CS345: Assignment 7

Q1. Show that if all the capacities of a flow network are integers, then there is a maximum flow assigning integral flow to each edge. Is the converse true? Justify.

Q2. Let $G = (A, B, E)$ be a bipartite graph. Design a flow network for G to find a maximum matching.

Q3. Given a bipartite graph $G = (A, B; E)$. Compute a subgraph H with maximum number of edges such that each A vertex has degree at most d_1 and each B vertex has degree at most d_2.

Q4. Let $G = (V, E)$ be a directed graph and $u, v \in V$. A u, v cut of G is any partition $(S, V \setminus S)$ of V such that $u \in S$ and $v \notin S$. The corresponding edge set is $\{(x, y) | x \in S, y \notin S\}$. A minimum u, v cut is that u, v cut which has minimum number of edges. Show that if the minimum u, v cut has k edges, then there are k edge-disjoint $u - v$ paths (paths that do not share any edge) in the graph.

Give a polynomial time algorithm that finds the maximum possible number of edge-disjoint paths that exist between u and v.

Q5. Let $G = (V, E)$ be an undirected graph and $u, v \in V$. A u, v cut is a vertex partition $(S, V \setminus S)$ such that $u \in S$ and $v \notin S$. The edge set of the cut is $\{\{x, y\} | x \in S, y \in V \setminus S\}$. Describe an algorithm to compute a u, v cut having minimum number of edges, for a given pair u, v.

Q6. As in the case of circulation, define minimum cost flow for a flow network (with one source and and one sink). Each (directed) edge is assigned a cost (positive or negative) and the cost of a flow is the sum of weighted flow along each edge.

Given an undirected graph and specific vertices x and y we want to compute the shortest path between them. Reduce this problem into a minimum weight flow problem.

Q7. Consider the problem of integral maximum multi-commodity flow, which is a generalization of the max flow problem, where we have multiple source-sink pairs, s_i-t_i for $1 \leq i \leq k$. Each source sends a different commodity to the corresponding sink and multiple commodities flow together in any edge. The sum of the volumes of all commodities in an edge must not exceed its capacity. Assume that each commodity has integral magnitude in each edge. The maximum multi-commodity flow problem asks for maximizing the total flow (sum of the flows of all commodities) in the graph. Note that the sum of the commodity j that flows into sink s_i, for $i \neq j$, must be equal to the sum of commodity j flows out of s_i.

Reduce the maximum matching problem in any (undirected) graph into an integral multi-commodity flow problem in a tree of height 1 (all vertices, other than the root, being leaf-nodes).

Show that for trees of height one and unit edge capacities, the problem of integral maximum multi-commodity flow is same as maximum matching.