \end{aligned}$$

Refer to Figure 1 for the Trellis Diagram demonstrating $forward(1, c_k)$

1.2 $forward(2, c_k)$

$$\begin{aligned} forward(2, c_1) &= forward(2, SUNNY) \\ &= [\{P(SUNNY|SUNNY) \times forward(1, SUNNY)\} + \\ &\quad \{P(SUNNY|RAIN) \times forward(1, RAIN)\}] \times P(STUDY|SUNNY) \\ &= [(0.6 \times 0.32) + (0.5 \times 0.02)] \times 0.2 \\ &= 0.04 \end{aligned}$$

$$
\begin{aligned}
forward(2, c_2) &= forward(2, RAIN) \\
&= [\{P(RAIN|SUNNY) \times forward(1, SUNNY)\} + \\
&\quad \{P(RAIN|RAIN) \times forward(1, RAIN)\}] \times P(STUDY|RAIN) \\
&= [(0.4 \times 0.32) + (0.5 \times 0.02)] \times 0.5 \\
&= 0.069
\end{aligned}
$$

Refer to Figure 2 for the Trellis Diagram demonstrating $forward(2, c_k)$

1.3 $forward(3, c_k)$

$$
\begin{aligned}
forward(3, c_1) &= forward(3, SUNNY) \\
&= [\{P(SUNNY|SUNNY) \times forward(2, SUNNY)\} + \\
&\quad \{P(SUNNY|RAIN) \times forward(2, RAIN)\}] \times P(PLAY|SUNNY) \\
&= [(0.6 \times 0.04) + (0.5 \times 0.069)] \times 0.4 \\
&= 0.0234
\end{aligned}
$$

$$
\begin{aligned}
forward(3, c_2) &= forward(3, RAIN) \\
&= [\{P(RAIN|SUNNY) \times forward(2, SUNNY)\} + \\
&\quad \{P(RAIN|RAIN) \times forward(2, RAIN)\}] \times P(PLAY|RAIN) \\
&= [(0.4 \times 0.04) + (0.5 \times 0.069)] \times 0.1 \\
&= 0.00505
\end{aligned}
$$

