D - Math

* $N = 1, 2, 3 \ldots\ldots$
* $W = 0, 1, 2, 3 \ldots\ldots$
* I or Z $\rightarrow -3, -2, -1, 0, 1, 2, 3, 4.$

$$Q = \frac{m}{n}$$

where $n \neq 0$

$Q^2 \rightarrow \sqrt{2}, \sqrt{3}, \ldots\ldots\ldots$

$\sqrt{-2}, 1\sqrt{2}\ i$

$z = 2 + 3i$

$a = 2$ Real part imginary

$b = 3$

$$R = \frac{\sqrt{1}}{4} = \frac{1}{2}$$

if $A = \{a, b, c\}$

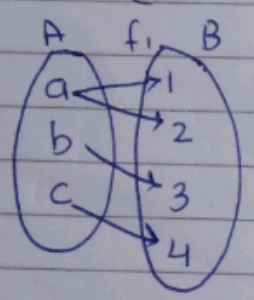
$B = \{1, 2, 3, 4\}$

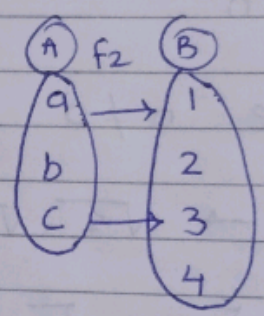
let, $F_1 = \{(a,1), (a,2), (b,3), (c,4)\}$

$f_2 = \{(a,1), (c,3)\}$

$f_3 = \{(a,1), (b,1), (c,3)\}$

→ $f_1 =$





∴ There are two images of element 'a'

f

$A = \{1, 2, 3\}$ $= n(A) = 3$

$B = \{a, b, c\}$ $= n(B) = 3$

* $\cup$ = Union $\rightarrow$ Collest / combine
* $\cap$ = intersection $\rightarrow$ Common elements.

$A \cup B = \{1, 2, 3, a, b, c\}$

$C = \{1, 2, 3, 4, 5\}$

$D = \{2, 4, 6\}$

$C \cup D = \{1, 2, 3, 4, 5, 6\}$

$C \cap D = \{2, 4\}$

$n(D) = 3$

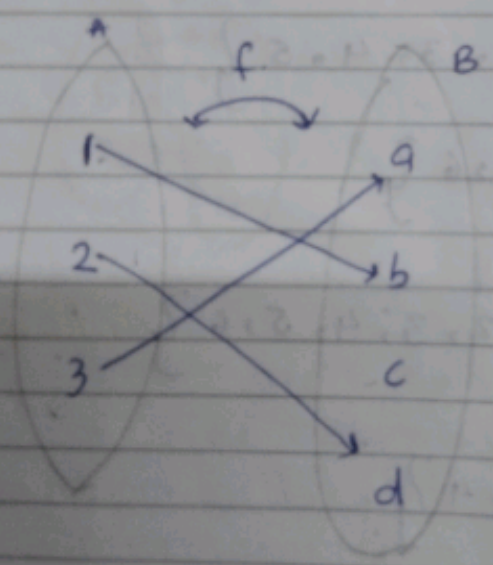
$A \cap B = \{\quad\}$ or ϕ is called empty Se

