17 ALGEBRA

Importance : Algebra based 2-3 questions are essentially asked in almost all competitive exams obviously this chapter should be given sufficient time and practice done.

Scope of questions : Questions based on different algebraic expressions, equations (e.g. quadratic or higher order, square root, cube root and inverse) or based on graphic representation of equations and the value of a variable is asked or an equation is required to be validated.

Way to success : Solution of questions of this chapter can be ensured by memorising the concerved formulae/rules and by regular practice.

Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

General Form : $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a polynomial in variable x, where $a_0, a_1, a_2, a_3 ... a_n$ are real numbers and n is non-negative integer.

Remainder Theorem : Let $f(x)$ be a polynomial of degree $n \geq 1$, and let a be any real number. When $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

Proof : Suppose that when $f(x)$ is divided by $(x - a)$, the quotient is $g(x)$ and the remainder is $r(x)$.

Then, degree $r(x)$ < degree $(x - a)$

$\Rightarrow$ degree $r(x) < 1$

$\Rightarrow$ degree $r(x) = 0$ [$\because$ degree of $(x - a) = 1$]

$\Rightarrow r(x)$ is constant, equal to r (say).

Thus, when $f(x)$ is divided by $(x - a)$, then the quotient is $g(x)$ and the remainder is r.

$\therefore f(x) = (x - a) . g(x) + r$ ... (i)

Putting $x = a$ in (i), we get $r = f(a)$.

Thus, when $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

Remarks

(i) If a polynomial $p(x)$ is divided by $(x + a)$, the remainder is the value of $p(x)$ at $x = -a$ i.e. $p(-a)$

$[\because x + a = 0 \Rightarrow x = -a]$

(ii) If a polynomial $p(x)$ is divided by $(ax - b)$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$.

$[\because ax - b = 0 \Rightarrow x = \frac{b}{a}]$

(iii) If a polynomial $p(x)$ is divided by $(ax + b)$, then remainder is the value of $p(x)$ at $x = -\frac{b}{a}$ i.e. $p\left(-\frac{b}{a}\right)$

$[\because ax + b = 0 \Rightarrow x = -\frac{b}{a}]$

(iv) If a polynomial $p(x)$ is divided by $b - ax$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$

$[\because b - ax = 0 \Rightarrow x = \frac{b}{a}]$

Factor Theorem

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$,

then $p(a) = 0$

$\Rightarrow p(x)$, when divided by $(x - a)$ gives remainder zero.

But by Remainder theorem,

$p(x)$ when divided by $(x - a)$ gives the remainder equal to $p(a)$.

$\therefore p(a) = 0$

Remarks

(i) $(x + a)$ is a factor of a polynomial iff (if and only if) $p(-a) = 0$

(ii) $(ax - b)$ is a factor of a polynomial if $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax + b)$ is a factor of a polynomial $p(x)$ if $p\left(-\frac{b}{a}\right) = 0$

(iv) $(x - a)(x - b)$ are factors of a polynomial $p(x)$ if $p(a) = 0$ and $p(b) = 0$

ALGEBRAIC IDENTITIES

An algebraic identity is an algebraic equation which is true for all values of the variable (s).

IMPORTANT FORMULAE

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 = (a - b)^2 + 4ab$
4. $(a - b)^2 = (a + b)^2 - 4ab$
5. $a^2 - b^2 = (a + b)(a - b)$
6. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
7. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
8. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
9. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
10. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
11. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

12. $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ac)$

$= (a + b + c)\,\frac{1}{2}\,(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac)$

$= \frac{1}{2}\,(a + b + c)\,[(a - b)^2 + (b - c)^2 + (c - a)^2]$

13. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

14. $(a+b+c)^3 = a^3 + b^3 + c^3 + 3\,(b+c)\,(c+a)\,(a+b)$

15. $a^2 + b^2 = (a + b)^2 - 2ab$

16. $a^2 + b^2 = (a - b)^2 + 2ab$

17. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

18. $a^4 + b^4 + a^2b^2 = (a^2 - ab + b^2)\,(a^2 + ab + b^2)$

GRAPHIC REPRESENTATION OF STRAIGHT LINES

Ordered Pair : A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b).

Note that $(a, b) \neq (b, a)$.

Thus, (2, 3) is one ordered pair and (3, 2) is another ordered pair.

CO-ORDINATE SYSTEM

Co-ordinate Axes : The position of a point in a plane is determined with reference to two fixed mutually perpendicular lines, called the coordinate axes.

Let us draw two lines X'OX and YOY', which are perpendicular to each other and intersect at the point O. These lines are called the coordinate axes or the axes of reference.

The horizontal line X'OX is called the x-axis.

The vertical line YOY' is called the y-axis.

The point O is called the origin.

The distance of a point from y-axis is called its x-co-ordinate or abscissa and the distance of the point from x-axis is called its y-co ordinate or ordinate.

If x and y, denote respectively the abscissa and ordinate of a point P, then (x, y) are called the coordinates of the point P.

The y–co-ordinate of every point on x-axis is zero. i.e. when a straight line intersects at x-axis, its y-co-ordinate is zero. So, the co-ordinates of any point on the x-axis are of the form (x, 0).

The x-co-ordinate of every point on y-axis is zero. So, the co-ordinates of any point on y-axis are of the form (0, y).

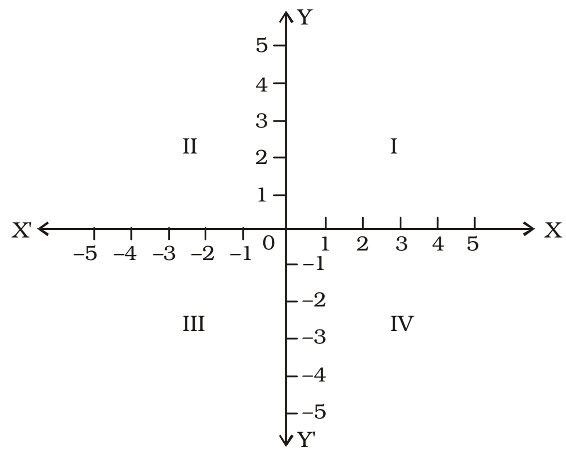
The co-ordinates of the origin are (0, 0).

y = a where a is constant denotes a straight line parallel to x-axis.

x = a where a is constant, denotes a straight line parallel to y-axis.

x = 0 denotes y-axis.

y = 0 denotes x-axis.



We can fix a convenient unit of length and taking the origin as zero, mark equal distances on the x-axis as well as on the y-axis.

Convention of Signs : The distances measured along OX and OY are taken as positive and those along OX' and OY' are taken as negative, as shown in the figure given above.

CO-ORDINATES OF A POINT IN A PLANE

Let P be a point in a plane.

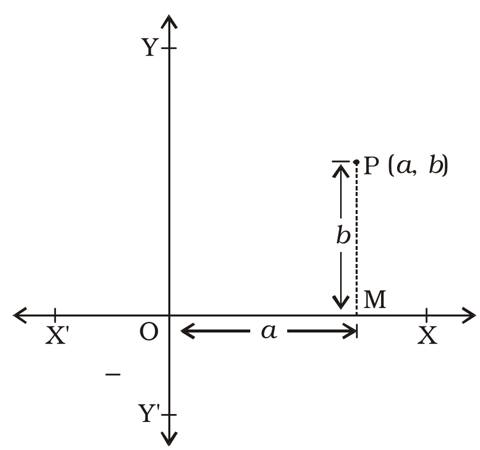
Let the distance of P from the y-axis = a units.

And, the distance of P from the x-axis = b units.

Then, we say that the co-ordinates of P are (a, b).

a is called the x-co-ordinate, or abscissa of P.

b is called the y co-ordinate, or ordinate of P.

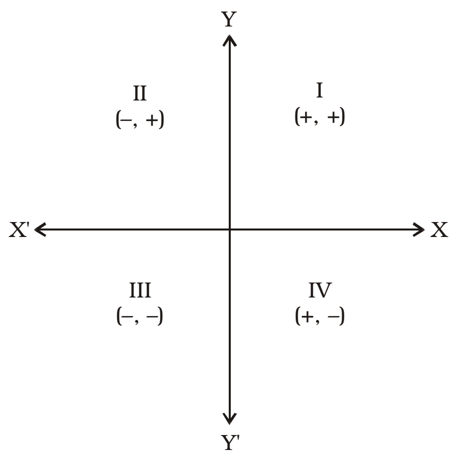


Quadrants : Let X' OX and YOY' be the co-ordinate axes.

These axes divide the plane of the paper into four regions, called quadrants. The regions XOY, YOX',X'OY' and Y'OX are respectively known as the first, second, third and fourth quadrants.

Using the convention of signs, we have the signs of the coordinates in various quadrants as given below.

Region	Quadrant	Nature of x and y	Signs of co-ordinates
XOY	I	$x > 0,\ y > 0$	(+, +)
YOX'	II	$x < 0,\ y > 0$	(–, +)
X'OY'	III	$x < 0,\ y < 0$	(–, –)
Y'OX	IV	$x > 0,\ y < 0$	(+, –)



Note : Any point lying on x-axis or y-axis does not lie in any quadrant.

Consistency and Inconsistency

A system of a pair of linear equations in two variables is said to be consistent if it has at least one solution.

A system of a pair of linear equations in two variables is said to be inconsistent if it has no solution.

The system of a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has :

(i) a unique solution (i.e. consistent) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. The graph of the linear equations intersect at only one point.

(ii) no solution (i.e. inconsistent) if $\frac{a_1}{b_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

The graph of the two linear equations are parallel to each other i.e. the lines do not intersect.

(iii) an infinite number of solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The graph of the linear equations are coincident.

Homogeneous equation of the form $ax + by = 0$ is a line passing through the origin. Therefore, this system is always consistent.

Rule 1. $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow$ $a^2 + b^2 = (a + b)^2 - 2ab$

$(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow$ $a^2 + b^2 = (a - b)^2 + 2ab$

Rule 2. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

Rule 3. $(a + b)^2 - (a - b)^2 = 4ab$

or, $(a + b)^2 = (a - b)^2 + 4ab$

or, $(a - b)^2 = (a + b)^2 - 4ab$

Rule 4. $(a^2 - b^2) = (a + b)(a - b)$

Rule 5. $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$ or, $\left(a - \frac{1}{a}\right)^2 + 2$

Rule 6. $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$

Rule 7. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

or, $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

Rule 8. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

or, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

Rule 9. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

or, $a^3 - b^3 = (a - b)^3 - 3ab(a - b)$.

Rule 10. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Rule 11. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Rule 12. $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$

Rule 13. $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3\left(a - \frac{1}{a}\right)$

Rule 14. If $a + \frac{1}{a} = 2$ then $a^n + \frac{1}{a^n} = 2$.

Rule 15. If $a + \frac{1}{a} = 2$ then, $a^n - \frac{1}{a^n} = 0$

(By putting a = 1)

Rule 16. If $a + \frac{1}{a} = 2$ then $a^m + \frac{1}{a^n} = 2$

(By putting a = 1), and $m \neq n$.

Rule 17. If $a + \frac{1}{a} = 2$ then $a^m - \frac{1}{a^n} = 0$

(By putting a = 1)

Rule 18. If $a + \frac{1}{a} = -2$, then $a^n + \frac{1}{a^n} = 2$ If n is even and $a^n + \frac{1}{a^n} = -2$, if n is odd.

(By putting a = –1)

Rule 19. If $a + \frac{1}{a} = -2$ then the value of $a^m \pm \frac{1}{a^n} = (-1)^m \pm \frac{1}{(-1)^n}$

Rule 20. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ or, $\frac{1}{2}(a + b + c)$ $[(a - b)^2 + (b - c)^2 + (c - a)^2]$

Rule 21. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Rule 22. If $a^3 + b^3 + c^3 = 3abc$, then $a + b + c = 0$ or $a = b = c$.

Proof $\because$ $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow$ $a^3 + b^3 + c^3 - 3abc = 0$

Now, $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$\Rightarrow$ $0 = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$\therefore$ Either $a + b + c = 0$ or, $(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$, i.e., $a - b = 0$

$\Rightarrow$ $a = b$, $b - c = 0$

$\Rightarrow$ $b = c$, $c - a = 0$

$\Rightarrow$ $c = a$

$\therefore$ $a = b = c$

Rule 23. If $a^2 + b^2 + c^2 = ab + bc + ca$, then $a = b = c$.

Rule 24. Componendo and Dividendo Rule, If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Rule 25. If $\frac{a+b}{a-b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{c+d}{c-d}$.

Rule 26. If $\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}$ where $x = n(n + 1)$

then $\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}} = (n+1)$

Rule 27. If $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}}$ where $x = n(n + 1)$ then,

$\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}} = n$.

Rule 28. $(a + b + c)^3 = a^3 + b^3 + c^3 - 3(a + b)(b + c)(c + a)$

Rule 29. $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

Rule 30. If $a + \frac{1}{a} = x$, then $a^3 + \frac{1}{a^3} = x^3 - 3x$.

Rule 31. If $a - \frac{1}{a} = x$, then $a^3 - \frac{1}{a^3} = x^3 + 3x$.

Rule 32. Binomial theorem :

$(a + b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_{n-1} a^1 b^{n-1} + {}^nC_n a^0 b^n$, where, n is a positive number and ${}^nC_r = \frac{n!}{r!(n-r)!}$

Permutation and Combination

Permutation : It is used where we have to arrange things. Out of total n things, *r* things (taken at a time) can be arranged as ${}^np_r$ or $P(n,r)$

$P(n,r) = {}^nP_r = \frac{n!}{(n-r)!}$ where $n \geq r$

Combination : It is used where we have to select things. It is written as ${}^nC_r$ or $C(n,r)$

$C(n,r) = \frac{n!}{(n-r)!\,r!}$ $n \geq r$

Some important results.

${}^nP_o = 1$; ${}^nP_n = n!$

${}^nC_o = {}^nC_n = 1$; ${}^nC_r = {}^nC_{n-r} = {}^nC_1 = {}^nC_{n-1} = n$.

Ex. ${}^7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7.6.5.4!}{4!} = 210$

${}^5C_2 = \frac{5!}{(5-2)!\,2!} = \frac{5.4.3!}{3! \times 2 \times 1} = 10$

n! (is called as n factorial)

5! = 5.4!

= 5.4.3!

= 5.4.3.2!

= 5.4.3.2.1!

$\boxed{5! = 120}$

Also $\boxed{0! = 1}$

COORDINATE GEOMETRY

Importance : Coordinate geometry is separate and important filled in mathematics but very rarely asked in competitive exams. However in two-dimensional (2–D) geometry introductory/easy questions should be practised for improving marks.

Scope of questions : Mostly questions are related to distance between two points, linear/non-linear these coplaner points, cutting a line a specific ratio by a given point.

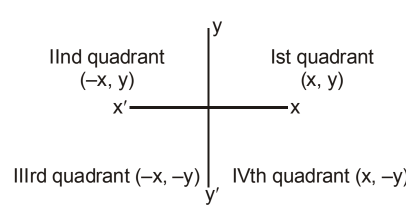
Way to success : The concept of coordinate geometry and practice of above mentioned questions is very important to solve questions.

Important Points :

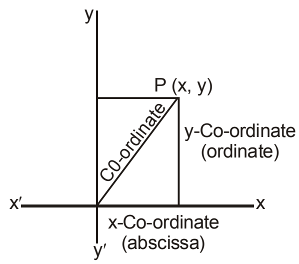
x–coordinate is called the abscissa of P, where (x, y) are co-ordinates of any point P.

y–co-ordinate is called the ordinate of P, where (x, y) are co–ordinates of any point P.

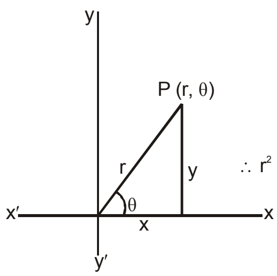
Quadrants :



Cartesian Co–ordinate System :



Polar Coordinate System :



$\therefore r^2 = x^2 + y^2$

RULE 1 : The distance between any two points in the plane is the length of the line segment joining them. The distance between two points P (x_1, y_1) and Q (x_2, y_2) is

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \text{ or,}$$

$$PQ = \sqrt{(\text{difference of abscissa})^2 + (\text{difference of ordinates})^2}$$

RULE 2 : The area of a triangle, the Co-ordinates of whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\text{Area } \Delta = \left(\frac{1}{2}\right) | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

$$= \left(\frac{1}{2}\right) \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If all three points are collinear,
then area of $\Delta = 0$

RULE 3 : The Co-ordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m : n are given by

$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$

RULE 4 : If P is the mid-point of AB, such that it divides AB in the ratio 1 : 1, then its Co-ordinates are (x,y) = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ also called mid point formula.

RULE 5 : The Co–ordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) externally in the ratio m : n, are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

RULE 6 : The Co-ordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

RULE 7 : The Co–ordinates of the in–centre of a triangle whose vertices are A (x_1, y_1), B(x_2, y_2), C(x_3, y_3) are given by

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right) \text{where a = BC,}$$

b = CA and c = AB.

Equation of straight line.

A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

RULE 8 : If (x_1, y_1) and (x_2, y_2) are the Co-ordinates of any two points on a line, then its slope is

$$(\tan\theta) = m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{\text{difference of ordinates}}{\text{difference of abscissa}}$$

RULE 9 : The angle θ between the lines having slopes m_1 and m_2 is given by $\tan\theta = \pm\frac{m_2 - m_1}{1 + m_1 m_2}$

RULE 10 : If two lines having slopes m_1 and m_2 are (i) parallel if $m_1 = m_2$ (ii) Perpendicular if $m_1 \times m_2 = -1$

RULE 11 : (Slope–Intercept) The equation of a line with slope m and making an intercept c on y-axis is y = mx + c.

RULE 12 : (Point-Slope form) The equation of a line which passes through the point (x_1, y_1) and has the slope 'm' is $(y - y_1) = m(x - x_1)$

RULE 13 : (Two-point form) The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_2}$$

RULE 14 : (Intercept form) The equation of a line which cuts off intercepts a and b respectively on the x and y–axes is

$$\frac{x}{a} + \frac{y}{b} = 1$$

RULE 15 : (i) The slope of a line whose general quation is given by Ax + By + C = 0 is $\frac{-A}{B}$

(ii) The intercepts of a line on x and y axes respectively whose general equation is Ax + By + C = 0 is given by :-

x-intercept = $\frac{-C}{A}$ and y-intercept = $\frac{-C}{B}$

RULE 16 : General equation of straight line is ax+ by + c = 0

$\therefore$ Now the area of the triangle made by the given straight line and its intercepts is

$$\Delta = \frac{1}{2} \times \left(\frac{-c}{a}\right) \times \left(\frac{-c}{b}\right) \text{ sq. units}$$

❑❑❑

QUESTIONS ASKED IN PREVIOUS SSC EXAMS

TYPE–I

1. If a * b = 2a – 3b + ab, then 3 * 5 + 5 * 3 is equal to :
(1) 22 (2) 24
(3) 26 (4) 28
(SSC CGL Prelim Exam. 04.07.1999 (First Sitting)

2. If p × q = p + q + $\frac{p}{q}$, the value of 8 × 2 is :
(1) 6 (2) 10
(3) 14 (4) 16
(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

3. Two numbers x and y (x > y) are such that their sum is equal to three times their difference. Then value of $\frac{3xy}{2\left(x^2-y^2\right)}$ will be:
(1) $\frac{2}{3}$ (2) 1
(3) $1\frac{1}{2}$ (4) $1\frac{2}{3}$
(SSC CGL Prelim Exam. 04.07.1999 (Second Sitting)

4. The value of
$$\left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)$$
is :
(1) $1+\frac{1}{x+4}$ (2) x+4
(3) $\frac{1}{x}$ (4) $\frac{x+4}{x}$
(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

5. If a * b = 2 (a + b), then 5 * 2 is equal to :
(1) 3 (2) 10
(3) 14 (4) 20
(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

6. If $\frac{2a+b}{a+4b}=3$, then find the value of $\frac{a+b}{a+2b}$
(1) $\frac{5}{9}$ (2) $\frac{2}{7}$
(3) $\frac{10}{9}$ (4) $\frac{10}{7}$
(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

7. If $x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$, then the value of $5x^2 - 5x - 1$ is
(1) 0 (2) 3
(3) 4 (4) 5
(SSC CGL Tier-1 Exam 26.06.2011 (Second Sitting)

8. If a * b = a + b + ab, then 3 * 4 – 2 * 3 is equal to :
(1) 6 (2) 8
(3) 10 (4) 12
(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

9. If $x = 7-4\sqrt{3}$, then the value of $\left(x+\frac{1}{x}\right)$ is :
(1) $3\sqrt{3}$ (2) $8\sqrt{3}$
(3) $14+8\sqrt{3}$ (4) 14
(SSC CGL Prelim Exam. 27.02.2000 (First Sitting)

10. If $x \quad y = 3x + 2y$, Then 2   3 + 3   4 is equal to
(1) 18 (2) 29
(3) 32 (4) 38
(SSC CGL Prelim Exam. 24.02.2002 (Middle Zone) & (SSC CGL Prelim Exam. 13.11.2005 (Ist Sitting)

11. If $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}$ then $\frac{a+b+c}{c}$ is equal to
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CPO S.I.Exam.12.01.2003

12. If $\frac{144}{0.144}=\frac{14.4}{x}$, then the value of x is
(1) 144 (2) 14.4
(3) 1.44 (4) 0.0144
(SSC CPO S.I.Exam.12.01.2003

13. If 1 < x < 2, then the value of $\sqrt{(x-1)^2}+\sqrt{(x-3)^2}$ is
(1) 1 (2) 2
(3) 3 (4) 2x–4
(SSC CPO S.I.Exam.12.01.2003

14. If a ⊗ b = (a × b) + b, then 5 ⊗ 7 equals to
(1) 12 (2) 35
(3) 42 (4) 50
(SSC CPO S.I.Exam.12.01.2003

15. Given that $10^{0.48} = x$, $10^{0.70} = y$, and $x^z = y^2$, then the value of z is close to
(1) 1.45 (2) 1.88
(3) 2.9 (4) 3.7
(SSC CPO S.I.Exam.12.01.2003)

16. If 47.2506 $= 4A+\frac{7}{B}+2C+\frac{5}{D}+6E$, then the value of 5A + 3B + 6C + D + 3E is
(1) 53.6003 (2) 53.603
(3) 153.6003 (4) 213.0003
(SSC CGL Prelim Exam.11.05.2003 (First Sitting)

17. If $x * y = x^2 + y^2 - xy$, then the value of 9 * 11 is
(1) 93 (2) 103
(3) 113 (4) 121
(SSC CGL Prelim Exam. 11.05.2003 (Second Sitting)

18. If $\frac{2p}{p^2-2p+1}=\frac{1}{4}, p \neq 0$, then the value of $p+\frac{1}{p}$ is
(1) 4 (2) 5
(3) 10 (4) 12
FCI Assistant Grade-III Exam.25.02.2012 (Paper-I) North Zone (Ist Sitting)

19. If $5^{5x+5} = 1$, then x equals
(1) 0 (2) – 1
(3) 1 (4) $-\frac{4}{5}$
(SSC CPO S.I. Exam. 07.09.2003)

20. If $3^{x+3} + 7 = 250$, then x is equal to
(1) 5 (2) 3
(3) 2 (4) 1
(SSC CPO S.I.Exam.07.09.2003)

21. If $\frac{1}{4}\times\frac{2}{6}\times\frac{3}{8}\times\frac{4}{10}\times\frac{5}{12}\times$ $\ldots\ldots\times\frac{31}{64}=\frac{1}{2^x}$, the value of x is

(1) 31 (2) 32
(3) 36 (4) 37

(SSC Section Officer (Commercial Audit) Exam.16.11.2003)

22. The value of

$\frac{(243)^{\frac{n}{5}}.3^{2n+1}}{9^n.3^{n-1}}$ is

(1) 1 (2) 9
(3) 3 (4) 3^n

(SSC CGL Prelim Exam. 08.02.2004 (First Sitting)

23. If x = 0.5 and y = 0.2, then value of $\sqrt{0.6}\times(3y)^x$ is equal to

(1) 1.0 (2) 0.5
(3) 0.6 (4) 1.1

(SSC CGL Prelim Exam. 08.02.2004 (Second Sitting)

24. If $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^x$, then x equals

(1) $\frac{4}{9}$ (2) $\frac{2}{3}$

(3) $\frac{9}{4}$ (4) $\frac{3}{2}$

(SSC CPO S.I. Exam. 05.09.2004)

25. If a = 7, b = 5 and c = 3, then the value of $a^2+b^2+c^2-ab-bc-ca$ is

(1) 12 (2) –12
(3) 0 (4) 8

(SSC CPO S.I. Exam.05.09.2004)

26. If $7^x=\frac{1}{343}$, then the value of x is

(1) 3 (2) –3

(3) $\frac{1}{3}$ (4) $\frac{1}{7}$

(SSC CPO S.I. Exam. 05.09.2004)

27. If $\frac{a}{2}=\frac{b}{3}=\frac{c}{5}$, then $\frac{a+b+c}{c}$ is equal to

(1) 2 (2) 4
(3) 5 (4) 6

(SSC Data Entry Operator Exam. 31.08.2008)

28. If 0.13 ÷ p^2 = 13, then p is equal to

(1) 10 (2) 0.01
(3) 0.1 (4) 100

(SSC CGL Prelim Exam. 13.11.2005 (Second Sitting)

29. If $\frac{a}{3}=\frac{b}{2}$, then value of $\frac{2a+3b}{3a-2b}$ is

(1) $\frac{12}{5}$ (2) $\frac{5}{12}$

(3) 1 (4) $\frac{12}{7}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

30. For what value(s) of a is $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square?

(1) $\pm\frac{1}{18}$ (2) $\frac{1}{8}$

(3) $-\frac{1}{5}$ (4) $\frac{1}{4}$

(SSC CPO S.I. Exam. 03.09.2006)

31. If $a\neq b$, then which of the following statements is true ?

(1) $\frac{a+b}{2}=\sqrt{ab}$

(2) $\frac{a+b}{2}<\sqrt{ab}$

(3) $\frac{a+b}{2}>\sqrt{ab}$

(4) All of the above

(SSC CPO S.I. Exam. 03.09.2006)

32. If $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then the value of

$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

(1) 1 (2) 2
(3) 3 (4) 4

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone) & (SSC GL Tier-I Exam. 19.05.2013)

33. If x, y are two positive real numbers and $x^{1/3}=y^{1/4}$, then which of the following relations is *true*?

(1) $x^3=y^4$ (2) $x^3=y$
(3) $x=y^4$ (4) $x^{20}=y^{15}$

(SSC Section Officer (Commercial Audit) Exam. 26.11.2006 (Second Sitting)

34. If $a^{2x+2}=1$, where a is a positive real number other than 1, then x is equal to

(1) –2 (2) –1
(3) 0 (4) 1

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

35. If x is real, then the minimum value of (x^2-x+1) is

(1) $\frac{3}{4}$ (2) 0

(3) 1 (4) $\frac{1}{4}$

(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

36. If $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$, then the value of a is

(1) $\frac{11}{3}$ (2) $-\frac{4}{3}$

(3) $\frac{4}{3}$ (4) $\frac{-4\sqrt{7}}{3}$

(SSC CPO S.I. Exam. 16.12.2007)

37. If $(125)^x$ = 3125, then the value of x is

(1) $\frac{1}{5}$ (2) $\frac{3}{5}$

(3) $\frac{5}{3}$ (4) $\frac{5}{7}$

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

38. If $5^{\sqrt{x}}+12^{\sqrt{x}}=13^{\sqrt{x}}$, then x is equal to

(1) $\frac{25}{4}$ (2) 4

(3) 9 (4) 16

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

39. If $2^{2x-y}=16$ and $2^{x+y}=32$, the value of xy is

(1) 2 (2) 4
(3) 6 (4) 8

(SSC CPO S.I. Exam. 06.09.2009)

40. If $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$, then x is equal to

(1) –2 (2) 2
(3) –1 (4) 1

(SSC CGL Tier-I Exam. 16.05.2010 (First Sitting)

41. If $\frac{2x-y}{x+2y}=\frac{1}{2}$, then value of $\frac{3x-y}{3x+y}$ is :

(1) $\frac{1}{5}$ (2) $\frac{3}{5}$

(3) $\frac{4}{5}$ (4) 1

(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone)

42. If a and b be positive integers such that $a^2 - b^2 = 19$, then the value of a is

(1) 19 (2) 20

(3) 9 (4) 10

(SSC CGL Tier-I Exam. 16.05.2010 (First Sitting)

43. $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}}=2$ then x is equal to

(1) $\frac{5}{12}$ (2) $\frac{12}{5}$

(3) $\frac{5}{7}$ (4) $\frac{7}{5}$

(SSC CGL Tier-I Exam. 16.05.2010 (First Sitting)

44. If $x+\frac{1}{x}=5$, then $\frac{2x}{3x^2-5x+3}$ is equal to

(1) 5 (2) $\frac{1}{5}$

(3) 3 (4) $\frac{1}{3}$

(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (East Zone)

45. If $x=\frac{\sqrt{3}}{2}$, then the value of $\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right)$ is

(1) $-\sqrt{3}$ (2) -1

(3) 1 (4) $\sqrt{3}$

(SSC SAS Exam. 26.06.2010 (Paper-1)

46. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, then value of x^2+y^2 is :

(1) 14 (2) 13

(3) 15 (4) 10

(SSC CGL Prelim Exam. 11.05.2003 (First Sitting)

47. If $4^{4x+1}=\frac{1}{64}$, then the value of x is

(1) $\frac{1}{2}$ (2) -1

(3) $-\frac{1}{2}$ (4) $-\frac{1}{6}$

(SSC CISF ASI Exam. 29.08.2010 (Paper-1)

48. If $\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}}=2$ then x is equal to

(1) 2.4 (2) 3.2

(3) 4 (4) 5

(SSC (South Zone) Investigator Exam.12.09.2010)

49. If $\sqrt{2^x}=256$, then the value of x is

(1) 14 (2) 16

(3) 18 (4) 20

(SSC CPO S.I. Exam. 12.12.2010 (Paper-I)

50. If $\left(\sqrt{5}\right)^7 \div \left(\sqrt{5}\right)^5 = 5^p$, then the value of p is

(1) 5 (2) 2

(3) $\frac{3}{2}$ (4) 1

(SSC CPO S.I. Exam. 12.12.2010 (Paper-I)

51. If $\sqrt{1-\frac{x^3}{100}}=\frac{3}{5}$, then x equals

(1) 2 (2) 4

(3) 16 (4) $(136)^{1/3}$

(SSC CGL Tier-1 Exam. 19.06.2011 (First Sitting)

52. If a $b = 2a + 3b - ab$, then the value of (3 5 + 5 3) is

(1) 10 (2) 6

(3) 4 (4) 2

(SSC CGL Tier-1 Exam. 19.06.2011 (First Sitting)

53. If $\sqrt{1+\frac{x}{9}}=\frac{13}{3}$, then the value of x is

(1) $\frac{1439}{9}$ (2) 160

(3) $\frac{1443}{9}$ (4) 169

(SSC CGL Tier-1 Exam. 19.06.2011 (Second Sitting)

54. If $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}=a+b\sqrt{6}$, then the values of a and b are respectively

(1) $\frac{9}{15},-\frac{4}{15}$ (2) $\frac{3}{11},\frac{4}{33}$

(3) $\frac{9}{10},\frac{2}{5}$ (4) $\frac{3}{5},\frac{4}{15}$

(SSC CGL Tier-1 Exam. 19.06.2011 (Second Sitting)

55. If $x + y = 2z$ then the value of $\frac{x}{x-z}+\frac{z}{y-z}$ is

(1) 1 (2) 3

(3) $\frac{1}{2}$ (4) 2

(SSC Delhi Police S.I.(SI) Exam. 19.08.2012)

56. If $a * b = a^b$, then the value of $5 * 3$ is

(1) 125 (2) 243

(3) 53 (4) 15

(SSC CGL Tier-1 Exam. 19.06.2011 (Second Sitting)

57. If $\sqrt{0.03 \times 0.3a} = 0.3 \times 0.3 \times \sqrt{b}$, value of $\frac{a}{b}$ is

(1) 0.009 (2) 0.03

(3) 0.9 (4) 0.08

(SSC CGL Tier-1 Exam 19.06.2011 (Second Sitting)

58. If $x * y = (x+3)^2 (y-1)$, then the value of $5 * 4$ is

(1) 192 (2) 182

(3) 180 (4) 172

(SSC CGL Tier-1 Exam 26.06.2011 (First Sitting)

59. If $9\sqrt{x}=\sqrt{12}+\sqrt{147}$, then x = ?

(1) 2 (2) 3

(3) 4 (4) 5

(SSC CGL Tier-1 Exam 26.06.2011 (First Sitting)

60. If $X * Y = X^2 + Y^2 - XY$ then $11 * 13$ is

(1) 117 (2) 147
(3) 290 (4) 433

(SSC CGL Tier-1 Exam 26.06.2011 (Second Sitting)

61. If $\sqrt{1+\frac{x}{961}} = \frac{32}{31}$, then the value of x is

(1) 63 (2) 61
(3) 65 (4) 64

(SSC CGL Tier-1 Exam 26.06.2011 (Second Sitting)

62. If $\sqrt{0.04 \times 0.4 \times a} = 0{\cdot}004 \times 0{\cdot}4 \times \sqrt{b}$, then the value of $\frac{a}{b}$ is

(1) 16×10^{-3} (2) 16×10^{-4}
(3) 16×10^{-5} (4) 16×10^{-6}

(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I)

63. The expression $x^4 - 2x^2 + k$ will be a perfect square when the value of k is

(1) 2 (2) 1
(3) –1 (4) –2

(SSC Graduate Level Tier-I Exam. 11.11.2012, Ist Sitting)

64. If $2^{x+3} = 32$, then the value of 3^{x+1} is equal to

(1) 27 (2) 81
(3) 72 (4) 9

FCI Assistant Grade-III Exam.25.02.2012 (Paper-I) North Zone (Ist Sitting)

65. The value of the expression $x^4 - 17x^3 + 17x^2 - 17x + 17$ at $x = 16$ is

(1) 0 (2) 1
(3) 2 (4) 3

FCI Assistant Grade-III Exam.05.02.2012 (Paper-I) East Zone (IInd Sitting)

66. If $\frac{x}{y} = \frac{3}{4}$, the value of $\frac{6}{7} + \frac{y-x}{y+x}$ is :

(1) 1 (2) $\frac{2}{7}$

(3) $\frac{3}{7}$ (4) $1\frac{3}{7}$

(SSC CPO S.I.Exam.26.05.2005)

67. If $n + \frac{2}{3}n + \frac{1}{2}n + \frac{1}{7}n = 97$ then the value of n is

(1) 40 (2) 42
(3) 44 (4) 46

(SSC Data Entry Operator Exam. 31.08.2008)

68. If $x^2 - 3x + 1 = 0$, then the vaule of $x + \frac{1}{x}$ is

(1) 0 (2) 1
(3) 2 (4) 3

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

69. If 1.5 a = 0.04 b then $\frac{b-a}{b+a}$ is equal to

(1) $\frac{73}{77}$ (2) $\frac{77}{33}$

(3) $\frac{2}{75}$ (4) $\frac{75}{2}$

(SSC CGL Tier-I Exam. 16.05.2010 (Second Sitting)

70. If $x = \left(\sqrt{2}+1\right)^{-\frac{1}{3}}$, the value of $\left(x^3 - \frac{1}{x^3}\right)$ is

(1) 0 (2) $-\sqrt{2}$

(3) –2 (4) $3\sqrt{2}$

(SSC SAS Exam. 26.06.2010 (Paper-1)

71. If $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{2}{3}$, then the value of $\left(x + \frac{1}{x}\right)$ is

(1) 4 (2) 5
(3) 6 (4) 8

(SSC CISF ASI Exam. 29.08.2010 (Paper-1)

72. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = 3$, then

$\frac{2a^2 + 3c^2 + 4e^2}{2b^2 + 3d^2 + 4f^2} = ?$

(1) 2 (2) 3
(3) 4 (4) 9

(SSC CGL Tier-1 Exam. 19.06.2011 (First Sitting)

73. If x, y and z are real numbers such that $(x-3)^2 + (y-4)^2 + (z-5)^2 = 0$ then $(x + y + z)$ is equal to

(1) –12 (2) 0
(3) 8 (4) 12

(SSC Data Entry Operator Exam. 31.08.2008)

74. If $x = 7 - 4\sqrt{3}$, then $\sqrt{x} + \frac{1}{\sqrt{x}}$ is equal to :

(1) 1 (2) 2
(3) 3 (4) 4

(SSC CPO S.I.Exam.26.05.2005)

75. If $(a-1)^2 + (b+2)^2 + (c+1)^2 = 0$, then the value of $2a - 3b + 7c$ is

(1) 12 (2) 3
(3) –11 (4) 1

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

76. If $2x + \frac{1}{3x} = 5$, find the value of $\frac{5x}{6x^2 + 20x + 1}$.

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$

(3) $\frac{1}{5}$ (4) $\frac{1}{7}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (North Zone)

77. If x varies inversely as $(y^2 - 1)$ and is equal to 24 when $y = 10$, then the value of x when $y = 5$ is

(1) 99 (2) 12
(3) 24 (4) 100

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone)

78. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{31} + y^{35}$ is

(1) –1 (2) 0
(3) 1 (4) 2

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (North Zone)

79. If $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$, then value of $\left(x + \frac{1}{x}\right)$ is :

(1) 2 (2) $\frac{1}{2}$

(3) $-\frac{1}{2}$ (4) –2

(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone)

80. If a, b, c are real and $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of $2a - 3b + 4c$ is
(1) –1 (2) 0
(3) 1 (4) 2
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone) & (SSC GL Tier-I Exam. 21.04.2013) & (SSC CHSL DEO & LDC Exam. 20.10.2013)

81. If $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$, then the value of $(3a + b + 2c)$ is equal to :
(1) 3 (2) –1
(3) 2 (4) 5
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone)

82. The value of the expression
$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$$ is :
(1) 0 (2) 3
(3) $\frac{1}{3}$ (4) 2
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone) & (SSC CHSL DEO & LDC Exam. 27.10.2013)

83. If $(a-3)^2 + (b - 4)^2 + (c - 9)^2 = 0$, then the value of $\sqrt{a+b+c}$ is :
(1) – 4 (2) 4
(3) ± 4 (4) ± 2
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone)

84. If $a^3b = abc = 180$, a, b, c are positive integers, then the value of c is
(1) 110 (2) 1
(3) 4 (4) 25
(SSC Graduate Level Tier-II Exam. 16.09.2012)

85. If $(x - 3)^2 + (y - 5)^2 + (z - 4)^2 = 0$, then the value of
$$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16}$$ is
(1) 12 (2) 9
(3) 3 (4) 1
(SSC Graduate Level Tier-I Exam. 19.05.2013)

86. If a, b are rational numbers and $(a-1)\sqrt{2} + 3 = b\sqrt{2} + a$, the value of $(a + b)$ is
(1) –5 (2) 3
(3) –3 (4) 5
(SSC Graduate Level Tier-II Exam. 16.09.2012)

87. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of
$$\frac{a^2+ab+b^2}{a^2-ab+b^2}$$ is
(1) $\frac{3}{4}$ (2) $\frac{4}{3}$
(3) $\frac{3}{5}$ (4) $\frac{5}{3}$
(SSC CGL Prelim Exam. 13.11.2005 (Second Sitting)

88. If $64^{x+1} = \frac{64}{4^x}$, then the value of x is
(1) 1 (2) 0
(3) $\frac{1}{2}$ (4) 2
(SSC Assistant Grade-III Exam. 11.11.2012 (IInd Sitting)

89. If $ax^2 + bx + c = a(x - p)^2$, then the relation among a, b, c would be
(1) $abc = 1$ (2) $b^2 = ac$
(3) $b^2 = 4ac$ (4) $2b = a + c$
(SSC Delhi Police S.I. (SI) Exam. 19.08.2012)

90. If $a + b + c + d = 1$, then the maximum value of $(1 + a)(1 + b)(1 + c)(1 + d)$ is
(1) 1 (2) $\left(\frac{1}{2}\right)^3$
(3) $\left(\frac{3}{4}\right)^3$ (4) $\left(\frac{5}{4}\right)^4$
(SSC Graduate Level Tier-I Exam. 11.11.2012, Ist Sitting)

91. x varies inversely as square of y. Given that $y = 2$ for $x = 1$, the value of x for $y = 6$ will be equal to
(1) 3 (2) 9
(3) $\frac{1}{3}$ (4) $\frac{1}{9}$
(SSC Multi-Tasking Staff Exam. 17.03.2013, Kolkata Region)

92. If $x = \frac{\sqrt{3}}{2}$, then
$$\frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$$ is equal to
(1) 1 (2) $2/\sqrt{3}$
(3) $2-\sqrt{3}$ (4) 2
(SSC CPO S.I. Exam. 03.09.2006)

93. If $a^2 + b^2 + c^2 + 3 = 2(a - b - c)$, then the value of 2 a – b + c is :
(1) 3 (2) 4
(3) 0 (4) 2
(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

94. If $x^2 - y^2 = 80$ and $x - y = 8$, then the average of x and y is
(1) 2 (2) 3
(3) 4 (4) 5
(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

95. If for non-zero, x, $x^2 - 4x - 1 = 0$, the value of $x^2 + \frac{1}{x^2}$ is
(1) 4 (2) 10
(3) 12 (4) 18
(SSC Section Officer (Commercial Audit) Exam. 26.11.2006 (Second Sitting)

96. The third proportional to $\left(\frac{x}{y} + \frac{y}{x}\right)$ and $\sqrt{x^2+y^2}$ is
(1) xy (2) $\sqrt{xy}$
(3) $\sqrt[3]{xy}$ (4) $\sqrt[4]{xy}$
(SSC Graduate Level Tier-I Exam. 21.04.2013)

97. If $\frac{4x}{3} + 2P = 12$ for what value of P, $x = 6$?
(1) 6 (2) 4
(3) 2 (4) 1
(SSC Graduate Level Tier-I Exam. 19.05.2013)

98. The value of $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$ is
(1) $5\sqrt{3}-8$ (2) $5\sqrt{3}+8$
(3) $8\sqrt{3}+5$ (4) $8\sqrt{3}-5$
(SSC Graduate Level Tier-I Exam. 19.05.2013)

99. Let

$a = \sqrt{6} - \sqrt{5}$, $b = \sqrt{5} - 2$,

$c = 2 - \sqrt{3}$

Then point out the correct alternative among the four alternatives given below.

(1) $b < a < c$ (2) $a < c < b$
(3) $b < c < a$ (4) $a < b < c$

(SSC CHSL DEO & LDC Exam. 20.10.2013)

100. If $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of

$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$ is

(1) 1 (2) 2
(3) $\sqrt{3}$ (4) $\sqrt{5}$

(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

101. If $x = 5 - \sqrt{21}$, then the value of

$\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$ is

(1) $\frac{1}{\sqrt{2}}\left(\sqrt{3}-\sqrt{7}\right)$

(2) $\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

(3) $\frac{1}{\sqrt{2}}\left(\sqrt{7}+\sqrt{3}\right)$

(4) $\frac{1}{\sqrt{2}}\left(7-\sqrt{3}\right)$

(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

102. If $6x - 5y = 13$, $7x + 2y = 23$ then $11x + 18y =$

(1) –15 (2) 51
(3) 33 (4) 15

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

103. The value of

$\left(x^{b+c}\right)^{b-c}\left(x^{c+a}\right)^{c-a}\left(x^{a+b}\right)^{a-b}$, $(x \neq 0)$ is

(1) 1 (2) 2
(3) –1 (4) 0

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

104. If $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$, then the value of $x - x^2$ is :

(1) $-a$ (2) $\frac{1}{a}$

(3) $-\frac{1}{a}$ (4) a

(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

105. If $x + \frac{1}{x} = 99$, find the value of

$\frac{100x}{2x^2+102x+2}$

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$

(3) $\frac{1}{3}$ (4) $\frac{1}{4}$

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

106. If $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$,

then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is

(1) 9 (2) 3
(3) 4 (4) 6

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

107. If $\frac{xy}{x+y} = a, \frac{xz}{x+z} = b$ and

$\frac{yz}{y+z} = c$, where a, b, c are all non-zero numbers, then x equals to

(1) $\frac{2abc}{ab+bc-ac}$

(2) $\frac{2abc}{ab+ac-bc}$

(3) $\frac{2abc}{ac+bc-ab}$

(4) $\frac{2abc}{ab+bc+ac}$

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

108. If $x = 3 + \sqrt{8}$, then $x^2 + \frac{1}{x^2}$ is equal to

(1) 38 (2) 36
(3) 34 (4) 30

(SSC CGL Prelim Exam. 04.02.2007 (Ist Sitting) & (SSC CGL Prelim Exam. 27.07.2008 (IInd Sitting) & (SSC Investigator Exam. 12.09.2010) (South Zone)

109. If x and y are positive real numbers and $xy = 8$, then the minimum value of $2x + y$ is

(1) 9 (2) 17
(3) 10 (4) 8

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

110. If the expression $x^2 + x + 1$ is written in the form $\left(x+\frac{1}{2}\right)^2 + q^2$, then the possible values of q are

(1) $\pm\frac{1}{3}$ (2) $\pm\frac{\sqrt{3}}{2}$

(3) $\pm\frac{2}{\sqrt{3}}$ (4) $\pm\frac{1}{2}$

(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

111. If $a^2 - 4a - 1 = 0$, then value of

$a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$ is

(1) 25 (2) 30
(3) 35 (4) 40

(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

112. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$,

then $c + \frac{1}{a}$ is equal to

(1) 0 (2) $\frac{1}{2}$

(3) 1 (4) 2

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

113. The minimum value of $(x-2)(x-9)$ is

(1) $-\frac{11}{4}$ (2) $\frac{49}{4}$

(3) 0 (4) $-\frac{49}{4}$

(SSC Graduate Level Tier-I Exam. 21.04.2013)

114. One of the factors of the expression

$4\sqrt{3}x^2+5x-2\sqrt{3}$ is :

(1) $4x+\sqrt{3}$ (2) $4x+3$

(3) $4x-3$ (4) $4x-\sqrt{3}$

(SSC CAPFs SI & CISF ASI Exam. 23.06.2013)

115. If $\sqrt{x}=\sqrt{3}-\sqrt{5}$, then the value of $x^2-16x+6$ is

(1) 0 (2) –2

(3) 2 (4) 4

(SSC Graduate Level Tier-II Exam. 29.09.2013

116. If $x-\frac{1}{x}=4$, then $\left(x+\frac{1}{x}\right)$ is equal to

(1) $5\sqrt{2}$ (2) $2\sqrt{5}$

(3) $4\sqrt{2}$ (4) $4\sqrt{5}$

(SSC CGL Prelim Exam. 27.07.2008 (First Sitting)

117. If $x=5+2\sqrt{6}$, then the value of

$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ is,

(1) $2\sqrt{2}$ (2) $3\sqrt{2}$

(3) $2\sqrt{3}$ (4) $3\sqrt{3}$

(SSC SAS Exam 26.06.2010 (Paper-1)

118. For $a>b$, if $a+b=5$ and $ab=6$, then the value of (a^2-b^2) is

(1) 1 (2) 3

(3) 5 (4) 7

(SSC (South Zone) Investigator Exam. 12.09.2010)

119. If $1.5x=0.04y$, then the value of $\frac{y^2-x^2}{y^2+2xy+x^2}$ is

(1) $\frac{730}{77}$ (2) $\frac{73}{77}$

(3) $\frac{73}{770}$ (4) $\frac{74}{77}$

(SSC CGL Tier-1 Exam. 19.06.2011 (Second Sitting)

120. If $a^{\frac{1}{3}}=11$, then the value of a^2-331a is

(1) 1331331 (2) 1331000

(3) 1334331 (4) 1330030

(SSC CGL Tier-1 Exam 26.06.2011 (Second Sitting)

121. If $x^2+y^2+\frac{1}{x^2}+\frac{1}{y^2}=4$, then the value of x^2+y^2 is

(1) 2 (2) 4

(3) 8 (4) 16

(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I)

122. If $x^2=y+z$, $y^2=z+x$, $z^2=x+y$, then the value of

$\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}$ is

(1) –1 (2) 1

(3) 2 (4) 4

(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I) & (SSC CHSL DEO & LDC Exam. 04.12.2011) (Ist Sitting) & (SSC CGL Tier-I Exam. 19.05.2013) (Ist Sitting)

123. If $a^2+b^2=2$ and $c^2+d^2=1$, then the value of

$(ad-bc)^2+(ac+bd)^2$ is

(1) $\frac{4}{9}$ (2) $\frac{1}{2}$

(3) 1 (4) 2

(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I)

124. If $a^2+b^2+c^2+3=2(a+b+c)$ then the value of (a + b + c) is

(1) 2 (2) 3

(3) 4 (4) 5

(FCI Assistant Grade-III Exam.25.02.2012 (Paper-I) North Zone (Ist Sitting)

125. If $x-\frac{1}{x}=5$,

then $x^2+\frac{1}{x^2}$ is :

(1) 5 (2) 25

(3) 27 (4) 23

(FCI Assistant Grade-III Exam.05.02.2012 (Paper-I) East Zone (IInd Sitting)

126. If $x=3+2\sqrt{2}$, then the value of

$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$ is :

(1) 1 (2) 2

(3) $2\sqrt{2}$ (4) $3\sqrt{3}$

(SSC CPO S.I. Exam. 12.01.2003) & (FCI Assistant Grade-III Exam.05.02.2012 (Paper-I) East Zone (IInd Sitting)

127. If $x=\sqrt{3}+\sqrt{2}$, then the value of

$\left(x^2+\frac{1}{x^2}\right)$ is :

(1) 4 (2) 6

(3) 9 (4) 10

(SSC CHSL DEO & LDC Exam. 27.11.2010)

128. If $x+\frac{9}{x}=6$, then the value of

$(x^2+\frac{9}{x^2})$ is

(1) 8 (2) 9

(3) 10 (4) 12

(SSC CHSL DEO & LDC Exam. 28.11.2010 (Ist Sitting)

129. If $x=\frac{4ab}{a+b}$ $(a\neq b)$, the value of

$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ is

(1) a (2) b

(3) 2 ab (4) 2

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

130. If $m+\frac{1}{m-2}=4$, find the value

of $(m-2)^2+\frac{1}{(m-2)^2}$.