Refer to Figure 3 for the Trellis Diagram demonstrating $forward(3, c_k)$

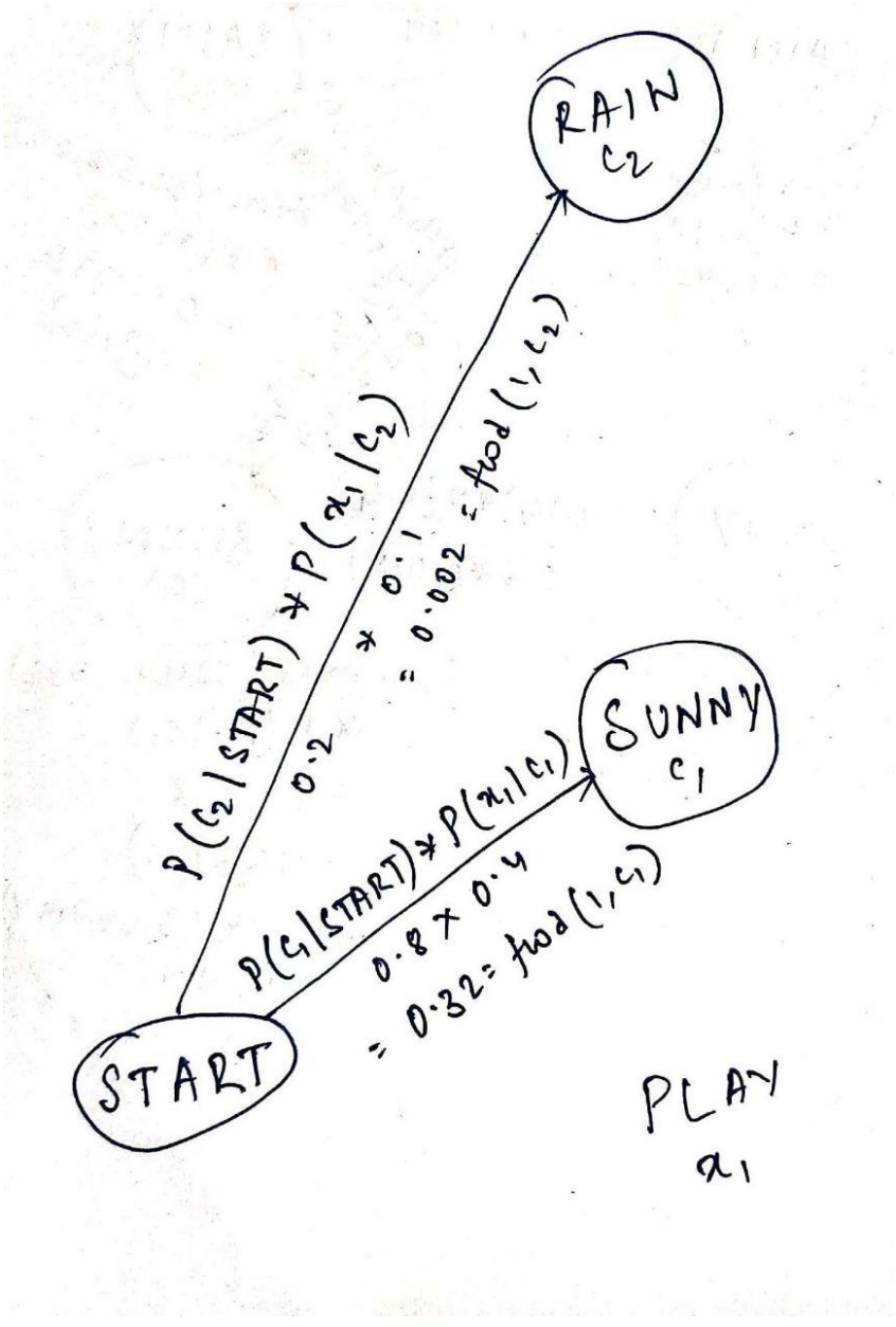

Figure 1: Trellis Diagram showing $forward(1, c_k)$ where $k \in \{1, 2\}$

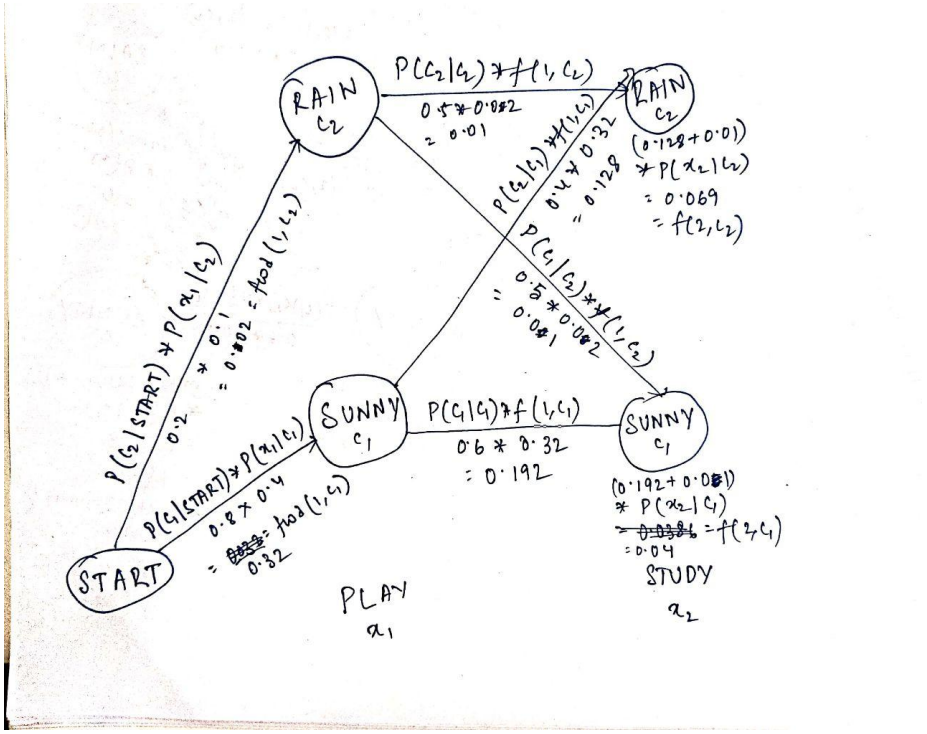

Figure 2: Trellis Diagram showing $forward(2, c_k)$ where $k \in \{1, 2\}$

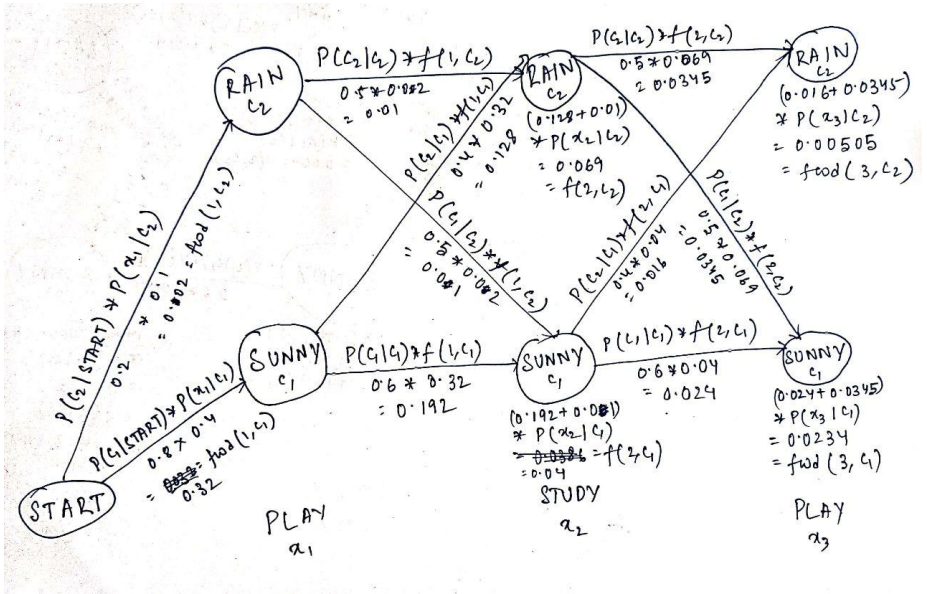

Figure 3: Trellis Diagram showing $forward(3, c_k)$ where $k \in \{1, 2\}$