4 POWER, INDICES AND SURDS

Importance : 1 or 2 questions from 'Surds and Indices' have essentially been asked in every exam. In order to accuracy in your calculations you will require complete practice of this chapter.

Scope of questions : Asked questions are based on basic concepts, completely arithmetic and without language like to evaluate/simply, greatest/lowest number, increasing/ decreasing order, square, cube, square root, cube root and higher powers starting from easier to tougher levels.

Way to success : Note that practice to solve these questions with full concentration and accuracy is essential. Only because of small mistake or not understanding the basic concepts many students are unable to solve these questions.

INDICES

In seventeenth century a French mathematician Reni Dakata' multiplied a number several times and showed the obtained product by a special rule, which called 'indices' and the converse of indices is called surds.

Rule 1 : If any number is multiplied by the same number 'n' times, then,

$a \times a \times a \times a$ .... $\times$ a (n times) = a^n

(1) where n and a are real numbers. (including fractions)
(ii) a is called base.
(iii) n is called indices.

Rule 2 : $a^m \times a^n = a^{m+n}$

and $a^m \times a^n \times a^p = a^{m+n+p}$

While multiplying. If base is same then powers get added.

Rule 3 : While multiplying, if bases are different but powers are same then,

$a^x \times b^x \times c^x = (abc)^x$

Rule 4 : While dividing, if base is same then powers get subtracted, as

$a^m \div a^n = a^{m-n}$

Rule 5 : If there is negative indices on a number, then

$a^{-m} = \frac{1}{a^m}$ or, $a^m = \frac{1}{a^{-m}}$

Rule 6 : If there are indices on indices, then indices are multiplied. as–

(i) $\left(a^m\right)^n = a^{mn}$ (ii) $\left(a^m\right)^{\frac{1}{n}} = a^{\frac{m}{n}}$

(iii) $\left\{\left(a^m\right)^n\right\}^p = a^{mnp}$

Rule 7 : (i) $a^{m^n} \neq \left(a^m\right)^n$

(ii) $a^{m^{\frac{1}{n}}} \neq \left(a^m\right)^{\frac{1}{n}}$ (NOTE)

(iii) $a^{m^{n^p}} \neq \left\{\left(a^m\right)^n\right\}^p$ (NOTE)

Rule 8 : Indices as fraction.

(i) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (ii) $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Rule 9 : If $a^x = a^y$ then x = y and
if $x^n = y^n$ then x = y

Rule 10 : If the indices on any number is zero, the value of that number is 1, as

$x^\circ = 1$, $5^\circ = 1$, $10^\circ = 1$, $(50000)^\circ = 1$

Rule 11 : If 'a' is a rational number and n is a positive integer, then, nth root of 'a', $a^{\frac{1}{n}}$ or $\sqrt[n]{a}$ is an irrational number, $\sqrt[n]{a}$ is called the surd of n indices, it means $\sqrt[n]{a}$ is a surd where,

(i) 'a' is a rational number. (ii) 'n' is a positive integer.
(iii) $\sqrt[n]{a}$ is an irrational number.

Rule 12 : If $\sqrt[n]{a}$ is a surd, then n is called surd indices and a is called 'Radicand'. Every surd can be an irrational number, but every irrational number can not be a surd.

Rule 13 : Mixed Surds– A surd having a rational co–efficient other than unity is called a mixed surd.

Rule 14 : Pure Surd : The surds whose one factor is 1 and other factor is an irrational number, then that type of surd is called pure surd or the surd which is completely under radical sign.

Rule 15 : Similar Surds–The surds whose irrational factor is same, that is called similar surds.

Rule 16 : Irrational numbers as – $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ ....etc. have infinite recurring decimals.

Rule 17 : $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

Rule 18 : $\left(\sqrt[n]{a}\right)^n = a$

Rule 19 : $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = (a)^{\frac{1}{n}} \times (b)^{\frac{1}{n}}$

Rule 20 : $\sqrt[n]{\sqrt[n]{a}} = \left((a)^{\frac{1}{n}}\right)^{\frac{1}{n}} = a^{n^{\frac{1}{2}}}$

Rule 21 : $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$

Rule 22 : $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Rule 23: $\sqrt{x\sqrt{x\sqrt{x\sqrt{x................n\text{ times}}}}} = x^{\left(1-\frac{1}{x^n}\right)}$

Rule 24 : If $\sqrt{x-\sqrt{x-\sqrt{x-.......\infty}}}$, where x=n(n + 1)

then, $\sqrt{x-\sqrt{x-\sqrt{x-.......\infty}}} = n$

Rule 25 : If $\sqrt{x+\sqrt{x+\sqrt{x+.......\infty}}}$ where, x=n(n + 1)

then $\sqrt{x+\sqrt{x+\sqrt{x+.......\infty}}} = (n + 1)$

Rule 26 : $\sqrt[a]{b}, \sqrt[x]{y}, \sqrt[n]{m}, \sqrt[p]{q}$

To find smallest or greatest out of these, we should equate all the indices and compare the base. ❑❑❑

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

TYPE–I

1. By how much does $\sqrt{12}+\sqrt{18}$ exceed $\sqrt{3}+\sqrt{2}$?

(1) $2\left(\sqrt{3}-\sqrt{2}\right)$ (2) $2\left(\sqrt{3}+\sqrt{2}\right)$

(3) $\sqrt{3}+2\sqrt{2}$ (4) $\sqrt{2}-4\sqrt{3}$

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

2. The value of

$\sqrt{5+2\sqrt{6}}-\dfrac{1}{\sqrt{5+2\sqrt{6}}}$ is :

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$

(3) $1+\sqrt{5}$ (4) $\sqrt{5}-1$

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

3. The value of $\sqrt{2^4}+\sqrt[3]{64}+\sqrt[4]{2^8}$ is :

(1) 12 (2) 16

(3) 18 (4) 24

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

4. $2\sqrt[3]{32}-3\sqrt[3]{4}+\sqrt[3]{500}$ is equal to :

(1) $4\sqrt[3]{6}$ (2) $3\sqrt{24}$

(3) $6\sqrt[3]{4}$ (4) 916

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

5. $\left(\sqrt{8}-\sqrt{4}-\sqrt{2}\right)$ equals :

(1) $2-\sqrt{2}$ (2) $\sqrt{2}-2$

(3) 2 (4) –2

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

6. $8^{2/3}$ is equal to :

(1) $5\frac{1}{2}$ (2) $21\frac{1}{3}$

(3) 4 (4) $3\frac{1}{3}$

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

7. The simplified form of $\left(16^{3/2}+16^{-3/2}\right)$ is :

(1) 0 (2) $\dfrac{4097}{64}$

(3) 1 (4) $\dfrac{16}{4097}$

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

8. $16^{3/4}$ is equal to :

(1) $4\sqrt{2}$ (2) 8

(3) $2\sqrt{2}$ (4) 16

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

9. $(0.01024)^{1/5}$ is equal to :

(1) 4.0 (2) 0.04

(3) 0.4 (4) 0.00004

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

10. $(16^{0.16}\times 2^{0.36})$ is equal to

(1) 2 (2) 16

(3) 32 (4) 64

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

11. The value of $(256)^{0.16}\times(16)^{0.18}$ is :

(1) 4 (2) – 4

(3) 16 (4) 256

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

12. The value of

$$\frac{(243)^{0.13}\times(243)^{0.07}}{(7)^{0.25}\times(49)^{0.075}\times(343)^{0.2}}$$

(1) $\dfrac{3}{7}$ (2) $\dfrac{7}{3}$

(3) $1\dfrac{3}{7}$ (4) $2\dfrac{2}{7}$

(SSC CPO S.I. Exam. 12.01.2003

13. The value of :

$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ is

(1) 1 (2) 2

(3) 3 (4) 8

(SSC CGL Prelim Exam. 11.05.2003 (First Sitting)

14. $\sqrt{\sqrt[3]{0.004096}}$ is equal to

(1) 4 (2) 0.4

(3) 0.04 (4) 0.004

(SSC CGL Prelim Exam. 11.05.2003 (Second Sitting)

15. The **approximate** value of

$\dfrac{3\sqrt{12}}{2\sqrt{28}}\div\dfrac{2\sqrt{21}}{\sqrt{98}}$ is

(1) 1.0727 (2) 1.0606

(3) 1.6026 (4) 1.6007

(SSC Section Officer (Commercial Audit) Exam. 16.11.2003)

16. The value of

$2+\sqrt{0.09}-\sqrt[3]{0.008}-75\%$ of 2.80 is :

(1) 0 (2) 0.01

(3) –1 (4) 0.001

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

17. The value of

$\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)\left\{\left(\sqrt[3]{3.5}\right)^2-\sqrt[3]{8.75}+\left(\sqrt[3]{2.5}\right)^2\right\}$ is :

(1) 5.375 (2) 1

(3) 6 (4) 5

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

18. The value of

$\left(3+2\sqrt{2}\right)^{-3}+\left(3-2\sqrt{2}\right)^{-3}$ is

(1) 189 (2) 180

(3) 108 (4) 198

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

19. $\dfrac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}-\dfrac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}+\dfrac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ is equal to :

(1) 0 (2) $2\sqrt{15}$

(3) $2\sqrt{10}$ (4) $2\sqrt{6}$

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

20. When $(4+\sqrt{7})$ is presented in the form of perfect square it will be equal to

(1) $(2+\sqrt{7})^2$ (2) $\left(\dfrac{\sqrt{7}}{2}+\dfrac{1}{2}\right)^2$

(3) $\left\{\dfrac{1}{\sqrt{2}}\left(\sqrt{7}+1\right)\right\}^2$ (4) $\left(\sqrt{3}+\sqrt{4}\right)^2$

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005)

21. The value of

$\dfrac{1}{\sqrt{3.25}+\sqrt{2.25}}+\dfrac{1}{\sqrt{4.25}+\sqrt{3.25}}+\dfrac{1}{\sqrt{5.25}+\sqrt{4.25}}+\dfrac{1}{\sqrt{6.25}+\sqrt{5.25}}$ is

(1) 1.00 (2) 1.25

(3) 1.50 (4) 2.25

(SSC CPO S.I. Exam. 05.09.2004)

22. The simplified form of

$$\frac{2}{\sqrt{7}+\sqrt{5}}+\frac{7}{\sqrt{12}-\sqrt{5}}-\frac{5}{\sqrt{12}-\sqrt{7}}$$

is :

(1) 5 (2) 2

(3) 1 (4) 0

(SSC CPO S.I. Exam. 26.05.2005)

23. $\left(\frac{1}{2}\right)^{-\frac{1}{2}}$ is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) $2\sqrt{2}$

(3) $-\sqrt{2}$ (4) $\sqrt{2}$

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005 & SSC HSL DEO & LDC Exam. 28.11.2010)

24. $\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$ is equal to

(1) $\sqrt{3}$ (2) $3\sqrt{3}$

(3) $3-\sqrt{3}$ (4) $5-\sqrt{3}$

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005)

25. $(16)^{0.16}\times(16)^{0.04}\times(2)^{0.2}$ is equal to :

(1) 1 (2) 2

(3) 4 (4) 16

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

26. $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(1) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$

(2) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(3) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$

(4) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

27. $\left(3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}+\frac{1}{\sqrt{3}-3}\right)$ is equal to

(1) 1 (2) 3

(3) $3+\sqrt{3}$ (4) $3-\sqrt{3}$

(SSC CGL Prelim Exam. 04.02.2007 (IInd Sitting) & SSC CGL Tier-I Exam. 19.06.2011 (IInd Sitting) & SSC (10+2) DEO & LDC Exam. 20.10.2013)

28. $\sqrt{8-2\sqrt{15}}$ is equal to :

(1) $\sqrt{5}+\sqrt{3}$ (2) $5-\sqrt{3}$

(3) $\sqrt{5}-\sqrt{3}$ (4) $3-\sqrt{5}$

(SSC CPO S.I. Exam. 16.12.2007)

29. $(0.04)^{-(1.5)}$ is equal to

(1) 25 (2) 125

(3) 60 (4) 5

(SSC CGL Prelim Exam. 27.07.2008 (Second Sitting)

30. The value of

$\sqrt[3]{1372}\times\sqrt[3]{1458}\div\sqrt[3]{343}$ is

(1) 18 (2) 15

(3) 13 (4) 12

(SSC CGL Prelim Exam. 27.07.2008 (Second Sitting)

31. $\left(\frac{2}{\sqrt{5}+\sqrt{3}}-\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{1}{\sqrt{6}+\sqrt{5}}\right)$ is equal to

(1) $2\sqrt{6}$ (2) $2\sqrt{5}$

(3) $2\sqrt{3}$ (4) 0

(SSC CPO S.I. Exam. 09.11.2008)

32. $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=$

(1) 5 (2) 4

(3) 3 (4) 2

(SSC CPO S.I.Exam. 06.09.2009) & SSC MTS (Non-Tech.) Exam. 20.02.2011)

33. $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ is equal to

(1) $5+2\sqrt{6}$ (2) $\frac{3+2\sqrt{6}}{2}$

(3) $5-2\sqrt{3}$ (4) $5+2\sqrt{3}$

(SSC CISF ASI Exam 29.08.2010 (Paper-1)

34. The value of

$\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$ is

(1) $16+\sqrt{3}$ (2) $4-\sqrt{3}$

(3) $2-\sqrt{3}$ (4) $2+\sqrt{3}$

(SSC CGL Tier-1 Exam 19.06.2011 (First Sitting)

35. The square root of $14+6\sqrt{5}$ is

(1) $2+\sqrt{5}$ (2) $3+\sqrt{5}$

(3) $5+\sqrt{3}$ (4) $3+2\sqrt{5}$

(SSC CGL Tier-1 Exam. 19.06.2011 (First Sitting)

36. The value of

$\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ is

(1) 4 (2) 0

(3) $\sqrt{2}$ (4) $3\sqrt{6}$

(SSC CGL Prelim Exam. 11.05.2003 (IInd Sitting) & SSC CPO S.I. 16.12.2007 & SSC CGL 27.07.2008 (Ist Sitting) & SSC CGL Tier-I Exam. 26.06.2011 (Ist Sitting) & SSC CGL Tier-II Exam. 29.09.2013)

37. Simplify : $\left(\dfrac{\frac{3}{2+\sqrt{3}}-\frac{2}{2-\sqrt{3}}}{2-5\sqrt{3}}\right)$

(1) $\frac{1}{2}-5\sqrt{3}$ (2) $2-5\sqrt{3}$

(3) 1 (4) 0

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

38. $(64)^{-\frac{2}{3}}\times\left(\frac{1}{4}\right)^{-2}$ is equal to :

(1) 1 (2) 2

(3) $\frac{1}{2}$ (4) $\frac{1}{16}$

(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

39. $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ simplifies to :

(1) $\sqrt{5}+\sqrt{6}$ (2) $2\sqrt{5}+\sqrt{6}$

(3) $\sqrt{5}-\sqrt{6}$ (4) $2\sqrt{5}-3\sqrt{6}$

(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

40. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ simplifies to :

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $16-\sqrt{3}$ (4) $40-\sqrt{3}$

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

41. $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2+\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$ is equal to :

(1) 64 (2) 62
(3) 66 (4) 68

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

42. The value of

$$\sqrt{\frac{\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{3}+\sqrt{2}\right)}{5+\sqrt{24}}}$$ is :

(1) $\sqrt{6}-\sqrt{2}$ (2) $\sqrt{6}+\sqrt{2}$
(3) $\sqrt{6}-2$ (4) $2-\sqrt{6}$

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

43. Simplify :

$$\left[64^{\frac{2}{3}}\times 2^{-2}\div 8^0\right]^{\frac{1}{2}}$$

(1) 0 (2) 1
(3) 2 (4) $\frac{1}{2}$

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

44. The value of

$$\frac{1}{\sqrt{\left(12-\sqrt{140}\right)}}-\frac{1}{\sqrt{\left(8-\sqrt{60}\right)}}-\frac{2}{\sqrt{10+\sqrt{84}}}$$

is :

(1) 0 (2) 1
(3) 2 (4) 3

(SSC CGL Prelim Exam. 24.02.2002 (IInd Sitting) & SSC CGL Exam. 13.11.2005 (IInd Sitting)

45. The value of

$$\sqrt{11+2\sqrt{30}}-\frac{1}{\sqrt{11+2\sqrt{30}}}$$ is

(1) $2\sqrt{5}$ (2) $2\sqrt{6}$
(3) $1+\sqrt{6}$ (4) $1+\sqrt{5}$

(SSC CGL Prelim Exam. 24.02.2002 (Middle Zone)

46. The value of $(243)^{0.16}\times(243)^{0.04}$ is equal to :

(1) 0.16 (2) 3
(3) $\frac{1}{3}$ (4) 0.04

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

47. $\frac{3^0+3^{-1}}{3^{-1}-3^0}$ is simplified to

(1) –2 (2) –1
(3) 1 (4) 2

(SSC CPO S.I. Exam. 05.09.2004)

48. Simplify

$$\frac{1}{\sqrt{100}-\sqrt{99}}-\frac{1}{\sqrt{99}-\sqrt{98}}+\frac{1}{\sqrt{98}-\sqrt{97}}-\frac{1}{\sqrt{97}-\sqrt{96}}+\ldots+\frac{1}{\sqrt{2}-\sqrt{1}}$$

(1) 0 (2) 9
(3) 10 (4) 11

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005)

49. $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$ in simplified form equals to :

(1) 1 (2) $\sqrt{2}$
(3) $\frac{1}{\sqrt{2}}$ (4) 0

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

50. $\left[\sqrt[3]{2}\times\sqrt{2}\times\sqrt[3]{3}\times\sqrt{3}\right]$ is equal to

(1) 6^5 (2) $6^{5/6}$
(3) 6
(4) None of these

(SSC CGL Prelim Exam. 13.11.2005 (Second Sitting)

51. The value of $(256)^{0.16}\times(256)^{0.09}$ is :

(1) 256.25 (2) 64
(3) 16 (4) 4

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

52. $\left[8-\left(\frac{4^{\frac{9}{4}}\sqrt{2.2^2}}{2\sqrt{2^{-2}}}\right)^{\frac{1}{2}}\right]$ is equal to

(1) 32 (2) 8
(3) 1 (4) 0

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

53. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{2\sqrt{6}}{\sqrt{3}+1}+\frac{2\sqrt{3}}{\sqrt{6}+2}$ is equal to

(1) 3 (2) 2
(3) 0 (4) $\sqrt{3}$

(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

54. $(4)^{0.5}\times(0.5)^4$ is equal to :

(1) 1 (2) 4
(3) $\frac{1}{8}$ (4) $\frac{1}{32}$

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

55. $\left[\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right]$ simplifies to

(1) $2\sqrt{6}$ (2) $4\sqrt{6}$
(3) $2\sqrt{3}$ (4) $3\sqrt{2}$

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

56. The value of $\sqrt{40+\sqrt{9\sqrt{81}}}$ is

(1) $\sqrt{111}$ (2) 9
(3) 7 (4) 11

(SSC CHSL DEO & LDC Exam. 20.10.2013)

57. $\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}$ is equal to

(1) 5 (2) 1
(3) 3 (4) 0

(SSC CGL Prelim Exam. 27.07.2008 (Second Sitting)

58. Simplified form of

$$\left[\left(\sqrt[5]{x^{-3/5}}\right)^{-5/3}\right]^5$$ is

(1) x^5 (2) x^{-5}
(3) x (4) $\frac{1}{x}$

(SSC CGL Tier-I Exam. 16.05.2010 (Second Sitting)

59. $\left[\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{2}-1}{\sqrt{2}+1}\right]$ is simplified to

(1) 10 (2) 12
(3) 14 (4) 18

(SSC (South Zone) Investigator Exam. 12.09.2010)

60. $\dfrac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is equal to

(1) 3 (2) $\sqrt{3}$

(3) $3\sqrt{2}$ (4) $2\sqrt{3}$

(SSC (South Zone) Investigator Exam. 12.09.2010)

61. $\left(\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ simplifies to

(1) $\sqrt{5}+\sqrt{6}$ (2) $2\sqrt{5}+\sqrt{6}$

(3) $\sqrt{5}-\sqrt{6}$ (4) $2\sqrt{5}-3\sqrt{6}$

FCI Assistant Grade-III Exam. 25.02.2012 (Paper-I) North Zone (Ist Sitting)

62. When simplified equal to $(256)^{-\left(4^{-\frac{3}{2}}\right)}$ is

(1) 8 (2) $\dfrac{1}{8}$

(3) 2 (4) $\dfrac{1}{2}$

FCI Assistant Grade-III Exam. 25.02.2012 (Paper-I) North Zone (Ist Sitting)

63. $\left\{(-2)^{(-2)}\right\}^{(-2)}$ is equal to :

(1) 16 (2) 8

(3) –8 (4) –1

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

64. $\left(\sqrt{2}+\sqrt{7-2\sqrt{10}}\right)$ is equal to

(1) $\sqrt{2}$ (2) $\sqrt{7}$

(3) $\sqrt{5}$ (4) $2\sqrt{5}$

(SSC Data Entry Operator Exam. 31.08.2008)

65. $(256)^{0.16}\times(4)^{0.36}$ is equal to

(1) 64 (2) 16

(3) 256.25 (4) 4

(SSC Data Entry Operator Exam. 02.08.2009)

66. By how much does $5\sqrt{7}-2\sqrt{5}$ exceed $3\sqrt{7}-4\sqrt{5}$?

(1) $5\left(\sqrt{7}+\sqrt{5}\right)$ (2) $\sqrt{7}+\sqrt{5}$

(3) $2\left(\sqrt{7}+\sqrt{5}\right)$ (4) $7\left(\sqrt{2}+\sqrt{5}\right)$

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

67. $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}+\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ is equal to :

(1) 12 (2) $6\sqrt{35}$

(3) 6 (4) $2\sqrt{35}$

(SSC HSL DEO & LDC Exam. 28.11.2010 (Ist Sitting)

68. $\left(\dfrac{2}{\sqrt{6}+2}+\dfrac{1}{\sqrt{7}+\sqrt{6}}+\dfrac{1}{\sqrt{8}-\sqrt{7}}+2-2\sqrt{2}\right)$ is equal to

(1) 0 (2) $2\sqrt{2}$

(3) $\sqrt{6}$ (4) $2\sqrt{7}$

(SSC HSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

69. By how much does $\left(\sqrt{12}+\sqrt{18}\right)$ exceed $\left(2\sqrt{3}+2\sqrt{2}\right)$?

(1) 2 (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) 3

(SSC CGL Prelim Exam. 27.07.2008 (Second Sitting)

70. The value of $\dfrac{1}{\sqrt{2}+1}+\dfrac{1}{\sqrt{3}+\sqrt{2}}+\dfrac{1}{\sqrt{4}+\sqrt{3}}+....+\dfrac{1}{\sqrt{100}+\sqrt{99}}$ is

(1) 1 (2) 9

(3) $\sqrt{99}$ (4) $\sqrt{99}-1$

(SSC Multi-Tasking (Non-Technical) Staff Exam. 27.02.2011)

71. $\left[\left\{\left(-\dfrac{1}{2}\right)^{2}\right\}^{-2}\right]^{-1}$ is equal to :

(1) $\dfrac{1}{16}$ (2) 16

(3) $-\dfrac{1}{16}$ (4) –16

(SSC HSL DEO & LDC Exam. 27.11.2010)

72. $2\sqrt[3]{40}-4\sqrt[3]{320}+3\sqrt[3]{625}-3\sqrt[3]{5}$ is equal to

(1) $-2\sqrt[3]{340}$ (2) 0

(3) $\sqrt[3]{340}$ (4) $\sqrt[3]{660}$

(SSC CGL Tier-II Exam.16.09.2012)

73. The value of $\sqrt[3]{0.000125}$ is

(1) 0.005 (2) 0.05

(3) 0.5 (4) 0.0005

(SSC Assistant Grade-III Exam. 11.11.2012 (IInd Sitting)

74. $\dfrac{0.3555\times0.5555\times2.025}{0.225\times1.7775\times0.2222}$ is equal to

(1) 5.4 (2) 4.58

(3) 4.5 (4) 5.45

(SSC CHSL DEO & LDC Exam. 04.11.2012, IInd Sitting)

75. The simplification of

$$\frac{0.06\times0.06\times0.06-0.05\times0.05\times0.05}{0.06\times0.06+0.06\times0.05+0.05\times0.05}$$

gives :

(1) 0.01 (2) 0.001

(3) 0.1 (4) 0.02

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

76. Simplify :

$$\frac{0.05\times0.05\times0.05-0.04\times0.04\times0.04}{0.05\times0.05+0.002+0.04\times0.04}$$

(1) 1 (2) 0.1

(3) 0.01 (4) 0.001

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

77. If $\dfrac{\left(x-\sqrt{24}\right)\left(\sqrt{75}+\sqrt{50}\right)}{\sqrt{75}-\sqrt{50}}=1$, then the value of x is

(1) $\sqrt{5}$ (2) 5

(3) $2\sqrt{5}$ (4) $3\sqrt{5}$

(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

78. Evaluate

$$\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\frac{4}{\sqrt{5}-\sqrt{3}}-\sqrt{81}$$

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) 0 (4) $2\sqrt{2}$

(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

79. Let

$$a=\frac{1}{2-\sqrt{3}}+\frac{1}{3-\sqrt{8}}+\frac{1}{4-\sqrt{15}}.$$

Then we have

(1) $a<18$ but $a\neq9$

(2) $a>18$

(3) $a=18$

(4) $a=9$

(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

80. If a, b are rationals and $a\sqrt{2}+b\sqrt{3}$ $=\sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$ then the values of a, b are respectively

(1) 1, 2 (2) 1, 3
(3) 2, 1 (4) 2, 3

(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

81. Let $\sqrt[3]{a}=\sqrt[3]{26}+\sqrt[3]{7}+\sqrt[3]{63}$. Then

(1) $a<729$ but $a>216$
(2) $a<216$
(3) $a>729$
(4) $a=729$

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

82. The value of

$$\frac{\sqrt{72}\times\sqrt{363}\times\sqrt{175}}{\sqrt{32}\times\sqrt{147}\times\sqrt{252}} \text{ is}$$

(1) $\frac{55}{42}$ (2) $\frac{45}{56}$
(3) $\frac{45}{28}$ (4) $\frac{55}{28}$

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

83. Simplify :

$$\frac{5.32\times56+5.32\times44}{(7.66)^2-(2.34)^2}$$

(1) 7.2 (2) 8.5
(3) 10 (4) 12

(SSC CGL Prelim Exam. 04.07.1999 (IInd Sitting) & (SSC SO Commercial Audit Exam. 16.11.2003)

84. $2+\frac{6}{\sqrt{3}}+\frac{1}{2+\sqrt{3}}+\frac{1}{\sqrt{3}-2}$ equals to

(1) $+\left(2\sqrt{3}\right)$ (2) $-\left(2+\sqrt{3}\right)$
(3) 1 (4) 2

(SSC Multi-Tasking Staff Exam. 10.03.2013, Ist Sitting : Patna)

85. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}}=\text{A}+\sqrt{\text{B}}$, then B – A is

(1) –13 (2) $2\sqrt{13}$
(3) 13 (4) $3\sqrt{3}-\sqrt{7}$

(SSC CGL Tier-I Exam. 21.04.2013 IInd Sitting)

86. Find the simplest value of $2\sqrt{50}+\sqrt{18}-\sqrt{72}$ (given $\sqrt{2}$ = 1.414).

(1) 4.242 (2) 9.898
(3) 10.312 (4) 8.484

(SSC CGL Tier-I Exam. 19.05.2013 Ist Sitting)

87. $(6.5\times6.5-45.5+3.5\times3.5)$ is equal to :

(1) 10 (2) 9
(3) 7 (4) 6

(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

88. $(7.5\times7.5+37.5+2.5\times2.5)$ is equal to :

(1) 100 (2) 80
(3) 60 (4) 30

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

89. Simplify :

$$\frac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$$

(1) 0 (2) 1
(3) 10 (4) 30

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

90. Simplify :

$$\frac{(6.25)^{\frac{1}{2}}\times(0.0144)^{\frac{1}{2}}+1}{(0.027)^{\frac{1}{3}}\times(81)^{\frac{1}{4}}}$$

(1) 0.14 (2) 1.4
(3) 1 (4) $1.\overline{4}$

(SSC CGL Prelim Exam. 24.02.2002 (Ist Sitting) & (SSC CGL Prelim Exam. 13.11.2005)

91. Simplify :

$$\frac{0.41\times0.41\times0.41+0.69\times0.69\times0.69}{0.41\times0.41-0.41\times0.69+0.69+0.69}$$

(1) 0.28 (2) 1.1
(3) 11 (4) 2.8

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

92. $\frac{10.3\times10.3\times10.3+1}{10.3\times10.3-10.3+1}$ is equal to :

(1) 9.3 (2) 10.3
(3) 11.3 (4) 12.3

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

93. $\frac{1.49\times14.9-0.51\times5.1}{14.9-5.1}$ is equal to :

(1) 0.20 (2) 20.00
(3) 2.00 (4) 22.00

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

94. $(0.04)^{-1.5}$ on simplification gives :

(1) 25 (2) 125
(3) 250 (4) 625

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

95. $\frac{(0.96)^3-(0.1)^3}{(0.96)^2+0.096+(0.1)^2}$ is simplified to :

(1) 1.06 (2) 0.95
(3) 0.86 (4) 0.97

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

96. The value of $\frac{64-0.008}{16+0.8+0.04}$ is :

(1) 2 (2) 3.8
(3) 0.6 (4) 4.2

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

97. The value of

$$\frac{0.796\times0.796-0.204\times0.204}{0.796-0.204} \text{ is :}$$

(1) 0.408 (2) 0.59
(3) 0.592 (4) 1

(SSC CPO S.I. Exam. 26.05.2005)

98. $\frac{(2.3)^3+0.027}{(2.3)^2-0.69+0.09}$ is equal to :

(1) 2.60 (2) 2.00
(3) 2.33 (4) 2.80

(SSC CPO S.I. Exam. 26.05.2005)

99. The value of

$$\frac{5.71\times5.71\times5.71-2.79\times2.79\times2.79}{5.71\times5.71+5.71\times2.79+2.79\times2.79}$$

in simplified form is :

(1) 8.5 (2) 8.6
(3) 2.82 (4) 2.92

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

100. The value of

$$\frac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5} \text{ is :}$$

(1) 0 (2) 1
(3) 10 (4) 30

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

101. $\left[\dfrac{(0.73)^3+(0.27)^3}{(0.73)^2+(0.27)^2-(0.73)\times(0.27)}\right]$ simplifies to

(1) 1 (2) 0.4087
(3) 0.73 (4) 0.27

(SSC CGL Prelim Exam. 13.11.2005 (Second Sitting)

102. 0.75 × 7.5 – 2 × 7.5 × 0.25 + 0.25 × 2.5 is equal to

(1) 250 2) 2500
(3) 2.5 (4) 25

(SSC CPO S.I. Exam. 03.09.2006)

103. $\left(\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+\frac{1}{13.16}\right)$ is equal to

(1) $\frac{1}{3}$ (2) $\frac{5}{16}$
(3) $\frac{3}{8}$ (4) $\frac{41}{7280}$

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

104. $\dfrac{137\times137+133\times133+18221}{137\times137\times137-133\times133\times133}$ is equal to

(1) 4 (2) 270
(3) $\frac{1}{4}$ (4) $\frac{1}{270}$

(SSC CGL Prelim Exam. 04.02.2007 (IInd Sitting) & (SSC CGL Prelim Exam. 27.07.2008) & (SSC DEO Exam. 31.08.2008)

105. $\left(\dfrac{2.75\times2.75\times2.75-2.25\times2.25\times2.25}{2.75\times2.75+2.75\times2.25+2.25\times2.25}\right)$ is equal to :

(1) – 5 (2) 0.5
(3) –0.5 (4) 5

(SSC CPO S.I. Exam. 16.12.2007)

106. $\dfrac{(5.624)^3+(4.376)^3}{5.624\times5.624-(5.624\times4.376)+4.376\times4.376}$ is equal to

(1) 10 (2) 1.248
(3) 20.44 (4) 1

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

107. The value of $\left[\dfrac{(0.337+0.126)^2-(0.337-0.126)^2}{0.337\times0.126}\right]$ is

(1) 4 (2) 0.211
(2) 0.463 (4) 0.4246

(SSC CPO S.I. Exam. 06.09.2009)

108. $\dfrac{256\times256-144\times144}{112}$ is equal to

(1) 420 (2) 400
(3) 360 (4) 320

(SSC CGL Tier-I Exam. 16.05.2010 (First Sitting)

109. [8.7 × 8.7 + 2 × 8.7 × 1.3 + 1.3 × 1.3] is equal to

(1) 1.69 (2) 10
(3) 75.69 (4) 100

(SSC (South Zone) Investigator Exam. 12.09.2010)

110. $\dfrac{(3.06)^3-(1.98)^3}{(3.06)^2+3.06\times1.98+(1.98)^2}$ is equal to

(1) 1.08 (2) 5.04
(3) 2.16 (4) 1.92

(SSC (South Zone) Investigator Exam. 12.09.2010)

111. $\dfrac{3.25\times3.25+1.75\times1.75-2\times3.25\times1.75}{3.25\times3.25-1.75\times1.75}$ is simplified to

(1) 0.5 (2) 0.4
(3) 0.3 (4) 0.2

(SSC CPO Sub-Inspector Exam. 12.12.2010 (Paper-I)

112. $\dfrac{(0.05)^2+(0.41)^2+(0.073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}$ is

(1) 10 (2) 100
(3) 1000 (4) None of these

(SSC CGL Tier-1 Exam 26.06.2011 (First Sitting)

113. $\dfrac{2.3\times2.3\times2.3-1}{2.3\times2.3+2.3+1}$ is equal to

(1) 1.3 (2) 3.3
(3) 0.3 (4) 2.2

(SSC CPO S.I. Exam. 07.09.2003)

114. Find the value of : $(0.98)^3+(0.02)^3+3\times0.98\times0.02-1$

(1) 1.98 (2) 1.09
(3) 1 (4) 0

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

115. $\dfrac{0.08\times0.08\times0.08+0.02\times0.02\times0.02}{0.08\times0.08-0.0016+0.02\times0.02}$ is simplified to :

(1) 0.001 (2) 0.1
(3) 0.0016 (4) 0.016

(SSC CHSL DEO & LDC Exam. 27.11.2010)

116. The value of 0.65 × 0.65 + 0.35 × 0.35 + 0.70 × 0.65 is

(1) 1.75 (2) 1.00
(3) 1.65 (4) 1.55

(SSC Constable (GD) Exam. 12.05.2013 Ist Sitting)

117. $(2.4\times10^3)\div(8\times10^{-2})$ equals

(1) 3×10^5 (2) 3×10^4
(3) 3×10^{-5} (4) 30

(SSC CPO S.I. Exam. 12.01.2003)

118. $[3-4(3-4)^{-1}]^{-1}$ is equal to :

(1) 7 (2) –7
(3) $\frac{1}{7}$ (4) $-\frac{1}{7}$

(SSC CPO S.I.Exam. 26.05.2005)

119. $\dfrac{(998)^2-(997)^2-45}{(98)^2-(97)^2}$ equals

(1) 1995 (2) 195
(3) 95 (4) 10

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

120. Evaluate : $\dfrac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$.

(1) 2 (2) 3
(3) 4 (4) 5

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

121. $55^3+17^3-72^3+201960$ is equal to

(1) –1 (2) 0
(3) 1 (4) 17

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

122. What is the value of $\dfrac{2.75\times2.75\times2.75-2.25\times2.25\times2.25}{2.75\times2.75+2.75\times2.25+2.25\times2.25}$ is

(1) 3 (2) $\frac{3}{2}$
(3) 1 (4) $\frac{1}{2}$

(SSC CGL Tier-I Exam. 26.10.2014)

123. The value of $\dfrac{(243)^{\frac{n}{5}}\times3^{2n+1}}{9^n\times3^{n-1}}$ is

(1) 3 (2) 9
(3) 6 (4) 12

(SSC CGL Tier-I Exam. 26.10.2014

124. The simplified value of $(\sqrt{3}+1)(10+\sqrt{12})(\sqrt{12}-2)(5-\sqrt{3})$ is

(1) 16 (2) 88
(3) 176 (4) 132

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014

125. The simplified value of $(0.2)^3 \times 200 \div 2000$ of $(0.2)^2$ is

(1) $\frac{1}{100}$ (2) $\frac{1}{50}$
(3) $\frac{1}{10}$ (4) 1

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

126. The simplified value of

$$\left(\sqrt{6}+\sqrt{10}-\sqrt{21}-\sqrt{35}\right)\left(\sqrt{6}-\sqrt{10}+\sqrt{21}-\sqrt{35}\right)$$ is

(1) 13 (2) 12
(3) 11 (4) 10

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

127. The value of

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$ is

(1) 2 (2) 0
(3) 4 (4) 1

(SSC CGL Tier-II Exam. 12.04.2015 (TF No. 567 TL 9)

128. The value of

$$\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{3-\sqrt{8}}$$ is

(1) 7 (2) 0
(3) 1 (4) 5

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

129. If $2+x\sqrt{3}=\frac{1}{2+\sqrt{3}}$, then the simplest vlaue of x is

(1) – 1 (2) 1
(3) – 2 (4) 2

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

130. The value of :

$$\sqrt{\frac{0.324\times 0.081\times 4.624}{1.5625\times 0.0289\times 72.9\times 64}}$$ is