(1) –2 (2) 0

(3) 2 (4) 4

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone) & (SSC GL Tier-I Exam. 21.04.2013)

131. If $a^2+b^2+2b+4a+5=0$, then

the value of $\frac{a-b}{a+b}$ is

(1) 3 (2) –3

(3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone)

132. If $x-y=\frac{x+y}{7}=\frac{xy}{4}$, the numerical value of xy is

(1) $\frac{4}{3}$ (2) $\frac{3}{4}$

(3) $\frac{1}{4}$ (4) $\frac{1}{3}$

(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (East Zone)

133. If $x+y+z=0$,

then $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$ = ?

(1) $(xyz)^2$ (2) $x^2+y^2+z^2$

(3) 9 (4) 3

(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (East Zone) & (SSC GL Tier-I Exam.19.05.2013 (Ist Sitting)

134. If $a + b + c = 0$, then the value of

$$\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} + \frac{1}{(c+a)(c+b)}$$ is:

(1) 1 (2) 0
(3) –1 (4) –2

(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone)

135. If $a + b + c = 0$, then the value of

$$\frac{a^2+b^2+c^2}{a^2-bc}$$ is

(1) 0 (2) 1
(3) 2 (4) 3

(SSC Graduate Level Tier-II Exam. 16.09.2012)

136. If $n = 7 + 4\sqrt{3}$, then the value of $\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)$ is

(1) $2\sqrt{3}$ (2) 4
(3) –4 (4) $-2\sqrt{3}$

(SSC Graduate Level Tier-II Exam. 16.09.2012)

137. If $x = \sqrt{3}+\sqrt{2}$, then the value of $\left(x+\frac{1}{x}\right)$ is

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$
(3) 2 (4) 3

(SSC CHSL DEO & LDC Exam. 21.10.2012 (Ist Sitting)

138. If $p + q = 10$ and $pq = 5$, then the numerical value of $\frac{p}{q}+\frac{q}{p}$ will be

(1) 16 (2) 20
(3) 22 (4) 18

(SSC CHSL DEO & LDC Exam. 21.10.2012 (Ist Sitting)

139. If $x = 3 + 2\sqrt{2}$ and $xy = 1$, then the value of $\frac{x^2+3xy+y^2}{x^2-3xy+y^2}$ is

(1) $\frac{30}{31}$ (2) $\frac{70}{31}$
(3) $\frac{35}{31}$ (4) $\frac{37}{31}$

(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

140. If $\frac{x}{b+c}=\frac{y}{c+a}=\frac{z}{a+b}$, then

(1) $\frac{x-y}{b-a}=\frac{y-z}{c-b}=\frac{z-x}{a-c}$

(2) $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$

(3) $\frac{x-y}{c}=\frac{y-z}{b}=\frac{z-x}{c}$

(4) None of the above is true

(SSC CHSL DEO & LDC Exam. 04.11.2012, Ist Sitting)

141. If $a + b + c = 0$, then the value of

$$\left(\frac{a+b}{c}+\frac{b+c}{a}+\frac{c+a}{b}\right)\left(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\right)$$ is :

(1) 8 (2) –3
(3) 9 (4) 0

(SSC Graduate Level Tier-I Exam. 21.04.2013)

142. If a, b, c are non-zero, $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$, then the value of abc is :

(1) – 1 (2) 3
(3) – 3 (4) 1

(SSC Graduate Level Tier-I Exam. 21.04.2013)

143. If $a + b + c = 2s$, then

$$\frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$$

is equal to

(1) $a^2 + b^2 + c^2$ (2) 0
(3) 1 (4) 2

(SSC Graduate Level Tier-I Exam. 21.04.2013)

144. If $x = 3 + 2\sqrt{2}$, the value of $x^2+\frac{1}{x^2}$ is

(1) 36 (2) 30
(3) 32 (4) 34

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

145. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$, then the value of $x^2+\frac{1}{x^2}$ is

(1) $2\frac{1}{9}$ (2) $2\frac{4}{9}$
(3) $3\frac{1}{9}$ (4) $3\frac{4}{9}$

(SSC Graduate Level Tier-I Exam. 19.05.2013)

146. If $x^2-3x+1=0$, then the value of $x^2+x+\frac{1}{x}+\frac{1}{x^2}$ is

(1) 10 (2) 2
(3) 6 (4) 8

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

147. If $a^2 + b^2 = 5ab$, then the value of $\left(\frac{a^2}{b^2}+\frac{b^2}{a^2}\right)$ is :

(1) 32 (2) 16
(3) 23 (4) –23

(SSC CAPFs SI & CISF ASI Exam. 23.06.2013)

148. If $xy + yz + zx = 0$, then

$$\left(\frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}\right)$$

$(x, y, z \neq 0)$ is equal to

(1) 3 (2) 1
(3) $x + y + z$ (4) 0

(SSC CHSL DEO & LDC Exam. 20.10.2013)

149. If $a + b + c = 9$ (where a, b, c are real numbers), then the minimum value of $a^2 + b^2 + c^2$ is

(1) 100 (2) 9
(3) 27 (4) 81

(SSC CHSL DEO & LDC Exam. 20.10.2013)

150. If $x + y + z = 13$ and $x^2 + y^2 + z^2 = 69$, then $xy + z(x + y)$ is equal to

(1) 70 (2) 40
(3) 50 (4) 60

(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

151. If a = 0.1039, then the value of $\sqrt{4a^2-4a+1}+3a$ is

(1) 0.1039 (2) 0.2078
(3) 1.1039 (4) 2.1039

(SSC CPO S.I. Exam.12.01.2003)

152. If a = 0.25, b = – 0.05, c = 0.5, then the value of

$$\frac{a^2-b^2-c^2-2bc}{a^2+b^2-2ab-c^2}$$ is

(1) $\frac{7}{8}$ (2) $\frac{14}{17}$

(3) 1 (4) $\frac{25}{16}$

(SSC CPO S.I. Exam. 12.01.2003)

153. If $a = 23$ and $b = -29$ then the value of $25a^2 + 40ab + 16b^2$ is :

(1) 1 (2) –1

(3) 0 (4) 2

FCI Assistant Grade-III Exam.05.02.2012 (Paper-I) East Zone (IInd Sitting)

154. If $x - y = 2$ and $x^2 + y^2 = 20$, then value of $(x + y)^2$ is

(1) 38 (2) 36

(3) 16 (4) 12

(SSC CHSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

155. If $x^2 + y^2 - 4x - 4y + 8 = 0$, then the value of $x - y$ is

(1) 4 (2) –4

(3) 0 (4) 8

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

156. If $x = b + c - 2a$, $y = c + a - 2b$, $z = a + b - 2c$, then the value of $x^2 + y^2 - z^2 + 2xy$ is

(1) 0 (2) $a + b + c$

(3) $a - b + c$ (3) $a + b - c$

(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

157. For real a, b, c if $a^2 + b^2 + c^2 = ab + bc + ca$, then value of $\frac{a+c}{b}$ is

(1) 1 (2) 2

(3) 3 (4) 0

(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (Delhi Zone) & (SSC CHSL DEO & LDC Exam. 10.11.2013)

158. If $x - y = 2$, $xy = 24$, then the value of $(x^2 + y^2)$ is :

(1) 25 (2) 36

(3) 63 (4) 52

(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

159. If the expression $\frac{x^2}{y^2}+tx+\frac{y^2}{4}$ is a perfect square, then the values of t is

(1) ± 1 (2) ± 2

(3) 0 (4) ± 3

(SSC CHSL DEO & LDC Exam. 28.10.2012 (Ist Sitting)

160. If $a = x + y$, $b = x - y$, $c = x + 2y$, then $a^2 + b^2 + c^2 - ab - bc - ca$ is

(1) $4y^2$ (2) $5y^2$

(3) $6y^2$ (4) $7y^2$

(SSC CHSL DEO & LDC Exam. 04.11.2012 (IInd Sitting)

161. If $a^2 + b^2 + c^2 = ab + bc + ca$, where a, b, c are non zero real numbers, then the value of

$\frac{a+b}{c}$ is

(1) 2 (2) 1

(3) 0 (4) –1

(SSC CHSL DEO & LDC Exam. 28.10.2012, Ist Sitting)

162. If $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$ and a, b, c are real, then the value of $(a^2 + b^2 + c^2)$ is

(1) 3 (2) $3\frac{1}{4}$

(3) 2 (4) $2\frac{1}{4}$

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

163. If $\frac{x-a^2}{b+c}+\frac{x-b^2}{c+a}+\frac{x-c^2}{a+b}$ $= 4(a + b + c)$, then x is equal to

(1) $(a + b + c)^2$

(2) $a^2 + b^2 + c^2$

(3) $ab + bc + ca$

(4) $a^2 + b^2 + c^2 - ab - bc - ca$

(SSC Graduate Level Tier-II Exam. 29.09.2013)

164. Number of solutions of the two equations $4x - y = 2$ and $2y - 8x + 4 = 0$ is

(1) zero (2) one

(3) two

(4) infinitely many

(SSC CHSL DEO & LDC Exam. 20.10.2013)

165. If $\frac{a}{b}=\frac{4}{5}$ and $\frac{b}{c}=\frac{15}{16}$, then

$$\frac{18c^2-7a^2}{45c^2+20a^2}$$ is equal to

(1) $\frac{1}{3}$ (2) $\frac{2}{5}$

(3) $\frac{3}{4}$ (4) $\frac{1}{4}$

(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

166. If $x \neq 0$, $y \neq 0$ and $z \neq 0$ and

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx},$$

then the relation among x, y, z is

(1) $x + y + z = 0$

(2) $x + y = z$

(3) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

(4) $x = y = z$

(SSC Graduate Level Tier-I Exam. 21.04.2013)

167. The term to be added to $121a^2 + 64b^2$ to make a perfect square is

(1) $176\ ab$ (2) $276\ a^2b$

(3) $178\ ab$ (4) $188\ b^2a$

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

168. If $a = 2 + \sqrt{3}$, then the value of

$\left(a^2+\frac{1}{a^2}\right)$ is

(1) 12 (2) 14

(3) 16 (4) 10

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

169. For what valsue (s) of k the expression $p+\frac{1}{4}\sqrt{p}+k^2$ is a perfect square ?

(1) $\pm\frac{1}{3}$ (2) $\pm\frac{1}{4}$

(3) $\pm\frac{1}{8}$ (4) $\pm\frac{1}{2}$

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

170. If $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}=1$ and $a - b + c \neq 0$ then which one of the following relations is true ?

(1) $\frac{1}{c}=\frac{1}{a}+\frac{1}{b}$ (2) $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$

(3) $\frac{1}{b}=\frac{1}{a}-\frac{1}{c}$ (4) $\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

171. If $a + b = 1$, $c + d = 1$ and $a - b = \frac{d}{c}$, then the value of $c^2 - d^2$ is

(1) $\frac{a}{b}$ (2) $\frac{b}{a}$

(3) 1 (4) –1

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

172. If $x = 3t$, $y = \frac{1}{2}(t+1)$, then the value of t for which $x = 2y$ is

(1) 1 (2) $\frac{1}{2}$

(3) –1 (4) $\frac{2}{3}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

173. If $x^2 + \frac{1}{5}x + a^2$ is a perfect square, then a is

(1) $\frac{1}{100}$ (2) $\pm\frac{1}{10}$

(3) $\frac{1}{10}$ (4) $-\frac{1}{10}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

174. Find the value of x for which the expression $2 - 3x - 4x^2$ has the greatest value.

(1) $-\frac{41}{16}$ (2) $\frac{3}{8}$

(3) $-\frac{3}{8}$ (4) $\frac{41}{16}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

175. The expression $x^4 - 2x^2 + k$ will be a perfect square if the value of k is

(1) 1 (2) 0

(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

176. If $(x - 1)$ and $(x + 3)$ are the factors of $x^2 + k_1x + k_2$ then

(1) $k_1 = -2$, $k_2 = -3$
(2) $k_1 = 2$, $k_2 = -3$
(3) $k_1 = 2$, $k_2 = 3$
(4) $k_1 = -2$, $k_2 = 3$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

177. If $\frac{5x}{2x^2+5x+1} = \frac{1}{3}$, then the value of $\left(x + \frac{1}{2x}\right)$ is

(1) 15 (2) 10

(3) 20 (4) 5

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

178. The reciprocal of $x + \frac{1}{x}$ is

(1) $\frac{x}{x^2+1}$ (2) $\frac{x}{x+1}$

(3) $x - \frac{1}{x}$ (4) $\frac{1}{x} + x$

(SSC CGL Tier-I Exam. 26.10.2014)

179. If a, b, c are positive and $a + b + c = 1$, then the least value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is

(1) 9 (2) 5

(3) 3 (4) 1

(SSC CGL Tier-I Exam. 26.10.2014)

180. If $a(2+\sqrt{3}) = b(2-\sqrt{3}) = 1$, then the value of

$\frac{1}{a^2+1} + \frac{1}{b^2+1}$ is

(1) –5 (2) 1

(3) 4 (4) 9

(SSC CGL Tier-I Exam. 26.10.2014)

181. If $\left(2+\sqrt{3}\right)a = \left(2-\sqrt{3}\right)b = 1$ then the value of $\frac{1}{a} + \frac{1}{b}$ is

(1) 1 (2) 2

(3) $2\sqrt{3}$ (4) 4

(SSC CGL Tier-II Exam. 21.09.2014)

182. If $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$ $(a \neq b \neq c)$, then the value of abc is

(1) ± 1 (2) ± 2

(3) 0 (4) $\pm\frac{1}{2}$

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014)

183. If $\frac{x}{y} = \frac{4}{5}$, then the value of $\left(\frac{4}{7} + \frac{2y-x}{2y+x}\right)$ is

(1) $\frac{3}{7}$ (2) $1\frac{1}{7}$

(3) 1 (4) 2

(SSC CHSL DEO & LDC Exam. 9.11.2014)

184. If $(x - 2)$ is a factor of $x^2 + 3Qx - 2Q$, then the value of Q is

(1) 2 (2) –2

(3) 1 (4) –1

(SSC CHSL DEO Exam. 02.11.2014 (Ist Sitting)

185. If $a + b = 12$, $ab = 22$, then $(a^2 + b^2)$ is equal to

(1) 188 (2) 144

(3) 34 (4) 100

(SSC CHSL DEO Exam. 02.11.2014 (Ist Sitting)

186. If $x = \sqrt{3} - \frac{1}{\sqrt{3}}$ and $y = \sqrt{3} + \frac{1}{\sqrt{3}}$, then the value of $\frac{x^2}{y} + \frac{y^2}{x}$ is

(1) $\sqrt{3}$ (2) $3\sqrt{3}$

(3) $16\sqrt{3}$ (4) $2\sqrt{3}$

(SSC CHSL DEO Exam. 02.11.2014 (Ist Sitting)

187. If $x^2 + ax + b$ is a perfect square, then which one of the following relations between a and b is true ?

(1) $a^2 = b$ (2) $a^2 = 4b$

(3) $b^2 = 4a$ (4) $b^2 = a$

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

188. If $a + b + c + d = 4$, then find the value of

$$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)} + \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}.$$

(1) 0 (2) 5

(3) 1 (4) 4

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

189. If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$, then a relation among a, b, c is

(1) $a + b + c = 0$
(2) $(a + b + c)^3 = 27abc$
(3) $a + b + c = 3abc$
(4) $a^3 + b^3 + c^3 = 0$

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

190. If $a = \sqrt{6} + \sqrt{5}$, $b = \sqrt{6} - \sqrt{5}$ then $2a^2 - 5ab + 2b^2 = ?$

(1) 38 (2) 39

(3) 40 (4) 41

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

191. If $a^2 + b^2 + c^2 = 2a - 2b - 2$, then the value of $3a - 2b + c$ is
(1) 0 (2) 3
(3) 5 (4) 2
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

192. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are all non-zero, then 'abc' is equal to
(1) $\frac{2}{3}$ (2) $\frac{3}{2}$
(3) $\frac{1}{2}$ (4) $\frac{1}{3}$
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

193. If $a^2 - 4a - 1 = 0$, $a \neq 0$, then the value of $a^2 + 3a + \frac{1}{a^2} - \frac{3}{a}$ is
(1) 24 (2) 26
(3) 28 (4) 30
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

194. If $x = 2 + \sqrt{3}$, then $x^2 + \frac{1}{x^2}$ is equal to
(1) 10 (2) 12
(3) –12 (4) 14
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

195. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of $(a + b + c)$ is
(1) 0 (2) 1
(3) – 1 (4) 2
(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting TF No. 333 LO 2)

196. If x is a prime number and $-1 \leq \frac{2x-7}{5} \leq 1$ then the number of values of x is
(1) 4 (2) 3
(3) 2 (4) 5
(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014, IInd Sitting TF No. 545 QP 6)

197. If $\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$, the value of $\frac{2}{x} + \frac{2}{y} + \frac{2}{z}$ is
(1) 20 (2) 5
(3) 10 (4) 15
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

198. If 2s = a + b + c, then the value of s(s – c) + (s – a) (s – b) is
(1) ab (2) abc
(3) 0 (4) $\frac{a+b+c}{2}$
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

199. If $\frac{2p}{p^r - 2p + 1} = \frac{1}{4}$, then the value of $\left(p + \frac{1}{p}\right)$ is
(1) 7 (2) $\frac{2}{5}$
(3) 1 (4) 10
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

200. If $\sqrt{1 + \frac{27}{169}} = 1 + \frac{x}{13,}$ then x equals
(1) 1 (2) 27
(3) 13 (4) $3\sqrt{3}$
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

201. If $2x = \sqrt{a} + \frac{1}{\sqrt{a}}$, $a > 0$, then the value of $\frac{\sqrt{x^2-1}}{x - \sqrt{x^2-1}}$ is
(1) $a + 1$ (2) $\frac{1}{2}(a + 1)$
(3) $\frac{1}{2}(a - 1)$ (4) $a - 1$
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

202. If a, b, c are real numbers and $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of $a + b + c$ is
(1) – 1 (2) 1
(3) 3 (4) 0
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

203. If $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a}$ and $a + b + c \neq 0$, then
(1) $a \neq b \neq c$ (2) $a = b = c$
(3) $a = b \neq c$ (4) $a \neq b = c$
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

204. If $bc + ab + ca = abc$, then the value of $\frac{b+c}{bc(a-1)} + \frac{a+c}{ac(b-1)} + \frac{a+b}{ab(c-1)}$ is
(1) 0 (2) $-\frac{1}{2}$
(3) $-\frac{3}{2}$ (4) 1
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

205. If $\frac{a^2-bc}{a^2+bc} + \frac{b^2-ca}{b^2+ca} + \frac{c^2-ab}{c^2+ab} = 1$, then the value of $\frac{a^2}{a^2+bc} + \frac{b^2}{b^2+ac} + \frac{c^2}{c^2+ab}$ is
(1) 0 (2) 1
(3) – 1 (4) 2
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

206. If $999x + 888y = 1332$
$888x + 999y = 555$,
then the value of $x + y$ is
(1) 888 (2) 555
(3) 1 (4) 999
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

207. If $a = \frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}}$, then the value of $(a^2 - ax)$ is
(1) 1 (2) 2
(3) – 1 (4) 0
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 IInd Sitting)

208. If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$, then the value of $8xy(x^2 + y^2)$ is

(1) 196 (2) 290
(3) 112 (4) 194

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 IInd Sitting)

209. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $\frac{a+c}{b}$ is

(1) 3 (2) 2
(3) 0 (4) 1

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 IInd Sitting)

210. If $\frac{m-a^2}{b^2+c^2} + \frac{m-b^2}{c^2+a^2} + \frac{m-c^2}{a^2+b^2} = 3$, then the value of m is

(1) $a^2 + b^2 - c^2$ (2) $a^2 + b^2$
(3) $a^2 + b^2 + c^2$ (4) $a^2 - b^2 - c^2$

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

211. If $x + \frac{1}{x} = 1$ then the value of $\frac{x^2+3x+1}{x^2+7x+1}$ is

(1) 1 (2) $\frac{3}{7}$
(3) $\frac{1}{2}$ (4) 2

(SSC CGL Tier-I Exam, 09.08.2015 (IInd Sitting) TF No. 4239378)

212. If $p = 99$ then, the value of $p(p^2 + 3p + 3)$ is :

(1) 989898 (2) 988899
(3) 999999 (4) 998889

(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

213. If $x = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $y = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ then the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2} = ?$

(1) $\frac{63}{61}$ (2) $\frac{67}{65}$
(3) $\frac{65}{63}$ (4) $\frac{69}{67}$

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

214. If $x + \frac{1}{x} = 1$ then the value of $\frac{2}{x^2-x+2} = ?$

(1) 2 (2) 4
(3) $\frac{2}{3}$ (4) 1

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

215. If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$, then $\frac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)}$ is equal to

(1) 1 (2) 0
(3) 2 (4) $\frac{1}{2}$

(SSC CGL Tier-I Re-Exam, 30.08.2015)

216. Let $x = \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$ and $y = \frac{1}{x}$, then the value of $3x^2 - 5xy + 3y^2$ is

(1) 1717 (2) 1177
(3) 1771 (4) 1171

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

217. If $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$, where $a \neq b \neq c \neq 0$, then the value of $a^2 b^2 c^2$ is

(1) –1 (2) abc
(3) 0 (4) 1

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

218. For real a, b, c if $a^2 + b^2 + c^2 = ab + bc + ca$, the value of $\frac{a+c}{b}$ is

(1) 3 (2) 1
(3) 2 (4) 0

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

219. $9x^2 + 25 - 30x$ can be expressed as the square of

(1) $-3x - 5$ (2) $3x + 5$
(3) $3x - 5$ (4) $3x^2 - 25$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

220. If $\frac{x}{3} + \frac{3}{x} = 1$ then the value of x^3 is

(1) 1 (2) 27
(3) 0 (4) –27

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

221. If $x + y = 2a$, then the value of $\frac{a}{x-a} + \frac{a}{y-a}$ is

(1) 2 (2) 0
(3) 1 (4) –1

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

222. If $\frac{x+1}{x-1} = \frac{a}{b}$ and $\frac{1-y}{1+y} = \frac{b}{a}$, then the value of $\frac{x-y}{1+xy}$ is

(1) $\frac{2ab}{a^2-b^2}$ (2) $\frac{a^2-b^2}{2ab}$
(3) $\frac{a^2+b^2}{2ab}$ (4) $\frac{a^2-b^2}{ab}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

223. If $\frac{a}{b} + \frac{b}{a} = 2$, then the value of $(a - b)$ is :

(1) 1 (2) 2
(3) –1 (4) 0

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

224. If $\sqrt{y} = 4x$, then $\frac{x^2}{y}$ is :

(1) 2 (2) $\frac{1}{16}$
(3) $\frac{1}{4}$ (4) 4

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

225. If $\frac{x}{y} = \frac{a+2}{a-2}$, then the value of $\frac{x^2-y^2}{x^2+y^2}$ is :

(1) $\frac{4a}{a^2+2}$ (2) $\frac{2a}{a^2+2}$

(3) $\frac{4a}{a^2+4}$ (4) $\frac{2a}{a^2+4}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

226. If $x(x+y+z) = 20$, $y(x+y+z) = 30$, and $z(x+y+z) = 50$, then the value of $2(x+y+z)$ is :

(1) 20 (2) –10

(3) 15 (4) 18

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (Ist Sitting) TF No. 1375232)

227. If $x + y = 4$, $x^2 + y^2 = 14$ and $x > y$, then the correct value of x and y is :

(1) $2+\sqrt{3}$, $2-\sqrt{3}$

(2) $2-\sqrt{2}$, $\sqrt{3}$

(3) 3, 1

(4) $2+\sqrt{3}$, $2\sqrt{2}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (Ist Sitting) TF No. 1375232)

228. If $a^2 + b^2 + c^2 = 2(a+b+c) - 3$, then the value of $a + b + c$ is :

(1) 2 (2) –1

(3) 1 (4) –2

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (Ist Sitting) TF No. 1375232)

229. If for non-zero x, $x^2 - 4x - 1 = 0$, the value of $x^2 + \frac{1}{x^2}$ is :

(1) 12 (2) 4

(3) 18 (4) 10

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

230. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$ then $c + \frac{1}{a}$ is equal to :

(1) $\frac{1}{2}$ (2) 2

(3) 1 (4) 0

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

231. If $\frac{a}{b} = \frac{25}{6}$, then the value of $\frac{a^2-b^2}{a^2+b^2}$ is

(1) $\frac{589}{661}$ (2) $\frac{661}{589}$

(3) $\frac{625}{36}$ (4) $\frac{589}{651}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 20.12.2015 (Ist Sitting) TF No. 9692918)

232. If $(x-2)(x-p) = x^2 - ax + 6$, then the value of $(a-p)$ is

(1) 0 (2) 1

(3) 2 (4) 3

(SSC CGL Tier-I (CBE) Exam.10.09.2016)

233. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$, $y = \sqrt{a} - \frac{1}{\sqrt{a}}$, $(a > 0)$, then the value of $(x^4 + y^4 - 2x^2y^2)$ is

(1) 16 (2) 20

(3) 10 (4) 5

(SSC CGL Tier-I (CBE) Exam.10.09.2016)

234. If $2x + \frac{1}{3x} = 5$, the value of $\frac{5x}{6x^2+20x+1}$ is

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$

(3) $\frac{1}{5}$ (4) $\frac{1}{7}$

(SSC CGL Tier-I (CBE) Exam.11.09.2016) (Ist Sitting)

235. If $a + b = 10$ and $ab = 21$, then the value of $(a-b)^2$ is

(1) 15 (2) 16

(3) 17 (4) 18

(SSC CGL Tier-I (CBE) Exam.11.09.2016) (Ist Sitting)

236. Let $0 < x < 1$. Then the correct inequality is

(1) $x < \sqrt{x} < x^2$ (2) $\sqrt{x} < x < x^2$

(3) $x^2 < x < \sqrt{x}$ (4) $\sqrt{x} < x^2 < x$

(SSC CGL Tier-II Online Exam.01.12.2016)

237. If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\frac{x^2+xy+y^2}{x^2-xy+y^2}$ is

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$

(3) $\frac{3}{5}$ (4) $\frac{5}{3}$

(SSC CGL Tier-II Online Exam.01.12.2016)

238. If $a + b + c = m$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$, then average of a^2, b^2 and c^2 is

(1) m^2 (2) $\frac{m^2}{3}$

(3) $\frac{m^2}{9}$ (4) $\frac{m^2}{27}$

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

239. If $x = \frac{8ab}{a+b}$ $(a \neq b)$, then the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$ is :

(1) 0 (2) 1

(3) 2 (4) 4

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

240. The value of $(2a+b)^2 - (2a-b)^2$ is :

(1) $8ab$ (2) $-8ab$

(3) $8a^2 + 2b^2$ (4) $8a^2 - 2b^2$

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

241. If $a + b + c = 0$ then the value of $\frac{a^2+b^2+c^2}{ab+bc+ca}$ is

(1) 2 (2) –2

(3) 0 (4) 4

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

242. If $a + b = 2c$, find $\frac{a}{a-c} + \frac{c}{b-c}$.

(1) 0 (2) 1

(3) 2 (4) –1

(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

243. If $2x+\frac{1}{4x}$ =1, then the value of $x^2+\frac{1}{64x^2}$ is

(1) 0 (2) 1

(3) $\frac{1}{4}$ (4) 2

(SSC CHSL (10+2) Tier-I (CBE) Exam. 08.09.2016) (Ist Sitting)

244. The value of $\frac{a}{a-b}+\frac{b}{b-a}$ is

(1) $\frac{(a+b)}{(a-b)}$ (2) –1

(3) $2ab$ (4) 1

(SSC CGL Tier-I (CBE) Exam. 09.09.2016) (Ist Sitting)

245. If $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$ then $c+\frac{1}{a}$ is equal to

(1) 1 (2) 0

(3) –1 (4) 2

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

246. If $\frac{a}{b}=\frac{1}{2}$, find the value of the expression $\frac{(2a-5b)}{(5a+3b)}$

(1) –32 (2) 11

(3) $\frac{-8}{11}$ (4) 17

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

247. If $\frac{1}{x^2}+x^2$ represents the radius of circle P and $\frac{1}{x}+x=17$, which of the following best approximates the circumference of circle P ?

(1) 287 π (2) 547 π

(3) 574 π (4) 278 π

(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

248. What is the value of m in the quadratic equation $x^2+mx+24=0$ if one of its roots is $\frac{3}{2}$

(1) $-\frac{45}{2}$ (2) 16

(3) $-\frac{21}{2}$ (4) $-\frac{35}{2}$

(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

249. If $ab=21$ and $\frac{(a+b)^2}{(a-b)^2}=\frac{25}{4}$, then the value of a^2+b^2+3ab is

(1) 115 (2) 121

(3) 125 (4) 127

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (Ist Sitting)

250. If $a+\frac{1}{a-2}=4$, then the value of $(a-2)^2+\left(\frac{1}{a-2}\right)^2$ is :

(1) 0 (2) 2

(3) –2 (4) 4

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (IInd Sitting)

251. If $x=\frac{6pq}{p+q}$, then the value of $\frac{x+3p}{x-3p}+\frac{x+3q}{x-3q}$ is

(1) 6 (2) 8

(3) 2 (4) 3

(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (IInd Sitting)

252. If $x+\frac{1}{9x}=4$, then the value $9x^2+\frac{1}{9x^2}$ is

(1) 140 (2) 142

(3) 144 (4) 146

(SSC CGL Tier-I (CBE) Exam. 28.08.2016) (IInd Sitting)

253. If $x\left(3-\frac{2}{x}\right)=\frac{3}{x}$, then the value of $x^2+\frac{1}{x^2}$ will be

(1) $3\frac{1}{9}$ (2) $3\frac{2}{9}$

(3) $2\frac{1}{9}$ (4) $2\frac{4}{9}$

(SSC CGL Tier-I (CBE) Exam. 28.08.2016) (IInd Sitting)

254. If $x^2+\frac{1}{x^2}=2$, then the value of $x-\frac{1}{x}$ is

(1) –2 (2) 0

(3) 1 (4) –1

(SSC CGL Tier-I (CBE) Exam. 29.08.2016) (IInd Sitting)

255. If $9x^2+16y^2=60$ and $3x+4y=6$, then the value of xy is

(1) –1 (2) 1

(3) –2 (4) 2

(SSC CGL Tier-I (CBE) Exam. 29.08.2016) (IInd Sitting)

256. If $p^2+q^2=7$ pq, then the value of $\frac{p}{q}+\frac{q}{p}$ is equal to

(1) 9 (2) 5

(3) 7 (4) 3

(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (Ist Sitting)

257. If $x=99$, then the value of $2(x^2+3x+3)$ is equal to

(1) 1000001 (2) 1000000

(3) 999999 (4) 9999999

(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (Ist Sitting)

258. If $\frac{2p}{p^2-2p+1}=\frac{1}{4}$, then the value of $p+\frac{1}{p}$ will be

(1) 8 (2) 10

(3) 12

(4) None of these

(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (Ist Sitting)

259. If $(a-b)=3$ and $(a^2+b^2)=25$, then the value of (ab) is

(1) 16 (2) 8

(3) 10 (4) 15

(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (Ist Sitting)

260. If $a+\frac{1}{a}=1$, then the value of $\frac{a^2-a+1}{a^2+a+1}$ is $(a\neq 0)$

(1) 1 (2) –1

(3) 0 (4) 2

(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (Ist Sitting)

261. If $x - \frac{1}{x} = 2$, then what is the value of $x^2 + \frac{1}{x^2}$?
(1) 4 (2) 5
(3) 3 (4) 6
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

262. If $a + b = 2c$, then the value of $\frac{a}{a-c} + \frac{c}{b-c}$ is equal to (where $a \neq b \neq c$)
(1) –1 (2) 1
(3) 0 (4) $\frac{1}{2}$
(SSC CGL Tier-I (CBE) Exam. 04.09.2016) (Ist Sitting)

263. If $x + \frac{1}{x} = 5$, then the value of $\frac{x}{1+x+x^2}$ is
(1) $\frac{1}{5}$ (2) $\frac{1}{6}$
(3) 5 (4) 6
(SSC CGL Tier-I (CBE) Exam. 04.09.2016) (Ist Sitting)

264. If $\frac{a^2}{b+c} = \frac{b^2}{c+a} = \frac{c^2}{a+b} = 1$ then find the value of $\frac{2}{1+a} + \frac{2}{1+b} + \frac{2}{1+c}$.
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-I (CBE) Exam. 04.09.2016) (Ist Sitting)

265. If $5x + \frac{1}{x} = 10$, then $x^2 + \frac{1}{25x^2}$ is equal to
(1) $2\frac{1}{5}$ (2) $3\frac{1}{5}$
(3) $3\frac{3}{5}$ (4) $2\frac{3}{5}$
(SSC CGL Tier-I (CBE) Exam. 06.09.2016) (Ist Sitting)

266. If $4r = h + \sqrt{r^2 + h^2}$ then $r : h$ is ? $(r \neq 0)$
(1) 17:8 (2) 8:17
(3) 8:15 (4) 15:8
(SSC CGL Tier-I (CBE) Exam. 06.09.2016) (Ist Sitting)

267. If p = 99, then the value of $p(p^2 + 3p + 3)$ will be
(1) 999999 (2) 1000000
(3) 1000001 (4) 999998
(SSC CGL Tier-I (CBE) Exam. 07.09.2016) (Ist Sitting)

268. If $\frac{x}{a+b} + 1 = \frac{x}{a-b} + \frac{a-b}{a+b}$, then x is equal to
(1) $2a - b$ (2) $a + b$
(3) $a - b$ (4) $2a + b$
(SSC CGL Tier-I (CBE) Exam. 07.09.2016) (Ist Sitting)

269. If $x^2 + y^2 = 29$ and $xy = 10$, where $x > 0$, $y > 0$, $x > y$ then the value of $\frac{x+y}{x-y}$ is
(1) $\frac{-7}{3}$ (2) $\frac{7}{3}$
(3) $\frac{3}{7}$ (4) $\frac{-3}{7}$
(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (IInd Sitting)

270. If $4x^2 - 12x + k$ is a perfect square, then the value of k is
(1) 2 (2) 9
(3) 12 (4) 10
(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (IInd Sitting)

271. The value of
$$\left(\frac{1}{(p-n)(n-q)} + \frac{1}{(n-q)(q-p)} + \frac{1}{(q-p)(p-n)}\right)$$
is
(1) 1 (2) 0
(3) $p + q + n$ (4) $\frac{2n}{p+q}$
(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (IInd Sitting)

272. If $\frac{a^2}{b+c} = \frac{b^2}{c+a} = \frac{c^2}{a+b} = 1$ then $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$ is
(1) 1 (2) 2
(3) 3 (4) 4
(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (IInd Sitting)

273. If $a^2 + 1 = 9a$, $(a \neq 0)$ then the value of $(a)^2 + \frac{1}{(a)^2}$ is
(1) 81 (2) 18
(3) 79 (4) 83
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

274. If $p = 99$, then the value of $p(p^2 + 3p + 3)$ is
(1) 9999 (2) 999999
(3) 99999 (4) 9999999
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

275. If $x + \frac{1}{x} = c + \frac{1}{c}$ then the value of x is
(1) $c\frac{1}{c}$ (2) c, c^2
(3) $c, 2c$ (4) 0, 1
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

276. If $x^2 + y^2 + 6x + 5 = 4(x - y)$ then $(x - y)$ is
(1) 1 (2) –1
(3) 0 (4) 4
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

277. If $\left(x - \frac{1}{3x}\right) = \frac{1}{3}$, the value of $3\left(x - \frac{1}{3x}\right)$ is :
(1) –1 (2) 1
(3) –2 (4) 2
(SSC CGL Tier-I (CBE) Exam. 28.08.2016 (IST Sitting)

278. If $\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q}$, find the value of $(pa + qb + rc)$.
(1) 0 (2) 1
(3) 2 (4) –1
(SSC CGL Tier-I (CBE) Exam. 29.08.2016 (IST Sitting)

279. If $\frac{3a+4b}{3c+4d} = \frac{3a-4b}{3c-4d}$, then
(1) $ab = cd$ (2) $ad = bc$
(3) $ac = bd$ (4) $a = b = c \neq d$
(SSC CGL Tier-I (CBE) Exam. 30.08.2016 (IIIrd Sitting)

280. If $\left(x + \frac{1}{x}\right) = 2$ then $\left(x^2 + \frac{1}{x^2}\right)$ is equal to
(1) 0 (2) 2
(3) 4 (4) 8
(SSC CGL Tier-I (CBE) Exam. 31.08.2016 (IIIrd Sitting)

281. If $a + b = 17$ and $a - b = 9$, then the value of $(4a^2 + 4b^2)$ is :
(1) 710 (2) 720
(3) 730 (4) 740
(SSC CGL Tier-I (CBE) Exam. 31.08.2016 (IIIrd Sitting)

282. If $x + y = \sqrt{3}$ and $x - y = \sqrt{2}$, then the value of $8xy(x^2 + y^2)$ is :
(1) 6 (2) $\sqrt{6}$
(3) 5 (4) $\sqrt{5}$
(SSC CGL Tier-I (CBE) Exam. 31.08.2016 (IIIrd Sitting)

283. If $a^2 + 1 = a$, then the value of a^3 is
(1) 0 (2) 1
(3) –1 (4) 2
(SSC CGL Tier-I (CBE) Exam. 01.09.2016 (IIIrd Sitting)

284. If $x + 3y = -3x + y$, then $\frac{x^2}{2y^2}$ is equal to
(1) $\frac{1}{8}$ (2) $\frac{1}{2}$
(3) $\frac{1}{4}$ (4) 4
(SSC CGL Tier-I (CBE) Exam. 01.09.2016 (IIIrd Sitting)

285. If $(a + b - 6)^2 + a^2 + b^2 + 1 + 2b = 2ab + 2a$, then the value of a is
(1) 7 (2) 6
(3) 3.5 (4) 2.5
(SSC CGL Tier-I (CBE) Exam. 01.09.2016 (IIIrd Sitting)

286. If $\left(a + \frac{1}{a}\right)^2 = 3$, then the value of $\left(a^2 + \frac{1}{a^2}\right)$ will be
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-I (CBE) Exam. 02.09.2016 (IInd Sitting)

287. If $\left\{\frac{1}{2}(a-b)\right\}^2 + ab = p\,(a + b)^2$, then the value of p is :
(1) $p = 4$ (2) $p = \frac{1}{2}$
(3) $p = \frac{1}{4}$ (4) $p = 2$
(SSC CGL Tier-I (CBE) Exam. 02.09.2016 (IInd Sitting)

288. The maximum value of $5 + 20x - 4x^2$, when x is a real number is :
(1) 1 (2) 5
(3) 25 (4) 30
(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IInd Sitting)

289. If $x = at^2$ and $y = 2at$ then
(1) $x^2 = 4ay$
(2) $y^2 = 4ax$
(3) $x^2 + y^2 = a^2$
(4) $x^2 - y^2 = a^2$
(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IInd Sitting)

290. If $\left(a + \frac{1}{b}\right) = 1$ and $\left(b + \frac{1}{c}\right) = 1$, then the value of $\left(c + \frac{1}{a}\right)$ is :
(1) 0 (2) 1
(3) −1 (4) 2
(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IInd Sitting)

291. If $(a - 2) + \frac{1}{(a+2)} = -1$, then the value of $(a + 2)^2 + \frac{1}{(a+2)^2}$ is :
(1) 7 (2) 11
(3) 23 (4) 27
(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IIIrd Sitting)

292. If $a^2 = b + c$, $b^2 = c + a$, $c^2 = a + b$, then the value of $3\left(\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}\right)$ is :
(1) 1 (2) $\frac{1}{3}$
(3) 3 (4) 4
(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IIIrd Sitting)

293. If $x^2 + 5x + 6 = 0$, then the value of $\frac{2x}{x^2 - 7x + 6}$ is :
(1) $\frac{1}{6}$ (2) $\frac{1}{3}$
(3) $-\frac{1}{6}$ (4) $-\frac{1}{3}$
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IInd Sitting)

294. If $a + b = 5$ and $a - b = 3$, then the value of $(a^2 + b^2)$ is :
(1) 17 (2) 18
(3) 19 (4) 20
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IInd Sitting)

295. If $\left(x + \frac{1}{x}\right) = 5$, then find the value of $\frac{6x}{x^2 + x + 1}$
(1) 3 (2) 2
(3) 1 (4) 0
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IIIrd Sitting)

296. If $\frac{3}{(x+2)(2x+1)} = \frac{a}{2x+1} + \frac{b}{x+2}$ be an identify, then the value of b is :
(1) 0 (2) −1
(3) 2 (3) 3
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IIIrd Sitting)

297. If $a + \frac{1}{b} = 1, b + \frac{1}{c} = 1$, then the value of (abc) is :
(1) 0 (2) −1
(3) 1 (4) ab
(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IInd Sitting)

298. If $2x - \frac{1}{2x} = 5, x \neq 0$ then the value of $\left(x^2 + \frac{1}{16x^2} - 2\right)$ is :
(1) $\frac{19}{4}$ (2) $\frac{23}{4}$
(3) $\frac{27}{4}$ (4) $\frac{31}{4}$
(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IInd Sitting)

299. If $a\,(x + y) = b(x - y) = 2ab$, then the value of $2\,(x^2 + y^2)$ is :
(1) $2\,(a^2 - b^2)$ (2) $2\,(a^2 + b^2)$
(3) $4\,(a^2 - b^2)$ (4) $4\,(a^2 + b^2)$
(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IIIrd Sitting)

300. If $\left(x + \frac{1}{x}\right) = 6$, then value of $\left(x^2 + \frac{1}{x^2}\right)$ is :
(1) 23 (2) 16
(3) 34 (4) 32
(SSC CGL Tier-I (CBE) Exam. 08.09.2016 (IIIrd Sitting)

301. If $x^2 - 3x + 1 = 0$, $(x \neq 0)$, then the value of $\left(x + \frac{1}{x}\right)$ is
(1) 1 (2) 0
(3) 3 (4) 2
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IInd Sitting)

302. If $\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$, then the value of $\left(\frac{ab+bc+ca}{abc}\right)$ is
(1) 2 (2) 1
(3) 0 (4) $\frac{1}{2}$
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IInd Sitting)

303. If $\left(x + \frac{1}{x}\right) = 5$, then the value of $\frac{5x}{x^2 + 5x + 1}$ is :
(1) $\frac{1}{3}$ (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) $\frac{1}{5}$
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IIIrd Sitting)

304. If $\left(p^2 + \frac{1}{p^2}\right) = 47$, the value of $\left(p + \frac{1}{p}\right)$ is :
(1) 5 (2) 6
(3) 7 (4) 8
(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IInd Sitting)

305. If $\frac{a}{1-2a}+\frac{b}{1-2b}+\frac{c}{1-2c}=\frac{1}{2}$, then the value of $\frac{1}{1-2a}+\frac{1}{1-2b}+\frac{1}{1-2c}$ is :

(1) 1 (2) 2
(3) 3 (4) 4

(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IIIrd Sitting)

306. If $\left(4x+\frac{1}{x}\right)=5$, $x\neq 0$, then the value of $\frac{5x}{4x^2+10x+1}$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$
(3) $\frac{2}{3}$ (4) 3

(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IIIrd Sitting)

307. If $(a+b)^2=100$ and $(a-b)=4$, then ab equals to :

(1) 116 (2) 84
(3) 21 (4) 53

(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

308. $\frac{x^2+3x+1}{x^2-3x+1}=\frac{1}{2}$, then the value of $\left(x+\frac{1}{x}\right)$ is :

(1) 9 (2) –9
(3) 1 (4) 2

(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

309. What should be added to $8(3x-4y)$ to obtain $(18x-18y)$?

(1) $6x-14y$ (2) $14y+6x$
(3) $14y-6x$ (4) $6xy$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

310. If $4(2x+3)>5-x$ and $5x-3(2x-7)>3x-1$, then x can take which of the following values?

(1) 6 (2) –1
(3) 5 (4) –6

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

311. If $5x-40=3x$, then the numerical value of $(2x-11)$ is

(1) 29 (2) 39
(3) 19 (4) 9

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

312. Which of the following equations has equal roots?

(1) $3x^2-6x+2=0$
(2) $3x^2-6x+3=0$
(3) $x^2-8x+8=0$
(4) $4x^2-8x+2=0$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

313. If $2x-3(4-2x)<4x-5<4x+\frac{2x}{3}$, then x can take which of the following values ?

(1) 2 (2) 8
(3) 0 (4) –8

(SSC CHSL (10+2) Tier-I (CBE) Exam. 16.01.2017) (IInd Sitting)

314. If $a-b=11$ and $ab=24$, then the value of (a^2+b^2) is

(1) 169 (2) 37
(3) 73 (4) 48

(SSC CHSL (10+2) Tier-I (CBE) Exam. 16.01.2017) (IInd Sitting)

315. The simplified form of $(x+(3)^2+(x-1)^2$ is

(1) (x^2+2x+5) (2) $2(x^2+2x+5)$
(3) (x^2-2x+5) (4) $2(x^2-2x+5)$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 16.01.2017) (IInd Sitting)

316. If $a+\frac{1}{b}=1$ and $b+\frac{1}{c}=1$, then the value of $c+\frac{1}{a}$ is

(1) 0 (2) 2
(3) 1 (4) 3

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

317. If $a+b+c+d=4$ then the value of $\frac{1}{(1-a)(1-b)(1-c)}+\frac{1}{(1-b)(1-c)(1-d)}+\frac{1}{(1-c)(1-d)(1-a)}+\frac{1}{(1-d)(1-a)(1-b)}$ is

(1) 0 (2) 1
(3) 4 (4) $1+abcd$

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

318. If $a=\frac{1}{a-5}$ $(a>0)$, then the value of $a+\frac{1}{a}$ is

(1) $\sqrt{29}$ (2) $-\sqrt{27}$
(3) $-\sqrt{29}$ (4) $\sqrt{27}$

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

319. If $a+\frac{1}{b}=b+\frac{1}{c}=c+\frac{1}{a}$ (where $a\neq b\neq c$), then abc is equal to

(1) +1 (2) –1
(3) +1 and –1
(4) None of the options

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

320. If $ax+by=1$ and $bx+ay=\frac{2ab}{a^2+b^2}$ then $(x^2+y^2)(a^2+b^2)$ is equal to

(1) 1 (2) 2
(3) 0.5 (4) 0

(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

TYPE–II

1. If $x=\sqrt{3}+\sqrt{2}$, then the value of $\left(x^3+\frac{1}{x^3}\right)$ is

(1) $6\sqrt{3}$ (2) $12\sqrt{3}$
(3) $18\sqrt{3}$ (4) $24\sqrt{3}$

(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

2. If $x+y=7$, then the value of x^3+y^3+21xy is

(1) 243 (2) 143
(3) 343 (4) 443

(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

3. If $x^{\frac{1}{3}}+y^{\frac{1}{3}}=z^{\frac{1}{3}}$, then $\{(x+y-z)^3+27xyz\}$ equals :

(1) –1 (2) 1
(3) 0 (4) 27

(SSC CPO S.I. Exam. 16.12.2007)

4. If $4b^2+\frac{1}{b^2}=2$, then the value of $8b^3+\frac{1}{b^3}$ is

(1) 0 (2) 1
(3) 2 (4) 5

(SSC CPO S.I. Exam. 09.11.2008)

5. If $2p+\frac{1}{p}=4$, then value of $p^3+\frac{1}{8p^3}$ is

(1) 4 (2) 5
(3) 8 (4) 15

(SSC CGL Tier-I Exam. 16.05.2010 (Second Sitting)

6. If $a^4+b^4=a^2b^2$, then (a^6+b^6) equals

(1) 0 (2) 1
(3) a^2+b^2 (4) $a^2b^4+a^4b^2$

(SSC CPO S.I.Exam.12.12.2010 (Paper-I)

7. If $x + \frac{1}{x} = \sqrt{3}$ then the value of $x^{18} + x^{12} + x^6 + 1$ is
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I)

8. If $x + \frac{1}{x} = 2, x \neq 0$ then value of $x^2 + \frac{1}{x^3}$ is equal to
(1) 1 (2) 2
(3) 3 (4) 4
FCI Assistant Grade-III Exam.25.02.2012 (Paper-I) North Zone (Ist Sitting)

9. If $\frac{a}{b} + \frac{b}{a} = 1, a \neq 0, b \neq 0$ the value of $a^3 + b^3$ is
(1) 0 (2) 1
(3) –1 (4) 2
(SSC CGL Prelim Exam. 04.02.2007 (IInd Sitting) & (FCI Assistant Grade-III Exam. 25.02.2012 (Paper-I) North Zone (Ist Sitting) & (SSC GL Tier-I Exam. 19.05.2013 (Ist Sitting)

