AVERAGE

1. Average or Arithmetic mean of n quantities
$$= \frac{\text{Sum of n quantities}}{n}$$

2. Average of n consecutive natural number
$$= \frac{n+1}{2}$$

3. Average of squares of n consecutive natural number $= \frac{(n+1)(2n+1)}{6}$

4. Average of cubes of n consecutive natural number $= \frac{n(n+1)^2}{4}$

5. Average of n consecutive even numbers $= n+1$

6. Average of squares of n consecutive even numbers $= \frac{2(n+1)(2n+1)}{3}$

7. Average of n consecutive odd numbers $= n$

8. Average Speed $= \frac{\text{Distance Travelled}}{\text{Time Taken}}$

9. Average of two different speeds x and y to travel same distance $= \frac{2xy}{x+y}$

10. Average of three different speeds x, y, z to travel same distance $= \frac{3xyz}{xy+yz+xz}$

11. If a person is replaced by another person by which

(i) average is increased, then

Age of the new comer = Age of person left + No. of persons × increase in average age

(ii) average is decreased, then

Age of new comer = Age of person left − No. of person × decrease in average age

12. If a person joins a group without replacing any person by which -

(i) average increased, then

Age of the new comer = Previous average age + No. of persons (including new comer × increase in avg age)

(OR) Age of the new comer = Increased avg. age + (No. of persons originally in the group × increased in average age.)

(ii) average decreased then

Age of the new comer = Previous average age
− No. of persons (including new comer)
× decrease in average age.

13. If a person leaves the group but nobody joins the group by which -

(i) average increased, then

Age of man left = Previous average age +
No. of present persons × Increase in average age.

(ii) average decreased, then

Age of man left = Previous average age +
No. of present persons × Decrease in average age.

14. If the average marks obtained by x candidates in an examination is n, If the average of marks of passed candidates is p and that of the failed candidates is q. Then -

(i) No. of passed candidates $= \frac{\text{Total Candidates} \times (\text{Total avg.} - \text{Failed avg})}{\text{Passed avg} - \text{Failed avg}}$.

(ii) No. of failed candidates $= \frac{\text{Total Candidates} \times (\text{Passed avg} - \text{Total avg})}{\text{Passed avg} - \text{Failed avg}}$.

15. The Geometric mean of numbers $x_1, x_2 \ldots\ldots x_n$ is given by $\sqrt[n]{x_1, x_2 \text{ ------} x_n}$

QUESTIONS -

1. The average of five numbers is 29. If one number is excluded the average becomes 27. What is the excluded number?

→ Excluded number = $(29 \times 5) - (27 \times 4)$

$= 145 - 108$

$= 37.$

2. Find the average of first 20 natural numbers?

→ Sum of first n natural numbers = $\frac{n(n+1)}{2}$.

So, avg of 20 natural numbers = $\frac{20(20+1)}{2}$

$= \frac{20 \times 21}{2}$

$= 210$

∴ Required average is = $\frac{210}{20} = 10.5$

3. Average age of 20 students is 16. When the teacher joins the class then the average increases by 1. Then what's the teacher's age?

→ No. of students + Average Age + 1 (Denotes the teacher)

= Answer

$= 20 + 16 + 1$

$= 37.$

4. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach the target of 282 runs?

→ Required run rate $= \left[\frac{282 - (3.2 \times 10)}{40}\right] = \frac{250}{40} = 6.25$

5. A family consists of two grandparents, two parents and three grandchildren. The average age of the grandparents is 67 years, that of the parents is 35 years and that of the grandchildren is 6 years. What is the average age of the family?

→ Required average $= \left(\frac{67 \times 2 + 35 \times 2 + 6 \times 3}{2+2+3}\right)$

$= \left(\frac{134 + 70 + 18}{7}\right)$

$= \frac{222}{7}$

$= 31\frac{5}{7}$ years.

6. The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero?

→ Average of 20 numbers = 0

∴ Sum of 20 numbers $(0 \times 20) = 0$

• For the sum of 20 numbers to be equal to zero, there may be all 19 numbers of them > 0 and only one number which is -ve of all the 19 numbers

Sum

7. The average weight of A, B and C is 45 Kg. If the average weight of A and B be 40 Kg and that of B and C be 43 Kg. then weight of B is :-

→ $A+B+C = 45 \times 3 = 135$ —— (i)

$A+B = 40 \times 2 = 80$ —— (ii)

$B+C = 43 \times 2 = 86$ —— (iii)

Adding (ii) & (iii);

$A+2B+C = 166$ —— (iv)

Subtracting (i) from (iv);

$B = 31$ Kg

8. A pupil's marks were wrongly entered as 83 instead of 63. Due to that, average marks for the class got increased by half (1/2). The number of pupil's in the class is :-

→ Let there be 'x' pupils in the class.

