Department of Mathematics & Statistics

MTH-102A Ordinary Differential Equations

Assignment VII

1. ⋆ Find the Laplace transform of the following functions.
(i) e^{at} for $a \neq 0$. (ii) $\cosh bt$. (iii) $e^{\lambda t} \cos\omega t$ for $\lambda, \omega \in \mathbb{R}$.
(iv) $\cos 2t + \sin 3t$ (v) $t^2 e^{3t} \sin 5t$.

2. ⋆ Find the Laplace transform of
(i) $f(t) = \begin{cases} e^{-t} & \text{for } 0 \leq t < 1 \\ e^{-2t} & \text{for } t \geq 1 \end{cases}$ (ii) $f(t) = \begin{cases} t & \text{for } 0 \leq t < 1 \\ 2-t & \text{for } 1 \leq t \leq 2 \end{cases}$ and $f(t+2) = f(t)$.
(iii)$f(t) = |\sin t|$

3. ⋆ Show that
(i)$L(\int_0^t f(\tau)d\tau) = \frac{1}{s}L(f)$ (ii) $L(\frac{1}{t}f(t)) = \int_s^\infty F(r)dr$.

4. ⋆ Find the inverse Laplace transform of the following functions.
(i)$F(s) = \frac{2+3s}{(s^2+1)(s+2)(s+1)}$ (ii) $F(s) = \frac{3s^2+2s+1}{(s^2+1)(s^2+2s+3)}$.

5. ⋆ Solve the following initial value problems.
(i)$2y'' + 3y' + y = 8e^{-2t}$, $y(0) = -4$ and $y'(0) = 2$.
(ii)$y'' + y = \sin 2t$, $y(0) = 0$ and $y'(0) = 1$.

6. Using the unit step function find the $L(f)$ if $f(t) = \begin{cases} \sin t & \text{if } 0 \leq t \leq \frac{\pi}{2} \\ \cos t - 3\sin t & \text{if } \frac{\pi}{2} \leq t < \pi \\ 3\cos t & \text{if } t \geq \pi \end{cases}$.

7. Find the inverse Laplace transform of $\frac{1}{s^2} - e^{-s}\left[\frac{1}{s^2} + \frac{2}{s}\right] + e^{-4s}\left[\frac{4}{s^3} + \frac{1}{s}\right]$.

8. Solve the following initial value problems:
. (i) $y'' + y = f$ where $f(t) = \begin{cases} \sin t & \text{if } 0 \leq t < \frac{\pi}{2} \\ \cos t & \text{if } \frac{\pi}{2} \leq t < \pi \\ 0 & \text{if } t \geq \pi \end{cases}$ and $y(0) = 2$, $y'(0) = -1$.
(ii) $y'' - 4y' + 4y = f$ where $f(t) = \begin{cases} e^{2t} & \text{if } 0 \leq t < 2 \\ -e^{2t} & \text{if } t \geq 2 \end{cases}$ and $y(0) = 0$, $y'(0) = -1$.

9. Using convolution method solve the equation $y(t) = 1 + 2\int_0^t e^{-2(t-\tau)}y(\tau)d\tau$.

10. Solve the initial value problem
$y'' + 6y' + 5y = 3e^{-2t} + 2\delta(t-1)$, $y(0) = -3$ and $y'(0) = 2$.