(1) 2.4 (2) 24
(3) 0.024 (4) 0.24

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

131. If $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a+\sqrt{7}\,b$, then the values of a and b are respectively

(1) $\sqrt{7}$, –1 (2) $\sqrt{7}$, 1
(3) 0, $-\frac{2}{3}$ (4) $-\frac{2}{3}$, 0

(SSC CGL Tier-I Re-Exam, 30.08.2015)

132. The value of

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots+\frac{1}{\sqrt{8}+\sqrt{9}}$$ is

(1) 1 (2) 0
(3) 2 (4) $\sqrt{2}$

(SSC CGL Tier-I Re-Exam, 30.08.2015)

133. If $\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}=\sqrt{3}$, then $a : b$ is equal to

(1) $2:\sqrt{3}$ (2) $\sqrt{3}:4$
(3) $\sqrt{3}:2$ (4) $4:\sqrt{3}$

(SSC CGL Tier-I Re-Exam, 30.08.2015)

134. The value of

$$\frac{(75.8)^2-(35.8)^2}{40}$$ is

(1) 121.6 (2) 40
(3) 160 (4) 111.6

(SSC Constable (GD) Exam, 04.10.2015, IInd Sitting)

135. The value of

$$\frac{(0.67\times 0.67\times 0.67)-(0.33\times 0.33\times 0.33)}{(0.67\times 0.67)-(0.67\times 0.33)-(0.33\times 0.33)}$$ is

(1) 11 (2) 1.1
(3) 3.4 (4) 0.34

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

136. The value of $\frac{1}{1+\sqrt{2}+\sqrt{3}}+\frac{1}{1-\sqrt{2}+\sqrt{3}}$ is :

(1) $\sqrt{2}$ (2) $\sqrt{3}$
(3) 1 (4) $4\left(\sqrt{3}+\sqrt{2}\right)$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (Ist Sitting) TF No. 1375232)

137. If $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of $\frac{a^2}{b}+\frac{b^2}{a}$ is :

(1) 1030 (2) 1025
(3) 970 (4) 930

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

138. If $1^3+2^3+\ldots..+10^3=3025$, then the value of $2^3+4^3+\ldots\ldots+20^3$ is :

(1) 7590 (2) 5060
(3) 24200 (4) 12100

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

139. The value of $\frac{(2.3)^3+0.027}{(2.3)^2-0.69+0.09}$ is :

(1) 2 (2) 2.27
(3) 2.33 (4) 2.6

(SSC CHSL (10+2) Tier-I (CBE) Exam. 08.09.2016) (Ist Sitting)

140. The value of $\left(1-\sqrt{2}\right)+\left(\sqrt{2}-\sqrt{3}\right)+\left(\sqrt{3}-\sqrt{4}\right)+\ldots\ldots+\left(\sqrt{15}-\sqrt{16}\right)$ is

(1) 0 (2) 1
(3) –3 (4) 4

(SSC CGL Tier-I (CBE) Exam. 09.09.2016) (Ist Sitting)

141. The simplified value of the following expression is :

$$\frac{1}{\sqrt{11-2\sqrt{30}}} - \frac{3}{\sqrt{7-2\sqrt{10}}} - \frac{4}{\sqrt{8+4\sqrt{3}}}$$

(1) 0 (2) 1

(3) $\sqrt{2}$ (4) $\sqrt{3}$

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 20.03.2016) (IInd Sitting)

142. Find the value of

$$\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}.$$

(1) 3 (2) 9
(3) 27 (4) 4

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

143. The value of $(d^{s+t} \div d^s) \div d^t$ would be

(1) $d^{2(s+t)}$ (2) 1
(3) 0 (4) d^{s-t}

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (Ist Sitting)

144. $(2^{51} + 2^{52} + 2^{53} + 2^{54} + 2^{55})$ is divisible by

(1) 23 (2) 58
(3) 124 (4) 127

(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (Ist Sitting)

145. If $\frac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}} = 2$, the value of x is

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$

(3) $\frac{8}{5}$ (4) $\frac{1}{5}$

(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (Ist Sitting)

146. The value of

$$\frac{3 \times 9^{n+1} + 9 \times 3^{2n-1}}{9 \times 3^{2n} - 6 \times 9^{n-1}}$$ is equal to

(1) $3\frac{3}{5}$ (2) $3\frac{2}{5}$

(3) $3\frac{1}{5}$ (4) 3

(SSC CGL Tier-I (CBE) Exam. 06.09.2016) (Ist Sitting)

147. The value of $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} - 4\sqrt{3}\right)^2$ is

(1) 36 (2) $36\sqrt{3}$
(3) 49 (4) $49 + \sqrt{3}$

(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (IInd Sitting)

148. Simplify :

$$\sqrt[3]{-2197} \times \sqrt[3]{-125} \div \sqrt[3]{\frac{27}{512}}$$

(1) $\frac{492}{7}$ (2) $\frac{520}{3}$

(3) $\frac{554}{7}$ (4) $\frac{571}{5}$

(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

149. On simplification the value of $1 - \frac{1}{1+\sqrt{2}} + \frac{1}{1-\sqrt{2}}$ is

(1) $2\sqrt{2} - 1$ (2) $1 - 2\sqrt{2}$
(3) $1 - \sqrt{2}$ (4) $-2\sqrt{2}$

(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (IInd Sitting)

150. The simplest value of

$$\frac{3\sqrt{8} - 2\sqrt{12} + \sqrt{20}}{3\sqrt{18} - 2\sqrt{27} + \sqrt{45}}$$ is :

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) 2

(SSC CGL Tier-I (CBE) Exam. 29.08.2016) (IST Sitting)

151. The simplified value of

$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$$ is

(1) 0 (2) 1
(3) 5 (4) 6

(SSC CGL Tier-I (CBE) Exam. 01.09.2016 (IIIrd Sitting)

152. Simplify :

$$\frac{(0.73)^3 + (0.27)^3}{(0.73)^2 + (0.27)^2 - (0.73) \times (0.27)}$$

(1) 1 (2) 0.4087
(3) 0.73 (4) 0.27

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IIIrd Sitting)

153. The simplified value of $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}-\sqrt{18}} - \frac{1}{3} \times \sqrt{27} - \frac{1}{2} \times \sqrt[3]{27}$ is closest to

(1) $\left(\sqrt{3}-1\right)$ (2) $\left(1-\sqrt{3}\right)$

(3) $-\left(-\sqrt{3}-1\right)$ (4) $-\left(\sqrt{3}+1\right)$

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

TYPE–II

1. Which one of the following is the least?

$$\sqrt{3}, \sqrt[3]{2}, \sqrt{2} \text{ and } \sqrt[3]{4}$$

(1) $\sqrt{2}$ (2) $\sqrt[3]{4}$

(3) $\sqrt{3}$ (4) $\sqrt[3]{2}$

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

2. Which of the following is the biggest ?

$$\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15}, \text{ and } \sqrt[12]{245},$$

(1) $\sqrt[3]{4}$ (2) $\sqrt[4]{6}$

(3) $\sqrt[6]{15}$ (4) $\sqrt[12]{245}$

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

3. Which of the following number is the least?

$$(0.5)^2, \sqrt{0.49}, \sqrt[3]{0.008}, 0.23$$

(1) $(0.5)^2$ (2) $\sqrt{0.49}$

(3) $\sqrt[3]{0.008}$ (4) 0.23

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

4. Arrange the following in descending order : $\sqrt[3]{4}$, $\sqrt{2}$, $\sqrt[6]{3}$, $\sqrt[4]{5}$

(1) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

(2) $\sqrt[4]{5} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt{2}$

(3) $\sqrt{2} > \sqrt[6]{3} > \sqrt[3]{4} > \sqrt[4]{5}$

(4) $\sqrt[6]{3} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt{2}$

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

5. The greatest of the numbers $(2.89)^{0.5}$, $2-(0.5)^2$, $1+\frac{0.5}{1-\frac{1}{2}}$, $\sqrt{3}$ is :

(1) $(2.89)^{0.5}$ (2) $2-(0.5)^2$

(3) $1+\frac{0.5}{1-\frac{1}{2}}$ (4) $\sqrt{3}$

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

6. Among $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{5}, \sqrt[3]{2}$ which one is the greatest ?

(1) $\sqrt[4]{5}$ (2) $\sqrt{2}$

(3) $\sqrt[3]{3}$ (4) $\sqrt[3]{2}$

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

7. The ascending order of $(2.89)^{0.5}$, $2-(0.5)^2$, $\sqrt{3}$ and $\sqrt[3]{0.008}$ is

(1) $2-(0.5)^2$, $\sqrt{3}$, $\sqrt[3]{0.008}$, $(2.89)^{0.5}$

(2) $\sqrt[3]{0.008}$, $(2.89)^{0.5}$, $\sqrt{3}$, $2-(0.5)^2$

(3) $\sqrt[3]{0.008}$, $\sqrt{3}$, $(2.89)^{0.5}$, $2-(0.5)^2$

(4) $\sqrt{3}$, $\sqrt[3]{0.008}$, $2-(0.5)^2$, $(2.89)^{0.5}$

(SSC CGL Prelim Exam. 11.05.2003 (First Sitting)

8. The greatest one of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[6]{6}$, $\sqrt[5]{5}$ is

(1) $\sqrt{2}$ (2) $\sqrt[3]{3}$
(3) $\sqrt[6]{6}$ (4) $\sqrt[5]{5}$

(SSC CPO S.I. Exam. 07.09.2003)

9. The smallest of $\sqrt{8}+\sqrt{5}$, $\sqrt{7}+\sqrt{6}$, $\sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ is :

(1) $\sqrt{8}+\sqrt{5}$ (2) $\sqrt{7}+\sqrt{6}$

(3) $\sqrt{10}+\sqrt{3}$ (4) $\sqrt{11}+\sqrt{2}$

(SSC CPO S.I. Exam. 26.05.2005)

10. Which of the following is the largest number ?

$\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}$

(1) $\sqrt{2}$ (2) $\sqrt[3]{3}$

(3) $\sqrt[4]{4}$ (4) $\sqrt[6]{6}$

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005) & SSC CGL Prelim Exam. 27.07.2008 (Ist Sitting)

11. Which is the greatest among $(\sqrt{19}-\sqrt{17})$, $(\sqrt{13}-\sqrt{11})$, $(\sqrt{7}-\sqrt{5})$ and $(\sqrt{5}-\sqrt{3})$?

(1) $\sqrt{19}-\sqrt{17}$ (2) $\sqrt{13}-\sqrt{11}$

(3) $\sqrt{7}-\sqrt{5}$ (4) $\sqrt{5}-\sqrt{3}$

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

12. The greatest number among $\sqrt[3]{2}$, $\sqrt{3}$, $\sqrt[3]{5}$ and 1.5 is :

(1) $\sqrt[3]{2}$ (2) $\sqrt[3]{5}$

(3) $\sqrt{3}$ (4) 1.5

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

13. The greatest of

$\sqrt{2}$, $\sqrt[6]{3}$, $\sqrt[3]{4}$, $\sqrt[4]{5}$ is

(1) $\sqrt{2}$ (2) $\sqrt[6]{3}$

(3) $\sqrt[3]{4}$ (4) $\sqrt[4]{5}$

(SSC CGL Prelim Exam. 13.11.2005 (IInd Sitting) & SSC (10+2) DEO & LDC Exam. 11.12.2011 (East Zone)

14. The greatest one of $\sqrt{4}$, $\sqrt[3]{4}$, $\sqrt[4]{6}$ and $\sqrt[6]{8}$ is

(1) $\sqrt{3}$ (2) $\sqrt[3]{4}$

(3) $\sqrt[4]{6}$ (4) $\sqrt[6]{8}$

(SSC Section Officer (Commercial Audit) Exam. 26.11.2006 (Second Sitting)

15. The greatest among

$\sqrt{7}-\sqrt{5}$, $\sqrt{5}-\sqrt{3}$, $\sqrt{9}-\sqrt{7}$, $\sqrt{11}-\sqrt{9}$ is

(1) $\sqrt{7}-\sqrt{5}$ (2) $\sqrt{5}-\sqrt{3}$

(3) $\sqrt{9}-\sqrt{7}$ (4) $\sqrt{11}-\sqrt{9}$

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

16. Greatest among the numbers

$\sqrt[3]{9}$, $\sqrt{3}$, $\sqrt[4]{16}$, $\sqrt[6]{80}$ is

(1) $\sqrt[3]{9}$ (2) $\sqrt{3}$

(3) $\sqrt[4]{16}$ (4) $\sqrt[6]{80}$

(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

17. The least one of $2\sqrt{3}$, $2\sqrt[4]{5}$, $\sqrt{8}$ and $3\sqrt{2}$ is

(1) $2\sqrt{3}$ (2) $2\sqrt[4]{5}$

(3) $\sqrt{8}$ (4) $3\sqrt{2}$

(SSC Section Officer (Commercial Audit) Exam. 30.09.2007 (Second Sitting)

18. Out of the numbers 0.3, 0.03, 0.9, 0.09 the number that is nearest to the value of $\sqrt{0.9}$ is

(1) 0.3 (2) 0.03

(3) 0.9 (4) 0.09

(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

19. The greatest number among 2^{60}, 3^{48}, 4^{36} and 5^{24} is

(1) 2^{60} (2) 3^{48}
(3) 4^{36} (4) 5^{24}

(SSC SAS Exam 26.06.2010 (Paper-1)

20. The greatest among the numbers $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$, $\sqrt[6]{6}$ is

(1) $\sqrt{2}$ (2) $\sqrt[3]{3}$

(3) $\sqrt[6]{6}$ (4) $\sqrt[4]{5}$

(SSC (South Zone) Investigator Exam 12.09.2010)

21. The smallest among $\sqrt[6]{12}$, $\sqrt[3]{4}$, $\sqrt[4]{5}$, $\sqrt{3}$ is

(1) $\sqrt[6]{12}$ (2) $\sqrt[3]{4}$

(3) $\sqrt{3}$ (4) $\sqrt[4]{5}$

(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I) & SSC (10+2) Data Entry Operator & LDC Exam 11.12.2011 (Delhi Zone)

22. The largest among the numbers 0.9, $(0.9)^2$, $\sqrt{0.9}$, $0.\bar{9}$ is :

(1) 0.9 (2) $(0.9)^2$

(3) $\sqrt{0.9}$ (4) $0.\bar{9}$

(SSC CHSL DEO & LDC Exam. 27.11.2010)

23. Among the numbers $\sqrt{2}$, $\sqrt[3]{9}$, $\sqrt[4]{16}$, $\sqrt[5]{32}$, the greatest one is

(1) $\sqrt{2}$ (2) $\sqrt[3]{9}$

(3) $\sqrt[4]{16}$ (4) $\sqrt[5]{32}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

24. The greatest among the numbers $\sqrt[4]{3}$, $\sqrt[5]{4}$, $\sqrt[10]{12}$, 1 is

(1) 1 (2) $\sqrt[5]{4}$

(3) $\sqrt[4]{3}$ (4) $\sqrt[10]{12}$

(SSC CHSL DEO & LDC Exam. 04.12.2011(IInd Sitting (North Zone)

25. The greatest among the numbers $3\sqrt{2}$, $3\sqrt{7}$, $6\sqrt{5}$, $2\sqrt{20}$ is

(1) $3\sqrt{2}$ (2) $3\sqrt{7}$

(3) $6\sqrt{5}$ (4) $2\sqrt{20}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

26. The greatest among the numbers $\sqrt{0.09}, \sqrt[3]{0.064}$, 0·5 and $\frac{3}{5}$ is

(1) $\sqrt{0.09}$ (2) $\sqrt[3]{0.064}$

(3) 0·5 (4) $\frac{3}{5}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone)

27. The largest number among $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$ is

(1) $\sqrt{2}$ (2) $\sqrt[3]{3}$

(3) $\sqrt[4]{4}$ (4) All are equal

(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (Delhi Zone)

28. The greatest of the following numbers 0.16, $\sqrt{0.16}$, $(0.16)^2$, 0.04 is

(1) 0.16 (2) $\sqrt{0.16}$

(3) 0.04 (4) $(0.16)^2$

(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

29. The smallest among the numbers 2^{250}, 3^{150}, 5^{100} and 4^{200}

(1) 4^{200} (2) 5^{100}

(3) 3^{150} (4) 2^{250}

(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

30. The greatest of the numbers $\sqrt[2]{8}$, $\sqrt[4]{13}$, $\sqrt[5]{16}$, $\sqrt[10]{41}$ is:

(1) $\sqrt[4]{13}$ (2) $\sqrt[5]{16}$

(3) $\sqrt[10]{41}$ (4) $\sqrt[2]{8}$

(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone)

31. Which is greater $\sqrt[3]{2}$ or $\sqrt{3}$?

(1) Cannot be compared

(2) $\sqrt[3]{2}$

(3) $\sqrt{3}$

(4) Equal

(SSC CHSL DEO & LDC Exam. 20.10.2013)

32. Arranging the following in descending order, we get

$\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$

(1) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

(2) $\sqrt[4]{5} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt{2}$

(3) $\sqrt{2} > \sqrt[6]{3} > \sqrt[3]{4} > \sqrt[4]{5}$

(4) $\sqrt[6]{3} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt{2}$

(SSC CGL Tier-I Exam. 19.10.2014

33. The greatest number among the following is

$\frac{4}{9}, \sqrt{\frac{9}{49}}, 0.\dot{4}\dot{7}, (0.7)^2$

(1) $\frac{4}{9}$ (2) $\sqrt{\frac{9}{49}}$

(3) $0.\dot{4}\dot{7}$ (4) $(0.7)^2$

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting (TF No. 333 LO 2)

34. The greatest number among 3^{50}, 4^{40}, 5^{30} and 6^{20} is

(1) 3^{50} (2) 4^{40}

(3) 5^{30} (4) 6^{20}

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

35. Which is the largest among the numbers $\sqrt{5}$, $3\sqrt{7}$, $4\sqrt{13}$

(1) $\sqrt{5}$ (2) $3\sqrt{7}$

(3) $4\sqrt{13}$

(4) All are equal

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

36. If the numbers $\sqrt[3]{9}$, $\sqrt[4]{20}$, $\sqrt[6]{25}$ are arranged in adcending order, then the right arrangement is

(1) $\sqrt[6]{25} < \sqrt[4]{20} < \sqrt[3]{9}$

(2) $\sqrt[3]{9} < \sqrt[4]{20} < \sqrt[6]{25}$

(3) $\sqrt[4]{20} < \sqrt[6]{25} < \sqrt[3]{9}$

(4) $\sqrt[6]{25} < \sqrt[3]{9} < \sqrt[4]{20}$

(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IInd Sitting)

TYPE–III

1. Given $\sqrt{2}$ = 1.414. The value of $\sqrt{8}+2\sqrt{32}-3\sqrt{128}+4\sqrt{50}$ is

(1) 8.484 (2) 8.526

(3) 8.426 (4) 8.876

(SSC CGL Prelim Exam. 11.05.2003 (First Sitting)

2. If $\sqrt{15} = 3.88$, then what is the value of $\sqrt{\frac{5}{3}}$

(1) $1.29\overline{3}$ (2) 1.2934

(3) 1.29 (4) 1.295

(SSC CGL Prelim Exam. 11.05.2003 (Second Sitting)

3. If $\sqrt{3}$ = 1.732, then what is the value of $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}}$ upto three places of decimal ?

(1) 0.023 (2) 0.464

(3) 2.464 (4) 3.023

(SSC Section Officer (Commercial Audit) Exam. 25.09.2005)

4. Given that $\sqrt{3}$ = 1.732, the value of

$$\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$$ is

(1) 4.899 (2) 2.551

(3) 1.414 (4) 1.732

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

5. Given that $\sqrt{5} = 2.236$ and $\sqrt{3} = 1.732$; the value of $\frac{1}{\sqrt{5}+\sqrt{3}}$ is

(1) 0.504 (2) 0.252

(3) 0.362 (4) 0.372

(SSC CPO S.I. Exam. 16.12.2007)

6. Given that $\sqrt{5} = 2.24$, then the value of $\frac{3\sqrt{5}}{2\sqrt{5}-0.48}$ is

(1) 0.168 (2) 1.68

(3) 16.8 (4) 168

(SSC CPO S.I. Exam. 09.11.2008)

7. Given that $\sqrt{2} = 1.414$;

the value of $\frac{1}{\sqrt{2}+1}$ is

(1) 0.414 (2) 2.414

(3) 3.414 (4) 5.414

(SSC CPO S.I. Exam. 09.11.2008)

8. Evaluate :

$16\sqrt{\frac{3}{4}}-9\sqrt{\frac{4}{3}}$ if $\sqrt{12}$ =3.46

(1) 3.46 (2) 10.38

(3) 13.84 (4) 24.22

(SSC CPO S.I. Exam. 06.09.2009)

9. If $\sqrt{2}$ = 1.4142, find the value of

$$2\sqrt{2}+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$$

(1) 1.4144 (2) 2.8284

(3) 28.284 (4) 2.4142

(SSC CGL Tier-1 Exam 26.06.2011 (Second Sitting)

10. If $\sqrt{3}$ =1.732, is given, then the value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ is

(1) 11.732 (2) 13.928
(3) 12.928 (4) 13.925

(SSC Data Entry Operator Exam. 31.08.2008)

11. If $\sqrt{2}=1.4142...$ is given, then the value of $\frac{7}{(3+\sqrt{2})}$ correct upto two decimal places is

(1) 1.59 (2) 1.60
(3) 2.58 (4) 2.57

(SSC CHSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

12. If $\sqrt{5329}$ = 73, then the value of $\sqrt{5329}$ + $\sqrt{53.29}$ + $\sqrt{0.5329}$ + $\sqrt{0.005329}$ + $\sqrt{0.00005329}$ is

(1) 81. 1003 (2) 81. 0113
(3) 81. 1103 (4) 81. 1013

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

13. If $\sqrt{33}$ = 5.745, then the value of $\sqrt{\frac{3}{11}}$ is approximately

(1) 1 (2) 0.5223
(3) 6.32 (4) 2.035

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

14. If $\sqrt{7}$ = 2.646, then the value of $\frac{1}{\sqrt{28}}$ up to three places of decimal is :

(1) 0.183 (2) 0.185
(3) 0.187 (4) 0.189

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

15. If $\sqrt{5}$ = 2.236, then what is the value of $\frac{\sqrt{5}}{2}+\frac{5}{3\sqrt{5}}-\sqrt{45}$?

(1) –8.571 (2) –4.845
(3) –2.987 (4) –6.261

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

16. If $\sqrt{3}$ = 1.732, then the value of $\frac{9+2\sqrt{3}}{\sqrt{3}}$ is :

(1) 7.169 (2) 7.196
(3) 5.198 (4) 7.296

(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IInd Sitting)

TYPE–IV

1. The rationalising factor of $3\sqrt{3}$ is

(1) $\frac{1}{3}$ (2) 3
(3) – 3 (4) $\sqrt{3}$

(SSC CPO S.I. Exam. 07.09.2003)

2. A rationalising factor of $\left(\sqrt[3]{9}-\sqrt[3]{3}+1\right)$ is

(1) $\sqrt[3]{3}-1$ (2) $\sqrt[3]{3}+1$
(3) $\sqrt[3]{9}+1$ (4) $\sqrt[3]{9}-1$

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

3. The total number of prime factors in $4^{10} \times 7^3 \times 16^2 \times 11 \times 10^2$ is

(1) 34 (2) 35
(3) 36 (4) 37

(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

4. The number of prime factors in $6^{333} \times 7^{222} \times 8^{111}$

(1) 1221 (2) 1222
(3) 1111 (4) 1211

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

5. The square root of $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$ is

(1) $\sqrt{3}+\sqrt{2}$ (2) $\sqrt{3}-\sqrt{2}$
(3) $\sqrt{2}\pm\sqrt{3}$ (4) $\sqrt{2}-\sqrt{3}$

(SSC CGL Tier-1 Exam 26.06.2011 (First Sitting)

6. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $(x + y)$ equals :

(1) 8 (2) 16
(3) $2\sqrt{15}$ (4) $2\left(\sqrt{5}+\sqrt{3}\right)$

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

7. If x, y are rational numbers and $\frac{5+\sqrt{11}}{3-2\sqrt{11}} = x + y\sqrt{11}$.

The values of x and y are

(1) $x = \frac{-14}{17}$, $y = \frac{-13}{26}$

(2) $x = \frac{4}{13}$, $y = \frac{11}{17}$

(3) $x = \frac{-27}{25}$, $y = \frac{-11}{37}$

(4) $x = \frac{-37}{35}$, $y = \frac{-13}{35}$

(SSC Constable (GD) Exam, 04.10.2015, IInd Sitting)

TYPE–V

1. Simplify : $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4$

(1) 5^2 (2) 5^4
(3) 5^8 (4) 5^{12}

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

2. If $27^{2x-1} = (243)^3$ then the value of x is :

(1) 3 (2) 6
(3) 7 (4) 9

(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

3. If $3^{x+8} = 27^{2x+1}$, the value of x is :

(1) 7 (2) 3
(3) –2 (4) 1

(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

4. $(36)^{\frac{1}{6}}$ is equal to :

(1) 1 (2) 6
(3) $\sqrt{6}$ (4) $\sqrt[3]{6}$

(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

5. $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$ simplifies to :

(1) $\frac{625}{16}$ (2) $\frac{625}{8}$
(3) $\frac{625}{32}$ (4) $\frac{16}{625}$

(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

6. If $(125)^{2/3} \times (625)^{-1/4} = 5^x$ the value of x is
(1) 3 (2) 2
(3) 0 (4) 1
(SSC CGL Prelim Exam. 24.02.2002 (Middle Zone)

7. If $(2000)^{10} = 1{\cdot}024 \times 10^k$, then the value of k is
(1) 33 (2) 30
(3) 34 (4) 31
(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I) (Middle Zone)

8. If $0.42 \times 100^k = 42$, then the value of k is
(1) 4 (2) 2
(3) 1 (4) 3
(SSC CISF Constable (GD) Exam. 05.06.2011)

9. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then the value of x is
(1) 42 (2) $\frac{15}{8}$
(3) $\frac{17}{8}$ (4) 39
(SSC Data Entry Operator Exam. 02.08.2009)

10. If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to
(1) 0 (2) 1
(3) $\frac{3}{2}$ (4) $-\frac{1}{2}$
(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (Delhi Zone)

11. If $a = 7 - 4\sqrt{3}$, the value of $a^{\frac{1}{2}} + a^{-\frac{1}{2}}$ is
(1) $3\sqrt{3}$ (2) 4
(3) 7 (4) $2\sqrt{3}$
(SSC FCI Assistant Grade-III Main Exam. 07.04.2013)

12. If $\left(\frac{3}{4}\right)^3\left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$, then x is :
(1) –2 (2) 2
(3) 5 (4) $2\frac{1}{2}$
(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

13. What is the product of the roots of the equation $x^2 - \sqrt{3} = 0$?
(1) $+\sqrt{3}$ (2) $\sqrt{3}\, i$
(3) $-\sqrt{3}\, i$ (4) $-\sqrt{3}$
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

14. If $2^{x-1} + 2^{x+1} = 320$, then the value of x is
(1) 6 (2) 8
(3) 5 (4) 7
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

15. $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by
(1) 17 (2) 3
(3) 11 (4) 13
(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

16. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is
(1) 4 (2) 5
(3) 6 (4) 8
(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

17. The value of
$\left(3+2\sqrt{2}\right)^{-3} + \left(3-2\sqrt{2}\right)^{-3}$ is
(1) 198 (2) 180
(3) 108 (4) 189
(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

18. Solve for x :
$3^x - 3^{x-1} = 486$.
(1) 7 (2) 9
(3) 5 (4) 6
(SSC CGL Tier-I Exam. 26.10.2014)

19. A tap is dripping at a constant rate into a container. The level (L cm) of the water in the container is given by the equation $L = 2 - 2^t$, where t is time taken in hours. Then the level of water in the container at the start is
(1) 0 cm (2) 1 cm
(3) 2 cm (4) 4 cm
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014)

20. Arranging the following in ascending order
$3^{34}, 2^{51}, 7^{17}$ we get
(1) $3^{34} > 2^{51} > 7^{17}$
(2) $7^{17} > 2^{51} > 3^{34}$
(3) $3^{34} > 7^{17} > 2^{51}$
(4) $2^{51} > 3^{34} > 7^{17}$
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

21. If $3^{2x-y} = 3^{x+y} = \sqrt{27}$, then the value of 3^{x-y} will be
(1) 3 (2) $\frac{1}{\sqrt{3}}$
(3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{27}}$
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

22. The value of $[(0.87)^2+(0.13)^2 + (0.87) \times (0.26)]^{2013}$ is
(1) 0 (2) 2013
(3) 1 (4) –1
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 IInd Sitting)

23. The mean of $1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3$ is
(1) 20 (2) 112
(3) 56 (4) 28
(SSC CGL Tier-I Re-Exam, 30.08.2015)

24. The unit digit in the product $(2467)^{153} \times (341)^{72}$ is
(1) 7 (2) 3
(3) 9 (4) 1
(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

25. The exponential form of $\sqrt{\sqrt{2} \times \sqrt{3}}$ is :
(1) 6 (2) $6^{\frac{1}{2}}$
(3) $6^{-\frac{1}{2}}$ (4) $6^{\frac{1}{4}}$
(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

26. The quotient when 10^{100} is divided by 5^{75} is :
(1) $2^{25} \times 10^{75}$ (2) 10^{25}
(3) 2^{75} (4) $2^{75} \times 10^{25}$
(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

27. If $m^n = 169$, what is the value of $(m+1)^{(n-1)}$?
(1) 14 (2) 13
(3) 196 (4) 170
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

28. If $a = b^p$, $b = c^q$, $c = a^r$ then pqr is
(1) 1 (2) 0
(3) –1 (4) abc
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

29. Which of the following numbers is not a factor of $5^p 7^q$ ($p \neq 0$, $q \neq 0$)
(1) 35 (2) 175
(3) 1225 (4) 735
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

30. What will be the remainder when $252^{126} + 244^{152}$ is divided by 10 ?
(1) 4 (2) 6
(3) 0 (4) 8
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

31. If $x = 3^{\frac{1}{3}} - 3^{-\frac{1}{3}}$, then the value of $(3x^3 + 9x)$ is :
(1) 8 (2) 9
(3) 27 (4) 16
(SSC CGL Tier-I (CBE) Exam. 30.08.2016 (IIIrd Sitting)

32. If $3^{10} \times 27^2 = 9^2 \times 3^n$ then the value of n is :
(1) 10 (2) 12
(3) 15 (4) 30
(SSC CGL Tier-I (CBE) Exam. 31.08.2016 (IIIrd Sitting)

33. If $2^{x+4} - 2^{x+2} = 3$, then the value of 'x' is :
(1) 0 (2) 2
(3) –1 (4) –2
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IInd Sitting)

34. If $\sqrt{3^n} = 2187$ then the value of n is :
(1) 13 (2) 14
(3) 15 (4) 16
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IIIrd Sitting)

35. If $\left(x + \frac{1}{x}\right) = 2$, then the value of $\left(x^{99} + \frac{1}{x^{99}} - 2\right)$ is :
(1) – 2 (2) 0
(3) 2 (4) 4
(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IInd Sitting)

36. The value of $\left(x^{\frac{1}{3}} + x^{\frac{-1}{3}}\right)\left(x^{\frac{2}{3}} - 1 + x^{\frac{-2}{3}}\right)$ is
(1) $x^{-1} + x^{\frac{2}{3}}$ (2) $x + x^{-\frac{1}{3}}$
(3) $x^{\frac{1}{3}} + x^{-1}$ (4) $x + x^{-1}$
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IInd Sitting)

37. If $(2^3)^2 = 4^x$ then 3^x is equal to
(1) 3 (2) 6
(3) 9 (4) 27
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IIIrd Sitting)

38. If $x = 3^{\frac{1}{3}} - 3^{-\frac{1}{3}}$, then $(3x^3 + 9x)$ is equal to
(1) 5 (2) 6
(3) 7 (4) 8
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IIIrd Sitting)

TYPE–VI

1. $\sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots\ldots}}}$
(1) equals 1
(2) lies between 0 and 1
(3) lies between 1 and 2
(4) is greater than 2
(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

2. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots\ldots}}}$ is equal to
(1) $\sqrt{2}$ (2) $2\sqrt{2}$
(3) 2 (4) 3
(SSC CPO S.I. Exam. 12.01.2003

3. $\sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots\ldots}}}$ is equal to
(1) 3 (2) 4
(3) 6 (4) 2
(SSC Section Officer (Commercial Audit) Exam. 30.09.2007 (Second Sitting)

4. $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots\ldots}}}$ is equal to
(1) 3 (2) 4
(3) 5 (4) 6
(SSC SAS Exam 26.06.2010 (Paper-1) & SSC CGL Tier-I Exam 26.06.2010 (Ist Sitting) & SSC CGL Prelim Exam. 27.02.2000 (IInd Sitting)

5. $\sqrt{3\sqrt{3\sqrt{3 \ldots\ldots}}}$ is equal to
(1) $\sqrt{3}$ (2) 3
(3) $2\sqrt{3}$ (4) $3\sqrt{3}$
(SSC HSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

6. Find the value of $\sqrt{30 + \sqrt{30 + \sqrt{30 + \ldots\ldots}}}$
(1) 5 (2) $3\sqrt{10}$
(3) 6 (4) 7
(SSC Graduate Level Tier-II Exam. 29.09.2013

7. The value of $\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4 \ldots\ldots}}}}}}$ is
(1) 2 (2) 2^2
(3) 2^3 (4) 2^5
(SSC Graduate Level Tier-II Exam. 29.09.2013

8. If $m = \sqrt{5 + \sqrt{5 + \sqrt{5 + \ldots\ldots}}}$ and $n = \sqrt{5 - \sqrt{5 - \sqrt{5 - \ldots\ldots}}}$, then among the following the relation between m and n holds is
(1) $m - n + 1 = 0$
(2) $m + n - 1 = 0$
(3) $m + n + 1 = 0$
(4) $m - n - 1 = 0$
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

9. The value of $\sqrt{72 + \sqrt{72 + \sqrt{72 + \ldots\ldots}}}$ is
(1) 9 (2) 8
(3) 18 (4) 12
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

10.
$$\frac{\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}}{\sqrt[3]{8}} = ?$$
(1) 4 (2) 2
(3) 8 (4) $\frac{1}{2}$
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

11. The value of the expression $\sqrt{6+\sqrt{6+\sqrt{6+...+\text{upto }\infty}}}$ is

(1) 5 (2) 3

(3) 2 (4) 30

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 20.12.2015 (Ist Sitting) TF No. 9692918)

12. The value of the following is :

$\sqrt{12+\sqrt{12+\sqrt{12+......}}}$

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$

(3) 2 (4) 4

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 20.03.2016) (IInd Sitting)

13. Find the value of

$\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$.