Total increase in marks $= x \times \frac{1}{2} = \frac{x}{2}$

$\therefore \frac{x}{2} = (83-63)$

$\Rightarrow \frac{x}{2} = 20$

$\Rightarrow \boxed{x = 40}$

9. The average of four numbers is 80. The first number is 1/3 of sum of three number. What would be the first number?

A. 65 B. 90 C. 85 D. 80

→

→ $P+Q+R+S = 320$ —(I)

$P = \frac{Q+R+S}{3}$ —(II)

From (I) and (II);

$4P = 240$

$P = 80$ (Ans) → (d)

10. The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52. find the sixth result?

→ Total of 11 results = 11 × 50
= 550

Total of first 6 results = 6×49 = 294

Total of last 6 results = 6×52 = 312.

∴ Sixth result = 294 + 312 − 550
= 56

{As, 6th result is common to both}

⇒ Direct formula.

6th result = $50+6\{(52-50)+(49-50)\}$
$= 50+6(2-1)$
$= 56.$

11. A man bought 13 shirts of Rs 50 each, 15 pants of Rs 60 each and 12 pairs of Rs 65 a pair. Find the average value of each article.

→ Average $= \frac{13 \times 50 + 15 \times 60 + 12 \times 65}{12+15+13}$ $= \text{Rs } 58.25$

12. Find the largest number if average of 7 consecutive numbers is 20.

→ Let, numbers be $x, x+1, x+2, x+3, x+4, x+5, x+6$

Then, $\frac{(x+(x+1)+(x+2)+(x+3)+(x+4)+(x+5)+(x+6))}{7} = 20$

$\Rightarrow 7x + 21 = 140$

$\Rightarrow 7x = 119$

$\Rightarrow \boxed{x = 17.}$ (Ans)

13. Out of the four numbers, whose average is 60, the first is one-fourth of the sum of the last three. The first number is –

(i) 17 (ii) 29 (iii) 36 (iv) 48 ✓

Let first number be x;

Sum of 4 no's $= x + 4x = 5x$

So, $\frac{5x}{4} = 60$

$\Rightarrow x = \frac{60 \times 4}{5}$

$\Rightarrow \boxed{x = 48}$

14. If the arithmetic mean of Seventy - five numbers is calculated, it is 85. If each number is increased by 5, then mean of new numbers is:

(A) 30 (B) ✓ 40 (C) 50 (D) 60

→ Let the numbers are

$x_1, x_2 \cdots x_{75}$.

By hypothesis;

$$\frac{x_1+x_2+\cdots x_{75}}{75} = 35 \cdots$$

Mean of new numbers

$$= \frac{(x_1+5)+(x_2+5)+\cdots(x_{75}+5)}{75}$$

$$= \frac{(x_1+x_2+\cdots x_{75})+5*75}{75}$$

$$= \frac{x_1+x_2+\cdots x_{75}}{75} + \frac{375}{75}$$

$$= 35+5 = 40. \quad \{\text{By hypothesis}\}.$$

15. A Batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17th inning?

(i) 40 (ii) ✓ 39 (iii) 52 (iv) 55

→ Let average after 17th innings = x

Then average after 16 innings = $x-3$

$\therefore$ $16(x-3)+87 = 17x$

$\Rightarrow$ $x = 39$

16. David obtained 76, 65, 82, 67 and 85 marks (out of 100) in english, maths, physics, chemistry and biology. What are his average marks.

→ (A) 65 (B) 69 (C) 75 (D) None of these.

Its very easy.

17. Average of ten positive numbers is x. If each number is increased by 10%., then x:—

(A.) Remains Unchanged

(B) May decrease

(C) May increase

(D) is increased by 10%.

→ Let 10 numbers be $x_1, x_2, x_3 \dots x_{10}$

A/C Ques;

$$(x_1 + x_2 + x_3 + \text{—} + x_{10}) / 10 = x$$

Now each number is increased by 10%. then;

$$y = (1.1x_1 + 1.1x_2 + 1.1x_3 + \dots 1.1x_{10}) / 10$$

$$\Rightarrow y = 1.1 \times ((x_1 + x_2 + \dots + x_{10})/10)$$

$$\Rightarrow y = 1.1x.$$

$\therefore y$ is 10% ↑d.

18. Average of age of boys in a class is 16 years and average age of girls is 15 years. What is the average age at all.

(A) 15.5 (B) 15 (C) 16 (D) Can't be computed

→ As number of girls and boys are not given, so result can't be computed.

19. The average of six numbers is X and average of three of these is Y. If the average of the remaining three is Z, then –

A. $x = y+z$

B. $2x = y+z$

C. $x = 2y+z$

D. $x = y+2z$

→ $x = \frac{3y+3z}{6}$

$\Rightarrow 2x = y+z$

20. A library has an average of 510 visitors on Sundays and 240 on other day. The average number of visitors in a month of 30 days starting with Sunday is—

(A) 280 (B) 285 (C) 290 (D) 295

→ As the month begin with Sunday, so there will be 5 Sundays in the month.

So, result $= \frac{510 \times 5 + 240 \times 25}{30}$

$= \frac{8550}{3}$

$= 285.$