10. If $x + \frac{1}{x} = 3$, then the value of $\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$ is :
(1) $\frac{3}{2}$ (2) $\frac{5}{2}$
(3) $\frac{7}{2}$ (4) $\frac{11}{2}$
(SSC CHSL DEO & LDC Exam. 27.11.2010)

11. If $a + \frac{1}{a} + 1 = 0 \ (a \neq 0)$ then the value of $(a^4 - a)$ is :
(1) 0 (2) 1
(3) 2 (4) – 1
(SSC CHSL DEO & LDC Exam. 27.11.2010)

12. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$, then the value of $x^4 + y^4 - 2x^2y^2$ is
(1) 24 (2) 18
(3) 16 (4) 12
(SSC CHSL DEO & LDC Exam. 28.11.2010 (IInd Sitting)

13. If $x + \frac{1}{2x} = 2$, find the value of $8x^3 + \frac{1}{x^3}$.
(1) 48 (2) 88
(3) 40 (4) 44
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

14. If for two real constants a and b, the expression $ax^3 + 3x^2 - 8x + b$ is exactly divisible by $(x + 2)$ and $(x - 2)$, then
(1) $a = 2, b = 12$
(2) $a = 12, b = 2$
(3) $a = 2, b = -12$
(4) $a = -2, b = 12$
(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (North Zone)

15. If $x^2 - 3x + 1 = 0$, then the value of $x^3 + \frac{1}{x^3}$ is
(1) 9 (2) 18
(3) 27 (4) 1
(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (North Zone)

16. If $x + \frac{1}{4x} = \frac{3}{2}$, find the value of $8x^3 + \frac{1}{8x^3}$.
(1) 18 (2) 36
(3) 24 (4) 16
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone)

17. If $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ $(x \neq 0, y \neq 0, x \neq y)$ then, the value of $x^3 - y^3$ is
(1) 0 (2) 1
(3) –1 (4) 2
(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (Delhi Zone)

18. If $x = a(b - c)$, $y = b(c - a)$ and $z = c(a - b)$, then
$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 =$$
(1) $\frac{xyz}{3abc}$ (2) $3\,xyzabc$
(3) $\frac{3xyz}{abc}$ (4) $\frac{xyz}{abc}$
(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (Delhi Zone)

19. If $xy(x + y) = 1$, then the value of $\frac{1}{x^3y^3} - x^3 - y^3$ is :
(1) 0 (2) 1
(3) 3 (4) –2
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone) & (SSC GL Tier-I Exam. 21.04.2013)

20. If $x^4 + \frac{1}{x^4} = 119$ and $x > 1$, then the value of $x^3 - \frac{1}{x^3}$ is
(1) 54 (2) 18
(3) 72 (4) 36
(SSC CHSL DEO & LDC Exam. 11.12.2011 (Ist Sitting (East Zone)

21. If $3x + \frac{1}{2x} = 5$, then the value of $8x^3 + \frac{1}{27x^3}$ is :
(1) $118\frac{1}{2}$ (2) $30\frac{10}{27}$
(3) 0 (4) 1
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone)

22. If $x + y = z$, then the expression $x^3 + y^3 - z^3 + 3xyz$ will be equal to :
(1) 0 (2) $3xyz$
(3) $-3xyz$ (4) z^3
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (East Zone)

23. If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $(x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1)$ is
(1) 1 (2) 2
(3) 3 (4) 4
(SSC Graduate Level Tier-II Exam. 16.09.2012)

24. If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$ is
(1) 0 (2) 1
(3) 84 (4) 206
(SSC Graduate Level Tier-II Exam. 16.09.2012)

25. If $a + \frac{1}{a} = \sqrt{3}$, then the value of $a^6 - \frac{1}{a^6} + 2$ will be

(1) 1 (2) 2

(3) $3\sqrt{3}$ (4) 5

(SSC CHSL DEO & LDC Exam. 21.10.2012 (Ist Sitting)

26. If $x^3 + y^3 = 35$ and $x + y = 5$, then the value of $\frac{1}{x} + \frac{1}{y}$ will be :

(1) $\frac{1}{3}$ (2) $\frac{5}{6}$

(3) 6 (4) $\frac{2}{3}$

(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

27. If $a^3 - b^3 = 56$ and $a - b = 2$, then value of $a^2 + b^2$ will be :

(1) 48 (2) 20

(3) 22 (4) 5

(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

28. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$, then $\frac{a}{b} + \frac{b}{a}$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $-\frac{1}{3}$ (4) $-\frac{2}{3}$

(SSC CHSL DEO & LDC Exam. 28.10.2012 (Ist Sitting)

29. If $x + \frac{1}{x} = 5$, then the value of $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$

(1) $\frac{43}{23}$ (2) $\frac{47}{21}$

(3) $\frac{41}{23}$ (4) $\frac{45}{21}$

(SSC CHSL DEO & LDC Exam. 28.10.2012 (Ist Sitting)

30. If x is real, $x + \frac{1}{x} \neq 0$ and $x^3 + \frac{1}{x^3} = 0$, then the value of $\left(x + \frac{1}{x}\right)^4$ is

(1) 4 (2) 9

(3) 16 (4) 25

(SSC Graduate Level Tier-I Exam. 11.11.2012 (Ist Sitting)

31. If $x + \frac{1}{x} = 3$, then the value of $\left(x^5 + \frac{1}{x^5}\right)$ is

(1) 322 (2) 126

(3) 123 (4) 113

(SSC Graduate Level Tier-I Exam. 11.11.2012 (Ist Sitting) & (SSC CHSL DEO & LDC Exam. 27.10.2013 (IInd Sitting)

32. If $x - \frac{1}{x} = 3$, then value of $x^3 - \frac{1}{x^3}$ is

(1) 32 (2) 36

(3) 40 (4) 49

(SSC Assistant Grade-III Exam. 11.11.2012 (IInd Sitting)

33. If $m^4 + \frac{1}{m^4} = 119$, then $m - \frac{1}{m} = ?$

(1) ± 3 (2) 4

(3) ± 2 (4) ± 1

(SSC Assistant Grade-III Exam. 11.11.2012 (IInd Sitting)

34. If $x + y + z = 6$, then the value of $(x - 1)^3 + (y - 2)^3 + (z - 3)^3$ is

(1) $3(x - 1)(y + 2)(z - 3)$

(2) $3(x + 1)(y - 2)(z - 3)$

(3) $3(x - 1)(y - 2)(z + 3)$

(4) $3(x - 1)(y - 2)(z - 3)$

(SSC Delhi Police S.I.(SI) Exam. 19.08.2012)

35. If $x^2 + 1 = 2x$, then the value of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$ is

(1) 0 (2) 1

(3) 2 (4) –2

(SSC Delhi Police S.I.(SI) Exam. 19.08.2012)

36. If $x = \sqrt{3} + \sqrt{2}$, then the value of $x^3 - \frac{1}{x^3}$ is :

(1) $14\sqrt{2}$ (2) $14\sqrt{3}$

(3) $22\sqrt{2}$ (4) $10\sqrt{2}$

(SSC CHSL DEO & LDC Exam. 04.11.2012, Ist Sitting)

37. If $x > 1$ and $x^2 + \frac{1}{x^2} = 83$, then $x^3 - \frac{1}{x^3}$ is

(1) 764 (2) 750

(3) 756 (4) 760

(SSC FCI Assistant Grade-III Main Exam. 07.04.2013)

38. If $\left(a + \frac{1}{a}\right)^2 = 3$, then $a^3 + \frac{1}{a^3} = ?$

(1) $2\sqrt{3}$ (2) 2

(3) $3\sqrt{3}$ (4) 0

(SSC FCI Assistant Grade-III Main Exam. 07.04.2013)

39. If $\frac{x}{x^2 - 2x + 1} = \frac{1}{3}$, then the value of $x^3 + \frac{1}{x^3}$ is :

(1) 81 (2) 110

(3) 125 (4) 27

(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

40. If, $\left(x + \frac{1}{x}\right) = 4$, then the value of $x^4 + \frac{1}{x^4}$ is :

(1) 64 (2) 194

(3) 81 (4) 124

(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

41. If $x + y + z = 6$ and $x^2 + y^2 + z^2 = 20$ then the value of $x^3 + y^3 + z^3 - 3xyz$ is

(1) 64 (2) 70

(3) 72 (4) 76

(SSC Graduate Level Tier-I Exam. 21.04.2013)

42. If $x = 1 - \sqrt{2}$, the value of $\left(x - \frac{1}{x}\right)^3$ is

(1) – 8 (2) 8

(3) $2\sqrt{2}$ (4) 1

(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

43. If $x = a - b$, $y = b - c$, $z = c - a$, then the numerical value of the algebraic expression $x^3 + y^3 + z^3 - 3xyz$ will be

(1) $a + b + c$ (2) 0

(3) $4(a + b + c)$

(4) $3abc$

(SSC CAPFs SI & CISF ASI Exam. 23.06.2013)

44. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of $x^3 + y^3$ is :
(1) 950 (2) 730
(3) 650 (4) 970
(SSC CAPFs SI & CISF ASI Exam. 23.06.2013)

45. If $(x - a)(x - b) = 1$ and $a - b + 5 = 0$, then the value of $(x-a)^3 - \frac{1}{(x-a)^3}$ is
(1) –125 (2) 1
(2) 125 (4) 140
(SSC Graduate Level Tier-II Exam. 29.09.2013

46. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of $4a - 3b + 5c$ is
(1) 2 (2) 3
(3) 5 (4) 6
(SSC Graduate Level Tier-II Exam. 29.09.2013

47. If $2x + \frac{2}{x} = 3$, then the value of $x^3 + \frac{1}{x^3} + 2$ is
(1) $-\frac{9}{8}$ (2) $-\frac{25}{8}$
(3) $\frac{7}{8}$ (4) 11
(SSC Graduate Level Tier-II Exam. 29.09.2013

48. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$ then the value of $a^3 + b^3 + c^3 - 3abc$
(1) 200 (2) 180
(3) 190 (4) 210
(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

49. If $a - b = 3$ and $a^3 - b^3 = 117$ then $|a+b|$ is equal to
(1) 3 (2) 5
(3) 7 (4) 9
(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

50. If $x + \frac{1}{x+1} = 1$, then $(x+1)^5 + \frac{1}{(x+1)^5}$ equals
(1) 1 (2) 2
(3) 4 (4) 8
(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

51. If $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$, then the value of $a^3 + b^3$ is
(1) 0 (2) –1
(3) 1 (4) 2
(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

52. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to
(1) $a + b + c$ (2) abc
(3) $2\,abc$ (4) $3\,abc$
(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

53. If a = 4,965, b = 2.343 and c = 2.622, then the value of $a^3 - b^3 - c^3 - 3abc$ is :
(1) – 2 (2) – 1
(3) 0 (4) 9.9^3
(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

54. If $a = 1.21$, $b = 2.12$ and $c = -3.33$, then the value of $a^3 + b^3 + c^3 - 3abc$ is
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Prelim Exam. 24.02.2002 (Middle Zone)

55. If $p = 999$, then the value of $\sqrt[3]{p\left(p^2+3p+3\right)+1}$ is
(1) 1000 (2) 999
(3) 998 (4) 1002
(SSC CGL Prelim Exam. 11.05.2003 & 27.07.2008 (Second Sitting)

56. If $a = 4\,.\,36$, $b = 2.39$ and $c = 1.97$, then the value of $a^3 - b^3 - c^3 - 3abc$ is
(1) 3.94 (2) 2.39
(3) 0 (4) 1
(SSC CGL Prelim Exam. 13.11.2005 (Second Sitting)

57. $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)\left(x^2+\frac{1}{x^2}+1\right)$ is equal to
(1) $x^6 + \frac{1}{x^6}$ (2) $x^8 + \frac{1}{x^8}$
(3) $x^8 - \frac{1}{x^8}$ (4) $x^6 - \frac{1}{x^6}$
(SSC CPO S.I. Exam. 03.09.2006)

58. If $a = 11$ and $b = 9$, then the value of $\left(\frac{a^2+b^2+ab}{a^3-b^3}\right)$ is
(1) $\frac{1}{2}$ (2) 2
(3) $\frac{1}{20}$ (4) 20
(SSC CGL Tier-I Exam. 16.05.2010 (First Sitting)

59. If $a = \sqrt{7+2\sqrt{12}}$ and $b = \sqrt{7-2\sqrt{12}}$, then $\left(a^3+b^3\right)$ is equal to
(1) 40 (2) 44
(3) 48 (4) 52
(SSC SAS Exam. 26.06.2010 (Paper-1)

60. If the sum of $\frac{a}{b}$ and its reciprocal is 1 and a ≠ 0, b ≠ 0, then the value of $a^3 + b^3$ is
(1) 2 (2) –1
(3) 0 (4) 1
(SSC CPO (SI, ASI & Intelligence Officer) Exam 28.08.2011 (Paper-I)

61. If $x = 2 - 2^{1/3} + 2^{2/3}$, then the value of $x^3 - 6x^2 + 18x + 18$ is
(1) 22 (2) 33
(3) 40 (4) 45
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

62. If $a^3 - b^3 - c^3 - 3abc = 0$, then
(1) $a = b = c$
(2) $a + b + c = 0$
(3) $a + c = b$
(4) $a = b + c$
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

63. If p, q, r are all real numbers, then $(p - q)^3 + (q - r)^3 + (r - p)^3$ is equal to
(1) $(p - q)(q - r)(r - p)$
(2) $3(p - q)(q - r)(r - p)$
(3) 0
(4) 1
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (East Zone) (IInd Sitting (North Zone)

64. If $a = 2{\cdot}361$, $b = 3{\cdot}263$ and $c = 5{\cdot}624$, then the value of $a^3 + b^3 - c^3 + 3abc$ is
(1) 35·621 (2) 0
(3) 19·277 (4) 1
(SSC CHSL DEO & LDC Exam. 04.12.2011 (IInd Sitting (East Zone)

65. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$ and $a^3 + b^3 + c^3 = 36$, then the value of abc is
(1) 3 (2) 6
(3) 9 (4) 12
(SSC Graduate Level Tier-II Exam. 16.09.2012)

66. If $a + b = 1$ and $a^3 + b^3 + 3ab = k$, then the value of k is
(1) 1 (2) 3
(3) 5 (4) 7
(SSC CHSL DEO & LDC Exam. 04.11.2012 (IInd Sitting)

67. If $a = 34$, $b = c = 33$, then the value of $a^3 + b^3 + c^3 - 3abc$ is
(1) 0 (2) 111
(3) 50 (4) 100
(SSC CHSL DEO & LDC Exam. 28.10.2012, Ist Sitting)

68. If $x = y = 333$ and $z = 334$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is
(1) 0 (2) 667
(3) 1000 (4) 2334
(SSC Graduate Level Tier-II Exam. 29.09.2013)

69. Out of the given responses, one of the factors of
$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is
(1) $(a + b)(a - b)$
(2) $(a + b)(a + b)$
(3) $(a - b)(a - b)$
(4) $(b - c)(b - c)$
(SSC Graduate Level Tier-II Exam. 29.09.2013)

70. If $a = \frac{b^2}{b-a}$ then the value of $a^3 + b^3$ is
(1) $6ab$ (2) 0
(3) 1 (4) 2
(SSC CHSL DEO & LDC Exam. 20.10.2013)

71. If p = 99, then value of $p(p^2 + 3p + 3)$ is
(1) 999 (2) 9999
(3) 99999 (4) 999999
(SSC CGL Prelim Exam. 04.02.2007 (Second Sitting)

72. If $p = 101$, then the value of $\sqrt[3]{p(p^2 - 3p + 3) - 1}$ is
(1) 100 (2) 101
(3) 102 (4) 1000
(SSC SAS Exam. 26.06.2010 (Paper-1)

73. If p = 124,
$\sqrt[3]{p(p^2 + 3p + 3) + 1} = ?$
(1) 5 (2) 7
(3) 123 (4) 125
(SSC CGL Tier-1 Exam. 19.06.2011 (First Sitting)

74. If $p - 2q = 4$, then the value of $p^3 - 8q^3 - 24pq - 64$ is :
(1) 2 (2) 0
(3) 3 (4) -1
(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

75. If $x = 19$ and $y = 18$, then the value of $\frac{x^2 + y^2 + xy}{x^3 - y^3}$ is
(1) 1 (2) 37
(3) 324 (4) 361
(SSC CISF ASI Exam. 29.08.2010 (Paper-1)

76. If $x + \frac{1}{x} = 2$ and x is real, then the value of $x^{17} + \frac{1}{x^{19}}$ is
(1) 1 (2) 0
(3) 2 (4) -2
(SSC CHSL DEO & LDC Exam. 04.12.2011 (Ist Sitting (North Zone)

77. The value of $(x + y + z)^3 - (y + z - x)^3 - (z + x - y)^3 - (x + y - z)^3$ is :
(1) $12xyz$ (2) $24xyz$
(3) $36xyz$ (4) 0
(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

78. If $x = -1$, then the value of
$$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$$
is
(1) 1 (2) 0
(3) -2 (4) -1
(SSC Multi-Tasking Staff Exam. 17.03.2013, Kolkata Region)

79. If $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$ and a, b, c are rational numbers, then $a + b + c$ is equal to
(1) 0 (2) 1
(3) 2 (4) 3
(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

80. If $x = \sqrt[3]{2 + \sqrt{3}}$, then the value of $x^3 + \frac{1}{x^3}$ is
(1) 8 (2) 9
(3) 2 (4) 4
(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

81. If $x = \sqrt[3]{5} + 2$, then the value of $x^3 - 6x^2 + 12x - 13$ is
(1) -1 (2) 1
(3) 2 (4) 0
(SSC Graduate Level Tier-II Exam. 29.09.2013

82. If $x + y = a$ and $xy = b^2$, then the value of $x^3 - x^2y - xy^2 + y^3$ in terms of a and b is :
(1) $(a^2 + 4b^2)\,a$ (2) $a^3 - 3b^2$
(3) $a^3 - 4b^2 a$ (4) $a^3 + 3b^2$
(SSC CHSL DEO & LDC Exam. 11.12.2011 (IInd Sitting (Delhi Zone)

83. If $x - \frac{1}{x} = 1$, then the value of $\frac{x^4 - \frac{1}{x^2}}{3x^2 + 5x - 3}$ is
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$
(3) $\frac{3}{4}$ (4) 0
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

84. If $x + y = 15$, then $(x - 10)^3 + (y - 5)^3$ is
(1) 25 (2) 125
(3) 625 (4) 0
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

85. If $x^2 + \frac{1}{x^2} = 66$, then the value of $\frac{x^2 - 1 + 2x}{x} = ?$
(1) ± 8 (2) 10, -6
(3) 6, -10 (4) ± 4
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

86. If $a^2 + a + 1 = 0$, then the value of a^9 is
(1) 2 (2) 3
(3) 1 (4) 0
(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

87. If $x+\frac{2}{x}=1$, then the value of $\frac{x^2+x+2}{x^2(1-x)}$ is

(1) 1 (2) –1
(3) 2 (4) –2

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

88. If $x = k^3 - 3k^2$ and $y = 1 - 3k$, then for what value of k, will be $x = y$?

(1) 0 (2) 1
(3) –1 (4) 2

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

89. Find the value of $\sqrt{(x^2+y^2+z)(x+y-3z)} \div \sqrt[3]{xy^3z^2}$ when $x = +1$, $y = -3$, $z = -1$.

(1) 1 (2) 0
(3) –1 (4) $\frac{1}{2}$

(SSC CGL Tier-I Re-Exam. (2013) 27.04.2014)

90. The simplest form of the expression

$\frac{p^2-p}{2p^3+6p^2} \div \frac{p^2-1}{p^2+3p} \div \frac{p^2}{p+1}$ is

(1) $2p^2$ (2) $\frac{1}{2p^2}$
(3) $p + 3$ (4) $\frac{1}{p+3}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

91. If $x+\frac{1}{x}=2$, then the value of $\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$ is

(1) 20 (2) 4
(3) 8 (4) 16

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

92. If a, b, c be all positive integers, then the least positive value of $a^3 + b^3 + c^3 - 3abc$ is

(1) 1 (2) 2
(3) 4 (4) 3

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

93. When $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $(3x + 2)$, then the remainder is

(1) 2 (2) 0
(3) –1 (4) 1

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

94. If $ab + bc + ca = 0$, then the value of

$\frac{1}{a^2-bc}+\frac{1}{b^2-ac}+\frac{1}{c^2-ab}$ is

(1) 2 (2) –1
(3) 0 (4) 1

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

95. If the equation $2x^2 - 7x + 12 = 0$ has two roots α and β, then the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is

(1) $\frac{7}{2}$ (2) $\frac{1}{24}$
(3) $\frac{7}{24}$ (4) $\frac{97}{24}$

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

96. If $x^3+\frac{3}{x} = 4\ (a^3 + b^3)$ and $3x+\frac{1}{x^3} = 4\ (a^3 - b^3)$, then $a^2 - b^2$ is equal to

(1) 4 (2) 0
(3) 1 (4) 2

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

97. If $x = 6+\frac{1}{x}$, then the value of $x^4+\frac{1}{x^4}$ is

(1) 1448 (2) 1442
(3) 1444 (4) 1446

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (IInd Sitting)

98. If $x+\frac{1}{x}=5$, then $x^6+\frac{1}{x^6}$ is

(1) 12098 (2) 12048
(3) 14062 (4) 12092

(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

99. If $x^2 - 3x + 1 = 0$, then the value of $\frac{x^6+x^4+x^2+1}{x^3}$ will be

(1) 18 (2) 15
(3) 21 (4) 30

(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

100. If $x^4+\frac{1}{x^4}=119$ and $x > 1$, then find the positive value of $x^3-\frac{1}{x^3}$.

(1) 25 (2) 27
(3) 36 (4) 49

(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

101. If $\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$ and $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}=0$, where p, q, r and a, b, c are non-zero, then the value of $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}$ is

(1) –1 (2) 0
(3) 1 (4) 2

(SSC CGL Tier-I Exam. 19.10.2014)

102. If x is a rational number and $\frac{(x+1)^3-(x-1)^3}{(x+1)^2-(x-1)^2} = 2$, then the sum of numerator and denominator of x is

(1) 3 (2) 4
(3) 5 (4) 7

(SSC CGL Tier-I Exam. 19.10.2014)

103. If $x = \sqrt{5} + 2$, then the value $\frac{2x^2-3x-2}{3x^2-4x-3}$ is equal to

(1) 0.185 (2) 0.525
(3) 0.625 (4) 0.785

(SSC CGL Tier-I Exam. 19.10.2014)

104. If $a = 2.234$, $b = 3.121$ and $c = -5.355$, then the value of $a^3+b^3 + c^3 - 3\ abc$ is

(1) –1 (2) 0
(3) 1 (4) 2

(SSC CGL Tier-I Exam. 19.10.2014)

105. If $x^2 + y^2 + 1 = 2x$, then the value of $x^3 + y^5$ is

(1) 2 (2) 0
(3) –1 (4) 1

(SSC CGL Tier-I Exam. 19.10.2014)

106. If $3(a^2 + b^2 + c^2) = (a + b + c)^2$, then the relation between a, b and c is

(1) $a = b = c$ (2) $a = b \neq c$
(3) $a < b < c$ (4) $a > b > c$

(SSC CGL Tier-I Exam. 19.10.2014)

107. If $x(x - 3) = -1$, then the value of $x^3(x^3 - 18)$ is

(1) -1 (2) 2
(3) 1 (4) 0

(SSC CGL Tier-I Exam. 26.10.2014)

108. If $a^2 + b^2 + c^2 = ab + bc + ac$ then the value of $\frac{a+C}{b}$ is

(1) 0 (2) 2
(3) 1 (4) -1

(SSC CGL Tier-II Exam. 21.09.2014)

109. If $ab + bc + ca = 0$ then the value of $\left(\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}\right)$ is

(1) 0 (2) 1
(3) 3 (4) $a + b + c$

(SSC CGL Tier-II Exam. 21.09.2014)

110. If $3x+\frac{3}{x}=1$ then $x^3+\frac{1}{x^3}+1$ is

(1) 0 (2) $\frac{1}{27}$
(3) $\frac{5}{27}$ (4) $\frac{28}{27}$

(SSC CGL Tier-II Exam. 21.09.2014)

111. The factors of $(a^2 + 4b^2 + 4b - 4ab - 2a - 8)$ are

(1) $(a - 2b - 4)(a - 2b + 2)$
(2) $(a - b + 2)(a - 4b - 4)$
(3) $(a + 2b - 4)(a + 2b + 2)$
(4) $(a + 2b - 1)(a - 2b + 1)$

(SSC CGL Tier-II Exam. 21.09.2014)

112. The value of

$$\frac{1}{a^2+ax+x^2}-\frac{1}{a^2-ax+x^2}+\frac{2ax}{a^4+a^2x^2+x^4}$$ is

(1) 2 (2) 1
(3) -1 (4) 0

(SSC CGL Tier-II Exam. 21.09.2014)

113. If $x = 11$, then the value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$ is

(1) 5 (2) 10
(3) 15 (4) 20

(SSC CGL Tier-II Exam. 21.09.2014)

114. If p = 99, then the value of $p(p^2 + 3p + 3)$ is

(1) 10000000 (2) 999000
(3) 999999 (4) 990000

(SSC CGL Tier-II Exam. 21.09.2014)

115. An example of an equality relation of two expressions in x, which is not an identity is

(1) $(x + 3)^2 = x^2 + 6x + 9$
(2) $(x + 2y)^3 = x^3 + 8y^3 + 6xy(x+2y)$
(3) $(x + 2)^2 = x^2 + 2x + 4$
(4) $(x + 3)(x - 3) = x^2 - 9$

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014)

116. The numerical value of

$$\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(c-a)(a-b)}+\frac{(c-a)^2}{(a-b)(b-c)}$$ is $(a \neq b \neq c)$

(1) 0 (2) 1
(3) $\frac{1}{3}$ (4) 3

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014)

117. If $\left(a+\frac{1}{a}\right)^2 = 3$, then the value of $a^3+\frac{1}{a^3}$ is

(1) 0 (2) 1
(3) 2 (4) 6

(SSC CHSL DEO & LDC Exam. 02.11.2014 (IInd Sitting)

118. If $a+\frac{1}{a}=\sqrt{3}$, then the value of $a^{18} + a^{12} + a^6 + 1$ is

(1) 0 (2) 1
(3) -1 (4) 4

(SSC CHSL DEO & LDC Exam. 02.11.2014 (IInd Sitting)

119. If $x = 997$, $y = 998$ and $z = 999$, then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ is

(1) 0 (2) 1
(3) -1 (4) 3

(SSC CHSL DEO & LDC Exam. 02.11.2014 (IInd Sitting)

120. If $x+\frac{1}{x}=3$, then the value of $\frac{3x^2-4x+3}{x^2-x+1}$ is

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$
(3) $\frac{5}{2}$ (4) $\frac{5}{3}$

(SSC CHSL DEO & LDC Exam. 9.11.2014)

121. If $x = 3 + 2\sqrt{2}$, then $\frac{x^6+x^4+x^2+1}{x^3}$ is equal to

(1) 216 (2) 192
(3) 198 (4) 204

(SSC CHSL CGLDEO & LDC Exam. 9.11.2014)

122. If $x = p + \frac{1}{p}$ and $y = p - \frac{1}{p}$ then the value of $x^4 - 2x^2y^2 + y^4$ is

(1) 24 (2) 4
(3) 16 (4) 8

(SSC CHSL DEO & LDC Exam. 9.11.2014)

123. If $a + b + c = 0$, then the value of $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$ is

(1) 0 (2) $8abc$
(3) $4(a^2 + b^2 + c^2)$
(4) $4(ab + bc + ca)$

(SSC CHSL DEO & LDC Exam. 16.11.2014)

124. If $p^3 + 3p^2 + 3p = 7$, then the value of $p^2 + 2p$ is

(1) 4 (2) 3
(3) 5 (4) 6

(SSC CHSL DEO & LDC Exam. 16.11.2014)

125. If $x = 2015$, $y = 2014$ and $z = 2013$, then value of $x^2 + y^2 + z^2 - xy - yz - zx$ is

(1) 3 (2) 4
(3) 6 (4) 2

(SSC CHSL DEO & LDC Exam. 16.11.2014)

126. If $3a^2 = b^2 \neq 0$, then the value of $\frac{(a+b)^3-(a-b)^3}{(a+b)^2+(a-b)^2}$ is

(1) $\frac{3b}{2}$ (2) b
(3) $\frac{b}{2}$ (4) $\frac{2b}{3}$

(SSC CHSL DEO & LDC Exam. 16.11.2014)

127. If $x > 1$ and $x+\frac{1}{x}=2\frac{1}{12}$, then the value of $x^4-\frac{1}{x^4}$ is

(1) $\frac{58975}{20736}$ (2) $\frac{59825}{20736}$

(3) $\frac{57985}{20736}$ (4) $\frac{57895}{20736}$

(SSC CHSL DEO & LDC Exam. 16.11.2014)

128. The value of $\frac{4x^3-x}{(2x+1)(6x-3)}$ when $x = 9999$ is

(1) 1111 (2) 2222

(3) 3333 (4) 6666

(SSC CHSL DEO Exam. 02.11.2014 (Ist Sitting)

129. If $a^3 + b^3 = 9$ and $a + b = 3$, then the value of $\frac{1}{a}+\frac{1}{b}$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) –1

(SSC CHSL DEO Exam. 02.11.2014 (Ist Sitting)

130. If $t^2 - 4t + 1 = 0$, then the value of $t^3+\frac{1}{t^3}$ is

(1) 44 (2) 48

(3) 52 (4) 64

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

131. If $\sqrt[3]{a}+\sqrt[3]{b}=\sqrt[3]{c}$, then the simplest value of $(a+b-c)^3+27abc$ is

(1) – 1 (2) 3

(3) – 3 (4) 0

(SSC CHSL DEO Exam. 16.11.2014 (Ist Sitting)

132. If $p=\frac{5}{18}$, then

$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

is equal to

(1) $\frac{4}{27}$ (2) $\frac{5}{27}$

(3) $\frac{8}{27}$ (4) $\frac{10}{27}$

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

133. If $x+\frac{1}{x}=2$, then

$x^{2013}+\frac{1}{x^{2014}}=?$

(1) 0 (2) 1

(3) –1 (4) 2

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

134. If $a = 331$, $b = 336$ and $c = -667$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

(1) 1 (2) 6

(3) 3 (4) 0

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam. 22.06.2014 TF No. 999 KP0)

135. If $a = 4.965$, $b = 2.343$ and $c = 2.622$, then the value of $a^3 - b^3 - c^3 - 3abc$ is

(1) –2 (2) –1

(3) 0 (4) 9.93

(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

136. If $x + y + z = 0$, then the value of $\frac{x^2+y^2+z^2}{x^2-yz}$ is

(1) –1 (2) 0

(3) 1 (4) 2

(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

137. If $x+\frac{1}{x}=0$, then the value of $x^5+\frac{1}{x^5}$ is

(1) 2 (2) – 1

(3) 1 (4) 0

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting TF No. 333 LO 2)

138. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then

(1) $a = b = c$ (2) $a \neq b = c$

(3) $a = b \neq c$ (4) $a \neq b \neq c$

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting TF No. 333 LO 2)

139. If $x^4+\frac{1}{x^4}=119$, then the values of $x^3+\frac{1}{x^3}$ are

(1) $\pm 10\sqrt{13}$ (2) $\pm\sqrt{13}$

(3) $\pm 16\sqrt{13}$ (4) $\pm 13\sqrt{13}$

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting TF No. 333 LO 2)

140. If $x+\frac{1}{x}=\sqrt{3}$, then the value of $x^{30} + x^{24} + x^{18} + x^{12} + x^6 + 1$ is

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) 1 (4) 0

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014 , Ist Sitting TF No. 333 LO 2)

141. If $m + n = -2$, then the value of $m^3 + n^3 - 6mn$ is

(1) 8 (2) 4

(3) –8 (4) –4

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014, IInd Sitting TF No. 545 QP 6)

142. If $u_n=\frac{1}{n}-\frac{1}{n+1}$ then the value of $u_1 + u_2 + u_3 + u_4 + u_5$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) $\frac{2}{5}$ (4) $\frac{5}{6}$

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014, IInd Sitting TF No. 545 QP 6)

143. If $x = 5$, $y = 6$ and $z = -11$, then the value of $x^3 + y^3 + z^3$ is

(1) –890 (2) –970

(3) –870 (4) –990

(SSC CHSL (10+2) DEO & LDC Exam. 16.11.2014, IInd Sitting TF No. 545 QP 6)

144. If $p + m = 6$ and $p^3 + m^3 = 72$, then the value of pm is

(1) 6 (2) 12

(3) 9 (4) 8

(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

145. If average of two numbers x and $\frac{1}{x}$ (where $x \neq 0$) is A, what will be the average of x^3 and $\frac{1}{x^3}$?

(1) $4A^3 - 2A$ (2) $4A^3 - 3A$

(3) $4A^3 - 4A$ (4) $4A^3 - A$

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

146. If $a = 2 + \sqrt{3}$, then the value of $\frac{a^6 + a^4 + a^2 + 1}{a^3}$ is

(1) 45 (2) 65
(3) 42 (4) 56

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

147. If $x = \sqrt{5} + \sqrt{3}$ and $y = \sqrt{5} - \sqrt{3}$, then the value of $(x^4 - y^4)$ is

(1) $64\sqrt{15}$ (2) 16
(3) 544 (4) $32\sqrt{15}$

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

148. If $x + y + z = 6$, then the value of $(x - 1)^3 + (y - 2)^3 + (z - 3)^3$ is

(1) $3(x - 1)(y - 2)(z - 3)$
(2) $3xyz$
(3) $(x - 1)(y - 2)(z - 3)$
(4) $2(x - 1)(y - 2)(z - 3)$

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

149. If $p^4 = 119 - \frac{1}{p^{4'}}$ then the value of $p^3 - \frac{1}{p^3}$ is

(1) 24 (2) 32
(3) 36 (4) 18

(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

150. If $x + \left(\frac{1}{x}\right) = 2$, then the value of $x^7 + \left(\frac{1}{x^5}\right)$ is

(1) 2^{12} (2) 2
(3) 2^5 (4) 2^7

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

151. If $x = 332$, $y = 333$, $z = 335$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is

(1) 10000 (2) 7000
(3) 8000 (4) 9000

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

152. If $m = -4$, $n = -2$, then the value of $m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$ is

(1) – 126 (2) 124
(3) – 124 (4) 126

(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

153. If $x + \frac{1}{x} = 2$ then the value of $x^{12} - \frac{1}{x^{12}}$ is

(1) 2 (2) – 4
(3) 0 (4) 4

(SSC CGL Tier-I Exam, 09.08.2015 (IInd Sitting) TF No. 4239378)

154. Given that $x^3 + y^3 = 72$ and $xy = 6$ with $x > y$. Then the value of $(x - y)$ is

(1) 4 (2) – 4
(3) 2 (4) – 2

(SSC CGL Tier-I Exam, 09.08.2015 (IInd Sitting) TF No. 4239378)

155. If $x = 2$ then the value of $x^3 + 27x^2 + 243x + 631$ is

(1) 1233 (2) 1211
(3) 1231 (4) 1321

(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

156. If $\frac{x^{24} + 1}{x^{12}} = 7$ then the value of $\frac{x^{72} + 1}{x^{36}}$ is

(1) 433 (2) 322
(3) 343 (4) 432

(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

157. The HCF of $x^8 - 1$ and $x^4 + 2x^3 - 2x - 1$ is :

(1) $x^2 + 1$ (2) $x^2 - 1$
(3) $x + 1$ (4) $x - 1$

(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

158. If $x^2 + y^2 + z^2 = 2(x + z - 1)$, then the value of :
$x^3 + y^3 + z^3 = ?$

(1) 2 (2) 0
(3) –1 (4) 1

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

159. If $x^2 + x = 5$ then the value of $(x + 3)^3 + \frac{1}{(x+3)^3}$ is :

(1) 140 (2) 110
(3) 130 (4) 120

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

160. If $x = z = 225$ and $y = 226$ then the value of :
$x^3 + y^3 + z^3 - 3xyz$ is

(1) 765 (2) 676
(3) 576 (4) 674

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

161. If $4a - \frac{4}{a} + 3 = 0$ then the value of : $a^3 - \frac{1}{a^3} + 3 = ?$

(1) $\frac{3}{16}$ (2) $\frac{7}{16}$
(3) $\frac{21}{64}$ (4) $\frac{21}{16}$

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

162. If $a + b - c = 0$ then the value of $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

(1) 7 (2) 0
(3) 14 (4) 28

(SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

163. If $\frac{p^2}{q^2} + \frac{q^2}{p^2} = 1$, then the value of $(p^6 + q^6)$ is

(1) 0 (2) 1
(3) 2 (4) 3

(SSC CGL Tier-I Re-Exam, 30.08.2015)

164. If $(m + 1) = \sqrt{n} + 3$, the value of $\frac{1}{2}\left(\frac{m^3 - 6m^2 + 12m - 8}{\sqrt{n}} - n\right)$ is

(1) 0 (2) 1
(3) 2 (4) 3

(SSC CGL Tier-I Re-Exam, 30.08.2015)

165. If $(3x - 2y) : (2x + 3y) = 5 : 6$, then one of the values of

$$\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$$ is

(1) $\frac{1}{5}$ (2) 5

(3) 25 (4) $\frac{1}{25}$

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

166. If $a - \frac{1}{a-3} = 5$, then the value of $(a-3)^3 - \frac{1}{(a-3)^3}$ is

(1) 5 (2) 7

(3) 2 (4) 14

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

167. If $\left(\frac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{3}} \div \left(\frac{p^6\,q^{-3}}{p^{-2}q^3}\right)^{\frac{1}{3}} = p^a q^b$, then the value of $a + b$, where p and q are different positive primes, is

(1) –1 (2) 2

(3) 1 (4) 0

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

168. If $a + b = 1$, find the value of $a^3 + b^3 - ab - (a^2 - b^2)^2$.

(1) –1 (2) 1

(3) 0 (4) 2

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

169. If $x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}, y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$ then value of $(x^4 - x^2y^2 - 1) + (y^4 - x^2y^2 + 1)$ is

(1) 16 (2) 13

(3) 12 (4) 14

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

170. If $x^2 + y^2 + z^2 = xy + yz + zx$, then the value of

$$\frac{3x^4+7y^4+5z^4}{5x^2y^2+7y^2z^2+3z^2x^2}$$ is

(1) 2 (2) 1

(3) 0 (4) –1

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

171. If $x - \sqrt{3} - \sqrt{2} = 0$ and $y - \sqrt{3} + \sqrt{2} = 0$, then the value of $\left(x^3 - 20\sqrt{2}\right) - \left(y^3 + 20\sqrt{2}\right)$ is

(1) 0 (2) 1

(3) 3 (4) 2

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

172. If $p^3 - q^3 = (p-q)\{(p-q)^2 - xpq\}$, then find the value of x

(1) 3 (2) –3

(3) 1 (4) –1

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 01.11.2015, IInd Sitting)

173. If $x + y + z = 6$ and $xy + yz + zx = 10$ then the value of $x^3 + y^3 + z^3 - 3xyz$ is :

(1) 36 (2) 48

(3) 42 (4) 40

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

174. If $x - \frac{1}{x} = 2$, then the value of $x^3 - \frac{1}{x^3}$ is :

(1) 15 (2) 2

(3) 14 (4) 11

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

175. If $a^2 + a + 1 = 0$, then the value of $a^5 + a^4 + 1$ is :

(1) a^2 (2) 1

(3) 0 (4) $a + 1$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

176. If $x = a(b - c)$, $y = b(c - a)$, $z = c(a - b)$, then the value of $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$ is :

(1) $\frac{2xyz}{abc}$ (2) $\frac{xyz}{abc}$

(3) 0 (4) $\frac{3xyz}{abc}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

177. If $x = y = z$, then $\frac{(x+y+z)^2}{x^2+y^2+z^2}$ is equal to

(1) 4 (2) 2

(3) 3 (4) 1

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (IInd Sitting) TF No. 7203752)

178. The simplified value of following is :

$$\left(\frac{3}{15}a^5b^6c^3 \times \frac{5}{9}ab^5c^4\right) \div \frac{10}{27}a^2bc^3$$

(1) $\frac{9a^2bc^4}{10}$ (2) $\frac{3ab^4c^3}{10}$

(3) $\frac{3a^4b^{10}c^4}{10}$ (4) $\frac{1a^4b^4c^{10}}{10}$

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

179. If $(2a-1)^2 + (4b-3)^2 + (4c+5)^2 = 0$, then the value of $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2}$ is

(1) $1\frac{3}{8}$ (2) $2\frac{3}{8}$

(3) $3\frac{3}{8}$ (4) 0

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (IInd Sitting) TF No. 3441135)

180. If $x + \frac{1}{x} = 3$, then the value of $x^5 + \frac{1}{x^5}$ is

(1) 110 (2) 132

(3) 122 (4) 123

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 20.12.2015 (Ist Sitting) TF No. 9692918)

181. When $2x + \frac{2}{x} = 3$, then value of $\left(x^3 + \frac{1}{x^3} + 2\right)$ is

(1) $\frac{2}{7}$ (2) $\frac{7}{8}$

(3) $\frac{7}{2}$ (4) $\frac{8}{7}$

(SSC CGL Tier-I (CBE) Exam.10.09.2016)

182. If $x = \sqrt[3]{x^2+11}-2$, then the value of $(x^3 + 5x^2 + 12x)$ is
(1) 0 (2) 3
(3) 7 (4) 11
(SSC CGL Tier-I (CBE) Exam.10.09.2016)

183. If x, y, and z are real numbers such that $(x-3)^2 + (y-4)^2 + (z-5)^2 = 0$ then, $(x + y + z)$ is equal to
(1) –12 (2) 0
(3) 8 (4) 12
(SSC CGL Tier-I (CBE) Exam.11.09.2016) (Ist Sitting)

184. If $(x - 4)(x^2 + 4x + 16) = x^3 - p$, then p is equal to
(1) 27 (2) 8
(3) 64 (4) 0
(SSC CGL Tier-I (CBE) Exam.11.09.2016) (Ist Sitting)

185. The simplified value of

$$\left(1-\frac{2xy}{x^2+y^2}\right)\div\left(\frac{x^3-y^3}{x-y}-3xy\right)$$ is

(1) $\frac{1}{x^2-y^2}$ (2) $\frac{1}{x^2+y^2}$
(3) $\frac{1}{x-y}$ (4) $\frac{1}{x+y}$
(SSC CGL Tier-II Online Exam.01.12.2016)

186. If $a + b + c = 0$ then the value of

$$\frac{1}{(a+b)(b+c)}+\frac{1}{(b+c)(c+a)}+\frac{1}{(c+a)(a+b)}$$ is

(1) 0 (2) 1
(3) 3 (4) 2
(SSC CGL Tier-II Online Exam.01.12.2016)

187. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{31} + y^{35}$ is
(1) –1 (2) 0
(3) 1 (4) 2
(SSC CGL Tier-II Online Exam.01.12.2016)

188. If $\left(x-\frac{1}{x}\right)^2 = 3$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ equals
(1) 90 (2) 100
(3) 110 (4) 120
(SSC CGL Tier-II Online Exam.01.12.2016)

189. If $x^4 + 2x^3 + ax^2 + bx + 9$ is a perfect square, where a and b are positive real numbers, then the values of a and b are
(1) $a = 5$, $b = 6$
(2) $a = 6$, $b = 7$
(3) $a = 7$, $b = 6$
(4) $a = 7$, $b = 8$
(SSC CGL Tier-II Online Exam.01.12.2016)

190. If $a^2 + b^2 + c^2 = 16$, $x^2 + y^2 + z^2 = 25$ and $ax + by + cz = 20$, then the value of $\frac{a+b+c}{x+y+z}$ is
(1) $\frac{3}{5}$ (2) $\frac{5}{3}$
(3) $\frac{4}{5}$ (4) $\frac{5}{4}$
(SSC CGL Tier-II Online Exam.01.12.2016)

191. The value of x which satisfies the equation $\frac{x+a^2+2c^2}{b+c}+\frac{x+b^2+2a^2}{c+a}+\frac{x+c^2+2b^2}{a+b}=0$ is
(1) $(a^2 + b^2 + c^2)$
(2) $-(a^2 + b^2 + c^2)$
(3) $(a^2 + 2b^2 + c^2)$
(4) $-(a^2 + b^2 + 2c^2)$
(SSC CGL Tier-II Online Exam.01.12.2016)

192. If $a^3 = 117 + b^3$ and $a = 3 + b$, then the value of $(a + b)$ is :
(1) ± 7 (2) ± 49
(3) ± 13 (4) 0
(SSC CGL Tier-II Online Exam.01.12.2016)

193. If $\left(a+\frac{1}{a}\right) = -2$, then the value of $a^{1000} + a^{-1000}$ is
(1) 2 (2) 0
(3) 1 (4) $\frac{1}{2}$
(SSC CGL Tier-II Online Exam.01.12.2016)

194. If $a^2 = b + c$, $b^2 = a + c$, $c^2 = b + a$, then what will be the value of $\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}$?
(1) –1 (2) 2
(3) 1 (4) 0
(SSC CPO SI, ASI Online Exam.05.06.2016) (IInd Sitting)

195. If a, b, c and d satisfy the equations
$a + 7b + 3c + 5d = 0$,
$8a + 4b + 6c + 2d = -4$
$2a + 6b + 4c + 8d = 4$,
$5a + 3b + 7c + d = -4$,
then $(a + d)/(b + c) = ?$
(1) 0 (2) 1
(3) –1 (4) –4
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

196. If $\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)}$ then $(x + y + z)$ is
(1) $a + b + c$ (2) 0
(3) $a^2 + b^2 + c^2$ (4) 2
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

197. If $a+\frac{1}{a} = 3$ then $a^3 +1\frac{1}{a^3}$ is
(1) 27 (2) 24
(3) 19 (4) 25
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

198. If $c+\frac{1}{c} = 3$, then the value of $(c-3)^7+\frac{1}{c^7}$ is
(1) 2 (2) 0
(3) 3 (4) 1
(SSC CHSL (10+2) Tier-I (CBE) Exam. 08.09.2016) (Ist Sitting)

199. If $x = \sqrt[3]{7}+3$ then the vlaue of $x^3 - 9x^2 + 27x - 34$ is :
(1) 0 (2) 1
(3) 2 (4) –1
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 20.03.2016) (IInd Sitting)

200. If $p(x + y)^2 = 5$ and $q(x - y)^2 = 3$, then the simplified value of $p^2(x + y)^2 + 4pqxy - q^2(x - y)^2$ is :
(1) $-(p + q)$ (2) $2(p + q)$
(3) $p + q$ (4) $-2(p + q)$
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 20.03.2016) (IInd Sitting)

201. If $x + \frac{1}{x} = -2$ then the vlaue of $x^p + x^q$ is :
(where p is an even number and q is an odd number)
(1) –2 (2) 2
(3) 1 (4) 0
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 20.03.2016) (IInd Sitting)

202. If $(2a - 3)^2 + (3b + 4)^2 + (6c + 1)^2 = 0$, then the value of $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2} + 3$ is :
(1) $abc + 3$ (2) 6
(3) 0 (4) 3
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

203. If $a + b + c = 1$, $ab + bc + ca = -1$ and $abc = -1$, then the value of $a^3 + b^3 + c^3$ is :
(1) 1 (2) –1
(3) 2 (4) –2
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

204. If for a non–zero x, $3x^2 + 5x + 3 = 0$, then the value of $x^3 + \frac{1}{x^3}$ is :
(1) $\frac{10}{27}$ (2) $-\left(\frac{10}{27}\right)$
(3) $\frac{2}{3}$ (4) $-\left(\frac{2}{3}\right)$
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

205. What will be the value of $x^3 + y^3 + z^3 - 3xyz$ when $x + y + z = 9$ and $x^2 + y^2 + z^2 = 31$?
(1) 27 (2) 3
(3) 54 (4) 9
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

206. What is
$$\frac{\left(x^2-y^2\right)^3+\left(y^2-z^2\right)^3+\left(z^2-x^2\right)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$$
(1) $\frac{(x+y)(y+z)}{(x+z)}$
(2) $(x + y)^3 (y + z)^3 (z + x)^3$
(3) $(x + y) (y + z) (z + x)$
(4) $(x + y) (y + z)$
(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

207. If $\frac{x^3+3y^2x}{y^3+3x^2y} = \frac{35}{19}$, what is $\frac{x}{y}$ =
(1) $\frac{7}{6}$ (2) $\frac{5}{6}$
(3) $\frac{5}{1}$ (4) $\frac{7}{1}$
(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

208. Given $(a - b) = 2$, $(a^3 - b^3) = 26$ then $(a + b)^2$ is
(1) 9 (2) 4
(3) 16 (4) 12
(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (Ist Sitting)

209. If $x + y + z = 9$ then the value of $(x - 4)^3 + (y - 2)^3 + (z - 3)^3 - 3(x - 4)(y - 2)(z - 3)$ is
(1) 6 (2) 9
(3) 0 (4) 1
(SSC CGL Tier-I (CBE) Exam. 27.08.2016) (Ist Sitting)

210. If $a = 2$, $b = -3$ then the value of $27a^3 - 54a^2b + 36ab^2 - 8b^3$ is
(1) 1562 (2) 1616
(3) 1676 (4) 1728
(SSC CGL Tier-I (CBE) Exam. 28.08.2016) (IInd Sitting)

211. If $a^3 + \frac{1}{a^3} = 2$, then value of $\frac{a^2+1}{a}$ is (a is a positive number.)
(1) 1 (2) 2
(3) 3 (4) 4
(SSC CGL Tier-I (CBE) Exam. 28.08.2016) (IInd Sitting)