(1) 6 (2) 10

(3) 8 (4) 4

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (IInd Sitting)

14. The value of

$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ is

(1) 2 (2) 4

(3) ±2 (4) –2

(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (Ist Sitting)

15. The value of $\sqrt{9+2\sqrt{16}+\sqrt[3]{512}}$ is :

(1) 6 (2) 5

(3) $2\sqrt{8}$ (4) $3\sqrt{6}$

(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IIIrd Sitting)

TYPE–VII

1. Which of the following is closest to $\sqrt{3}$?

(1) $\frac{9}{5}$ (2) 1.75

(3) $\frac{173}{100}$ (4) 1.69

(SSC CGL Prelim Exam. 13.11.2005 (First Sitting)

2. If $a=\frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+a}+\sqrt{1-a}$ is

(1) $\sqrt{3}$ (2) $\frac{\sqrt{3}}{2}$

(3) $2+\sqrt{3}$ (4) $2-\sqrt{3}$

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

3. If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\left(\frac{a^2+ab+b^2}{a^2-ab+b^2}\right)$ is

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$

(3) $\frac{3}{5}$ (4) $\frac{5}{3}$

(SSC Section Officer (Commercial Audit) Exam. 30.09.2007 (Second Sitting)

4. If $x=1+\sqrt{2}+\sqrt{3}$, then the value of $\left(x+\frac{1}{x-1}\right)$ is

(1) $1+2\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $3+\sqrt{2}$ (4) $2\sqrt{3}-1$

(SSC CGL Prelim Exam. 27.07.2008 (Second Sitting)

5. If $x+\frac{1}{x}=-2$ then the value of $x^{2n+1}+\frac{1}{x^{2n+1}}$ where n is a positive integer, is

(1) 0 (2) 2

(3) –2 (4) –5

(SSC CPO S.I. Exam. 09.11.2008)

6. If m and $n\ (n>1)$ are whole numbers such that $m^n = 121$,the value of $(m-1)^{n+1}$ is

(1) 1 (2) 10

(3) 121 (4) 1000

(SSC CPO S.I. Exam. 09.11.2008)

7. The number, which when multiplied with $(\sqrt{3}+\sqrt{2})$ gives $(\sqrt{12}+\sqrt{18})$, is

(1) $3\sqrt{2}-2\sqrt{3}$ (2) $3\sqrt{2}+2\sqrt{3}$

(3) $\sqrt{6}$ (4) $2\sqrt{3}-3\sqrt{2}$

(SSC CHSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

8. If the product of first fifty positive consecutive integers be divisible by 7^n, where n is an integer, then the largest possible value of n is

(1) 7 (2) 8

(3) 10 (4) 5

(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

9. If $9\sqrt{x}=\sqrt{12}+\sqrt{147}$, then x = ?

(1) 5 (2) 3

(3) 2 (4) 4

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014, IInd Sitting TF No. 545 QP 6)

10. A man is born in the year 1896 A.D. If in the year x^2 A.D. his age is $x-4$, the value of x is

(1) 40 (2) 44

(3) 36 (4) 42

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

11. Choose the incorrect relation(s) from the following:

(i) $\sqrt{6}+\sqrt{2}=\sqrt{5}+\sqrt{3}$

(ii) $\sqrt{6}+\sqrt{2}<\sqrt{5}+\sqrt{3}$

(iii) $\sqrt{6}+\sqrt{2}>\sqrt{5}+\sqrt{3}$

(1) (ii) and (iii) (2) (i)

(3) (ii) (4) (i) and (iii)

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

12. If $x=\frac{1}{\sqrt{2}+1}$ then $(x+1)$ equals to

(1) 2 (2) $\sqrt{2}$

(3) $\sqrt{2}+1$ (4) $\sqrt{2}-1$

(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

13. If $p = 5+2\sqrt{6}$ then $\frac{\sqrt{p}-1}{\sqrt{p}}$ is

(1) $1+\sqrt{2}-\sqrt{3}$

(2) $1-\sqrt{2}+\sqrt{3}$

(3) $-1+\sqrt{2}-\sqrt{3}$

(4) $1-\sqrt{2}-\sqrt{3}$

(SSC CPO Exam. 06.06.2016) (Ist Sitting)

14. If $\sqrt{x}-\sqrt{y} = 1$, $\sqrt{x}+\sqrt{y} = 17$ then $\sqrt{xy} = ?$

(1) $\sqrt{72}$ (2) 72
(3) 32 (4) 24

(SSC CHSL (10+2) Tier-I (CBE) Exam. 08.09.2016) (Ist Sitting)

15. If $x = \sqrt{3}+\frac{1}{\sqrt{3}}$, then the value of $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)$

$\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$ is

(1) $5\frac{\sqrt{3}}{6}$ (2) $\frac{2\sqrt{3}}{3}$

(3) $\frac{5}{6}$ (4) $\frac{2}{3}$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 08.09.2016) (Ist Sitting)

16. If $4x = \sqrt{5}+2$, then the value of $\left(x-\frac{1}{16x}\right)$ is

(1) 1 (2) –1
(3) 4 (4) $2\sqrt{5}$

(SSC CGL Tier-I (CBE) Exam. 09.09.2016) (Ist Sitting)

17. What is x, if $x^3 = 1.5^3 - 0.9^3 - 2.43$

(1) –0.5 (2) 0.6
(3) –0.7 (4) –1.6

(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

18. If $\left(\frac{1}{5}\right)^{3y} = 0.008$, then the value of $(0.25)^y$ is :

(1) 0.25 (2) 6.25
(3) 2.5 (4) 53

(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

19. If $x = 1+\sqrt{2}+\sqrt{3}$, then find the value of x^2-2x+4.

(1) $2\left(7+\sqrt{6}\right)$ (2) $2\left(4+\sqrt{6}\right)$

(3) $2\left(3+\sqrt{7}\right)$ (4) $\left(4+\sqrt{6}\right)$

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (IInd Sitting)

20. If $x = \sqrt{2}+1$, then the value of $x^4-\frac{1}{x^4}$ is

(1) $8\sqrt{2}$ (2) $18\sqrt{2}$

(3) $6\sqrt{2}$ (4) $24\sqrt{2}$

(SSC CGL Tier-I (CBE) Exam. 29.08.2016) (IInd Sitting)

21. $\frac{1}{\sqrt{a}}-\frac{1}{\sqrt{b}} = 0$, then the value of $\frac{1}{a}+\frac{1}{b}$ is :

(1) $\frac{1}{\sqrt{ab}}$ (2) $\sqrt{ab}$

(3) $\frac{2}{\sqrt{ab}}$ (4) $\frac{1}{2\sqrt{ab}}$

(SSC CGL Tier-I (CBE) Exam. 03.09.2016) (IInd Sitting)

22. If $x = (0.25)^{\frac{1}{z}}$, $y = (0.4)^2$, $z = (0.216)^{\frac{1}{3}}$, then

(1) $y > x > z$ (2) $x > y > z$
(3) $z > x > y$ (4) $x > z > y$

(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (IInd Sitting)

23. If $a+\frac{1}{a} = 2$, then the value of $\left(a^5+\frac{1}{a^5}\right)$ will be

(1) 0 (2) 1
(3) 3 (4) 2

(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (IInd Sitting)

24. If $x = 2+\sqrt{3}$, then the value of $\frac{x^2-x+1}{x^2+x+1}$ is :

(1) $\frac{2}{3}$ (2) $\frac{3}{4}$

(3) $\frac{4}{5}$ (4) $\frac{3}{5}$

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IInd Sitting)

25. If $3a = 4b = 6c$ and $a+b+c = 27\sqrt{29}$ then $\sqrt{a^2+b^2+c^2}$ is equal to

(1) 87 (2) $3\sqrt{29}$
(3) 82 (4) 83

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IInd Sitting)

26. If $\left(\sqrt{3}+1\right)^2 = x+\sqrt{3}y$, then the value of $(x+y)$ is

(1) 2 (2) 4
(3) 6 (4) 8

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IIIrd Sitting)

27. If $p = 9$, $q = \sqrt{17}$ then the value of $\left(p^2-q^2\right)^{\frac{-1}{3}}$ is equal to

(1) 4 (2) $\frac{1}{4}$

(3) 3 (4) $\frac{1}{3}$

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IIIrd Sitting)

28. If $\sqrt{1+\frac{x}{144}} = \frac{13}{12}$, then x equals to

(1) 1 (2) 13
(3) 27 (4) 25

(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IIIrd Sitting)

29. If $a = \sqrt{2}+1$ and $b = \sqrt{2}-1$, then the value of $\frac{1}{a+1}+\frac{1}{b+1}$ will be

(1) 0 (2) 1
(3) 2 (4) –1

(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IIIrd Sitting)

30. If $x = \frac{1}{(\sqrt{2}+1)}$ then the value of $(x^2 + 2x - 1)$ is :

(1) $2\sqrt{2}$ (2) 4
(3) 0 (4) 2

(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IIIrd Sitting)

31. If $x + \frac{1}{x} = \sqrt{13}$, then $\frac{3x}{(x^2-1)}$ equals to

(1) $3\sqrt{13}$ (2) $\frac{\sqrt{13}}{3}$
(3) 1 (4) 3

(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IIIrd Sitting)

32. If $a = \sqrt{2} + 1$, $b = \sqrt{2} - 1$, then the value of $\left(\frac{1}{a+1} + \frac{1}{b+1}\right)$ is :

(1) 0 (2) 1
(3) 2 (4) 3

(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IIIrd Sitting)

33. If $x + \sqrt{5} = 5 + \sqrt{y}$ and x, y are positive integers, then the value of $\frac{\sqrt{x}+y}{x+\sqrt{y}}$ is :

(1) 1 (2) 2
(3) $\sqrt{5}$ (4) 5

(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IInd Sitting)

34. If $c + \frac{1}{c} = \sqrt{3}$, then the value of $c^3 + \frac{1}{c^3}$ is equal to

(1) 0 (2) $3\sqrt{3}$
(3) $\frac{1}{\sqrt{3}}$ (4) $6\sqrt{3}$

(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IIIrd Sitting)

SHORT ANSWERS

TYPE-I

1. (3)	**2.** (1)	**3.** (1)	**4.** (3)
5. (2)	**6.** (3)	**7.** (2)	**8.** (2)
9. (3)	**10.** (1)	**11.** (1)	**12.** (1)
13. (2)	**14.** (2)	**15.** (2)	**16.** (1)
17. (3)	**18.** (4)	**19.** (1)	**20.** (3)
21. (1)	**22.** (4)	**23.** (4)	**24.** (3)
25. (2)	**26.** (2)	**27.** (2)	**28.** (3)
29. (2)	**30.** (1)	**31.** (1)	**32.** (1)
33. (1)	**34.** (1)	**35.** (2)	**36.** (2)
37. (3)	**38.** (1)	**39.** (3)	**40.** (3)
41. (2)	**42.** (3)	**43.** (3)	**44.** (1)
45. (1)	**46.** (2)	**47.** (1)	**48.** (4)
49. (3)	**50.** (2)	**51.** (4)	**52.** (4)
53. (3)	**54.** (3)	**55.** (2)	**56.** (3)
57. (1)	**58.** (3)	**59.** (1)	**60.** (2)
61. (3)	**62.** (4)	**63.** (1)	**64.** (3)
65. (4)	**66.** (3)	**67.** (1)	**68.** (4)
69. (3)	**70.** (2)	**71.** (1)	**72.** (2)
73. (2)	**74.** (3)	**75.** (1)	**76.** (3)
77. (2)	**78.** (3)	**79.** (1)	**80.** (1)
81. (1)	**82.** (4)	**83.** (3)	**84.** (4)
85. (3)	**86.** (2)	**87.** (2)	**88.** (1)
89. (3)	**90.** (4)	**91.** (2)	**92.** (3)
93. (3)	**94.** (2)	**95.** (3)	**96.** (2)
97. (4)	**98.** (1)	**99.** (4)	**100.** (3)
101. (1)	**102.** (3)	**103.** (2)	**104.** (3)
105. (2)	**106.** (1)	**107.** (1)	**108.** (2)
109. (4)	**110.** (1)	**111.** (3)	**112.** (2)
113. (1)	**114.** (4)	**115.** (2)	**116.** (2)
117. (2)	**118.** (3)	**119.** (4)	**120.** (2)
121. (2)	**122.** (4)	**123.** (2)	**124.** (3)
125. (2)	**126.** (4)	**127.** (1)	**128.** (4)
129. (1)	**130.** (3)	**131.** (3)	**132.** (3)
133. (4)	**134.** (4)	**135.** (4)	**136.** (3)
137. (3)	**138.** (3)	**139.** (4)	**140.** (3)
141. (1)	**142.** (2)	**143.** (2)	**144.** (3)
145. (3)	**146.** (1)	**147.** (3)	**148.** (2)
149. (2)	**150.** (2)	**151.** (1)	**152.** (1)
153. (*)			

TYPE-II

1. (4)	**2.** (1)	**3.** (3)	**4.** (1)
5. (3)	**6.** (1)	**7.** (2)	**8.** (2)
9. (4)	**10.** (2)	**11.** (4)	**12.** (3)
13. (3)	**14.** (1)	**15.** (2)	**16.** (1)
17. (3)	**18.** (3)	**19.** (2)	**20.** (4)
21. (4)	**22.** (4)	**23.** (2)	**24.** (2)
25. (3)	**26.** (4)	**27.** (2)	**28.** (2)
29. (2)	**30.** (4)	**31.** (3)	**32.** (1)
33. (4)	**34.** (2)	**35.** (3)	**36.** (4)

TYPE-III

1. (1)	**2.** (1)	**3.** (3)	**4.** (4)
5. (2)	**6.** (2)	**7.** (1)	**8.** (1)
9. (2)	**10.** (2)	**11.** (1)	**12.** (3)
13. (2)	**14.** (4)	**15.** (2)	**16.** (2)

TYPE-IV

1. (4)	**2.** (2)	**3.** (3)	**4.** (1)
5. (1)	**6.** (1)	**7.** (4)	

TYPE-V

1. (2)	**2.** (1)	**3.** (4)	**4.** (4)
5. (1)	**6.** (4)	**7.** (1)	**8.** (3)
9. (3)	**10.** (1)	**11.** (2)	**12.** (3)
13. (4)	**14.** (4)	**15.** (1)	**16.** (1)
17. (1)	**18.** (4)	**19.** (2)	**20.** (1)
21. (3)	**22.** (3)	**23.** (2)	**24.** (1)
25. (4)	**26.** (4)	**27.** (1)	**28.** (1)
29. (4)	**30.** (3)	**31.** (1)	**32.** (2)
33. (4)	**34.** (2)	**35.** (2)	**36.** (4)
37. (4)	**38.** (4)		

TYPE-VI

1. (3)	**2.** (3)	**3.** (2)	**4.** (1)
5. (2)	**6.** (3)	**7.** (1)	**8.** (4)
9. (1)	**10.** (2)	**11.** (2)	**12.** (4)
13. (4)	**14.** (1)	**15.** (2)	

TYPE-VII

1. (3)	**2.** (1)	**3.** (2)	**4.** (1)
5. (3)	**6.** (4)	**7.** (3)	**8.** (2)
9. (2)	**10.** (2)	**11.** (4)	**12.** (2)
13. (1)	**14.** (2)	**15.** (3)	**16.** (1)
17. (2)	**18.** (1)	**19.** (2)	**20.** (4)
21. (3)	**22.** (3)	**23.** (4)	**24.** (4)
25. (1)	**26.** (3)	**27.** (2)	**28.** (4)
29. (2)	**30.** (3)	**31.** (3)	**32.** (2)
33. (1)	**34.** (1)		

EXPLANATIONS

TYPE-I

1. (3) $\left(\sqrt{12}+\sqrt{18}\right)-\left(\sqrt{3}+\sqrt{2}\right)$

$=\left(2\sqrt{3}-\sqrt{3}\right)+\left(3\sqrt{2}-\sqrt{2}\right)$

$=\sqrt{3}+2\sqrt{2}$

2. (1) $\sqrt{5+2\sqrt{6}}$

$=\sqrt{3+2+2\times\sqrt{3}\times\sqrt{2}}$

$=\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}=\sqrt{3}+\sqrt{2}$

$\therefore\ \dfrac{1}{\sqrt{5+2\sqrt{6}}}=\sqrt{3}-\sqrt{2}$

Hence, Expression

$=\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2}=2\sqrt{2}$

3. (1) $?=\sqrt{2^4}+\sqrt[3]{64}+\sqrt[4]{2^8}$

$=2^{4\times\frac{1}{2}}+4^{3\times\frac{1}{3}}+2^{8\times\frac{1}{4}}$

$=2^2+4+2^2$

$=4+4+4=12$

4. (3) Expression

$=2\sqrt[3]{8\times4}-3\sqrt[3]{4}+\sqrt[3]{125\times4}$

$=2\times2\sqrt[3]{4}-3\sqrt[3]{4}+5\sqrt[3]{4}=6\sqrt[3]{4}$

5. (2) $?=\left(\sqrt{8}-\sqrt{4}-\sqrt{2}\right)$

$=\left(2\sqrt{2}-2-\sqrt{2}\right)$

$=\sqrt{2}-2$

6. (3) $8^{\frac{2}{3}}=\left(2^3\right)^{\frac{2}{3}}$

$=2^{3\times\frac{2}{3}}=2^2=4$

7. (2) $\left(16^{\frac{3}{2}}+16^{\frac{-3}{2}}\right)$

$=\left(4^2\right)^{\frac{3}{2}}+\dfrac{1}{(16)^{\frac{3}{2}}}$

$=4^{2\times\frac{3}{2}}+\dfrac{1}{4^{2\times\frac{3}{2}}}=4^3+\dfrac{1}{4^3}$

$=64+\dfrac{1}{64}=\dfrac{4096+1}{64}=\dfrac{4097}{64}$

8. (2) $(16)^{\frac{3}{4}}=\left(4^2\right)^{\frac{3}{4}}$

$=4^{2\times\frac{3}{4}}=4^{\frac{3}{2}}=2^{2\times\frac{3}{2}}=2^3=8$

9. (3) $(0.01024)^{\frac{1}{5}}$

$=\left[(0.4)^5\right]^{1/5}=0.4^{5\times1/5}=0.4$

10. (1) $\left(16^{0.16}\times2^{0.36}\right)$

$=\left(16^{\frac{16}{100}}\times2^{\frac{36}{100}}\right)$

$=\left(2^{4\times\frac{16}{100}}\times2^{\frac{36}{100}}\right)$

$=\left(2^{\frac{64}{100}+\frac{36}{100}}\right)=\left(2^{\frac{100}{100}}\right)$

$=2$

11. (1) Expression

$=(256)^{0.16}\times(16)^{0.18}$

$=(4)^{4\times0.16}\times(4)^{2\times0.18}$

$=(4)^{0.64}\times(4)^{0.36}$

$=(4)^{0.64+0.36}=(4)^1=4$

12. (1) Expression

$=\dfrac{(243)^{0.13+0.07}}{(7)^{0.25}\times(7\times7)^{0.075}\times(7\times7\times7)^{0.2}}$

$=\dfrac{\left(3^5\right)^{0.2}}{(7)^{0.25}\times(7)^{0.075\times2}\times(7)^{3\times0.2}}$

$=\dfrac{(3)^{5\times0.2}}{(7)^{0.25+0.15+0.6}}=\dfrac{3}{7}$

13. (2) $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+2\times2\times\sqrt{3}}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{(2)^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+8\left(2+\sqrt{3}\right)}}$

$=\sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$

$=\sqrt{-\sqrt{3}+\sqrt{\left(\sqrt{3}\right)^2+(4)^2+2\times4\times\sqrt{3}}}$

$=\sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$

$=\sqrt{-\sqrt{3}+4+\sqrt{3}}=\sqrt{4}=2$

14. (2) $\left(\sqrt[3]{0.004096}\right)^{\frac{1}{2}}$

$=(0.004096)^{\frac{1}{3}\times\frac{1}{2}}$

$=\left(\dfrac{4096}{1000000}\right)^{\frac{1}{6}}=\left\{\left(\dfrac{4}{10}\right)^6\right\}^{\frac{1}{6}}$

$=\left(\dfrac{4}{10}\right)^{6\times\frac{1}{6}}=\dfrac{4}{10}=0.4$

15. (2) Expression

$=\dfrac{3\times\sqrt{12}}{2\times\sqrt{28}}\times\dfrac{\sqrt{98}}{2\times\sqrt{21}}$

$=\dfrac{3\times2\times\sqrt{3}}{2\times2\times\sqrt{7}}\times\dfrac{7\times\sqrt{2}}{2\times\sqrt{3}\times\sqrt{7}}$

$=\dfrac{3\sqrt{2}}{4}=\dfrac{3\times1.414}{4}=1.0605$

≈1.0606

16. (1) Expression

$=2+\sqrt{0.09}-\sqrt[3]{(0.2)^3}-75\%\ of\ 2.80$

$=2+0.3-0.2-\dfrac{75}{100}\times2.80$

$=2.3-0.2-2.1$

$=2.3-2.3=0$

17. (3) Let $\sqrt[3]{3.5} = a$ and $\sqrt[3]{2.5} = b$

$\therefore$ Expression

$= (a+b)(a^2 - ab + b^2)$

$= a^3 + b^3$

$= \left(\sqrt[3]{3.5}\right)^3 + \left(\sqrt[3]{2.5}\right)^3$

$= 3.5 + 2.5 = 6$

18. (4) We know that

$a^3 + b^3$

$= (a + b)^3 - 3ab\,(a + b)$

Now,

$\left(3+2\sqrt{2}\right)^{-3} + \left(3-2\sqrt{2}\right)^{-3}$

$= \frac{1}{\left(3+2\sqrt{2}\right)^3} + \frac{1}{\left(3-2\sqrt{2}\right)^3}$

$= \frac{\left(3-2\sqrt{2}\right)^3 + \left(3+2\sqrt{2}\right)^3}{\left(3+2\sqrt{2}\right)^3 \times \left(3-2\sqrt{2}\right)^3}$

$$= \frac{\left(3-2\sqrt{2}+3+2\sqrt{2}\right)^3 - 3\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}+3+2\sqrt{2}\right)}{\left[\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)\right]^3}$$

$= \frac{(6)^3 - 3(9-8)(6)}{1}$

$= 216 - 18 = 198$

19. (1)

$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{5}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$

$= \frac{\sqrt{15}-\sqrt{10}}{3-2} = \sqrt{15}-\sqrt{10}$

$\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{3\sqrt{3}\left(\sqrt{5}-\sqrt{2}\right)}{5-2} = \sqrt{15}-\sqrt{6}$

$\frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$

$= \frac{2\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3} = \sqrt{10}-\sqrt{6}$

$\therefore$ Expression

$= \left(\sqrt{15}-\sqrt{10}\right) - \left(\sqrt{15}-\sqrt{6}\right) + \left(\sqrt{10}-\sqrt{6}\right)$

$= \sqrt{15} - \sqrt{10} - \sqrt{15} + \sqrt{6} + \sqrt{10} - \sqrt{6}$

$= 0$

20. (3) $4 + \sqrt{7} = \frac{8 + 2\sqrt{7}}{2}$

$= \frac{7 + 1 + 2 \times \sqrt{7} \times 1}{2}$

$= \frac{\left(\sqrt{7}\right)^2 + (1)^2 + 2 \times \sqrt{7} \times 1}{2}$

$= \frac{\left(\sqrt{7}+1\right)^2}{\left(\sqrt{2}\right)^2} = \left\{\frac{1}{\sqrt{2}}\left(\sqrt{7}+1\right)\right\}^2$

21. (1) $\frac{1}{\sqrt{3.25}+\sqrt{2.25}}$

$= \frac{1}{\left(\sqrt{3.25}+\sqrt{2.25}\right)} \times \frac{\sqrt{3.25}-\sqrt{2.25}}{\sqrt{3.25}-\sqrt{2.25}}$

$= \frac{\sqrt{3.25}-\sqrt{2.25}}{3.25-2.25} = \sqrt{3.25}-\sqrt{2.25}$

Similarly,

$\frac{1}{\sqrt{4.25}+\sqrt{3.25}} = \sqrt{4.25}-\sqrt{3.25}$

$\frac{1}{\sqrt{5.25}+\sqrt{4.25}} = \sqrt{5.25}-\sqrt{4.25}$

$\frac{1}{\sqrt{6.25}+\sqrt{5.25}} = \sqrt{6.25}-\sqrt{5.25}$

$\therefore$ Expression

$= \sqrt{3.25} - \sqrt{2.25} + \sqrt{4.25} - \sqrt{3.25} + \sqrt{5.25} - \sqrt{4.25} + \sqrt{6.25} - \sqrt{5.25}$

$= \sqrt{6.25} - \sqrt{2.25} = 2.5 - 1.5 = 1$

22. (4) First term $= \frac{2}{\sqrt{7}+\sqrt{5}}$

$= \frac{2 \times \left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)}$

$= \frac{2\left(\sqrt{7}-\sqrt{5}\right)}{7-5} = \sqrt{7}-\sqrt{5}$

Second term $= \frac{7}{\sqrt{12}-\sqrt{5}}$

$= \frac{7\left(\sqrt{12}+\sqrt{5}\right)}{\left(\sqrt{12}-\sqrt{5}\right)\left(\sqrt{12}+\sqrt{5}\right)}$

$= \frac{7\left(\sqrt{12}+\sqrt{5}\right)}{12-5}$

$= \frac{7(\sqrt{12}+\sqrt{5})}{7} = \sqrt{12}+\sqrt{5}$

Third term $= \frac{5}{\sqrt{12}-\sqrt{7}}$

$= \frac{5\left(\sqrt{12}+\sqrt{7}\right)}{\left(\sqrt{12}-\sqrt{7}\right)\left(\sqrt{12}+\sqrt{7}\right)}$

$= \frac{5\left(\sqrt{12}+\sqrt{7}\right)}{12-7} = \sqrt{12}+\sqrt{7}$

$\therefore$ Expression

$= \left(\sqrt{7}-\sqrt{5}\right) + \left(\sqrt{12}+\sqrt{5}\right) - \left(\sqrt{12}+\sqrt{7}\right)$

$= \sqrt{7} - \sqrt{5} + \sqrt{12} + \sqrt{5} - \sqrt{12} - \sqrt{7} = 0$

23. (4) $\left(\frac{1}{2}\right)^{-\frac{1}{2}} = (2)^{\frac{1}{2}} = \sqrt{2}$

24. (3) $\frac{1}{\sqrt{3}+\sqrt{4}}$

$= \frac{1}{\sqrt{3}+\sqrt{4}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}$

$= \frac{\sqrt{4}-\sqrt{3}}{4-3} = \sqrt{4}-\sqrt{3}$

Similarly,

$\frac{1}{\sqrt{4}+\sqrt{5}} = \sqrt{5}-\sqrt{4}\ldots$ so on

$\therefore$ Expression

$= \sqrt{4} - \sqrt{3} + \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6} + \sqrt{8} - \sqrt{7} + \sqrt{9} - \sqrt{8}$

$= \sqrt{9} - \sqrt{3} = 3 - \sqrt{3}$

25. (2) $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2}$

$= (2^4)^{0.16} \times (2^4)^{0.04} \times (2)^{0.2}$

$= 2^{0.64} \times 2^{0.16} \times 2^{0.2}$

$= (2)^{0.64+0.16+0.2} = 2$

26. (2) Expression

$$= \frac{12}{3+\sqrt{5}+2\sqrt{2}}$$

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{\left[(3+\sqrt{5})+2\sqrt{2}\right]\left[(3+\sqrt{5})-2\sqrt{2}\right]}$$

[Rationalising the denominator]

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{(3+\sqrt{5})^2-(2\sqrt{2})^2}$$

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{9+5+6\sqrt{5}-8}$$

$$= \frac{12(3+\sqrt{5}-2\sqrt{2})}{6\sqrt{5}+6}$$

$$= \frac{2(3+\sqrt{5}-2\sqrt{2})}{\sqrt{5}+1}$$

$$= \frac{2(3+\sqrt{5}-2\sqrt{2})(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$$

$$= \frac{2(3\sqrt{5}+5-2\sqrt{10}-3-\sqrt{5}+2\sqrt{2})}{5-1}$$

$$= \frac{2\left(2\sqrt{5}+2\sqrt{2}-2\sqrt{10}+2\right)}{4}$$

$$= \frac{2\times 2(\sqrt{5}+\sqrt{2}-\sqrt{10}+1)}{4}$$

$$= 1+\sqrt{5}+\sqrt{2}-\sqrt{10}$$

27. (2) Expression

$$= 3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}+\frac{1}{\sqrt{3}-3}$$

$$= 3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}-\frac{1}{3-\sqrt{3}}$$

$$= 3+\frac{1}{\sqrt{3}}+\left(\frac{3-\sqrt{3}-3-\sqrt{3}}{(3+\sqrt{3})(3-\sqrt{3})}\right)$$

$$= 3+\frac{1}{\sqrt{3}}+\frac{-2\sqrt{3}}{9-3} = 3+\frac{1}{\sqrt{3}}-\frac{\sqrt{3}}{3}$$

$$= 3+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{3}} = 3$$

28. (3) Expression $= \sqrt{8-2\sqrt{15}}$

$$= \sqrt{5+3-2\sqrt{5}\times\sqrt{3}}$$

$$= \sqrt{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2-2\sqrt{5}\times\sqrt{3}}$$

$$= \sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2} = \sqrt{5}-\sqrt{3}$$

29. (2) Expression $= (0.04)^{-1.5}$

$$= \frac{1}{(0.04)^{1.5}} = \frac{1}{(0.04)^{\frac{3}{2}}}$$

$$= \frac{1}{(0.04\times 0.04\times 0.04)^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{0.0000064}}$$

$$= \frac{1}{0.008} = \frac{1000}{8} = 125$$

30. (1) Expression

$$= \frac{\sqrt[3]{1372}\times\sqrt[3]{1458}}{\sqrt[3]{343}}$$

$$= \sqrt[3]{\frac{1372\times 1458}{343}}$$

$$= \sqrt[3]{5832}$$

$$= \sqrt[3]{18\times 18\times 18} = 18$$

31. (1)

$$\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

(Rationalising the denominator)

$$= \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3} = \sqrt{5}-\sqrt{3}$$

Similarly,

$$\frac{3}{\sqrt{6}-\sqrt{3}} = \frac{3\left(\sqrt{6}+\sqrt{3}\right)}{6-3} = \sqrt{6}+\sqrt{3}$$

$$\frac{1}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}-\sqrt{5}}{6-5} = \sqrt{6}-\sqrt{5}$$

$\therefore$ Expression

$$= \sqrt{5}-\sqrt{3}+\sqrt{6}+\sqrt{3}+\sqrt{6}-\sqrt{5}$$

$$= 2\sqrt{6}$$

32. (1) Here,

$$\frac{1}{3-\sqrt{8}} = \frac{\left(3+\sqrt{8}\right)}{\left(3-\sqrt{8}\right)\left(3+\sqrt{8}\right)}$$

$$= \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

$$\frac{1}{\sqrt{8}-\sqrt{7}} = \frac{\sqrt{8}+\sqrt{7}}{\left(\sqrt{8}-\sqrt{7}\right)\left(\sqrt{8}+\sqrt{7}\right)}$$

$= \sqrt{8}+\sqrt{7}$ AND.... so on

Expression

$$= \left(3+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$= 3 + 2 = 5$

33. (1) Expression $= \dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

Rationalising the denominator,

$$= \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$= \frac{\left(3\sqrt{2}+2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$= \frac{18+12+2\times 3\sqrt{2}\times 2\sqrt{3}}{18-12}$$

$$= \frac{30+12\sqrt{6}}{6}$$

$$= \frac{6\left(5+2\sqrt{6}\right)}{6} = 5+2\sqrt{6}$$

34. (1) Expression

$$= \left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)+\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \left[\frac{\left(2+\sqrt{3}\right)^2+\left(2-\sqrt{3}\right)^2}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}\right]+\frac{\left(\sqrt{3}+1\right)^2}{3-1}$$

$$= \frac{2(4+3)}{4-3} + \frac{3+1+2\sqrt{3}}{2}$$

$$\left[\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)\right]$$

$$= 14 + 2 + \sqrt{3} = 16 + \sqrt{3}$$

35. (2) $14 + 6\sqrt{5} = 14 + 2 \times 3 \times \sqrt{5}$

$$= 9 + 5 + 2 \times 3 \times \sqrt{5}$$

$$= (3)^2 + (\sqrt{5})^2 + 2 \times 3 \times \sqrt{5}$$

$$= (3 + \sqrt{5})^2$$

$$\therefore \sqrt{14 + 6\sqrt{5}} = \sqrt{(3+\sqrt{5})^2}$$

$$= 3 + \sqrt{5}$$

36. (2) Expression

$$= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} + \frac{\sqrt{6}}{(\sqrt{3}+\sqrt{2})} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(6-2)} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{3-2}$$

$$= \sqrt{2}(\sqrt{6}-\sqrt{3}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) + \sqrt{6}(\sqrt{3}-\sqrt{2})$$

$$= \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6} + \sqrt{18} - \sqrt{12}$$

$$= 0$$

37. (3) Expression

$$= \frac{\dfrac{3(2-\sqrt{3}) - 2(2+\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}}{2-\sqrt{3}}$$

$$= \frac{6 - 3\sqrt{3} - 4 - 2\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})(2-5\sqrt{3})}$$

$$= \frac{2-5\sqrt{3}}{2-5\sqrt{3}} = 1$$

38. (1) $(64)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-2}$

$$= \frac{1}{(64)^{\frac{2}{3}}} \times (4)^2$$

$$= \frac{1}{(4)^{3\times\frac{2}{3}}} \times 4^2 \qquad = \frac{1}{4^2} \times 4^2 = 1$$

39. (3) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3}) + (1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

[Using $(a + b)(a - b) = a^2 - b^2$]

$$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3} = \frac{2(\sqrt{5}-\sqrt{6})}{2}$$

$$= \sqrt{5}-\sqrt{6}$$

40. (3) Given expression

$$= \left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

$$= \left[\frac{(2+\sqrt{3})^2 + (2-\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} + \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}\right]$$

$$= \left[\frac{4+3+4\sqrt{3}+4+3-4\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}\right]$$

$$= \left[\frac{14}{4-3} + \frac{3+1-2\sqrt{3}}{3-1}\right]$$

$$= \left[14 + \frac{2(2-\sqrt{3})}{2}\right] = 16 - \sqrt{3}$$

41. (2) Given expression :

$$\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$$

Now,

$$\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2 = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})^2}$$

$$= \frac{(\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5}\times\sqrt{3}}{(\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}\times\sqrt{3}}$$

$$= \frac{5+3+2\sqrt{15}}{5+3-2\sqrt{15}}$$

$$= \frac{8+2\sqrt{15}}{8-2\sqrt{15}} = \frac{4+\sqrt{15}}{4-\sqrt{15}}$$

Similarly,

$$\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2 = \frac{4-\sqrt{15}}{4+\sqrt{15}}$$

Therefore, given expression

$$= \frac{4+\sqrt{15}}{4-\sqrt{15}} + \frac{4-\sqrt{15}}{4+\sqrt{15}}$$

$$= \frac{16+15+8\sqrt{15}+16+15-8\sqrt{15}}{16-15}$$

$$= 62$$

42. (3) $\sqrt{\dfrac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}}$

$$= \sqrt{\frac{\sqrt{36}-\sqrt{24}+\sqrt{24}-\sqrt{16}}{5+\sqrt{24}}}$$

$$= \sqrt{\frac{6-4}{5+\sqrt{24}}} = \sqrt{\frac{2}{5+\sqrt{24}}}$$

$$= \sqrt{\frac{2}{5+\sqrt{6\times4}}} = \sqrt{\frac{2}{5+2\sqrt{6}}}$$

$$= \sqrt{\frac{2}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}}}$$

$$= \sqrt{\frac{2(5-2\sqrt{6})}{25-24}} = \sqrt{2(5-2\sqrt{6})}$$

$$= \sqrt{2\left(3+2+2\sqrt{6}\right)}$$

$$= \sqrt{2\left[\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\sqrt{3}\sqrt{2}\right]}$$

$$= \sqrt{2\left(\sqrt{3}-\sqrt{2}\right)^2} = \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$$

$$= \sqrt{6}-2$$

43. (3) $\left[64^{\frac{2}{3}} \times 2^{-2} \div 8^{\circ}\right]^{\frac{1}{2}}$

$$= \left[\left(64^{\frac{1}{3}}\right)^2 \times \frac{1}{2^2} \div 1\right]^{\frac{1}{2}}$$

$$= \left[\left(\sqrt[3]{64}\right)^2 \times \frac{1}{4}\right]^{\frac{1}{2}} = \left[(4)^2 \times \frac{1}{4}\right]^{\frac{1}{2}}$$

$$= [4]^{\frac{1}{2}} = \sqrt{4} = 2$$

44. (1) Expression

$$= \frac{1}{\sqrt{12-\sqrt{140}}} - \frac{1}{\sqrt{8-\sqrt{60}}} - \frac{2}{\sqrt{10+\sqrt{84}}}$$

$$= \frac{1}{\sqrt{12-\sqrt{4\times35}}} - \frac{1}{\sqrt{8-\sqrt{4\times15}}} - \frac{2}{\sqrt{10+\sqrt{4\times21}}}$$

$$= \frac{1}{\sqrt{12-2\times\sqrt{7}\times\sqrt{5}}} - \frac{1}{\sqrt{8-2\times\sqrt{5}\times\sqrt{3}}}$$

$$- \frac{2}{\sqrt{10+2\times\sqrt{7}\times\sqrt{3}}}$$

$$= \frac{1}{\sqrt{\left(\sqrt{7}\right)^2+\left(\sqrt{5}\right)^2-2\times\sqrt{7}\times\sqrt{5}}}$$

$$- \frac{1}{\sqrt{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2-2\times\sqrt{5}\times\sqrt{3}}}$$

$$- \frac{2}{\sqrt{\left(\sqrt{7}\right)^2+\left(\sqrt{3}\right)^2-2\times\sqrt{7}\times\sqrt{3}}}$$

$$= \frac{1}{\sqrt{\left(\sqrt{7}-\sqrt{5}\right)^2}} - \frac{1}{\sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2}} - \frac{2}{\sqrt{\left(\sqrt{7}+\sqrt{3}\right)^2}}$$

$$= \frac{1}{\sqrt{7}-\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{3}} - \frac{2}{\sqrt{7}+\sqrt{3}}$$

$$= \frac{\sqrt{7}+\sqrt{5}}{\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2} - \frac{\sqrt{5}+\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$- \frac{2\left(\sqrt{7}-\sqrt{3}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$$

[Rationalizing each term by respective conjugates]

$$= \frac{\sqrt{7}+\sqrt{5}}{2} - \frac{\sqrt{5}+\sqrt{3}}{2} - \frac{2\left(\sqrt{7}-\sqrt{3}\right)}{4}$$

$$= \frac{\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{3}-\sqrt{7}+\sqrt{3}}{2} = 0$$

45. (1) $\sqrt{11+2\sqrt{30}}$

$$= \sqrt{5+6+2\times\sqrt{5}\times\sqrt{6}}$$

$$= \sqrt{\left(\sqrt{5}+\sqrt{6}\right)^2} = \sqrt{6}+\sqrt{5}$$

$$\therefore \frac{1}{\sqrt{11+2\sqrt{30}}} = \sqrt{6}-\sqrt{5}$$

$\therefore$ Expression

$$= \sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5} = 2\sqrt{5}$$

46. (2) $(243)^{0.16} \times (243)^{0.04}$
$= (243)^{0.16+0.04}$
$= (243)^{0.2} = (243)^{1/5}$
$= (3^5)^{1/5} = 3$

47. (1) Expression $= \dfrac{3^{\circ}+3^{-1}}{3^{-1}-3^{\circ}}$

$$= \frac{1+\frac{1}{3}}{\frac{1}{3}-1} = \frac{\frac{3+1}{3}}{\frac{1-3}{3}} = \frac{4}{3} \times \frac{-3}{2} = -2$$

