PHY305A
Exercise Set 6

1. Use the equation for hydrostatic equilibrium to determine the pressure as a function of height for the Earth's atmosphere. In this case, we can neglect the curvature of the Earth and the equation becomes

$$\frac{dP}{dz} = -\rho g\,,$$

where z is the vertical distance above the surface. For small z, we can treat g as constant. Eliminate ρ from this equation by using the gas law. Show that the solution can be written as

$$P(z) = P_0 \exp\left[-\frac{g\mu m_H}{k}\int_0^z \frac{dz'}{T(z')}\right]\,,$$

where P_0 is the pressure at the surface of the Earth. As a very rough approximation for small z ($z < 100$ Km), assume that $T(z) = T_0 - \beta z$, where $T_0 = 300$K and $\beta = 1$K/Km, that is, the temperature changes by 1K per Km. Determine $P(z)$ in this case.

2. The pressure, $P(z)$ near the Earth's surface ($z < 100$ Km) can be approximated as

$$P(z) \approx P_0 e^{-z/z_0}\,,$$

where z is the altitude, $z_0 \approx 7.3$ Km, and $P_0 \approx 101,000$ Pa ($Pa = N/m^2$). Assume that in some region the ambient temperature at the surface is $T_0 = 300$K and decreases with height at the rate of 15K/Km. Determine whether the air is stable or unstable to convection in this region. Assume $\gamma = 7/5$, applicable for diatomic gases, N_2 and O_2.

3. Determine the numerical value of the Thomson cross section. You can use the relationship

$$\alpha = \frac{e^2}{\hbar c}$$

to replace e^2 in terms of $\hbar$ and c. Note that $\alpha \approx 1/137$ is the fine structure constant.

4. Make an estimate of the Rosseland mean opacity at very high temperature assuming that it is dominated by Compton scattering. Assume that the medium is dominantly Hydrogen. How does your answer compare with the result given in class? Repeat this calculation for $X = 0.7$ and $Y = 0.3$.

5. The potential energy of a star of radius R is given by

$$U_g = -4\pi G\int_0^R dr M(r)\rho r$$

Determine U_g by assuming uniform density.

6. Determine the mean molecular weight by including the contribution due to all metals. Assume that all metals are ionized and replace $\bar{Z} = Z + 1$. Furthermore, assume that $Z + 1 \approx A/2$. With these approximations, show that

$$\mu = \frac{1}{2X + 3Y/4 + Z'/2}, \tag{1}$$

where Z' is now interpreted as the total mass fraction of all elements with $Z > 2$.