212. If $pq(p + q) = 1$, then the value of $\frac{1}{p^3q^3} - p^3 - q^3$ is equal to
(1) 1 (2) 2
(3) 3 (4) 4
(SSC CGL Tier-I (CBE) Exam. 29.08.2016) (IInd Sitting)

213. If $x + \frac{1}{x} = \sqrt{3}$, then the value of $x^3 + \frac{1}{x^3}$ is equal to
(1) 1 (2) $3\sqrt{3}$
(3) 0 (4) 3
(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (Ist Sitting)

214. If $\frac{a}{b} + \frac{b}{a} = 1$, the value of $a^3 + b^3$ is equal to
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (Ist Sitting)

215. If $l + m + n = 9$ and $l^2 + m^2 + n^2 = 31$, then the value of $(lm + mn + nl)$ will be
(1) 22 (2) 50
(3) 25 (4) –25
(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (Ist Sitting)

216. If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is
(1) 0 (2) 1
(3) 2 (4) –1
(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (Ist Sitting)

217. If $x = \frac{3}{2}$, then the value of $27x^3 - 54x^2 + 36x - 11$ is
(1) $11\frac{3}{8}$ (2) $11\frac{5}{8}$
(3) $12\frac{3}{8}$ (4) $12\frac{5}{8}$
(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (Ist Sitting)

218. If $a + b + c = 6$ and $ab + bc + ca = 11$, then the value of $bc(b + c) + ca(c + a) + ab(a + b) + 3abc$ is
(1) 33 (2) 66
(3) 55 (4) 23
(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (Ist Sitting)

219. If $\left(a + \frac{1}{a}\right)^2 = 3$, then the value of $a^6 - \frac{1}{a^6}$ will be
(1) 1 (2) 3
(3) 0 (4) 2
(SSC CGL Tier-I (CBE) Exam. 01.09.2016) (Ist Sitting)

220. If $m + n = 1$, then the value of $m^3 + n^3 + 3mn$ is equal to
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (Ist Sitting)

221. If $x^4 + \frac{1}{x^4} = 119$, then the value of $\left(x - \frac{1}{x}\right)$ is
(1) 6 (2) 12
(3) 11 (4) 3
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (Ist Sitting)

222. If $x^3 + \frac{1}{x^3} = 110$, then find the value of $x + \frac{1}{x}$.
(1) 2 (2) 3
(3) 4 (4) 5
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

223. If $x^2 + y^2 + z^2 = 14$ and $xy + yz + zx = 11$, then the value of $(x + y + z)^2$ is
(1) 16 (2) 25
(3) 36 (4) 49
(SSC CGL Tier-I (CBE) Exam. 03.09.2016) (IInd Sitting)

224. If $x = \sqrt[3]{28}$, $y = \sqrt[3]{27}$, then the value of $x + y - \frac{1}{x^2 + xy + y^2}$ is
(1) 8 (2) 7
(3) 6 (4) 5
(SSC CGL Tier-I (CBE) Exam. 03.09.2016) (IInd Sitting)

225. If $x = 12$ and $y = 4$, then the value of $(x + y)^{\frac{x}{y}}$ is
(1) 48 (2) 1792
(3) 4096 (4) 570
(SSC CGL Tier-I (CBE) Exam. 03.09.2016) (IInd Sitting)

226. If $2x + \frac{2}{x} = 3$, then the value of $x^3 + \frac{1}{x^3} + 2$ is
(1) $\frac{3}{4}$ (2) $\frac{4}{5}$
(3) $\frac{5}{8}$ (4) $\frac{7}{8}$
(SSC CGL Tier-I (CBE) Exam. 04.09.2016) (Ist Sitting)

227. If $a + b = 3$, then the value of $a^3 + b^3 + 9ab - 27$ is
(1) 24 (2) 25
(3) 0 (4) 27
(SSC CGL Tier-I (CBE) Exam. 06.09.2016) (Ist Sitting)

228. If $x + \frac{1}{x} = 2$, then the value of $x^2 + \frac{2}{x^6}$ is equal to ?
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-I (CBE) Exam. 06.09.2016) (Ist Sitting)

229. If $\frac{a}{b} + \frac{b}{a} = 1$, then the value of $a^3 + b^3$ will be
(1) 1 (2) 0
(3) –1 (4) 2
(SSC CGL Tier-I (CBE) Exam. 07.09.2016) (Ist Sitting)

230. If $a - b = 1$ and $a^3 - b^3 = 61$, then the value of ab will be
(1) –20 (2) 20
(3) 30 (4) 60
(SSC CGL Tier-I (CBE) Exam. 07.09.2016) (Ist Sitting)

231. If $p^3 - q^3 = (p - q)\{(p + q)^2 - x\,p\,q\}$ then the value of x is
(1) 1 (2) –1
(3) 2 (4) –2
(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (IInd Sitting)

232. If $a^2 = by + cz$, $b^2 = cz + ax$, $c^2 = ax + by$, then the value of $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$ is
(1) 1 (2) $a + b + c$
(3) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (4) 0
(SSC CGL Tier-I (CBE) Exam. 30.08.2016) (IInd Sitting)

233. If $p^3 - q^3 = (p - q)\{(p - q)^2 + x\,p\,q\}$ then value of x is
(1) 1 (2) –1
(3) 3 (4) 2
(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (IInd Sitting)

234. If $\left(a + \frac{1}{a}\right)^2 = 3$, then the value of $a^{18} + a^{12} + a^6 + 1$ is
(1) 3 (2) 1
(3) 0 (4) 2
(SSC CGL Tier-I (CBE) Exam. 31.08.2016) (IInd Sitting)

235. If $x + 5 + \frac{1}{x+1} = 6$, then the value of $(x + 1)^3 + \frac{1}{(x+1)^3}$ is
(1) 2 (2) 0
(3) –2 (4) 4
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

236. If $a + b + c = 15$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{71}{abc}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is
(1) 160 (2) 180
(3) 200 (4) 220
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

237. If k is the largest possible real number such that $p^4 + q^4 = (p^2 + kpq + q^2)(p^2 - kpq + q^2)$, then the value of k is
(1) 1 (2) $-\sqrt{2}$
(3) 2 (4) $\sqrt{2}$
(SSC CGL Tier-I (CBE) Exam. 02.09.2016) (IInd Sitting)

238. A complete factorisation of $(x^4 + 64)$ is
(1) $(x^2 + 8)^2$
(2) $(x^2 + 8)(x^2 - 8)$
(3) $(x^2 - 4x + 8)(x^2 - 4x - 8)$
(4) $(x^2 + 4x + 8)(x^2 - 4x + 8)$
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

239. If $a + b = 1$, then $a^4 + b^4 - a^3 - b^3 - 2a^2b^2 + ab$ is equal to
(1) 1 (2) 2
(3) 4 (4) 0
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

240. If $a = 299$, $b = 298$, $c = 297$ then the value of $2a^3 + 2b^3 + 2c^3 - 6abc$ is
(1) 5154 (2) 5267
(3) 5364 (4) 5456
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

241. If $x + \frac{1}{x} = \sqrt{3}$ the value of $(x^{18} + x^{12} + x^6 + 1)$ is
(1) 0 (2) 1
(3) 2 (4) 3
(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

242. If $x = 1 + \sqrt{2} + \sqrt{3}$, then the value of $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$ is

(1) $2\sqrt{2}$ (2) $3\sqrt{3}$

(3) $5\sqrt{5}$ (4) $6\sqrt{6}$

(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

243. If $x + y = 1 + xy$, then $x^3 + y^3 - x^3y^3$ is equal to :

(1) 0 (2) 1

(3) –1 (4) 2

(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

244. If $p = 3 + \frac{1}{p}$, the value of $\left(p^4 + \frac{1}{p^4}\right)$ is :

(1) 81 (2) 27

(3) 120 (4) 119

(SSC CGL Tier-I (CBE) Exam. 28.08.2016 (IST Sitting)

245. If $x^2 - xy + y^2 = 2$ and $x^4 + x^2y^2 + y^4 = 6$, then the value of $(x^2 + xy + y^2)$ is :

(1) 1 (2) 12

(3) 3 (4) 36

(SSC CGL Tier-I (CBE) Exam. 28.08.2016 (IST Sitting)

246. If $\left(a + \frac{1}{a}\right)^2 = 3$, the value of $\left(a^3 + \frac{1}{a^3}\right)$ is :

(1) 0 (2) $3\left(a + \frac{1}{a}\right)$

(3) $3\left(a^2 + \frac{1}{a^2}\right)$

(4) 1

(SSC CGL Tier-I (CBE) Exam. 29.08.2016 (IST Sitting)

247. If $\frac{a^2 + b^2}{c^2} = \frac{b^2 + c^2}{a^2} = \frac{c^2 + a^2}{b^2} = \frac{1}{k}$, $(k \neq 0)$ then k = ?

(1) 2 (2) 1

(3) 0 (4) $\frac{1}{2}$

(SSC CGL Tier-I (CBE) Exam. 29.08.2016 (IST Sitting)

248. If $\left(2x + \frac{2}{9x}\right) = 4$, then the value of $\left(27x^3 + \frac{1}{27x^3}\right)$ is :

(1) 180 (2) 198

(3) 234 (4) 252

(SSC CGL Tier-I (CBE) Exam. 29.08.2016 (IST Sitting)

249. If $xy (x + y) = m$, then the value of $(x^3 + y^3 + 3m)$ is :

(1) $\frac{m^3}{xy}$ (2) $\frac{m^3}{(x+y)^3}$

(3) $\frac{m^3}{x^3y^3}$ (4) mx^3y^3

(SSC CGL Tier-I (CBE) Exam. 30.08.2016 (IIIrd Sitting)

250. If $p + \frac{1}{p+2} = 1$, then the value of $(p + 2)^3 + \frac{1}{(p+2)^3} - 3$ is :

(1) 12 (2) 16

(3) 18 (4) 15

(SSC CGL Tier-I (CBE) Exam. 30.08.2016 (IIIrd Sitting)

251. If $\left(x + \frac{1}{x}\right) \neq 0$ and $\left(x^3 + \frac{1}{x^3}\right) = 0$ then the value $\left(x + \frac{1}{x}\right)^4$ is

(1) 9 (2) 12

(3) 15 (4) 16

(SSC CGL Tier-I (CBE) Exam. 31.08.2016 (IIIrd Sitting)

252. If $2x - \frac{2}{x} = 1 (x \neq 0)$, then the value of $\left(x^3 - \frac{1}{x^3}\right)$ is

(1) $\frac{13}{4}$ (2) $\frac{13}{8}$

(3) $\frac{17}{4}$ (4) $\frac{17}{8}$

(SSC CGL Tier-I (CBE) Exam. 02.09.2016 (IInd Sitting)

253. Sum of the factors of $4b^2c^2 - (b^2 + c^2 - a^2)^2$ is :

(1) $a + b + c$ (2) $2 (a + b + c)$

(3) 0 (4) 1

(SSC CGL Tier-I (CBE) Exam. 02.09.2016 (IInd Sitting)

254. If $(4a - 3)^2 = 0$, then the value of $64a^3 - 48a^2 + 12a + 13$ is :

(1) 0 (2) 11

(3) 22 (4) 33

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IInd Sitting)

255. If $a = 101$, then the value of $a (a^2 - 3a + 3)$ is :

(1) 1000000 (2) 1010101

(3) 1000001 (4) 999999

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IInd Sitting)

256. If $\left(x + \frac{1}{x}\right) = -2$, then the value of $\left(x^7 + \frac{1}{x^7}\right)$ is

(1) 1 (2) –1

(3) 0 (4) –2

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IIIrd Sitting)

257. If $a^2 + b^2 + c^2 = 14$ and $a + b + c = 6$, then the value of $(ab + bc + ca)$ is,

(1) 11 (2) 12

(3) 13 (4) 14

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IIIrd Sitting)

258. If $\frac{a}{b} + \frac{b}{a} = 1$, then the value of $(a^3 + b^3)$ is :

(1) 1 (2) 0

(3) –1 (4) 2

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IIIrd Sitting)

259. If $(a + b) = 5$, then the value of $(a - 3)^7 + (b - 2)^7$ is :

(1) 2^7 (2) 3^7

(3) 1 (4) 0

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IInd Sitting)

260. If $(x^2 - 2x + 1) = 0$, then the value of $\left(x^4 + \frac{1}{x^4}\right)$ is :

(1) 0 (2) 1

(3) 2 (4) 3

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IInd Sitting)

261. If $a^2 + b^2 + c^2 = 83$ and $a + b + c = 15$, then the value of $(ab + bc + ca)$ is :

(1) 69 (2) 70

(3) 71 (4) 72

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IIIrd Sitting)

262. If $m - n = 2$ and $mn = 15$, $(m, n > 0)$ then the value of $(m^2 - n^2)(m^3 - n^3)$ is :

(1) 1856 (2) 1658

(3) 1586 (4) 1568

(SSC CGL Tier-I (CBE) Exam. 04.09.2016 (IIIrd Sitting)

263. If $xy + yz + zx = 1$, then the value of $\frac{1 + y^2}{(x+y)(y+z)}$ is :

(1) 2 (2) 3

(3) 4 (4) 1

(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IInd Sitting)

264. If $x^2 - 4x + 1 = 0$, then the value of $\left(\frac{x^6 + 1}{x^3}\right)$ is :

(1) 48 (2) 52

(3) 55 (4) 58

(SSC CGL Tier-I (CBE) Exam. 06.09.2016 (IIIrd Sitting)

265. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$, then the value of $x^4 + y^4 - 2x^2y^2$ is :
(1) 4 (2) 8
(3) 16 (4) 64
(SSC CGL Tier-I (CBE) Exam. 07.09.2016 (IInd Sitting)

266. If $a^3 - b^3 = 56$ and $a - b = 2$, what is the value of $(a^2 + b^2)$?
(1) 12 (2) 20
(3) 28 (4) 32
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IInd Sitting)

267. If $x + y + z = 1$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ and $xyz = -1$, then $x^3 + y^3 + z^3$ is equal to
(1) –1 (2) 1
(3) –2 (4) 2
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IInd Sitting)

268. If $\frac{1}{a}(a^2 + 1) = 3$, then the value of $\left(\frac{a^6+1}{a^3}\right)$ is :
(1) 9 (2) 18
(3) 27 (4) 1
(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IIIrd Sitting)

269. The third proportional of the following numbers $(x - y)^2$, $(x^2 - y^2)^2$ is :
(1) $(x + y)^3 (x - y)^2$
(2) $(x + y)^4 (x - y)^2$
(3) $(x + y)^2 (x - y)^2$
(4) $(x + y)^2 (x-y)^3$
(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IInd Sitting)

270. If $(x - 5)^2 + (y - 2)^2 + (z - 9)^2 = 0$, then value of $(x + y - z)$ is :
(1) 16 (2) – 1
(3) – 2 (4) 12
(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IIIrd Sitting)

271. If $\left(x + \frac{1}{x}\right) = 3$ then $\left(x^8 + \frac{1}{x^8}\right)$ is equal to
(1) 2201 (2) 2203
(3) 2207 (4) 2213
(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IIIrd Sitting)

272. If $x = 999$, $y = 1000$, $z = 1001$, then the value of
$$\frac{x^3 + y^3 + z^3 - 3xyz}{x - y + z}$$ is :
(1) 1000 (2) 9000
(3) 1 (4) 9
(SSC CGL Tier-I (CBE) Exam. 10.09.2016 (IIIrd Sitting)

273. If $a + b + c = 0$, then the value of $(a^3 + b^3 + c^3)$ is
(1) abc (2) $2abc$
(3) $3abc$ (4) 0
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IInd Sitting)

274. If, $\frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$, then the value of $(p^3 - q^3)$ is
(1) $p - q$ (2) pq
(3) 1 (4) 0
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IInd Sitting)

275. If $x = 93$, $y = 93$, $z = 94$ then the value of $(x^2 - y^2 + 10xz + 10yz)$ is
(1) 104784 (2) 147840
(3) 174840 (4) 184740
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IInd Sitting)

276. If $x = 222$, $y = 223$, $z = 225$ then the value of $(x^3 + y^3 + z^3 + 3xyz)$ is :
(1) 4590 (2) 4690
(3) 4950 (4) 4960
(SSC CGL Tier-I (CBE) Exam. 11.09.2016 (IIIrd Sitting)

277. If $\frac{a}{b} + \frac{b}{a} = 1$, then the value of $a^3 + b^3 - 2$ is
(1) 0 (2) –2
(3) –1 (4) 2
(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

278. If $x + \frac{1}{x} = \sqrt{3}$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is :
(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$
(3) 0 (4) 1
(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

279. If $a + b = 3$, then the value of $a^3 + b^3 + 9ab$ is :
(1) 27 (2) 9
(3) 16 (4) 81
(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

280. If $6x^2 - 12x + 1 = 0$, then the value of $27x^3 + \frac{1}{8x^3}$ is
(1) 162 (2) 189
(3) 207 (4) 225
(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

281. If $x^2 + \frac{1}{x^2} = 98$ $(x > 0)$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is
(1) 970 (2) 1030
(3) –970 (4) –1030
(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

282. If $x = y + z$ then $x^3 - y^3 - z^3$ is
(1) 0 (2) $3xyz$
(3) $-3xyz$ (4) 1
(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

283. If $x = 11$, the value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$ is
(1) 11 (2) 10
(3) 12 (4) –10
(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

284. If x, y, z are the three factors of $a^3 - 7a - 6$, then value of $(x + y + z)$ will be
(1) $3a$ (2) 3
(3) 6 (4) a
(SSC CGL Tier-II (CBE) Exam. 12.01.2017)

TYPE–III

1. If $(2^x)(2^y) = 8$ and $(9^x)(3^y) = 81$, then (x, y) is :
(1) (1,2) (2) (2, 1)
(3) (1,1) (4) (2, 2)
FCI Assistant Grade-III Exam.05.02.2012 (Paper-I) East Zone (IInd Sitting)

2. The lines $2x + y = 5$ and $x + 2y = 4$ intersect at the point :
(1) (1,2) (2) (2,1)
(3) $(\frac{5}{2},0)$ (4) (0,2)
FCI Assistant Grade-III Exam. 05.02.2012 (Paper-I) East Zone (IInd Sitting)

3. The graph of the linear equation $3x + 4y = 24$ is a straight line intersecting x-axis and y-axis at the points A and B respectively. P(2, 0) and Q $\left(0, \frac{3}{2}\right)$ are two points on the sides OA and OB respectively of Δ OAB, where O is the origin of the co-ordinate system. Given that AB = 10 cm, then PQ =
(1) 20 cm (2) 2.5 cm
(3) 40 cm (4) 5 cm
(SSC Graduate Level Tier-II Exam. 16.09.2012)

4. The length of the intercept of the graph of the equation $9x - 12y = 108$ between the two axes is
(1) 15 units (2) 9 units
(3) 12 units (4) 18 units
(SSC Graduate Level Tier-II Exam. 16.09.2012)

5. The x-intercept on the graph of $7x - 3y = 2$ is
(1) $\frac{3}{4}$ (2) $\frac{3}{7}$
(3) $\frac{2}{5}$ (4) $\frac{2}{7}$
(SSC CHSL DEO & LDC Exam. 21.10.2012 (Ist Sitting)

6. If $2x + y = 6$ and $x = 2$ are two linear equations, then graph of two equations meet at a point :
(1) (2,0) (2) (0,2)
(3) (2,2) (4) (1,2)
(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

7. An equation whose graph passes through the origin, out of the given equations $2x + 3y = 2$, $2x - 3y = 3$, $-2x + 3y = 5$ and $2x + 3y = 0$ is :
(1) $2x - 3y = 3$
(2) $-2x + 3y = 5$
(3) $2x + 3y = 0$
(4) $2x + 3y = 2$
(SSC CHSL DEO & LDC Exam. 21.10.2012 (IInd Sitting)

8. If a linear equation is of the form $x = k$ where k is a constant, then graph of the equation will be
(1) a line parallel to x-axis
(2) a line cutting both the axes
(3) a line making positive acute angle with x-axis
(4) a line parallel to y-axis
(SSC CHSL DEO & LDC Exam. 28.10.2012 (Ist Sitting)

9. The graph of the equation $2x - 3y = 6$ intersects the y-axis at the point
(1) (–2, 0) (2) (0, –2)
(3) (2, 3) (4) (2, –3)
(SSC CHSL DEO & LDC Exam. 28.10.2012 (Ist Sitting)

10. The graph of the equations $25x + 75y = 225$ and $x = 9$ meet at the point
(1) (0,9) (2) (9,0)
(3) (3,0) (4) (0,3)
(SSC CHSL DEO & LDC Exam. 04.11.2012 (IInd Sitting)

11. The area bounded by the lines $x = 0$, $y = 0$, $x + y = 1$, $2x + 3y = 6$ (in square units) is
(1) 2 (2) $2\frac{1}{3}$
(3) $2\frac{1}{2}$ (4) 3
(SSC Graduate Level Tier-I Exam. 11.11.2012 (Ist Sitting

12. The graph of the equation $4x - 5y = 20$ intersects the x-axis at the point
(1) (2, 0) (2) (5, 0)
(3) (4, 5) (4) (0, 5)
(SSC Delhi Police S.I.(SI) Exam. 19.08.2012)

13. The graph of $2x + 1 = 0$ and $3y - 9 = 0$ intersect at the point
(1) $(-\frac{1}{2}, -3)$ (2) $(-\frac{1}{2}, 3)$
(3) $(\frac{1}{2}, -3)$ (4) None of these
(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

14. An equation of the form $ax + by + c = 0$ where $a \neq 0$, $b \neq 0$, $c = 0$ represents a straight line which passes through
(1) (0, 0) (2) (3, 2)
(3) (2, 4) (4) None of these
(SSC Graduate Level Tier-I Exam. 19.05.2013 Ist Sitting)

15. The linear equation such that each point on its graph has an ordinate four times its abscissa is :
(1) $y + 4x = 0$ (2) $y = 4x$
(3) $x = 4y$ (4) $x + 4y = 0$
(SSC CAPFs SI & CISF ASI Exam. 23.06.2013)

16. If the graph of the equations $3x + 2y = 18$ and $3y - 2x = 1$ intersect at the point (p, q), then the value of $p + q$ is
(1) 7 (2) 6
(3) 5 (4) 4
(SSC CHSL DEO & LDC Exam. 27.10.2013 IInd Sitting)

17. If the graph of the equations $x + y = 0$ and $5y + 7x = 24$ intersect at (m, n), then the value of $m + n$ is
(1) 2 (2) 1
(3) 0 (4) –1
(SSC CHSL DEO & LDC Exam. 10.11.2013, Ist Sitting)

18. The area of the triangle formed by the graph of $3x + 4y = 12$, x-axis and y-axis (in sq. units) is
(1) 4 (2) 12
(3) 6 (4) 8
(SSC CHSL DEO & LDC Exam. 10.11.2013, IInd Sitting)

19. Equation of the straight line parallel to x-axis and also 3 units below x-axis is :
(1) $x = -3$ (2) $y = 3$
(3) $y = -3$ (4) $x = 3$
(SSC Graduate Level Tier-I Exam. 21.04.2013, Ist Sitting)

20. The straight line $2x + 3y = 12$ passes through :
(1) 1st, 2nd and 3rd quadrant
(2) 1st, 2nd and 4th quadrant
(3) 2nd, 3rd and 4th quadrant
(4) 1st, 3rd and 4th quadrant
(SSC Graduate Level Tier-I Exam. 19.05.2013)

21. The graphs of $x = a$ and $y = b$ intersect at
(1) (a, b) (2) (b, a)
(3) $(-a, b)$ (4) $(a, -b)$
(SSC CGL Tier-I Exam. 19.10.2014 (Ist Sitting)

22. The area in sq. unit. of the triangle formed by the graphs of $x = 4$, $y = 3$ and $3x + 4y = 12$ is
(1) 12 (2) 8
(3) 10 (4) 6
(SSC CGL Tier-I Exam. 19.10.2014)

23. The equations
$3x + 4y = 10$
$-x + 2y = 0$
have the solution (a, b). The value of $a + b$ is
(1) 1 (2) 2
(3) 3 (4) 4
(SSC CGL Tier-I Exam. 19.10.2014)

24. Area of the triangle formed by the graph of the straight lines $x - y = 0$, $x + y = 2$ and the x– axis is
(1) 1 sq unit (2) 2 sq units
(3) 4 sq units (4) None of these
(SSC CGL Tier-II Exam. 21.09.2014)

25. If $2\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 7 = 0$, then two values of x are
(1) 1, 2 (2) $2, -\frac{1}{2}$
(3) 0, 1 (4) $\frac{1}{2}, 1$
(SSC CHSL DEO & LDC Exam. 02.11.2014 (IInd Sitting)

26. The total area (in sq. unit) of the triangles formed by the graph of $4x + 5y = 40$, x - axis, y - axis and $x = 5$ and $y = 4$ is
(1) 10 (2) 20
(3) 30 (4) 40
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

27. For what value of k, the system of equations $kx + 2y = 2$ and $3x + y = 1$ will be coincident ?
(1) 2 (2) 3
(3) 5 (4) 6
(SSC CGL Tier-I Exam. 19.10.2014 TF No. 022 MH 3)

28. The area (in square units) of the triangle formed by the graphs of the equations $x = 4$, $y = 3$ and $3x + 4y = 12$; is
(1) 24 (2) 12
(3) 6 (4) 3
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9 and SSC CGL Tier-I Exam, 16.08.2015 (IInd Sitting) TF No. 2176783)

29. If the ordinate and abscissa of the point $(k, 2k-1)$ be equal, then the value of k is
(1) 0 (2) -1
(3) 1 (4) $\frac{1}{2}$
(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

30. The graph of $3x + 4y - 24 = 0$ forms a triangle OAB with the coordinate axes, where O is the origin. Also the graph of $x + y + 4 = 0$ forms a triangle OCD with the coordinate axes. Then the area of ΔOCD is equal to
(1) $\frac{1}{2}$ of area of ΔOAB
(2) $\frac{1}{3}$ of area of ΔOAB
(3) $\frac{2}{3}$ of area of ΔOAB
(4) the area of ΔOAB
(SSC CGL Tier-II Exam, 2014 12.04.2015 (Kolkata Region) TF No. 789 TH 7)

31. The angle between the graph of the linear equation $239x - 239y + 5 = 0$ and the x – axis is
(1) 0° (2) 60°
(3) 30° (4) 45°
(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

32. The length of the portion of the straight line $3x + 4y = 12$ intercepted between the axes is
(1) 5 (2) 3
(3) 4 (4) 7
(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

33. $2x-ky+7 = 0$ and $6x-12y + 15 = 0$ has no solution for
(1) $k = -1$ (2) $k = -4$
(3) $k = 4$ (4) $k = 1$
(SSC CGL Tier-I Exam, 09.08.2015 (Ist Sitting) TF No. 1443088)

34. Among the equations $x + 2y + 9 = 0$; $5x - 4 = 0$; $2y - 13 = 0$; $2x - 3y = 0$, the equation of the straight line passing through origin is
(1) $2x - 3y = 0$
(2) $x + 2y + 9 = 0$
(3) $5x - 4 = 0$
(4) $2y - 13 = 0$
(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

35. If the number of vertices, edges and faces of a rectangualr parallelopiped are denoted by v, e and f respectively, the value of $(v-e+f)$ is
(1) 0 (2) 2
(3) 4 (4) 1
(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

36. The area of the triangle formed by the graphs of the equations $x = 0$, $2x+3y = 6$ and $x+y = 3$ is :
(1) 3 sq. unit (2) $4\frac{1}{2}$ sq. unit
(3) $1\frac{1}{2}$ sq. unit (4) 1 sq. unit
(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

37. If $5x + 9y = 5$ and $125x^3 + 729y^3 = 120$ then the vlaue of the product of x and y is
(1) $\frac{1}{9}$ (2) $\frac{1}{135}$
(3) 45 (4) 135
(SSC CGL Tier-I Exam, 16.08.2015 (Ist Sitting) TF No. 3196279)

38. A point in the 4th quadrant is 6 unit away from x–axis and 7 unit away from y–axis. The point is at
(1) (7, –6) (2) (–7, 6)
(3) (–6, –7) (4) (–6, 7)
(SSC CGL Tier-I Re-Exam, 30.08.2015)

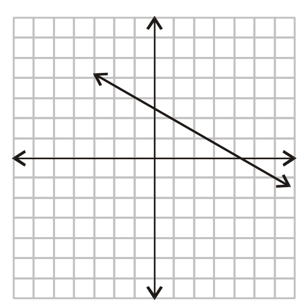
39. The straight line $y = 3x$ must pass through the point :
(1) (0, 0) (2) (0, 1)
(3) (1, 2) (4) (2, 0)
(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 06.12.2015 (Ist Sitting) TF No. 1375232)

40. If (2, 0) is a solution of the linear equation $2x + 3y = k$, then the value of k is
(1) 6 (2) 5
(3) 2 (4) 4
(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 20.12.2015 (Ist Sitting) TF No. 9692918)

41. The graph of linear equation $y = x$ passes through the point
(1) $\left(0, \frac{3}{2}\right)$ (2) (1, 1)
(3) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{2}, -\frac{3}{2}\right)$
(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 20.12.2015 (Ist Sitting) TF No. 9692918)

42. What is the area of the region bounded by straight line $9x + 4y = 36$, x - axis and the y - axis ?
(1) 12 sq. units
(2) 18 sq. units
(3) 16 sq. units
(4) 15 sq. units
(SSC CPO Exam. 06.06.2016) (Ist Sitting)

43. The slope of the given line is:



(1) Positive (2) Negative
(3) Undefined
(4) Zero
(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

44. What is the area of the triangle formed by points (0,0), (3,4), (4,3) ?

(1) 4 units^2 (2) $\frac{7}{2}$ units^2

(3) $\frac{5}{2}$ units^2 (4) $\frac{5}{3}$ units^2

(SSC CPO SI & ASI, Online Exam. 06.06.2016) (IInd Sitting)

45. The area of a triangle with vertices A (0, 8) , O (0,0) and B (5, 0) is :

(1) 8 sq. units (2) 13 sq. units
(3) 20 sq. units (4) 40 sq. units

(SSC CGL Tier-I (CBE) Exam. 09.09.2016 (IIIrd Sitting)

46. What is the equation of the line whose y-intercept is $\frac{-3}{4}$ and making an angle of 45° with the positive x–axis?

(1) $4x - 4y = 3$ (2) $4x - 4y = -3$
(3) $3x - 3y = 4$ (4) $3x - 3y = -4$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

47. In what ratio does the point T (3, 0) divide the segment joining the points S (4, –2) and U (1, 4)?

(1) 2 : 1 (2) 1 : 2
(3) 2 : 3 (4) 3 : 2

(SSC CHSL (10+2) Tier-I (CBE) Exam. 15.01.2017) (IInd Sitting)

48. P (4, (2) and R (–2, 0) are vertices of a rhombus PQRS. What is the equation of diagonal QS ?

(1) $x - 3y = -2$ (2) $3x + y = 4$
(3) $3x + y = -4$ (4) $x - 3y = 2$

(SSC CHSL (10+2) Tier-I (CBE) Exam. 16.01.2017) (IInd Sitting)

49. Point P is the midpoint of segment AB. Co-ordinates of point P are (2,1) and that of point A are (11,5). The co-ordinates of point B are

(1) (-7,-(3) (2) (6.5,(3)
(3) (7,(3) (4) (-6.5,-(3)

(SSC CHSL (10+2) Tier-I (CBE) Exam. 16.01.2017) (IInd Sitting)

TYPE–IV

1. If $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$, then the ratio $\frac{a+b}{b+c}$ equal to :

(1) $\frac{20}{27}$ (2) $\frac{27}{20}$
(3) $\frac{6}{8}$ (4) $\frac{8}{6}$

(SSC CGL Prelim Exam. 27.02.2000 (Second Sitting)

2. If a : b = 2 : 3 and b : c = 4 : 5, find $a^2 : b^2$: bc

(1) 4 : 9 : 45 (2) 16 : 36 : 45
(3) 16 : 36 : 20 (4) 4 : 36 : 40

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

3. If A : B $=\frac{1}{2}:\frac{3}{8}$,

$B : C = \frac{1}{3}:\frac{5}{9}$ and $C : D = \frac{5}{6}:\frac{3}{4}$

then the ratio A : B : C : D is
(1) 6 : 4 : 8 : 10
(2) 6 : 8 : 9 : 10
(3) 8 : 6 : 10 : 9
(4) 4 : 6 : 8 : 10

(SSC CGL Prelim Exam. 24.02.2002 (First Sitting)

4. If x : y = 3 : 2, then the ratio $2x^2 + 3y^2 : 3x^2 - 2y^2$ is equal to :

(1) 12 : 5 (2) 6 : 5
(3) 30 : 19 (4) 5 : 3

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

5. If A : B : C = 2 : 3 : 4, then $\frac{A}{B}:\frac{B}{C}:\frac{C}{A}$ is equal to :

(1) 8 : 9 : 16 (2) 8 : 9 : 12
(3) 8 : 9 : 24 (4) 4 : 9 : 16

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

6. If A : B = 1 : 2. B : C = 3 : 4 and C : D = 5 : 6, find D : C : B : A

(1) 6 : 5 : 4 : 2
(2) 6 : 3 : 2 : 1
(3) 6 : 4 : 2 : 1
(4) 48 : 40 : 30 : 15

(SSC CGL Prelim Exam. 24.02.2002 (Second Sitting)

7. If $\frac{2a-5b}{3a+6b} = \frac{4}{7}$ then $a : b$ is equal to

(1) 21 : 36 (2) 2 : 59
(3) 59 : 2 (4) 36 : 21

(SSC CGL Prelim Exam. 24.02.2002 (Middle Zone)

8. If $\frac{a}{b}=\frac{7}{9},\frac{b}{c}=\frac{3}{5}$, then the value of a : b : c is

(1) 7 : 9 : 15 (2) 7 : 9 : 5
(3) 21 : 35 : 45 (4) 7 : 3 : 15

(SSC CPO S.I.Exam.12.01.2003

9. If x : y = 7 : 3, then the value of $\frac{xy+y^2}{x^2-y^2}$ is

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$
(3) $\frac{3}{7}$ (4) $\frac{7}{3}$

(SSC CPO S.I.Exam.12.01.2003

10. If $\frac{3a+5b}{3a-5b}$ = 5, then $a : b$ is equal to :

(1) 2 : 1 (2) 5 : 3
(3) 3 : 2 (4) 5 : 2

(SSC CPO S.I. Exam. 26.05.2005)

11. If $p : q = r : s = t : u = 2 : 3$, then $(mp + nr + ot) : (mq + ns + ou)$ equals :

(1) 3 : 2 (2) 2 : 3
(3) 1 : 3 (4) 1 : 2

(SSC CPO S.I.Exam.26.05.2005)

12. If $x : y = 3 : 4$, then $(7x + 3y) : (7x - 3y)$ is equal to :

(1) 5 : 2 (2) 4 : 3
(3) 11 : 3 (4) 37 : 19

(SSC CPO S.I. Exam. 26.05.2005)

13. If $a : b : c = (y - z) : (z - x) : (x - y)$ then the value of $ax+ by+ cz$ is

(1) 1 (2) 3
(3) 0 (4) –1

(SSC (South Zone) Investigator Exam. 12.09.2010)

14. If 50% of (p – q) =30% of (p + q), then p : q is equal to

(1) 5 : 3 (2) 4 : 1
(3) 3 : 5 (4) 1 : 4

(SSC (South Zone) Investigator Exam.12.09.2010)

15. If $x : y = 2 : 1$, then $(5x^2 - 13xy + 6y^2)$ is equal to

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$

(3) 0 (4) $\frac{55}{4}$

(SSC CPO Sub-Inspector Exam. 12.12.2010 (Paper-I)

16. If $y : x = 4 : 15$, then the value of $\left(\frac{x-y}{x+y}\right)$ is

(1) $\frac{11}{19}$ (2) $\frac{19}{11}$

(3) $\frac{4}{11}$ (4) $\frac{15}{19}$

FCI Assistant Grade-III Exam.25.02.2012 (Paper-I) North Zone (Ist Sitting)

17. If $x : y = 3 : 4$, then the value of $\frac{5x-2y}{7x+2y} =$

(1) $\frac{7}{25}$ (2) $\frac{7}{23}$

(3) $\frac{7}{29}$ (4) $\frac{7}{17}$

(SSC Multi-Tasking (Non-Technical) Staff Exam. 20.02.2011)

18. If $x^2 + 9y^2 = 6xy$, then $x : y$ is

(1) 1 : 3 (2) 3 : 2

(3) 3 : 1 (4) 2 : 3

(SSC Constable (GD) Exam. 12.05.2013 Ist Sitting)

19. If $a + b + c = 4\sqrt{3}$ and $a^2 + b^2 + c^2 = 16$, then the ratio $a : b : c$ is

(1) 1 : 1 : 1 (2) $1 : \sqrt{2} : \sqrt{3}$

(3) 1 : 2 : 3 (4) None of these

(SSC CGL Tier-I Re-Exam. (2013) 20.07.2014 (Ist Sitting)

20. If $4x + 5y = 83$ and $3x : 2y = 21 : 22$, then $(y - x)$ equals

(1) 3 (2) 4

(3) 7 (4) 11

(SSC CGL Tier-II Exam. 21.09.2014)

21. If $\frac{x}{xa+yb+zc} = \frac{y}{ya+zb+xc} = \frac{z}{za+xb+yc}$ and $x + y + z \neq 0$, then each ratio is

(1) $\frac{1}{a-b-c}$ (2) $\frac{1}{a+b-c}$

(3) $\frac{1}{a-b+c}$ (4) $\frac{1}{a+b+c}$

(SSC CHSL DEO & LDC Exam. 9.11.2014)

22. If $x : y = 3 : 2$, then the value of $\frac{x+y}{x-y}$ is

(1) 5 : 1 (2) 1 : 3

(3) 1 : 5 (4) 3 : 1

(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

23. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, Then $a : b : c$ is :

(1) 1 : 1 : 2 (2) 1 : 1 : 1

(3) 1 : 2 : 1 (4) 2 : 1 : 1

(SSC CHSL (10+2) LDC, DEO & PA/SA Exam, 15.11.2015 (Ist Sitting) TF No. 6636838)

24. If $a^2 + 13b^2 + c^2 - 4ab - 6bc = 0$, then $a : b : c$ is

(1) 1 : 2 : 3 (2) 2 : 3 : 1

(3) 2 : 1 : 3 (4) 1 : 3 : 2

(SSC CGL Tier-I (CBE) Exam. 28.08.2016 (IST Sitting)

25. If $(2x - y)^2 + (3y - 2z)^2 = 0$, then the ratio $x : y : z$ is :

(1) 1 : 3 : 2 (2) 1 : 2 : 3

(3) 3 : 1 : 2 (4) 3 : 2 : 1

(SSC CGL Tier-I (CBE) Exam. 03.09.2016 (IInd Sitting)

TYPE–V

1. In how many ways can a committee schedule three speakers for three different meetings if they are all available on any of five possible dates?

(1) 10 (2) 36

(3) 60 (4) 120

(SSC CPO S.I. Exam. 05.09.2004)

2. How many even three-digit numbers can be formed from the digits 1, 2, 5, 6 and 9 without repeating any of the digits ?

(1) 120 (2) 48

(3) 40 (4) 24

(SSC CPO S.I. Exam. 07.09.2003)

3. If ten friends shake hands mutually, then the total number of hand shakes is

(1) 45 (2) 50

(3) 90 (4) 100

(SSC CPO S.I. Exam. 05.09.2004)

4. The total number of integers between 200 and 400, each of which either begins with 3 or ends with 3 or both, is

(1) 10 (2) 100

(3) 110 (4) 120

(SSC CGL Prelim Exam. 04.02.2007 (First Sitting)

TYPE–VI

1. If $[p]$ means the greatest integer less than or equal to p, then $\left[-\frac{1}{4}\right] + \left[4\frac{1}{4}\right] + [3]$ is equal to

(1) 4 (2) 5

(3) 6 (4) 7

(SSC Section Officer (Commercial Audit) Exam.16.11.2003)

2. If $\oplus$ is an operation such that

$a \oplus b = 2a$ when $a > b$

$= a + b$ when $a < b$

$= a^2$ when $a = b$,

then, $\left[\frac{(5 \oplus 7) + (4 \oplus 4)}{3(5 \oplus 5) - (15 \oplus 11) - 3}\right]$ is equal to :

(1) $\frac{1}{3}$ (2) $\frac{14}{23}$

(3) $\frac{2}{3}$ (4) $\frac{14}{13}$

(SSC CPO S.I. Exam. 16.12.2007)

3. If $\circledast$ is an operation such that $a \circledast b = a + b$ when $a > 0, b > 0$

$a \circledast b = \sqrt{a^2 + b^2}$ for all other values of a and b. The value of $\frac{8 \circledast (7-13) - (3 \circledast 1)}{(3-6) \circledast (9-5)}$ is

(1) $\frac{1}{5}$ (2) $\frac{4}{5}$

(3) $\frac{6}{5}$ (4) $\frac{2}{5}$

(SSC CPO S.I. Exam. 09.11.2008)

4. The expression $x^4 - 2x^2 + k$ will be a perfect square when the value of k is

(1) 2 (2) 1

(3) –1 (4) –2

(SSC Graduate Level Tier-I Exam. 11.11.2012 (Ist Sitting)

5. If $x = \sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}$, then $x^3 + 3bx$ is equal to

(1) 0 (2) a

(3) $2a$ (4) 1

(SSC Graduate Level Tier-I Exam. 21.04.2013 IInd Sitting)

6.

$$\frac{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}-3\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{5}+\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}}{\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{5}-\left(\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{1}{5}+\frac{1}{5}\cdot\frac{1}{3}\right)}$$

is equal to :

(1) $\frac{2}{3}$ (2) $\frac{3}{4}$

(3) $\frac{47}{60}$ (4) $\frac{49}{60}$

(SSC CGL Prelim Exam. 08.02.2004 (Ist Sitting) & (SSC Delhi Police S.I. Exam. 19.08.2012)

7. When x^m is multiplied by x^n, product is 1. The relation between m and n is

(1) $mn = 1$ (2) $m = n$

(3) $m + n = 1$ (4) $m = -n$

(SSC CGL Tier-II Exam. 12.04.2015 TF No. 567 TL 9)

8. The term, that should be added to $(4x^2 + 8x)$ so that resulting expression be a perfect square, is

(1) 2 (2) 4

(3) $2x$ (4) 1

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

9. The mean of x and $\frac{1}{x}$ is N. Then the mean of x^2 and $\frac{1}{x^2}$ is

(1) N^2 (2) $2N^2-1$

(3) N^2-2 (4) $4N^2-2$

(SSC CAPFs SI, CISF ASI & Delhi Police SI Exam, 21.06.2015 (Ist Sitting) TF No. 8037731)

10. If $3(a^2 + b^2 + c^2) = (a + b + c)^2$, then the relation between a, b and c is

(1) $a \ne b \ne c$ (2) $a = b \ne c$

(3) $a \ne b = c$ (4) $a = b = c$

(SSC CGL Tier-II Exam, 25.10.2015, TF No. 1099685)

11. What is the digit in the unit's place in the number $\frac{15!}{100}$.

(1) 5 (2) 7

(3) 3 (4) 0

(SSC CAPFs (CPO) SI & ASI, Delhi Police Exam. 05.06.2016) (Ist Sitting)

12. Three numbers are in Arithmetic Progression (A.P.) whose sum is 30 and the product is 910. Then the greatest number in the A.P. is

(1) 17 (2) 15

(3) 13 (4) 10

(SSC CGL Tier-II (CBE) Exam. 30.11.2016)

13. If $U_n = \frac{1}{n} - \frac{1}{n+1}$, then the value of $U_1 + U_2 + U_3 + U_4 + U_5$ is :

(1) $\frac{1}{4}$ (2) $\frac{5}{6}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

(SSC CGL Tier-I (CBE) Exam. 27.10.2016 (Ist Sitting)

SHORT ANSWERS

TYPE-I

1. (1)	**2.** (3)	**3.** (2)	**4.** (4)
5. (3)	**6.** (3)	**7.** (3)	**8.** (2)
9. (4)	**10.** (2)	**11.** (3)	**12.** (4)
13. (2)	**14.** (3)	**15.** (3)	**16.** (3)
17. (2)	**18.** (3)	**19.** (2)	**20.** (3)
21. (3)	**22.** (2)	**23.** (3)	**24.** (3)
25. (1)	**26.** (2)	**27.** (1)	**28.** (3)
29. (1)	**30.** (2)	**31.** (3)	**32.** (4)
33. (4)	**34.** (2)	**35.** (1)	**36.** (2)
37. (3)	**38.** (2)	**39.** (3)	**40.** (3)
41. (2)	**42.** (4)	**43.** (2)	**44.** (2)
45. (4)	**46.** (1)	**47.** (2)	**48.** (4)
49. (2)	**50.** (4)	**51.** (2)	**52.** (1)
53. (2)	**54.** (4)	**55.** (1)	**56.** (1)
57. (3)	**58.** (1)	**59.** (2)	**60.** (2)
61. (1)	**62.** (3)	**63.** (2)	**64.** (1)
65. (2)	**66.** (1)	**67.** (2)	**68.** (4)
69. (1)	**70.** (3)	**71.** (2)	**72.** (4)
73. (4)	**74.** (4)	**75.** (4)	**76.** (4)
77. (1)	**78.** (1)	**79.** (2)	**80.** (3)
81. (1)	**82.** (2)	**83.** (3)	**84.** (2)
85. (3)	**86.** (4)	**87.** (2)	**88.** (2)
89. (3)	**90.** (4)	**91.** (4)	**92.** (2)
93. (4)	**94.** (4)	**95.** (4)	**96.** (1)
97. (3)	**98.** (1)	**99.** (4)	**100.** (2)
101. (2)	**102.** (2)	**103.** (1)	**104.** (4)
105. (3)	**106.** (3)	**107.** (3)	**108.** (3)
109. (4)	**110.** (2)	**111.** (2)	**112.** (3)
113. (4)	**114.** (4)	**115.** (3)	**116.** (2)
117. (3)	**118.** (3)	**119.** (2)	**120.** (2)
121. (1)	**122.** (2)	**123.** (4)	**124.** (2)
125. (3)	**126.** (2)	**127.** (4)	**128.** (3)
129. (4)	**130.** (3)	**131.** (3)	**132.** (1)
133. (4)	**134.** (2)	**135.** (3)	**136.** (2)
137. (2)	**138.** (4)	**139.** (4)	**140.** (1)
141. (3)	**142.** (1)	**143.** (3)	**144.** (4)
145. (2)	**146.** (1)	**147.** (3)	**148.** (4)
149. (3)	**150.** (3)	**151.** (3)	**152.** (1)
153. (1)	**154.** (2)	**155.** (3)	**156.** (1)
157. (2)	**158.** (4)	**159.** (1)	**160.** (4)
161. (1)	**162.** (4)	**163.** (1)	**164.** (4)
165. (4)	**166.** (4)	**167.** (1)	**168.** (2)
169. (3)	**170.** (3)	**171.** (2)	**172.** (2)
173. (3)	**174.** (3)	**175.** (1)	**176.** (2)
177. (4)	**178.** (1)	**179.** (1)	**180.** (2)
181. (4)	**182.** (1)	**183.** (3)	**184.** (4)
185. (4)	**186.** (2)	**187.** (2)	**188.** (1)
189. (2)	**190.** (2)	**191.** (3)	**192.** (2)
193. (4)	**194.** (4)	**195.** (3)	**196.** (4)
197. (3)	**198.** (1)	**199.** (4)	**200.** (1)
201. (3)	**202.** (1)	**203.** (2)	**204.** (4)
205. (4)	**206.** (3)	**207.** (3)	**208.** (3)
209. (2)	**210.** (3)	**211.** (3)	**212.** (3)
213. (1)	**214.** (1)	**215.** (1)	**216.** (1)
217. (4)	**218.** (3)	**219.** (3)	**220.** (4)
221. (2)	**222.** (1)	**223.** (4)	**224.** (2)
225. (3)	**226.** (1)	**227.** (1)	**228.** (2)
229. (3)	**230.** (3)	**231.** (1)	**232.** (3)
233. (1)	**234.** (4)	**235.** (2)	**236.** (3)
237. (2)	**238.** (2)	**239.** (3)	**240.** (1)

241. (2)	**242.** (2)	**243.** (1)	**244.** (4)
245. (1)	**246.** (3)	**247.** (3)	**248.** (4)
249. (2)	**250.** (2)	**251.** (3)	**252.** (2)
253. (4)	**254.** (2)	**255.** (1)	**256.** (3)
257. (*)	**258.** (2)	**259.** (2)	**260.** (3)
261. (4)	**262.** (2)	**263.** (2)	**264.** (3)
265. (3)	**266.** (3)	**267.** (1)	**268.** (3)
269. (2)	**270.** (2)	**271.** (2)	**272.** (1)
273. (3)	**274.** (2)	**275.** (1)	**276.** (1)
277. (2)	**278.** (1)	**279.** (2)	**280.** (2)
281. (4)	**282.** (3)	**283.** (3)	**284.** (1)
285. (3)	**286.** (2)	**287.** (3)	**288.** (4)
289. (2)	**290.** (2)	**291.** (1)	**292.** (3)
293. (3)	**294.** (1)	**295.** (3)	**296.** (2)
297. (2)	**298.** (1)	**299.** (4	**300.** (3)
301. (3)	**302.** (4)	**303.** (3)	**304.** (3)
305. (4)	**306.** (2)	**307.** (3)	**308.** (2)
309. (3)	**310.** (3)	**311.** (1)	**312.** (2)
313. (3)	**314.** (1)	**315.** (2)	**316.** (3)
317. (1)	**318.** (1)	**319.** (3)	**320.** (1)