48. (4) $\dfrac{1}{\sqrt{100}-\sqrt{99}}$

$$= \frac{\sqrt{100}+\sqrt{99}}{\left(\sqrt{100}-\sqrt{99}\right)\left(\sqrt{100}+\sqrt{99}\right)}$$

$$= \sqrt{100}+\sqrt{99}$$

Similarly, $\dfrac{1}{\sqrt{99}-\sqrt{98}}$

$= \sqrt{99}+\sqrt{98}$ ... and so on

$\therefore$ Expression

$$= \sqrt{100}+\sqrt{99}-\sqrt{99}-\sqrt{98}+$$
$$\sqrt{98}+\sqrt{97}\ldots.+\sqrt{2}+1$$

$$= \sqrt{100}+1 = 10+1 = 11$$

49. (3) $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$

$$= \left[\frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\right] \times \frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2+3+2\sqrt{6}-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$$

Similarly, $\dfrac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$

$$= \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{\left[\left(\sqrt{2}-\sqrt{3}\right)-\sqrt{5}\right]\left[\left(\sqrt{2}-\sqrt{3}\right)+\sqrt{5}\right]}$$

$$= \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{-2\sqrt{6}}$$

$\therefore$ Expression

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}} - \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}-\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$

$$= \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$$

50. (2) Expression

$$= \sqrt[3]{2} \times \sqrt{2} \times \sqrt[3]{3} \times 3$$

$$= 2^{\frac{1}{3}} \times 2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 3^{\frac{1}{2}}$$

$$= 2^{\frac{5}{6}} \times 3^{\frac{5}{6}} = (6)^{\frac{5}{6}}$$

51. (4) $(256)^{16/100} \times (256)^{9/100}$

$$= (256)^{25/100} = (256)^{1/4} = \left(4^4\right)^{\frac{1}{4}}$$

$$= (4)^{4\times\frac{1}{4}} = 4$$

52. (4) Expression

$$= \left[8 - \left(\frac{4^{\frac{9}{4}}\sqrt{2\times2^2}}{2\sqrt{2^{-2}}}\right)^{\frac{1}{2}}\right]$$

$$= \left[8 - \left(\frac{(2)^{2\times\frac{9}{4}} \times 2^{\frac{3}{2}}}{2 \times (2^{-2})^{\frac{1}{2}}}\right)^{\frac{1}{2}}\right]$$

$$= \left[8 - \left(\frac{2^{\frac{9}{2}} \times 2^{\frac{3}{2}}}{2^1 \times 2^{-1}}\right)^{\frac{1}{2}}\right]$$

$$= \left[8 - \left(\frac{2^{\frac{9}{2}+\frac{3}{2}}}{2^{1-1}}\right)^{\frac{1}{2}}\right]$$

$$= \left[8 - (2^6)^{\frac{1}{2}}\right] = (8 - 2^3) = 8 - 8 = 0$$

53. (3) Expression

$$= \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{2\sqrt{6}}{\sqrt{3}+1} + \frac{2\sqrt{3}}{\sqrt{6}+2}$$

Now,

$$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{6-3} = \sqrt{2}(\sqrt{6}-\sqrt{3})$$

$$= \sqrt{12} - \sqrt{6} = 2\sqrt{3} - \sqrt{6}$$

$$= \frac{2\sqrt{6}}{\sqrt{3}+1} = \frac{2\sqrt{6}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \sqrt{6}(\sqrt{3}-1) = \sqrt{18} - \sqrt{6}$$

$$= 3\sqrt{2} - \sqrt{6}$$

$$= \frac{2\sqrt{3}}{\sqrt{6}+2} = \frac{2\sqrt{3}(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$= \frac{2\sqrt{3}(\sqrt{6}-2)}{6-4} = \sqrt{3}(\sqrt{6}-2)$$

$$= \sqrt{3}\times\sqrt{6} - 2\sqrt{3} = 3\sqrt{2} - 2\sqrt{3}$$

$\therefore$ Expression

$$= (2\sqrt{3}-\sqrt{6}) - (3\sqrt{2}-\sqrt{6})$$

$$+ (3\sqrt{2} - 2\sqrt{3})$$

$$= 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6}$$

$$+ 3\sqrt{2} - 2\sqrt{3} = 0$$

54. (3) $(4)^{0.5} \times (0.5)^4$

$$= \left(2^2\right)^{\frac{1}{2}} \times \left(\frac{5}{10}\right)^4 = 2 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{2}{2\times2\times2\times2} = \frac{1}{2\times2\times2} = \frac{1}{8}$$

55. (2) Expression

$$= \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)^2 - \left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$$

$$= \frac{3+2+2\sqrt{6}-3-2+2\sqrt{6}}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{4\sqrt{6}}{3-2} = 4\sqrt{6}$$

56. (3) $\sqrt{40+\sqrt{9\sqrt{81}}}$

$$= \sqrt{40+\sqrt{9\times9}}$$

$$= \sqrt{40+9} = \sqrt{49} = 7$$

57. (1)

$$\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{\sqrt{9}+\sqrt{8}}{9-8}$$

$$= \sqrt{9}+\sqrt{8}$$

Similarly, $\frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}+\sqrt{7}$

... and so on

Expression =

$$\left(\sqrt{9}+\sqrt{8}\right) - \left(\sqrt{8}+\sqrt{7}\right) + \left(\sqrt{7}+\sqrt{6}\right) -$$

$$\left(\sqrt{6}+\sqrt{5}\right) + \left(\sqrt{5}+\sqrt{4}\right)$$

$$= \sqrt{9}+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}$$

$$-\sqrt{5}+\sqrt{5}+\sqrt{4}$$

$$= \sqrt{9}+\sqrt{4} = 3+2 = 5$$

58. (3) $\left[\left(\sqrt[5]{x^{-3/5}}\right)^{\frac{-5}{3}}\right]^5$

$$= \left(x^{-\frac{3}{5}}\right)^{\frac{1}{5}\times\frac{-5}{3}\times5}$$

$$= x^{-\frac{3}{5}\times\frac{-5}{3}} = x$$

59. (1)

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}+1\right)^2}{3-1}$$

$$= \frac{3+1+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$\therefore \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$$

$$\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}$$

$$= \frac{2+1+2\sqrt{2}}{2-1} = 3 + 2\sqrt{2}$$

$$\therefore \frac{\sqrt{2}-1}{\sqrt{2}+1} = 3 - 2\sqrt{2}$$

$\therefore$ Expression $= 2 + \sqrt{3} + 3 + 2\sqrt{2} + 2 - \sqrt{3} + 3 - 2\sqrt{2}$

$= 10$

60. (2) Expression

$$= \frac{3+\sqrt{6}}{5\sqrt{3} - 2\sqrt{2\times2\times3} - \sqrt{2\times2\times2\times2\times2} + \sqrt{5\times5\times2}}$$

$$= \frac{3+\sqrt{6}}{5\sqrt{3} - 4\sqrt{3} - 4\sqrt{2} + 5\sqrt{2}}$$

$$= \frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}} = \sqrt{3}$$

61. (3) Expression

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{\left(1+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{3}\right) + \left(\sqrt{5}+\sqrt{3}\right)\left(1-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$= \frac{\sqrt{5}+\sqrt{10}-\sqrt{3}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{2\sqrt{5}-2\sqrt{6}}{2} = \frac{2\left(\sqrt{5}-\sqrt{6}\right)}{2}$$

$$= \sqrt{5}-\sqrt{6}$$

62. (4)

$$(256)^{-\left(4^{\frac{-3}{2}}\right)} = (256)^{-\left(\frac{1}{4^{3/2}}\right)}$$

$$= (256)^{-\frac{1}{8}} = \frac{1}{(256)^{\frac{1}{8}}} = \frac{1}{(2^8)^{\frac{1}{8}}} = \frac{1}{2}$$

63. (1) $\left\{(-2)^{(-2)}\right\}^{(-2)} = \dfrac{1}{\left\{(-2)^{(-2)}\right\}^2}$

$$= \frac{1}{(-2)^{-4}} = (-2)^4 = 16$$

64. (3) Expression

$$= \sqrt{2} + \sqrt{7 - 2\sqrt{10}}$$

$$= \sqrt{2} + \sqrt{7 - 2 \times \sqrt{5} \times \sqrt{2}}$$

$$= \sqrt{2} + \sqrt{\left(\sqrt{5}\right)^2 + \left(\sqrt{2}\right)^2 - 2 \times \sqrt{5} \times \sqrt{2}}$$

$$= \sqrt{2} + \sqrt{\left(\sqrt{5} - \sqrt{2}\right)^2}$$

$$= \sqrt{2} + \sqrt{5} - \sqrt{2} = \sqrt{5}$$

65. (4) $(256)^{0.16} \times (4)^{0.36}$

$= (4^4)^{0.16} \times 4^{0.36} = 4^{0.64} \times 4^{0.36}$

$= (4)^{0.64 + 0.36} = 4$

66. (3) $? = 5\sqrt{7} - 2\sqrt{5} - 3\sqrt{7} + 4\sqrt{5}$

$$= 2\sqrt{7} + 2\sqrt{5} = 2\left(\sqrt{7} + \sqrt{5}\right)$$

67. (1) $\dfrac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} + \dfrac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}}$

$$= \frac{\left(\sqrt{7} - \sqrt{5}\right)^2 + \left(\sqrt{7} + \sqrt{5}\right)^2}{\left(\sqrt{7} + \sqrt{5}\right)\left(\sqrt{7} - \sqrt{5}\right)}$$

$$= \frac{2\left[\left(\sqrt{7}\right)^2 + \left(\sqrt{5}\right)^2\right]}{\left(\sqrt{7}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{2(7+5)}{7-5} = 12$$

68. (4) $\dfrac{2}{\sqrt{6} + 2} = \dfrac{2}{\sqrt{6} + 2} \times \dfrac{\sqrt{6} - 2}{\sqrt{6} - 2}$

$$= \frac{2\left(\sqrt{6} - 2\right)}{6 - 4} = \sqrt{6} - 2$$

Similarly, $\dfrac{1}{\sqrt{7} + \sqrt{6}} = \sqrt{7} - \sqrt{6}$

and $\dfrac{1}{\sqrt{8} - \sqrt{7}} = \dfrac{\sqrt{8} + \sqrt{7}}{\left(\sqrt{8} - \sqrt{7}\right)\left(\sqrt{8} + \sqrt{7}\right)}$

$$= \frac{\sqrt{8} + \sqrt{7}}{8 - 7} = \sqrt{8} + \sqrt{7}$$

$$= 2\sqrt{2} + \sqrt{7}$$

$\therefore$ Expression $= \sqrt{6} - 2 + \sqrt{7} - \sqrt{6} + 2\sqrt{2} + \sqrt{7} + 2 - 2\sqrt{2} = 2\sqrt{7}$

69. (3) $\sqrt{12} + \sqrt{18}$

$$= \sqrt{3 \times 2 \times 2} + \sqrt{2 \times 3 \times 3}$$

$$= 2\sqrt{3} + 3\sqrt{2}$$

$\therefore$ Required difference

$$= 2\sqrt{3} + 3\sqrt{2} - 2\sqrt{3} - 2\sqrt{2} = \sqrt{2}$$

70. (2) $\dfrac{1}{\sqrt{2} + 1} = \dfrac{1}{\sqrt{2} + 1} \times \dfrac{\sqrt{2} - 1}{\sqrt{2} - 1}$

$$= \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1$$

$\therefore$ Expression

$$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots\ldots + \sqrt{99} - \sqrt{98} + \sqrt{100} - \sqrt{99}$$

$$= \sqrt{100} - 1 = 10 - 1 = 9$$

71. (1) $\left\{\left(\dfrac{-1}{2}\right)^2\right\}^{-2\times(-1)} = \left(\dfrac{-1}{2}\right)^{2\times 2}$

$$= \left(-\frac{1}{2}\right)^4 = \frac{1}{16}$$

72. (2) $2.\sqrt[3]{40} = 2.\sqrt[3]{2 \times 2 \times 2 \times 5}$

$= 4\sqrt[3]{5}$

$4.\sqrt[3]{320}$

$= 4.\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5}$

$= 16.\sqrt[3]{5}$

$3.\sqrt[3]{625}$

$= 3.\sqrt[3]{5 \times 5 \times 5 \times 5} = 15.\sqrt[3]{5}$

$\therefore$ Expression $= 4.\sqrt[3]{5} - 16\sqrt[3]{5} + 15.\sqrt[3]{5} - 3.\sqrt[3]{5}$

$= 19.\sqrt[3]{5} - 19.\sqrt[3]{5} = 0$

73. (2) $\sqrt[3]{0.000125}$

$= \sqrt[3]{0.05 \times 0.05 \times 0.05} = 0.05$

74. (3) Expression

$$= \frac{0.3555 \times 0.5555 \times 2.025}{0.225 \times 1.7775 \times 0.2222}$$

$$= \frac{3555 \times 5555 \times 2025}{225 \times 17775 \times 2222} = 4.5$$

75. (1)

$$? = \frac{0.06 \times 0.06 \times 0.06 - 0.05 \times 0.05 \times 0.05}{0.06 \times 0.06 + 0.06 \times 0.05 + 0.05 \times 0.05}$$

We know that

$$\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

[$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$\therefore$ Required answer

$= 0.06 - 0.05 = 0.01$

76. (3) Using the formula

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$? = \frac{0.05 \times 0.05 \times 0.05 - 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 + 0.002 + 0.04 \times 0.04}$$

$$= \frac{(0.05)^3 - (0.04)^3}{(0.05)^2 + 0.05 \times 0.04 + (0.04)^2}$$

$= 0.05 - 0.04 = 0.01$

77. (2) $\dfrac{\left(x - \sqrt{24}\right)\left(\sqrt{75} + \sqrt{50}\right)}{\sqrt{75} - \sqrt{50}} = 1$

$$\Rightarrow \frac{\left(x - 2\sqrt{6}\right)\left(5\sqrt{3} + 5\sqrt{2}\right)}{5\sqrt{3} - 5\sqrt{2}} = 1$$

$$\Rightarrow \frac{\left(x - 2\sqrt{6}\right)\left(\sqrt{3} + \sqrt{2}\right)}{\sqrt{3} - \sqrt{2}} = 1$$

Now, $x - 2\sqrt{6} = \dfrac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$$= \frac{\left(\sqrt{3} - \sqrt{2}\right)^2}{\left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right)}$$

$$= 3 + 2 - 2\sqrt{6}$$

$$\Rightarrow x - 2\sqrt{6} = 5 - 2\sqrt{6} \Rightarrow x = 5$$

78. (3) $\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\dfrac{4}{\sqrt{5}-\sqrt{3}}-\sqrt{81}$

$= 2\sqrt{5}+2\sqrt{3}+9-\dfrac{4}{\left(\sqrt{5}-\sqrt{3}\right)}\times\dfrac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}-9$

$= 2\sqrt{5}+2\sqrt{3}+9-\dfrac{4\left(\sqrt{5}+\sqrt{3}\right)}{5-3}-9$

$= 2\sqrt{5}+2\sqrt{3}+9-2\sqrt{5}-2\sqrt{3}-9$

$= 0$

79. (1) $a = \dfrac{1}{2-\sqrt{3}}+\dfrac{1}{3-\sqrt{8}}+\dfrac{1}{4-\sqrt{15}}$

$= \dfrac{1}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}+\dfrac{1}{3-\sqrt{8}}\times\dfrac{3+\sqrt{8}}{3+\sqrt{8}}+\dfrac{1}{4-\sqrt{15}}\times\dfrac{4+\sqrt{15}}{4+\sqrt{15}}$

$= \dfrac{2+\sqrt{3}}{4-3}+\dfrac{3+\sqrt{8}}{9-8}+\dfrac{4+\sqrt{15}}{16-15}$

$= 2+\sqrt{3}+3+2\sqrt{2}+4+\sqrt{15}$

$= 9+\sqrt{3}+2\sqrt{2}+\sqrt{15}$

$= 9 < 9+\sqrt{3}+2\sqrt{2}+\sqrt{15} < 18$

[Illustration : $\sqrt{3}=1.7$

$\sqrt{2}=1.4$

$\sqrt{15}=3.9$

$= 9 + 1.7 + 2.8 + 3.9 = 17.4<18$]

80. (1) $a\sqrt{2}+b\sqrt{3}$

$= \sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$

$\Rightarrow a\sqrt{2}+b\sqrt{3}$

$= \sqrt{7\times7\times2}+\sqrt{2\times2\times3\times3\times3}$

$-\sqrt{2\times2\times2\times2\times3}-\sqrt{2\times2\times2\times3\times3}$

$\Rightarrow a\sqrt{2}+b\sqrt{3}$

$= 7\sqrt{2}+6\sqrt{3}-4\sqrt{3}-6\sqrt{2}$

$= \sqrt{2}+2\sqrt{3}$

$\therefore a = 1,\ b = 2$

81. (1) $\sqrt[3]{a} = \sqrt[3]{26}+\sqrt[3]{7}+\sqrt[3]{63}$

$\Rightarrow \sqrt[3]{a} < \sqrt[3]{27}+\sqrt[3]{8}+\sqrt[3]{64}$

$\Rightarrow \sqrt[3]{a} < 3+2+4$

$\Rightarrow \sqrt[3]{a} < 9$

$\Rightarrow a < 9^3 = 729$

82. (4) $\dfrac{\sqrt{72}\times\sqrt{363}\times\sqrt{175}}{\sqrt{32}\times\sqrt{147}\times\sqrt{252}}$

$= \dfrac{\sqrt{2\times2\times2\times3\times3}\times\sqrt{3\times11\times11}\times\sqrt{5\times5\times7}}{\sqrt{2\times2\times2\times2\times2}\times\sqrt{7\times7\times3}\times\sqrt{2\times2\times3\times3\times7}}$

$= \dfrac{6\sqrt{2}\times11\sqrt{3}\times5\sqrt{7}}{4\sqrt{2}\times7\sqrt{3}\times6\sqrt{7}}$

$= \dfrac{6\times11\times5}{4\times7\times6} = \dfrac{55}{28}$

83. (3) Expression

$= \dfrac{5.32\ (56+44))}{(7.66+2.34)\ (7.66-2.34)}$

$= \dfrac{532}{10\times5.32} = 10$

84. (4) Expression

$= 2+\dfrac{6}{\sqrt{3}}+\dfrac{1}{2+\sqrt{3}}+\dfrac{1}{\sqrt{3}-2}$

$= 2+\dfrac{6}{\sqrt{3}}+\dfrac{1}{2+\sqrt{3}}-\dfrac{1}{2-\sqrt{3}}$

$= 2+2\sqrt{3}+\left(\dfrac{2-\sqrt{3}-2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}\right)$

$= 2+2\sqrt{3}-2\sqrt{3} = 2$

85. (3) $\sqrt{7+4\sqrt{3}} = \sqrt{7+2\times2\times\sqrt{3}}$

$= \sqrt{4+3+2\times2\times\sqrt{3}}$

$= \sqrt{\left(2+\sqrt{3}\right)^2} = 2+\sqrt{3}$

$\therefore \dfrac{4+3\sqrt{3}}{2+\sqrt{3}} = A+\sqrt{B}$

$\Rightarrow \dfrac{\left(4+3\sqrt{3}\right)\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} = A+\sqrt{B}$

$\Rightarrow \dfrac{8-4\sqrt{3}+6\sqrt{3}-9}{4-3} = A+\sqrt{B}$

$\Rightarrow 2\sqrt{3}-1 = A+\sqrt{B}$

$\Rightarrow A = -1$ and $\sqrt{B} = 2\sqrt{3}$

$\Rightarrow B = 2\sqrt{3}\times2\sqrt{3} = 12$

$\therefore B - A = 12 + 1 = 13$

86. (2) Expression

$= 2\sqrt{50}+\sqrt{18}-\sqrt{72}$

$= 2\sqrt{2\times5\times5}+\sqrt{3\times3\times2}$

$-\sqrt{2\times2\times2\times3\times3}$

$= 10\sqrt{2}+3\sqrt{2}-6\sqrt{2}$

$= (10 \times 1.414) + (3 \times 1.414) - (6 \times 1.414) = 7 \times 1.414 = 9.8\ 98$

87. (2) $(6.5 \times 6.5 - 45.5 + 3.5 \times 3.5)$

$= [(6.5)^2 - 2 \times 6.5 \times 3.5 + (3.5)^2]$

$= (6.5 - 3.5)^2 = (3)^2 = 9$

88. (1) $? = (7.5 \times 7.5 + 37.5 + 2.5 \times 2.5)$

$= [(7.5)^2 + 2 \times 7.5 \times 2.5 + (2.5)^2]$

$= [7.5 + 2.5]^2 = (10)^2 = 100$

89. (3)

$\dfrac{(1.5)^2+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$

$= \dfrac{(1.5+4.7+3.8)\left(\begin{array}{l}1.5^2+4.7^2+3.8^2-1.5\times4.7\\-4.7\times3.8-3.8\times1.5\end{array}\right)}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$

$= 1.5 + 4.7 + 3.8 = 10$

90. (4) $\dfrac{(6.25)^{\frac{1}{2}}\times(0.0144)^{\frac{1}{2}}+1}{(0.027)^{\frac{1}{3}}\times(81)^{\frac{1}{4}}}$

$= \dfrac{2.5\times0.12+1}{0.3\times3} = \dfrac{0.3+1}{0.9} = \dfrac{1.3}{0.9}$

$= 1.4444 = 1.\overline{4}$

91. (2) Let 0.41 = x and 0.69 = y

$\therefore$ Expression $= \dfrac{\left(x^3+y^3\right)}{\left(x^2-xy+y^2\right)}$

$= \dfrac{(x+y)\left(x^2-xy+y^2\right)}{\left(x^2-xy+y^2\right)}$

$= x + y = 0.41 + 0.69 = 1.10$

92. (3) Expression

$= \frac{10.3 \times 10.3 \times 10.3 + 1 \times 1 \times 1}{10.3 \times 10.3 - 10.3 \times 1 + 1 \times 1}$

Let 10.3 = a and 1 = b,

Then,

Expression $= \frac{a^3 + b^3}{a^2 - ab + b^2}$

$= \frac{(a+b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$

$= a + b = 10.3 + 1 = 11.3$

93. (3) Let, 1.49 = a and 0.51 = b

$\therefore \frac{a^2 - b^2}{a - b}$

$= \frac{(a+b)(a-b)}{(a-b)} = a + b$

$\therefore$ 1.49 + 0.51 = 2

94. (2) $(0.04)^{-1.5} = \frac{1}{(0.04)^{1.5}}$

$= \frac{1}{[(0.2)^2]^{\frac{3}{2}}} = \frac{1}{(0.2)^{2\times\frac{3}{2}}} = \frac{1}{(0.2)^3}$

$= \frac{1}{0.008} = \frac{1000}{8} = 125$

95. (3) Let 0.96 = a and 0.1 = b,

$\therefore$ Expression

$= \frac{a^3 - b^3}{a^2 + ab + b^2} = \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2}$

$= a - b = 0.96 - 0.1 = 0.86$

96. (2) Expression

$= \frac{(4)^3 - (0.2)^3}{(4)^2 + 4 \times 0.2 + (0.2)^2}$

Let 4 = a, 0.2 = b

$\therefore$ Expression

$= \frac{a^3 - b^3}{a^2 + ab + b^2}$

$= \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2}$

$= a - b = 4 - 0.2 = 3.8$

97. (4) Let 0.796 = a

and 0.204 = b

$\therefore$ Expression $= \frac{a \times a - b \times b}{a - b}$

$= \frac{a^2 - b^2}{a - b} = \frac{(a+b)(a-b)}{a - b}$

$= a + b = 0.796 + 0.204 = 1$

98. (1) Expression

$= \frac{(2.3)^3 + (0.3)^3}{(2.3)^2 - 2.3 \times 0.3 + 0.3 \times 0.3}$

Let 2.3 = a and 0.3 = b

$\therefore$ Expression $= \frac{a^3 + b^3}{a^2 - ab + b^2}$

$= a + b$

$= 2.3 + 0.3 = 2.6$

99. (4) Let 5.71 = a and

2.79 = b

$\therefore$ Expression

$= \frac{a^3 - b^3}{a^2 + ab + b^2}$

$= \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2}$

$= a - b = 5.71 - 2.79 = 2.92$

100. (3) Let 1.5 = a, 4.7 = b, 3.8 = c

$\therefore$ Expression

$= \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$

$= a + b + c$

$= 1.5 + 4.7 + 3.8 = 10$

101. (1) Let 0.73 = a and 0.27 = b

$\therefore$ Expression

$= \frac{a^3 + b^3}{a^2 + b^2 - ab}$

$= \frac{(a+b)(a^2 + b^2 - ab)}{a^2 + b^2 - ab}$

$= a + b = 0.73 + 0.27 = 1$

102. (3) Expression

= 0.75 × 7.5 – 2 × 7.5 × 0.25 + 0.25 × 2.5

= 0.75 × 0.75 × 10 –2 × 0.75 × 0.25 × 10 + 0.25 × 0.25 ×10

$= 10((0.75)^2 - 2 \times 0.75 \times 0.25 + (0.25)^2)$

$= 10(0.75 - 0.25)^2 = 10 \times 0.25 = 2.5$

103. (2) Expression

$= \frac{1}{1\times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16}$

$= \frac{1}{3}\left(\left(\frac{3}{1\times 4}\right) + \left(\frac{3}{4\times 7}\right) + \left(\frac{3}{7\times 10}\right) + \left(\frac{3}{10\times 13}\right) + \left(\frac{3}{13\times 16}\right)\right)$

$= \frac{1}{3}\left[\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) + \left(\frac{1}{10} - \frac{1}{13}\right) + \left(\frac{1}{13} - \frac{1}{16}\right)\right]$

$= \frac{1}{3}\left(1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} + \frac{1}{13} - \frac{1}{16}\right)$

$= \frac{1}{3}\left(1 - \frac{1}{16}\right) = \frac{1}{3}\left(\frac{16-1}{16}\right)$

$= \frac{1}{3} \times \frac{15}{16} = \frac{5}{16}$

104. (3) Expression

$= \frac{137 \times 137 + 133 \times 133 + 137 \times 133}{137 \times 137 \times 137 - 133 \times 133 \times 133}$

[$\because$ 137×133=18221]

Let 137 = a and 133 = b

$\therefore$ Expression $= \frac{a^2 + b^2 + ab}{a^3 - b^3}$

$= \frac{a^2 + b^2 + ab}{(a-b)(a^2 + b^2 + ab)}$

$= \frac{1}{a - b} = \frac{1}{137 - 133} = \frac{1}{4}$

105. (2) Let 2.75 = a

and 2.25 = b

$\therefore$ Expression $= \frac{a^3 - b^3}{a^2 + ab + b^2}$

$= \frac{(a-b)(a^2 + ab + b^2)}{a^2 + ab + b^2}$

$= a - b = 2.75 - 2.25 = 0.5$

106. (1) Let 5.624 = a

and 4.376 = b

$\therefore$ Given expression $= \frac{a^3 + b^3}{a^2 - ab + b^2}$

$= \frac{(a+b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$

$= a + b = 5.624 + 4.376 = 10$

107. (1) Let 0.337 = a and 0.126 = b.

$\therefore$ Expression

$= \frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$

108. (2) $\boxed{\text{Tricky approach}}$

If 256 = a and 144 = b, then

Expression = $\dfrac{a^2-b^2}{a-b}$

$[a - b = 256 - 144 = 112]$

$= \dfrac{(a+b)(a-b)}{(a-b)} = a+b$

= 256 + 144 = 400

109. (4) Expression

$= (8.7 + 1.3)^2$

$[a^2 + 2ab + b^2 = (a + b)^2]$

$= 10^2 = 100$

110. (1) Let 3.06 = a and 1.98 = b

$\therefore$ Expression

$= \dfrac{a^3-b^3}{a^2+ab+b^2}$

$= \dfrac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}$

$= a - b = 3.06 - 1.98 = 1.08$

111. (3) If 3.25 = a and 1.75 = b then,

Expression = $\dfrac{a^2+b^2-2ab}{a^2-b^2}$

$= \dfrac{(a-b)^2}{(a+b)(a-b)} = \dfrac{a-b}{a+b}$

$= \dfrac{3.25-1.75}{3.25+1.75} = \dfrac{1.5}{5} = 0.3$

112. (2) Let 0.05 = a $\therefore$ 0.005 = $\dfrac{a}{10}$

0.41 = b $\therefore$ 0.041 = $\dfrac{b}{10}$

and 0.073 = c $\therefore$ 0.0073 = $\dfrac{c}{10}$

$\therefore$ Expression

$= \dfrac{a^2+b^2+c^2}{\left(\frac{a}{10}\right)^2+\left(\frac{b}{10}\right)^2+\left(\frac{c}{10}\right)^2}$

$= \dfrac{a^2+b^2+c^2}{\frac{1}{100}\left(a^2+b^2+c^2\right)} = 100$

113. (1) If 2.3 = a and 1 = b,

Expression = $\dfrac{a^3-b^3}{a^2+ab+b^2}$

$= a - b = 2.3 - 1 = 1.3$

114. (4)

$(0.98)^3 + (0.02)^3 + 3 \times 0.98 \times 0.2 - 1$

$= 0.941192 + 0.000008 + 0.0588 - 1$

$= 1 - 1 = 0$

115. (2) If 0.08 = a and 0.02 = b then

Expression = $\dfrac{a^3+b^3}{a^2-ab+b^2}$

$= \dfrac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2} = a + b$

= 0.08 + 0.02 = 0.1

116. (2) Expression = 0.65 × 0.65 + 0.35 × 0.35 + 2 × 0.35 × 0.65

$= (0.65+0.35)^2 = (1)^2 = 1$

$[\because\ a^2 + b^2 + 2ab = (a + b)^2]$

117. (2) $\dfrac{2.4\times10^3}{8\times10^{-2}} = \dfrac{24\times10^3}{8\times10\times10^{-2}}$

$= \dfrac{24\times10^3\times10}{8} = 3 \times 10^4$

118. (3) Given expression

$= [3 - 4(3 - 4)^{-1}]^{-1}$

$= [3 - 4(-1)^{-1}]^{-1}$

$= \left[3-\dfrac{4}{-1}\right]^{-1} = (3+4)^{-1}$

$= (7)^{-1} = \dfrac{1}{7}$

119. (4) Expression

$= \dfrac{\left[(998)^2-(997)^2\right]-45}{(98)^2-(97)^2}$

$= \dfrac{(998+997)(998-997)-45}{(98+97)(98-97)}$

$= \dfrac{1995-45}{195} = \dfrac{1950}{195} = 10$

120. (2) Expression = $\dfrac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$

$= \dfrac{\sqrt{2\times2\times6}+\sqrt{6}}{\sqrt{2\times2\times6}-\sqrt{6}} = \dfrac{2\sqrt{6}+\sqrt{6}}{2\sqrt{6}-\sqrt{6}}$

$= \dfrac{\sqrt{6}(2+1)}{\sqrt{6}(2-1)} = 3$

121. (2) $a = 55$, $b = 17$, $c = -72$

$a + b + c = 55 + 17 - 72 = 0$

$\therefore\ a^3 + b^3 + c^3 - 3abc = 0$

122. (4) Let 2.75 = a and 2.25 = b

$\therefore$ Expression = $\dfrac{a^3-b^3}{a^2+ab+b^2}$

$= \dfrac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}$

$= a - b = 2.75 - 2.25$

$= 0.50 = \dfrac{1}{2}$

123. (2) Expression

$= \dfrac{(243)^{\frac{n}{5}}\times3^{2n+1}}{9^n\times3^{n-1}}$

$= \dfrac{(3^5)^{\frac{n}{5}}\times3^{2n+1}}{(3^2)^n\times3^{n-1}} = \dfrac{(3)^{5\times\frac{n}{5}}\times3^{2n+1}}{3^{2n}\times3^{n-1}}$

$= \dfrac{3^n\times3^{2n+1}}{3^{2n}\times3^{n-1}} = \dfrac{3^{3n+1}}{3^{3n-1}}$

$= 3^{3n+1-3n+1} = 3^2 = 9$

$[a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $(a^m)^n = a^{mn}]$

124. (3) Expression = $(\sqrt{3}+1)(10+\sqrt{12})(\sqrt{12}-2)(5-\sqrt{3})$

$= (\sqrt{3}+1)(10+2\sqrt{3})(2\sqrt{3}-2)(5-\sqrt{3})$

$= (\sqrt{3}+1)\times2(5+\sqrt{3})\times2(\sqrt{3}-1)(5-\sqrt{3})$

$= 4(\sqrt{3}+1)(\sqrt{3}-1)(5+\sqrt{3})(5-\sqrt{3})$

$= 4(3-1)(25-3)$

$[(a+b)(a-b) = a^2 - b^2]$

$= 4 \times 2 \times 22 = 176$

125. (2) Expression

$= (0.2)^3 \times 200 \div 2000$ of $(0.2)^2$

$= (0.2)^3 \times 200 \div (2000 \times 0.2 \times 0.2)$

$= \dfrac{0.2\times0.2\times0.2\times200}{2000\times0.2\times0.2}$

$= \dfrac{2\times2\times2\times200}{2000\times2\times2\times10}$

$= \dfrac{2}{100} = \dfrac{1}{50}$

126. (4) $\left(\sqrt{6}+\sqrt{10}-\sqrt{21}-\sqrt{35}\right)$

$\left(\sqrt{6}-\sqrt{10}+\sqrt{21}-\sqrt{35}\right)$

$= \left\{\left(\sqrt{6}-\sqrt{35}\right)+\left(\sqrt{10}-\sqrt{21}\right)\right\}$

$$\left\{\left(\sqrt{6}-\sqrt{35}\right)-\left(\sqrt{10}-\sqrt{21}\right)\right\}$$

$$=\left(\sqrt{6}-\sqrt{35}\right)^2-\left(\sqrt{10}-\sqrt{21}\right)^2$$

$$=\left(6+35-2\sqrt{210}\right)$$

$$-\left(10+21-2\sqrt{210}\right)$$

$= 41 - 2\sqrt{210} - 31 + 2\sqrt{210}$

$= 41 - 31 = 10$

127. (1) $\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$

Similarly, $\frac{1}{\sqrt{3}+\sqrt{2}}$

$=\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$=\frac{\sqrt{3}-\sqrt{2}}{3-2}$

$=\sqrt{3}-\sqrt{2}$

$\therefore$ Expression $=\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\sqrt{7}-\sqrt{6}+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}$

$=\sqrt{9}-1$

$= 3 - 1 = 2$

128. (4) $\frac{1}{\sqrt{7}-\sqrt{6}}$

$=\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$

(rationalising the denominator)

$=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\sqrt{7}+\sqrt{6}$

Similarly,

$\frac{1}{\sqrt{6}-\sqrt{5}}=\sqrt{6}+\sqrt{5}$;

$\frac{1}{\sqrt{5}-2}=\sqrt{5}+2$

$\frac{1}{\sqrt{8}-\sqrt{7}}=\sqrt{8}+\sqrt{7}$,

$\frac{1}{3-\sqrt{8}}=3+\sqrt{8}$

$\therefore$ Expression

$=\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+2\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(3+\sqrt{8}\right)$

$=\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2-\sqrt{8}-\sqrt{7}+3+\sqrt{8}=2+3=5$

129. (1) $2+x\sqrt{3}$

$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$

(On rationalising the denominator)

$\Rightarrow 2+x\sqrt{3}=2-\sqrt{3}$

$\Rightarrow x\sqrt{3}=-\sqrt{3}\Rightarrow x=-1$

130. (3) Expression

$=\sqrt{\frac{0.324\times0.081\times4.624}{1.5625\times0.0289\times72.9\times64}}$

$=\sqrt{\frac{324\times81\times4624}{15625\times289\times729\times64}}$

$=\frac{18\times9\times68}{125\times17\times27\times8}=0.024$

131. (3) $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a+\sqrt{7}\,b$

$\Rightarrow\frac{\left(\sqrt{7}-1\right)^2-\left(\sqrt{7}+1\right)^2}{\left(\sqrt{7}+1\right)\left(\sqrt{7}-1\right)}=a+\sqrt{7}\,b$

$\Rightarrow\frac{-4\times\sqrt{7}\times1}{7-1}=a+\sqrt{7}\,b$

[$\because$ $(a-b)^2-(a+b)^2=-4ab$]

$\Rightarrow\frac{-4\sqrt{7}}{6}=a+\sqrt{7}\,b$

$\Rightarrow 0-\frac{2}{3}\sqrt{7}=a+\sqrt{7}\,b$

$\Rightarrow a=0,\ b=-\frac{2}{3}$

132. (3) $\frac{1}{\sqrt{2}+1}=\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}$

(Rationalising the denominator)

$=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$

Similarly,

$\frac{1}{\sqrt{2}+\sqrt{3}}=\sqrt{3}-\sqrt{2}$;

$\frac{1}{\sqrt{4}+\sqrt{3}}=\sqrt{4}-\sqrt{3}$ .....;

$\frac{1}{\sqrt{8}+\sqrt{9}}=\sqrt{9}-\sqrt{8}$

$\therefore$ Expression

$=\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\sqrt{7}-\sqrt{6}+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}=\sqrt{9}-1=3-1=2$

133. (4) $\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}=\frac{\sqrt{3}}{1}$

By componendo and dividendo,

$\frac{\sqrt{a+2b}+\sqrt{a-2b}+\sqrt{a+2b}-\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}-\sqrt{a+2b}+\sqrt{a-2b}}$

$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow\frac{\sqrt{a+2b}}{\sqrt{a-2b}}=\frac{\sqrt{3}+1}{\sqrt{3}-1}$

