Problem 4.1: Assume that the magnitude $f(r)$ of electric field due to a positive unit charge is given as $f(r) = \frac{1}{r^{2+\varepsilon}}$ $(0 < \varepsilon \ll 1)$ and it points away from it.

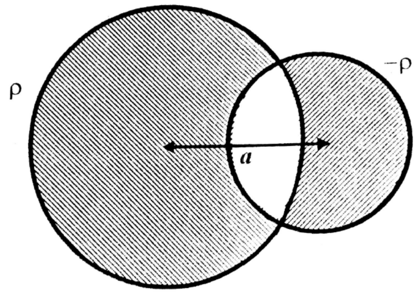
(a) Calculate the electric field $\mathbf{E}(r)$ on the axis of a uniformly charged ring of radius R carrying a total charge Q.

(b) Using the result of part (a), find the electric field inside and outside a uniformly charged thin spherical shell of radius R and carrying a total charge Q correct to first order in ε. Show that the field inside the shell points towards its centre.

(c) Show that field near the centre of the shell varies linearly with r.

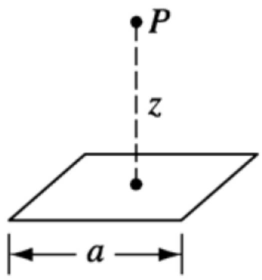
(d) Plot $\mathbf{E}(r)$ versus r inside and outside the shell.

Problem 4.2: What is the electric field in a cavity formed by the intersection of two spheres as shown in the figure? The spheres carry uniform charge densities of ρ and $-\rho$ respectively and the distance between the spheres is a.

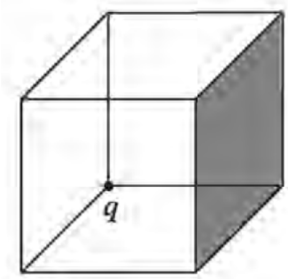


Problem 4.3: Use the result of problem above to find the surface charge density on a spherical surface that gives a uniform field inside the spherical cavity enclosed by the surface. Hence find the surface charge density induced on a metallic sphere put in a uniform electric field.

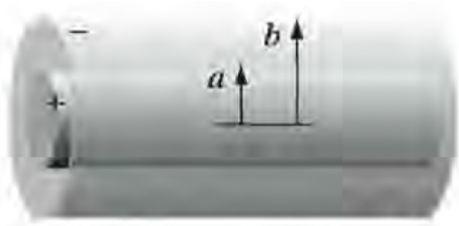
Problem 4.4: Find the electric field a distance z above the centre of a square loop (side a) carrying uniform line charge λ.



Problem 4.5: A charge q sits at the back corner of a cube as shown in the figure. What is the flux of $\mathbf{E}$ through the shaded side?



Problem 4.6: A long coaxial cable (see figure) carries a uniform volume charge density ρ on the inner cylinder (radius a) and a uniform surface charge density on the outer cylindrical shell (radius b). this surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the cable ($s > b$). Also plot $|\mathbf{E}|$ as a function of s.



Problem 4.7: Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct field. Sketch $V(r)$.

Problem 4.8: Find the potential a distance s from an infinitely long straight wire that carries a uniform line charge λ. Compute the gradient of your potential, and check that it yields the correct field.