TYPE-II

1. (3)	**2.** (3)	**3.** (3)	**4.** (1)
5. (2)	**6.** (1)	**7.** (1)	**8.** (2)
9. (1)	**10.** (3)	**11.** (1)	**12.** (3)
13. (3)	**14.** (3)	**15.** (2)	**16.** (1)
17. (1)	**18.** (3)	**19.** (3)	**20.** (4)
21. (2)	**22.** (1)	**23.** (1)	**24.** (1)
25. (2)	**26.** (2)	**27.** (2)	**28.** (2)
29. (1)	**30.** (2)	**31.** (3)	**32.** (2)
33. (1)	**34.** (4)	**35.** (4)	**36.** (3)
37. (3)	**38.** (4)	**39.** (2)	**40.** (2)
41. (3)	**42.** (2)	**43.** (2)	**44.** (4)
45. (4)	**46.** (1)	**47.** (3)	**48.** (2)
49. (3)	**50.** (2)	**51.** (1)	**52.** (4)
53. (3)	**54.** (1)	**55.** (1)	**56.** (3)
57. (4)	**58.** (1)	**59.** (4)	**60.** (3)
61. (3)	**62.** (4)	**63.** (2)	**64.** (2)
65. (2)	**66.** (1)	**67.** (4)	**68.** (3)
69. (1)	**70.** (2)	**71.** (4)	**72.** (1)
73. (4)	**74.** (2)	**75.** (1)	**76.** (3)
77. (2)	**78.** (3)	**79.** (1)	**80.** (4)
81. (4)	**82.** (3)	**83.** (2)	**84.** (4)
85. (2)	**86.** (3)	**87.** (1)	**88.** (2)
89. (3)	**90.** (2)	**91.** (2)	**92.** (3)
93. (4)	**94.** (3)	**95.** (2)	**96.** (3)
97. (2)	**98.** (1)	**99.** (3)	**100.** (3)
101. (3)	**102.** (2)	**103.** (3)	**104.** (2)
105. (4)	**106.** (1)	**107.** (1)	**108.** (2)
109. (1)	**110.** (2)	**111.** (1)	**112.** (4)
113. (2)	**114.** (3)	**115.** (3)	**116.** (4)
117. (1)	**118.** (1)	**119.** (4)	**120.** (3)
121. (4)	**122.** (3)	**123.** (3)	**124.** (2)
125. (1)	**126.** (1)	**127.** (1)	**128.** (3)
129. (2)	**130.** (3)	**131.** (4)	**132.** (3)
133. (4)	**134.** (4)	**135.** (3)	**136.** (4)
137. (4)	**138.** (1)	**139.** (1)	**140.** (4)
141. (3)	**142.** (4)	**143.** (4)	**144.** (4)
145. (2)	**146.** (4)	**147.** (1)	**148.** (1)
149. (3)	**150.** (2)	**151.** (2)	**152.** (1)
153. (1)	**154.** (3)	**155.** (1)	**156.** (2)
157. (2)	**158.** (1)	**159.** (2)	**160.** (2)
161. (3)	**162.** (2)	**163.** (1)	**164.** (1)
165. (3)	**166.** (4)	**167.** (*)	**168.** (3)
169. (1)	**170.** (2)	**171.** (1)	**172.** (2)
173. (1)	**174.** (3)	**175.** (3)	**176.** (4)
177. (3)	**178.** (3)	**179.** (4)	**180.** (4)
181. (2)	**182.** (2)	**183.** (4)	**184.** (3)
185. (2)	**186.** (1)	**187.** (1)	**188.** (3)
189. (3)	**190.** (3)	**191.** (2)	**192.** (1)
193. (1)	**194.** (3)	**195.** (3)	**196.** (2)
197. (3)	**198.** (2)	**199.** (1)	**200.** (2)
201. (4)	**202.** (4)	**203.** (1)	**204.** (1)
205. (3)	**206.** (3)	**207.** (3)	**208.** (3)
209. (3)	**210.** (4)	**211.** (2)	**212.** (3)
213. (3)	**214.** (1)	**215.** (3)	**216.** (1)
217. (4)	**218.** (2)	**219.** (3)	**220.** (2)
221. (4)	**222.** (4)	**223.** (3)	**224.** (3)
225. (3)	**226.** (4)	**227.** (3)	**228.** (4)
229. (2)	**230.** (2)	**231.** (1)	**232.** (1)
233. (3)	**234.** (3)	**235.** (1)	**236.** (2)
237. (4)	**238.** (4)	**239.** (4)	**240.** (3)
241. (1)	**242.** (4)	**243.** (2)	**244.** (4)
245. (3)	**246.** (1)	**247.** (4)	**248.** (2)
249. (3)	**250.** (4)	**251.** (1)	**252.** (2)
253. (2)	**254.** (3)	**255.** (3)	**256.** (4)
257. (1)	**258.** (2)	**259.** (4)	**260.** (3)
261. (3)	**262.** (4)	**263.** (4)	**264.** (2)
265. (3)	**266.** (2)	**267.** (2)	**268.** (2)
269. (2)	**270.** (3)	**271.** (3)	**272.** (4)
273. (3)	**274.** (4)	**275.** (3)	**276.** (2)
277. (2)	**278.** (3)	**279.** (1)	**280.** (2)
281. (1)	**282.** (2)	**283.** (2)	**284.** (1)

TYPE-III

1. (1)	**2.** (2)	**3.** (2)	**4.** (1)
5. (4)	**6.** (3)	**7.** (3)	**8.** (4)
9. (2)	**10.** (2)	**11.** (3)	**12.** (2)
13. (2)	**14.** (1)	**15.** (2)	**16.** (1)
17. (3)	**18.** (3)	**19.** (3)	**20.** (2)
21. (1)	**22.** (4)	**23.** (3)	**24.** (1)
25. (2)	**26.** (2)	**27.** (4)	**28.** (3)
29. (3)	**30.** (2)	**31.** (4)	**32.** (1)
33. (3)	**34.** (1)	**35.** (2)	**36.** (3)
37. (2)	**38.** (1)	**39.** (1)	**40.** (4)
41. (2)	**42.** (2)	**43.** (2)	**44.** (2)
45. (3)	**46.** (1)	**47.** (2)	**48.** (2)
49. (1)			

TYPE-IV

1. (1)	**2.** (2)	**3.** (3)	**4.** (3)
5. (3)	**6.** (4)	**7.** (3)	**8.** (1)
9. (1)	**10.** (4)	**11.** (2)	**12.** (3)
13. (3)	**14.** (2)	**15.** (3)	**16.** (1)
17. (3)	**18.** (3)	**19.** (1)	**20.** (2)
21. (4)	**22.** (1)	**23.** (2)	**24.** (3)
25. (2)			

TYPE-V

1. (3)	**2.** (4)	**3.** (1)	**4.** (3)

TYPE-VI

1. (3)	**2.** (3)	**3.** (3)	**4.** (2)
5. (3)	**6.** (3)	**7.** (4)	**8.** (2)
9. (2)	**10.** (4)	**11.** (4)	**12.** (3)
13. (2)			

EXPLANATIONS

TYPE-I

1. (1) a * b = 2a – 3b + ab
$\Rightarrow 3 * 5 = 2 \times 3 - 3 \times 5 + 3 \times 5 = 6$
$5 * 3 = 2 \times 5 - 3 \times 3 + 3 \times 5$
$= 10 - 9 + 15 = 16$
Therefore, 3 * 5 + 5 *3
$= 6 + 16 = 22$

2. (3) $p \times q = p + q + \frac{p}{q}$

$\therefore\ 8 \times 2 = 8 + 2 + \frac{8}{2}$

$= 10 + 4 = 14$

3. (2) $(x + y) = 3(x - y) = 3x - 3y$
$\Rightarrow 3y + y = 3x - x$
$\Rightarrow 2x = 4y$
$\Rightarrow x = 2y$

$\Rightarrow \frac{x}{y} = \frac{2}{1}$

$\therefore x = 2,\ y = 1$

$$\frac{3xy}{2(x^2 - y^2)} = \frac{3 \times 2 \times 1}{2 \times (4-1)} = \frac{6}{6} = 1$$

4. (4) Given expression

$$= \left(1+\frac{1}{x}\right)\left(1+\frac{1}{x+1}\right)\left(1+\frac{1}{x+2}\right)\left(1+\frac{1}{x+3}\right)$$

$$= \frac{x+1}{x} \times \frac{x+2}{x+1} \times \frac{x+3}{x+2} \times \frac{x+4}{x+3}$$

$$= \frac{x+4}{x}$$

5. (3) a * b = 2 (a + b)
$\therefore$ 5 * 2 = 2 (5 + 2)
$= 2 \times 7 = 14$

6. (3) $\frac{2a+b}{a+4b} = 3$ (Given)

$\Rightarrow 2a + b = 3a + 12b$
$\Rightarrow 3a - 2a = b - 12b$
$\Rightarrow a = -11b$

Then, $\frac{a+b}{a+2b} = \frac{-11b+b}{-11b+2b}$

$= \frac{-10b}{-9b} = \frac{10}{9}$

7. (3)

$$x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}} = \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}}$$

$$= \sqrt{\frac{(\sqrt{5}+1)^2}{4}} = \frac{\sqrt{5}+1}{2}$$

$\therefore\ 5x^2 - 5x - 1$

$$= 5\left(\frac{(\sqrt{5}+1)}{2}\right)^2 - 5\frac{(\sqrt{5}+1)}{2} - 1$$

$$= 5\left(\frac{5+1+2\sqrt{5}}{4}\right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= 5\left(\frac{3+\sqrt{5}}{2}\right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= \frac{15+5\sqrt{5}-5\sqrt{5}-5-2}{2}$$

$$= \frac{8}{2} = 4$$

8. (2) Given a * b = a + b + ab
$\therefore$ 3 * 4 – 2 * 3
$= (3 + 4 + 3 \times 4) - (2 + 3 + 2 \times 3)$
$= (7 + 12) - (5 + 6) = 19 - 11 = 8$

9. (4) $x = 7 - 4\sqrt{3}$

$\therefore\ \frac{1}{x} = \frac{1}{7 - 4\sqrt{3}}$

$$= \frac{1(7+4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$= \frac{7+4\sqrt{3}}{49-48} = 7 + 4\sqrt{3}$$

$\therefore\ x + \frac{1}{x}$

$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = 14$

10. (2) $\because x * y = 3x + 2y$
2 * 3 + 3 * 4
$= 3 \times 2 + 2 \times 3 + 3 \times 3 + 2 \times 4$
$= 6 + 6 + 9 + 8 = 29$

11. (3) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$ (Let)

$a = 3k,\ b = 4k,\ c = 7k$

$\therefore\ \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k}$

$= \frac{14k}{7k} = 2$

12. (4) $\frac{144}{0.144} = \frac{14.4}{x}$

$\Rightarrow 144 \times x = 14.4 \times 0.144$

$\Rightarrow x = \frac{14.4 \times 0.144}{144}$

$= \frac{144 \times 144}{144 \times 10000} = 0.0144$

13. (2) Since 1 < x < 2, we have
x – 1 > 0 and
x – 3 < 0
or, 3 – x > 0

$\therefore \sqrt{(x-1)^2} + \sqrt{(x-3)^2}$

$= \sqrt{(x-1)^2} + \sqrt{(3-x)^2}$

$[\because (x-3)^2 = (3-x)^2]$
$= x - 1 + 3 - x = 2$

14. (3) It is given that

$a \otimes b = (a \times b) + b$

$\therefore 5 \otimes 7 = (5 \times 7) + 7 = 35 + 7 = 42$

15. (3) We have,
$10^{0.48} = x,\ 10^{0.70} = y$
$\therefore x^z = y^2$
$\Rightarrow (10^{0.48})^z = (10^{0.70})^2$
$\Rightarrow 10^{0.48z} = 10^{1.4}$
$\Rightarrow 0.48z = 1.4$

$\Rightarrow z = \frac{1.4}{0.48} = 2.9$

16. (3) $4A + \frac{7}{B} + 2C + \frac{5}{D} + 6E$
$= 47.2506$

$= 40 + 7 + \frac{2}{10} + \frac{5}{100} + \frac{6}{10000}$

$4A = 40 \Rightarrow A = 10$

$\frac{7}{B} = 7 \Rightarrow 7B = 7 = \Rightarrow B = 1$

$2C = \frac{2}{10} \Rightarrow C = 0.1$

$\frac{5}{D} = \frac{5}{100} \Rightarrow D = 100$

$6E = \frac{6}{10000} \Rightarrow E = 0.0001$

$5A + 3B + 6C + D + 3E$
$= 5 \times 10 + 3 \times 1 + 6 \times 0.1 + 100 + 3 \times 0.0001$
$= 50 + 3 + 0.6 + 100 + 0.0003$
$= 153.6003$

17. (2) $x * y = x^2 + y^2 - xy$ (Given)
$\Rightarrow 9 * 11 = 9^2 + 11^2 - 9 \times 11$
$= 81 + 121 - 99$
$= 202 - 99 = 103$

18. (3) $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$

$\Rightarrow \frac{p^2 - 2p + 1}{2p} = 4$

$\Rightarrow \frac{p^2 - 2p + 1}{p} = 8$

$\Rightarrow \frac{p^2}{p} - \frac{2p}{p} + \frac{1}{p} = 8$

$\Rightarrow p + \frac{1}{p} = 8 + 2 = 10$

19. (2) $5^{5x+5}=1$

$\Rightarrow 5^{5x}\times 5^{5}=1$

$\Rightarrow 5^{5x}=\frac{1}{5^5}$

$\Rightarrow 5^{5x}=5^{-5} \Rightarrow 5x=-5$

$\Rightarrow x=-1$

Method 2 :

$5^{5x+5}=1$

$\Rightarrow 5^{5x+5}=5^{\circ}$

$\Rightarrow 5x+5=0 \Rightarrow x=-1$

20. (3) $3^{x+3}+7=250$

$\Rightarrow 3^{x+3}=243 \quad \Rightarrow 3^{x+3}=3^5$

$\Rightarrow x+3=5 \Rightarrow x=2$

21. (3) $\frac{1}{4}\times\frac{2}{6}\times\frac{3}{8}\times\frac{4}{10}\times\frac{5}{12}\ldots\times\frac{31}{64}$

$=\frac{1}{2^x}$

$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times \ldots$ to 30 terms

$\times\frac{1}{64}=\frac{1}{2^x}$

$\Rightarrow \frac{1}{2^{30}}\times\frac{1}{2^6}=\frac{1}{2^x}$

$\Rightarrow \frac{1}{2^{36}}=\frac{1}{2^x}\Rightarrow x=36$

22. (2) Expression

$=\frac{(243)^{\frac{n}{5}}\times 3^{2n+1}}{9^n\times 3^{n-1}}$

$=\frac{(3^5)^{\frac{n}{5}}\times 3^{2n+1}}{(3^2)^n\times 3^{n-1}}=\frac{3^n\times 3^{2n+1}}{3^{2n}\times 3^{n-1}}$

$=\frac{3^{n+2n+1}}{3^{2n+n-1}}=\frac{3^{3n+1}}{3^{3n-1}}$

$=3^{3n+1-3n+1}=3^2=9$

23. (3) $x=0.5$ and $y=0.2$ (Given)

$\therefore \sqrt{0.6}\times(3y)^x$

$=\sqrt{0.6}\times(3\times 0.2)^{0.5}$

$=\sqrt{0.6}\times(0.6)^{\frac{1}{2}}$

$=\sqrt{0.6\times 0.6}=0.6$

24. (3) $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^x$

$\Rightarrow x^{x.x^{\frac{1}{2}}}=\left(x\times x^{\frac{1}{2}}\right)^x$

$\Rightarrow x^{x^{1+\frac{1}{2}}}=\left(x^{1+\frac{1}{2}}\right)^x$

$x^{x^{3/2}}=\left(x^{3/2}\right)^x=x^{\frac{3x}{2}}$

$\Rightarrow x^{\frac{3}{2}}=\frac{3x}{2}\Rightarrow x^{\frac{3}{2}}-\frac{3x}{2}=0$

$\Rightarrow x\left(x^{\frac{1}{2}}-\frac{3}{2}\right)=0$

$\Rightarrow x=0$ or $x^{\frac{1}{2}}=\frac{3}{2}$

$\Rightarrow x=\left(\frac{3}{2}\right)^2=\frac{9}{4}$

$x=0$ given indeterminate value.

$\therefore x=\frac{9}{4}$

25. (1) $a^2+b^2+c^2-ab-bc-ca$

$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$=\frac{1}{2}\left[(7-5)^2+(5-3)^2+(3-7)^2\right]$

$=\frac{1}{2}(4+4+16)$

$=\frac{1}{2}\times 24=12$

26. (2) $7^x=\frac{1}{343}$

$\Rightarrow 7^x=\frac{1}{7^3}=7^{-3}$

$\Rightarrow x=-3$

27. (1) $\frac{a}{2}=\frac{b}{3}=\frac{c}{5}=k$ (Let)

$\therefore a=2k,\ b=3k,\ c=5k$

$\therefore \frac{a+b+c}{c}=\frac{2k+3k+5k}{5k}$

$=\frac{10k}{5k}=2$

28. (3) $0.13\div p^2=13$

$\Rightarrow \frac{0.13}{p^2}=13$

$\Rightarrow p^2=\frac{0.13}{13}=\frac{1}{100}$

$\Rightarrow p=\frac{1}{10}=0.1$

29. (1) $\frac{a}{3}=\frac{b}{2}\Rightarrow\frac{a}{b}=\frac{3}{2}$

$\therefore \frac{2a+3b}{3a-2b}=\frac{2\times\frac{a}{b}+3}{3\times\frac{a}{b}-2}$

$=\frac{2\times\frac{3}{2}+3}{3\times\frac{3}{2}-2}=\frac{6}{\frac{9-4}{2}}=\frac{12}{5}$

30. (2) $x+\frac{1}{4}\sqrt{x}+a^2$

$=\left(\sqrt{x}\right)^2+2.\sqrt{x}.\frac{1}{8}+(a)^2$

Clearly $a=\frac{1}{8}$.

Then, expression $=\left(\sqrt{x}+\frac{1}{8}\right)^2$

31. (3) Arithmetic mean (AM) $=\frac{a+b}{2}$

Geometric mean (GM) $=\sqrt{ab}$

As AM > GM

$\frac{a+b}{2}>\sqrt{ab}$

32. (4) **Tricky Approach**

$\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$

$\Rightarrow\left(\frac{a}{1-a}+1\right)+\left(\frac{b}{1-b}+1\right)+\left(\frac{c}{1-c}+1\right)$

$=3+1=4$

$\Rightarrow \frac{a+1-a}{1-a}+\frac{b+1-b}{1-b}+\frac{c+1-c}{1-c}=4$

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$

33. (4) $x^{\frac{1}{3}}=y^{\frac{1}{4}}$

$\Rightarrow\left(x^{\frac{1}{3}}\right)^{12}=\left(y^{\frac{1}{4}}\right)^{12}\Rightarrow x^4=y^3$

$\Rightarrow (x^4)^5=(y^3)^5\Rightarrow x^{20}=y^{15}$

34. (2) We know that $a^{\circ}=1$

$\therefore a^{2x+2}=1=a^{\circ}$

$\Rightarrow 2x+2=0$

$\Rightarrow x=\frac{-2}{2}=-1$

35. (1) For expression ax^2+bx+c, $a>0$, the minimum value is given by

$\frac{4ac-b^2}{4a}$

Here, for $x^2 - x + 1$
$a = 1$, $b = -1$, $c = 1$
$\therefore$ Minimum value

$$= \frac{4 \times 1 \times 1 - 1}{4 \times 1} = \frac{3}{4}$$

36. (2) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = \frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$

(Rationalising the denominator)

$$= \frac{(\sqrt{7}-2)^2}{7-4} = \frac{7+4-4\sqrt{7}}{3}$$

$$= \frac{11}{3} - \frac{4\sqrt{7}}{3}$$

$$\therefore \frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$$

$$\Rightarrow \frac{11}{3} - \frac{4}{3}\sqrt{7} = a\sqrt{7}+b$$

Clearly,

$a = -\frac{4}{3}$ and $b = \frac{11}{3}$

37. (3) $(125)^x = 3125$
$\Rightarrow (5^3)^x = 5^5 \Rightarrow 5^{3x} = 5^5$
$\Rightarrow 3x = 5$

$$\Rightarrow x = \frac{5}{3}$$

38. (2) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$
We know that $5^2 + 12^2 = 13^2$
[Pythagorean Triplet]

$\therefore \sqrt{x} = 2 \Rightarrow x = 2^2 = 4$

39. (3) $2^{2x-y} = 16 = 2^4$
$\Rightarrow 2x - y = 4$ .... (i)
$2^{x+y} = 32 = 2^5$
$\Rightarrow x + y = 5$ .... (ii)
On adding equations (i) and (ii),
$3x = 9 \Rightarrow x = 3$
From equation (ii),
$y = 5 - x = 5 - 3 = 2$
$\therefore xy = 3 \times 2 = 6$

40. (3) $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$

$$\Rightarrow \left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-3}\left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^0\left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2x-1}$$

$\Rightarrow 2x - 1 = -3$
$\Rightarrow 2x = -3 + 1 = -2$
$\Rightarrow x = -1$

Method : 2

$$\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{-6+3} = \left(\frac{3}{5}\right)^{2x-1}$$

$\Rightarrow -3 = 2x - 1$
$\Rightarrow -2 = 2x$
$\Rightarrow x = -1$

41. (2) $\frac{2x-y}{x+2y} = \frac{1}{2}$

$\Rightarrow 4x - 2y = x + 2y$
$\Rightarrow 3x = 4y$

$$\Rightarrow \frac{x}{y} = \frac{4}{3}$$

$$\therefore \frac{3x-y}{3x+y} = \frac{y\left(3\frac{x}{y}-1\right)}{y\left(3\frac{x}{y}+1\right)}$$

$$= \frac{3 \times \frac{4}{3} - 1}{3 \times \frac{4}{3} + 1}$$

$$= \frac{4-1}{4+1} = \frac{3}{5}$$

42. (4) $\boxed{\text{Tricky approach}}$
$a^2 - b^2 = 19$
$\Rightarrow 10^2 - 9^2 = 19$
$\Rightarrow a = 10$

43. (2) $\boxed{\text{Tricky approach}}$

$$\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}} = \frac{2}{1}$$

By componendo and dividendo,

$$\Rightarrow \frac{2\sqrt{3+x}}{2\sqrt{3-x}} = \frac{2+1}{2-1} = 3$$

Squaring on both sides, we get

$$\frac{3+x}{3-x} = 9$$

$\Rightarrow 3 + x = 27 - 9x$
$\Rightarrow 9x + x = 27 - 3 = 24$

$$\Rightarrow x = \frac{24}{10} = \frac{12}{5}$$

44. (2) $x + \frac{1}{x} = 5$

$\Rightarrow x^2 - 5x + 1 = 0$
$\Rightarrow 3x^2 - 15x + 3 = 0$

$$\therefore \frac{2x}{3x^2-5x+3} = \frac{2x}{15x-5x}$$

$$= \frac{2x}{10x} = \frac{1}{5}$$

45. (4) $x = \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{x} = \frac{2}{\sqrt{3}}$

By componendo and dividendo,

$$\frac{1+x}{1-x} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{1+x}{1-x} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{4-3}$$

$$\Rightarrow \frac{1+x}{1-x} = (2+\sqrt{3})^2$$

$$\therefore \frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{2+\sqrt{3}}{1}$$

By componendo and dividendo

$$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \frac{2+\sqrt{3}+1}{2+\sqrt{3}-1}$$

$$= \frac{3+\sqrt{3}}{\sqrt{3}+1} = \frac{\sqrt{3}(\sqrt{3}+1)}{\sqrt{3}+1} = \sqrt{3}$$

46. (1) $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2}$$

$$= \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Similarly,

$$y = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$$

$$\therefore x^2 + y^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$$

$= 4 + 3 + 4\sqrt{3} + 4 + 3 - 4\sqrt{3}$
$= 14$

47. (2) $4^{4x+1} = \frac{1}{64} = \frac{1}{4^3}$
$\Rightarrow 4^{4x+1} = 4^{-3} \Rightarrow 4x + 1 = -3$
$\Rightarrow 4x = -4 \qquad \Rightarrow x = -1$

48. (4) $\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}}=\frac{2}{1}$

By componendo and dividendo,

$\frac{2\sqrt{x+4}}{2\sqrt{x-4}}=\frac{3}{1}$

On squaring,

$\frac{x+4}{x-4}=\frac{9}{1}$

$\Rightarrow 9x - 36 = x + 4$

$\Rightarrow 9x - x = 36 + 4$

$\Rightarrow 8x = 40$

$\Rightarrow x = 5$

49. (2) $\sqrt{2^x} = 256$

$\Rightarrow 2^{\frac{x}{2}} = 2^8$

$\Rightarrow \frac{x}{2} = 8 \Rightarrow x = 16$

50. (4) $\frac{(\sqrt{5})^7}{(\sqrt{5})^5} = 5^p$

$\Rightarrow (\sqrt{5})^{7-5} = 5^p$

$\Rightarrow (\sqrt{5})^2 = 5^p$

$\Rightarrow 5^1 = 5^P \Rightarrow p = 1$

51. (2) $\sqrt{1-\frac{x^3}{100}}=\frac{3}{5}$

Squaring both sides,

$1-\frac{x^3}{100}=\frac{9}{25}$

$\Rightarrow \frac{x^3}{100}=1-\frac{9}{25}=\frac{25-9}{25}=\frac{16}{25}$

$\Rightarrow x^3 = \frac{16}{25}\times 100 = 64$

$\therefore x = \sqrt[3]{64} = \sqrt[3]{4\times4\times4} = 4$

52. (1) Given that

$a \star b = 2a + 3b - ab$

$\therefore 3 \star 5 + 5 \star 3$

$= (2 \times 3 + 3 \times 5 - 3 \times 5) + (5 \times 2 + 3 \times 3 - 5 \times 3)$

$= (6 + 15 - 15) + (10 + 9 - 15)$

$= 6 + 4 = 10$

53. (2) $\sqrt{1+\frac{x}{9}}=\frac{13}{3}$

Squaring both sides,

$1+\frac{x}{9}=\frac{169}{9}$

$\Rightarrow \frac{x}{9}=\frac{169}{9}-1=\frac{160}{9}$

$\Rightarrow x = \frac{160}{9}\times 9 = 160$

54. (4) L.H.S. $= \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$

$= \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$

(Rationalising the denominator)

$= \frac{16\times3-12\sqrt{6}+20\sqrt{6}-15\times2}{(4\sqrt{3})^2-(3\sqrt{2})^2}$

$= \frac{48+8\sqrt{6}-30}{48-18}$

$= \frac{18+8\sqrt{6}}{30}=\frac{9}{15}+\frac{4\sqrt{6}}{15}$

$= \frac{3}{5}+\frac{4\sqrt{6}}{15}$

Now, $\frac{3}{5}+\frac{4\sqrt{6}}{15}=a+b\sqrt{6}$

$\therefore a = \frac{3}{5}$ and $b = \frac{4}{15}$

55. (1) $x + y = 2z$

$\Rightarrow x = 2z - y$

$\Rightarrow x - z = 2z - y - z = z - y$

$\therefore \frac{x}{x-z}+\frac{z}{y-z}$

$= \frac{x}{x-z}-\frac{z}{z-y}$

$= \frac{x}{x-z}-\frac{z}{x-z}=\frac{x-z}{x-z}=1$

56. (1) $a * b = a^b$

$\therefore 5 * 3 = 5^3 = 5 \times 5 \times 5 = 125$

57. (3) $\sqrt{0.03\times0.3\times a}$

$= 0.3 \times 0.3\ \sqrt{b}$

On squaring,

$0.03 \times 0.3 \times a = 0.09 \times 0.09 \times b$

$\Rightarrow \frac{a}{b}=\frac{0.09\times0.09}{0.03\times0.3}=0.9$

58. (1) $x \star y = (x + 3)^2 (y - 1)$

$\therefore 5 \star 4 = (5 + 3)^2 (4 - 1)$

$= 64 \times 3 = 192$

59. (2)

$9\sqrt{x}=\sqrt{3\times2\times2}+\sqrt{3\times7\times7}$

$\Rightarrow 9\sqrt{x}=2\sqrt{3}+7\sqrt{3}=9\sqrt{3}$

$\therefore x = 3$

60. (2) $X \star\star Y = X^2 + Y^2 - XY$

$\therefore 11 \star\star 13 = 11^2 + 13^2 - 11 \times 13$

$= 121 + 169 - 143 = 147$

61. (1) $\sqrt{1+\frac{x}{961}}=\frac{32}{31}$

Squaring both sides,

$1+\frac{x}{961}=\left(\frac{32}{31}\right)^2=\frac{1024}{961}$

$\Rightarrow \frac{x}{961}=\frac{1024}{961}-1=\frac{1024-961}{961}=\frac{63}{961}$

$\Rightarrow x = 63$

62. (3) $\sqrt{0.04\times0.4\times a}$

$= 0.004 \times 0.4 \times \sqrt{b}$

Squaring both sides,

$0.04 \times 0.4 \times a$

$= 0.004 \times 0.4 \times 0.004 \times 0.4 \times b$

$\Rightarrow \frac{a}{b}$

$= \frac{0.004\times0.004\times0.4\times0.4}{0.04\times0.4}$

$= 0.00016$

$= \frac{16}{100000} = 16 \times 10^{-5}$

63. (2) Using Rule 1,

$(a - b)^2 = a^2 - 2ab + b^2$

$x^4 - 2x^2 + k = (x^2)^2 - 2.x^2.1 + k$

$\therefore k = (1)^2 = 1$

64. (1) $2^{x+3} = 32 = 2^5$

$\Rightarrow x + 3 = 5 \Rightarrow x = 5 - 3 = 2$

$\therefore 3^{x+1} = 3^3 = 27$

65. (2) $x^4 - 17x^3 + 17x^2 - 17x + 17$

$= x^4 - 16x^3 + 16x^2 - 16x - x^3 + x^2 - x + 17$

When $x = 16$,

Expression $= 16^4 - 16^4 + 16^3 - 16^2 - 16^3 + 16^2 - 16 + 17 = 1$

66. (1) Given, $\frac{x}{y}=\frac{3}{4}$

Now, $\frac{6}{7}+\frac{y-x}{y+x}=\frac{6}{7}+\frac{1-\frac{x}{y}}{1+\frac{x}{y}}$

[Dividing N^r and D^r by y]

$= \frac{6}{7}+\frac{1-\frac{3}{4}}{1+\frac{3}{4}} = \frac{6}{7}+\frac{4-3}{4+3}$

$= \frac{6}{7} + \frac{1}{7} = 1$

67. (2) $n + \frac{2n}{3} + \frac{n}{2} + \frac{n}{7} = 97$

$\Rightarrow \frac{42n + 28n + 21n + 6n}{42} = 97$

$\Rightarrow \frac{97n}{42} = 97 \Rightarrow n = \frac{97 \times 42}{97} = 42,$

$\therefore n = 42$

68. (4) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow \frac{x^2 + 1}{x} = \frac{3x}{x}$

$\Rightarrow x + \frac{1}{x} = 3$

69. (1) $\boxed{\text{Tricky Approach}}$

$1.5a = 0.04b$

$\frac{b}{a} = \frac{1.5}{0.04}$

By componendo and dividendo,

$\frac{b - a}{b + a} = \frac{1.5 - 0.04}{1.5 + 0.04} = \frac{1.46}{1.54} = \frac{73}{77}$

70. (3) $x = \left(\sqrt{2} + 1\right)^{-\frac{1}{3}}$

$\Rightarrow x^{-3} = \sqrt{2} + 1$

$\Rightarrow \frac{1}{x3} = \sqrt{2} + 1$

and $x^3 = \frac{1}{\sqrt{2} + 1} = \frac{1\left(\sqrt{2} - 1\right)}{\left(\sqrt{2} + 1\right)\left(\sqrt{2} - 1\right)}$

$= \sqrt{2} - 1$

$\therefore x^3 - \frac{1}{x^3}$

$= \sqrt{2} - 1 - \sqrt{2} - 1 = -2$

71. (2) $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{2}{3}$

$\Rightarrow \frac{x^2 + 1 - x}{x^2 + 1 + x} = \frac{2}{3}$

Dividing numerator and denominator by x,

$\frac{\left(x + \frac{1}{x}\right) - 1}{\left(x + \frac{1}{x}\right) + 1} = \frac{2}{3}$

$\Rightarrow 3\left(x + \frac{1}{x}\right) - 3 = 2\left(x + \frac{1}{x}\right) + 2$

$\Rightarrow x + \frac{1}{x} = 2 + 3 = 5$

72. (4) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = 3$

$\Rightarrow a = 3b$; $c = 3d$; $e = 3f$

$\therefore \frac{2a^2 + 3c^2 + 4e^2}{2b^2 + 3d^2 + 4f^2}$

$= \frac{2 \times 9b^2 + 3 \times 9d^2 + 4 \times 9f^2}{2b^2 + 3d^2 + 4f^2}$

$= \frac{9\left(2b^2 + 3d^2 + 4f^2\right)}{2b^2 + 3d^2 + 4f^2} = 9$

73. (4) $(x - 3)^2 + (y - 4)^2 + (z - 5)^2 = 0$

$\Rightarrow x - 3 = 0, \; y - 4 = 0$

and $z - 5 = 0$

$\Rightarrow x = 3, y = 4$ and $z = 5$

$\therefore x + y + z = 3 + 4 + 5 = 12$

74. (4) $x = 7 - 4\sqrt{3}$

$\therefore \sqrt{x} = \sqrt{7 - 4\sqrt{3}}$

$= \sqrt{7 - 2 \times 2 \times \sqrt{3}}$

$= \sqrt{4 + 3 - 2 \times 2 \times \sqrt{3}}$

$= \sqrt{(2 - \sqrt{3})^2} = 2 - \sqrt{3}$

$\therefore \frac{1}{\sqrt{x}} = \frac{1}{2 - \sqrt{3}}$

$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3}$

$= 2 + \sqrt{3}$

$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

75. (4) $(a-1)^2 + (b+2)^2 + (c+1)^2 = 0$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1;$

$b + 2 = 0 \Rightarrow b = -2$

$c + 1 = 0 \Rightarrow c = -1$

$\therefore 2a - 3b + 7c$

$= 2 - 3(-2) + 7(-1)$

$= 2 + 6 - 7 = 1$

76. (4) $2x + \frac{1}{3x} = 5$

$\Rightarrow 6x^2 + 1 = 15x$

$\Rightarrow 6x^2 + 20x + 1 = 15x + 20x$

$= 35x$

$\Rightarrow \frac{5x}{6x^2 + 20x + 1} = \frac{5x}{35x} = \frac{1}{7}$

77. (1) $x \propto \frac{1}{y^2 - 1}$

$\Rightarrow x = \frac{k}{y^2 - 1}$

Where k is a constant.

When $y = 10$, $x = 24$, then

$\therefore 24 = \frac{k}{10^2 - 1} \Rightarrow 24 = \frac{k}{99}$

$\Rightarrow k = 24 \times 99$

When $y = 5$, then

$x = \frac{k}{y^2 - 1} = \frac{24 \times 99}{5^2 - 1} = \frac{24 \times 99}{24} = 99$

78. (1) Using Rule 1,

$x^2 + y^2 + 2x + 1 = 0$

$\Rightarrow x^2 + 2x + 1 + y^2 = 0$

$\Rightarrow (x + 1)^2 + y^2 = 0$

$\Rightarrow x + 1 = 0 \Rightarrow x = -1$ and $y = 0$

$\therefore x^{31} + y^{35} = -1$

79. (2) $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$

$\Rightarrow 2x^2 + 5x + 2 = 6x$

$\Rightarrow 2x^2 + 2 = 6x - 5x = x$

$\Rightarrow x^2 + 1 = \frac{x}{2}$

On dividing by x,

$\Rightarrow x + \frac{1}{x} = \frac{1}{2}$

80. (3) $a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$\Rightarrow a^2 - 2a + b^2 + 2b + c^2 + 2c + 1 + 1 + 1 = 0$

$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1$

$\Rightarrow b + 1 = 0 \Rightarrow b = -1$

and $c + 1 = 0 \Rightarrow c = -1$

$\therefore 2a - 3b + 4c = 2 + 3 - 4 = 1$

81. (1) $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$

$\Rightarrow 3a + 1 = 0$

$\Rightarrow 3a = -1$

$b - 1 = 0$

$\Rightarrow b = 1$

$2c - 3 = 0$

$\Rightarrow 2c = 3$

$\therefore 3a + b + 2c = -1 + 1 + 3 = 3$

82. (2) $\frac{(a - b)^2}{(b - c)(c - a)} +$

$\frac{(b - c)^2}{(a - b)(c - a)} + \frac{(c - a)^2}{(a - b)(b - c)}$

$= \frac{(a - b)^3}{(a - b)(b - c)(c - a)} +$

$\frac{(b - c)^3}{(a - b)(b - c)(c - a)} +$

$$\frac{(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$$

$[\because [a-b+b-c+c-a=0]]$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

$$\left[\begin{array}{r} \text{If } a+b+c=0, \\ \because a^3+b^3+c^3=3abc \end{array}\right]$$

83. (3) $(a-3)^2+(b-4)^2+(c-9)^2=0$

$\Rightarrow a-3=0 \Rightarrow a=3$

$b-4=0 \Rightarrow b=4$

and $c-9=0 \Rightarrow c=9$

$\therefore \sqrt{a+b+c} = \sqrt{3+4+9}$

$= \sqrt{16} = \pm 4$

84. (2) $180 = 2\times 2\times 3\times 3\times 5$

$a^3b = abc$

$\Rightarrow a^2 = c$

$\therefore a^3b = abc = 180 = 1^2 \times 180 \times 1$

$= 1^3 \times 180$

$\Rightarrow c = 1$

85. (3) $(x-3)^2+(y-5)^2+(z-4)^2=0$

$\Rightarrow x-3=0 \quad \Rightarrow x=3$

$y-5=0 \quad \Rightarrow y=5$

$z-4=0 \quad \Rightarrow z=4$

$$\therefore \frac{x^2}{9}+\frac{y^2}{25}+\frac{z^2}{16}$$

$$= \frac{9}{9}+\frac{25}{25}+\frac{16}{16}$$

$= 1+1+1 = 3$

86. (4) $(a-1)\sqrt{2}+3 = b\sqrt{2}+a$

$\Rightarrow a=3\ ;\ a-1=b$

$\Rightarrow 3-1=b \Rightarrow b=2$

$\therefore a+b = 3+2 = 5$

87. (2) $a = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} = \dfrac{\sqrt{5}+1}{\sqrt{5}-1}\times\dfrac{\sqrt{5}+1}{\sqrt{5}+1}$

$$= \frac{\left(\sqrt{5}+1\right)^2}{5-1} = \frac{5+1+2\sqrt{5}}{4}$$

$$= \frac{3+\sqrt{5}}{2}$$

$$\therefore b = \frac{\sqrt{5}-1}{2} = \frac{3-\sqrt{5}}{2}$$

$\therefore a+b$

$$= \frac{3+\sqrt{5}}{2}+\frac{3-\sqrt{5}}{2} = 3$$

and $ab = \dfrac{\sqrt{5}+1}{\sqrt{5}-1}\times\dfrac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$\therefore$ Expression

$$= \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{(a+b)^2-ab}{(a+b)^2-3ab}$$

$$= \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$$

88. (2) $(64)^{x+1} = \dfrac{64}{4^x}$

$\Rightarrow (4^3)^{x+1}\times 4^x = 64$

$\Rightarrow 4^{3x+3+x} = 4^3$

$\Rightarrow 4^{4x+3} = 4^3$

$\Rightarrow 4x+3 = 3$

$\Rightarrow x = 0$

89. (3) $ax^2+bx+c = a(x-p)^2$

$\Rightarrow ax^2+bx+c = a(x^2-2px+p^2)$

$\Rightarrow ax^2+bx+c = ax^2-2apx+ap^2$

Comparing the corresponding coefficients,

$b = -2ap$ and $c = ap^2$

$\Rightarrow b^2 = 4a^2p^2$ and $p^2 = \dfrac{c}{a}$

$$\Rightarrow p^2 = \frac{b^2}{4a^2};$$

$$\therefore \frac{b^2}{4a^2} = \frac{c}{a} \Rightarrow b^2 = 4ac$$

90. (4) For maximum value,

$$a = b = c = d = \frac{1}{4}$$

$\therefore (1+a)(1+b)(1+c)(1+d)$

$$= \left(\frac{5}{4}\right)^4$$

91. (4) $x \,\alpha\, \dfrac{1}{y^2}$

$\Rightarrow x = \dfrac{k}{y^2}$ where k is a constant of proportionality.

When, $x=1$, $y=2$

$$\Rightarrow 1 = \frac{k}{4} \Rightarrow k = 4$$

$$\therefore x = \frac{4}{y^2}$$

When $y = 6$,

$$x = \frac{4}{6\times 6} = \frac{1}{9}$$

92. (2) Given $x = \dfrac{\sqrt{3}}{2}$

Given expression

$$= \frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$$

$$= \frac{\sqrt{1+x}}{1+\sqrt{1+x}}\times\frac{1-\sqrt{1+x}}{1-\sqrt{1+x}}$$

$$+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}\times\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}$$

$$= \frac{\sqrt{1+x}-1-x}{1-1-x}+\frac{\sqrt{1-x}+1-x}{1-1+x}$$

$$= \frac{\sqrt{1-x}+1-x}{x}-\frac{\sqrt{1+x}-1-x}{x}$$

$$= \frac{\sqrt{1-x}+1-x-\sqrt{1+x}+1+x}{x}$$

$$= \frac{2+\sqrt{1-x}-\sqrt{1+x}}{x}$$

$$= \frac{2+\sqrt{1-\frac{\sqrt{3}}{2}}-\sqrt{1+\frac{\sqrt{3}}{2}}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2+\sqrt{\frac{2-\sqrt{3}}{2}}-\sqrt{\frac{2+\sqrt{3}}{2}}}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2+\frac{\sqrt{4-2\sqrt{3}}}{2}-\frac{\sqrt{4+2\sqrt{3}}}{2}}{\frac{\sqrt{3}}{2}}$$

$$\left[\because \sqrt{4-2\sqrt{3}} = \sqrt{3+1-2\sqrt{3}}\right.$$

$$\left.= \sqrt{(\sqrt{3}-1)^2} = \sqrt{3}-1\right] \text{ and}$$

$$\left[\sqrt{4+2\sqrt{3}} = \sqrt{3+1+2\sqrt{3}}\right.$$

$= \sqrt{(\sqrt{3}+1)^2} = \sqrt{3}+1]$

$= \frac{4+\sqrt{3}-1-\sqrt{3}-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

93. (4) $a^2 + b^2 + c^2 + 3$
$= 2a - 2b - 2c$
$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$
$\Rightarrow (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$
$\therefore a - 1 = 0 \Rightarrow a = 1$
$b + 1 = 0 \Rightarrow b = -1$
$c + 1 = 0 \Rightarrow c = -1$
$\therefore 2a - b + c = 2 + 1 - 1 = 2$

94. (4) $x^2 - y^2 = 80$
$x - y = 8$

$\therefore x + y = \frac{x^2-y^2}{x-y} = \frac{80}{8} = 10$

$\therefore$ Required average

$= \frac{x+y}{2} = \frac{10}{2} = 5$

95. (4) $x^2 - 4x - 1 = 0$
$\Rightarrow x^2 - 1 = 4x$
Dividing by x,

$x - \frac{1}{x} = 4$

On squaring both sides,

$\left(x - \frac{1}{x}\right)^2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 16 + 2 = 18$

Aliter :
Using Rule 5,
Here, $x^2 - 4x - 1 = 0$
$\Rightarrow x^2 - 1 = 4x$

$\Rightarrow x^2 - \frac{1}{x} = 4$

We know that

$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$= 4^2 + 2 = 18$

96. (1) Third proportional of a and b

$= \frac{b^2}{a}$

$= \frac{\left(\sqrt{x^2+y^2}\right)^2}{\frac{x}{y}+\frac{y}{x}} = \frac{x^2+y^2}{\frac{x^2+y^2}{xy}} = xy$

97. (3) When $x = 6$,

$\frac{4 \times 6}{3} + 2P = 12$

$\Rightarrow 8 + 2P = 12$
$\Rightarrow 2P = 12 - 8 = 4$
$\Rightarrow P = 2$

98. (1) Expression $= \frac{4+3\sqrt{3}}{7+4\sqrt{3}}$

Rationalising the denominator,

$= \frac{(4+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$

$= \frac{28-16\sqrt{3}+21\sqrt{3}-12\times 3}{49-48}$

$= 28 + 5\sqrt{3} - 36 = 5\sqrt{3} - 8$

99. (4) $\frac{1}{a} = \frac{1}{\sqrt{6}-\sqrt{5}}$

$= \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6}+\sqrt{5}$

Similarly,

$\frac{1}{b} = \sqrt{5}+2;\ \frac{1}{c} = 2+\sqrt{3}$

$\therefore \frac{1}{a} > \frac{1}{b} > \frac{1}{c} \Rightarrow a < b < c$

100. (2) $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$

$= \frac{4\sqrt{15}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$

$= \frac{4\sqrt{15}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$

$= 2\sqrt{15}\left(\sqrt{5}-\sqrt{3}\right) = 10\sqrt{3} - 6\sqrt{5}$

$\therefore \frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$

$= \frac{10\sqrt{3}-6\sqrt{5}+2\sqrt{5}}{10\sqrt{3}-6\sqrt{5}-2\sqrt{5}} + \frac{10\sqrt{3}-6\sqrt{5}+2\sqrt{3}}{10\sqrt{3}-6\sqrt{5}-2\sqrt{3}}$

$= \frac{10\sqrt{3}-4\sqrt{5}}{10\sqrt{3}-8\sqrt{5}} + \frac{12\sqrt{3}-6\sqrt{5}}{8\sqrt{3}-6\sqrt{5}}$

$= \frac{5\sqrt{3}-2\sqrt{5}}{5\sqrt{3}-4\sqrt{5}} + \frac{6\sqrt{3}-3\sqrt{5}}{4\sqrt{3}-3\sqrt{5}}$

$= \frac{60-15\sqrt{15}-8\sqrt{15}+30+90 \quad -15\sqrt{15}-24\sqrt{15}+60}{\left(5\sqrt{3}-4\sqrt{5}\right)\left(4\sqrt{3}-3\sqrt{5}\right)}$

$= \frac{240-62\sqrt{15}}{60-15\sqrt{15}-16\sqrt{15}+60}$

$= \frac{240-62\sqrt{15}}{120-31\sqrt{15}}$

$= \frac{2\left(120-31\sqrt{15}\right)}{120-31\sqrt{15}} = 2$

101. (2) $\sqrt{x} = \sqrt{5-\sqrt{21}}$

$\Rightarrow \sqrt{x} = \frac{\sqrt{10-2\sqrt{21}}}{\sqrt{2}}$

$= \frac{\sqrt{7+3-2\times\sqrt{7}\times\sqrt{3}}}{\sqrt{2}}$

$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$

$\sqrt{32-2x} = \sqrt{32-2\left(5-\sqrt{21}\right)}$

$= \sqrt{32-10+2\sqrt{21}}$

$= \sqrt{22+2\sqrt{21}}$

$= \sqrt{21+1+2\times\sqrt{21}\times 1}$

$= \sqrt{21}+1$

$\therefore$ Expression

$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}\left(\sqrt{21}+1-\sqrt{21}\right)}$

$= \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$

102. (2) $6x - 5y = 13$ ...(i)
$7x + 2y = 23$ ...(ii)
By equation (i) × 2 + (ii) × 5,
$12x - 10y = 26$
$35x + 10y = 115$

$47x = 141$
$\Rightarrow x = 3$

From equation (i),

$6 \times 3 - 5y = 13$

$\Rightarrow 18 - 5y = 13$

$\Rightarrow 5y = 5$

$\Rightarrow y = 1$

$\therefore 11x + 18y = 11 \times 3 + 18 \times 1$

$= 33 + 18 = 51$

103. (1) $(x^{b+c})^{b-c} . (x^{c+a})^{c-a} . (x^{a+b})^{a-b}$

$= x^{b^2-c^2} . x^{c^2-a^2} . x^{a^2-b^2}$

$= x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$

104. (4) $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$

$\Rightarrow \frac{x}{a} = \frac{x-a}{ax}$

$\Rightarrow x^2 = x - a$

$\Rightarrow x - x^2 = a$

105. (3) $x + \frac{1}{x} = 99$

$\therefore \frac{100x}{2x^2 + 102x + 2}$

$= \frac{100x}{2x^2 + 2 + 102x}$

On dividing by x,

$= \frac{100}{2x + \frac{2}{x} + 102}$

$= \frac{100}{2\left(x + \frac{1}{x}\right) + 102}$

$= \frac{100}{2 \times 99 + 102} = \frac{100}{300} = \frac{1}{3}$

106. (3) $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$

$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$

$\Rightarrow \frac{3}{x} + \frac{3}{y} + \frac{3}{z} = 4 + 4 + 4 = 12$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{12}{3} = 4$

107. (3) $\frac{xy}{x+y} = a \Rightarrow \frac{x+y}{xy} = \frac{1}{a}$

$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{a}$ ...(i)