On squaring,

$\frac{a+2b}{a-2b}=\frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}=\frac{4+2\sqrt{3}}{4-2\sqrt{3}}$

$\Rightarrow\frac{a+2b}{a-2b}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$

By componendo and dividendo,

$\frac{a+2b+a-2b}{a+2b-a+2b}=\frac{2+\sqrt{3}+2-\sqrt{3}}{2+\sqrt{3}+2+\sqrt{3}}$

$\Rightarrow\frac{2a}{4b}=\frac{4}{2\sqrt{3}}\Rightarrow\frac{a}{2b}=\frac{2}{\sqrt{3}}$

$\Rightarrow\frac{a}{b}=\frac{4}{\sqrt{3}}=4:\sqrt{3}$

134. (4) Expression

$$= \frac{(75.8)^2 - (35.8)^2}{40}$$

$$= \frac{(75.8+35.8)(75.8-35.8)}{40}$$

$$= \frac{111.6 \times 40}{40} = 111.6$$

135. (4) Let, $0.67 = a$ and $0.33 = b$

$$\therefore \text{ Expression} = \frac{a^3 - b^3}{\left(a^2 + ab + b^2\right)}$$

$$= \frac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}$$

$= a - b = 0.67 - 0.33 = 0.34$

136. (3) Expression

$$= \frac{1}{\left(1+\sqrt{3}\right)+\sqrt{2}} + \frac{1}{\left(1+\sqrt{3}\right)-\sqrt{2}}$$

$$= \frac{1+\sqrt{3}-\sqrt{2}+1+\sqrt{3}+\sqrt{2}}{\left(1+\sqrt{3}+\sqrt{2}\right)\left(1+\sqrt{3}-\sqrt{2}\right)}$$

$$= \frac{2+2\sqrt{3}}{\left(1+\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{2\left(1+\sqrt{3}\right)}{1+3+2\sqrt{3}-2}$$

$$= \frac{2\left(1+\sqrt{3}\right)}{2\left(1+\sqrt{3}\right)} = 1$$

137. (3) $a = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

(Rationalising the denominator)

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{3+2-2\times\sqrt{3}\times\sqrt{2}}{3-2}$$

$= 5 - 2\sqrt{6}$

$$\therefore b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5 + 2\sqrt{6}$$

$\therefore a + b = 5 - 2\sqrt{6} + 5 + 2\sqrt{6}$
$= 10$ and

$$ab = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 1$$

$$\therefore \frac{a^2}{b} + \frac{b^2}{a} = \frac{a^3+b^3}{ab}$$

$= a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
$= (10)^3 - 3 \times 10 = 1000 - 30$
$= 970$

138. (3) $1^3 + 2^3 + \ldots\ldots + 10^3 = 3025$
$\therefore 2^3 + 4^3 + \ldots\ldots + 20^3$
$= 2^3 (1 + 2^3 + \ldots\ldots + 10^3)$
$= 8 \times 3025 = 24200$

139. (4) Expression

$$= \frac{(2.3)^3 + 0.027}{(2.3)^2 - 0.69 + 0.09}$$

$$= \frac{(2.3)^3 + (0.3)^3}{(2.3)^2 - 2.3\times 0.3 + (0.3)^2}$$

If $2.3 = a$ and $0.3 = b$, then Expression

$$= \frac{a^3+b^3}{a^2-ab+b^2}$$

$$= \frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2}$$

$= a + b = 2.3 + 0.3 = 2.6$

140. (3) Expression $= \left(1-\sqrt{2}\right) + \left(\sqrt{2}-\sqrt{3}\right) + \left(\sqrt{3}-\sqrt{4}\right) + \ldots. + \left(\sqrt{15}-\sqrt{16}\right)$

$= 1 - \sqrt{2} + \sqrt{2} - \sqrt{3} + \sqrt{3} - \sqrt{4} + \ldots. + \sqrt{15} - \sqrt{16}$
$= 1 - \sqrt{16} = 1 - 4 = -3$

141. (1) $\dfrac{1}{\sqrt{11-2\sqrt{30}}}$

$$= \frac{1}{\sqrt{6+5-2\times\sqrt{6}\times\sqrt{5}}}$$

$$= \frac{1}{\sqrt{(6)^2 + \left(\sqrt{5}\right)^2 - 2\times\sqrt{6}\times\sqrt{5}}}$$

$$= \frac{1}{\sqrt{\left(\sqrt{6}-\sqrt{5}\right)^2}}$$

$$= \frac{1}{\sqrt{6}-\sqrt{5}}$$

$$= \frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}+\sqrt{5}\right)}$$

$= \sqrt{6}+\sqrt{5}$

$$\frac{3}{\sqrt{7-2\sqrt{10}}}$$

$$= \frac{3}{\sqrt{5+2-2\times\sqrt{5}\times\sqrt{2}}}$$

$$= \frac{3}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)}$$

$$= \frac{3\left(\sqrt{5}+\sqrt{2}\right)}{5-2} = \sqrt{5}+\sqrt{2}$$

$$\frac{4}{\sqrt{8+4\sqrt{3}}} = \frac{4}{\sqrt{8+2\sqrt{12}}}$$

$$= \frac{4}{\sqrt{6+2+2\times\sqrt{6}\times\sqrt{2}}}$$

$$= \frac{4}{\sqrt{\left(\sqrt{6}+\sqrt{2}\right)^2}}$$

$$= \frac{4}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$$

$$= \frac{4\left(\sqrt{6}-\sqrt{2}\right)}{6-2} = \sqrt{6} - \sqrt{2}$$

$\therefore$ Expression

$$= \left(\sqrt{6}+\sqrt{5}\right) - \left(\sqrt{5}+\sqrt{2}\right) - \left(\sqrt{6}-\sqrt{2}\right)$$

$= \sqrt{6}+\sqrt{5}-\sqrt{5}-\sqrt{2} - \sqrt{6} + \sqrt{2}$
$= 0$

142. (2) $\dfrac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$

$= \dfrac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}}$

$= \dfrac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \dfrac{3^{n+2n+1}}{3^{2n+n-1}}$

$= \dfrac{3^{3n+1}}{3^{3n-1}}$

$= 3^{3n+1-(3n-1)} = 3^2 = 9$

143. (2) $(d^{s+t} \div d^s) \div d^t$

$= d^{s+t-s} \div d^t$

$= d^t \div d^t = 1$

144. (3) $2^{51} + 2^{52} + 2^{53} + 2^{54} + 2^{55}$

$= 2^{51} (1 + 2 + 2^2 + 2^3 + 2^4)$

$= 2^{51} (1 + 2 + 4 + 8 + 16)$

$= 2^{51} \times 31$

$= 2^{49} \times 4 \times 31$

$= 2^{49} \times 124$

145. (3) $\dfrac{\sqrt{2+x} + \sqrt{2-x}}{\sqrt{2+x} - \sqrt{2-x}} = \dfrac{2}{1}$

By componendo and dividendo,

$\dfrac{\sqrt{2+x} + \sqrt{2-x} + \sqrt{2+x} - \sqrt{2-x}}{\sqrt{2+x} + \sqrt{2-x} - \sqrt{2+x} + \sqrt{2-x}}$

$= \dfrac{2+1}{2-1}$

$\Rightarrow \dfrac{2\sqrt{2+x}}{2\sqrt{2-x}} = 3$

On squaring,

$\dfrac{2+x}{2-x} = 9 \Rightarrow 2 + x = 18 - 9x$

$\Rightarrow 10x = 18 - 2 \Rightarrow 10x = 16$

$\Rightarrow x = \dfrac{16}{10} = \dfrac{8}{5}$

146. (1) Expression

$= \dfrac{3 \times 9^{n+1} + 9 \times 3^{2n-1}}{9 \times 3^{2n} - 6 \times 9^{n-1}}$

$= \dfrac{3 \times (3^2)^{n+1} + 3^2 \times 3^{2n-1}}{3^2 \times 3^{2n} - 6 \times (3^2)^{n-1}}$

$= \dfrac{3^{2n+2+1} + 3^{2n-1+2}}{3^{2n+2} - 6 \times 3^{2n-2}}$

$= \dfrac{3^{2n+3} + 3^{2n+1}}{3^{2n+2} - 6 \times 3^{2n-2}}$

$= \dfrac{3^{2n+1}(3^2 + 1)}{3^{2n-2}(3^4 - 6)}$

$= 3^{2n+1-2n+2}\left(\dfrac{10}{75}\right)$

$= \dfrac{3^3 \times 10}{75} = \dfrac{27 \times 10}{75}$

$= \dfrac{18}{5} = 3\dfrac{3}{5}$

147. (3) Expression

$= \left(\dfrac{2+\sqrt{3}}{2-\sqrt{3}} - 4\sqrt{3}\right)^2$

$= \left\{\left(\dfrac{2+\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}\right) - 4\sqrt{3}\right\}^2$

$= \left\{\dfrac{(2+\sqrt{3})^2}{4-3} - 4\sqrt{3}\right\}^2$

$= \left(4 + 4\sqrt{3} + 3 - 4\sqrt{3}\right)^2$

$= 7^2 = 49$

148. (2) Expression

$= \sqrt[3]{-2197} \times \sqrt[3]{-125} \div \sqrt[3]{\dfrac{27}{512}}$

$= \sqrt[3]{-13 \times -13 \times -13} \times \sqrt[3]{-5 \times -5 \times -5}$

$\div \sqrt[3]{\dfrac{3 \times 3 \times 3}{8 \times 8 \times 8}}$

$= -13 \times -5 \div \dfrac{3}{8}$

$= \dfrac{65 \times 8}{3} = \dfrac{520}{3}$

149. (2) $1 - \dfrac{1}{1+\sqrt{2}} + \dfrac{1}{1-\sqrt{2}}$

$= 1 - \left(\dfrac{1}{1+\sqrt{2}} - \dfrac{1}{1-\sqrt{2}}\right)$

$= 1 - \left(\dfrac{1}{\sqrt{2}+1} + \dfrac{1}{\sqrt{2}-1}\right)$

$= 1 - \left(\dfrac{\sqrt{2} - 1 + \sqrt{2} + 1}{(\sqrt{2}+1)(\sqrt{2}-1)}\right)$

$= 1 - \dfrac{2\sqrt{2}}{2-1} = 1 - 2\sqrt{2}$

150. (2) Expression

$= \dfrac{3\sqrt{8} - 2\sqrt{12} + \sqrt{20}}{3\sqrt{18} - 2\sqrt{27} + \sqrt{45}}$

$= \dfrac{3\sqrt{2\times2\times2} - 2\sqrt{2\times2\times3} + \sqrt{2\times2\times5}}{3\sqrt{3\times3\times2} - 2\sqrt{3\times3\times3} + \sqrt{3\times3\times5}}$

$= \dfrac{6\sqrt{2} - 4\sqrt{3} + 2\sqrt{5}}{9\sqrt{2} - 6\sqrt{3} + 3\sqrt{5}}$

$= \dfrac{2(3\sqrt{2} - 2\sqrt{3} + \sqrt{5})}{3(3\sqrt{2} - 2\sqrt{3} + \sqrt{5})} = \dfrac{2}{3}$

151. (1) $\dfrac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}}$

$= \dfrac{3\sqrt{7}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

(Rationalising the denominator)

$= \dfrac{3\sqrt{7}(\sqrt{5}-\sqrt{2})}{5-2}$

$= \sqrt{7}(\sqrt{5}-\sqrt{2})$

$= \sqrt{35} - \sqrt{14}$

Similarly,

$\dfrac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} = \dfrac{5\sqrt{5}(\sqrt{7}-\sqrt{2})}{(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})}$

$= \dfrac{5\sqrt{5}(\sqrt{7}-\sqrt{2})}{7-2}$

$= \sqrt{5}(\sqrt{7}-\sqrt{2}) = \sqrt{35} - \sqrt{10}$

$\dfrac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}} = \dfrac{2\sqrt{2}(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}$

$= \dfrac{2\sqrt{2}(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{2}\left(\sqrt{7}-\sqrt{5}\right) = \sqrt{14}-\sqrt{10}$

$\therefore$ Expression

$= \left(\sqrt{35}-\sqrt{14}\right)-\left(\sqrt{35}-\sqrt{10}\right)+\left(\sqrt{14}-10\right)$

$= \sqrt{35}-\sqrt{14}-\sqrt{35}+\sqrt{10}+\sqrt{14}-\sqrt{10}$

$= 0$

152. (1) Let 0.73 = a and 0.27 = b

$\therefore$ Expression $= \dfrac{a^3+b^3}{a^2-ab+b^2}$

$= \dfrac{(a+b)\left(a^2-ab+b^2\right)}{a^2-ab+b^2} = a + b$

$= 0.73 + 0.27 = 1$

153. (*) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{12}-\sqrt{18}} - \dfrac{1}{3} \times \sqrt{27} - \dfrac{1}{2} \times \sqrt[3]{27}$

$= \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{4\times3}-\sqrt{3\times3\times2}} - \dfrac{1}{3}$

$\sqrt{3\times3\times3} - \dfrac{1}{2} \times \sqrt[3]{3\times3\times3}$

$= \dfrac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}-3\sqrt{2}} - \dfrac{1}{3} \times 3\sqrt{3} - \dfrac{1}{2} \times 3$

$= \dfrac{\left(\sqrt{3}-\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)}{\left(2\sqrt{3}-3\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)} - \sqrt{3} - \dfrac{3}{2}$

$= \dfrac{2\times3-2\sqrt{6}+3\sqrt{6}-6}{\left(2\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2} - \sqrt{3} - \dfrac{3}{2}$

$= \dfrac{\sqrt{6}}{12-18} - \sqrt{3} - \dfrac{3}{2}$

$= \dfrac{\sqrt{6}}{-6} - \sqrt{3} - \dfrac{3}{2}$

$= \dfrac{-\sqrt{6}-6\sqrt{3}-9}{6}$

TYPE-II

1. (4) $\sqrt{3}, \sqrt[3]{2}, \sqrt{2}$ and $\sqrt[3]{4}$

LCM of 2 and 3 = 6

$\therefore \sqrt{3} = (3)^{\frac{1}{2}} = 3^{\frac{3}{6}} = \left(3^3\right)^{\frac{1}{6}} = \sqrt[6]{27}$

$\sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4}$

$\sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$

$\sqrt[3]{4} = \sqrt[6]{4^2} = \sqrt[6]{16}$

2. (1) $\sqrt[3]{4} = \sqrt[12]{256}$, $\sqrt[4]{6} = \sqrt[12]{216}$,

$\sqrt[6]{15} = \sqrt[12]{225}$, $\sqrt[12]{245}$

3. (3) $(0.5)^2 = 0.25$

$\sqrt{0.49} = 0.7$

$\sqrt[3]{0.008} = 0.2$

$0.23 = 0.23$

$\therefore \sqrt{0.49} > (0.5)^2 > 0.23 > \sqrt[3]{0.008}$

4. (1) $\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$

L.C.M. of 3, 2, 6, 4, = 12

$\sqrt[3]{4} = (4)^{\frac{1}{3}} = (4)^{\frac{4}{12}}$

$= \left(4^4\right)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$

$\sqrt{2} = (2)^{\frac{1}{2}} = (2)^{\frac{6}{12}}$

$= \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$

$\sqrt[6]{3} = (3)^{\frac{1}{6}} = (3)^{\frac{2}{12}} = \left(3^2\right)^{\frac{1}{12}}$

$= (9)^{\frac{1}{12}}$

$\sqrt[4]{5} = (5)^{\frac{1}{4}} = (5)^{\frac{3}{12}} = \left(5^3\right)^{\frac{1}{12}}$

$= (125)^{\frac{1}{12}}$

$\therefore (256)^{\frac{1}{12}} > (125)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (9)^{\frac{1}{12}}$

or, $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

5. (3) $(2.89)^{0.5} = (2.89)^{\frac{5}{10}}$

$= \sqrt{2.89} = 1.7$

$= 2 - (0.5)^2 = 2 - 0.25 = 1.75$

$1 + \dfrac{0.5}{1-\dfrac{1}{2}} = 1 + \dfrac{0.5}{\dfrac{1}{2}}$

$= 1 + \dfrac{0.5}{0.5} = 1 + 1 = 2$

$\sqrt{3} = 1.732$

6. (1) LCM of 2, 3, 4, 3 = 12

Thus $\sqrt{2} = \left(2^6\right)^{\frac{1}{12}} = \sqrt[12]{64}$

$\sqrt[3]{3} = \left(3^4\right)^{\frac{1}{12}} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$

Obviously, $\sqrt[4]{5}$ is the greatest

= 0.05

7. (2) $(2.89)^{0.5} = (2.89)^{\frac{1}{2}} = 1.7$,

$2 - (0.5)^2 = 2 - 0.25 = 1.75$,

$\sqrt{3} = 1.732$

and $\sqrt[3]{0.008}$

$= \sqrt[3]{0.2\times0.2\times0.2} = 0.2$

Obviously,

$0.2 < 1.7 < 1.732 < 1.75$

$\therefore \sqrt[3]{0.008} < (2.89)^{0.5} < \sqrt{3} < 2-(0.5)^2$

8. (2) $\sqrt{2}, \sqrt[3]{3}, \sqrt[6]{6}, \sqrt[5]{5}$

LCM of 2, 3, 6 & 5 = 30

$2^{\frac{1}{2}} = 2^{\frac{15}{30}} = \sqrt[30]{2^{15}} = 32768$

$3^{1/3} = 3^{\frac{10}{30}} = \sqrt[30]{3^{10}} = 59049$

$6^{1/6} = 6^{5/30} = \sqrt[30]{6^5} = 7776$

$5^{1/5} = 5^{6/30} = \sqrt[30]{5^5} = 15625$

Therefore, $\sqrt[3]{3}$ is the greatest.

9. (4) Here, $\left(\sqrt{8}+\sqrt{5}\right)^2$

$= \left(\sqrt{8}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{8}\times\sqrt{5}$

$= 8 + 5 + 2 \times \sqrt{8\times5}$

$= 13 + 2\sqrt{40}$

Similarly,

$\left(\sqrt{7}+\sqrt{6}\right)^2 = 7 + 6 + 2 \times \sqrt{7\times6}$

$= 13 + 2\sqrt{42}$,

$\left(\sqrt{10}+\sqrt{3}\right)^2$

$= 10 + 3 + 2 \times \sqrt{10\times3}$

$= 13 + 2\sqrt{30}$,

Similarly, $\left(\sqrt{11}+\sqrt{2}\right)^2$

$= 11 + 2 + 2\sqrt{11\times2}$

$= 13 + 2\sqrt{22}$

Clearly, $13 + 2\sqrt{22}$ is the smallest among these.

$\therefore \sqrt{11}+\sqrt{2}$ is the smallest.

10. (2) LCM of 2, 3, 4 and 6 = 12

$\therefore\ \sqrt{2} = (2)^{\frac{1}{2}} = (2)^{\frac{6}{12}}$

$= \left(2^6\right)^{\frac{1}{12}} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{4} = \sqrt[12]{4^4} = \sqrt[12]{64}$

$\sqrt[6]{6} = \sqrt[12]{6^2} = \sqrt[12]{36}$

11. (4) $\sqrt{19} - \sqrt{17}$

$$= \frac{\left(\sqrt{19} - \sqrt{17}\right) \times \left(\sqrt{19} \times \sqrt{17}\right)}{\sqrt{19} + \sqrt{17}}$$

$$= \frac{19 - 17}{\sqrt{19} + \sqrt{17}} = \frac{2}{\sqrt{19} + \sqrt{17}}$$

Similarly, $\sqrt{13} - \sqrt{11}$

$$= \frac{2}{\sqrt{13} + \sqrt{11}},$$

$$\sqrt{7} - \sqrt{5} = \frac{2}{\sqrt{7} + \sqrt{5}}$$

$$\sqrt{5} - \sqrt{3} = \frac{2}{\sqrt{5} + \sqrt{3}}$$

Clearly, $\sqrt{5} - \sqrt{3}$ is the greatest.

(Smaller the denominator, greater the no.)

12. (3) LCM of 3 and 2 = 6.

$\therefore\ \sqrt[3]{2} = \sqrt[6]{2^2} = \sqrt[6]{4};$

$\sqrt{3} = \sqrt[6]{27}; \sqrt[3]{5} = \sqrt[6]{25}$

$1.5 = \sqrt{2.25} = \sqrt[6]{(2.25)^3}$

13. (3) LCM of 2, 6, 3, 4 = 12

$\therefore\ \sqrt{2} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[6]{3} = \sqrt[12]{3^2} = \sqrt[12]{9}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Clearly,

$\sqrt[12]{9} < \sqrt[12]{64} < \sqrt[12]{125} < \sqrt[12]{256}$

$\therefore\ \sqrt[6]{3} < \sqrt{2} < \sqrt[4]{5} < \sqrt[3]{4}$

14. (1)

$$\sqrt{3} = (3)^{\frac{1}{2} \times \frac{6}{6}} = \left(3^6\right)^{\frac{1}{12}} = (729)^{\frac{1}{12}}$$

$$\sqrt[3]{4} = (4)^{\frac{1}{3} \times \frac{4}{4}} = \left(4^4\right)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$$

$$\sqrt[4]{6} = (6)^{\frac{1}{4} \times \frac{3}{3}} = \left(6^3\right)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$$

$$\sqrt[6]{8} = (8)^{\frac{1}{6} \times \frac{2}{2}} = \left(8^2\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$$

Now, it is clear that $\sqrt{3}$ is the greatest.

15. (2) $\dfrac{1}{\sqrt{7} - \sqrt{5}}$

$$= \frac{\sqrt{7} + \sqrt{5}}{(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5})}$$

$$= \frac{\sqrt{7} + \sqrt{5}}{7 - 5} = \frac{\sqrt{7} + \sqrt{5}}{2},$$

$$\frac{1}{\sqrt{5} - \sqrt{3}} = \frac{\sqrt{5} + \sqrt{3}}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$$

$$= \frac{\sqrt{5} + \sqrt{3}}{5 - 3} = \frac{\sqrt{5} + \sqrt{3}}{2}$$

Similarly,

$$\frac{1}{\sqrt{9} - \sqrt{7}} = \frac{\sqrt{9} + \sqrt{7}}{2}$$

$$\frac{1}{\sqrt{11} - \sqrt{9}} = \frac{\sqrt{11} + \sqrt{9}}{2}$$

Clearly, $\dfrac{\sqrt{5} + \sqrt{3}}{2}$ is the smallest.

$\therefore\ \dfrac{1}{\sqrt{5} - \sqrt{3}}$ is the smallest.

$\therefore\ \sqrt{5} - \sqrt{3}$ is the greatest.

16. (1) The orders of the given surds are 3, 2, 4 and 6.

Their LCM = 12

Now we convert each surd into a surd of order 12.

$\sqrt[3]{9} = (9)^{\frac{1}{3}} = (9)^{\frac{4}{12}} = (9^4)^{\frac{1}{12}}$

$= \sqrt[12]{6561}$

Similarly,

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\sqrt[4]{16} = \sqrt[12]{16^3} = \sqrt[12]{4096}$

$\sqrt[6]{80} = \sqrt[12]{80^2} = \sqrt[12]{6400}$

Clearly,

$\sqrt[12]{729} < \sqrt[12]{4096} < \sqrt[12]{6400} < \sqrt[12]{6561}$

$\therefore\ \sqrt[3]{9}$ is the greatest number.

17. (3) The orders of the surds are 2, 4, 2 and 2. Their LCM = 4

We convert each surd into a surd of order 4.

$2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12} = \sqrt[4]{(12)^2} = \sqrt[4]{144}$

$2\sqrt[4]{5} = \sqrt[4]{2^4 \times 5} = \sqrt[4]{80}$

$3\sqrt{2} = \sqrt{18} = \sqrt[4]{(18)^2} = \sqrt[4]{324}$

$\sqrt{8} = \sqrt[4]{64}$

Hence, the least number $= \sqrt{8}$

18.(3)

$$\begin{array}{r|l|l}
9 & 0.\overline{90}\ \overline{00} & 0.94 \\
9 & 81 & \\
\hline
184 & 900 & \\
4 & 736 & \\
\hline
188 & 164 &
\end{array}$$

$\therefore\ \sqrt{0.9} = 0.94 \approx 0.9$

19. (2) $2^{60} = \left(2^5\right)^{12} = (32)^{12}$

$5^{24} = \left(5^2\right)^{12} = (25)^{12}$

$\therefore\ 2^{60} > 5^{24}$

$3^{48} = \left(3^4\right)^{12} = (81)^{12}$

$\therefore\ 3^{48} > 2^{60}$

$4^{36} = \left(4^3\right)^{12} = (64)^{12}$

$\therefore\ 3^{48}$ is the largest number.

20. (4) LCM of orders 2,3,4, 6 = 12

$\therefore\ (2)^{\frac{1}{2}} = 2^{\frac{6}{12}} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt[6]{6} = \sqrt[12]{6^2} = \sqrt[12]{36}$

The greatest number = $\sqrt[4]{5}$

21. (4) LCM of indices of surds
= LCM of 6, 3, 4 and 2 = 12

$\therefore\ \sqrt[6]{12} = \sqrt[12]{12^2} = \sqrt[12]{144}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\therefore$ The smallest surd = $\sqrt[4]{5}$

22. (4) $(0.9)^2 = 0.81;$

$\sqrt{0.9} = 0.95$

$0.\overline{9} = \frac{9}{9} = 1$

23. (2) $(16)^{\frac{1}{4}} = \left(2^4\right)^{\frac{1}{4}} = 2$

$\sqrt[5]{32} = (32)^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}} = 2$

$\sqrt[3]{9} > 2, \sqrt{2} < 2$

24. (2) LCM of indices of surds = 20

$\therefore\ \sqrt[4]{3} = \sqrt[20]{3^5} = \sqrt[20]{243}$

$\sqrt[5]{4} = \sqrt[20]{4^4} = \sqrt[20]{256}$

$\sqrt[10]{12} = \sqrt[20]{144}$

25. (3) $3\sqrt{2} = 3 \times 1.4 = 4.2$

$3\sqrt{7} = 3 \times 2.6 = 7.8$

$6\sqrt{5} = 6 \times 2.2 = 13.2$

$2\sqrt{20} = 2 \times 4.5 = 9$

26. (4) $\sqrt{0.09} = 0.3\ ;\ \sqrt[3]{0.064}$
= 0.4; 0.5;

$\frac{3}{5} = 0.6$

Clearly, $\sqrt{0.09} < \sqrt[3]{0.064} < 0.5 < \frac{3}{5}$

27. (2) LCM of power of surds = 12

$\therefore\ \sqrt{2} = (2)^{\frac{1}{2}} = (2^6)^{\frac{1}{12}}$

$= \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$

$\because$ Since 81 is the largest, hence, $\sqrt[3]{3}$ is the largest number.

28. (2) $\sqrt{0.16} = 0.4;\ (0.16)^2$
= 0.0256
Clearly,

$0.0256 < 0.04 < 0.16 < \sqrt{0.16}$

29. (2) $2^{250} = (2^5)^{50} = (32)^{50}$

$3^{150} = (3^3)^{50} = (27)^{50}$

$5^{100} = (5^2)^{50} = (25)^{50}$

$4^{200} = (4^4)^{50} = (256)^{50}$

$\therefore$ The smallest number
$= (5)^{100}$

30. (4) LCM of 2, 4, 5 and 10 = 20

$\therefore \sqrt[2]{8} = \sqrt[20]{8^{10}}\ ;\ \sqrt[4]{13} = \sqrt[20]{13^5}$

$\sqrt[5]{16} = \sqrt[20]{16^4}\ ;\ \sqrt[10]{41} = \sqrt[20]{41^2}$

Clearly, $\sqrt[2]{8}$ is the largest.

31. (3) $\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{2}{6}} = \sqrt[6]{4}$

$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{3}{6}} = \sqrt[6]{27}$

32. (1) LCM of indices = LCM of 3, 6, 4 and 2 = 12

$\therefore\ \sqrt[3]{4} = (4)^{\frac{1}{3}} = (4)^{\frac{1}{12}} = \sqrt[12]{4^4}$

$= \sqrt[12]{256}$

$\sqrt{2} = (2)^{\frac{1}{2}} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[6]{3} = \sqrt[12]{3^2} = \sqrt[12]{9}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Clearly, $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

33. (4) Decimal equivalents of fractions :

$\frac{4}{9} = 0.44\ ;\ \sqrt{\frac{9}{49}} = \frac{3}{7} = 0.43;$

$(0.7)^2 = 0.49$

34. (2) $3^{50} = (3^5)^{10} = (243)^{10}$

$4^{40} = (4^4)^{10} = (256)^{10}$

$5^{30} = (5^3)^{10} = (125)^{10}$

$6^{20} = (6^2)^{10} = (36)^{10}$

$\therefore$ Largest number = 4^{40}

35. (3) $\sqrt{5}$,

$3\sqrt{7} = \sqrt{9 \times 7} = \sqrt{63}$

$4\sqrt{13} = \sqrt{4 \times 4 \times 13} = \sqrt{208}$

Clearly, $\sqrt{5} < 3\sqrt{7} < 4\sqrt{13}$

36. (4) Making each surd of the same order :
LCM of 3, 4 and 6 = 12

$\therefore\ \sqrt[3]{9} = (9)^{\frac{1}{3}} = (9)^{\frac{4}{12}} = \left(9^4\right)^{\frac{1}{12}}$

$= \sqrt[12]{9^4} = \sqrt[12]{6561}$

$\sqrt[4]{20} = \sqrt[12]{20^3} = \sqrt[12]{8000}$

$\sqrt[6]{25} = \sqrt[12]{25^2} = \sqrt[12]{625}$

$\therefore\ \sqrt[12]{625} < \sqrt[12]{6561} < \sqrt[12]{8000}$

$\Rightarrow\ \sqrt[6]{25} < \sqrt[3]{9} < \sqrt[4]{20}$

TYPE-III

1. (1) $\sqrt{8} + 2\sqrt{32} - 3\sqrt{128} + 4\sqrt{50}$

$= 2\sqrt{2} + 8\sqrt{2} - 3 \times 8\sqrt{2} + 4 \times 5\sqrt{2}$

$= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2}$

$= (2 + 8 - 24 + 20)\sqrt{2}$

$= 6\sqrt{2} = 6 \times 1.414 = 8.484$

2. (1) $\sqrt{15} = 3.88$ (Given)

Now, $\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3}$

$= \frac{3.88}{3} = 1.29\overline{3}$

3. (3)

$\frac{4 + 3\sqrt{3}}{\sqrt{7 + 4\sqrt{3}}} = \frac{4 + 3\sqrt{3}}{\sqrt{4 + 3 + 2 \times 2 \times \sqrt{3}}}$

$= \frac{4 + 3\sqrt{3}}{\sqrt{\left(2 + \sqrt{3}\right)^2}} = \frac{4 + 3\sqrt{3}}{2 + \sqrt{3}}$

$= \frac{\left(4 + 3\sqrt{3}\right)\left(2 - \sqrt{3}\right)}{\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)}$

$= 8 - 4\sqrt{3} + 6\sqrt{3} - 9$

$= 2\sqrt{3} - 1 = 2 \times 1.732 - 1$

= 3.464 – 1 = 2.464

4. (4) Expression

$$=\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$$

$$=\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{2\times2\times3}-\sqrt{2\times2\times2\times2\times2}+\sqrt{2\times5\times5}}$$

$$=\frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}}$$

$$=\frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}=\frac{(3+\sqrt{6})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

[On rationalising the denominator]

$$=\frac{3\sqrt{3}+\sqrt{18}-3\sqrt{2}-\sqrt{12}}{3-2}$$

$$=3\sqrt{3}+3\sqrt{2}-3\sqrt{2}-2\sqrt{3}$$

$$=\sqrt{3}=1.732$$

5. (2) Expression $=\frac{1}{\sqrt{5}+\sqrt{3}}$

$$=\frac{1}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

(Rationalising the denominator)

$$=\frac{\sqrt{5}-\sqrt{3}}{5-3}=\frac{2.236-1.732}{2}$$

$$=\frac{0.504}{2}=0.252$$

6. (2) Expression

$$=\frac{3\sqrt{5}}{2\sqrt{5}-0.48}$$

$$=\frac{3\times2.24}{2\times2.24-0.48}=\frac{6.72}{4.48-0.48}$$

$$=\frac{6.72}{4}=1.68$$

7. (1) Expression

$$=\frac{1}{\sqrt{2}+1}=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}$$

$$=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$$

$= 1.414 - 1 = 0.414$

8. (1) Expression

$$=16\sqrt{\frac{3\times4}{4\times4}}-9\sqrt{\frac{4\times3}{3\times3}}$$

$$=\frac{16\sqrt{12}}{4}-\frac{9\sqrt{12}}{3}$$

$$=4\sqrt{12}-3\sqrt{12}$$

$$=\sqrt{12}=3.46$$

9. (2) Expression

$$=2\sqrt{2}+\sqrt{2}+\frac{1}{2+\sqrt{2}}+\frac{1}{\sqrt{2}-2}$$

$$=2\sqrt{2}+\sqrt{2}+\left(\frac{1}{2+\sqrt{2}}-\frac{1}{2-\sqrt{2}}\right)$$

$$=2\sqrt{2}+\sqrt{2}+\left(\frac{2-\sqrt{2}-2-\sqrt{2}}{(2+\sqrt{2})(2-\sqrt{2})}\right)$$

$$=2\sqrt{2}+\sqrt{2}+\frac{-2\sqrt{2}}{4-2}$$

$$=2\sqrt{2}+\sqrt{2}-\sqrt{2}=2\sqrt{2}$$

$= 2 \times 1.4142 = 2.8284$

10. (2) Expression

$$=\frac{2+\sqrt{3}}{2-\sqrt{3}}=\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

[On rationalising the denominator]

$$=\frac{(2+\sqrt{3})^2}{4-3}=(2+\sqrt{3})^2$$

$$=2^2+(\sqrt{3})^2+2\times2\times\sqrt{3}$$

$= 4 + 3 + 4\sqrt{3}$
$= 7 + 4 \times 1.732 = 7 + 6.928$
$= 13.928$

11. (1) $\frac{7}{3+\sqrt{2}}=\frac{7(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$

[Rationalising the denominator]

$$=\frac{7(3-\sqrt{2})}{9-2}\quad[(a+b)(a-b)=a^2-b^2]$$

$= 3-\sqrt{2}$
$= 3 - 1.4142 = 1.5858$
$= 1.59$ (correct to two decimal places)

12. (3) $\sqrt{5329} = 73$

$\therefore \sqrt{5329}+\sqrt{53.29}+\sqrt{0.5329}+\sqrt{0.005329}+\sqrt{0.00005329}$
$= 73 + 7.3 + 0.73 + 0.073 + 0.0073$
$= 81.1103$

13. (2) $\sqrt{33} = 5.745$ (Given)

$$\therefore\sqrt{\frac{3}{11}}=\sqrt{\frac{3\times11}{11\times11}}=\frac{\sqrt{33}}{11}$$

$$=\frac{5.745}{11}$$

≈ 0.5223

14. (4) $\frac{1}{\sqrt{28}}=\frac{1}{2\sqrt{7}}$

$$=\frac{\sqrt{7}}{2\sqrt{7}\times\sqrt{7}}=\frac{\sqrt{7}}{14}$$

$$=\frac{2.646}{14}=0.189$$

15. (2) $\frac{\sqrt{5}}{2}+\frac{5}{3\sqrt{5}}-\sqrt{45}$

$$=\frac{\sqrt{5}}{2}+\frac{5\times\sqrt{3}}{3\times5}-3\sqrt{5}$$

$$=\frac{\sqrt{5}}{2}+\frac{\sqrt{5}}{3}-3\sqrt{5}$$

$$=\frac{3\sqrt{5}+2\sqrt{5}-18\sqrt{5}}{6}$$

$$=\frac{-13\sqrt{5}}{6}=\frac{-13\times2.236}{6}$$

$$=\frac{-29.068}{6}=-4.845$$

16. (2) Expression $=\frac{9+2\sqrt{3}}{\sqrt{3}}$

$$=\frac{(9+2\sqrt{3})\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$=\frac{9\sqrt{3}+6}{3}=3\sqrt{3}+2$$

$= 3 \times 1.732 + 2 = 5.196 + 2$
$= 7.196$

TYPE-IV

1. (4) $3\sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$

$\therefore$ Required rationalising factor is $\sqrt{3}$.