$n(A \cap B) = 0$ if $A \cap B = \{0\}$

$n(A \cap B) = 1$

$a = \{0, 1, 2\}$ $\therefore A \cap B = \{0\}$

$B = \{0, 4\}$ $\therefore n(A \cap B) = 1$



Pre-image
Domain

image
co-Domain

Pairing = $\{(1, b), (2, d), (3, a)\}$

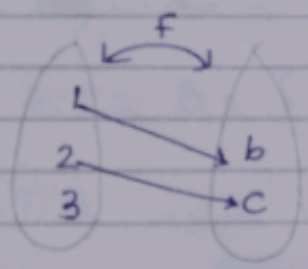
Range = $\{a, b, d\}$

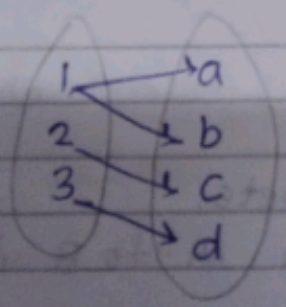
Co-Domain $\{(c)\}$

fi $A \to B$

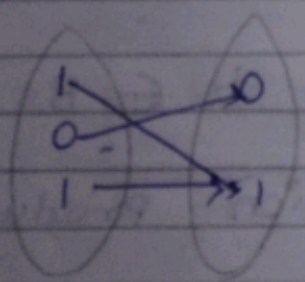
$A = \{1, 2, 3\}$

$B = \{a, b, c, d\}$





X Wrong Co-Domain



< Biggest

Domain

$f(x) = x^2$

$x \in \mathbb{Z} \rightarrow$

$\mathbb{Z} = \{-1, 0, 1\}$

$\therefore -3, -2, -1, 0, +1, +2,$

$f(-1) = (-1)^2 = 1$

$f(0) = 0$

$f(1) = (1)^2 = 1$

→ ✓ Right

✓ × Wrong.

Relation.

* Relation.

let A and B be Sets.
A binargralation form A to B is a Subset of A X B

let $a \in A = b \in B$

$A \times B \rightarrow$ cartesion Product.

$(a, b) \in R$ We say that small a $\in$ to small b, We the is realated to b by
+2, releted aRb.

If, A, B

$(a, b) \in R$ B say

Example

① let $A = \{1, 2, 3\}$

$B = \{a, b, c\}$

then.

$R = \{(1, a), (1, b), (2, c)\}$

is relation from A to B

② let $A = B$ $\{1, 2, 3, 4\}$

be a relation on A defined. (a, b) $a = 1, 2, 3, 4$ $b = 1, 2, 3, 4$

as aRb iff $a > b$.

i.R $= \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

Example
2) bRa iff b > a

let $A = \{1, 2, 3\}$
$B = \{2, 3, 4\}$

then

$$R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$$

is relation form B to A.

Domain of Relation.

* Domain of Relation are is a set of all first element of order pair which dented by Dom (R)

$$R = \{(1,2), (1,3), (2,4), (2,1), (2,3)\}$$

Domain $= \{1, 2\}$

Co-Domain $= \{1, 2, 3, 4\}$

1) Set
2) Subset
3) Union , intersection
4) $n(A)$, $n(B)$
5) Cartesion Product
6) Relation

* ① Equal Set

$A = \{1, 2\}$

$B = \{2, 1\}$ or $\{1, 2\}$

$C = \{1, 2, 3\}$

$A = B \neq C$

* ② Equivalemt Set

$A = \{1, 2\}$ and $B = \{a, b\}$

$n(A) = 2$, $n(B) = 2$

$\therefore$ A and B sets are equivalent set

* ③ Empty Set or multiple Set

Denated by $\{\ \}$ or $\emptyset$, ph. Empty

e.g.

$A = \{1, 2\}$

$B = \{a, b\}$

$A \cup B = \{1, 2, a, b\}$

$A \cap B = \{\ \}$ or $\emptyset$

$n(A) = 2$, $n(B) 2$

$A(A \cup B) = 4$, $n(A \cap B) = 0$

e.g A $\{0\}$, $n(A) = 1$

* ④ Singletion Set

$\{a\}$, $\{b\}$, $\{0\}$

$\{1\}$

* 5 Complement of Set

let, $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 3, 5, 7\}$

$B = \{2, 4, 6\}$

To find $A \cdot\cdot B$ $(A \cup B)$ and $(A \cap B)$

$A' \mid A^c$ → Complement of Sets

$A' = \{2, 4, 6\}$

$B' = \{1, 3, 5, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$(A \cup B)' = \{\ \}$ Empty Set

$(A \cap B) = \{\ \}$ Empty Set.