$\frac{xz}{x+z} = b \Rightarrow \frac{x+z}{xz} = \frac{1}{b}$

$\Rightarrow \frac{1}{z} + \frac{1}{x} = \frac{1}{b}$ ...(ii)

Similarly,

$\frac{1}{z} + \frac{1}{y} = \frac{1}{c}$

$\Rightarrow \frac{1}{y} = \frac{1}{c} - \frac{1}{z}$ ...(iii)

By substitutio method

From equations (i) and (iii),

$\frac{1}{a} - \frac{1}{x} = \frac{1}{c} - \frac{1}{z}$

$\Rightarrow \frac{1}{a} - \frac{1}{x} = \frac{1}{c} - \frac{1}{b} + \frac{1}{x}$

[From equation (ii)]

$\Rightarrow \frac{1}{x} + \frac{1}{x} = \frac{1}{a} - \frac{1}{c} + \frac{1}{b}$

$\Rightarrow \frac{2}{x} = \frac{bc - ab + ac}{abc}$

$\Rightarrow x = \frac{2abc}{bc + ac - ab}$

108. (3) $x = 3 + \sqrt{8}$

$\therefore \frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})}$

$= \frac{3-\sqrt{8}}{9-8} = 3 - \sqrt{8}$

Now, $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (3 + \sqrt{8} + 3 - \sqrt{8})^2 - 2$

$= 36 - 2 = 34$

109. (4) $xy = 8 = 1 \times 8 = 2 \times 4$

$= \frac{1}{2} \times 16 = \frac{1}{3} \times 24$

$\therefore$ Minimum value of $2x + y$

$= 2 \times 2 + 4 = 8$

110. (2) $x^2 + x + 1$

$= x^2 + 2.x.\frac{1}{2} + \frac{1}{4} + \frac{3}{4}$

$= \left(x + \frac{1}{2}\right)^2 + \left(\pm\frac{\sqrt{3}}{2}\right)^2$

$\therefore \left(x + \frac{1}{2}\right)^2 + \left(\pm\frac{\sqrt{3}}{2}\right)^2$

$= \left(x + \frac{1}{2}\right)^2 + q^2$

$\Rightarrow q = \pm\frac{\sqrt{3}}{2}$

111. (2) $a^2 - 4a - 1 = 0$

$\Rightarrow a^2 - 1 = 4a$

On dividing by a, we have

$a - \frac{1}{a} = 4$

$\therefore a^2 + \frac{1}{a^2} + 3\left(a - \frac{1}{a}\right)$

$= \left(a - \frac{1}{a}\right)^2 + 2 + 3\left(a - \frac{1}{a}\right)$

$= 16 + 2 + 3\ (4) = 30$

112. (3) $a + \frac{1}{b} = 1 \Rightarrow a = 1 - \frac{1}{b} = \frac{b-1}{b}$

$\Rightarrow \frac{1}{a} = \frac{b}{b-1}$ and

$b + \frac{1}{c} = 1 \Rightarrow \frac{1}{c} = 1 - b \Rightarrow c = \frac{1}{1-b}$

$\therefore c + \frac{1}{a} = \frac{1}{1-b} + \frac{b}{b-1}$

$= \frac{1}{1-b} - \frac{b}{1-b} = \frac{1-b}{1-b} = 1$

113. (4) Expression $= (x - 2)\ (x - 9)$

$= x^2 - 11x + 18 = ax^2 + bx + c$

Minimum value $= \frac{4ac - b^2}{4a}$

$= \frac{4 \times 1 \times 18 - 121}{4} = \frac{-49}{4}$

114. (4) $4\sqrt{3}\ x^2 + 5x - 2\sqrt{3}$

$= 4\sqrt{3}\ x^2 + 8x - 3x - 2\sqrt{3}$

$= 4x\ (\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (4x - \sqrt{3})\ (\sqrt{3}x + 2) \Rightarrow$ factors

115. (3) $\sqrt{x} = \sqrt{3} - \sqrt{5}$

On squaring both sides,

$x = 3 + 5 - 2\sqrt{15}$

$\Rightarrow x - 8 = -2\sqrt{15}$

Squaring again,

$x^2 - 16x + 64 = 60$

$\Rightarrow x^2 - 16x + 4 = 0$

$\therefore x^2 - 16x + 6 = 2$

116. (2) $x - \frac{1}{x} = 4$ (Given)

$\therefore \left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4$

$= (4)^2 + 4 = 20$

$\Rightarrow x + \frac{1}{x} = \sqrt{20} = 2\sqrt{5}$

117. (3) $x = 5 + 2\sqrt{6}$

$\therefore \frac{1}{x} = \frac{1}{5+2\sqrt{6}} = \frac{5-2\sqrt{6}}{(5+2\sqrt{6})(5-2\sqrt{6})}$

$= \frac{5-2\sqrt{6}}{25-24} = 5 - 2\sqrt{6}$

$\therefore \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2$

$= 5 + 2\sqrt{6} + 5 - 2\sqrt{6} + 2 = 12$

$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{12} = 2\sqrt{3}$

118. (3) $(a - b)^2 = (a + b)^2 - 4ab$

$= 5^2 - 4 \times 6 = 1$

$\Rightarrow a - b = 1$

$\therefore (a^2 - b^2) = (a + b)(a - b) = 5$

119. (2) $1.5x = 0.04y$

$\Rightarrow \frac{x}{y} = \frac{0.04}{1.5} = \frac{4}{150} = \frac{2}{75}$

$\Rightarrow \frac{y}{x} = \frac{75}{2}$

Now, $\frac{y^2 - x^2}{y^2 + 2xy + x^2}$

$= \frac{(y-x)(y+x)}{(y+x)^2}$

$= \frac{y-x}{y+x} = \frac{\frac{y}{x} - 1}{\frac{y}{x} + 1}$

$= \frac{\frac{75}{2} - 1}{\frac{75}{2} + 1} = \frac{73}{77}$

120. (2) $a^{\frac{1}{3}} = 11 \Rightarrow a = 11^3 = 1331$

$\therefore a^2 - 331a = a(a - 331)$

$= 1331\,(1331 - 331)$

$= 1331 \times 1000 = 1331000$

121. (1) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} - 4 = 0$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 + y^2 + \frac{1}{y^2} - 2 = 0$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + \left(y - \frac{1}{y}\right)^2 = 0$

$\Rightarrow x - \frac{1}{x} = 0$

$\Rightarrow x^2 - 1 = 0 \Rightarrow x = 1$

Similarly,

$y = 1$

$\therefore x^2 + y^2 = 1 + 1 = 2$

122. (2) **Tricky Approach**

$x^2 = y + z$

$\Rightarrow x^2 + x = x + y + z$

$\Rightarrow x(x + 1) = x + y + z$ ....(i)

Similarly,

$y(y + 1) = x + y + z$ ....(ii)

and, $z(z + 1) = x + y + z$ ...(iii)

$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$= \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$

$\Rightarrow \frac{x+y+z}{x+y+z} = 1$

123. (4) $(ad - bc)^2 + (ac + bd)^2$

$= a^2d^2 + b^2c^2 - 2abcd + a^2c^2 + b^2d^2 + 2abcd$

$= a^2d^2 + b^2c^2 + a^2c^2 + b^2d^2$

$= a^2d^2 + b^2d^2 + b^2c^2 + a^2c^2$

$= d^2(a^2 + b^2) + c^2(b^2 + a^2)$

$= (a^2 + b^2)(c^2 + d^2)$

$= 2 \times 1 = 2$

124. (2) $a^2 + b^2 + c^2 + 3$

$= 2a + 2b + 2c$

$\Rightarrow a^2 - 2a + 1 + b^2 - 2b + 1 + c^2 - 2c + 1 = 0$

$\Rightarrow (a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 0$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1;$

$b - 1 = 0 \Rightarrow b = 1$

and, $c - 1 = 0 \Rightarrow c = 1$

$\therefore a + b + c = 3$

125. (3) $x - \frac{1}{x} = 5$

On squaring both sides,

$x^2 + \frac{1}{x^2} - 2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 27$

Aliter :

Using Rule 5,

Here, $x - \frac{1}{x} = 5$

We know that

$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$= 5^2 + 2 = 27$

126. (2) $x = 3 + 2\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{3+2\sqrt{2}}$

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$

$\therefore \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$

$= 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2$

$= 4$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

127. (4) $x = \sqrt{3} + \sqrt{2}$

$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}}$

$= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \sqrt{3} - \sqrt{2}$

$\therefore x + \frac{1}{x} = 2\sqrt{3}$

$\therefore \left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2$

$= (2\sqrt{3})^2 - 2 = 12 - 2 = 10$

128. (3) $x + \frac{9}{x} = 6$

$\Rightarrow x^2 - 6x + 9 = 0$

$\Rightarrow (x - 3)^2 = 0 \Rightarrow x = 3$

$\therefore \left(x^2 + \frac{9}{x^2}\right) = \left(9 + \frac{9}{9}\right) = 10$

129. (4) $x = \frac{4ab}{a+b} \Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

By componendo and dividendo,

$\frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}$

Similarly, $\frac{x}{2b} = \frac{2a}{a+b}$

$\Rightarrow \frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b}$

$= \frac{3a+b}{a-b}$

$\therefore \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$

$= \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

$= \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a}$

$= \frac{2(b-a)}{b-a} = 2$

130. (3) $m + \frac{1}{m-2} = 4$

$\Rightarrow m + \frac{1}{m-2} - 2 = 4 - 2$

$\Rightarrow (m-2) + \frac{1}{(m-2)} = 4 - 2 = 2$

On squaring both sides,

$(m-2)^2 + \frac{1}{(m-2)^2} +$

$2(m-2)\left(\frac{1}{m-2}\right) = 4$

$\Rightarrow (m-2)^2 + \frac{1}{(m-2)^2} = 4 - 2 = 2$

Aliter :

Using Rule 14,

$m + \frac{1}{m-2} = 4$

$\Rightarrow m - 2 + \frac{1}{m-2} = 4 - 2$

$\Rightarrow m - 2 + \frac{1}{m-2} = 2$

$\Rightarrow (m-2)^2 + \frac{1}{(m-2)^2} = 2$

131. (3) Using Rule 1,

$a^2 + b^2 + 2b + 4a + 5 = 0$

$\Rightarrow a^2 + 4a + b^2 + 2b + 5 = 0$

$\Rightarrow a^2 + 4a + 4 + b^2 + 2b + 1 = 0$

$\Rightarrow (a+2)^2 + (b+1)^2 = 0$

It is possible only when

$a + 2 = 0 \Rightarrow a = -2$

and, $b + 1 = 0 \Rightarrow b = -1$

$\therefore \frac{a-b}{a+b} = \frac{-2+1}{-2-1}$

$= \frac{-1}{-3} = \frac{1}{3}$

132. (1) $x - y = \frac{x+y}{7} = \frac{xy}{4} = k$

$\Rightarrow x - y = k$

$x + y = 7k$

$\therefore (x+y)^2 - (x-y)^2 = 49k^2 - k^2$

$\Rightarrow 4xy = 48k^2$

$\Rightarrow 16k = 48k^2$

$\Rightarrow k = \frac{1}{3}$

$\therefore xy = 4k = 4 \times \frac{1}{3} = \frac{4}{3}$

133. (4) $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$

$= \frac{x^3+y^3+z^3}{xyz} = \frac{3xyz}{xyz} = 3$

134. (2) $\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)}$

$+ \frac{1}{(c+a)(c+b)}$

$= \frac{c+a+b+c+a+b}{(a+b)(b+c)(c+a)}$

$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$

$= 0$ [$\because a + b + c = 0$]

135. (3) Using Rule 1,

$a + b + c = 0$

$\Rightarrow b + c = -a$

On squaring both sides,

$\Rightarrow (b+c)^2 = a^2$

$\Rightarrow b^2 + c^2 + 2bc = a^2$

$\Rightarrow a^2 + b^2 + c^2 + 2bc = 2a^2$

$\Rightarrow a^2 + b^2 + c^2 = 2a^2 - 2bc$

$= 2(a^2 - bc)$

$\therefore \frac{a^2+b^2+c^2}{a^2-bc} = \frac{2(a^2-bc)}{a^2-bc} = 2$

136. (2) $n = 7 + 4\sqrt{3} = 7 + 2 \times 2 \times \sqrt{3}$

$= 4 + 3 + 2 \times 2 \times \sqrt{3}$

$= \left(2+\sqrt{3}\right)^2$

$\therefore \sqrt{n} = 2 + \sqrt{3}$

$\therefore \frac{1}{\sqrt{n}} = \frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2 - \sqrt{3}$

$\therefore \sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

137. (2) $x = \sqrt{3} + \sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \sqrt{3} - \sqrt{2}$

$\therefore x + \frac{1}{x}$

$= \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} = 2\sqrt{3}$

138. (4) $\frac{p}{q} + \frac{q}{p} = \frac{p^2+q^2}{pq}$

$= \frac{(p+q)^2 - 2pq}{pq}$

$= \frac{100 - 2 \times 5}{5} = \frac{90}{5} = 18$

139. (4) $x = 3 + 2\sqrt{2}$

$xy = 1$

$\Rightarrow y = \frac{1}{3+2\sqrt{2}}$

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

$\therefore x + y$

$= 3+2\sqrt{2}+3-2\sqrt{2} = 6$

$\therefore \frac{x^2+3xy+y^2}{x^2-3xy+y^2}$

$= \frac{(x+y)^2+xy}{(x+y)^2-5xy}$

$= \frac{36+1}{36-5} = \frac{37}{31}$

140. (1) $\frac{x}{b+c} = \frac{y}{c+a}$

$= \frac{x-y}{b+c-c-a} = \frac{x-y}{b-a}$;

$\frac{y}{c+a} = \frac{z}{a+b}$

$= \frac{y-z}{c+a-a-b} = \frac{y-z}{c-b}$;

$\frac{z}{a+b} = \frac{x}{b+c}$

$= \frac{z-x}{a+b-b-c} = \frac{z-x}{a-c}$

$\therefore \frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$

141. (3) $a + b + c = 0$

$\Rightarrow a + b = -c$; $b + c = -a$, $c + a = -b$

$\therefore \frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}$

$= -1 - 1 - 1 = -3$

$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$

$= -1 - 1 - 1 = -3$

$\therefore$ Expression = $(-3) \times (-3) = 9$

142. (1) $a + \frac{1}{b} = 1 \Rightarrow ab + 1 = b$

$\Rightarrow ab = b - 1$ .....(i)

Again,

$b + \frac{1}{c} = 1$

$\frac{1}{c} = 1 - b \Rightarrow c = \frac{1}{1-b}$ .....(ii)

On multiplying (i) & (ii)

$abc = \frac{b-1}{1-b} = -1$

143. (3) Expression

$= \frac{(s-a)^2+(s-b)^2+(s-c)^2+s^2}{a^2+b^2+c^2}$

$= \frac{s^2-2sa+a^2+s^2+b^2-2sb+s^2-2sc+c^2+s^2}{a^2+b^2+c^2}$

$= \frac{4s^2+a^2+b^2+c^2-2s(a+b+c)}{a^2+b^2+c^2}$

$= \frac{4s^2+a^2+b^2+c^2-4s^2}{a^2+b^2+c^2} = 1$

144. (4) $x = 3+2\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{3+2\sqrt{2}}$

$= \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{9-8}$

$= 3-2\sqrt{2}$

$x + \frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2} = 6$

$\therefore x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$

$= (6)^2 - 2 = 36 - 2 = 34$

145. (2) $3x - 2 = \frac{3}{x}$

$\Rightarrow 3x - \frac{3}{x} = 2$

$\Rightarrow x - \frac{1}{x} = \frac{2}{3}$

On squaring both sides

$\left(x-\frac{1}{x}\right)^2 = \frac{4}{9}$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$

$\Rightarrow x^2 + \frac{1}{x^2}$

$= \frac{4}{9} + 2 = \frac{22}{9} = 2\frac{4}{9}$

146. (1) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

Dividing both sides by x,

$\Rightarrow x + \frac{1}{x} = 3$

$\therefore x^2 + x + \frac{1}{x} + \frac{1}{x^2}$

$= \left(x^2+\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right)$

$= \left(x+\frac{1}{x}\right)^2 - 2 + \left(x+\frac{1}{x}\right)$

$= 9 - 2 + 3 = 10$

147. (3) $a^2 + b^2 = 5ab$

$\Rightarrow \frac{a^2+b^2}{ab} = 5$

$\Rightarrow \frac{a}{b} + \frac{b}{a} = 5$

On squaring both sides,

$\therefore \left(\frac{a}{b}+\frac{b}{a}\right)^2 = 25$

$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 = 25$

$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{a^2} = 25 - 2 = 23$

148. (4) $x^2 - yz = x^2 + xy + zx$

$= x(x + y + z)$

$\left[\because \begin{array}{l} xy+yz+zx=0 \\ \Rightarrow yz=-xy-zx \end{array}\right]$

Similarly,

$y^2 - zx = y(x + y + z)$

$z^2 - xy = x(x + y + z)$

$\therefore$ Expression

$= \frac{1}{x(x+y+z)} + \frac{1}{y(x+y+z)} + \frac{1}{z(x+y+z)}$

$= \frac{yz+zx+xy}{xyz(x+y+z)} = 0$

149. (3) $a + b + c = 9$

$a^2 + b^2 + c^2$

$= (a + b + c)^2 - 2(ab + bc + ca)$

[$ab + bc + ca$ will be maximum if $a = b = c$]

$a^2 + b^2 + c^2 = 9^2 - 2 \times 27$

$= 81 - 54 = 27$

150. (3) $x + y + z = 13$

$x^2 + y^2 + z^2 = 69$

$(x + y + z)^2$
$= x^2 + y^2 + z^2 + 2(xy + yz + zx)$
$\Rightarrow (13)^2 = 69 + 2(xy + yz + zx)$
$\Rightarrow 2(xy + yz + zx)$
$= 169 - 69 = 100$

$\Rightarrow xy + yz + zx = \frac{100}{2} = 50$

151. (3) a = 0.1039 (Given)

Now, $\sqrt{4a^2 - 4a + 1} + 3a$

$= \sqrt{(1-2a)^2} + 3a$
$= 1 - 2a + 3a$
$= 1 + a = 1 + 0.1039$
$= 1.1039$

152. (1) $\frac{a^2 - b^2 - c^2 - 2bc}{a^2 + b^2 - 2ab - c^2}$

$= \frac{a^2 - (b^2 + c^2 + 2bc)}{(a^2 + b^2 - 2ab) - c^2}$

$= \frac{a^2 - (b+c)^2}{(a-b)^2 - c^2}$

$= \frac{(a+b+c)(a-b-c)}{(a-b+c)(a-b-c)}$

$= \frac{a+b+c}{a-b+c} = \frac{0.25 - 0.05 + 0.5}{0.25 + 0.05 + 0.5}$

$= \frac{0.7}{0.8} = \frac{7}{8}$

153. (1) Using Rule 1,
$25a^2 + 40ab + 16b^2$
$= (5a + 4b)^2$
$= (5 \times 23 - 29 \times 4)^2$
$= (115 - 116)^2 = 1$

154. (2) Using Rule 1,
$(x - y)^2 = x^2 + y^2 - 2xy$
$\Rightarrow 2^2 = 20 - 2xy$
$\Rightarrow 2xy = 20 - 4 = 16$
$\therefore (x + y)^2 = x^2 + y^2 + 2xy$
$= 20 + 16 = 36$

155. (3) Using Rule 1,
$x^2 + y^2 - 4x - 4y + 8 = 0$
$\Rightarrow x^2 - 4x + 4 + y^2 - 4y + 4 = 0$
$\Rightarrow (x - 2)^2 + (y - 2)^2 = 0$
$\Rightarrow x = 2$ and $y = 2$
$\therefore x - y = 2 - 2 = 0$

156. (1) $x^2 + y^2 - z^2 + 2xy$
$= x^2 + y^2 + 2xy - z^2$
$= (x + y)^2 - z^2 = (x+y+z)(x+y-z)$
$= (b + c - 2a + c + a - 2b + a + b - 2c)(x + y - z) = 0$

157. (2) $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow 2a^2 + 2b^2 + 2c^2$
$= 2ab + 2bc + 2ca$
$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ac + a^2 = 0$
$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2$
$= 0$
$\Rightarrow a - b = 0 \Rightarrow a = b$
$b - c = 0 \Rightarrow b = c$
$c - a = 0 \Rightarrow c = a$
$\Rightarrow a = b = c$

$\therefore \frac{a+c}{b} = \frac{a+a}{a} = 2$

158. (4) $x^2 + y^2 = (x - y)^2 + 2xy$
$= 4 + 2 \times 24 = 52$

159. (1) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

If $a = \frac{x}{y}$; $b = \frac{y}{2}$

then,

$\pm 2ab = \pm 2 \times \frac{x}{y} \times \frac{y}{2} = \pm x$

$\therefore tx = \pm x$
$\Rightarrow t = \pm 1$

160. (4) $a - b = x + y - x + y = 2y$
$b - c = x - y - x - 2y = -3y$
$c - a = x + 2y - x - y = y$
Now,
$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$= \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$= \frac{1}{2}((2y)^2 + (-3y)^2 + y^2)$

$= \frac{1}{2} \times 14y^2 = 7y^2$

161. (1) $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow 2a^2 + 2b^2 + 2c^2 = 2ab + 2bc + 2ca$
$\Rightarrow a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ac + a^2 = 0$
$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$
$\therefore a - b = 0 \Rightarrow a = b$
$b - c = 0 \Rightarrow b = c$
$c - a = 0 \Rightarrow c = a$
$\therefore a = b = c$

$\therefore \frac{a+b}{c} = \frac{a+a}{a} = 2$

162. (4) $a^2 + b^2 + 4c^2 = 2a + 2b - 4c - 3$
$\Rightarrow a^2 + b^2 + 4c^2 - 2a - 2b + 4c + 3 = 0$
$\Rightarrow a^2 - 2a + 1 + b^2 - 2b + 1 + 4c^2 + 4c + 1 = 0$
$\Rightarrow (a - 1)^2 + (b - 1)^2 + (2c + 1)^2 = 0$
$\therefore a - 1 = 0 \Rightarrow a = 1;$
$b - 1 = 0 \Rightarrow b = 1;$

$2c + 1 = 0 \Rightarrow c = -\frac{1}{2}$

$\therefore a^2 + b^2 + c^2 = 1 + 1 + \frac{1}{4} = 2\frac{1}{4}$

163. (1) Check through option
When $x = (a + b + c)^2$,

$\frac{x - a^2}{b+c} + \frac{x - b^2}{c+a} + \frac{x - c^2}{a+b}$

$= \frac{(a+b+c)^2 - a^2}{b+c} + \frac{(a+b+c)^2 - b^2}{c+a} + \frac{(a+b+c)^2 - c^2}{a+b}$

$= \frac{(2a+b+c)(b+c)}{b+c} + \frac{(a+2b+c)(c+a)}{c+a} + \frac{(a+b+2c)(a+b)}{a+b}$

$= 2a + b + c + a + 2b + c + a + b + 2c$
$= 4a + 4b + 4c = 4(a + b + c) =$ RHS.

164. (4) $4x - y = 2$ ....(i)
$2y - 8x + 4 = 0$
$\Rightarrow 8x - 2y = 4$ .....(ii)
For simultaneous linear equations
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$ if
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, there are infinite solutions.

165. (4) $\frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{15}{16}$

$\frac{a}{c} = \frac{3}{4}$

$\Rightarrow a = \frac{3}{4}c$

Put in the given equation,

$= \frac{18c^2 - 7\left(\frac{3}{4}c\right)^2}{45c^2 + 20\left(\frac{3}{4}c\right)^2}$

$= \frac{18c^2 - \frac{63}{16}c^2}{45c^2 + \frac{180}{16}c^2} = \frac{1}{4}$

166. (4) Check through options.
If $x = y = z$, then

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{3}{x^2}$$

and

$$\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}$$

$$=\frac{1}{x^2}+\frac{1}{x^2}+\frac{1}{x^2}=\frac{3}{x^2}$$

167. (1) Using Rule 1,
$121a^2 + 64b^2$
$= (11a)^2 + (8b)^2$
$\because (x + y)^2 = x^2 + y^2 + 2xy$
$\therefore$ Required expression
$= 2 \times 11a \times 8b = 176ab$

168. (2) $a = 2+\sqrt{3}$

$$\frac{1}{a} = \frac{1}{2+\sqrt{3}}=\frac{1}{\left(2+\sqrt{3}\right)}\times\frac{2-\sqrt{3}}{\left(2-\sqrt{3}\right)}$$

$$= \frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

$$\therefore\ a^2+\frac{1}{a^2}=\left(a+\frac{1}{a}\right)^2-2$$

$$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^2-2$$

$$= 16-2=14$$

169. (3) $p+\frac{1}{4}\sqrt{p}+k^2$

$$=\left(\sqrt{p}\right)^2+2.\sqrt{p}.\frac{1}{8}+\left(\frac{1}{8}\right)^2-\left(\frac{1}{8}\right)^2+k^2$$

$$\Rightarrow k^2=\left(\frac{1}{8}\right)^2\Rightarrow k=\pm\frac{1}{8}$$

170. (3) $\frac{b-c}{a}+\frac{a+c}{b}+\frac{a-b}{c}=1$

$$\Rightarrow\frac{b-c}{a}+\frac{a-b}{c}+\frac{a+c}{b}-1=0$$

$$\Rightarrow\frac{b-c}{a}+\frac{a-b}{c}+\frac{a+c-b}{b}=0$$

$$\Rightarrow\frac{c-b}{a}+\frac{b-a}{c}=\frac{a+c-b}{b}$$

$$\Rightarrow\frac{c^2-bc+ab-a^2}{ac}=\frac{a+c-b}{b}$$

$$\Rightarrow\frac{(c^2-a^2)-(bc-ab)}{ac}=\frac{a+c-b}{b}$$

$$\Rightarrow\frac{(c-a)(c+a)-b(c-a)}{ac}$$

$$=\frac{a+c-b}{b}$$

$$\Rightarrow\frac{(c-a)(c+a-b)}{ac}=\frac{a+c-b}{b}$$

$$\Rightarrow\frac{c-a}{ac}=\frac{1}{b}$$

$$\Rightarrow\frac{c}{ac}-\frac{a}{ac}=\frac{1}{b}$$

$$\Rightarrow\frac{1}{a}-\frac{1}{c}=\frac{1}{b}$$

171. (2) $\frac{d}{c}=a-b$

$$\Rightarrow\frac{c}{d}=\frac{1}{a-b}=\frac{a+b}{a-b}$$

$$\Rightarrow\frac{c+d}{c-d}=\frac{a+b+a-b}{a+b-a+b}=\frac{a}{b}$$

(By componendo and dividendo)

$$\Rightarrow\frac{1}{c-d}=\frac{a}{b}$$

$$\Rightarrow (c-d)=\frac{b}{a}$$

$$\Rightarrow c^2-d^2=(c+d)(c-d)=\frac{b}{a}$$

172. (2) $x = 2y$

$\Rightarrow 3t = 2 \times \frac{1}{2}(t + 1)$

$\Rightarrow 3t = t + 1 \Rightarrow 3t - t = 1$

$\Rightarrow 2t = 1 \Rightarrow t = \frac{1}{2}$

173. (3) $x^2+\frac{1}{5}x+a^2$

$$=x^2+2.x.\frac{1}{10}+\left(\frac{1}{10}\right)^2-\left(\frac{1}{10}\right)^2+a^2$$

$$\therefore\ a^2-\left(\frac{1}{10}\right)^2=0\Rightarrow a^2=\left(\frac{1}{10}\right)^2$$

$$\Rightarrow a=\frac{1}{10}$$

174. (3) Expression $= 2 - 3x - 4x^2$
$= -(4x^2 + 3x - 2)$

$$=-\left[(2x)^2+2\times2x\times\frac{3}{4}+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-2\right]$$

$$=-\left[\left(2x+\frac{3}{4}\right)^2\right]+\left(\frac{3}{4}\right)^2+2$$

The value of expression will be maximum if,

$$2x+\frac{3}{4}=0$$

$$\Rightarrow 2x=-\frac{3}{4}$$

$$\Rightarrow x=-\frac{3}{8}$$

175. (1) Expression $= x^4 - 2x^2 + k$
$= (x^2)^2 - 2.x^2.1 + (1)^2 - (1)^2 + k$
For a perfect square,
$-1 + k = 0 \Rightarrow k = 1$

176. (2) $f(x) = x^2 + k_1x + k_2$
$(x - 1)$ is a factor of $f(x)$.
$\therefore f(1) = 0$
$\Rightarrow 1 + k_1 + k_2 = 0$
$\Rightarrow k_1 + k_2 = -1$ ... (i)
Again,
$f(-3) = 0$
$\Rightarrow (-3)^2 + k_1(-3) + k_2 = 0$
$\Rightarrow 9 - 3k_1 + k_2 = 0$
$\Rightarrow 3k_1 - k_2 = 9$ ...(ii)
On adding both equations,
$4k_1 = 8 \Rightarrow k_1 = 2$
From equation (i),
$k_1 + k_2 = -1$
$\Rightarrow 2 + k_2 = -1$
$\Rightarrow k_2 = -1 - 2 = -3$

177. (4) $\frac{5x}{2x^2+5x+1}=\frac{1}{3}$

Dividing Numerator and Denominator by x,

$$\frac{5}{2x+5+\frac{1}{x}}=\frac{1}{3}$$

On dividing N^r and D^r by 2,

$$\frac{\frac{5}{2}}{x+\frac{5}{2}+\frac{1}{2x}}=\frac{1}{3}$$

$$\Rightarrow\left(x+\frac{1}{2x}\right)+\frac{5}{2}=\frac{15}{2}$$

$$\Rightarrow x+\frac{1}{2x}=\frac{15}{2}-\frac{5}{2}=\frac{10}{2}=5$$

178. (1) $x+\frac{1}{x}=\frac{x^2+1}{x}$

$\therefore$ Its reciprocal $= \dfrac{x}{x^2+1}$

179. (1) The value of $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$ will be minimum, if values of a, b and c be maximum.

$a + b + c = 1$

$\therefore$ Values of a, b and c will be maximum if

$a = b = c$

$\therefore a = b = c = \dfrac{1}{3}$

$\therefore \dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c} = 3 + 3 + 3 = 9$

180. (2) $a(2+\sqrt{3}) = b(2-\sqrt{3}) = 1$

$\Rightarrow a = \dfrac{1}{2+\sqrt{3}} = \dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

and $b = \dfrac{1}{2-\sqrt{3}}$

$= \dfrac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = \dfrac{2+\sqrt{3}}{4-3}$

$= 2+\sqrt{3}$

$\therefore a^2 + 1 = (2-\sqrt{3})^2 + 1$

$= 4 + 3 - 4\sqrt{3} + 1 = 8 - 4\sqrt{3}$

$b^2 + 1 = (2+\sqrt{3})^2 + 1$

$= 4 + 3 + 4\sqrt{3} + 1 = 8 + 4\sqrt{3}$

$\therefore \dfrac{1}{a^2+1} + \dfrac{1}{b^2+1}$

$= \dfrac{1}{8-4\sqrt{3}} + \dfrac{1}{8+4\sqrt{3}}$

$= \dfrac{8+4\sqrt{3}+8-4\sqrt{3}}{(8-4\sqrt{3})(8+4\sqrt{3})}$

$= \dfrac{16}{64-16\times 3} = \dfrac{16}{64-48}$

$= \dfrac{16}{16} = 1$

181. (4) $(2+\sqrt{3})a = (2-\sqrt{3})b = 1$

$\Rightarrow a = \dfrac{1}{2+\sqrt{3}}$

$\therefore \dfrac{1}{a} = 2+\sqrt{3}$

Similarly,

$b = \dfrac{1}{2-\sqrt{3}}$

$\dfrac{1}{b} = 2-\sqrt{3}$

$\therefore \dfrac{1}{a}+\dfrac{1}{b} = 2+\sqrt{3}+2-\sqrt{3} = 4$

182. (1) $a+\dfrac{1}{b} = b+\dfrac{1}{c} = c+\dfrac{1}{a} = \pm 1$

$\Rightarrow a+\dfrac{1}{b} = 1$

$\Rightarrow ab + 1 = b \Rightarrow ab = b - 1$

$b+\dfrac{1}{c} = 1, \Rightarrow \dfrac{1}{c} = 1 - b$

$\Rightarrow c = \dfrac{1}{1-b}$

$\therefore abc = \dfrac{b-1}{1-b} = -1$

Again, $a+\dfrac{1}{b} = -1$

$\Rightarrow ab + 1 = -b \Rightarrow ab = -b - 1$

$b+\dfrac{1}{c} = -1 \Rightarrow \dfrac{1}{c} = -1 - b$

$\Rightarrow c = \dfrac{1}{-1-b}$

$\therefore abc = 1$

$\therefore abc = \pm 1$

183. (3) $\dfrac{x}{y} = \dfrac{4}{5}$ (Given)

Expression $= \dfrac{4}{7} + \dfrac{2y-x}{2y+x}$

$= \dfrac{4}{7} + \dfrac{\dfrac{2y}{y}-\dfrac{x}{y}}{\dfrac{2y}{y}+\dfrac{x}{y}}$

$= \dfrac{4}{7} + \dfrac{2-\dfrac{x}{y}}{2+\dfrac{x}{y}} = \dfrac{4}{7} + \dfrac{2-\dfrac{4}{5}}{2+\dfrac{4}{5}}$

$= \dfrac{4}{7} + \dfrac{\dfrac{10-4}{5}}{\dfrac{10+4}{5}} = \dfrac{4}{7} + \dfrac{6}{14}$

$= \dfrac{4}{7} + \dfrac{3}{7} = \dfrac{7}{7} = 1$

184. (4) $P(x) = x^2 + 3Qx - 2Q$

$\because (x-2)$ is a factor of $P(x)$.

$\therefore P(2) = 0$

$\Rightarrow (2)^2 + 3Q \times 2 - 2Q = 0$

$\Rightarrow 4 + 6Q - 2Q = 0$

$\Rightarrow 4Q + 4 = 0$

$\Rightarrow 4Q = -4 \Rightarrow Q = -1$

185. (4) Using Rule 1,

$a + b = 12$, $ab = 22$

$\therefore a^2 + b^2 = (a + b)^2 - 2ab$

$= (12)^2 - 2 \times 22$

$= 144 - 44 = 100$

186. (2) $x = \sqrt{3} - \dfrac{1}{\sqrt{3}}$

$y = \sqrt{3} + \dfrac{1}{\sqrt{3}}$

$x + y = \sqrt{3} - \dfrac{1}{\sqrt{3}} + \sqrt{3} + \dfrac{1}{\sqrt{3}}$

$= 2\sqrt{3}$

$xy = \left(\sqrt{3}-\dfrac{1}{\sqrt{3}}\right)\left(\sqrt{3}+\dfrac{1}{\sqrt{3}}\right)$

$= 3 - \dfrac{1}{3} = \dfrac{9-1}{3} = \dfrac{8}{3}$

$\therefore \dfrac{x^2}{y} + \dfrac{y^2}{x} = \dfrac{x^3+y^3}{xy}$

$= \dfrac{(x+y)^3 - 3xy(x+y)}{xy}$

$= \dfrac{(2\sqrt{3})^3 - 3\times\dfrac{8}{3}(2\sqrt{3})}{\dfrac{8}{3}}$

$= \dfrac{24\sqrt{3}-16\sqrt{3}}{\dfrac{8}{3}}$

$= \frac{8\sqrt{3}\times 3}{8} = 3\sqrt{3}$

187. (2) $ax^2 + bx + c$ will be a perfect square, if $b^2 = 4ac$

$\therefore$ $x^2 + ax + b$ will be a perfect square if $a^2 = 4b$

Look : $x^2 + 2\sqrt{b}\,x + b$

$= x^2 + 2.x.\sqrt{b} + \left(\sqrt{b}\right)^2$

$= (x + \sqrt{b})^2$

188. (1) $a + b + c + d = 4$ (Given)

Expression

$= \frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)}$

$+ \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}$

$= \frac{1-d+1-a+1-b+1-c}{(1-a)(1-b)(1-c)(1-d)}$

$= \frac{4-(a+b+c+d)}{(1-a)(1-b)(1-c)(1-d)}$

$= \frac{4-4}{(1-a)(1-b)(1-c)(1-d)} = 0$

189. (2) If $a + b + c = 0$,

$a^3 + b^3 + c^3 = 3abc$

$\therefore$ If $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}} = 0$

$\Rightarrow \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{1}{3}}\right)^3 + \left(c^{\frac{1}{3}}\right)^3$

$= 3.a^{\frac{1}{3}}.b^{\frac{1}{3}}.c^{\frac{1}{3}}$

$\Rightarrow a + b + c = 3a^{\frac{1}{3}}.b^{\frac{1}{3}}.c^{\frac{1}{3}}$

$\Rightarrow (a + b + c)^3$

$= 3^3.\left(a^{\frac{1}{3}}.b^{\frac{1}{3}}.c^{\frac{1}{3}}\right)^3 = 27abc$

190. (2) $a = \sqrt{6} + \sqrt{5}$, $b = \sqrt{6} - \sqrt{5}$ a

$- b = \sqrt{6} + \sqrt{5} - \sqrt{6} + \sqrt{5}$

$= 2\sqrt{5}$

$ab = \left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)$

$= 6 - 5 = 1$

$\therefore 2a^2 - 5ab + 2b^2$

$= 2\left(a^2 - \frac{5}{2}ab + b^2\right)$

$= 2\left(a^2 - 2ab + b^2 - \frac{1}{2}ab\right)$

$= 2\ (a^2 - 2ab + b^2) - ab$

$= 2\ (a - b)^2 - ab$

$= 2 \times \left(2\sqrt{5}\right)^2 - 1$

$= 2 \times 4 \times 5 - 1 = 40 - 1 = 39$

191. (3) $a^2 + b^2 + c^2 = 2a - 2b - 2$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + c^2 = 0$

$\Rightarrow a - 1 = 0 \Rightarrow a = 1;$

$\Rightarrow b + 1 = 0 \Rightarrow b = -1;$

and $c = 0$

$\therefore 3a - 2b + c = 3 \times 1 - 2\ (-1) + 0$

$= 3 + 2 = 5$

192. (2) $a + b + c = 3$; $a^2 + b^2 + c^2 = 6$

$\therefore$ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow 3^2 = 6 + 2\ (ab + bc + ca)$

$\Rightarrow 9 - 6 = 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = \frac{3}{2}$

$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

$\Rightarrow \frac{bc + ac + ab}{abc} = 1$

$\Rightarrow abc = ab + bc + ca = \frac{3}{2}$

193. (4) $a^2 - 4a - 1 = 0$

$\Rightarrow a^2 - 1 = 4a$

On dividing both sides by a,

$\frac{a^2 - 1}{a} = \frac{4a}{a}$

$\Rightarrow a - \frac{1}{a} = 4$ ...(i)

Expression $= a^2 + 3a + \frac{1}{a^2} - \frac{3}{a}$

$= a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$

$= \left(a - \frac{1}{a}\right)^2 + 2 + 3\left(a - \frac{1}{a}\right)$

$= (4)^2 + 2 + 3 \times 4$

$= 16 + 2 + 12 = 30$

194. (4) $x = 2 + \sqrt{3}$

$\therefore \frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2+\sqrt{3})\ (2-\sqrt{3})}$

$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

$\therefore\ x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (2 + \sqrt{3} + 2 - \sqrt{3})^2 - 2$

$= 4^2 - 2 = 16 - 2 = 14$

195. (3) $a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$\Rightarrow a = 1,\ b = -1,\ c = -1$

$\therefore\ a + b + c = 1 - 1 - 1 = -1$

196. (4) $-1 \le \frac{2x-7}{5} \le 1$

$\Rightarrow -5 \le 2x - 7 \le 5$

$\Rightarrow -5 + 7 \le 2x - 7 + 7 \le 5 + 7$

$\Rightarrow 2 \le 2x \le 12$

$\Rightarrow 1 \le x \le 6$

197. (3)

$\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$

$\Rightarrow \frac{3}{2x} - \frac{5x}{2x} + \frac{3}{2y} - \frac{5y}{2y} + \frac{3}{2z} - \frac{5z}{2z} = 0$

$\Rightarrow \frac{3}{2x} + \frac{3}{2y} + \frac{3}{2z} - \frac{5}{2} - \frac{5}{2} - \frac{5}{2} = 0$

$\Rightarrow \frac{3}{2x} + \frac{3}{2y} + \frac{3}{2z} = \frac{3\times 5}{2}$

$\Rightarrow \frac{1}{2x} + \frac{1}{2y} + \frac{1}{2z} = \frac{5}{2}$

$\Rightarrow \frac{4}{2x} + \frac{4}{2y} + \frac{4}{2z} = \frac{4\times 5}{2}$

$\Rightarrow \frac{2}{x} + \frac{2}{y} + \frac{2}{z} = 10$

198. (1) $2s = a + b + c$

$\therefore\ s\ (s - c) = \left(\frac{a+b+c}{2}\right)\left(\frac{a+b+c}{2} - c\right)$

$= \frac{(a+b+c)(a+b-c)}{4}$

Again, $(s - a)\ (s - b)$

$= \frac{1}{4}\ (2s - 2a)\ (2s - 2b)$

$= \frac{1}{4}$ $(a+b+c-2a)$ $(a+b+c-2b)$

$= \frac{1}{4}$ $(b+c-a)$ $(a+c-b)$

$\therefore$ $s(s-c)+(s-a)(s-b)$

$= \frac{1}{4}$ $[(a+b+c)(a+b-c)+(b+c-a)(a+c-b)]$

$= \frac{1}{4}$ $[(a+b)^2-c^2+ab+ac-a^2+bc+c^2-ac-b^2-bc+ab]$

$= \frac{1}{4}$ $(a^2+b^2+2ab-c^2+ab+ac-a^2+bc+c^2-ac-b^2-bc+ab)$

$= \frac{1}{4} \times 4ab = ab$

199. (4) $\frac{2p}{p^2-2p+1} = \frac{1}{4}$

On dividing numerator and denominator by p, we get,

$\frac{2}{p-2+\frac{1}{p}} = \frac{1}{4}$

$\Rightarrow p + \frac{1}{p} - 2 = 8$

$\Rightarrow p + \frac{1}{p} = 8 + 2 = 10$

200. (1) $\sqrt{1+\frac{27}{169}} = 1 + \frac{x}{13}$

$\Rightarrow \sqrt{\frac{169+27}{169}} = 1 + \frac{x}{13}$

$\Rightarrow \sqrt{\frac{196}{169}} = 1 + \frac{x}{13}$

$\Rightarrow \frac{14}{13} = 1 + \frac{x}{13}$

$\Rightarrow 1 + \frac{1}{13} = 1 + \frac{x}{13}$

$\Rightarrow x = 1$

201. (3) $2x = \sqrt{a} + \frac{1}{\sqrt{a}}$

On squaring both sides,

$4x^2 = a + \frac{1}{a} + 2$

$\Rightarrow 4x^2 - 4 = a + \frac{1}{a} + 2 - 4$

$= a + \frac{1}{a} - 2$

$\therefore \sqrt{4x^2-4} = \sqrt{\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)^2}$

$= \sqrt{a} - \frac{1}{\sqrt{a}}$

$\therefore \sqrt{x^2-1} = \frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)$

$\therefore$ Expression $= \frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$

$= \frac{\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}{\frac{1}{2}\left(\sqrt{a}+\frac{1}{\sqrt{a}}\right)-\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}$

$= \frac{\frac{1}{2}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)}{\frac{1}{\sqrt{a}}} = \frac{1}{2}\sqrt{a}\left(\sqrt{a}-\frac{1}{\sqrt{a}}\right)$

$= \frac{1}{2}(a-1)$

202. (1) $a^2+b^2+c^2 = 2a-2b-2c-3$

$\Rightarrow a^2-2a+1+b^2+2b+1+c^2+2c+1 = 0$

$\Rightarrow (a-1)^2+(b+1)^2+(c+1)^2 = 0$

$\Rightarrow a-1 = 0,\ b+1 = 0,\ c+1 = 0$

$\Rightarrow a = 1,\ b = -1,\ c = -1$

$\therefore a+b+c = 1-1-1 = -1$

203. (2) $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c}$

$= \frac{c+a-b}{c+a}$

$\Rightarrow \frac{a+b}{a+b} - \frac{c}{a+b} = \frac{b+c}{b+c} - \frac{a}{b+c}$

$= \frac{c+a}{c+a} - \frac{b}{c+a}$

$\Rightarrow 1 - \frac{c}{a+b} = 1 - \frac{a}{b+c}$

$= 1 - \frac{b}{c+a}$

$\Rightarrow \frac{c}{a+b} = \frac{a}{b+c} = \frac{b}{c+a}$

$\Rightarrow \frac{a+b}{c} = \frac{b+c}{a} = \frac{c+a}{b}$

$\Rightarrow \frac{a+b}{c}+1 = \frac{b+c}{a}+1 = \frac{c+a}{b}+1$

$\Rightarrow \frac{a+b+c}{c} = \frac{b+c+a}{a}$

$= \frac{c+a+b}{b}$

$\Rightarrow \frac{1}{c} = \frac{1}{a} = \frac{1}{b} \Rightarrow a = b = c$

204. (4) Given, $bc+ab+ca = abc$

$\therefore$ $bc+ab = abc-ac$

$ab+ca = abc-bc$

$bc+ca = abc-ab$

$\therefore$ Expression $= \frac{b+c}{abc-bc} + \frac{a+c}{abc-ac} + \frac{a+b}{abc-ab}$

$= \frac{b+c}{ab+ac} + \frac{a+c}{bc+ab} + \frac{a+b}{bc+ca}$

$= \frac{b+c}{a(b+c)} + \frac{a+c}{b(c+a)} + \frac{a+b}{c(a+b)}$

$= \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$= \frac{bc+ac+ab}{abc}$

$= \frac{abc}{abc} = 1$

205. (4) $\frac{a^2-bc}{a^2+bc} + \frac{b^2-ca}{b^2+ca} + \frac{c^2-ab}{c^2+ab} = 1$

$\Rightarrow \left(\frac{a^2-bc}{a^2+bc}+1\right) + \left(\frac{b^2-ca}{b^2+ca}+1\right) + \left(\frac{c^2-ab}{c^2+ab}+1\right) = 4$

$\Rightarrow \dfrac{a^2-bc+a^2+bc}{a^2+bc}$ +

$\dfrac{b^2-ca+b^2+ca}{b^2+ca}+\dfrac{c^2-ab+c^2+ab}{c^2+ab}=4$

$\Rightarrow \dfrac{2a^2}{a^2+bc}+\dfrac{2b^2}{b^2+ca}+\dfrac{2c^2}{c^2+ab}=4$

$\Rightarrow \dfrac{a^2}{a^2+bc}+\dfrac{b^2}{b^2+ca}+\dfrac{c^2}{c^2+ab}$

$=\dfrac{4}{2}=2$

206. (3) $999x + 888y = 1332$
$888x + 999y = 555$
On adding,
$1887x + 1887y = 1887$
$\Rightarrow 1887(x+y) = 1887$

$\Rightarrow x+y = \dfrac{1887}{1887} = 1$

207. (3) $a = \dfrac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}}$

By componendo and dividendo,

$\dfrac{a+1}{a-1}$

$= \dfrac{\sqrt{x+2}+\sqrt{x-2}+\sqrt{x+2}-\sqrt{x-2}}{\sqrt{x+2}+\sqrt{x-2}-\sqrt{x+2}+\sqrt{x-2}}$

$\Rightarrow \dfrac{a+1}{a-1}$

$= \dfrac{2\sqrt{x+2}}{2\sqrt{x-2}} = \dfrac{\sqrt{x+2}}{\sqrt{x-2}}$

On squaring both sides,

$\dfrac{a^2+2a+1}{a^2-2a+1} = \dfrac{x+2}{x-2}$

$\Rightarrow \dfrac{a^2+1+2a}{a^2+1-2a} = \dfrac{x+2}{x-2}$

By componendo and dividendo,

$\dfrac{2(a^2+1)}{4a} = \dfrac{2x}{4}$

$\Rightarrow \dfrac{a^2+1}{a} = x$

$\Rightarrow a^2 + 1 = ax$
$\Rightarrow a^2 - ax = -1$

208. (3) $x = \dfrac{1}{2+\sqrt{3}}$

$= \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = \dfrac{2-\sqrt{3}}{4-3}$

$= 2-\sqrt{3}$

$\therefore y = \dfrac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$\therefore x+y = 2-\sqrt{3}+2+\sqrt{3} = 4$

$xy = (2-\sqrt{3})(2+\sqrt{3})$
$= 4 - 3 = 1$
$\therefore 8xy(x^2+y^2)$
$= 8xy[(x+y)^2 - 2xy]$
$= 8 \times 1 (4^2 - 2 \times 1)$
$= 8(16-2) = 8 \times 14 = 112$

209. (2) $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$
$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca = 0$
$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$
$\Rightarrow a-b=0,\ b-c=0,\ c-a=0$
$\Rightarrow a=b,\ b=c,\ c=a$
$\Rightarrow$ a = b = c