2. (2) $\sqrt[3]{9} - \sqrt[3]{3} + 1 = (3)^{\frac{2}{3}} - (3)^{\frac{1}{3}} + (1)^{\frac{2}{3}}$

$\therefore \left(\sqrt[3]{3}+1\right)\left(\sqrt[3]{9} - \sqrt[3]{3} + 1\right) = (3^{\frac{1}{3}})^3 + 1$

= 3 + 1 = 4

[$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$\therefore$ Rationalising factor $= \sqrt[3]{3} + 1$

3. (3) $4^{10} \times 7^3 \times 16^2 \times 11 \times 10^2$

$= (2^2)^{10} \times (7)^3 \times (2^4)^2 \times 11 \times (2 \times 5)^2$

$= 2^{20} \times 7^3 \times 2^8 \times 11 \times 2^2 \times 5^2$

$= (2)^{20+8+2} \times 5^2 \times 7^3 \times 11^1$

$= (2)^{30} \times 5^2 \times 7^3 \times 11^1$

$\therefore$ Total number of prime factors $= 30 + 2 + 3 + 1 = 36$

4. (1) $(6)^{333} \times (7)^{222} \times (8)^{111}$

$\therefore (2 \times 3)^{333} \times (7)^{222} \times (2^3)^{111}$

$\therefore 2^{333} \times 3^{333} \times 7^{222} \times 2^{333}$

$\therefore 2^{666} \times 3^{333} \times 7^{222}$

$\therefore$ Number of prime factors

= 666 + 333 + 222 = 1221

5. (1) Expression $= \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Rationalising the denominator,

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)}$$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{3-2} = \left(\sqrt{3}+\sqrt{2}\right)^2$$

$$\therefore \sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}} = \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}$$

$= \sqrt{3} + \sqrt{2}$

6. (1) $x = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3} = \frac{5+3+2\sqrt{15}}{2}$$

$$= \frac{8+2\sqrt{15}}{2} = 4 + \sqrt{15}$$

$$\therefore y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 4 - \sqrt{15}$$

$\therefore x + y$

$= 4 + \sqrt{15} + 4 - \sqrt{15} = 8$

7. (4) Expression $= \dfrac{5+\sqrt{11}}{3-2\sqrt{11}}$

$$= \frac{(5+\sqrt{11})(3+2\sqrt{11})}{(3-2\sqrt{11})(3+2\sqrt{11})}$$

(On rationalising the denominator)

$$= \frac{15+22+10\sqrt{11}+3\sqrt{11}}{9-4\times 11}$$

$$= \frac{37+13\sqrt{11}}{-35}$$

$$\therefore x + y\sqrt{11} = \frac{-37}{35} - \frac{13}{35}\sqrt{11}$$

$\therefore x = \dfrac{-37}{35}$,

$y = -\dfrac{13}{35}$

TYPE-V

1. (2) $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{5^9}}\right]^4$

$$= \left[5^{9\times\frac{1}{6}\times\frac{1}{3}}\right]^4 \left[5^{9\times\frac{1}{6}\times\frac{1}{3}}\right]^4$$

$$= \left[5^{\frac{1}{2}\times 4}\right]\left[5^{\frac{1}{2}\times 4}\right] = 5^2 \times 5^2 = 5^4$$

2. (1) $27^{2x-1} = (243)^3$

$\Rightarrow (3^3)^{2x-1} = (3^5)^3$

$\Rightarrow (3)^{3(2x-1)} = (3)^{5\times 3}$

$\Rightarrow 3(2x-1) = 5 \times 3$

or $2x - 1 = 5 \therefore x = 3$

3. (4) $3^{x+8} = 3^{3(2x+1)}$

$\Rightarrow x + 8 = 6x + 3$

$\Rightarrow 5x = 5$

$\therefore x = 1$

4. (4) $(36)^{\frac{1}{6}} = \left(6^2\right)^{\frac{1}{6}}$

$= (6)^{\frac{2}{6}} = (6)^{\frac{1}{3}} = \sqrt[3]{6}$

5. (1) $\left(\dfrac{8}{125}\right)^{-\frac{4}{3}} = \left(\dfrac{2^3}{5^3}\right)^{-\frac{4}{3}}$

$$= \left[\left(\frac{2}{5}\right)^3\right]^{-\frac{4}{3}} = \left(\frac{2}{5}\right)^{-\frac{4}{3}\times 3}$$

$$= \left(\frac{5}{2}\right)^4 = \frac{625}{16}$$

6. (4) $(125)^{2/3} \times (625)^{-1/4} = 5^x$

$\Rightarrow 5^{3\times\frac{2}{3}} \times 5^{4\times -\frac{1}{4}} = 5^x$

$\Rightarrow 5^2 \times 5^{-1} = 5^x$

$\Rightarrow 5^1 = 5^x \Rightarrow x = 1$

7. (1) $(2000)^{10} = 1.024 \times 10^k$

$\Rightarrow (2 \times 10^3)^{10} = \dfrac{1024}{1000} \times 10^k$

$\Rightarrow 2^{10} \times 10^{30} = 1024 \times 10^{k-3}$

$\Rightarrow 2^{10} \times 10^{30} = 2^{10} \times 10^{k-3}$

$\Rightarrow 30 = k - 3 \Rightarrow k = 33$

8. (3) $0.42 \times 100^k = 42$

$\Rightarrow \dfrac{42}{100} \times 100^k = 42$

$\Rightarrow 100^k = \dfrac{42 \times 100}{42} = 100^1$

$\Rightarrow k = 1$

9. (3) $3^{x+y} = 81$

$\Rightarrow 3^{x+y} = 3^4$

$\Rightarrow x + y = 4$ ...(i)

and $81^{x-y} = 3$

$\Rightarrow (3^4)^{x-y} = 3$

$\Rightarrow 3^{4x-4y} = 3^1 \Rightarrow 4x - 4y = 1$ ...(ii)

By equation (i) × 4 + (ii) we have,

$4x + 4y = 16$

$4x - 4y = 1$

$8x = 17 \Rightarrow x = \dfrac{17}{8}$

10. (1) $2^x = 3^y = 6^{-z} = k$

$\Rightarrow 2 = k^{\frac{1}{x}}; 3 = k^{\frac{1}{y}}; 6 = k^{-\frac{1}{z}}$

$\because 2 \times 3 = 6$

$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{-\frac{1}{z}}$

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{y} = -\dfrac{1}{z} \Rightarrow \dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 0$

11. (2) $a = 7 - 4\sqrt{3}$

$\therefore \frac{1}{a} = \frac{1}{7-4\sqrt{3}}$

$= \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$

$= 7 + 4\sqrt{3}$

$\therefore \left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)^2 = a + \frac{1}{a} + 2$

$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3} + 2 = 16$

$\Rightarrow \sqrt{a} + \frac{1}{\sqrt{a}} = 4$

12. (3) $\left(\frac{3}{4}\right)^3 \times \left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow \left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^7 = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow \left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^{2x}$

$\Rightarrow 2x = 10 \Rightarrow x = 5$

13. (4) $x^2 - \sqrt{3} = 0$

$\Rightarrow x^2 - (3)^{\frac{1}{2}} = 0$

$\Rightarrow x^2 - \left(3^{\frac{1}{4}}\right)^2 = 0$

$\Rightarrow \left(x + 3^{\frac{1}{4}}\right)\left(x - 3^{\frac{1}{4}}\right) = 0$

$\therefore x = 3^{\frac{1}{4}}$ or $-3^{\frac{1}{4}}$

$\therefore$ Product of roots

$= 3^{\frac{1}{4}} \times -3^{\frac{1}{4}} = -\sqrt{3}$

Note : Product of the roots of $ax^2 + bx + c = 0$ is $\frac{c}{a}$

$\therefore$ Product of the roots of $x^2 - b.0 - \sqrt{3} = 0$ is $-\sqrt{3}$

14. (4) $2^{x-1} + 2^{x+1} = 320$

$\Rightarrow 2^{x-1}\left(1 + 2^2\right) = 320$

$\Rightarrow 2^{x-1} \times 5 = 320$

$\Rightarrow 2^{x-1} = \frac{320}{5} = 64 \Rightarrow 2^{x-1} = 2^6$

$\Rightarrow x - 1 = 6 \Rightarrow x = 7$

15. (1) $4^{61} + 4^{62} + 4^{63} + 4^{64}$

$= 4^{61}(1 + 4 + 4^2 + 4^3)$

$= 4^{61}(1 + 4 + 16 + 64)$

$= 4^{61} \times 85$ which is divisible by 17.

16. (1) $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$

$\Rightarrow 5 \times 5^{\frac{1}{2}} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$

$\Rightarrow 5^{1+\frac{1}{2}+3+\frac{3}{2}} = 5^{a+2}$

$\Rightarrow 5^6 = 5^{a+2} \Rightarrow a + 2 = 6$

$\Rightarrow a = 6 - 2 = 4$

$[a^m \times a^n = a^{m+n},]$

$[a^m \div a^n = a^{m-n}]$

17. (1) $(3 + 2\sqrt{2})(3 - 2\sqrt{2})$

$= (3)^2 - (2\sqrt{2})^2 = 9 - 8 = 1$

$\therefore 3 + 2\sqrt{2} = \frac{1}{3-2\sqrt{2}}$

$(x + y)^3 + (x - y)^3 = x^3 + y^3 + 3x^2y + 3xy^2 + x^3 - y^3 - 3x^2y + 3xy^2$

$= 2x^3 + 6xy^2$

$\therefore (3 + 2\sqrt{2})^{-3} + (3 - 2\sqrt{2})^{-3}$

$= \left(\frac{1}{3+2\sqrt{2}}\right)^3 + \left(\frac{1}{3-2\sqrt{2}}\right)^3$

$= (3 - 2\sqrt{2})^3 + (3 + 2\sqrt{2})^3$

$= 2 \times (3)^3 + 6 \times 3 \times (2\sqrt{2})^2$

$= 2 \times 27 + 18 \times 8$

$= 54 + 144 = 198$

18. (4) $3^x - 3^{x-1} = 486$

$\Rightarrow 3^{x-1}(3 - 1) = 486$

$\Rightarrow 3^{x-1} \times 2 = 486$

$\Rightarrow 3^{x-1} = \frac{486}{2} = 243$

$\Rightarrow 3^{x-1} = 3^5 \Rightarrow x - 1 = 5$

$\Rightarrow x = 5 + 1 = 6$

19. (2) $L = 2 - 2^t$

At the start, $t = 0$

$\therefore L = 2 - 2^{\circ} = 2 - 1 = 1$ cm

20. (1) $3^{34} = (3^2)^{17} = 9^{17}$

$2^{51} = (2^3)^{17} = 8^{17}$

$7^{17} = 7^{17}$

Clearly, $9^{17} > 8^{17} > 7^{17}$

i.e., $3^{34} > 2^{51} > 7^{17}$

21. (3) $3^{2x-y} = 3^{x+y} = \sqrt{27} = (3)^{\frac{3}{2}}$

$\Rightarrow 2x - y = \frac{3}{2}$

$\Rightarrow 4x - 2y = 3$ ....(i)

and, $3^{x+y} = (3)^{\frac{3}{2}}$

$\Rightarrow x + y = \frac{3}{2}$

$\Rightarrow 2x + 2y = 3$ ....(ii)

From equations (i) and (ii)

$4x - 2y + 2x + 2y = 3 + 3$

$\Rightarrow 6x = 6 \Rightarrow x = 1$

From equation (i),

$4 - 2y = 3$

$\Rightarrow 2y = 1 \Rightarrow y = \frac{1}{2}$

$\therefore 3^{x-y} = 3^{1-\frac{1}{2}} = \sqrt{3}$

22. (3) Expression

$= [(0.87)^2 + (0.13)^2 + 0.87 \times 0.26]^{2013}$

$= (0.87 + 0.13)^{2013} = 1^{2013} = 1$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

23. (2) Sum of the cubes of first n natural numbers

$= \left(\frac{n(n+1)}{2}\right)^2$

Their average $= \frac{n(n+1)^2}{4}$

$\therefore$ Required average when $n = 7$,

$= \frac{7(7+1)^2}{4} = \frac{7 \times 8 \times 8}{4} = 112$

24. (1) $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$

i.e. after index 4, the unit's digit is repeated.

$\therefore$ On dividing 153 by 4, remainder = 1

$\therefore$ Unit's digit in the expansion of $(2467)^{153} = 7^1 = 7$ and unit's digit in the expansion of $(341)^{72} = 1$

$\therefore$ Required unit's digit $= 7 \times 1 = 7$

25. (4) Expression $= \sqrt{\sqrt{2} \times \sqrt{3}}$

$= \left(\sqrt{2} \times \sqrt{3}\right)^{\frac{1}{2}} = \left(2^{\frac{1}{2}} \times 3^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$= (6)^{\frac{1}{2} \times \frac{1}{2}} = (6)^{\frac{1}{4}}$

26. (4) Expression $= \dfrac{(10)^{100}}{(5)^{75}}$

$= \dfrac{(2 \times 5)^{100}}{(5)^{75}}$

$= \dfrac{(2)^{100} \times (5)^{100}}{(5)^{75}}$

$= 2^{100} \times 5^{25}$

$= 2^{25} \times 5^{25} \times 2^{75}$

$= (10)^{25} \times 2^{75}$

27. (1) $m^n = 169 = 13^2$

$\Rightarrow m = 13,\ n = 2$

$\therefore (m + 1)^{n-1} = (13 + 1)^{2-1} = 14$

28. (1) $a = (b)^p$ and

$b = (c)^q$

$\therefore c = a^r = (b^p)^r = (b)^{pr} = (c^q)^{pr} = c^{pqr}$

$\Rightarrow pqr = 1$

29. (4) $35 = 5 \times 7$

$175 = 5 \times 5 \times 7$

$1225 = 5 \times 5 \times 7 \times 7$

$735 = 5 \times 7 \times 7 \times 3$

Clearly, 735 is not a factor of 5^p7^q.

30. (3) Unit's digit in the expansion of $(252)^{126}$

$= 2^2 = 4$ ($\because$ Remainder on dividing 126 by 4 = 2)

Unit's digit in the expansion of $(244)^{152} = 6$

$\therefore$ Unit's digit in the expansion of $252^{126} + 244^{152} = 0$

$\therefore$ Required remainder = 0

31. (1) $x = (3)^{\frac{1}{3}} - (3)^{-\frac{1}{3}}$

On cubing both sides,

$x^3 = \left(3^{\frac{1}{3}} - (3)^{-\frac{1}{3}}\right)^3$

$= \left((3)^{\frac{1}{3}}\right)^3 - \left(3^{-\frac{1}{3}}\right)^3 - 3 \times 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}}\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$

$= 3 - 3^{-1} - 3 \times x$

$= 3 - \dfrac{1}{3} - 3x$

$\Rightarrow x^3 + 3x = 3 - \dfrac{1}{3} = \dfrac{9-1}{3}$

$\Rightarrow x^3 + 3x = \dfrac{8}{3}$

$\Rightarrow 3x^3 + 9x = \dfrac{8}{3} \times 3 = 8$

32. (2) $3^{10} \times 27^2 = 9^2 \times 3^n$

$\Rightarrow 3^{10} \times (3^3)^2 = (3^2)^2 \times 3^n$

$\Rightarrow 3^{10} \times 3^6 = 3^4 \times 3^n$

$\Rightarrow 3^{10+6} = 3^4 \times 3^n$

$\Rightarrow 3^{16} = 3^{4+n}$

$\Rightarrow 4 + n = 16$

$\Rightarrow n = 16 - 4 = 12$

33. (4) $2^{x+4} - 2^{x+2} = 3$

$\Rightarrow 2^{x+2}(2^2 - 1) = 3$

$\Rightarrow 2^{x+2} \times 3 = 3$

$\Rightarrow 2^{x+2} = 1 = 2^{\circ}$

$\Rightarrow x + 2 = 0 \Rightarrow x = -2$

34. (2) $\sqrt{3^n} = 2187$

$\Rightarrow 3^{\frac{n}{2}} = (3)^7$

$\Rightarrow \dfrac{n}{2} = 7$

$\Rightarrow n = 2 \times 7 = 14$

35. (2) $x + \dfrac{1}{x} = 2$

$\Rightarrow \dfrac{x^2 + 1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0 \Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

$\therefore x^{99} + \dfrac{1}{x^{99}} - 2 = 1 + 1 - 2 = 0$

36. (4) $(a + b)(a^2 - ab + b^2) = a^3 + b^3$

$\therefore \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right)\left(x^{\frac{2}{3}} - 1 + x^{-\frac{2}{3}}\right)$

$= \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right)$

$\left(\left(x^{\frac{1}{3}}\right)^2 - x^{\frac{1}{3}}.x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right)$

$= \left(x^{\frac{1}{3}}\right)^3 + \left(x^{-\frac{1}{3}}\right)^3$

$= x + x^{-1} = x + \dfrac{1}{x}$

37. (4) $\left(2^3\right)^2 = \left(2^2\right)^x$

$\Rightarrow 2^6 = 2^{2x} \Rightarrow 2x = 6$

$\Rightarrow x = \dfrac{6}{2} = 3$

$\therefore 3^x = 3^3 = 3 \times 3 \times 3 = 27$

38. (4) $x = 3^{\frac{1}{3}} - 3^{-\frac{1}{3}}$

On cubing both sides,

$x^3 = \left((3)^{\frac{1}{3}}\right)^3 - \left((3)^{-\frac{1}{3}}\right)^3 - 3 \times 3^{\frac{1}{3}} \times 3^{-\frac{1}{3}}$

$\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$

$\Rightarrow x^3 = 3 - 3^{-1} - 3x$

$\Rightarrow x^3 + 3x = 3 - \dfrac{1}{3}$

$\Rightarrow x^3 + 3x = \dfrac{9-1}{3} = \dfrac{8}{3}$

$\Rightarrow 3x^3 + 9x = 8$

TYPE-VI

1. (3) Let $x = \sqrt{1+\sqrt{1+\sqrt{1+.....}}}$

On squaring both sides

$$x^2 = 1+\sqrt{1+\sqrt{1+\sqrt{1+......}}}$$

$\Rightarrow x^2 = 1 + x$

$\Rightarrow x^2 - x - 1 = 0$

$$\Rightarrow x = \frac{+1 \pm \sqrt{1+4}}{2} = \frac{+1 \pm \sqrt{5}}{2}$$

But sum of + ve numbers can't be negative.

$$\therefore x = \frac{1+\sqrt{5}}{2} = \frac{1+2.236}{2}$$

$$= \frac{3.236}{2} = 1.618$$

Thus 1 < 1.618 < 2

2. (3) Let $x = \sqrt{2+\sqrt{2+\sqrt{2+}}}.....$

$$\therefore x^2 = 2+\sqrt{2+\sqrt{2+....}}$$

$\Rightarrow x^2 = 2 + x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x (x - 2) + 1 (x - 2) = 0$

$\Rightarrow (x - 2) (x + 1) = 0$

$\Rightarrow x = 2$ or $- 1$

But sum of positive numbers can't be negative.

$\therefore x = 2$

3. (2) Let $x = \sqrt{12+\sqrt{12+\sqrt{12+....}}}$

On squaring both sides,

$$x^2 = 12 + \sqrt{12+\sqrt{12+\sqrt{12+....}}}$$

$\Rightarrow x^2 = 12 + x$

$\Rightarrow x^2 - x - 12 = 0$

$\Rightarrow x^2 - 4x + 3x - 12 = 0$

$\Rightarrow x (x - 4) + 3 (x - 4) = 0$

$\Rightarrow (x - 4) (x + 3) = 0$

$\Rightarrow x = 4, - 3$

$\because$ The given expression is positive.

$\therefore x = 4$

Aliter :

Using Rule 25

$$\sqrt{12+\sqrt{12+\sqrt{12}}} = 4$$

It is because

$12 = 3 \times 4 = n (n + 1)$

4. (1) Let $x = \sqrt{6+\sqrt{6+\sqrt{6+}}}....$

Squaring on both sides,

$$x^2 = 6+\sqrt{6+\sqrt{6+....}}$$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x (x - 3) + 2 (x - 3) = 0$

$\Rightarrow (x + 2) (x- 3) = 0$

$\therefore x = 3$ because $x \neq -2$

Aliter :

$$\sqrt{6+\sqrt{6+\sqrt{6+.....}}} = 3$$

It is because

$6 = 2 \times 3 = n (n + 1)$

5. (2) Let $x = \sqrt{3\sqrt{3\sqrt{3.....}}}$

Squaring both sides,

$$x^2 = 3\sqrt{3\sqrt{3\sqrt{3.....}}} = 3x$$

$\Rightarrow x^2 - 3x = 0$

$\Rightarrow x (x - 3) = 0$

$\therefore x = 3$ because $x \neq 0$

Aliter :

Using Rule 23

$$\sqrt{3+\sqrt{3+\sqrt{3+.....\infty}}} = 3$$

It is because, here

$n = \infty$ and $x = 3$

$$\sqrt{3+\sqrt{3+\sqrt{3+.....\infty}}} = 3^{\left(1-\frac{1}{3\infty}\right)}$$

$$= 3^{(1-0)} \qquad \because \frac{\text{something}}{\infty} = 0$$

$= 3$

6. (3) Let $x = \sqrt{30+\sqrt{30+\sqrt{30+......}}}$

On squaring both sides,

$$x^2 = 30 + \sqrt{30+\sqrt{30+\sqrt{30+......}}}$$

$\Rightarrow x^2 = 30 + x$

$\Rightarrow x^2 - x - 30 = 0$

$\Rightarrow x^2 - 6x + 5x - 30 = 0$

$\Rightarrow x (x - 6) + 5 (x - 6) = 0$

$\Rightarrow (x - 6) (x + 5) = 0$

$\Rightarrow x = 6$ because $x \neq -5$

Aliter :

Using Rule 25

$$\sqrt{30+\sqrt{30+\sqrt{30+.....}}} = 6$$

It is because

$30 = 5 \times 6 = n (n +1)$

7. (1) $x = \sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4................}}}}$

On squaring

$$x^2 = 2\sqrt[3]{4\sqrt{2\sqrt[3]{4}}} ....$$

On cubing,

$x^6 = 8 \times 4x$

$\Rightarrow x^5 = 32 = 2^5 \Rightarrow x = 2$

8. (4) $m = \sqrt{5+\sqrt{5+\sqrt{5+.........}}}$

On squaring both sides,

$m^2 = 5 + m \Rightarrow m^2 - m = 5$ ....(i)

Again,

$$n = \sqrt{5-\sqrt{5-\sqrt{5.........}}}$$

On squaring both sides,

$n^2 = 5 - n$

$\Rightarrow n^2 + n = 5$ ....(ii)

$\therefore m^2 - m = n^2 + n$

$\Rightarrow (m^2 - n^2) = m + n$

$\Rightarrow (m + n) (m - n) - (m + n) = 0$

$\Rightarrow (m + n) (m-n-1) = 0$

9. (1) $x = \sqrt{72+\sqrt{72+\sqrt{72+.......}}}$

On squaring both sides,

$$x^2 = 72 + \sqrt{72+\sqrt{72+\sqrt{72+.......}}}$$

$\Rightarrow x^2 = 72 + x$

$\Rightarrow x^2 - x - 72 = 0$

$\Rightarrow x^2 - 9x + 8x - 72 = 0$

$\Rightarrow x (x - 9) + 8 (x - 9) = 0$

$\Rightarrow (x + 8) (x - 9) = 0$

$\Rightarrow x = 9$ because $x \neq -8$

Aliter :

Using Rule 25

$$\sqrt{72+\sqrt{72+\sqrt{72+....}}} = 9$$

It is because $72 = 8 \times 9 = n (n + 1)$

10. (2)

$$? = \frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+15}}}}}{\sqrt[3]{2\times2\times2}}$$

$$= \frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}}{2}$$

$$= \frac{\sqrt{10+\sqrt{25+\sqrt{108+13}}}}{2}$$

$$= \frac{\sqrt{10+\sqrt{25+\sqrt{121}}}}{2}$$

$$= \frac{\sqrt{10+\sqrt{25+11}}}{2}$$

$$= \frac{\sqrt{10+\sqrt{36}}}{2} = \frac{\sqrt{10+6}}{2}$$

$$= \frac{\sqrt{16}}{2} = \frac{4}{2} = 2$$

11. (2) Let, $x = \sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$

On squaring,

$x^2 = 6 + \sqrt{6+\sqrt{6+\sqrt{6+\ldots\infty}}}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x-3) + 2(x-3) = 0$

$\Rightarrow (x+2)(x-3) = 0$

$\Rightarrow x = 3$ because $x \neq -2$

Aliter :

Using Rule 25

$\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} = 3$

It is because, 6 = 2 × 3 =n (n+1)

12. (4) $x = \sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$

On squaring both sides,

$x^2 = 12 + \sqrt{12+\sqrt{12+\sqrt{12+\ldots}}}$

$\Rightarrow x^2 = 12 + x$

$\Rightarrow x^2 - x - 12 = 0$

$\Rightarrow x^2 - 4x + 3x - 12 = 0$

$\Rightarrow x(x-4) + 3(x-4) = 0$

$\Rightarrow (x-4)(x+3) = 0$

$\Rightarrow x = 4$ because $x \neq -3$

13. (4) Expression

$$= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+15}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+\sqrt{169}}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{108+13}}}$$

$$= \sqrt{10+\sqrt{25+\sqrt{121}}}$$

$$= \sqrt{10+\sqrt{25+11}}$$

$$= \sqrt{10+6} = \sqrt{16} = 4$$

14. (1) Expression

$$= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+2\times2\times\sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{(2)^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{(4)^2+\left(\sqrt{3}\right)^2+2\times4\times\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$$

$$= \sqrt{-\sqrt{3}+4+\sqrt{3}} = \sqrt{4} = 2$$

15. (2) Expression

$$= \sqrt{9+2\sqrt{16}+\sqrt[3]{512}}$$

$$= \sqrt{9+2\sqrt{4\times4}+\sqrt[3]{8\times8\times8}}$$

$$= \sqrt{9+2\times4+8}$$

$$= \sqrt{25} = 5$$

TYPE-VII

1. (3) $\sqrt{3} = 1.732$

$$\therefore \frac{173}{100} = 1.73 \approx 1.732$$

2. (1) $a = \frac{\sqrt{3}}{2}$

$\therefore \sqrt{1+a} + \sqrt{1-a}$

$$= \sqrt{1+\frac{\sqrt{3}}{2}} + \sqrt{1-\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2}} + \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}}$$

$$= \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{2}\times\sqrt{2}} + \frac{\sqrt{4-2\sqrt{3}}}{\sqrt{2}\times\sqrt{2}}$$

$$= \frac{\sqrt{(\sqrt{3}+1)^2}}{2} + \frac{\sqrt{(\sqrt{3}-1)^2}}{2}$$

$$= \frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}$$

$$= \frac{\sqrt{3}+1+\sqrt{3}-1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

3. (2) It is given that

$$a = \frac{\sqrt{5}+1}{\sqrt{5}-1} \text{ and } b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$$

Now, $a + b = \frac{\sqrt{5}+1}{\sqrt{5}-1} + \frac{\sqrt{5}-1}{\sqrt{5}+1}$

$$= \frac{\left(\sqrt{5}+1\right)^2+\left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}$$

$$= \frac{2\left[\left(\sqrt{5}\right)^2+(1)^2\right]}{\left(\sqrt{5}\right)^2-(1)^2}$$

$[\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)]$

$$= \frac{2(5+1)}{5-1} = \frac{2\times6}{4} = 3$$

and $a.\,b = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

Expression $= \frac{a^2+ab+b^2}{a^2-ab+b^2}$

$$= \frac{(a+b)^2-ab}{(a+b)^2-3ab} = \frac{(3)^2-1}{(3)^2-3\times1}$$

$$= \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$$

4. (1) $x = 1 + \sqrt{2} + \sqrt{3}$ (Given)

$$\therefore x + \frac{1}{x} = 1 + \sqrt{2} + \sqrt{3} + \frac{1}{\sqrt{2}+\sqrt{3}}$$

$$=1+\sqrt{2}+\sqrt{3}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$=1+\sqrt{2}+\sqrt{3}+\frac{\sqrt{3}-\sqrt{2}}{(3-2)}$$

$$=1+\sqrt{2}+\sqrt{3}+\sqrt{3}-\sqrt{2}$$

$$=1+2\sqrt{3}$$

5. (3) $x+\frac{1}{x}=-2$ ...(i)

$$\therefore \left(x-\frac{1}{x}\right)^2=\left(x+\frac{1}{x}\right)^2-4$$

$= (-2)^2 - 4 = 0$

$\Rightarrow x-\frac{1}{x}=0$ ...(ii)

Solving equations (i) and (ii), we have

$\therefore x = -1$

$$\therefore x^{2n+1}+\frac{1}{x^{2n+1}}$$

$$=(-1)^{2n+1}+\frac{1}{(-1)^{2n+1}}$$

$= -1-1 = -2$

6. (4) $m^n = 121 = (11)^2$

$\Rightarrow m = 11,\ n = 2$

$\therefore (m-1)^{n+1} = (11-1)^{2+1}$

$= 10^3$

$= 1000$

7. (3) Required number

$$=\frac{\sqrt{12}+\sqrt{18}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\sqrt{2\times2\times3}+\sqrt{3\times3\times2}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

(Rationalising the denominator)

$$=\frac{2\sqrt{3}\times\sqrt{3}+3\sqrt{2}\times\sqrt{3}-2\sqrt{3}\times\sqrt{2}-3\sqrt{2}\times\sqrt{2}}{3-2}$$

$= 6 + 3\sqrt{6} - 2\sqrt{6} - 6 = \sqrt{6}$

8. (2) All multiples of 7 upto 50

$\Rightarrow$ 7, 14, 21, 28, 35, 42 and 49

$\Rightarrow 7,\ 2\times7,\ 3\times7,\ 4\times7,\ 5\times7,\ 6\times7$ and 7×7

$\therefore 7^n = 7^8 \Rightarrow n = 8$

9. (2) $9\sqrt{x} = \sqrt{12} + \sqrt{147}$

$\Rightarrow 9\sqrt{x} = 2\sqrt{3} + 7\sqrt{3}$

$= \sqrt{3}\ (2+7)$

$\Rightarrow 9\sqrt{x} = 9\sqrt{3} \Rightarrow x = 3$

10. (2) $43^2 < \sqrt{1896} < 44^2$

$\therefore 44 \times 44 = 1936$

$\therefore x = 44$

11. (4) $\left(\sqrt{6}+\sqrt{2}\right)^2$

$= 6 + 2 + 2\sqrt{12}$

$= 8 + 2\sqrt{12}$

$\left(\sqrt{5}+\sqrt{3}\right)^2 = 5 + 3 + 2\sqrt{15}$

$= 8 + 2\sqrt{15}$

Clearly, $\sqrt{15} > \sqrt{12}$

Hence, $\sqrt{6} + \sqrt{2} < \sqrt{5} + \sqrt{3}$

12. (2) $x=\frac{1}{\sqrt{2}+1}$

$$=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1}$$

$= \sqrt{2} - 1$

$\therefore x + 1 = \sqrt{2} - 1 + 1$

$= \sqrt{2}$

13. (1) $p = 5+2\sqrt{6}$

$= 5+2\times\sqrt{3}\times\sqrt{2}$

$= 3+2+2\times\sqrt{3}\times\sqrt{2}$

$= \left(\sqrt{3}+\sqrt{2}\right)^2$

$\therefore \sqrt{p} = \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}$

$= \sqrt{3}+\sqrt{2}$

$$\therefore \frac{\sqrt{p}-1}{\sqrt{p}}=\frac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{\left(\sqrt{3}+\sqrt{2}-1\right)\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}$$

$$=\frac{3+\sqrt{6}-\sqrt{3}-\sqrt{6}-2+\sqrt{2}}{3-2}$$

$= 1+\sqrt{2}-\sqrt{3}$

14. (2) $\sqrt{x}-\sqrt{y} = 1,$

$\sqrt{x}+\sqrt{y} = 17$

$\therefore \left(\sqrt{x}+\sqrt{y}\right)^2-\left(\sqrt{x}-\sqrt{y}\right)^2$

$= 17^2 - 1$

$\Rightarrow 4\sqrt{xy} = 289 - 1 = 288$

$\Rightarrow 4\sqrt{xy} = 288$

$\Rightarrow \sqrt{xy} = \frac{288}{4} = 72$

15. (3) Given,

$x = \sqrt{3}+\frac{1}{\sqrt{3}}$

$\Rightarrow x-\sqrt{3} = \frac{1}{\sqrt{3}}$ or, $x-\frac{1}{\sqrt{3}}$

$= \sqrt{3}$

Expression

$$=\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$$

$$=\left(x-\frac{\sqrt{42\times3}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2}{\sqrt{3}}}\right)$$

$$=\left(x-\sqrt{3}\right)\left(x-\frac{1}{\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{3}}}\right)$$

$$=\frac{1}{\sqrt{3}}\left(x-\frac{1}{\sqrt{3}-\frac{1}{\sqrt{3}}}\right)$$

$= \frac{1}{\sqrt{3}}\left(x - \frac{\sqrt{3}}{3-1}\right)$

$= \frac{1}{\sqrt{3}}\left(x - \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{\sqrt{3}}\left(\sqrt{3} + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}\right)$

$= \frac{1}{\sqrt{3}}\left(\frac{6+2-3}{2\sqrt{3}}\right)$

$= \frac{1}{\sqrt{3}} \times \frac{5}{2\sqrt{3}} = \frac{5}{6}$

16. (1) Given,

$4x = \sqrt{5} + 2$

$\Rightarrow 16x = 4\left(\sqrt{5}+2\right)$

$= 4\sqrt{5} + 8$

$\therefore \frac{1}{16x} = \frac{1}{4\sqrt{5}+8}$

$\Rightarrow \frac{1}{16x} = \frac{4\sqrt{5}-8}{\left(4\sqrt{5}+8\right)\left(4\sqrt{5}-8\right)}$

[Rationalising the denominator]

$= \frac{4\sqrt{5}-8}{80-64} = \frac{4\sqrt{5}-8}{16}$

$= \frac{4\left(\sqrt{5}-2\right)}{16} = \frac{\sqrt{5}-2}{4}$

$\therefore x - \frac{1}{16x} = \frac{\sqrt{5}+2}{4} - \frac{\sqrt{5}-2}{4}$

$= \frac{\sqrt{5}+2-\sqrt{5}+2}{4} = \frac{4}{4} = 1$

17. (2) $x^3 = 1.5^3 - 0.9^3 - 2.43$

$= (1.5)^3 - (0.9)^3 - 3 \times 1.5 \times 0.9$ $(1.5 - 0.9)$

$= (1.5 - 0.9)^3 = (0.6)^3$

$\Rightarrow x = 0.6$

18. (1) $\left(\frac{1}{5}\right)^{3y} = 0.008 = \frac{8}{1000}$

$\Rightarrow \left(\frac{1}{5}\right)^{3y} = \frac{1}{125} = \left(\frac{1}{5}\right)^3$

$\Rightarrow 3y = 3 \Rightarrow y = 1$

$\therefore (0.25)^y = 0.25$

19. (2) $x = 1 + \sqrt{2} + \sqrt{3}$

$\Rightarrow x - 1 = \sqrt{2} + \sqrt{3}$

On squaring both sides,

$(x-1)^2 = \left(\sqrt{2}+\sqrt{3}\right)^2$

$\Rightarrow x^2 - 2x + 1 = 2 + 3 + 2\sqrt{6}$

$\Rightarrow x^2 - 2x + 4 = 5 + 2\sqrt{6} + 3$

$\Rightarrow x^2 - 2x + 4 = 8 + 2\sqrt{6}$

$= 2\left(4+\sqrt{6}\right)$

20. (4) $x = \sqrt{2} + 1$

$\therefore \frac{1}{x} = \frac{1}{\sqrt{2}+1}$

$= \frac{\sqrt{2}-1}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}$

$= \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$

$\therefore x^2 = \left(\sqrt{2}+1\right)^2 = 2 + 1 + 2\sqrt{2}$

$= 3 + 2\sqrt{2}$ and $\frac{1}{x^2} = \left(\sqrt{2}-1\right)^2$

$= 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$

$\therefore x^4 - \frac{1}{x^4}$

$= \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$

$= \left(3+2\sqrt{2}+3-2\sqrt{2}\right)$

$\left(3+2\sqrt{2}-3+2\sqrt{2}\right)$

$= 6 \times 4\sqrt{2} = 24\sqrt{2}$

21. (3) $\because \frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}} = 0$

