

PHY305A  
End Sem Exam

time: 3 hours

max. marks 120

Instructions:

1. Calculators and mobiles are not allowed. You may leave your final answer in the form of a numerical expression. However all numerical substitutions must be done and you are required to simplify the final result as much as possible.
2. Please answer all parts of a question adjacent to one another. We will grade an answer only at the first place where it appears in the answer sheet.

- 1a. Determine the condition under which a cloud of gas and dust is unstable to gravitational collapse.
- 1b. Briefly explain why a cloud of gas and dust gives rise to a cluster of stars. What physical process determines the minimum size of objects formed in the collapse of such a cloud?
- 1c. What is the upper and lower limit on the mass of Main Sequence stars.  
(marks 6+4+2)
- 2a. Make a clear, labelled plot of the potential energy of a two nuclei system undergoing nuclear fusion.
- 2b. Consider a target which consists of  $n$  particles per unit volume. Let the beam flux be  $F$ . An experimentalist observes that the reaction rate per unit volume (of target particles) is  $R$ . Determine the reaction cross section between the beam and target particles?
- 2c. The reaction rate of the first process in the PP1 chain is given by

$$R_1 = \lambda_1 \frac{N_H^2}{2}$$

Briefly explain why the factor of 2 arises in the denominator.

- 2d. In estimating the energy released in nuclear fusion reactions we use atomic masses although the particles involved in the process are nuclei. Provide a justification for this procedure.  
(marks 3+3+3+3)
- 3a. Make a clear, labelled plot of the mass fractions  $X$  and  $Y$  of Hydrogen and Helium respectively as a function of the distance from the center for Sun, as obtained in the standard solar model. Explain the main features seen in this plot.
- 3b. Write down the chain of reactions corresponding to the PP1 chain. Determine the equilibrium concentration of  ${}^3\text{He}$  in terms of  $\lambda_1, \lambda_2, \lambda_3$  and  $N_H$  where the symbols have their standard meaning.  
(marks 6+6)

4a. Define the Rosseland mean opacity

4b. Make a clear, labelled plot of the dependence of Rosseland mean opacity on temperature  $T$  for fixed value of  $\rho/T_6^3$  relevant for solar interiors.

4c. Using the result of 4b, make a guess for the dependence of the Rosseland mean opacity on  $r$ , the distance from the center of the Sun. Show this dependence on a plot. Indicate what is the main source of opacity in different regions of the Sun. Also explain what is the main source of opacity near the surface.

(marks 3+3+6)

5. The reaction rate between two nuclei in stellar interiors can be expressed as

$$R_{12} = N_1 N_2 \int \sigma(v) v \left( \frac{m_1}{2\pi kT} \right)^{3/2} \left( \frac{m_2}{2\pi kT} \right)^{3/2} \exp \left( -\frac{m_1 v_1^2}{2kT} - \frac{m_2 v_2^2}{2kT} \right) d^3 v_1 d^3 v_2.$$

where the symbols have their standard meaning. After making a suitable change of variables obtain the final expression for  $R_{12}$ . Give a clear justification for all the steps.

You may use

$$\int \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} d^3 v = 1$$

(marks 10)

6a. Use the equation for hydrostatic equilibrium to determine the pressure as a function of height for the Earth's atmosphere. In this case, we can neglect the curvature of the Earth and the equation becomes

$$\frac{dP}{dz} = -\rho g,$$

where  $z$  is the vertical distance above the surface. For small  $z$ , we can treat  $g$  as constant. Let the temperature near the surface of Earth vary as  $T(z) = T_0 - \beta z$ . Determine  $P(z)$  given that the mean molecular weight is  $\mu$ . Use the symbol  $m_H$  for mass of Hydrogen atom,  $P_0$  for surface pressure and  $k$  for the Boltzmann constant.

6b. The equation for radiative transport can be written in the form,

$$\frac{\cos \theta}{\bar{\kappa}(r)} \frac{dI}{dr} = -\rho I + J(r).$$

Starting from this obtain the final equation for temperature gradient  $dT/dr$  in regions where radiative transport dominates.

(marks 8+8)

7a. Determine the mean molecular weight by including the contribution due to all metals. Assume that all metals are ionized and replace  $\bar{Z} = Z + 1$ . Furthermore, assume that  $Z + 1 \approx A/2$ .

$$x_i = \frac{N_i A_i m_H}{M}$$

7b. Give a brief description of the spectrum produced by molecules. What is the fundamental reason for this spectra?

(marks 6+4)

8a. Explain the process of formation of absorption lines.

8b. Using the surface temperature of the Sun, make a rough estimate of the broadening of Balmer H-alpha line at  $\lambda = 6563$  Angstroms. The Boltzmann constant  $k = 1.38 \times 10^{-23}$   $\text{Kg m}^2/(\text{K} \cdot \text{s}^2)$  and mass of Hydrogen atom  $m_H = 1.67 \times 10^{-27}$  Kg.

(marks 3+6)

9a. Give a brief description of Globular clusters, including their physical properties, age and distribution in the Milky Way.

9b. Explain how one can use the color-magnitude diagram to obtain distance and age of open clusters. Recall that  $L \propto M^{3.5}$ , where the symbols have their standard meaning.

(marks 4+6)

10a. Consider a planet which is locked in resonance with Sun such that only a particular hemisphere always faces the Sun while the other always points away from the Sun. Hence only one of the hemisphere of the planet gets heated up by the Sun and the other hemisphere remains very cold and emits negligible radiation. Determine the mean temperature of the hot hemisphere of the planet in steady state. You may take  $R$  as the radius of the planet,  $r$  the planet-Sun distance and  $L_S$  as the luminosity of the Sun.

10b. In J2000 equatorial system, the supergalactic pole is located at  $\delta_p = 15.7^\circ$  and  $\alpha_p = 283.8^\circ$ . The reference direction for longitude is located at  $\delta_c = 59.5^\circ$  and  $\alpha_c = 42.3^\circ$ . Let  $(x, y, z)$  denote the equatorial coordinate system and  $(x', y', z')$  the supergalactic coordinate system. In the transformation from equatorial to supergalactic coordinates the first step involves rotation about the  $z$  axis in order to align the  $x$ -axis with the line of intersection of the  $x-y$  and  $x'-y'$  planes. (i) Determine the angle of rotation about the  $z$ -axis that would accomplish this alignment. Briefly explain your answer. (ii) Explain the subsequent two rotations required to complete the transformation. You only need to explain what are the transformations without working out the details.

(marks 7+10)

