Department of Mathematics & Statistics

MTH-102A Ordinary Differential Equations

Assignment V

1. $\star$ Show that the substitution $x = e^t$ transforms the Euler's equation $ax^2y'' + bxy' + cy = 0$ for $x > 0$, in to constant coefficient differential equation.

2. $\star$ Find the power series

   (a) in x for the general solution of $(1 + 2x^2)y'' + 6xy' + 2y = 0$

   (b) in $x - 1$ for the general solution of $(2 + 4x - 2x^2)y'' - 12(x - 1)y' - 12y = 0$.

3. $\star$ Find the power series in $x - x_0$ for the general solution of the differential equations

   (a) $y'' - y = 0, \quad x_0 = 3.$

   (b) $(1 - 4x + 2x^2)y'' + 10(x - 1)y' + 6y = 0, \quad x_0 = 1.$

4. $\star$ Find $a_0, \ldots, a_n$ for at least 7 terms in the power series $y = \sum_{n=0}^{\infty} a_n(x - x_0)^n$ for the soltuion of the initial value problems

   (a) $y'' + (x - 3)y' + 3y = 0, \quad y(3) = -2, \quad y'(3) = 3.$

   (b) $(4x^2 - 24x + 37)y'' + y = 0, \quad y(3) = 4, \quad y'(3) = -6.$

5. $\star$ Find a fundamental set of Frobenius solutions of

$$x^2(3 + x)y'' + 5x(x + 1)y' - (1 - 4x)y = 0.$$

6. Find a fundamental set of Frobenius solutions of

   (a) $4x^2y'' + x(7 + 2x + 4x^2)y' - (1 - 4x - 2x^2)y = 0,$

   (b) $x^2(5 + x + 10x^2)y'' + x(4 + 3x + 8x^2)y' + (x + 36x^2)y = 0,$

   (c) $2x^2y'' + x(3 + 2x)y' - (1 - x)y = 0$, and

   (d) $x^2(8 + x)y'' + x(2 + 3x)y' + (1 + x)y = 0.$