$\Rightarrow \frac{\sqrt{b}-\sqrt{a}}{\sqrt{ab}} = 0$

$\Rightarrow \sqrt{b} - \sqrt{a} = 0$

On squaring,

$\left(\sqrt{b}-\sqrt{a}\right)^2 = 0$

$\Rightarrow b + a - 2\sqrt{ab} = 0$

$\Rightarrow b + a = 2\sqrt{ab}$

On dividing by ab,

$\frac{b+a}{ab} = \frac{2\sqrt{ab}}{ab}$

$\Rightarrow \frac{b}{ab} + \frac{a}{ab} = \frac{2}{\sqrt{ab}}$

$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{2}{\sqrt{ab}}$

22. (3) $x = (0.25)^{\frac{1}{2}} = (0.5)^{2\times\frac{1}{2}} = 0.5$

$y = (0.4)^2 = 0.16$

$z = (0.216)^{\frac{1}{3}} = \left(0.6^3\right)^{\frac{1}{3}} = 0.6$

Clearly, $z > x > y$

23. (4) $a + \frac{1}{a} = 2 \Rightarrow a^2 + 1 = 2a$

$\Rightarrow a^2 - 2a + 1 = 0$

$\Rightarrow (a-1)^2 = 0$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1$

$\therefore a^5 + \frac{1}{a^5} = 1 + 1 = 2$

24. (4) $x = 2 + \sqrt{3}$

$\therefore x^2 = \left(2+\sqrt{3}\right)^2 = 4 + 3 + 4\sqrt{3}$

$= 7 + 4\sqrt{3}$

$\therefore \frac{x^2 - x + 1}{x^2 + x + 1}$

$= \frac{7+4\sqrt{3}-\left(2+\sqrt{3}\right)+1}{7+4\sqrt{3}+2+\sqrt{3}+1}$

$= \frac{8+4\sqrt{3}-2-\sqrt{3}}{10+5\sqrt{3}}$

$= \frac{6+3\sqrt{3}}{10+5\sqrt{3}}$

$$= \frac{3\left(2+\sqrt{3}\right)}{5\left(2+\sqrt{3}\right)} = \frac{3}{5}$$

$\boxed{\text{OR}}$

$x = 2 + \sqrt{3}$

$$\therefore \frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$$

$$\therefore \frac{x^2-x+1}{x^2+x+1} = \frac{x\left(x-1+\frac{1}{x}\right)}{x\left(x+1+\frac{1}{x}\right)}$$

$$= \frac{\left(x+\frac{1}{x}\right)-1}{x+\frac{1}{x}+1} = \frac{2+\sqrt{3}+2-\sqrt{3}-1}{2+\sqrt{3}+2-\sqrt{3}+1}$$

$$= \frac{3}{5}$$

25. (1) $3a = 4b = 6c$

$$\Rightarrow \frac{3a}{12} = \frac{4b}{12} = \frac{6c}{12}$$

$$\Rightarrow \frac{a}{4} = \frac{b}{3} = \frac{c}{2} = k$$

$\Rightarrow a = 4k;\ b = 3k;\ c = 2k$

$\therefore a + b + c = 27\sqrt{29}$

$\Rightarrow 4k + 3k + 2k = 27\sqrt{29}$

$\Rightarrow 9k = 27\sqrt{29}$

$\Rightarrow k = 3\sqrt{29}$

$\therefore \sqrt{a^2+b^2+c^2}$

$= \sqrt{16k^2+9k^2+4k^2}$

$= \sqrt{29\,k^2} = \sqrt{29}\,k$

$= \sqrt{29} \times 3\sqrt{29} = 29 \times 3 = 87$

26. (3) $\left(\sqrt{3}+1\right)^2 = x + \sqrt{3}\,y$

$\Rightarrow 3 + 1 + 2\sqrt{3} = x + \sqrt{3}\,y$

$\Rightarrow 4 + 2\sqrt{3} = x + \sqrt{3}\,y$

$\Rightarrow x = 4;\ y = 2$

$\therefore x + y = 4 + 2 = 6$

27. (2) $p = 9,\ q = \sqrt{17}$

$\therefore p^2 - q^2 = (9)^2 - \left(\sqrt{17}\right)^2$

$= 81 - 17 = 64$

$$\therefore \left(p^2-q^2\right)^{\frac{-1}{3}} = \frac{1}{\left(p^2-q^2\right)^{\frac{1}{3}}}$$

$$= \frac{1}{(64)^{\frac{1}{3}}} = \frac{1}{\left(4^3\right)^{\frac{1}{3}}} = \frac{1}{4}$$

28. (4) $\sqrt{1+\frac{x}{144}} = \frac{13}{12}$

On squaring both sides,

$$1+\frac{x}{144} = \left(\frac{13}{12}\right)^2 = \frac{169}{144}$$

$$\Rightarrow \frac{x}{144} = \frac{169}{144} - 1$$

$$\Rightarrow \frac{x}{144} = \frac{169-144}{144} = \frac{25}{144}$$

$\Rightarrow x = 25$

29. (2) $a = \sqrt{2} + 1$

$\therefore a + 1 = \sqrt{2} + 2$

Again, $b = \sqrt{2} - 1$

$\therefore b + 1 = \sqrt{2} - 1 + 1 = \sqrt{2}$

$$\therefore \frac{1}{a+1} + \frac{1}{b+1}$$

$$= \frac{1}{\sqrt{2}+2} + \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+\sqrt{2}+2}{\sqrt{2}\left(\sqrt{2}+2\right)} = \frac{2+2\sqrt{2}}{2+2\sqrt{2}} = 1$$

30. (3) $x = \dfrac{1}{\sqrt{2}+1}$

$$= \frac{\sqrt{2}-1}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)} = \sqrt{2} - 1$$

$\therefore x + 1 = \sqrt{2}$

$\Rightarrow x^2 + 2x + 1 = 2$

$\therefore x^2 + 2x - 1 = x^2 + 2x + 1 - 2$

$= 2 - 2 = 0$

31. (3) $x + \dfrac{1}{x} = \sqrt{13}$

$$\therefore \left(x-\frac{1}{x}\right)^2 = \left(x+\frac{1}{x}\right)^2 - 4$$

$= 13 - 4 = 9$

$$\therefore x - \frac{1}{x} = \sqrt{9} = 3$$

$$\therefore \text{Expression} = \frac{3x}{x^2-1}$$

$$= \frac{3x}{x\left(x-\frac{1}{x}\right)} = \frac{3}{3} = 1$$

32. (2) $a = \sqrt{2} + 1$

$\Rightarrow a + 1 = \sqrt{2} + 2$

$b = \sqrt{2} - 1$

$\Rightarrow b + 1 = \sqrt{2}$

$$\therefore \frac{1}{a+1} + \frac{1}{b+1}$$

$$= \frac{1}{\sqrt{2}+2} + \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+\sqrt{2}+2}{\sqrt{2}\left(\sqrt{2}+2\right)} = \frac{2+2\sqrt{2}}{2+2\sqrt{2}} = 1$$

33. (1) $x + \sqrt{5} = 5 + \sqrt{y}$

$\Rightarrow x = 5;\ y = 5$

$$\therefore \frac{\sqrt{x}+y}{x+\sqrt{y}} = \frac{\sqrt{5}+5}{5+\sqrt{5}} = 1$$

34. (1) $c + \dfrac{1}{c} = \sqrt{3}$ (Given)

On cubing both sides,

$$\left(c+\frac{1}{c}\right)^3 = \left(\sqrt{3}\right)^3$$

$$\Rightarrow c^3 + \frac{1}{c^3} + 3\left(c+\frac{1}{c}\right) = 3\sqrt{3}$$

$$\Rightarrow c^3 + \frac{1}{c^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow c^3 + \frac{1}{c^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

□□□

TEST YOURSELF

1. If $x = 1-\sqrt{2}$, find the value of $\left(x-\frac{1}{x}\right)^3$.

(1) 12 (2) 16
(3) 6 (4) 8

2. If $a = 7-4\sqrt{3}$, find the value of $\sqrt{a}+\frac{1}{\sqrt{a}}$.

(1) –6 (2) 6
(3) 4 (4) –4

3. If both a and b are rational numbers, find the values of a and b in the following equation :

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$$

(1) $a = 2,\ b = -1$
(2) $a = -2,\ b = 1$
(3) $a = -3,\ b = 1$
(4) $a = 3,\ b = -1$

4. Find the value of a and b in the following equaton.

$$\frac{5+\sqrt{3}}{7-4\sqrt{3}} = 47a+\sqrt{3}\,b$$

(1) $a = -27,\ b = 47$
(2) $a = -47,\ b = -27$
(3) $a = 47,\ b = 27$
(4) $a = 27,\ b = 47$

5. Simplify the following equation :

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$$

(1) $\frac{42}{11}$ (2) $\frac{-42}{11}$
(3) $\frac{-43}{22}$ (4) 11

6. $\frac{\sqrt{5}-2}{\sqrt{5}+2}+\frac{\sqrt{5}+2}{\sqrt{5}-2} = ?$

(1) $8\sqrt{5}$ (2) $-8\sqrt{5}$
(3) $8\sqrt{3}$ (4) $-8\sqrt{2}$

7. $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = ?$

(1) 11 (2) –11
(3) 12 (4) –12

8. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}} = ?$

(1) –2 (2) 2
(3) –1 (4) 1

9. Simplify :

$$\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

(1) 2 (2) –1
(3) 0 (4) 1

10. Simplify :

$$\frac{4\sqrt{18}}{\sqrt{12}}-\frac{8\sqrt{75}}{\sqrt{32}}+\frac{9\sqrt{2}}{\sqrt{3}}$$

(1) 0 (2) –1
(3) 1 (4) 2

11. If

$$\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}} = a+7\sqrt{5}\,b,$$

determine the rational number.

(1) $a = -2,\ b = \frac{2}{11}$

(2) $a = 0,\ b = \frac{1}{11}$

(3) $a = -1,\ b = \frac{1}{11}$

(4) $a = -2,\ b = -11$

12. $2\times\frac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}} = ?$

(1) 1 (2) $\frac{1}{3}$
(3) 2 (4) $\frac{1}{2}$

13. Evaluate :

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$

(1) 4 (2) 0
(3) 2 (4) –2

14. Given

$\sqrt{2} = 1.4142$, find correct to three places of decimal the value of

$$\frac{4}{3\sqrt{3}-2\sqrt{2}}+\frac{3}{3\sqrt{3}+2\sqrt{2}}.$$

(1) 2.063 (2) 2.036
(3) 2.306 (4) 2.36

15. Evaluate

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

it being given that

$\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.

(1) 5.938 (2) 5.398
(3) 5.893 (4) 5.839

16. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and

$y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ find the value of $x^3 + y^3$

(1) 807 (2) 907
(3) 970 (4) 870

17. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

find the value of $x^2 + xy - y^2$.

(1) $8\sqrt{2}+1$ (2) $8\sqrt{3}+1$
(3) $7\sqrt{3}+1$ (4) $8\sqrt{3}+2$

18. If $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$, then find the value of $bx^2 - ax + b$.

(1) 2 (2) 1
(3) 0 (4) 6

19. If $a = \frac{1}{3+2\sqrt{2}}$,

$b = \frac{1}{3-2\sqrt{2}}$ then $a^2b + ab^2$ =?

(1) –5 (2) 5
(3) –6 (4) 6

20. If $x=\sqrt{\dfrac{5+2\sqrt{6}}{5-2\sqrt{6}}}$ find the value of $x^2\left(x-10\right)^2$.

(1) 1 (2) –1
(3) 2 (4) –2

21. If $x=5-\sqrt{24}$, find the value of

$$\left(x^3+\frac{1}{x^3}\right)-10\left(x^2+\frac{1}{x^2}\right)+4\left(x+\frac{1}{x}\right)-30$$

(1) 1 (2) 0
(3) –1 (4) 2

22. a, b, c, p are rational numbers where p is a not a perfect cube.

If $a+bp^{\frac{1}{3}}+cp^{\frac{2}{3}}=0$, which of the following relations is correct ?

(1) $a = b = c = 2$
(2) $a \neq b = c$
(3) $a = b = c = 0$
(4) $a \neq b \neq c \neq 0$

23. Simplify : $\sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}}$

(1) $3^{15/16}$ (2) $3^{33/32}$
(3) $3^{21/32}$ (4) $3^{31/32}$

24. If $\sqrt{\sqrt[3]{0.000001\times x}}=0.5$ then find the value of x.

(1) 15625 (2) 15.625
(3) 16625 (4) 16.625

25. If $x=\sqrt{5+\sqrt{5+\sqrt{5+....}}}\infty$ and $y=\sqrt{5-\sqrt{5-\sqrt{5-....}}}\infty$, then find the value of x.

(1) $-y$
(2) $y + 1$
(3) $-y$ or $y + 1$
(4) None of these

26. If $x=\dfrac{1}{2-\sqrt{3}}$ find the value of $x^3 - 2x^2 - 7x + 5$.

(1) 2 (2) 8
(3) 4 (4) 3

27. Find the square root of $5+2\sqrt{6}$.

(1) $\sqrt{3}+\sqrt{2}$ (2) $\sqrt{3}+2$
(3) $2+\sqrt{3}$ (4) $\sqrt{3}-\sqrt{2}$

28. Find the positive square root of $14\sqrt{5}-30$.

(1) $\sqrt{5}(3-\sqrt{5})$
(2) $\sqrt[4]{5}(3-\sqrt{5})$
(3) $\sqrt{3}+2\sqrt{5}$
(4) $\sqrt{3}-2\sqrt{5}$

29. Evaluate : $\dfrac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}$

(1) 21 (2) $-\dfrac{4}{3}$
(3) $\dfrac{4}{3}$ (4) $\dfrac{1}{3}$

30. Simplify : $\sqrt{\left(\dfrac{6+2\sqrt{3}}{33-19\sqrt{3}}\right)}$

(1) $5-2\sqrt{3}$ (2) $5+2\sqrt{3}$
(3) $5-3\sqrt{3}$ (4) $5+3\sqrt{3}$

31. Find the value of

$$\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}-\sqrt{3-2\sqrt{2}}$$

(1) 1 (2) –1
(3) 2 (4) –2

32. Simplify :

$$\frac{4\sqrt{3}}{2-\sqrt{2}}-\frac{30}{4\sqrt{3}-\sqrt{18}}-\frac{\sqrt{18}}{3-2\sqrt{3}}$$

(1) $2\sqrt{6}$ (2) $4\sqrt{6}$
(3) $3\sqrt{6}$ (4) $-4\sqrt{6}$

33. Show that

$$\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{\sqrt{7-2\sqrt{10}}}-\frac{4}{\sqrt{8+4\sqrt{3}}}=0$$

(1) –2 (2) 2
(3) 0 (4) –1

34. Simplify : $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}$

(1) 1 (2) 2
(3) –2 (4) 3

35. Find the value of

$$\left(28+10\sqrt{3}\right)^{\frac{1}{2}}-\left(7-4\sqrt{3}\right)^{-\frac{1}{2}}.$$

(1) –3 (2) 3
(3) 2 (4) 4

36. Evaluate

$$(28-10\sqrt{3})^{\frac{1}{2}}-(7+4\sqrt{3})^{-\frac{1}{2}}+\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$$

(1) $4\frac{1}{2}$ (2) $2\frac{1}{2}$
(3) $3\frac{1}{2}$ (4) 3

37. Evaluate :

$$\frac{26-15\sqrt{3}}{\left[5\sqrt{2}-\sqrt{38+5\sqrt{3}}\right]^2}+\frac{\sqrt{10}+\sqrt{18}}{\sqrt{8}+\sqrt{\left(\sqrt{3}-\sqrt{5}\right)}}$$

(1) $4\frac{1}{2}$ (2) $2\frac{1}{4}$
(3) $3\frac{1}{2}$ (4) $2\frac{1}{3}$

38. Simplify :

$$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$$

(1) 2 (2) –2
(3) 3 (4) –3

39. Simplify :

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}}\times\left\{\left(\frac{25}{9}\right)^{-\frac{3}{2}}\div\left(\frac{5}{2}\right)^{-3}\right\}$$

(1) 1 (2) 2
(3) 0 (4) 4

40. Simplify :

$$\left(\frac{1}{4}\right)^{-2} - 3\,(8)^{\frac{2}{3}}(4)^0 + \left(\frac{9}{16}\right)^{\frac{-1}{2}}$$

(1) $4\frac{1}{3}$ (2) $5\frac{1}{3}$

(3) $2\frac{1}{3}$ (4) $6\frac{1}{3}$

41. $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} = ?$

(1) $\frac{512}{3375}$ (2) $\frac{512}{3275}$

(3) $\frac{3375}{512}$ (4) $\frac{3475}{512}$

42. $\dfrac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}} = ?$

(1) $\frac{4}{5}$ (2) $\frac{3}{4}$

(3) $\frac{2}{3}$ (4) $\frac{1}{2}$

43. Simplify :

$$\frac{\left(x^{a+b}\right)^2\left(x^{b+c}\right)^2\left(x^{c+a}\right)^2}{\left(x^a x^b x^c\right)^4}$$

(1) $2x$ (2) 0
(3) 1 (4) $a + b + c$

44. If $25^{x-1} = 5^{2x-1} - 100$, find the value of x.
(1) 4 (2) 2
(3) 1 (4) 0

45. If $\dfrac{9^n \times 3^2 \times \left(3^{\frac{-n}{2}}\right)^{-2} - (27)^n}{3^{3m} \times 2^3}$

$= \frac{1}{27}$, then $m - n = ?$

(1) 2 (2) 0
(3) 1 (4) 4

46. $\dfrac{a^{-1}}{a^{-1}+b^{-1}} + \dfrac{a^{-1}}{a^{-1}-b^{-1}} = ?$

(1) $\frac{b^2}{b^2-a^2}$ (2) $\frac{b^2}{b^2+a^2}$

(3) $\frac{2b^2}{b^2+a^2}$ (4) $\frac{2b^2}{b^2-a^2}$

47. Assuming that x is a positive real number and a, b, c are rational numbers, then

$$\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ac}} = ?$$

(1) 1 (2) 2
(3) 0 (4) 4

48. $\left(\dfrac{x^a}{x^b}\right)^{a+b}\left(\dfrac{x^b}{x^c}\right)^{b+c}\left(\dfrac{x^c}{x^a}\right)^{c+a} = ?$

(1) $\frac{1}{2}$ (2) 2

(3) 0 (4) 1

49. $\left(\dfrac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\dfrac{x^b}{x^c}\right)^{b^2+bc+c^2}\left(\dfrac{x^c}{x^a}\right)^{c^2+ca+a^2} = ?$

(1) 3 (2) 2
(3) 1 (4) 0

50. If x, y, z are positive real numbers, then

$$\sqrt{x^{-1}y}\,.\,\sqrt{y^{-1}z}\,.\,\sqrt{z^{-1}x} = ?$$

(1) 1 (2) 0
(3) –1 (4) –2

51. Find the simplest value of

$$\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-3\sqrt{2}} - \frac{3\sqrt{2}}{3-2\sqrt{3}}.$$

(1) $4\sqrt{2}$ (2) $4\sqrt{3}$

(3) $4\sqrt{6}$ (4) $5\sqrt{6}$

52. Find the value of n, if

$(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$

(1) 12 (2) 13
(3) 14 (4) 15

53. Find the value of

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

if $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$.
(1) 5.498 (2) 5.398
(3) 6.398 (4) 3.498

54. $\left(28-10\sqrt{3}\right)^{1/2} - \left(7+4\sqrt{3}\right)^{-1/2}$ is equal to
(1) 4 (2) 5
(3) 3 (4) 4.3

SHORT ANSWERS

1. (4)	**2.** (3)	**3.** (1)	**4.** (3)
5. (1)	**6.** (2)	**7.** (1)	**8.** (4)
9. (3)	**10.** (1)	**11.** (2)	**12.** (1)
13. (3)	**14.** (1)	**15.** (2)	**16.** (3)
17. (2)	**18.** (3)	**19.** (4)	**20.** (1)
21. (2)	**22.** (3)	**23.** (4)	**24.** (1)
25. (3)	**26.** (4)	**27.** (1)	**28.** (2)
29. (3)	**30.** (4)	**31.** (1)	**32.** (2)
33. (3)	**34.** (1)	**35.** (2)	**36.** (3)
37. (4)	**38.** (1)	**39.** (1)	**40.**(2)
41. (3)	**42.** (4)	**43.** (3)	**44.** (2)
45. (3)	**46.** (4)	**47.** (1)	**48.** (4)
49. (3)	**50.** (1)	**51.** (3)	**52.** (3)
53. (2)	**54.** (3)		

EXPLANATIONS

1. (4) Here, $x = 1-\sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{1-\sqrt{2}}$$

$$= \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1+\sqrt{2}}{1-2} = -\left(1+\sqrt{2}\right)$$

$$\therefore x - \frac{1}{x} = \left(1-\sqrt{2}\right) - \left\{-\left(1+\sqrt{2}\right)\right\}$$

$$= 1-\sqrt{2}+1+\sqrt{2} = 2$$

$$\therefore \left(x-\frac{1}{x}\right)^3 = 2^3 = 8$$

2. (3) We have $a = 7-4\sqrt{3}$

$$\therefore \frac{1}{a} = \frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$$

$$= \frac{7+4\sqrt{3}}{(7)^2-\left(4\sqrt{3}\right)^2} = \frac{7+4\sqrt{3}}{49-48}$$

$$= 7+4\sqrt{3}$$

Now, $\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)^2 = a+\frac{1}{a}+2$

$$= 7-4\sqrt{3}+7+4\sqrt{3}+2 = 16$$

$$\therefore \sqrt{a}+\frac{1}{\sqrt{a}} = 4$$

3. (1) Multiplying the numerator and denominator by the conjugate of $\sqrt{3}+1$, we have

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$= \frac{\left(\sqrt{3}\right)^2+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$\therefore \frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$$

$$\Rightarrow 2-\sqrt{3} = a+b\sqrt{3}$$

$$\Rightarrow a+b\sqrt{3} = 2+(-1)\sqrt{3}$$

On equating rational and irrational parts

$a = 2$ and $b = -1$

4. (3) Multiplying the numerator and denominator by the conjugate of $7-4\sqrt{3}$, we have

$$\frac{5+\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{5+\sqrt{3}}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}}$$

$$= \frac{35+5\times 4\sqrt{3}+7\times\sqrt{3}+\sqrt{3}\times 4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{35+20\sqrt{3}+7\sqrt{3}+4\left(\sqrt{3}\right)^2}{49-16\left(\sqrt{3}\right)^2}$$

$$= \frac{35+27\sqrt{3}+12}{49-48} = 47+27\sqrt{3}$$

$$\therefore \frac{5+\sqrt{3}}{7-4\sqrt{3}} = a+b\sqrt{3}$$

$$\Rightarrow 47+27\sqrt{3} = a+b\sqrt{3}$$

On equating rational and irrational parts.

$a = 47$ and $b = 27$

5. (1) Rationalising the denominator of each term, we have

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{\left(4+\sqrt{5}\right)^2}{\left(4-\sqrt{5}\right)\left(4+\sqrt{5}\right)}+\frac{\left(4-\sqrt{5}\right)^2}{\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)}$$

$$= \frac{\left(4+\sqrt{5}\right)^2}{16-5}+\frac{\left(4-\sqrt{5}\right)^2}{16-5}$$

$$= \frac{\left(4+\sqrt{5}\right)^2+\left(4-\sqrt{5}\right)^2}{11}$$

$$= \frac{2\left[(4)^2+\left(\sqrt{5}\right)^2\right]}{11}$$

$[\because (a+b)^2+(a-b)^2 = 2(a^2+b^2)]$

$$= \frac{2(16+5)}{11} = \frac{42}{11}$$

6. (2) $\dfrac{\sqrt{5}-2}{\sqrt{5}+2}-\dfrac{\sqrt{5}+2}{\sqrt{5}-2}$

$$= \frac{\sqrt{5}-2}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}\times\frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{\left(\sqrt{5}-2\right)^2}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}-\frac{\left(\sqrt{5}+2\right)^2}{\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)}$$

$$= \frac{\left(\sqrt{5}\right)^2+2^2-2\times 2\times\sqrt{5}}{\left(\sqrt{5}\right)^2-2^2}$$

$$- \frac{\left(\sqrt{5}\right)^2+2^2+2\times 2\times\sqrt{5}}{\left(\sqrt{5}\right)^2-2^2}$$

$$= \frac{5+4-4\sqrt{5}}{5-4}-\frac{5+4+4\sqrt{5}}{5-4}$$

$$= \left(9-4\sqrt{5}\right)-\left(9+4\sqrt{5}\right)$$

$$= 9-4\sqrt{5}-9-4\sqrt{5} = -8\sqrt{5}$$

7. (1) 1st term $= \dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$

Rationalising the denominator, we have

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$= \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$$

$$= \frac{\left(3\sqrt{2}-2\sqrt{3}\right)^2}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$= \frac{\left(3\sqrt{2}\right)^2+\left(2\sqrt{3}\right)^2-2\times 3\sqrt{2}\times 2\sqrt{3}}{18-12}$$

$$= \frac{18+12-12\sqrt{6}}{6} = \frac{30-12\sqrt{6}}{6}$$

$$= \frac{6\left(5-2\sqrt{6}\right)}{6} = 5-2\sqrt{6}$$

2nd term

$$= \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{4\times 3}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

[Rationalising the denominator]

$$= \frac{2\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{6+2\sqrt{6}}{3-2} = 6+2\sqrt{6}$$

$$\therefore \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$= 5-2\sqrt{6}+6+2\sqrt{6} = 11$$

8. (4) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{80}+\sqrt{48}-\sqrt{45}-\sqrt{27}}$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{16\times5}+\sqrt{16\times3}-\sqrt{9\times5}-\sqrt{9\times3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{4\sqrt{5}+4\sqrt{3}-3\sqrt{5}-3\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{(4-3)\sqrt{5}+(4-3)\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 1$$

9. (3) Rationalising the denominator of each term, we have

1st term

$$= \frac{6}{2\sqrt{3}-\sqrt{6}} \times \frac{2\sqrt{3}+\sqrt{6}}{2\sqrt{3}+\sqrt{6}}$$

$$= \frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{\left(2\sqrt{3}\right)^2-\left(\sqrt{6}\right)^2}$$

$$= \frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{12-6}$$

$$= \frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{6} = 2\sqrt{3}+\sqrt{6}$$

2nd term

$$= \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\sqrt{18}-\sqrt{12}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{9\times2}-\sqrt{4\times3}}{3-2} = 3\sqrt{2}-2\sqrt{3}$$

3rd term

$$= \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$= \frac{4\sqrt{18}+4\sqrt{6}}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{4\left(\sqrt{9\times2}+\sqrt{6}\right)}{6-2} = 3\sqrt{2}+\sqrt{6}$$

$\therefore$ Given expression =

$$2\sqrt{3}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}-3\sqrt{2}-\sqrt{6}$$

= 0

10. (1) $\dfrac{4\sqrt{18}}{\sqrt{12}} - \dfrac{8\sqrt{75}}{\sqrt{32}} + \dfrac{9\sqrt{2}}{\sqrt{3}}$

$$= \frac{4\sqrt{9\times2}}{\sqrt{4\times3}} - \frac{8\sqrt{25\times3}}{\sqrt{16\times2}} + \frac{9\sqrt{2}}{\sqrt{3}}$$

$$= \frac{12\sqrt{2}}{2\sqrt{3}} - \frac{40\sqrt{3}}{4\sqrt{2}} + \frac{9\sqrt{2}}{\sqrt{3}}$$

$$= \frac{6\sqrt{2}}{\sqrt{3}} - \frac{10\sqrt{3}}{\sqrt{2}} + \frac{9\sqrt{2}}{\sqrt{3}}$$

Now rationalising the denominator of each term, we have

$$= \frac{6\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{10\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{9\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{6\sqrt{6}}{3} - \frac{10\sqrt{6}}{2} + \frac{9\sqrt{6}}{3}$$

$$= 2\sqrt{6}-5\sqrt{6}+3\sqrt{6} = 0$$

11. (2) Rationalising the denominator of each term, we get

$$\text{L.H.S.} = \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}}$$

$$= \frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$= \frac{\left(7+\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2} - \frac{\left(7-\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{7^2+\left(\sqrt{5}\right)^2+2\times7\times\sqrt{5}}{49-5}$$

$$- \frac{7^2+\left(\sqrt{5}\right)^2-2\times7\times\sqrt{5}}{49-5}$$

$$= \frac{49+5+14\sqrt{5}}{44} - \frac{49+5-14\sqrt{5}}{44}$$

$$= \frac{54+14\sqrt{5}-54+14\sqrt{5}}{44}$$

$$= \frac{28\sqrt{5}}{44} = \frac{7\sqrt{5}}{11}$$

Now $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} - \dfrac{7-\sqrt{5}}{7+\sqrt{5}}$

$$= a+7\sqrt{5}\,b$$

$$\Rightarrow \frac{7\sqrt{5}}{11} = a+7\sqrt{5}\,b$$

$$\Rightarrow 0+7\sqrt{5}.\frac{1}{11} = a+7\sqrt{5}\,b$$

$\Rightarrow$ $a = 0$ and $b = \dfrac{1}{11}$

12. (1) $\dfrac{16\times2^{n+1}-4\times2^n}{16\times2^{n+2}-2\times2^{n+2}}$

$$= \frac{2^4\times2^{n+1}-2^2\times2^n}{2^4\times2^{n+2}-2\times2^{n+2}}$$

$$= \frac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$$

$$= \frac{2^{n+5}-2^{n+2}}{2(2^{n+5}-2^{n+2})} = \frac{1}{2}$$

$\therefore$ Expression $= 2 \times \dfrac{1}{2} = 1$

13. (3) Rationalising the denominator of each term, the given expression becomes

$$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4}$$

$$+ \frac{\sqrt{4}-\sqrt{5}}{4-5} + \ldots\ldots + \frac{\sqrt{8}-\sqrt{9}}{8-9}$$

$$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4}$$

$$- \sqrt{4} + \sqrt{5} - \sqrt{5} + \sqrt{6} - \sqrt{6}$$

$$+ \sqrt{7} - \sqrt{7} + \sqrt{8} - \sqrt{8} + \sqrt{9}$$

$$= -1 + \sqrt{9} = -1 + 3 = 2$$

14. (1) Rationalising the denominator of each term, we have

$$= \frac{4}{3\sqrt{3}-2\sqrt{2}} \times \frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$$

$$+ \frac{3}{3\sqrt{3}+2\sqrt{2}} \times \frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$$

$$= \frac{4\left(3\sqrt{3}+2\sqrt{2}\right)}{\left(3\sqrt{3}\right)^2 - \left(2\sqrt{2}\right)^2} + \frac{3\left(3\sqrt{3}-2\sqrt{2}\right)}{\left(3\sqrt{3}\right)^2 - \left(2\sqrt{2}\right)^2}$$

$$= \frac{12\sqrt{3}+8\sqrt{2}+9\sqrt{3}-6\sqrt{2}}{27-8}$$

$$= \frac{(12+9)\sqrt{3}+(8-6)\sqrt{2}}{19}$$

$$= \frac{21}{19}\sqrt{3} + \frac{2}{19}\sqrt{2}$$

$$= \frac{21}{19} \times 1.7321 + \frac{2}{19} \times 1.4142$$

$$= \frac{1}{19}(21 \times 1.7321 + 2 \times 1.4142)$$

$$= \frac{1}{19}(36.3741 + 2.8284)$$

$$= \frac{39.2025}{19} = 2.0632 = 2.063$$

Method 2 :

$$\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$$

$$= \frac{4\left(3\sqrt{3}+2\sqrt{2}\right) + 3\left(3\sqrt{3}-2\sqrt{2}\right)}{\left(3\sqrt{3}-2\sqrt{2}\right)\left(3\sqrt{3}+2\sqrt{2}\right)}$$

$$= \frac{12\sqrt{3}+8\sqrt{2}+9\sqrt{3}-6\sqrt{2}}{\left(3\sqrt{3}\right)^2 - \left(2\sqrt{2}\right)^2}$$

$$= \frac{21\sqrt{3}+2\sqrt{2}}{27-8}$$

$$= \frac{21}{19}\sqrt{3} + \frac{2}{19}\sqrt{2}$$

$$= \frac{21}{19} \times 1.7321 + \frac{2}{19} \times 1.4142$$

$$= \frac{1}{19}(21 \times 1.7321 + 2 \times 1.4142)$$

$$= \frac{1}{19}(36.3741 + 2.8284)$$

$$= \frac{39.2028}{19} = 2.0632 = 2.063$$

15. (2) We have

$$\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5} - \sqrt{80}$$

$$= \sqrt{10} + \sqrt{2^2 \times 5} + \sqrt{2^2 \times 10}$$

$$- \sqrt{5} - \sqrt{2^4 \times 5}$$

$$= \sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} - 4\sqrt{5}$$

$$= (1+2)\sqrt{10} + (2-1-4)\sqrt{5}$$

$$= 3\sqrt{10} - 3\sqrt{5} = 3\left(\sqrt{10} - \sqrt{5}\right)$$

$$\therefore \quad \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

$$= \frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)} = \frac{15}{\sqrt{10}-\sqrt{5}}$$

$$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{\left(\sqrt{10}-\sqrt{5}\right)\left(\sqrt{10}+\sqrt{5}\right)}$$

$$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{10-5}$$

$= \sqrt{10} + \sqrt{5} = 3.162 + 2.236 = 5.398$

16. (3) Rationalising the denominators, we have

$$x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{\left(\sqrt{3}-\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{3+2-2\sqrt{3}\times\sqrt{2}}{3-2} = 5 - 2\sqrt{6}$$

and, $y = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{3+2+2\sqrt{3}\times\sqrt{2}}{3-2} = 5 + 2\sqrt{6}$$

$\therefore\ x + y$

$$= 5 - 2\sqrt{6} + 5 + 2\sqrt{6} = 10$$

and, $xy = \left(5 - 2\sqrt{6}\right)\left(5 + 2\sqrt{6}\right)$

$$= 5^2 - \left(2\sqrt{6}\right)^2 = 25 - 24 = 1$$

$x^3 + y^3 = (x + y)^3 - 3xy\,(x + y)$

$= 10^3 - 3 \times 10 = 1000 - 30 = 970$

17. (2) We have

$$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2 - (1)^2} = \frac{3+1+2\sqrt{3}}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$$

Similarly, $y = 2 - \sqrt{3}$

$\therefore\ x + y = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$x - y$

$$= \left(2 + \sqrt{3}\right) - \left(2 - \sqrt{3}\right) = 2\sqrt{3}$$

$xy = \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$

$= 4 - 3 = 1$

Hence, $x^2 + xy - y^2$

$= x^2 - y^2 + xy$

$= (x + y)\,(x - y) + xy$

$= 4 \times 2\sqrt{3} + 1 = 8\sqrt{3} + 1$

18. (3) We have

$$x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$$

$$\times \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}}$$

$$= \frac{\left(\sqrt{a+2b}+\sqrt{a-2b}\right)^2}{\left(\sqrt{a+2b}\right)^2-\left(\sqrt{a-2b}\right)^2}$$

$$= \frac{a+2b+a-2b+2\sqrt{a+2b}\sqrt{a-2b}}{(a+2b)-(a-2b)}$$

$$= \frac{2a+2\sqrt{a^2-4b^2}}{4b}$$

$$\therefore\ x = \frac{a+\sqrt{a^2-4b^2}}{2b}$$

$\Rightarrow 2bx = a + \sqrt{a^2-4b^2}$

$\Rightarrow 2bx - a = \sqrt{a^2-4b^2}$

$\Rightarrow (2bx - a)^2 = \left(\sqrt{a^2-4b^2}\right)^2$

[On squaring both sides]

$\Rightarrow 4b^2x^2 + a^2 - 4abx$

$= a^2 - 4b^2$

$\Rightarrow 4b^2x^2 - 4abx + 4b^2 = 0$

$\Rightarrow 4b\,(bx^2 - ax + b) = 0$

$\Rightarrow bx^2 - ax + b = 0$

19. (4) $a = \dfrac{1}{3+2\sqrt{2}}$

$$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$$

$$b = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{(3)^2-\left(2\sqrt{2}\right)^2}$$

$$= \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$$

Now, $a^2b + ab^2 = ab\,(a + b)$

$\therefore\ a + b$

$= 3-2\sqrt{2}+3+2\sqrt{2} = 6$

and, $ab = \left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)$

$= 9 - 8 = 1$

Hence, $a^2b + ab^2 = ab\,(a + b)$

$= 1 \times 6 = 6$

20. (1) $x = \sqrt{\dfrac{5+2\sqrt{6}}{5-2\sqrt{6}}}$

On rationalising, we have

$$x = \sqrt{\frac{5+2\sqrt{6}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}}$$

$$= \sqrt{\frac{\left(5+2\sqrt{6}\right)^2}{(5)^2-\left(2\sqrt{6}\right)^2}}$$

$$= \sqrt{\frac{\left(5+2\sqrt{6}\right)^2}{25-24}} = 5+2\sqrt{6}$$

$\therefore\ x^2\,(x - 10)^2$

$$= \left(5+2\sqrt{6}\right)^2\left(5+2\sqrt{6}-10\right)^2$$

$$= \left(5+2\sqrt{6}\right)^2\left(2\sqrt{6}-5\right)^2$$

$$= \left(25+24+20\sqrt{6}\right)\left(24+25-20\sqrt{6}\right)$$

$$= \left(49+20\sqrt{6}\right)\left(49-20\sqrt{6}\right)$$

$$= (49)^2-\left(20\sqrt{6}\right)^2$$

$= 2401 - 2400 = 1$

21. (2) Here, $x = 5-\sqrt{24}$

$$\therefore\ \frac{1}{x} = \frac{1}{5-\sqrt{24}}$$

$$= \frac{1}{5-\sqrt{24}} \times \frac{5+\sqrt{24}}{5+\sqrt{24}}$$

$$= \frac{5+\sqrt{24}}{25-24} = 5+\sqrt{24}$$

$$\therefore\ x^3 + \frac{1}{x^3}$$

$$= \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$$

$= \left(5-\sqrt{24}+5+\sqrt{24}\right)^3$

$-3\left(5-\sqrt{24}+5+\sqrt{24}\right)$

$= 10^3 - 3 \times 10$

$= 1000 - 30 = 970$

$$x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$= \left(5-\sqrt{24}+5+\sqrt{24}\right)^2 - 2$$

$= 100 - 2 = 98$

$$x + \frac{1}{x}$$

$= 5-\sqrt{24}+5+\sqrt{24} = 10$

$\therefore$ The given expression

$= 970 - 10 \times 98 + 4 \times 10 - 30$

$= 970 - 980 + 40 - 30 = 0$

22. (3) $a + bp^{\frac{1}{3}} + cp^{\frac{2}{3}} = 0$ --(i)

On multiplying both sides by $p^{\frac{1}{3}}$,

$ap^{\frac{1}{3}} + bp^{\frac{2}{3}} + cp = 0$ ...(ii)

Equation (i) × b – (ii) × c,

$\left(ab + b^2p^{\frac{1}{3}} + bc\,p^{\frac{2}{3}}\right) -$

$$\left(acp^{\frac{1}{3}} + bc\,p^{\frac{2}{3}} + c^2p\right) = 0$$

$\Rightarrow\ (b^2 - ac)p^{\frac{1}{3}} + ab - c^2p = 0$

$\Rightarrow\ b^2 - ac = 0$ and $ab - c^2p = 0$

$\Rightarrow\ b^2 = ac$ and $ab = c^2p$

$\Rightarrow\ b^2 = ac$ and $a^2b^2 = c^4p^2$

$\Rightarrow\ a^2\,(ac) = c^4p^2$

$\Rightarrow\ a^3c - p^2c^4 = 0$

$\Rightarrow (a^3 - p^2c^3)\,c = 0$

$\Rightarrow\ a^3 - p^2c^3 = 0$ or $c = 0$

$a^3 - p^2c^3 = 0$

$p^2 = \dfrac{a^3}{c^3} \Rightarrow \left(p^2\right)^{\frac{1}{3}} = \dfrac{a}{c}$

$\Rightarrow \left(p^{\frac{1}{3}}\right)^{\frac{1}{2}} = \dfrac{a}{c}$, which is impossible as $(p)^{\frac{1}{3}}$ is irrational.