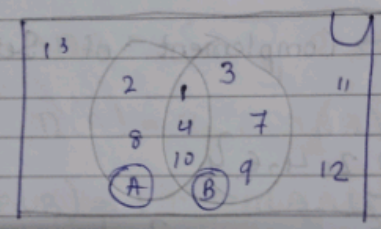
$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7\}$

* (6) Vern Diagram.

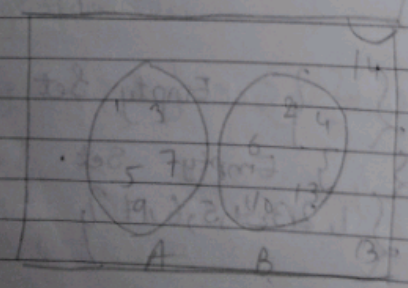
let :- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$

$A = \{1, 2, 4, 8, 10\}$

$B = \{1, 3, 4, 7, 9, 10\}$



* (7) Disjoint Set.



this is a Disjoint Set

* (8) formula Set

(1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(2) $n(A \cup B) = n(A) + n(B)$
if Sets are disjoints.

(3) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(A \cap C)$
$+ n(A \cap B \cap C)$.

e.g If $n(A) = 12$
$n(B) = 15$
$n(A \cup B)$ 20

Then to find : $n(A \cap B)$

We have.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$20 = 12 + 15 - n(A \cap B)$

$20 = 27 - n(A \cap B)$

$n(A \cap B) = 27 - 20$

$\boxed{n(A \cap B) = 7}$

① Subset :

here : B is a Super Set

eg $A = \{1, 2, 3\}$

$B = \{1, 2, 3, a, b, c\}$

$A \subseteq B$.

② Proper Subset

$A \subset B$

e.g $\therefore A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

$A \subset B$.

③ Power Set

example ① $P'(A)$

eg : $= \{1, 2\}$

$$P(A) = \{\{\ \}, \{1\}, \{2\}, \{1,2\}\}$$

$$n(P(A)) = 2^m$$

$\therefore$ where m is a no of elements in the set

$$\boxed{n(P(A)) = 2^2 \quad 4}$$

Example ②.

$$A = \{1, 2, 3\}$$

find $P(A) = ?$

$$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$$

* ④ Difference Of a set $\therefore$

$$A - B = \{x \mid x \in A, x \notin B\}$$

$$\& \ B - A = \{x \mid x \notin B, x \notin A\}$$

$$e.g = A = \{1, 2, 3, 5, 7\}$$

$B = \{1, 2, 3, 6, 8\}$

To find :- $A - B$ and $B - A$

$A - B = \{5, 7\}$

$B - A = \{6, 8\}$

Law of Set's

commutative law

① $A \cap B = B \cap A \longrightarrow$ Intersection

$A \cup B = B \cup A \longrightarrow$ Union.

② Associative Law

$A \cap (B \cap C) = (A \cap B) \cap C.$

$A \cup (B \cup C) = (A \cup B) \cup C.$

* ③ Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

* Example

$2(a+b) = 2a+2$

* ④ Demorgan's law

① $(A \cup B)^c = A^c \cap B^c$

② $(A \cap B)^c = A^c \cup B^c$

* ⑤ Identity law

$A \cup \phi = A$

$A \cap \phi = \phi$

* ⑥ Complement law

$(A^c)^c = A$

* (8) Idempotent law

$A \cup A = A$

$A \cap A = A$

* (9) Universal Bound law

$A \cup U = U$

$A \cap U = A$

* (10) Absorption law :

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

* (11) Set Difference law

$A - B = A \cap B^c$

$B - A = B \cap A^c$

Example :

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$A = \{1, 2, 3, 5, 7\}$

$B = \{1, 2, 3, 6, 8\}$

$A - B = \{5, 7\}$ — ①

* Thermon number One.