$\therefore \dfrac{a+c}{b} = \dfrac{2a}{a} = 2$

210. (3) $\dfrac{m-a^2}{b^2+c^2}+\dfrac{m-b^2}{c^2+a^2}+$

$\dfrac{m-c^2}{a^2+b^2}-3=0$

$\Rightarrow \dfrac{m-a^2}{b^2+c^2}-1+\dfrac{m-b^2}{c^2+a^2}-1$

$+\dfrac{m-c^2}{a^2+b^2}-1=0$

$\Rightarrow \dfrac{m-a^2-b^2-c^2}{b^2+c^2}+$

$\dfrac{m-b^2-c^2-a^2}{c^2+a^2}+\dfrac{m-c^2-a^2-b^2}{a^2+b^2}$
$=0$

$\Rightarrow \dfrac{m-(a^2+b^2+c^2)}{b^2+c^2}+$

$\dfrac{m-(a^2+b^2+c^2)}{c^2+a^2}+$

$\dfrac{m-(a^2+b^2+c^2)}{a^2+b^2}=0$

$\therefore$ Each term $= 0$

$\therefore \dfrac{m-(a^2+b^2+c^2)}{b^2+c^2}=0$

$\Rightarrow m - (a^2+b^2+c^2) = 0$
$\Rightarrow m = a^2+b^2+c^2$

211. (3) $x+\dfrac{1}{x} = 1$ (Given)

Expression $= \dfrac{x^2+3x+1}{x^2+7x+1}$

$= \dfrac{x+\frac{1}{x}+3}{x+\frac{1}{x}+7}$

(Dividing numerator and denominator by x)

$= \dfrac{1+3}{1+7} = \dfrac{4}{8} = \dfrac{1}{2}$

212. (3) Using Rule 8,
p = 99 (Given)
Expression = $p(p^2+3p+3)$
$= p^3 + 3p^2 + 3p$
$= p^3 + 3p^2 + 3p + 1 - 1$
$= (p+1)^3 - 1$
$= (99+1)^3 - 1 = (100)^3 - 1$
$= 1000000 - 1 = 999999$

213. (1) $x = \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$,

$y = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$\therefore x+y = \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$= \dfrac{\left(\sqrt{5}-\sqrt{3}\right)^2+\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}+\sqrt{3}\right)\times\left(\sqrt{5}-\sqrt{3}\right)}$

$= \dfrac{2\left(\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2\right)}{5-3}$

$= 5 + 3 = 8$

$xy = \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 1$

$\therefore \dfrac{x^2+xy+y^2}{x^2-xy+y^2}$

$= \dfrac{(x+y)^2-xy}{(x+y)^2-3xy}$

$= \dfrac{8^2-1}{8^2-3} = \dfrac{64-1}{64-3} = \dfrac{63}{61}$

214. (1) $x + \dfrac{1}{x} = 1$

$\Rightarrow x^2 + 1 = x \Rightarrow x^2 - x + 1 = 0$

$\therefore \dfrac{2}{x^2-x+2} = \dfrac{2}{x^2-x+1+1}$

$= \dfrac{2}{0+1} = 2$

215. (1) $\dfrac{x}{1} = \dfrac{a-b}{a+b}$

By componendo and dividendo,

$\dfrac{1-x}{1+x} = \dfrac{1-\dfrac{a-b}{a+b}}{1+\dfrac{a-b}{a+b}}$

$= \dfrac{a+b-a+b}{a+b+a-b} = \dfrac{b}{a}$

Similarly,

$\dfrac{1-y}{1+y} = \dfrac{c}{b}$; $\dfrac{1-z}{1+z} = \dfrac{a}{c}$

$\therefore$ Expression$= \dfrac{(1-x)(1-y)(1-z)}{(1+x)(1+y)(1+z)}$

$= \dfrac{b}{a} \times \dfrac{c}{b} \times \dfrac{a}{c} = 1$

216. (1) $x = \dfrac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

On rationalising the denominator,

$= \dfrac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} \times \dfrac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$

$= \dfrac{\left(\sqrt{13}+\sqrt{11}\right)^2}{\left(\sqrt{13}\right)^2-\left(\sqrt{11}\right)^2}$

$= \dfrac{13+11+2\sqrt{143}}{13-11}$

$= \dfrac{24+2\sqrt{143}}{2} = 12 + \sqrt{143}$

$\therefore y = \dfrac{1}{x} = \dfrac{1}{12+\sqrt{143}}$

$= \dfrac{1}{12+\sqrt{143}} \times \dfrac{12-\sqrt{143}}{12-\sqrt{143}}$

$= \dfrac{12-\sqrt{143}}{144-143} = 12 - \sqrt{143}$

$\therefore x - y = 12 + \sqrt{143} - 12 + \sqrt{143} = 2\sqrt{143}$ and

$xy = \left(12+\sqrt{143}\right)\left(12-\sqrt{143}\right)$

$= 144 - 143 = 1$

$\therefore 3x^2 - 5xy + 3y^2 = 3x^2 - 6xy + 3y^2 + xy$

$= 3(x-y)^2 + xy$

$= 3\left(2\sqrt{143}\right)^2 + 1$

$= 3 \times 4 \times 143 + 1 = 1716 + 1$

$= 1717$

217. (4) $a + \dfrac{1}{b} = b + \dfrac{1}{c} = c + \dfrac{1}{a}$

$\Rightarrow \dfrac{abc+c}{bc} = \dfrac{abc+a}{ac}$

$= \dfrac{abc+b}{ab}$

$\Rightarrow \dfrac{c}{bc} = \dfrac{a}{ac} = \dfrac{b}{ab}$

$\Rightarrow \dfrac{1}{b} = \dfrac{1}{c} = \dfrac{1}{a}$

$\Rightarrow a = b = c = 1$

$\therefore a^2 b^2 c^2 = 1$

218. (3) $a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\therefore a-b = 0 \Rightarrow a = b$

$b-c = 0 \Rightarrow b = c$

$c-a = 0 \Rightarrow c = a$

$\therefore a = b = c$

$\therefore \dfrac{a+c}{b} = \dfrac{2a}{a} = 2$

219. (3) $9x^2 + 25 - 30x$

$= (3x)^2 + (5)^2 - 2 \times 3x \times 5$

$= (3x-5)^2$

220. (4) $\dfrac{x}{3} + \dfrac{3}{x} = 1$

$\Rightarrow \dfrac{x^2+9}{3x} = 1$

$\Rightarrow x^2 + 9 = 3x$

$\Rightarrow x^2 - 3x + 9 = 0$

$\therefore x^3+3^3 = (x+3)(x^2 - 3x + 9) = 0$

$\Rightarrow x^3 = -27$

221. (2) $x + y = 2a = a + a$

$\Rightarrow x - a = a - y$

Expression $= \dfrac{a}{x-a} + \dfrac{a}{y-a}$

$= \dfrac{a}{x-a} - \dfrac{a}{a-y}$

$= \dfrac{a}{x-a} - \dfrac{a}{x-a} = 0$

222. (1) $\dfrac{x+1}{x-1} = \dfrac{a}{b}$

By componendo and dividendo,

$\dfrac{x+1+x-1}{x+1-x+1} = \dfrac{a+b}{a-b}$

$\Rightarrow \dfrac{2x}{2} = \dfrac{a+b}{a-b}$

$\Rightarrow x = \dfrac{a+b}{a-b}$

Again,

$\dfrac{1-y}{1+y} = \dfrac{b}{a}$

$\Rightarrow \dfrac{1+y}{1-y} = \dfrac{a}{b}$

$\Rightarrow \dfrac{1+y+1-y}{1+y-1+y} = \dfrac{a+b}{a-b}$

$\Rightarrow \dfrac{2}{2y} = \dfrac{a+b}{a-b}$

$\Rightarrow y = \dfrac{a-b}{a+b}$

$\therefore x - y = \dfrac{a+b}{a-b} - \dfrac{a-b}{a+b}$

$= \frac{(a+b)^2-(a-b)^2}{(a+b)(a-b)} = \frac{4ab}{a^2-b^2}$

$xy = \frac{a+b}{a-b} \times \frac{a-b}{a+b} = 1$

$\therefore$ Expression

$= \frac{x-y}{1+xy} = \frac{\frac{4ab}{a^2-b^2}}{1+1}$

$= \frac{4ab}{2(a^2-b^2)} = \frac{2ab}{a^2-b^2}$

223. (4) $\frac{a}{b}+\frac{b}{a} = 2$

$\Rightarrow \frac{a^2+b^2}{ab} = 2$

$\Rightarrow a^2 + b^2 = 2ab$

$\Rightarrow a^2 + b^2 - 2ab = 0$

$\Rightarrow (a-b)^2 = 0 \Rightarrow a - b = 0$

224. (2) $\sqrt{y} = 4x \Rightarrow y = (4x)^2 = 16x^2$

$\therefore \frac{x^2}{y} = \frac{x^2}{16x^2} = \frac{1}{16}$

225. (3) $\frac{x}{y} = \frac{a+2}{a-2}$

On squaring both sides,

$\frac{x^2}{y^2} = \frac{(a+2)^2}{(a-2)^2}$

By componendo and dividendo,

$\frac{x^2-y^2}{x^2+y^2} = \frac{(a+2)^2-(a-2)^2}{(a+2)^2+(a-2)^2}$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \frac{4 \times a \times 2}{2(a^2+4)}$

$= \frac{4a}{a^2+4}$

[$\because$ $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$; $(a + b)^2 - (a-b)^2 = 4ab$]

226. (1) $x (x + y + z) = 20$

$\Rightarrow x^2 + xy + xz = 20$ --- (i)

Again, $y (x + y + z) = 30$

$\Rightarrow xy + y^2 + yz = 30$ --- (ii)

and, $z (x + y + z) = 50$

$\Rightarrow xz + yz + z^2 = 50$ --- (iii)

On adding all three equations,

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 20 + 30 + 50$

$\Rightarrow (x + y + z)^2 = 100$

$\Rightarrow x + y + z = 10$

$\Rightarrow 2 (x + y + z) = 20$

227. (1) $x + y = 4$ --- (i)

$x^2 + y^2 = 14$ --- (ii)

$\therefore$ $(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 16 = 14 + 2xy$

$\Rightarrow 2xy = 16 - 14 = 2$

$\Rightarrow xy = 1$ --- (iii)

$\therefore$ $(x - y)^2 = (x + y)^2 - 4xy$

$= (4)^2 - 4 = 16 - 4 = 12$

$\Rightarrow x - y = \sqrt{12} = 2\sqrt{3}$ --- (iv)

$\therefore$ On adding equations (i) and (iv)

$x + y = 4$

$x - y = 2\sqrt{3}$

$2x = 4 + 2\sqrt{3}$

$\Rightarrow x = 2+\sqrt{3}$

From equation (i),

$2 + \sqrt{3} + y = 4$

$\Rightarrow y = 4 - 2 - \sqrt{3} = 2-\sqrt{3}$

228. (2) $a^2 + b^2 + c^2 = 2 (a - b - c) - 3$

$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$

$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$

$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

$\therefore$ $a - 1 = 0 \Rightarrow a = 1$

$b + 1 = 0 \Rightarrow b = -1$

$c + 1 = 0 \Rightarrow c = -1$

[If $x^2 + y^2 + z^2 = 0 \Rightarrow x = 0, y = 0, z = 0$]

$\therefore$ $a + b + c = 1 - 1 - 1 = -1$

229. (3) $x^2 - 4x - 1 = 0$

$\Rightarrow x^2 - 1 = 4x$

$\Rightarrow \frac{x^2-1}{x} = \frac{4x}{x}$

$\Rightarrow x - \frac{1}{x} = 4$

On squaring both sides,

$\left(x-\frac{1}{x}\right)^2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 16 + 2 = 18$

230. (3) $a + \frac{1}{b} = 1 \Rightarrow a = \frac{1}{2}$; $b = 2$

$b + \frac{1}{c} = 1 \Rightarrow b = 2, c = -1$

$\therefore c + \frac{1}{a} = -1 + 2 = 1$

231. (1) $\frac{a}{b} = \frac{25}{6}$

$\Rightarrow \frac{a^2}{b^2} = \frac{25^2}{6^2} = \frac{625}{36}$

By componendo and dividendo,

$\frac{a^2-b^2}{a^2+b^2} = \frac{625-36}{625+36}$

$= \frac{589}{661}$

232. (3) $(x - 2) (x - p) = x^2 - ax + 6$

$\Rightarrow x (x - p) -2 (x - p)$

$= x^2 - ax + 6$

$\Rightarrow x^2 - px - 2x + 2p = x^2 - ax + 6$

$\Rightarrow x^2 - x (p + 2) + 2p$

$= x^2 - ax + 6$

$\therefore p + 2 = a$

(comparing respective co-efficients)

$\Rightarrow a - p = 2$

233. (1) $x = \sqrt{a} + \frac{1}{\sqrt{a}}$

$y = \sqrt{a} - \frac{1}{\sqrt{a}}$

$\therefore x + y = \sqrt{a} + \frac{1}{\sqrt{a}} + \sqrt{a} - \frac{1}{\sqrt{a}}$

$= 2\sqrt{a}$

$x - y = \sqrt{a} + \frac{1}{\sqrt{a}} - \sqrt{a} + \frac{1}{\sqrt{a}}$

$= \frac{2}{\sqrt{a}}$

Now,

$x^4 + y^4 - 2x^2 y^2$
$= (x^2 - y^2)^2$
$= \{(x + y)(x - y)\}^2$

$$\left(2\sqrt{a} \times \frac{2}{\sqrt{a}}\right)^2 = 4^2 = 16$$

234. (4) $2x + \frac{1}{3x} = 5$

$$\Rightarrow \frac{6x^2+1}{3x} = 5$$

$\Rightarrow 6x^2 + 1 = 15x$

$$\therefore \frac{5x}{6x^2+20x+1}$$

$$= \frac{5x}{6x^2+1+20x}$$

$$= \frac{5x}{15x+20x}$$

$$= \frac{5x}{35x} = \frac{1}{7}$$

235. (2) $a + b = 10$;
$ab = 21$
$\therefore (a - b)^2 = (a + b)^2 - 4ab$
$= (10)^2 - 4 \times 21$
$= 100 - 84 = 16$

236. (3) Given,
$0 < x < 1$
$\Rightarrow x.0 < x.x < 1.x$
$\Rightarrow 0 < x^2 < x$
Again, $x < 1$
$\Rightarrow \sqrt{x} < 1$
$\therefore x^2 < x < \sqrt{x}$

237. (2) $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

$$= \frac{\left(\sqrt{5}+1\right)^2}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}$$

(Rationalising the denominator)

$$= \frac{5+1+2\sqrt{5}}{5-1} = \frac{6+2\sqrt{5}}{4}$$

$$= \frac{3+\sqrt{5}}{2}$$

$$\therefore y = \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{3-\sqrt{5}}{2}$$

$$\therefore x + y = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2}$$

$$= \frac{3+\sqrt{5}+3-\sqrt{5}}{2} = 3$$

$$xy = \frac{3+\sqrt{5}}{2} \times \frac{3-\sqrt{5}}{2}$$

$$= \frac{9-5}{4} = 1$$

$$\therefore \frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{(x+y)^2-xy}{(x+y)^2-3xy}$$

$$= \frac{(3)^2-1}{(3)^2-3} = \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$$

238. (2) $a + b + c = m$

and, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$$\Rightarrow \frac{bc+ac+ab}{abc} = 0$$

$\Rightarrow bc + ac + ab = 0$
$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow m^2 = a^2 + b^2 + c^2 + 2 \times 0$
$\Rightarrow a^2 + b^2 + c^2 = m^2$
$\therefore$ Required average

$$= \frac{a^2+b^2+c^2}{3} = \frac{m^2}{3}$$

239. (3) $x = \frac{8ab}{a+b}$

$$\Rightarrow \frac{x}{4a} = \frac{2b}{a+b}$$

By componendo and dividendo,

$$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b}$$

$$= \frac{a+3b}{b-a} \quad \text{..... (i)}$$

Again,

$$x = \frac{8ab}{a+b}$$

$$\frac{x}{4b} = \frac{2a}{a+b}$$

By componendo and dividendo,

$$\frac{x+4b}{x-4b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$$

$$\therefore \frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$$

$$= \frac{a+3b}{b-a} + \frac{3a+b}{a-b}$$

$$= \frac{a+3b}{b-a} - \frac{3a+b}{b-a}$$

$$= \frac{a+3b-3a-b}{b-a}$$

$$= \frac{2b-2a}{b-a} = \frac{2(b-a)}{b-a} = 2$$

240. (1) $x^2 - y^2 = (x + y)(x - y)$
$\therefore (2a + b)^2 - (2a - b)^2 = (2a + b + 2a - b)(2a + b - 2a + b)$
$= 4a \times 2b = 8ab$

241. (2) $a + b + c = 0$
$\therefore (a + b + c)^2 = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$
$\Rightarrow a^2 + b^2 + c^2 = -2ab - 2bc - 2ca$

$$\therefore \frac{a^2+b^2+c^2}{ab+bc+ca}$$

$$= \frac{-2(ab+bc+ca)}{ab+bc+ca} = -2$$

242. (2) $a + b = 2c$
$\Rightarrow a - c = c - b$

$$\therefore \frac{a}{a-c} + \frac{c}{b-c}$$

$$= \frac{a}{c-b} + \frac{c}{b-c}$$

$$= \frac{a}{c-b} - \frac{c}{c-b} = \frac{a-c}{c-b}$$

$$= \frac{c-b}{c-b} = 1$$

243. (1) $2x + \frac{1}{4x} = 1$

On dividing by 2, we get

$$x + \frac{1}{8x} = \frac{1}{2}$$

On squaring both sides, we get

$$\left(x+\frac{1}{8x}\right)^2 = \frac{1}{4}$$

$$\Rightarrow x^2+\frac{1}{64x^2}+2\times x\times\frac{1}{8x}$$

$$= \frac{1}{4}$$

$$\Rightarrow x^2+\frac{1}{64x^2}+\frac{1}{4} = \frac{1}{4}$$

$$\Rightarrow x^2+\frac{1}{64x^2} = \frac{1}{4}-\frac{1}{4} = 0$$

244. (4) Expression

$$= \frac{a}{a-b} + \frac{b}{b-a}$$

$$= \frac{a}{a-b} - \frac{b}{a-b}$$

$$= \frac{a-b}{a-b} = 1$$

245. (1) $a + \frac{1}{b} = 1$

$$\Rightarrow a = 1 - \frac{1}{b} = \frac{b-1}{b}$$

$$\therefore \frac{1}{a} = \frac{b}{b-1}$$

Again, $b + \frac{1}{c} = 1$

$$\Rightarrow \frac{1}{c} = 1 - b$$

$$\Rightarrow c = \frac{1}{1-b}$$

$$\therefore c + \frac{1}{a} = \frac{1}{1-b} + \frac{b}{b-1}$$

$$= \frac{1}{1-b} - \frac{b}{1-b} = \frac{1-b}{1-b} = 1$$

246. (3) $\frac{a}{b} = \frac{1}{2}$

$$\therefore \frac{2a-5b}{5a+3b} = \frac{2\left(\frac{a}{b}\right)-5}{5\left(\frac{a}{b}\right)+3}$$

$$= \frac{2\times\frac{1}{2}-5}{5\times\frac{1}{2}+3}$$

$$= \frac{1-5}{\frac{5}{2}+3} = \frac{-4\times 2}{5+6} = \frac{-8}{11}$$

247. (3) $x+\frac{1}{x}=17$

On squaring both sides,

$$\left(x+\frac{1}{x}\right)^2=17^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=289$$

$$\Rightarrow x^2+\frac{1}{x^2}=289-2=287$$

= radius of the circle

$\therefore$ Circumferece of circle $= 2\pi r$

$= 2 \times 287 \times \pi$

$= 574\pi$ units

248. (4) Putting $x = \frac{3}{2}$ in $x^2+mx+24$ $= 0$

$$\left(\frac{3}{2}\right)^2+m\times\frac{3}{2}+24=0$$

$$\Rightarrow \frac{9}{4}+\frac{3m}{2}+24=0$$

$$\Rightarrow \frac{3m}{2}=-\left(24+\frac{9}{4}\right)$$

$$\Rightarrow \frac{3m}{2}=-\left(\frac{96+9}{4}\right)$$

$$\Rightarrow \frac{3m}{2}=-\left(\frac{105}{4}\right)$$

$$\Rightarrow m=-\left(\frac{105}{4}\times\frac{2}{3}\right)=-\frac{35}{2}$$

249. (2) $\frac{(a+b)^2}{(a-b)^2}=\frac{25}{4}$

By componendo and dividendo,

$$\frac{(a+b)^2+(a-b)^2}{(a+b)^2-(a-b)^2}=\frac{25+4}{25-4}$$

$$\Rightarrow \frac{2(a^2+b^2)}{4ab}=\frac{29}{21}$$

$$\Rightarrow \frac{a^2+b^2}{2ab}=\frac{29}{21}$$

$$\Rightarrow \frac{a^2+b^2}{2\times 21} = \frac{29}{21}$$

$\Rightarrow a^2 + b^2 = 2 \times 29 = 58$

$\therefore a^2 + b^2 + 3ab = 58 + 3 \times 21$
$= 58 + 63 = 121$

250. (2) $a+\frac{1}{a-2}=4$

$$\Rightarrow (a-2)+\frac{1}{(a-2)}=4-2=2$$

On squaring both sides,

$$\left[(a-2)+\frac{1}{(a-2)}\right]^2=4$$

$$\Rightarrow (a-2)^2+\frac{1}{(a-2)^2}+$$

$$2\times (a-2)\times\frac{1}{(a-2)}=4$$

$$\Rightarrow (a-2)^2+\frac{1}{(a-2)^2}=4-2=2$$

251. (3) $x=\frac{6pq}{p+q}=\frac{3p\times 2q}{p+q}$

$$\Rightarrow \frac{x}{3p}=\frac{2q}{p+q}$$

$$\Rightarrow \frac{x+3p}{x-3p}=\frac{2q+p+q}{2q-p-q}$$

(By componendo and dividendo)

$$\Rightarrow \frac{x+3p}{x-3p}=\frac{3q+p}{q-p} \quad .....(i)$$

Again, $x=\frac{6pq}{p+q}=\frac{2p\times 3q}{p+q}$

$$\Rightarrow \frac{x}{3q}=\frac{2p}{p+q}$$

$$\Rightarrow \frac{x+3q}{x-3q}=\frac{2p+p+q}{2p-p-q}$$

(By componendo and dividendo)

$$\Rightarrow \frac{x+3q}{x-3q} = \frac{3p+q}{p-q} \quad ....(ii)$$

$$\therefore \frac{x+3p}{x-3p}+\frac{x+3q}{x-3q}=\frac{3q+p}{q-p}+\frac{3p+q}{p-q}$$

$$=\frac{3q+p}{q-p}-\frac{3p+q}{q-p}$$

$$=\frac{3q+p-3p-q}{q-p}=\frac{2q-2p}{q-p}$$

$$=\frac{2(q-p)}{q-p}=2$$

252. (2) $x+\frac{1}{9x} = 4$

On multiplying by 3,

$$3x+\frac{1}{3x} = 12$$

On squaring both sides,

$\left(3x+\frac{1}{3x}\right)^2 = (12)^2$

$\Rightarrow 9x^2 + \frac{1}{9x^2} + 2 \times 3x \times \frac{1}{3x}$
$= 144$

$\Rightarrow 9x^2 + \frac{1}{9x^2} = 144 - 2 = 142$

253. (4) $x\left(3-\frac{2}{x}\right) = \frac{3}{x}$

$\Rightarrow 3x - 2 = \frac{3}{x}$

$\Rightarrow 3x - \frac{3}{x} = 2$

On dividing by 3,

$x - \frac{1}{x} = \frac{2}{3}$

On squaring both sides,

$x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$

$\Rightarrow x^2 + \frac{1}{x^2} = 2 + \frac{4}{9}$

$= 2\frac{4}{9}$

254. (2) $x^2 + \frac{1}{x^2} = 2$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 + 2x \times \frac{1}{x} = 2$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = 2 - 2 = 0$

$\Rightarrow x - \frac{1}{x} = 0$

255. (1) $9x^2 + 16y^2 = 60$ .... (i)
and $3x + 4y = 6$
On squaring,
$9x^2 + 16y^2 + 2 \times 3x \times 4y = 36$
$\Rightarrow 60 + 24xy = 36$
$\Rightarrow 24xy = 36 - 60 = -24$

$\Rightarrow xy = -\frac{24}{24} - 1$

256. (3) $p^2 + q^2 = 7\ pq$

$\Rightarrow \frac{p^2+q^2}{pq} = 7$

$\Rightarrow \frac{p^2}{pq} + \frac{q^2}{pq} = 7$

$\Rightarrow \frac{p}{q} + \frac{q}{p} = 7$

257. (*) $x^2 + 3x + 3$
$= x^2 + 2x + 1 + x + 2$
$= (x + 1)^2 + x + 2$
$= (99 + 1)^2 + 99 + 2$
$= (100)^2 + 101$
$= 10000 + 101 = 10101$
$\therefore 2(x^2 + 3x + 3) = 2 \times 10101$
$= 20202$

258. (2) $\frac{2p}{p^2-2p+1} = \frac{1}{4}$

$\Rightarrow \frac{p^2-2p+1}{2p} = 4$

$\Rightarrow \frac{p^2}{p} - \frac{2p}{p} + \frac{1}{p} = 8$

$\Rightarrow p + \frac{1}{p} = 8 + 2 = 10$

259. (2) $a - b = 3$
On squaring both sides,
$(a - b)^2 = 9$
$\Rightarrow a^2 + b^2 - 2ab = 9$
$\Rightarrow 25 - 2ab = 9$
$\Rightarrow 2ab = 25 - 9 = 16$

$\Rightarrow ab = \frac{16}{2} = 8$

260. (3) $a + \frac{1}{a} = 1$

$\Rightarrow a^2 + 1 = a \Rightarrow a^2 - a + 1 = 0$

$\therefore \frac{a^2-a+1}{a^2+a+1} = \frac{0}{a^2+a+1} = 0$

261. (4) $x - \frac{1}{x} = 2$

On squaring both sides,

$x^2 + \frac{1}{x^2} - 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 6$

262. (2) $a + b = 2c$
$\Rightarrow a - c = c - b$

$\therefore \frac{a}{a-c} + \frac{c}{b-c}$

$= \frac{a}{a-c} - \frac{c}{a-c}$

$= \frac{a-c}{a-c} = 1$

263. (2) $x + \frac{1}{x} = 5$ (Given)

$\therefore \frac{x}{1+x+x^2} = \frac{x}{x\left(\frac{1}{x}+1+x\right)}$

$= \frac{1}{x+\frac{1}{x}+1} = \frac{1}{5+1} = \frac{1}{6}$

264. (3) $\frac{a^2}{b+c} = \frac{b^2}{c+a} = \frac{c^2}{a+b} = 1$

$\Rightarrow \frac{a^2}{b+c} = 1 \Rightarrow a^2 = b + c$
$\Rightarrow a^2 + a = a + b + c$
$\Rightarrow a\,(a + 1) = a + b + c$

$\Rightarrow a + 1 = \frac{a+b+c}{a}$

$\Rightarrow \frac{1}{a+1} = \frac{a}{a+b+c}$

Similarly,

$\frac{b^2}{c+a} = 1$

$\Rightarrow \frac{1}{b+1} = \frac{b}{a+b+c}$

and, $\frac{c^2}{a+b} = 1$

$\Rightarrow \frac{1}{c+1} = \frac{c}{a+b+c}$

$\therefore \frac{2}{1+a} + \frac{2}{1+b} + \frac{2}{1+c}$

$= 2\left(\frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c}\right)$

$= 2\left(\frac{a+b+c}{a+b+c}\right) = 2$

265. (3) $5x + \frac{1}{x} = 10$

On dividing by 5,

$x + \frac{1}{5x} = 2$

On squaring both sides,

$\left(x+\frac{1}{5x}\right)^2 = 4$

$\Rightarrow x^2 + \frac{1}{25x^2} + 2x \times \frac{1}{5x} = 4$

$\Rightarrow x^2 + \frac{1}{25x^2} = 4 - \frac{2}{5}$

$= \frac{20-2}{5} = \frac{18}{5} = 3\frac{3}{5}$

266. (3) $4r = h + \sqrt{r^2+h^2}$

$\Rightarrow 4r - h = \sqrt{r^2+h^2}$

On squaring both sides,

$(4r - h)^2 = \left(\sqrt{r^2 + h^2}\right)^2$

$\Rightarrow 16r^2 + h^2 - 8rh = r^2 + h^2$
$\Rightarrow 16r^2 - r^2 = 8\,rh \Rightarrow 15r^2 = 8rh$

$\Rightarrow 15r = 8h \Rightarrow \frac{r}{h} = \frac{8}{15}$

267. (1) $p\,(p^2 + 3p + 3)$
$= p^3 + 3p^2 + 3p$
$= p^3 + 3p^2 + 3p + 1 - 1$
$= (p + 1)^3 - 1$
$= (99 + 1)^3 - 1$
$= (100)^3 - 1 = 1000000 - 1$
$= 999999$

268. (3) $\frac{x}{a+b} + 1 = \frac{x}{a-b} + \frac{a-b}{a+b}$

$\Rightarrow \frac{x}{a+b} - \frac{a-b}{a+b} = \frac{x}{a-b} - 1$

$\Rightarrow \frac{x-a+b}{a+b} = \frac{x-a+b}{a-b}$

$\Rightarrow (x - a + b)\left(\frac{1}{a+b} - \frac{1}{a-b}\right)$
$= 0$
$\Rightarrow x - a + b = 0$
$\Rightarrow x = a - b$

269. (2) $x^2 + y^2 = 29;$
$xy = 10$
$\therefore\ (x + y)^2 = x^2 + y^2 + 2xy$
$= 29 + 2 \times 10 = 49$
$\Rightarrow x + y = \pm 7$
Again, $(x - y)^2 = x^2 + y^2 - 2xy$
$= 29 - 2 \times 10 = 9$
$\therefore\ x - y = \pm 3$

$\therefore\ \frac{x+y}{x-y} = \frac{\pm 7}{\pm 3} = \frac{7}{3}$

270. (2) $(a - b)^2 = a^2 - 2ab + b^2$
$\therefore\ 4x^2 - 12x + k = (2x)^2 - 2 \times 2x \times 3 + k$
$\therefore\ k = (3)^2 = 9$

271. (2)

$\frac{1}{(p-n)(n-q)} + \frac{1}{(n-q)(q-p)} + \frac{1}{(q-p)(p-n)}$

$= \frac{(q-p)+(p-n)+(n-q)}{(p-n)\,(n-q)\,(q-p)}$

$= \frac{0}{(p-n)\,(n-q)\,(q-p)} = 0$

272. (1) $\frac{a^2}{b+c} = \frac{b^2}{c+a} = \frac{c^2}{a+b} = 1$

$\Rightarrow \frac{a^2}{b+c} = 1$
$\Rightarrow a^2 = b + c$
$\Rightarrow a^2 + a = a + b + c$
$\Rightarrow a\,(a + 1) = a + b + c$

$\Rightarrow \frac{1}{a+1} = \frac{a}{a+b+c}$

Similarly,

$\frac{b^2}{c+a} = 1 \Rightarrow b^2 = c + a$
$\Rightarrow b^2 + b = a + b + c$
$\Rightarrow b\,(b + 1) = a + b + c$

$\Rightarrow \frac{1}{b+1} = \frac{b}{a+b+c}$

and $\frac{c^2}{a+b} = 1 \Rightarrow c^2 = a + b$
$\Rightarrow c^2 + c = a + b + c$
$\Rightarrow c\,(c + 1) = a + b + c$

$\Rightarrow \frac{1}{c+1} = \frac{c}{a+b+c}$

$\therefore\ \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$

$= \frac{a}{a+b+c} + \frac{b}{a+b+c} + \frac{c}{a+b+c}$

$= \frac{a+b+c}{a+b+c} = 1$

273. (3) $a^2 + 1 = 9a$

$\Rightarrow \frac{a^2+1}{a} = 9$

$\Rightarrow a + \frac{1}{a} = 9$

On squaring both sides,

$a^2 + \frac{1}{a^2} + 2 = 81$

$\Rightarrow a^2 + \frac{1}{a^2} = 81 - 2 = 79$

274. (2) Expression $= p\,(p^2 + 3p + 3)$
$= p^3 + 3p^2 + 3p + 1 - 1$
$= (p + 1)^3 - 1$
$= (99 + 1)^3 - 1 = (100)^3 - 1$
$= 1000000 - 1 = 999999$

275. (1) $x + \frac{1}{x} = c + \frac{1}{c}$

$\Rightarrow x - c = \frac{1}{c} - \frac{1}{x}$

$\Rightarrow x - c = \frac{x-c}{xc}$

$\Rightarrow (x - c) - \frac{x-c}{xc} = 0$

$\Rightarrow (x - c)\left(1 - \frac{1}{xc}\right) = 0$

$\Rightarrow x - c = 0 \Rightarrow x = c$

or, $1 - \frac{1}{xc} = 0$

$\Rightarrow \frac{1}{xc} = 1 \Rightarrow xc = 1$

$\Rightarrow x = \frac{1}{c}$

$\Rightarrow x = c,\ \frac{1}{c}$

276. (1) $x^2 + y^2 + 6x + 5 = 4x - 4y$
$\Rightarrow x^2 + y^2 + 6x - 4x + 4y + 5 = 0$
$\Rightarrow x^2 + 2x + 1 + y^2 + 4y + 4 = 0$
$\Rightarrow (x + 1)^2 + (y + 2)^2 = 0$
$\therefore\ x + 1 = 0 \Rightarrow x = -1$
$y + 2 = 0 \Rightarrow y = -2$
$\therefore\ x - y = -1 + 2 = 1$

277. (2) $x - \frac{1}{3x} = \frac{1}{3}$

$\therefore\ 3\left(x - \frac{1}{3x}\right)$

$= 3 \times \frac{1}{3} = 1$

278. (1)

$\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q} = k$ (let)

$\Rightarrow\ a = k\,(q - r);$
$b = k\,(r - p);$
$c = k\,(p - q)$
$\therefore\ pa + qb + rc$
$= k\,[p\,(q - r) + q\,(r - p) + r\,(p - q)]$
$= k\,(pq - pr + qr - pq + rp - qr)$
$= k \times 0 = 0$

279. (2) $\frac{3a+4b}{3c+4d} = \frac{3a-4b}{3c-4d}$

$\Rightarrow \frac{3a+4b}{3a-4b} = \frac{3c+4d}{3c-4d}$

By componendo and dividendo,

$\frac{3a+4b+3a-4b}{3a+4b-3a+4b}$

$= \frac{3c+4d+3c-4d}{3c+4d-3c+4d}$

$\Rightarrow \frac{6a}{8b} = \frac{6c}{8d}$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

$\Rightarrow ad = bc$

280. (2) $x + \frac{1}{x} = 2$

On squaring both sides,

$\left(x + \frac{1}{x}\right)^2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow\ x^2+\frac{1}{x^2}=4-2=2$

281. (4) $a + b = 17$
$a - b = 9$
$\therefore\ (a + b)^2 + (a - b)^2 = 17^2 + 9^2$
$\Rightarrow\ 2\,(a^2 + b^2) = 289 + 81 = 370$
$\Rightarrow\ 4\,(a^2 + b^2) = 2 \times 370 = 740$

282. (3) $x + y = \sqrt{3}$
$x - y = \sqrt{2}$
$\therefore\ (x + y)^2 + (x - y)^2 = 3 + 2$
$\Rightarrow\ 2\,(x^2 + y^2) = 5$ ...(i)
Again,
$(x + y)^2 - (x - y)^2 = 3 - 2$
$\Rightarrow\ 4xy = 1$ ...(ii)
$\therefore\ 8xy\,(x^2 + y^2) = 5 \times 1 = 5$

283. (3) $a^2 + 1 = a$
$\Rightarrow\ a^2 - a + 1 = 0$
$\Rightarrow\ (a + 1)\,(a^2 - a + 1) = 0$
$\Rightarrow\ a^3 + 1 = 0$
$\Rightarrow\ a^3 = -1$

284. (1) $x + 3y = -3x + y$
$\Rightarrow\ x + 3x = -3y + y$
$\Rightarrow\ 4x = -2y$
$\Rightarrow\ 2x = -y$
$\Rightarrow\ \frac{x}{y}=-\frac{1}{2}$
$\therefore\ \frac{x^2}{y^2}=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$
$\therefore\ \frac{x^2}{2y^2}=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

285. (3) $(a + b - 6)^2 + a^2 + b^2 + 1 + 2b = 2ab + 2a$
$\Rightarrow (a + b - 6)^2 + a^2 + b^2 + 1 + 2b - 2ab - 2a = 0$
$\Rightarrow (a + b - 6)^2 + (a)^2 + (-b)^2 + (-1)^2 + 2a\,(-b) + 2\,(-b)\,(-1) + 2\,(a)\,(-1) = 0$
$\Rightarrow (a + b - 6)^2 + (a - b - 1)^2 = 0$
$\Rightarrow a + b - 6 = 0$ and $a - b - 1 = 0$
$\Rightarrow a + b = 6$ and $a - b = 1$
On adding these two equations,
$a + b + a - b = 6 + 1$
$\Rightarrow 2a = 7$
$\Rightarrow a = \frac{7}{2} = 3.5$

286. (2) $\left(a+\frac{1}{a}\right)^2=3$
$\Rightarrow\ a^2+\frac{1}{a^2}+2=3$
$\Rightarrow\ a^2+\frac{1}{a^2}=3-2=1$

287. (3) $\left\{\frac{1}{2}(a-b)\right\}^2+ab=p(a+b)^2$
$\Rightarrow\ \frac{1}{4}(a^2+b^2-2ab)+ab$
$= p\,(a + b)^2$
$\Rightarrow\ \frac{1}{4}(a^2+b^2-2ab+4ab)$
$= p\,(a + b)^2$
$\Rightarrow\ \frac{1}{4}(a+b)^2=p(a+b)^2$
$\Rightarrow\ p=\frac{1}{4}$

288. (4) For $y = ax^2 + bx + c$
Maximum value $= c-\frac{b^2}{4a}$
Here, $c = 5,\ b = 20,\ a = -4$
$\therefore$ Maximum value
$= 5-\frac{20\times20}{4\times-4}=5+5\times5=30$

289. (2) $x = at^2$
$y = 2at$
$\Rightarrow\ y^2 = 4a^2t^2$
$= 4a.at^2 = 4ax$

290. (2) $a+\frac{1}{b}=1$
$\Rightarrow\ a=1-\frac{1}{b}=\frac{b-1}{b}$
$\Rightarrow\ \frac{1}{a}=\frac{b}{b-1}$ ..... (i)
Again, $b+\frac{1}{c}=1$
$\Rightarrow\ \frac{1}{c}=1-b$
$\Rightarrow\ c=\frac{1}{1-b}$ ...(ii)
$\therefore\ c+\frac{1}{a}=\frac{1}{1-b}+\frac{b}{b-1}$
$=\frac{1}{1-b}-\frac{b}{1-b}$
$=\frac{1-b}{1-b}=1$

291. (1) $a-2+\frac{1}{a+2}=-1$
$\Rightarrow\ (a-2+4)+\frac{1}{a+2}=4-1$
$\Rightarrow\ (a+2)+\frac{1}{(a+2)}=3$
On squaring both sides,
$(a+2)^2+\frac{1}{(a+2)^2}+2\times(a+2)\times\frac{1}{(a+2)}=9$
$\Rightarrow\ (a+2)^2+\frac{1}{(a+2)^2}$
$= 9 - 2 = 7$

292. (3) $a^2 = b + c$
$\Rightarrow a^2 + a = a + b + c$
$\Rightarrow a\,(a + 1) = a + b + c$
$\Rightarrow\ \frac{1}{a+1}=\frac{a}{a+b+c}$
Again,
$b^2 = c + a$
$\Rightarrow\ b^2 + b = a + b + c$
$\Rightarrow\ b\,(b + 1) = a + b + c$
$\Rightarrow\ \frac{1}{b+1}=\frac{b}{a+b+c}$
$c^2 = a + b$
$\Rightarrow\ c^2 + c = a + b + c$
$\Rightarrow\ c\,(c + 1) = a + b + c$
$\Rightarrow\ \frac{1}{c+1}=\frac{c}{a+b+c}$
$\therefore\ 3\left(\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}\right)$
$=3\left(\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}\right)$
$=3\left(\frac{a+b+c}{a+b+c}\right)=3$

293. (3) Given, $x^2 + 5x + 6 = 0$
$\therefore$ Expression $=\frac{2x}{x^2-7x+6}$
$=\frac{2x}{x^2+5x+6-12x}=\frac{2}{-12}$
$=\frac{-1}{6}$

294. (1) $a + b = 5$
$a - b = 3$
$\because\ (a + b)^2 + (a - b)^2$
$= 2\,(a^2 + b^2)$
$\Rightarrow\ 2\,(a^2 + b^2) = 5^2 + 3^2$
$= 25 + 9 = 34$
$\Rightarrow\ a^2 + b^2 = \frac{34}{2} = 17$

295. (3) It is given, $x+\frac{1}{x}=5$
Expression $=\frac{6x}{x^2+x+1}$
$=\frac{6x}{x\left(x+1+\frac{1}{x}\right)}=\frac{6}{\left(x+\frac{1}{x}+1\right)}$
$=\frac{6}{5+1}=\frac{6}{6}=1$

296. (2) $\frac{3}{(x+2)(2x+1)}$

$= \frac{a}{2x+1} + \frac{b}{x+2}$

$\Rightarrow \frac{3}{(x+2)(2x+1)}$

$= \frac{a(x+2)+b(2x+1)}{(2x+1)(x+2)}$

$\Rightarrow 3 = ax + 2a + 2bx + b$

$\Rightarrow 3 = ax + 2bx + 2a + b$

$\Rightarrow 3 = x(a + 2b) + (2a + b)$

On comparing the respective co–efficients,

$a + 2b = 0$

$\Rightarrow a = -2b$ ..... (i)

and, $2a + b = 3$

$2(-2b) + b = 3$

$\Rightarrow -4b + b = 3$

$\Rightarrow -3b = 3 \Rightarrow b = \frac{-3}{3} = -1$

297. (2) $a + \frac{1}{a} = 1$

$\Rightarrow a = 1 - \frac{1}{b} = \frac{b-1}{b}$

Again,

$b + \frac{1}{c} = 1$

$\Rightarrow b = 1 - \frac{1}{c} = \frac{c-1}{c}$

$\therefore a = \frac{b-1}{b} = \frac{\frac{c-1}{c} - 1}{\frac{c-1}{c}}$

$= \frac{c-1-c}{c-1} = \frac{-1}{c-1}$

$\therefore abc = \frac{-1}{c-1} \times \frac{c-1}{c} \times c = -1$

298. (1) $2x - \frac{1}{2x} = 5$

On dividing by 2,

$x - \frac{1}{4x} = \frac{5}{2}$

On squaring both sides

$\left(x - \frac{1}{4x}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$\Rightarrow x^2 + \frac{1}{16x^2} - 2 \times x \times \frac{1}{4x} = \frac{25}{4}$

$\Rightarrow x^2 + \frac{1}{16x^2} = \frac{25}{4} + \frac{1}{2}$

$= \frac{25+2}{4} = \frac{27}{4}$

$\Rightarrow x^2 + \frac{1}{16x^2} - 2$

$= \frac{27}{4} - 2 = \frac{27-8}{4} = \frac{19}{4}$

299. (4) $a(x + y) = b(x - y)$

$\Rightarrow ax - bx = -by - ay$

$\Rightarrow bx - ax = ay + by$

$\Rightarrow x(b - a) = y(a + b)$

$\Rightarrow \frac{x}{a+b} = \frac{y}{b-a}$

$= \frac{x^2+y^2}{(a+b)^2+(b-a)^2} = \frac{x^2+y^2}{2\left(a^2+b^2\right)}$

$\therefore 2(x^2 + y^2) = 4(a^2 + b^2)$

300. (3) $x + \frac{1}{x} = 6$

On squaring both sides,

$\left(x + \frac{1}{x}\right)^2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} = 36 - 2 = 34$

301. (3) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

On dividing by x,

$\frac{x^2+1}{x} = \frac{3x}{x}$

$\Rightarrow x + \frac{1}{x} = 3$

302. (4) $\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$

$\Rightarrow \frac{2}{a} + 1 + \frac{2}{b} + 1 + \frac{2}{c} + 1 = 4$

$\Rightarrow \frac{2}{a} + \frac{2}{b} + \frac{2}{c} = 4 - 3 = 1$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$

$\Rightarrow \frac{bc+ca+ab}{abc} = \frac{1}{2}$

303. (3) It is given, $x + \frac{1}{x} = 5$

Expression $= \frac{5x}{x^2+5x+1}$

$= \frac{5x}{x\left(x+5+\frac{1}{x}\right)}$

$= \frac{5}{\left(x+\frac{1}{x}\right)+5}$

$= \frac{5}{5+5} = \frac{5}{10} = \frac{1}{2}$

304. (3) $p^2 + \frac{1}{p^2} = 47$

$\Rightarrow \left(p + \frac{1}{p}\right)^2 - 2 = 47$

$\Rightarrow \left(p + \frac{1}{p}\right)^2 = 47 + 2 = 49$

$\Rightarrow p + \frac{1}{p} = \sqrt{49} = 7$

305. (4) $\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c}$

$= \frac{1}{2}$

$\Rightarrow \frac{2a}{1-2a} + \frac{2b}{1-2b} + \frac{2c}{1-2c}$

$= \frac{2}{2} = 1$

$\Rightarrow \left(\frac{2a}{1-2a}+1\right) + \left(\frac{2b}{1-2b}+1\right) + \left(\frac{2c}{1-2c}+1\right)$

$= 4$

$\Rightarrow \frac{2a+1-2a}{1-2a} + \frac{2b+1-2b}{1-2b}$

$+ \frac{2c+1-2c}{1-2c} = 4$

$\Rightarrow \frac{1}{1-2a} + \frac{1}{1-2b} + \frac{1}{1-2c}$

$= 4$

306. (2) $4x + \frac{1}{x} = 5$

Expression

$= \frac{5x}{4x^2+1+10x}$

$= \frac{5x}{x(4x+\frac{1}{x}+10)}$

$= \frac{5}{5+10} = \frac{5}{15} = \frac{1}{3}$

307. (3) We know that,

$4ab = (a + b)^2 - (a - b)^2$

$\Rightarrow 4ab = 100 - (4)^2 = 100 - 16$

$\Rightarrow 4ab = 84$

$\Rightarrow ab = \frac{84}{4} = 21$

308. (2) $\frac{x^2+3x+1}{x^2-3x+1} = \frac{1}{2}$

$\Rightarrow 2x^2 + 6x + 2 = x^2 - 3x + 1$

$\Rightarrow 2x^2 - x^2 + 2 - 1 = -6x - 3x$

$\Rightarrow x^2 + 1 = -9x$

$\Rightarrow \frac{x^2+1}{x} = -9$

$\Rightarrow x + \frac{1}{x} = -9$

309. (3) Required answer

$= (18x - 18y) - 8(3x - 4y)$

$= 18x - 18y - 24x + 32y$

$= 14y - 6x$

310. (3) $4(2x + 3) > 5 - x$

$\Rightarrow 8x + 12 > 5 - x$

$\Rightarrow 8x + x > 5 - 12$

$\Rightarrow 9x > -7$

$\Rightarrow x > \frac{-7}{9}$

Again,

$5x - 3(2x - 7) > 3x - 1$

$\Rightarrow 5x - 6x + 21 > 3x - 1$

$\Rightarrow -x + 21 > 3x - 1$

$\Rightarrow -x - 3x > -21 - 1$

$\Rightarrow -4x > -22$

$\Rightarrow 4x < 22$

$\Rightarrow x < \frac{22}{4}$ i.e., $x < 5.5$

$\therefore$ Required value of $x = 5$

311. (1) $5x - 40 = 3x$

$\Rightarrow 5x - 3x = 40$

$\Rightarrow 2x = 40 \Rightarrow x = \frac{40}{2} = 20$

$\therefore 2x - 11 = 2 \times 20 - 11$

$= 40 - 11 = 29$

312. (2) The roots of quadratic equation $ax^2 + bx + c = 0$ will be equal if $b^2 - 4ac = 0$

Option (1),

$3x^2 - 6x + 2 = 0$

$a = 3, b = -6, c = 2$

$\therefore b^2 - 4ac = (-6)^2 - 4 \times 3 \times 2$

$= 36 - 24 = 12 \neq 0$

Option (2),

$3x^2 - 6x + 3 = 0$

$a = 3, b = -6, c = 3$

$\therefore b^2 - 4ac = (-6)^2 - 4 \times 3 \times 3$

$= 36 - 36 = 0$

Option (3),

$x^2 - 8x + 8 = 0$

$\therefore b^2 - 4ac = (-8)^2 - 4 \times 8$

$= 64 - 32 = 32 \neq 0$

Option (4),

$4x^2 - 8x + 2 = 0$

$\therefore b^2 - 4ac = (-8)^2 - 4 \times 4 \times 2$

$= 64 - 32$

$= 32 \neq 0$

313. (3) $2x - 3(4 - 2x) < 4x - 5 < 4x + \frac{2x}{3}$

$\Rightarrow 2x - 12 + 6x < 4x - 5 < \frac{12x + 2x}{3}$

$\Rightarrow 8x - 12 < 4x - 5 < \frac{14x}{3}$

$\Rightarrow 24x - 36 < 12x - 15 < 14x$

When $x = 0$,

$-36 < -15 < 0$

314. (1) $a - b = 11$ and $ab = 24$

$\therefore (a - b)^2 = 11^2$

$\Rightarrow a^2 + b^2 - 2ab = 121$

$\Rightarrow a^2 + b^2 - 2 \times 24 = 121$

$\Rightarrow a^2 + b^2 = 121 + 48 = 169$

315. (2) $(x + (3)^2 + (x - 1)^2$

$= x^2 + 2 \times x \times 3 + 3^2 + x^2 - 2 \times x \times 1 + 1^2$

$= x^2 + 6x + 9 + x^2 - 2x + 1$

$= 2x^2 + 4x + 10 = 2(x^2 + 2x + 5)$

316. (3) $a + \frac{1}{b} = 1$

$\Rightarrow a = 1 - \frac{1}{b} = \frac{b-1}{b}$

$\Rightarrow \frac{1}{a} = \frac{b}{b-1}$

Again, $b + \frac{1}{c} = 1$

$\Rightarrow \frac{1}{c} = 1 - b$

$\Rightarrow c = \frac{1}{1-b}$

$\therefore c + \frac{1}{a} = \frac{1}{1-b} + \frac{b}{b-1}$

$= \frac{1}{1-b} - \frac{b}{1-b} = \frac{1-b}{1-b} = 1$

317. (1) a + b + c + d = 4

$\Rightarrow 4 - a - b - c - d = 0$ ...(i)

Expression

$= \frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)}$

$+ \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}$

$= \frac{(1-d)+(1-a)+(1-b)+(1-c)}{(1-a)(1-b)(1-c)(1-d)}$

$= \frac{4-a-b-c-d}{(1-a)(1-b)(1-c)(1-d)} = 0$

318. (1) $a = \frac{1}{a-5}$

$\Rightarrow a^2 - 5a = 1$

$\Rightarrow a^2 - 5a - 1 = 0$

$\therefore a = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times (-1)}}{2}$

$\left(\text{If } ax^2 + bx + c = 0, \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$

$= \frac{5 \pm \sqrt{25+4}}{2}$

$= \frac{5 \pm \sqrt{29}}{2}$

If $a = \frac{5+\sqrt{29}}{2}$, then

$\frac{1}{a} = \frac{2}{5+\sqrt{29}}$

$= \frac{2}{5+\sqrt{29}} \times \frac{5-\sqrt{29}}{5-\sqrt{29}}$

$= \frac{2(5-\sqrt{29})}{25-29} = \frac{5-\sqrt{29}}{-2}$

$\therefore a + \frac{1}{a} = \frac{5+\sqrt{29}}{2} - \frac{5-\sqrt{29}}{2}$

$= \frac{5+\sqrt{29}-5+\sqrt{29}}{2} = \sqrt{29}$

319. (3) $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$

$= \pm 1$ (let)

$\Rightarrow a + \frac{1}{b} = 1$

$\Rightarrow ab + 1 = b \Rightarrow ab = b - 1$

$b + \frac{1}{c} = 1 \Rightarrow \frac{1}{c} = 1 - b$

$c = \frac{1}{1-b}$

$\therefore abc = \frac{b-1}{1-b} = -1$

Again, $a + \frac{1}{b} = -1$

$\Rightarrow ab + 1 = -b \Rightarrow ab = -b - 1$

$b + \frac{1}{c} = -1 \Rightarrow \frac{1}{c} = -1 - b$

$c = \frac{1}{-1-b}$

$\therefore abc = 1$

$\therefore abc = \pm 1$

320. (1) $ax + by - 1 = 0$

$bx + ay - \frac{2ab}{a^2+b^2} = 0$

By cross-multiplication.