$\therefore\ c = 0$

Putting $c = 0$ in $b^2 = ac$,

$b = 0$

$\therefore\ a + bp^{\frac{1}{3}} + cp^{\frac{2}{3}} = 0$

$\Rightarrow\ a = 0$

$\therefore\ a = b = c = 0$

23. (4) Let $x = \sqrt{3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}}$

On squaring both sides, we have,

$\Rightarrow\ x^2 = 3\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}$

On squaring again,

$\Rightarrow x^4 = 3^2 \times 3\sqrt{3\sqrt{3\sqrt{3}}}$

On squaring again,

$\Rightarrow x^8 = 3^4 \times 3^2 \times 3\sqrt{3\sqrt{3}}$

On squaring again, $\Rightarrow x^{16}$

$= 3^8 \times 3^4 \times 3^2 \times 3\sqrt{3}$

On squaring again

$\Rightarrow x^{32} = 3^{16} \times 3^8 \times 3^4 \times 3^2 \times 3$

$x^{32} = 3^{16+8+4+2+1}$

$\Rightarrow\ x^{32} = 3^{31}$

$\Rightarrow\ x = \left(3^{31}\right)^{\frac{1}{32}} = 3^{\frac{31}{32}}$

24. (1) The given expression

$\sqrt{\sqrt[3]{0.000001 \times x}} = 0.5$

$\Rightarrow\ \left(\sqrt[3]{0.000001 \times x}\right)^{\frac{1}{2}} = 0.5$

$\Rightarrow\ (0.000001 \times x)^{\frac{1}{2}\times\frac{1}{3}} = 0.5$

$\Rightarrow\ (0.000001 \times x)^{\frac{1}{6}} = 0.5$

$\Rightarrow\ \left(10^{-6} \times x\right)^{\frac{1}{6}} = 0.5$

$\Rightarrow\ 10^{-1} \times x^{\frac{1}{6}} = 0.5$

$\Rightarrow\ x^{\frac{1}{6}} = \dfrac{0.5}{0.1} = 5$

$\Rightarrow x = 5^6 = 15625$

25. (3) Here,

$x = \sqrt{5 + \sqrt{5 + \sqrt{5 + \ldots\ldots}}}\ \infty$

On squaring we have $x^2 = 5 + x$ ....(i)

and, $y = \sqrt{5 - \sqrt{5 - \sqrt{5 - \ldots\ldots}}}\ \infty$

$\therefore\ y^2 = 5 - y$ ...(ii)

From equations (i) and (ii)

$x^2 - x = y^2 + y$

$\Rightarrow x^2 - y^2 = x + y$

$\Rightarrow (x + y)\,(x - y) = (x + y)$

$\Rightarrow (x + y)\,(x - y) - (x + y) = 0$

$\Rightarrow (x + y)\,(x - y - 1) = 0$

Thus, either $x + y = 0$ or, $(x - y - 1) = 0$

or, $x = -y$ or, $x = y + 1$

26. (4) We have

$x = \dfrac{1}{2 - \sqrt{3}}$

$= \dfrac{1}{2 - \sqrt{3}} \times \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$

$= \dfrac{2 + \sqrt{3}}{2^2 - \left(\sqrt{3}\right)^2}$

$= \dfrac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$

Now, $x = 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$

$\Rightarrow (x - 2)^2 = \left(\sqrt{3}\right)^2$

$\Rightarrow x^2 - 4x + 4 = 3$

$\Rightarrow x^2 - 4x + 1 = 0$ ....(i)

Now,

$x^2 - 4x + 1\ )\ x^3 - 2x^2 - 7x + 5\ (\,x + 2$

$x^3 - 4x + x$

$-\ \ +\ \ -$

$2x^2 - 8x + 5$

$2x^2 - 8x + 2$

$-\ \ +\ \ -$

3

$\therefore\ x^3 - 2x^2 - 7x + 5 = (x + 2)\,(x^2 - 4x + 1) + 3$

$= (x + 2) \times 0 + 3 = 3$

27. (1) $5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{3}\sqrt{2}$

$= \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2 \times \sqrt{3} \times \sqrt{2}$

$= \left(\sqrt{3} + \sqrt{2}\right)^2$

$\therefore\ \sqrt{5 + 2\sqrt{6}}$

$= \sqrt{\left(\sqrt{3} + \sqrt{2}\right)^2} = \sqrt{3} + \sqrt{2}$

Note : Express the given number in the form of

$x + y + 2\sqrt{xy}$

$= \left(\sqrt{x}\right)^2 + \left(\sqrt{y}\right)^2 + 2\sqrt{x} \times \sqrt{y}$

$= \left(\sqrt{x} + \sqrt{y}\right)^2$

28. (2) $14\sqrt{5} - 30$

$= \sqrt{5}\left(14 - 6\sqrt{5}\right)$

$= \sqrt{5}\left(14 - 2 \times 3 \times \sqrt{5}\right)$

$= \sqrt{5}\left(9 + 5 - 2 \times \sqrt{9} \times \sqrt{5}\right)$

$= \sqrt{5}\left[\left(\sqrt{9}\right)^2+\left(\sqrt{5}\right)^2-2\sqrt{9}\sqrt{5}\right]$

$= \sqrt{5}\left(3-\sqrt{5}\right)^2$

$\therefore \sqrt{14\sqrt{5}-30}$

$= \sqrt{\sqrt{5}\left(3-\sqrt{5}\right)^2} = \sqrt[4]{5}\left(3-\sqrt{5}\right)$

29. (3) Let the required value be x, i.e.,

$x = \dfrac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}$ then,

$x^2 = \left[\dfrac{2\left(\sqrt{2}+\sqrt{6}\right)}{3\sqrt{2+\sqrt{3}}}\right]^2$

$x^2 = \dfrac{4\left(\sqrt{2}+\sqrt{6}\right)^2}{\left(3\sqrt{2+\sqrt{3}}\right)^2}$

$\Rightarrow x^2 = \dfrac{4\left(\sqrt{2}+\sqrt{6}\right)^2}{9\left(2+\sqrt{3}\right)}$

$x^2 = \dfrac{4\left(2+6+2\times\sqrt{2}\times\sqrt{6}\right)}{9\left(2+\sqrt{3}\right)}$

$\Rightarrow x^2 = \dfrac{4\left(8+2\sqrt{12}\right)}{9\left(2+\sqrt{3}\right)}$

$x^2 = \dfrac{4\left(8+2\sqrt{2^2\times 3}\right)}{9\left(2+\sqrt{3}\right)}$

$\Rightarrow x^2 = \dfrac{4\left(8+4\sqrt{3}\right)}{9\left(2+\sqrt{3}\right)}$

$x^2 = \dfrac{16\left(2+\sqrt{3}\right)}{9\left(2+\sqrt{3}\right)} = \dfrac{16}{9}$

$\Rightarrow x = \dfrac{4}{3}$

30. (4) $\sqrt{\left(\dfrac{6+2\sqrt{3}}{33-19\sqrt{3}}\right)}$

$= \sqrt{\dfrac{\sqrt{3}\left(2\sqrt{3}+2\right)}{\sqrt{3}\left(11\sqrt{3}-19\right)}}$

$= \sqrt{\dfrac{2\left(\sqrt{3}+1\right)\left(11\sqrt{3}+19\right)}{\left(11\sqrt{3}-19\right)\left(11\sqrt{3}+19\right)}}$

$= \sqrt{\dfrac{2\left(33+19\sqrt{3}+11\sqrt{3}+19\right)}{\left(11\sqrt{3}\right)^2-(19)^2}}$

$= \sqrt{\dfrac{2\left(52+30\sqrt{3}\right)}{363-361}}$

$= \sqrt{52+30\sqrt{3}}$

$= \sqrt{52+2\times 15\times\sqrt{3}}$

$= \sqrt{52+2\times\sqrt{225}\times\sqrt{3}}$

$= \sqrt{52+2\times\sqrt{225\times 3}}$

$= \sqrt{52+2\times\sqrt{25\times 9\times 3}}$

$= \sqrt{25+27+2\times\sqrt{25}\times\sqrt{27}}$

$= \sqrt{\left(\sqrt{25}+\sqrt{27}\right)^2}$

$= \sqrt{25}+\sqrt{27} \quad = 5+3\sqrt{3}$

31. (1) It will be convenient to solve this problem part-wise.

Let, $x = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Then,

$x^2 = \left[\dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}\right]^2$

$= \dfrac{\left[\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}\right]^2}{\left[\sqrt{\sqrt{5}+1}\right]^2}$

$= \dfrac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{\sqrt{5}+2}\sqrt{\sqrt{5}-2}}{\sqrt{5}+1}$

$= \dfrac{2\sqrt{5}+2\sqrt{\left(\sqrt{5}\right)^2-(2)^2}}{\sqrt{5}+1}$

$= \dfrac{2\sqrt{5}+2\sqrt{5-4}}{\sqrt{5}+1}$

$= \dfrac{2\sqrt{5}+2}{\sqrt{5}+1} = \dfrac{2\left(\sqrt{5}+1\right)}{\sqrt{5}+1} = 2$

$x^2 = 2$

$\therefore x = \sqrt{2}$

Also,

$\sqrt{3-2\sqrt{2}}$

$= \sqrt{2+1-2\times\sqrt{2}\times 1}$

$= \sqrt{\left(\sqrt{2}\right)^2+(1)^2-2\times\sqrt{2}\times 1}$

$= \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2}-1$

Hence,

$\dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$

$= \sqrt{2}-\left(\sqrt{2}-1\right) = 1$

32. (2)

$\dfrac{4\sqrt{3}}{2-\sqrt{2}} - \dfrac{30}{4\sqrt{3}-\sqrt{18}} - \dfrac{\sqrt{18}}{3-2\sqrt{3}}$

On rationalising the denominators of each term, we get

$= \dfrac{4\sqrt{3}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}$

$$-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{\left(4\sqrt{3}-\sqrt{18}\right)\left(4\sqrt{3}+\sqrt{18}\right)}$$

$$-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)}$$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{(2)^2-\left(\sqrt{2}\right)^2}-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{\left(4\sqrt{3}\right)^2-\left(\sqrt{18}\right)^2}$$

$$-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{(3)^2-\left(2\sqrt{3}\right)^2}$$

[$\because (a+b)(a-b)=a^2-b^2$]

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{4-2}$$

$$-\frac{30\left(4\sqrt{3}+\sqrt{18}\right)}{48-18}-\frac{\sqrt{18}\left(3+2\sqrt{3}\right)}{9-12}$$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{2}-\frac{30\left(4\sqrt{3}+\sqrt{9\times 2}\right)}{30}$$

$$-\frac{\sqrt{9\times 2}\left(3+2\sqrt{3}\right)}{-3}$$

$$=2\sqrt{3}\left(2+\sqrt{2}\right)-\left(4\sqrt{3}+3\sqrt{2}\right)$$

$$-\frac{3\sqrt{2}\left(3+2\sqrt{3}\right)}{-3}$$

$$=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{2}-\frac{30\left(4\sqrt{3}+\sqrt{9\times 2}\right)}{30}$$

$$-\frac{\sqrt{9\times 2}\left(3+2\sqrt{3}\right)}{-3}$$

$$=2\sqrt{3}\left(2+\sqrt{2}\right)-\left(4\sqrt{3}+3\sqrt{2}\right)$$

$$-\frac{3\sqrt{2}\left(3+2\sqrt{3}\right)}{-3}$$

$$=4\sqrt{3}+2\sqrt{6}-4\sqrt{3}-3\sqrt{2}+3\sqrt{2}+2\sqrt{6}$$

$$=2\sqrt{6}+2\sqrt{6}=4\sqrt{6}$$

33. (3) The given expression consists of three parts. Now, we solve each part separately.

Part I $=\dfrac{1}{\sqrt{11-2\sqrt{30}}}$

or $\sqrt{\dfrac{1}{\left(11-2\sqrt{30}\right)}}$

On rationalising the denominator by its conjugate we have expression

$$=\sqrt{\frac{1\times\left(11+2\sqrt{30}\right)}{\left(11-2\sqrt{30}\right)\left(11+2\sqrt{30}\right)}}$$

$$=\sqrt{\frac{11+2\sqrt{30}}{(11)^2-\left(2\sqrt{30}\right)^2}}$$

[$\because (a+b)(a-b)=a^2-b^2$]

$$=\sqrt{\frac{11+2\sqrt{30}}{121-120}}$$

$$=\sqrt{11+2\sqrt{30}}$$

$$=\sqrt{11+2\times\sqrt{6}\times\sqrt{5}}$$

$$=\sqrt{6+5+2\times\sqrt{6}\times\sqrt{5}}$$

$$=\sqrt{\left(\sqrt{6}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{6}\times\sqrt{5}}$$

$$=\sqrt{\left(\sqrt{6}+\sqrt{5}\right)^2}=\sqrt{6}+\sqrt{5}$$

[$\because a^2+b^2+2ab=(a+b)^2$]

Part II

$$=\frac{3}{\sqrt{7-2\sqrt{10}}}=\sqrt{\frac{9}{7-2\sqrt{10}}}$$

$$=\sqrt{\frac{9\times\left(7+2\sqrt{10}\right)}{\left(7-2\sqrt{10}\right)\left(7+2\sqrt{10}\right)}}$$

$$=\sqrt{\frac{9\times\left(7+2\sqrt{10}\right)}{7^2-\left(2\sqrt{10}\right)^2}}$$

$$=\sqrt{\frac{9\times\left(7+2\sqrt{10}\right)}{49-40}}$$

$$=\sqrt{\frac{9\times\left(7+2\sqrt{10}\right)}{9}}$$

$$=\sqrt{7+2\sqrt{10}}$$

$$=\sqrt{7+2\times\sqrt{5}\times\sqrt{2}}$$

$$=\sqrt{5+2+2\times\sqrt{5}\times\sqrt{2}}$$

$$=\sqrt{\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\times\sqrt{5}\times\sqrt{2}}$$

$$=\sqrt{\left(\sqrt{5}+\sqrt{2}\right)^2}=\sqrt{5}+\sqrt{2}$$

Part III

$$=\frac{4}{\sqrt{8+4\sqrt{3}}}=\sqrt{\frac{16}{8+4\sqrt{3}}}$$

$$=\sqrt{\frac{16\times\left(8-4\sqrt{3}\right)}{\left(8+4\sqrt{3}\right)\left(8-4\sqrt{3}\right)}}$$

$$=\sqrt{\frac{16\times\left(8-4\sqrt{3}\right)}{8^2-\left(4\sqrt{3}\right)^2}}$$

$$=\sqrt{\frac{16\times\left(8-4\sqrt{3}\right)}{64-48}}$$

$$=\sqrt{\frac{16\times\left(8-4\sqrt{3}\right)}{16}}$$

$$=\sqrt{8-4\sqrt{3}}$$

$$=\sqrt{8-2\times 2\times\sqrt{3}}$$

$$=\sqrt{8-2\times\sqrt{2\times 2\times 3}}$$

$$=\sqrt{8-2\times\sqrt{6}\times\sqrt{2}}$$

$$=\sqrt{6+2-2\times\sqrt{6}\times\sqrt{2}}$$

$$=\sqrt{\left(\sqrt{6}\right)^2+\left(\sqrt{2}\right)^2-2\times\sqrt{6}\times\sqrt{2}}$$

$$=\sqrt{\left(\sqrt{6}-\sqrt{2}\right)^2}=\sqrt{6}-\sqrt{2}$$

Hence, the given expression
= Part I – Part II – Part III

$$=\left(\sqrt{6}+\sqrt{5}\right)-\left(\sqrt{5}+\sqrt{2}\right)-\left(\sqrt{6}-\sqrt{2}\right)$$

$$=\sqrt{6}+\sqrt{5}-\sqrt{5}-\sqrt{2}-\sqrt{6}+\sqrt{2}=0$$

34. (1) $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}$

$$=\frac{\sqrt{8-2\sqrt{7}}}{\sqrt{16+6\sqrt{7}}-4}$$

[On Multiplying Numerator and Denominator by $\sqrt{2}$]

$$=\frac{\sqrt{7+1-2\times\sqrt{7}\times 1}}{\sqrt{16+2\times 3\times\sqrt{7}}-4}$$

$$=\frac{\sqrt{\left(\sqrt{7}\right)^2+1-2\times\sqrt{7}\times 1}}{\sqrt{9+7+2\times 3\times\sqrt{7}}-4}$$

$$=\frac{\sqrt{\left(\sqrt{7}-1\right)^2}}{\sqrt{\left(3+\sqrt{7}\right)^2}-4}$$

$$=\frac{\sqrt{7}-1}{3+\sqrt{7}-4}=\frac{\sqrt{7}-1}{\sqrt{7}-1}=1$$

35. (2) It will be convenient to solve the given problem part-wise.

Part I $=\left(28+10\sqrt{3}\right)^{\frac{1}{2}}$

$$=\left(28+2\times 5\times\sqrt{3}\right)^{\frac{1}{2}}$$

$$=\left(28+2\times\sqrt{25}\times\sqrt{3}\right)^{\frac{1}{2}}$$

$$=\left(25+3+2\times\sqrt{25}\times\sqrt{3}\right)^{\frac{1}{2}}$$

$$=\left[(5)^2+\left(\sqrt{3}\right)^2+2\times\sqrt{25}\times\sqrt{3}\right]^{\frac{1}{2}}$$

$$=\left[\left(5+\sqrt{3}\right)^2\right]^{\frac{1}{2}}=5+\sqrt{3}$$

Part II

$$=\left(7-4\sqrt{3}\right)^{-\frac{1}{2}}$$

$$=\left(7-2\times 2\times\sqrt{3}\right)^{-\frac{1}{2}}$$

$$=\left(4+3-2\times 2\times\sqrt{3}\right)^{-\frac{1}{2}}$$

$$=\left[(2)^2+\left(\sqrt{3}\right)^2-2\times 2\times\sqrt{3}\right]^{-\frac{1}{2}}$$

$$=\left[\left(2-\sqrt{3}\right)^2\right]^{-\frac{1}{2}}$$

$$=\left(2-\sqrt{3}\right)^{2\times-\frac{1}{2}}$$

$$=\left(2-\sqrt{3}\right)^{-1}=\frac{1}{\left(2-\sqrt{3}\right)}$$

$$=\frac{1\times\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$$

$$=\frac{\left(2+\sqrt{3}\right)}{4-3}=2+\sqrt{3}$$

Hence, our given expression
= Part I – Part II

$$=\left(5+\sqrt{3}\right)-\left(2+\sqrt{3}\right)$$

$$=5+\sqrt{3}-2-\sqrt{3}=3$$

36. (3) Part I = $(28-10\sqrt{3})^{\frac{1}{2}}$

$$=\left(25+3-2\times 5\times\sqrt{3}\right)^{\frac{1}{2}}$$

$$=\left[\left(5-\sqrt{3}\right)^2\right]^{\frac{1}{2}}=5-\sqrt{3}$$

Part II $=(7+4\sqrt{3})^{-\frac{1}{2}}$

$$=\left(\frac{1}{7+4\sqrt{3}}\right)^{\frac{1}{2}}$$

$$=\left(\frac{1}{4+3+2\times 2\times\sqrt{3}}\right)^{\frac{1}{2}}$$

$$=\left[\frac{1}{\left(2+\sqrt{3}\right)^2}\right]^{\frac{1}{2}}=\frac{1}{2+\sqrt{3}}$$

$$=\frac{1\times\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$$

(on Rationalising)

$$=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

Part III

$$=\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$$

$$=\frac{\sqrt{7}\left(\sqrt{16+6\sqrt{7}}+\sqrt{16-6\sqrt{7}}\right)}{\left(\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}\right)\left(\sqrt{16+6\sqrt{7}}+\sqrt{16-6\sqrt{7}}\right)}$$

$$=\frac{\sqrt{7}\left(\sqrt{16+6\sqrt{7}}+\sqrt{16-6\sqrt{7}}\right)}{\left(16+6\sqrt{7}\right)-\left(16-6\sqrt{7}\right)}$$

$$=\frac{\sqrt{7}\left(\sqrt{9+7+2\times 3\times\sqrt{7}}+\sqrt{9+7-2\times 3\times\sqrt{7}}\right)}{16+6\sqrt{7}-16+6\sqrt{7}}$$

$$=\frac{\sqrt{7}\left(\sqrt{\left(3+\sqrt{7}\right)^2}+\sqrt{\left(3-\sqrt{7}\right)^2}\right)}{12\sqrt{7}}$$

$$=\frac{3+\sqrt{7}+3-\sqrt{7}}{12}=\frac{1}{2}$$

Hence the given expression = Part I – Part II + Part III

$$=\left(5-\sqrt{3}\right)-\left(2-\sqrt{3}\right)+\frac{1}{2}$$

$$= 5 - \sqrt{3} - 2 + \sqrt{3} + \frac{1}{2}$$

$$= 5 - 2 + \frac{1}{2} = 3 + \frac{1}{2} = 3\frac{1}{2}$$

37. (4) Part I

$$= \frac{26 - 15\sqrt{3}}{\left[5\sqrt{2} - \sqrt{38 + 5\sqrt{3}}\right]^2}$$

$$= \frac{26 - 15\sqrt{3}}{\left(5\sqrt{2}\right)^2 + \left(\sqrt{38 + 5\sqrt{3}}\right)^2 - 2 \times 5\sqrt{2} \times \sqrt{38 + 5\sqrt{3}}}$$

[$\because (a - b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{26 - 15\sqrt{3}}{50 + 38 + 5\sqrt{3} - 10\sqrt{76 + 10\sqrt{3}}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{76 + 10\sqrt{3}}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{75 + 1 + 2 \times 5\sqrt{3} \times 1}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\sqrt{\left(5\sqrt{3} + 1\right)^2}}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 10\left(5\sqrt{3} + 1\right)}$$

$$= \frac{26 - 15\sqrt{3}}{88 + 5\sqrt{3} - 50\sqrt{3} - 10}$$

$$= \frac{26 - 15\sqrt{3}}{78 - 45\sqrt{3}}$$

$$= \frac{26 - 15\sqrt{3}}{3\left(26 - 15\sqrt{3}\right)} = \frac{1}{3}$$

Part II

$$= \frac{\sqrt{10} + \sqrt{18}}{\sqrt{8} + \sqrt{\left(\sqrt{3} - \sqrt{5}\right)}}$$

[Rationalising the denominator]

$$= \frac{\left(\sqrt{10} + \sqrt{18}\right)\left(\sqrt{8} - \sqrt{3 - \sqrt{5}}\right)}{\left(\sqrt{8} + \sqrt{3 - \sqrt{5}}\right)\left(\sqrt{8} - \sqrt{3 - \sqrt{5}}\right)}$$

$$= \frac{\left(\sqrt{10} + \sqrt{18}\right)\left(\sqrt{8} - \sqrt{3 - \sqrt{5}}\right)}{\left(\sqrt{8}\right)^2 - \left(\sqrt{3 - \sqrt{5}}\right)^2}$$

$$= \frac{\sqrt{80} + \sqrt{8 \times 18} - \sqrt{30 - 10\sqrt{5}} - \sqrt{54 - 18\sqrt{5}}}{8 - 3 + \sqrt{5}}$$

$$= \frac{\sqrt{16 \times 5} + \sqrt{12 \times 12} - \sqrt{25 + 5 - 2 \times 5 \times \sqrt{5}} - \sqrt{9\left(6 - 2\sqrt{5}\right)}}{5 + \sqrt{5}}$$

$$= \frac{4\sqrt{5} + 12 - \sqrt{(5)^2 + \left(\sqrt{5}\right)^2 - 2 \times 5 \times \sqrt{5}} - 3\sqrt{5 + 1 - 2 \times \sqrt{5} \times 1}}{5 + \sqrt{5}}$$

$$= \frac{4\sqrt{5} + 12 - \sqrt{\left(5 - \sqrt{5}\right)^2} - 3\sqrt{\left(\sqrt{5} - 1\right)^2}}{5 + \sqrt{5}}$$

$$= \frac{4\sqrt{5} + 12 - \left(5 - \sqrt{5}\right) - 3\left(\sqrt{5} - 1\right)}{5 + \sqrt{5}}$$

$$= \frac{4\sqrt{5} + 12 - 5 + \sqrt{5} - 3\sqrt{5} + 3}{5 + \sqrt{5}}$$

$$= \frac{10 + 2\sqrt{5}}{5 + \sqrt{5}} = \frac{2\left(5 + \sqrt{5}\right)}{\left(5 + \sqrt{5}\right)} = 2$$

Hence the given expression = Part I + Part II

$$= \frac{1}{3} + 2 = 2\frac{1}{3}$$

38. (1) The given expression

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{(2)^2 + \left(\sqrt{3}\right)^2 + 2 \times 2 \times \sqrt{3}}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{\left(2 + \sqrt{3}\right)^2}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\left(2 + \sqrt{3}\right)}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 16 + 8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{16 + 3 + 2 \times 4 \times \sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{(4)^2 + \left(\sqrt{3}\right)^2 + 2 \times 4 \times \sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{\left(4 + \sqrt{3}\right)^2}}$$

$$= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = \sqrt{4} = 2$$

39. (1)

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left\{\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{16}{81}\right)^{\frac{3}{4}} \times \left\{\left(\frac{9}{25}\right)^{\frac{3}{2}} \div \left(\frac{2}{5}\right)^{3}\right\}$$

$$= \left\{\left(\frac{2}{3}\right)^{4}\right\}^{\frac{3}{4}} \times \left\{\left(\frac{3}{5}\right)^{2 \times \frac{3}{2}} \div \left(\frac{2}{5}\right)^{3}\right\}$$

$$= \left(\frac{2}{3}\right)^{4 \times \frac{3}{4}} \times \left\{\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right\}$$

$$=\left(\frac{2}{3}\right)^3\times\left(\frac{3^3}{5^3}\times\frac{5^3}{2^3}\right)$$

$$=\frac{2^3}{3^3}\times\frac{3^3}{2^3}=1$$

40. (2)

$$\left(\frac{1}{4}\right)^{-2}-3\,(8)^{\frac{2}{3}}(4)^0+\left(\frac{9}{16}\right)^{\frac{-1}{2}}$$

$$=\left[\left(\frac{1}{2}\right)^2\right]^{-2}-3\left[(2)^3\right]^{\frac{2}{3}}\times 1+$$

$$\left[\left(\frac{3}{4}\right)^2\right]^{\frac{-1}{2}}$$

$$=\left(\frac{1}{2}\right)^{-4}-3\times 2^2+\left(\frac{3}{4}\right)^{-1}$$

$$=16-12+\frac{4}{3}$$

$$=\frac{48-36+4}{3}=\frac{16}{3}=5\frac{1}{3}$$

41. (3) $\dfrac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$

$$=\frac{\left(5^2\right)^{\frac{3}{2}}\times\left(3^5\right)^{\frac{3}{5}}}{\left(2^4\right)^{\frac{5}{4}}\times\left(2^3\right)^{\frac{4}{3}}}$$

$$=\frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}}=\frac{5^3\times 3^3}{2^5\times 2^4}$$

$$=\frac{125\times 27}{32\times 16}=\frac{3375}{512}$$

42. (4) $\dfrac{16\times 2^{n+1}-4\times 2^n}{16\times 2^{n+2}-2\times 2^{n+2}}$

$$=\frac{2^4\times 2^{n+1}-2^2\times 2^n}{2^4\times 2^{n+2}-2\times 2^{n+2}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2\times 2^{n+5}-2\times 2^{n+2}}$$

$$=\frac{2^{n+5}-2^{n+2}}{2\left(2^{n+5}-2^{n+2}\right)}=\frac{1}{2}$$

43. (3) $\dfrac{\left(x^{a+b}\right)^2\left(x^{b+c}\right)^2\left(x^{c+a}\right)^2}{\left(x^a x^b x^c\right)^4}$

$$=\frac{x^{2(a+b)}.x^{2(b+c)}.x^{2(c+a)}}{(x^a)^4(x^b)^4(x^c)^4}$$

$$=\frac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}}$$

$$=\frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}$$

$$=\frac{x^{4a+4b+4c}}{x^{4a+4b+4c}}=1$$

44. (2) $25^{x-1}=5^{2x-1}-100$

$\Rightarrow (5^2)^{x-1}=5^{2x-1}-100$

$\Rightarrow 5^{2x-2}-5^{2x-1}=-100$

$\Rightarrow 5^{2x-2}-5^{2x-2}\times 5^1=-100$

$\Rightarrow 5^{2x-2}(1-5)=-100$

$\Rightarrow 5^{2x-2}\times -4=-100$

$\Rightarrow 5^{2x-2}=25$

$\Rightarrow 5^{2x-2}=5^2$

On equating the indices, we have,

$\Rightarrow 2x-2=2$

$\Rightarrow 2x=2+2$

$\Rightarrow 2x=4$

$\therefore x=2$

45. (3) $\dfrac{9^n\times 3^2\times\left(3^{\frac{-n}{2}}\right)^{-2}-(27)^n}{3^{3m}\times 2^3}$

$$=\frac{1}{27}$$

$$\Rightarrow\frac{\left(3^2\right)^n\times 3^2\times 3^{-\frac{n}{2}\times -2}-\left(3^3\right)^n}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow\frac{3^{2n}\times 3^2\times 3^n-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow\frac{3^{2n+2+n}-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow\frac{3^{3n+2}-3^{3n}}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow\frac{3^{3n}\left(3^2-1\right)}{3^{3m}\times 2^3}=\frac{1}{27}$$

$$\Rightarrow\frac{3^{3n}\times 8}{3^{3m}\times 8}=\frac{1}{27}$$

$\Rightarrow 3^{3n-3m}=\dfrac{1}{3^3}$

$\Rightarrow 3^{3n-3m}=3^{-3}$

(On equating the exponents)

$\Rightarrow 3n-3m=-3$

$\Rightarrow n-m=-1$

$\Rightarrow m-n=1$

46. (4) $\dfrac{a^{-1}}{a^{-1}+b^{-1}}+\dfrac{a^{-1}}{a^{-1}-b^{-1}}$

$$=\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}}+\frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}}$$

$$=\frac{\frac{1}{a}}{\frac{b+a}{ab}}+\frac{\frac{1}{a}}{\frac{b-a}{ab}}$$

$$=\frac{1}{a}.\frac{ab}{b+a}+\frac{1}{a}.\frac{ab}{b-a}$$

$$=\frac{b}{b+a}+\frac{b}{b-a}$$

$$=\frac{b\,(b-a)+b\,(b+a)}{(b+a)\,(b-a)}$$

$$=\frac{b^2-ba+b^2+ab}{b^2-a^2}$$

$$=\frac{2b^2}{b^2-a^2}$$

47. (1) $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\left(\frac{x^c}{x^a}\right)^{\frac{1}{ac}}$

$= \left(x^{a-b}\right)^{\frac{1}{ab}}.\left(x^{b-c}\right)^{\frac{1}{bc}}.\left(x^{c-a}\right)^{\frac{1}{ac}}$

$= x^{(a-b)/ab}.x^{(b-c)/bc}.x^{(c-a)/ac}$

$= x^{\frac{1}{b}-\frac{1}{a}}.x^{\frac{1}{c}-\frac{1}{b}}.x^{\frac{1}{a}-\frac{1}{c}}$

$= x^{\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}} = x^0 = 1$

48. (4) $\left(\frac{x^a}{x^b}\right)^{a+b}\left(\frac{x^b}{x^c}\right)^{b+c}\left(\frac{x^c}{x^a}\right)^{c+a}$

$= \left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a}$

$= x^{(a-b)(a+b)}.x^{(b-c)(b+c)}.x^{(c-a)(c+a)}$

$= x^{a^2-b^2}.x^{b^2-c^2}.x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

49. (3)

$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2}\left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2}$
$\left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$

$= \left(x^{a-b}\right)^{a^2+ab+b^2}.\left(x^{b-c}\right)^{b^2+bc+c^2}$
$\left(x^{c-a}\right)^{c^2+ca+a^2}$

$= x^{(a-b)\left(a^2+ab+b^2\right)}.x^{(b-c)\left(b^2+bc+c^2\right)}$
$.x^{(c-a)\left(c^2+ca+a^2\right)}$

$= x^{a^3-b^3}.x^{b^3-c^3}.x^{c^3-a^3}$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1$

50. (1) The given expression

$= \sqrt{x^{-1}y}.\sqrt{y^{-1}z}.\sqrt{z^{-1}x}$

$= \sqrt{\frac{y}{x}}.\sqrt{\frac{z}{y}}.\sqrt{\frac{x}{z}}$

$= \left(\frac{y}{x}\right)^{\frac{1}{2}}.\left(\frac{z}{y}\right)^{\frac{1}{2}}.\left(\frac{x}{z}\right)^{\frac{1}{2}}$

$= \left(\frac{y}{x}\times\frac{z}{y}\times\frac{x}{z}\right)^{\frac{1}{2}} = 1$

51. (3) $\frac{4\sqrt{3}}{2-\sqrt{2}} = \frac{4\sqrt{3}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$

$= \frac{4\sqrt{3}(2+\sqrt{2})}{4-2}$

$= 2\sqrt{3}(2+\sqrt{2}) = 4\sqrt{3}+2\sqrt{6}$

$\frac{30}{4\sqrt{3}-3\sqrt{2}}$

$= \frac{30(4\sqrt{3}+3\sqrt{2})}{(4\sqrt{3}-3\sqrt{2})(4\sqrt{3}+\sqrt{2})}$

$= \frac{30(4\sqrt{3}+3\sqrt{2})}{48-18}$

$= 4\sqrt{3}+3\sqrt{2}$

$\frac{3\sqrt{2}}{3-2\sqrt{3}} = \frac{3\sqrt{2}(3+2\sqrt{3})}{(3-2\sqrt{3})(3+2\sqrt{3})}$

$= \frac{3\sqrt{2}(3+2\sqrt{3})}{9-12}$

$= -3\sqrt{2}-2\sqrt{6}$

$\therefore$ Expression

$= 4\sqrt{3}+2\sqrt{6}-4\sqrt{3}-3\sqrt{2}+3\sqrt{2}$
$+ 2\sqrt{6}$

$= 4\sqrt{6}$

52. (3) $(10^{12}+25)^2-(10^2-25)^2 = 10^n$

$\because (a+b)^2-(a-b)^2 = 4ab$

$\therefore 4 \times 10^{12} \times 25 = 10^n$

$\Rightarrow 10^{14} = 10^n$

$\Rightarrow n = 14$

53. (2) $\sqrt{10} + \sqrt{20} + \sqrt{40} - \sqrt{5}$
$- \sqrt{80}$

$= \sqrt{10} + \sqrt{4\times5} + \sqrt{4\times10} -$
$\sqrt{5} - \sqrt{16\times5}$

$= \sqrt{10} + 2\sqrt{5} + 2\sqrt{10} - \sqrt{5} -$
$4\sqrt{5}$

$= 3\sqrt{10} + \sqrt{5} - 4\sqrt{5}$

$= 3\sqrt{10} - 3\sqrt{5} = 3\left(\sqrt{10}-\sqrt{5}\right)$

$\therefore$ Expression $= \frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)}$

$= \frac{5}{\sqrt{10}-\sqrt{5}}$

$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{\left(\sqrt{10}-\sqrt{5}\right)\left(\sqrt{10}+\sqrt{5}\right)}$

Rationalising the denominator

$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{10-5}$

$= \sqrt{10} + \sqrt{5} = 3.162 + 2.236$
$= 5.398$

54. (3) Let $\sqrt{28-10\sqrt{3}} = \sqrt{x}-\sqrt{y}$

$\Rightarrow 28-10\sqrt{3} = x+y-2\sqrt{xy}$

$\Rightarrow x+y = 28,\ xy = 75$

$\therefore (x-y)^2 = (x+y)^2-4xy$

$= 784-300=484$

$\Rightarrow x-y = 22$

$\therefore x = 25,\ y = 3$

$\Rightarrow 28-10\sqrt{3} = \sqrt{25}-\sqrt{3}$

Again, let $\sqrt{7+4\sqrt{3}} = \sqrt{x}+\sqrt{y}$

$\Rightarrow 7+4\sqrt{3} = x+y+2\sqrt{xy}$

$\Rightarrow x+y = 7,\ xy=12$

$\therefore x-y = (7)^2-4\times12=1$

$\Rightarrow x = 4,\ y = 3$

$\therefore \sqrt{7+4\sqrt{3}} = \sqrt{4}+\sqrt{3}$

Now,

$\left(28-10\sqrt{3}\right)^{\frac{1}{2}}-\left(7+4\sqrt{3}\right)^{-\frac{1}{2}}$

$= \sqrt{25}-\sqrt{3}-\frac{1}{\sqrt{4}+\sqrt{3}}$

$= \sqrt{25}-\sqrt{3}-\frac{\sqrt{4}-\sqrt{3}}{1}$

$= \sqrt{25}-\sqrt{4} = 5-2 = 3$

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