1] prove that if $A \subseteq B$ & $B \subseteq C$ then $A \subseteq C$

proof :- let $x \in A$

$\therefore A \subseteq B$

$\therefore x \subset B$

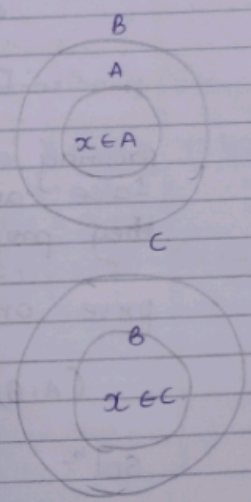
$\therefore B \subseteq C$

$\therefore x \in C$

$\therefore A \subseteq C$.

$a = b = c$

$a = c$.



2) prove that :-
if $A \subseteq B$ then $P(A) \subseteq P(B)$

proof :- Given :- $A \subseteq B$

let $x \in P(A)$ be any arbitary Set ①

then by difitection power Set

$x = \subseteq A$

We get , $x \subseteq B$

$\therefore \subseteq P(B)$ —— ②

$P(A) = \subseteq P(B)$.

proofing any propertity or a result is false and that is called disprouing they propertity

prove or disprove :-

$(A.B) \cup (B - C) = A - C$

Sol^n:- let $A = \{1, 2, 3, 5\}$

$B = \{2, 4, 5, 6\}$

$C = \{3, 5, 6, 7\}$

$A - B = \{1, 3\}$

$B - C = \{2, 4\}$

$(A - B) \cup (B - C) = \{1, 2, 3, 4\}$ —— (I)

and $A - C = \{1, 2\}$ — (II)

from (I) and (II)

$(A - B) \cup (B - C) \neq A - C$.

Boolean Algebra {Operation}

$+ : X \rightarrow x - y = yx$

$x + y = y + x$

$x + (y + z) = (x + y) + z$

=> The laws of thought is logic iff this are Express in Symbole instade of words then it is called mathematics logics and when it express in algebric Structure it is called boolean Algebra.

a) Name of properting

a) (I) Additional

Closeder :. $x + y \in B$.

2) commutative :- $x+y = y+x$

3) Associativety :- $x+(y+z) = (x+y)+z$

4) Distributive : (+ Over X)

$$x+(y.z) = (x+y)-(x+z)$$

5) Existence of identity

∴ Example :- $2+0=2$

$x+0=x$

6) Existence of compliment.

$\{ \therefore 2+(-2)=0 \}$

$x+x^{-1}=1$ $\qquad$ $3+(-2)=1$

*

* properties of boolean Algebra with respect to multiply

1) Closure

$x + y \in B$, $x - y \in B$

2) Commulative

$x + y = y + x$, $x \cdot y = y \cdot x$

3) Associativity

$x(y \cdot z) = (x \cdot y) z$

4) Distribution $\cdot$ over $+$

$x(y + z) =$

5) Existence of identity :

$x : 1 = x$

6) Existence of compliment

$x \cdot x' = 0$

* Duality :-

$$+ \rightleftarrows \cdot$$

$$0 \rightleftarrows 1$$

$$\{ (x+y) \cdot z = (x \cdot y) + z \}$$

* ① $(1 \cdot x) + 0 = x$

Dual of statement is.

Example $(0+x) \cdot 1 = x$

* $a(b+c) = (a \cdot b) + (a \cdot c)$

Dual of S.

* $(0 + x)' = 1 \cdot x'$

* $x \cdot y' = 0$ iff $x \cdot y = x$.

* $a(b+c) = (a \cdot b) + (a \cdot c)$

Dual of S.

$a + (b \cdot c) = (a+b) \cdot (a+c)$

* $(0+x)' = 1 \cdot x'$

$(1 \cdot x)' = 0 + x'$

* $x \cdot y' = 0$ iff $x \cdot y = x$

$x + y' = 1$

iff

$x + y = x$

Q1) Let B be a Boolean Algerbra then prove that $x+y=x$

proof :-

$R.H.S = x$

$= x + 0$ (Identity law)

$= x + (x.x')$ = ((compliment law)

$= (x+x).1.(x+x')$ (Distributive law)

$= (x+x).1$ (compliment law)

