Section 52: Problem 4 Solution

> Working problems is a crucial part of learning mathematics. No one can learn topology merely by poring over the definitions, theorems, and examples that are worked out in the text. One must work part of it out for oneself. To provide that opportunity is the purpose of the exercises.
>
> — *James R. Munkres*

Let $A \subset X$; suppose $r : X \to A$ is a continuous map such that $r(a) = a$ for each $a \in A$. (The map r is called a ***retraction*** of X onto A .) If $a_0 \in A$, show that

$$r_* : \pi_1(X, a_0) \to \pi_1(A, a_0)$$

is surjective.

By definition, $r_*([f]) = [r \circ f]$, so if $[f] \in \pi_1(A, a_0)$, i.e. f is a loop in A based at a_0 , then $r \circ f = f$, and $r_*([f]) = [f]$.