$= x+x$

$= L.H.S$

Q2 Write down the duals of following

1) $\left[x.\left(y+y'-(x+y')\right)\right] x^2 \equiv 0$

$\rightarrow \left[\left(x+(y.y'-(x.y'))\right)\right] x^2 \equiv 0$

2) $(x+y)-(y+z).(x'+y') = x+y.z$

$\rightarrow (x.y)-(y.z)+(x'.y') = x.y.z$

3) $(x' + y' + z') + (x \cdot y \cdot z') + (x' \cdot y \cdot z)$

$= y(x+z)$

$\rightarrow (x' \cdot y' \cdot z') \cdot (x + y + z') \cdot (x' + y + z)$

$= y(x \cdot z).$

* Similarly do that

$$\left[(x_1' \cdot x_2) + x_3\right] x_1 = x_3 \cdot x_1$$

proof :-

$$L.H.S = \left[(x_1' - x_2) + x_3\right] x_1$$

$$= \left[(x_1' + x_3) \cdot (x_2 + x_3)\right] x_1$$

$$= \left[x_1' (x_2 + x_3) + x_3 (x_2 + x_3)\right] x_1$$

$$= x_1 x_1' (x_2 + x_3) + (x_1 \cdot x_3)(x_2 + x_3)$$

$$= 0 (x + x_3) + x_1 x_2 (x_2 + x_3)$$

$$= 0 + x_1 \cdot x_2 - (x_2 + x_3).$$

Quantifiers.

$\forall = \forall \rightarrow$ for all

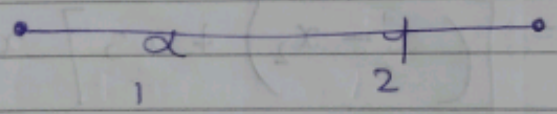
* Intervals :-

i) Open internal ( )

$\{x \in A \,/\, a < x < b\}$

$(1, 2)$

$x = 1.1, 1.2, 1.3, \ldots 1.9$



(2.3) $\quad 2 < x < 3$

$2.1, 2.2, 2.3 \ldots\ldots 2.9$

* Closed interval [ ]

a) $\{x \in A \,/\, a \leq x \leq b\}$

e.g $1 \leq x \leq 2$

$x = 1, 1.1, 1.2 \ldots\ldots 2$

* Semi Open

$\left\{ \begin{array}{l} (\;] = \text{Semi Open} \\ [\;) = \text{Semi closed} \end{array} \right\}$

3) $\{ x \in A \,/\, a < x \leq b \}$

$[a, b]$

$[1, 2]$

$x = 1.1, 1.2, 1.3, \ldots \ldots 1.9 \cdot 2$

* Semi Closed

4) $\{ x / a \leq x \leq , b \}$

$[a, b)$

$[2, 3)$

$x = 2, 2.1, 2.2, 2.3, \ldots 2.9$

5) $x \geq a \quad [a, \infty]$

6) $x > a \Rightarrow (a, \infty)$

7) $x \leq a \Longrightarrow (-\infty, a)$

8) $x < a \rightarrow ((-\infty, a)$

4) Exclusive OR
$\oplus$

let, p & q be a two Statement

$p \oplus q$

condition if both Values are Same then Value of $p \oplus q$ is f

i.e $T \oplus T$ is f (Otherwise T or ~~True~~ True)

$F \oplus F = F$.

Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

5) Conditional or implication

if ..... then

$(\longrightarrow)$

$p \longrightarrow q$

Cond^n :- $T \longrightarrow F = F$

otherwise $= T$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

6) Double implication or Biconditional Statement

=> $(\longleftrightarrow)$

$p (\longleftrightarrow) q$

cond^n – if both p & q are same or equal then $p \longleftrightarrow q$ is true otherwise F

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

4) laws of logic Statement

a) ~~comecative~~ Commentive law

a) $p \vee q \equiv q \wedge p$

b) $p \vee q \equiv q \vee p$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

6) Double implication or Biconditional Statement

=> $(\longleftrightarrow)$

$p (\longleftrightarrow) q$

cond^n – if both p & q are same or equal then $p \longleftrightarrow q$ is true otherwise F

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

4) laws of logic Statement

a) ~~comecative~~ Commentive law

a) $p \vee q \equiv q \wedge p$

b) $p \vee q \equiv q \vee p$

a) Truth Table

I	II	III	IV
p	q	$p \cap q$	$q \cap p$
T	T	T	T
F	F	F	F
F	T	F	F
F	F	F	F

Therefore, from column III & IV are $p \cap q \equiv q \cap p$ are True.

3) Associative law

a) $p \cap (q \cap r) \equiv (p \cap q) \cap r$

b).

p	q	r	$q \cap r$	$p \cap (q \cap r)$	$p \cap q$	$(p \cap q) \cap r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

3) Distributive law :- 2 Operation hoga

a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

from above table column no 5 & 8

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

4) Demorgan's law)

a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \cap q$	$\sim(p \cap q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

1. Tautology

If final Values are truth (T) then it can be say that it is tautology

2. Contradination

If final Values are false (F) then it is contradiction.

3. Construct truth table for the following problem

$$[p \rightarrow (q \rightarrow r)] \longrightarrow [(p \cap q \longrightarrow r)]$$

p	q	r	$(q \to r)$	$p \to (q \to r)$	$(p \cap q)$	$(p \cap q) \to r$	$[p \to (q \to r)] \to [(p \cap q) \to r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Justify

$p \to (q \to r) \equiv (p \to q) \to r$

$p \cap (q \cup r) \equiv \sim (p \cap r) \to (p \vee r)$.

p	q	r	$q \cup r$	$p \cap (q \cup r)$	$p \cap r$	$\sim (p \cap r)$	$p \vee r$	$2 (p \cap r) \to (p \vee r)$
T	T	T	T	T	T	F	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	T	F	T	T
T	F	F	F	F	F	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	F	F
F	F	T	T	F	F	T	T	T
F	F	F	F	F	F	T	F	F

* $p \cap (q \cup r) \equiv \sim(p \cap r) \rightarrow (p \cup r)$

from column (5) & (9)

$\therefore p \cap (q \cup r) \not\equiv \sim(p \cap r) \rightarrow p \cup r)$.

(D) math

Ex Define : $a + b = L.C.m \{a, b\}$ $a, b \in D$

$\therefore ab = G.C.D \{a.b\}$ $a, b \in D$

show that

$(D, +, \cdot)$ is ~~Bolle~~ Boolean Algebra

Solⁿ.

Since $\therefore$ D is the set of positive divisor of 15

$D = \{1, 3, 5, 15\}$

L.C.m $\{1, 3\}$

$1 \times 3 = 3$

L.C.m $= \{2, 4\} = 2$

$2 \rightarrow 1 \times 2$

$4 \rightarrow 2 \times 4$

① $a + b = L.C.M \{a, b\}$

First are prove to show $(D, +, \cdot)$ is Boolean Algebra

+	1	3	5	15
1	1	3	5	15
3	3	3	15	15
5	5	15	5	15
15	15	15	15	15

Hence for all a, b, c, D

at b.

$\therefore$ D is a.B.A.

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+	1	3	5	15
1	1	1	1	1
3	1	3	1	3
5	1	1	5	5
15	1	3	5	15

Hence for all $a, b, \in D, a$

$\therefore (D, \cdot)$ is a , B, A

① AND $\longrightarrow \wedge$

② OR $\longrightarrow \vee$

let. p & q be a two statement

(i) if $p \wedge q \longrightarrow$ Conjunction

T A T $\longrightarrow$ T

Otherwise F

(ii) Disjunction

$p \vee q$

F V F $\rightarrow$ F

Otherwise T

p	q	$p \wedge q$	$p \vee q$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

$$\left\{ \begin{array}{l} T \Rightarrow 1 \\ F \Rightarrow 0 \end{array} \right\}$$

p	q	p∧q	p∨q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

True or false Table

③ Negation (~)

p	~p	p	~p
T	F	1	0
F	T	0	1