$\frac{x}{b \times \frac{-2ab}{a^2+b^2} - a \times -1}$

$= \frac{-y}{a \times \frac{-2ab}{a^2+b^2} - b \times -1} = \frac{1}{a \times a - b \times b}$

TYPE-II

1. (3) Using Rule 8,

$x = \sqrt{3}+\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}}$

$= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$

$\therefore x+\frac{1}{x} = \sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}$

$= 2\sqrt{3}$

Now,

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= (2\sqrt{3})^3 - 3(2\sqrt{3})$

$= 24\sqrt{3}-6\sqrt{3} = 18\sqrt{3}$

2. (3) Using Rule 8,
Given, $x + y = 7$
Now, $x^3 + y^3 + 21xy$
$= (x + y)^3 - 3xy\,(x + y) + 21xy$
$= (7)^3 - 3xy\,(7) + 21xy$
$= 343 - 21xy + 21xy = 343$

3. (3) Using Rule 8,

$x^{\frac{1}{3}}+y^{\frac{1}{3}} = z^{\frac{1}{3}}$ ......(i)

Cubing both sides,

$\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right)^3 = z$

$\Rightarrow x+y+3\,x^{\frac{1}{3}}.y^{\frac{1}{3}}\left(x^{\frac{1}{3}}+y^{\frac{1}{3}}\right) = z$

[$\because$ $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$]

$\Rightarrow x + y - z$

$= -3.\ x^{\frac{1}{3}}.y^{\frac{1}{3}}.z^{\frac{1}{3}}$ ......(ii)

[From equation (i)]

$\therefore$ $(x + y - z)^3 + 27\,xyz$

$= \left(-3x^{\frac{1}{3}}.y^{\frac{1}{3}}.z^{\frac{1}{3}}\right)^3 + 27\,xyz$

[From equation (ii)]

$= -27xyz + 27xyz = 0$

4. (1) Using Rule 1,

$\left(2b+\frac{1}{b}\right)^2$

$= 4b^2+\frac{1}{b^2}+2\times 2b\times\frac{1}{b} = 2+4 = 6$

$\Rightarrow 2b+\frac{1}{b} = \sqrt{6}$

$\therefore 8b^3+\frac{1}{b^3}$

$= \left(2b+\frac{1}{b}\right)^3 - 3\times 2b\times\frac{1}{b}\left(2b+\frac{1}{b}\right)$

$= \left(\sqrt{6}\right)^3 - 6\left(\sqrt{6}\right)$

$= 6\sqrt{6}-6\sqrt{6} = 0$

5. (2) Using Rule 8,

$2p + \frac{1}{p} = 4$

$\Rightarrow p + \frac{1}{2p} = 2$

$\therefore \left(p+\frac{1}{2p}\right)^3$

$= p^3 + \frac{1}{8p^3} + 3.p.\frac{1}{2p}\left(p+\frac{1}{2p}\right)$

$\Rightarrow 8 = p^3 + \frac{1}{8p^3} + \frac{3}{2}\times 2$

$\Rightarrow p^3 + \frac{1}{8p^3} = 8 - 3 = 5$

6. (1) $a^4 + b^4 - a^2b^2 = 0$ ..... (i)
We know, $a^6+b^6 = (a^2)^3 + (b^2)^3$
$= (a^2+b^2)\,(a^4-a^2b^2 + b^4)$
$= (a^2+b^2) \times 0 = 0$

[From equation (i)]

7. (1) Using Rule 8,

$x+\frac{1}{x} = \sqrt{3}$

Cubing both sides,

$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = \left(\sqrt{3}\right)^3$

$\Rightarrow x^3+\frac{1}{x^3}+3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3} = 0$

Now, $x^{18} + x^{12} + x^6 + 1$
$= x^{12}\,(x^6 + 1) + 1\,(x^6 + 1)$
$= (x^{12} + 1)\,(x^6 + 1)$

$= (x^{12} + 1)\,.\,x^3\left(x^3+\frac{1}{x^3}\right) = 0$

8. (2) $x+\frac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x \Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0$
$\Rightarrow x = 1$

$\therefore x^2+\frac{1}{x^3} = 1+1 = 2$

Aliter :
Using Rule 16,

Here, $x+\frac{1}{x} = 2$

$\Rightarrow x+\frac{1}{x^3} = 2$

9. (1) $\frac{a}{b}+\frac{b}{a} = 1$

$\Rightarrow \frac{a^2+b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$
$\Rightarrow a^2 + b^2 - ab = 0$
$\therefore$ $a^3 + b^3$
$= (a + b)\,(a^2 - ab + b^2) = 0$

10. (3) $\dfrac{x^3+\frac{1}{x}}{x^2-x+1} = \dfrac{x^2+\frac{1}{x^2}}{x-1+\frac{1}{x}}$

$= \dfrac{\left(x+\frac{1}{x}\right)^2-2}{\left(x+\frac{1}{x}\right)-1} = \frac{9-2}{3-1} = \frac{7}{2}$

11. (1) $a+\frac{1}{a}+1 = 0$

$\Rightarrow a^2 + a + 1 = 0$
$\Rightarrow a^4 - a = a\,(a^3 - 1)$
$= a\,(a - 1)\,(a^2 + a + 1) = 0$

12. (3) $x^4 + y^4 - 2x^2y^2$
$= (x^2 - y^2)^2$

$= \left[(x+y)\,(x-\ y)\right]^2$

$= \left[\left(a+\frac{1}{a}+a-\frac{1}{a}\right)\left(a+\frac{1}{a}-a+\frac{1}{a}\right)\right]^2$

$= \left(2a\times\frac{2}{a}\right)^2 = 16$

13. (3) Using Rule 8,

$x + \frac{1}{2x} = 2$

$\Rightarrow 2x + \frac{2}{2x} = 4$

$\Rightarrow 2x + \frac{1}{x} = 4$

On cubing both sides,

$8x^3 + \frac{1}{x^3} + 3.2x.\frac{1}{x}\left(2x + \frac{1}{x}\right)$

$= 64$

$\Rightarrow 8x^3 + \frac{1}{x^3} + 6 \times 4 = 64$

$\Rightarrow 8x^3 + \frac{1}{x^3} = 64 - 24 = 40$

14. (3) $P(x) = ax^3 + 3x^2 - 8x + b$

$[\because P(x)$ is div. by $(x+2)$ & $(x-2)]$

$\therefore P(-2) = -8a + 12 + 16 + b = 0$

$\Rightarrow -8a + b + 28 = 0$ ...(i)

$\Rightarrow P(2) = 8a + 12 - 16 + b = 2$

$\Rightarrow 8a + b - 4 = 0$ ...(ii)

By equation (i) + (ii)

$2b + 24 = 0$

$\Rightarrow b = -\frac{24}{2} = -12$

From equation (i),

$-8a - 12 + 28 = 0$

$\Rightarrow -8a = -16$

$\Rightarrow a = 2$

15. (2) Using Rule 8,

$x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow x + \frac{1}{x} = 3$

$\therefore x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= 27 - 3 \times 3 = 18$

16. (1) Using Rule 8,

$x + \frac{1}{4x} = \frac{3}{2}$

Multiplying both sides by 2

$\Rightarrow 2x + \frac{1}{2x} = 3$

Cubing both sides,

$8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x}$

$\times\left(2x + \frac{1}{2x}\right) = 27$

$\Rightarrow 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$

$\Rightarrow 8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$

17. (1) $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$

$\Rightarrow (x+y)^2 = xy$

$\Rightarrow x^2 + 2xy + y^2 = xy$

$\Rightarrow x^2 + xy + y^2 = 0$

$\therefore x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$= 0$

18. (3)

$\frac{x}{a} = b - c;\ \frac{y}{b} = c - a;\ \frac{z}{c} = a - b$

Again, $b - c + c - a + a - b = 0$

$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

$= (b-c)^3 + (c-a)^3 + (a-b)^3$

$= 3(b-c)(c-a)(a-b)$

$= \frac{3xyz}{abc}$

19. (3) $xy(x+y) = 1$

$\Rightarrow x + y = \frac{1}{xy}$

Cubing both sides,

$x^3 + y^3 + 3xy(x+y) = \frac{1}{x^3y^3}$

$\Rightarrow x^3 + y^3 + 3xy \times \frac{1}{xy} = \frac{1}{x^3y^3}$

$\Rightarrow \frac{1}{x^3y^3} - x^3 - y^3 = 3$

20. (4) Using Rule1 and 8,

$x^4 + \frac{1}{x^4} = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 121$

$\Rightarrow x^2 + \frac{1}{x^2} = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 9 \Rightarrow x - \frac{1}{x} = 3$

Cubing both sides,

$\left(x - \frac{1}{x}\right)^3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} = 27 + 9 = 36$

21. (2) Using Rule 8,

$3x + \frac{1}{2x} = 5$

On multiplying both sides by $\frac{2}{3}$,

$2x + \frac{1}{3x} = \frac{10}{3}$

Cubing both sides,

$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x}$

$\left(2x + \frac{1}{3x}\right) = \frac{1000}{27}$

$\Rightarrow 8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$

$\Rightarrow 8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000 - 180}{27} = \frac{820}{27} = 30\frac{10}{27}$

22. (1) Using Rule 20,

$x + y = z \Rightarrow x + y + (-z) = 0$

$\therefore x^3 + y^3 - z^3 + 3xyz$

$= x^3 + y^3 + (-z)^3 - 3x.y(-z) = 0$

23. (1) Using Rule 8,

$\left(x + \frac{1}{x}\right)^2 = 3$

$\Rightarrow x + \frac{1}{x} = \sqrt{3}$

On cubing both sides,

$$\left(x+\frac{1}{x}\right)^3 = 3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3} = 0 \Rightarrow x^6+1 = 0$$

$\therefore x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1$
$= (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^6 + (x^6)^4 + x^6 + 1$
$= 1 - 1 - 1 + 1 + 1 + 0 = 1$

24. (1) Using Rule 8,

$$\left(x+\frac{1}{x}\right)^2 = 3$$

$$\Rightarrow x+\frac{1}{x} = \sqrt{3}$$

On cubing both sides,

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3} = 3\sqrt{3}-3\sqrt{3} = 0$$

$\Rightarrow x^6 + 1 = 0$
$\therefore x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$
$= x^{200}(x^6+1) + x^{84}(x^6+1) + x^{12}(x^6+1) + (x^6+1)$
$= 0$

25. (2) Using Rule 8,

(2) $a+\frac{1}{a} = \sqrt{3}$

On cubing both sides,

$$a^3+\frac{1}{a^3}+3a.\frac{1}{a}\left(a+\frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3+\frac{1}{a^3}+3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow a^3+\frac{1}{a^3} = 0 \qquad ....(i)$$

$$\Rightarrow a^6-\frac{1}{a^6}+2$$

$$= \left(a^3\right)^2-\left(\frac{1}{a^3}\right)^2+2$$

$$= \left(a^3+\frac{1}{a^3}\right)\left(a^3-\frac{1}{a^3}\right)+2 = 2$$

26. (2) Using Rule 8,
$(x + y)^3 = x^3 + y^3 + 3(xy)(x + y)$
$\Rightarrow 125 = 35 + 3(5)xy$
$\Rightarrow 15xy = 125 - 35 = 90$

$$\Rightarrow xy = \frac{90}{15} = 6$$

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{y}+\frac{1}{x} = \frac{5}{6}$$

27. (2) Using Rule 9,
$a^3 - b^3 = 56$
$\Rightarrow (a - b)(a^2 + ab + b^2) = 56$
$\Rightarrow a^2 + ab + b^2 = 28$
$\Rightarrow (a - b)^2 + 3ab = 28$
$\Rightarrow 4 + 3ab = 28$
$\Rightarrow 3ab = 28 - 4 = 24$
$\Rightarrow ab = 8$
$\therefore a^2 + b^2 = (a - b)^2 + 2ab$
$= 4 + 16 = 20$

28. (2) $(a^2 + b^2)^3 = (a^3 + b^3)^2$
$\Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2)$
$= a^6 + b^6 + 2a^3b^3$
$\Rightarrow 3(a^2 + b^2) = 2ab$

$$\Rightarrow \frac{a^2+b^2}{ab} = \frac{2}{3}$$

$$\Rightarrow \frac{a}{b}+\frac{b}{a} = \frac{2}{3}$$

29. (1) Using Rule1 and 8,

$$x+\frac{1}{x} = 5$$

On squaring both sides,

$$x^2+\frac{1}{x^2}+2 = 25$$

$$\Rightarrow x^2+\frac{1}{x^2} = 25 - 2 = 23 \quad ...(i)$$

Expression

$$= \frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$$

$$= \frac{x^4+1+3x^3+3x+5x^2}{x^4+1}$$

$$= \frac{x^2\left(x^2+\frac{1}{x^2}\right)+3x^2\left(x+\frac{1}{x}\right)+5x^2}{x^2\left(x^2+\frac{1}{x^2}\right)}$$

$$= \frac{\left(x^2+\frac{1}{x^2}\right)+3\left(x+\frac{1}{x}\right)+5}{x^2+\frac{1}{x^2}}$$

$$= \frac{23+3\times5+5}{23} = \frac{43}{23}$$

30. (2) Using Rule 8,

$$\left(x+\frac{1}{x}\right)^3$$

$$= x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$= 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 3$$

$$\therefore \left(x+\frac{1}{x}\right)^4 = 3 \times 3 = 9$$

31. (3) Using Rule 1 and 8,

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\Rightarrow x^2+\frac{1}{x^2} = 9 - 2 = 7$$

Again,

$$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow 27 = x^3+\frac{1}{x^3}+3\times3$$

$$\Rightarrow x^3+\frac{1}{x^3} = 18$$

$$\therefore \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)$$

$= 7 \times 18 = 126$

$$\Rightarrow x^5+\left(x+\frac{1}{x}\right)+\frac{1}{x^5} = 126$$

$$\Rightarrow x^5+\frac{1}{x^5} = 126–3 = 123$$

32. (2) Using Rule 9,

$$x-\frac{1}{x} = 3$$

On cubing both sides,

$$\left(x-\frac{1}{x}\right)^3 = 27$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right) = 27$$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} = 27 + 9 = 36$

33. (1) $m^4 + \frac{1}{m^4} = 119$

$\Rightarrow \left(m^2 + \frac{1}{m^2}\right)^2 - 2 = 119$

$\Rightarrow \left(m^2 + \frac{1}{m^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow m^2 + \frac{1}{m^2} = 11$

$\Rightarrow \left(m - \frac{1}{m}\right)^2 + 2 = 11$

$\Rightarrow \left(m - \frac{1}{m}\right)^2 = 11 - 2 = 9$

$\Rightarrow m - \frac{1}{m} = \pm 3$

34. (4) Using Rule 21,
$x + y + z = 6$
$\Rightarrow x + y + z - 6 = 0$
$\Rightarrow (x - 1) + (y - 2) + (z - 3) = 0$
If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
$\therefore (x - 1)^3 + (y - 2)^3 + (z - 3)^3 = 3(x - 1)(y - 2)(z - 3)$

35. (4) $x^2 + 1 = 2x$ (Given)

$\Rightarrow x + \frac{1}{x} = 2$ ...(i)

Expression

$= \frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1} = \frac{\frac{x^6 + 1}{x^2}}{(x^2 - 3x + 1)}$

$= \frac{x^6 + 1}{(x^2 + 1 - 3x).x^2}$

$= \frac{x^6 + 1}{(2x - 3x)x^2} = \frac{x^6 + 1}{-x^3}$

$= -\left(\frac{x^6 + 1}{x^3}\right) = -\left(\frac{x^6}{x^3} + \frac{1}{x^3}\right)$

$= -\left(x^3 + \frac{1}{x^3}\right)$

$= -\left[\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)\right]$

$= -[2^3 - 3 \times 2]$
$= -2$

36. (3) $x = \sqrt{3} + \sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{\sqrt{3} + \sqrt{2}}$

$= \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}$

$= \sqrt{3} - \sqrt{2}$

$\therefore x - \frac{1}{x} = \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2}$

$= 2\sqrt{2}$

Cubing both sides,

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = 16\sqrt{2}$

$x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$= 16\sqrt{2}$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 2\sqrt{2} = 16\sqrt{2}$

$\Rightarrow x^3 - \frac{1}{x^3} = 16\sqrt{2} + 6\sqrt{2} = 22\sqrt{2}$

37. (3) $x^2 + \frac{1}{x^2} = 83$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 83$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 83 - 2 = 81 = 9^2$

$\Rightarrow x - \frac{1}{x} = 9$

Cubing both sides,

$\left(x - \frac{1}{x}\right)^3 = 9^3 = 729$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 729$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 9 = 729$

$\Rightarrow x^3 - \frac{1}{x^3} = 729 + 27 = 756$

38. (4) Using Rule 8,

$\left(a + \frac{1}{a}\right)^2 = 3 = \left(\sqrt{3}\right)^2$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

Cubing both sides,

$\left(a + \frac{1}{a}\right)^3 = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 0$

39. (2) $\frac{x}{x^2 - 2x + 1} = \frac{1}{3}$

$\Rightarrow \frac{x^2 - 2x + 1}{x} = 3$

$\Rightarrow x - 2 + \frac{1}{x} = 3$

$\Rightarrow x + \frac{1}{x} = 5$

On cubing both sides

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 3 \times 5 = 110$

40. (2) Using Rule 1,

$\left(x + \frac{1}{x}\right) = 4$

On squaring both sides

$x^2 + \frac{1}{x^2} + 2 = 16$

$\Rightarrow x^2 + \frac{1}{x^2} = 14$

On squaring again

$x^4 + \frac{1}{x^4} + 2 = 196$

$\Rightarrow x^4 + \frac{1}{x^4} = 194$

41. (3) $x + y + z = 6$
On squaring,
$x^2 + y^2 + z^2 + 2xy + 2zy + 2zx = 36$
$\Rightarrow 20 + 2(xy + yz + zx) = 36$
$\Rightarrow xy + yz + zx = 8$
$\therefore x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= 6(20 - 8)$
$= 72$

42. (2) $x = 1 - \sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$= -1 - \sqrt{2}$

$\therefore \left(x - \frac{1}{x}\right)^3$

$= (1 - \sqrt{2} + 1 + \sqrt{2})^3$
$= 2^3 = 8$

43. (2) $x + y + z = a - b + b - c + c - a = 0$
$\therefore x^3 + y^3 + z^3 - 3xyz = 0$

44. (4) $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$

$= 3 + 2 - 2\sqrt{3}.\sqrt{2}$

$= 5 - 2\sqrt{6}$

$\therefore y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 5 + 2\sqrt{6}$

$\therefore x + y$

$= 5 - 2\sqrt{6} + 5 + 2\sqrt{6} = 10$

$xy = (5 - 2\sqrt{6})(5 + 2\sqrt{6})$

$= 25 - 24 = 1$
$\therefore x^3 + y^3 = (x + y)^3 - 3xy(x + y)$
$= (10)^3 - 3(10)$
$= 1000 - 30 = 970$

45. (4) $(x-a)^3 - \frac{1}{(x-a)^3}$

$= \left(x - a - \frac{1}{x-a}\right)^3 + 3\left(x - a - \frac{1}{x-a}\right)$

$= (x - a - x + b)^3 + 3(x - a - x + b)$
$= (b - a)^3 + 3(b - a)$
$= 5^3 + 3 \times 5 = 125 + 15 = 140$

46. (1) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$
$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$
$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$
$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$
$\therefore a - 1 = 0 \Rightarrow a = 1$
$b + 1 = 0 \Rightarrow b = -1$
$c + 1 = 0 \Rightarrow c = -1$
$\therefore 4a - 3b + 5c = 4 \times 1 - 3 \times (-1) + 5(-1) = 4 + 3 - 5 = 2$

47. (3) $2x + \frac{2}{x} = 3 \Rightarrow x + \frac{1}{x} = \frac{3}{2}$

On cubing,

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$

$= \frac{27-36}{8} = -\frac{9}{8}$

$\therefore x^3 + \frac{1}{x^3} + 2 = 2 - \frac{9}{8} = \frac{7}{8}$

48. (2) $a + b + c = 15$
$\therefore (a + b + c)^2 = 225$
$\therefore a^2 + b^2 + c^2 + 2(ab + bc + ca) = 225$
$\Rightarrow 2(ab + bc + ca) = 225 - 83 = 142$
$\Rightarrow ab + bc + ca = 142 \div 2 = 71$
$\therefore a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= 15(83 - 71) = 15 \times 12 = 180$

49. (3) $a - b = 3$
$a^3 - b^3 = 117$
$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
$\Rightarrow 117 = 27 + 3ab(3)$
$\Rightarrow 9ab = 117 - 27 = 90$
$\Rightarrow ab = 10$
$\therefore (a + b)^2 = (a - b)^2 + 4ab$
$= 9 + 40 = 49$
$\therefore |a + b| = 7$

50. (2) $x + \frac{1}{x+1} = 1$

$\Rightarrow (x + 1) + \frac{1}{x+1} = 2$

On squaring,

$(x+1)^2 + \frac{1}{(x+1)^2} + 2 = 4$

$\Rightarrow (x+1)^2 + \frac{1}{(x+1)^2} = 2$ ...(i)

Again, cubing $(x+1) + \frac{1}{(x+1)} = 2$,

$(x+1)^3 + \frac{1}{(x+1)^3}$

$+ 3\left((x+1) + \frac{1}{(x+1)}\right) = 8$

$\Rightarrow (x+1)^3 + \frac{1}{(x+1)^3}$

$= 8 - 3 \times 2 = 2$

$\therefore \left((x+1)^2 + \frac{1}{(x+1)^2}\right)$

$\left((x+1)^3 + \frac{1}{(x+1)^3}\right)$

$= 2 \times 2 = 4$

$\Rightarrow (x+1)^5 + \frac{1}{(x+1)} + \frac{1}{(x+1)^5}$

$+ (x + 1) = 4$

$\therefore (x+1)^5 + \frac{1}{(x+1)^5}$

$= 4 - 2 = 2$

Aliter :
Using Rule 14,

Here, $x + \frac{1}{x+1} = 1$

$\Rightarrow x + 1 + \frac{1}{x+1} = 2$

$\therefore (x+1)^2 + \frac{1}{(x+1)^2} = 2$

51. (1) $\frac{1}{a} - \frac{1}{b} = \frac{1}{a-b}$

$\Rightarrow \frac{b-a}{ab} = \frac{1}{a-b}$

$\Rightarrow (a - b)(a - b) = -ab$
$\Rightarrow a^2 - 2ab + b^2 = -ab$
$\Rightarrow a^2 - ab + b^2 = 0$
$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b)$
$= 0$

52. (4) Using Rule 21,
$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
If $a + b + c = 0$, then
$a^3 + b^3 + c^3 = 3abc$

53. (3) Using Rule 21,
If a + b + c = 0
then $a^3 + b^3 + c = 3abc$
$\therefore$ When a – b – c = 0,
$a^3 - b^3 - c^3 = 3abc$
i.e., $a^3 - b^3 - c^3 - 3abc = 0$
Here,
a = 4.965, b = 2.343,
c = 2.6222
$\therefore$ a – b – c = 4.965 – 2.343 – 2.622 = 0
Hence, $a^3 - b^3 - c^3 - 3abc = 0$

54. (1) Using Rule 21,
Here, a + b + c
= 1.21 + 2.12 – 3.33 = 0
$a^3 + b^3 + c^3 - 3abc = 0$
($\because$ $a + b + c = 0$)

55. (1) P = 999 (Given)
Now, $\sqrt[3]{P\left(P^2 + 3P + 3\right) + 1}$
$\sqrt[3]{P^3 + 3P^2 + 3P + 1}$
$= \sqrt[3]{(P+1)^3} = P + 1$
= 999 + 1 = 1000

56. (3) Using Rule 21,
Here, $a - b - c$
= 4.36 – 2.39 – 1.97 = 0
$\therefore$ $a^3 - b^3 - c^3 = 3abc$
$\Rightarrow a^3 - b^3 - c^3 - 3abc = 0$

57. (4) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$
$\left(x^2 + \frac{1}{x^2} - 1\right)\left(x^2 + \frac{1}{x^2} + 1\right)$
$= \left(x^2 - \frac{1}{x^2}\right)\left[\left(x^2 + \frac{1}{x^2}\right)^2 - 1\right]$
$= \left(x^2 - \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4} + 1\right)$
$= x^6 - \frac{1}{x^6}$

58. (1) $\frac{a^2 + b^2 + ab}{a^3 - b^3}$
$= \frac{a^2 + b^2 + ab}{(a - b)(a^2 + b^2 + ab)}$
$= \frac{1}{a - b}$
$= \frac{1}{11 - 9} = \frac{1}{2}$

59. (4) $a = \sqrt{7 + 2 \times \sqrt{4} \times \sqrt{3}}$
$= \sqrt{4 + 3 + 2 \times 2 \times \sqrt{3}}$
$= \sqrt{\left(2 + \sqrt{3}\right)^2} = 2 + \sqrt{3}$
$\therefore$ $b = \sqrt{7 - 2\sqrt{12}} = 2 - \sqrt{3}$
$\Rightarrow a + b = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$
$ab = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$
$\therefore a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
= 64 – 3 × 4 = 52

60. (3) According to question,
$\frac{a}{b} + \frac{b}{a} = 1$
$\Rightarrow a^2 + b^2 = ab$
$\Rightarrow a^2 - ab + b^2 = 0$
$\therefore a^3 + b^3$
$= (a + b)(a^2 - ab + b^2) = 0$

61. (3) $x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$
$\Rightarrow x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$
On cubing both sides,
$x^3 - 3x^2 \times 2 + 3x \times 4 - 8$
$= \left(2^{\frac{2}{3}}\right)^3 - \left(2^{\frac{1}{3}}\right)^3$
$- 3 . 2^{\frac{2}{3}} . 2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$
$\Rightarrow x^3 - 6x^2 + 12x - 8$
$= 4 - 2 - 6(x - 2)$
$\Rightarrow x^3 - 6x^2 + 12x - 8$
$= 2 - 6x + 12$
$\Rightarrow x^3 - 6x^2 + 18x + 18$
$= 2 + 12 + 8 + 18 = 40$

62. (4) Using Rule 21,
$a^3 + b^3 + c^3 - 3abc = 0$
If $a + b + c = 0$
$a^3 - b^3 - c^3 - 3abc = 0$
$\Rightarrow a - b - c = 0$
$\Rightarrow a = b + c$

63. (2) Using Rule 21,
Here, $p - q + q - r + r - p = 0$
$\therefore$ $(p - q)^3 + (q - r)^3 + (r - p)^3$
$= 3(p - q)(q - r)(r - p)$
[Formula : If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

64. (2) Using Rule 21,
$a + b + (-c) = 2.361 + 3.263 - 5.624 = 0$
$\therefore a^3 + b^3 + (-c^3 - 3ab(-c) = 0$
i.e. $a^3 + b^3 - c^3 + 3abc = 0$

65. (2) $(a + b + c)^2$
$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 36 = 14 + 2(ab + bc + ca)$
$\Rightarrow ab + bc + ca = (36 - 14) \div 2$
$\Rightarrow ab + bc + ca = 11$ ....(i)
$\therefore$ $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$\Rightarrow 36 - 3abc = 6(14 - 11)$ [By (i)]
$\Rightarrow 36 - 3abc = 84 - 66 = 18$
$\Rightarrow 3abc = 36 - 18 = 18$
$\Rightarrow abc = 6$

66. (1) Using Rule 8,
$a + b = 1$
Cubing both sides,
$(a + b)^3 = 1$
$\Rightarrow a^3 + b^3 + 3ab(a + b) = 1$
$\Rightarrow a^3 + b^3 + 3ab = 1 = k$
$\Rightarrow k = 1$

67. (4) Using Rule 22,
$a^3 + b^3 + c^3 - 3abc$
$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$
$= \frac{1}{2} \times 100(1 + 0 + 1) = 100$

68. (3) Using Rule 22,
$x^3 + y^3 + z^3 - 3xyz$
$= \frac{1}{2}(x + y + z)\left[(x - y)^2 + (y - z)^2 + (z - x)^2\right]$
$= \frac{1}{2}(333 + 333 + 334)(0 + 1 + 1)$
= 1000

69. (1) $a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$
$\therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$
$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$
[If $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$]
$= 3$ $\boldsymbol{(a + b)(a - b)}$ $(b + c)(b - c)(c + a)(c - a)$

70. (2) $a = \frac{b^2}{b - a} \Rightarrow ab - a^2 = b^2$
$\Rightarrow a^2 + b^2 - ab = 0$
$\therefore a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
$= (a + b) \times 0 = 0$

71. (4) Using Rule 8,
Expression = $p(p^2 + 3p + 3)$
$= (p^3 + 3p^2 + 3p + 1) - 1$
$= (p + 1)^3 - 1 = (99 + 1)^3 - 1$
$= (100)^3 - 1 = 1000000 - 1$
= 999999

72. (1) Using Rule 9,
Expression

$= \sqrt[3]{p\left(p^2-3p+3\right)-1}$

$= \sqrt[3]{p^3-3p^2+3p-1}$

$\sqrt[3]{(p-1)^3} = p - 1 = 101 - 1 = 100$

73. (4) Using Rule 8,
Expression

$= \sqrt[3]{p\left(p^2+3p+3\right)+1}$

$= \sqrt[3]{p^3+3p^2+3p+1}$

$= \left[(p+1)^3\right]^{\frac{1}{3}} = (p+1)^{3\times\frac{1}{3}}$

= p +1
When $p = 124$,
$p + 1 = 124 + 1 = 125$

74. (2) Using Rule 9,
$p - 2q = 4$
On cubing both sides,
$(p - 2q)^3 = 64$
$\Rightarrow p^3 - 8q^3 + 3p \,.\, 4q^2 - 3p^2 \,.\, 2q = 64$
$\Rightarrow p^3 - 8q^3 + 12pq^2 - 6p^2q = 64$
$\Rightarrow p^3 - 8q^3 - 6pq\,(p - 2q) = 64$
$\Rightarrow p^3 - 8q^3 - 6pq \times 4 = 64$
$\Rightarrow p^3 - 8q^3 - 24pq - 64 = 0$

75. (1) Expression $= \dfrac{x^2+y^2+xy}{x^3-y^3}$

$= \dfrac{x^2+y^2+xy}{(x-y)(x^2+y^2+xy)} = \dfrac{1}{x-y}$

$= \dfrac{1}{19-18} = 1$

76. (3) $x + \dfrac{1}{x} = 2$

$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$

$\therefore x^{17} + \dfrac{1}{x^{19}} = 1 + 1 = 2$

Aliter :
Using Rule 16,

Here, $x + \dfrac{1}{x} = 2$

$\Rightarrow x^{17} + \dfrac{1}{x^{19}} = 2$

77. (2) If $x = y = z = 1$, then
Expression
$= (3)^3 - (1)^3 - (1)^3 - (1)^3$
$= 27 - 3 = 24 = 24\,xyz$

78. (3) $\dfrac{1}{x^{99}} = \dfrac{1}{(-1)^{99}} = -1$

$\dfrac{1}{x^{98}} = \dfrac{1}{(-1)^{98}} = 1$ and so on.

$\therefore$ Expression = – 1 + 1 – 1 + 1 – 1 + 1 – 1 – 1 = – 2

79. (1) $\dfrac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}$

$= a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$\Rightarrow \dfrac{1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}$

$= a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\Rightarrow \dfrac{\left(2^{\frac{1}{3}}-1\right)}{\left(2^{\frac{1}{3}}-1\right)\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}+1\right)}$

$= a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\Rightarrow \dfrac{2^{\frac{1}{3}}-1}{2-1} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$

$\left[\because (a-b)(a^2+ab+b^2) = a^3-b^3\right]$

$\Rightarrow a = 0,\ b = 1,\ c = -1$
$\therefore a + b + c = 0 + 1 - 1 = 0$

80. (4) $x = \sqrt[3]{2+\sqrt{3}}$

$\Rightarrow x^3 = 2+\sqrt{3}$

$\dfrac{1}{x^3} = \dfrac{1}{2+\sqrt{3}}$

$= \dfrac{1}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$\therefore x^3 + \dfrac{1}{x^3} = 2+\sqrt{3}+2-\sqrt{3} = 4$

81. (4) $x = \sqrt[3]{5}+2$

$\Rightarrow x - 2 = \sqrt[3]{5}$
On cubing,

$x^3 - 3x^2 \times 2 + 3x.(-2)^2 - 2^3 = 5$
$\Rightarrow x^3 - 6x^2 + 12x - 8 = 5$
$\Rightarrow x^3 - 6x^2 + 12x - 13 = 0$

82. (3) $x^3 - x^2y - xy^2 + y^3$
$= x^3 + y^3 - x^2y - xy^2$
$= (x + y)^3 - 3xy\,(x + y) - xy(x + y)$
$= (x + y)^3 - 4xy\,(x + y) = a^3 - 4b^2a$

83. (2) Expression $= \dfrac{x^4-\dfrac{1}{x^2}}{3x^2+5x-3}$

Dividing numerator and denominator by x,

$= \dfrac{x^3-\dfrac{1}{x^3}}{3x+5-\dfrac{3}{x}} = \dfrac{x^3-\dfrac{1}{x^3}}{3\left(x-\dfrac{1}{x}\right)+5}$

$= \dfrac{\left(x-\dfrac{1}{x}\right)^3+3\left(x-\dfrac{1}{x}\right)}{3\left(x-\dfrac{1}{x}\right)+5}$

$= \dfrac{1+3}{3+5} = \dfrac{4}{8} = \dfrac{1}{2}$

84. (4) $x + y = 15$
$\Rightarrow (x - 10) + (y - 5) = 0$
$\therefore (x - 10)^3 + (y - 5)^3$
$= (x - 10 + y - 5)^3 - 3\,(x - 10)\,(y - 5)\,(x - 10 + y - 5) = 0$
$[a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)]$

85. (2) Using Rule 5,

$x^2 + \dfrac{1}{x^2} = 66$

$\Rightarrow \left(x-\dfrac{1}{x}\right)^2 + 2 = 66$

$\Rightarrow \left(x-\dfrac{1}{x}\right)^2 = 66 - 2 = 64$

$\Rightarrow x - \dfrac{1}{x} = \pm 8$

$\therefore$ Expression $= \dfrac{x^2-1+2x}{x}$

$= \dfrac{x^2}{x} - \dfrac{1}{x} + 2 = x - \dfrac{1}{x} + 2$

Putting the value of $x - \dfrac{1}{x}$

= 8 + 2 or – 8 + 2 = 10 or –6

86. (3) Using Rule 9,
$a^2 + a + 1 = 0$
$\Rightarrow (a - 1)\,(a^2 + a + 1) = 0$
$\Rightarrow a^3 - 1 = 0$
$\Rightarrow a^3 = 1 \Rightarrow a = 1$
$\therefore a^9 = 1$

87. (1) Given, $x + \frac{2}{x} = 1$

Expression

$= \frac{x^2 + x + 2}{x^2(1-x)} = \frac{x + 1 + \frac{2}{x}}{x(1-x)}$

(Dividing numerator and denominator by x)

$= \frac{x + \frac{2}{x} + 1}{x(1-x)} = \frac{1+1}{x \times \frac{2}{x}} = \frac{2}{2} = 1$

88. (2) Using Rule 9,
$x = k^3 - 3k^2$
$y = 1 - 3k$
When $x = y$, then
$k^3 - 3k^2 = 1 - 3k$
$\Rightarrow k^3 - 3k^2 + 3k - 1 = 0$
$\Rightarrow (k-1)^3 = 0 \Rightarrow k - 1 = 0$
$\Rightarrow k = 1$

89. (3) Expression

$= \frac{\sqrt{(x^2 + y^2 + z)(x + y - 3z)}}{\sqrt[3]{xy^3z^2}}$

Putting $x = 1$, $y = -3$, $z = -1$

$= \frac{\sqrt{(1 + 9 - 1)(1 - 3 + 3)}}{\sqrt[3]{1 \times -27 \times 1}}$

$= \frac{3}{-3} = -1$

Note : Original question is :

$\sqrt{(x^2 + y^2 + z)(x - y - 3z)} \div \sqrt[3]{xy^3z^2}$

which gives answer $= -\sqrt{7}$ which is not in options.

90. (2) Expression

$= \frac{p^2 - p}{2p^3 + 6p^2} \div \frac{p^2 - 1}{p^2 + 3p} \div \frac{p^2}{p+1}$

$= \frac{p(p-1)}{2p^2(p+3)} \div \frac{(p+1)(p-1)}{p(p+3)} \div \frac{p^2}{p+1}$

$= \frac{p(p-1)}{2p^2(p+3)} \times \frac{p(p+3)}{(p+1)(p-1)} \times \frac{(p+1)}{p^2}$

$= \frac{1}{2p^2}$

91. (2) $x + \frac{1}{x} = 2$

On squaring both sides,

$x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow x^2 + \frac{1}{x^2} = 4 - 2 = 2$

Again, $x + \frac{1}{x} = 2$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = 8$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 2 = 8$

$\Rightarrow x^3 + \frac{1}{x^3} = 8 - 6 = 2$

$\therefore \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$

$= 2 \times 2 = 4$

Aliter :

Using Rule 14,

Here, $x + \frac{1}{x} = 2$

$x^2 + \frac{1}{x^2} = 2$ and $x^3 + \frac{1}{x^3} = 2$

$\therefore \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$

$= 2 \times 3 = 4$

92. (3) $a^3 + b^3 + c^3 - 3abc$ will be minimum if $a = b = 1$, $c = 2$

$\therefore$ Least value $= 1 + 1 + 8 - 3 \times 1 \times 1 \times 2 = 10 - 6 = 4$

93. (4) By remainder theorem,

Remainder $= f\left(-\frac{2}{3}\right)$

$\because f(x) = 12x^3 - 13x^2 - 5x + 7$

$\therefore f\left(-\frac{2}{3}\right) = 12\left(\frac{-2}{3}\right)^3 - 13\left(\frac{-2}{3}\right)^2 - 5\left(\frac{-2}{3}\right) + 7$

$= -\frac{12 \times 8}{27} - \frac{13 \times 4}{9} + \frac{10}{3} + 7$

$= -\frac{32}{9} - \frac{52}{9} + \frac{10}{3} + 7$

$= \frac{-32 - 52 + 30 + 63}{9} = \frac{9}{9} = 1$

Second Method

$$\begin{array}{r|l} 3x+2 & 12x^3 - 13x^2 - 5x + 7 \;(4x^2 - 7x + 3 \\ & \underline{12x^3 \pm 8x^2} \\ & \quad -21x^2 - 5x \\ & \quad \underline{\mp 21x^2 \mp 14x} \\ & \qquad 9x + 7 \\ & \qquad \underline{9x \pm 6} \\ & \qquad\quad 1 \end{array}$$

94. (3) $ab + bc + ca = 0$
$\Rightarrow ab + ca = -bc$
$\therefore a^2 - bc = a^2 + ab + ca$
$= a(a + b + c)$
Similarly,
$b^2 - ac = b(a + b + c)$
$c^2 - ab = c(a + b + c)$

$\therefore \frac{1}{a^2 - bc} + \frac{1}{b^2 - ac} + \frac{1}{c^2 - ab}$

$= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$

$= \frac{bc + ac + ab}{abc(a+b+c)} = 0$

95. (2) $2x^2 - 7x + 12 = 0$

$\therefore \alpha + \beta = \frac{7}{2}$

$\alpha\beta = \frac{12}{2} = 6$

[In equation $ax^2 + bx + c = 0$,]

$\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$]

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{7}{2}\right)^2 - 2 \times 6}{6}$

$= \frac{\frac{49}{4} - 12}{6}$

$= \frac{49 - 48}{4 \times 6} = \frac{1}{24}$

96. (3) $x^3 + \frac{3}{x} = 4(a^3 + b^3)$

$3x + \frac{1}{x^3} = 4(a^3 - b^3)$

On adding,

$x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = 8a^3$

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (2a)^3$

$\Rightarrow x + \frac{1}{x} = 2a \Rightarrow a = \frac{1}{2}\left(x + \frac{1}{x}\right)$

Similarly,

$x^3 + \frac{3}{x} - 3x - \frac{1}{x^3} = 8b^3$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = (2b)^3$

$\Rightarrow b = \frac{1}{2}\left(x - \frac{1}{x}\right)$

$\therefore a^2 - b^2$

$= \frac{1}{4}\left[\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2\right]$

$= \frac{1}{4} \times 4 = 1$

97. (2) $x = 6 + \frac{1}{x}$

$\Rightarrow x - \frac{1}{x} = 6$

On squaring both sides,

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 36$

$\Rightarrow x^2 + \frac{1}{x^2} = 36 + 2 = 38$

On squaring again,

$x^4 + \frac{1}{x^4} + 2 = 1444$

$\Rightarrow x^4 + \frac{1}{x^4} = 1444 - 2 = 1442$

98.(1) $x + \frac{1}{x} = 5$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = 5^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\,x\,.\,\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= 125$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$

$\Rightarrow x^3 + \frac{1}{x^3} = 125 - 15 = 110$

On squaring both sides,

$x^6 + \frac{1}{x^6} + 2\,.\,x^3\,.\,\frac{1}{x^3}$

$= 12100$

$\Rightarrow x^6 + \frac{1}{x^6} = 12100 - 2$

$= 12098$

99. (3) $x^2 - 3x + 1 = 0$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow \frac{x^2 + 1}{x} = 3$

$\Rightarrow x + \frac{1}{x} = 3$ .... (i)

$\therefore \frac{x^6 + x^4 + x^2 + 1}{x^3}$

$= \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= \left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^3 - 3.\,x\,.\,\frac{1}{x}\left(x + \frac{1}{x}\right)$

$+ \left(x + \frac{1}{x}\right)$

$= 3^3 - 3 \times 3 + 3 = 27 - 9 + 3$

$= 21$

100. (3) $x^4 + \frac{1}{x^4} = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 11^2$

$\Rightarrow x^2 + \frac{1}{x^2} = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 11 - 2 = 9 = 3^2$

$\Rightarrow x - \frac{1}{x} = 3$

On cubing both sides,

$\left(x - \frac{1}{x}\right)^3 = 3^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\,x\,.\,\frac{1}{x}\left(x - \frac{1}{x}\right)$

$= 27$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 3 = 27$

$\Rightarrow x^3 - \frac{1}{x^3} = 27 + 9 = 36$

101. (3) Let $\frac{p}{a} = x,\ \frac{q}{b} = y,\ \frac{r}{c} = z$

$\therefore x + y + z = 1$

and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow \frac{yz + xz + xy}{xyz} = 0$

$\Rightarrow xy + yz + zx = 0$

$\therefore x + y + z = 1$

On squaring both sides

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 1$

$\Rightarrow x^2 + y^2 + z^2 + 0 = 1$

$\Rightarrow x^2 + y^2 + z^2 = 1$

102. (2) $\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2$

$\Rightarrow \frac{(x^3 + 3x^2 + 3x + 1) - (x^3 - 3x^2 + 3x - 1)}{(x^2 + 2x + 1) - (x^2 - 2x + 1)}$

$= 2$

$\Rightarrow \frac{x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1}{x^2 + 2x + 1 - x^2 + 2x - 1}$

$= 2$

$\Rightarrow \frac{6x^2 + 2}{4x} = 2$

$\Rightarrow \frac{3x^2 + 1}{4x} = 1 \Rightarrow 3x^2 + 1 = 4x$

$\Rightarrow 3x^2 - 4x + 1 = 0$
$\Rightarrow 3x^2 - 3x - x + 1 = 0$
$\Rightarrow 3x(x - 1) - 1(x - 1) = 0$
$\Rightarrow (3x - 1)(x - 1) = 0$
$\Rightarrow 3x - 1 = 0$, or, $x - 1 = 0$

$\Rightarrow x = \frac{1}{3}$ or 1

Hence, sum of the numerator and denominator = 1 + 3 = 4
or, 1 + 1 = 2

103. (3) Expression $= \dfrac{2x^2 - 3x - 2}{3x^2 - 4x - 3}$

$$= \frac{2(\sqrt{5}+2)^2 - 3(\sqrt{5}+2) - 2}{3(\sqrt{5}+2)^2 - 4(\sqrt{5}+2) - 3}$$

$$= \frac{2(5+4+4\sqrt{5}) - 3(\sqrt{5}+2) - 2}{3(5+4+4\sqrt{5}) - 4(\sqrt{5}+2) - 3}$$

$$= \frac{18 + 8\sqrt{5} - 3\sqrt{5} - 6 - 2}{27 + 12\sqrt{5} - 4\sqrt{5} - 8 - 3}$$

$$= \frac{10 + 5\sqrt{5}}{16 + 8\sqrt{5}} = \frac{5(2+\sqrt{5})}{8(2+\sqrt{5})} = \frac{5}{8}$$

$= 0.625$

104. (2) Using Rule 21,
$a = 2.234$, $b = 3.121$ and
$c = -5.355$
$a + b + c = 2.234 + 3.121 - 5.355 = 0$
$\therefore a^3 + b^3 + c^3 - 3abc = 0$

105. (4) $x^2 + y^2 + 1 = 2x$
$\Rightarrow x^2 + y^2 + 1 - 2x = 0$
$\Rightarrow x^2 - 2x + 1 + y^2 = 0$
$\Rightarrow (x - 1)^2 + y^2 = 0$
$\Rightarrow x - 1 = 0$
$\Rightarrow x = 1$ and $y = 0$
$\therefore x^3 + y^5 = 1 + 0 = 1$

106. (1) $3(a^2 + b^2 + c^2) = (a + b + c)^2$
$\Rightarrow 3a^2 + 3b^2 + 3c^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$
$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca = 0$
$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$
$\Rightarrow a - b = 0 \Rightarrow a = b$
[If $x^2 + y^2 + z^2 = 0$, $x = 0$, $y = 0$, $z = 0$]
$b - c = 0 \Rightarrow b = c$
$c - a = 0 \Rightarrow c = a$
$\therefore a = b = c$

107. (1) $x(x - 3) = -1$
$\Rightarrow x^2 - 3x = -1$
$\Rightarrow x^2 - 3x + 1 = 0$

Expression $= x^3(x^3 - 18)$
$= x^6 - 18x^3$
On dividing $x^6 - 18x^3$ by $x^2 - 3x + 1$

$$\begin{array}{l} x^2 - 3x + 1 \,\big)\, x^6 - 18x^3 \,\big(\, x^4 + 3x^3 + 8x^2 + 3x \\ \qquad\qquad\quad \underline{-x^6 \mp 3x^5 \pm x^4} \\ \qquad\qquad\quad 3x^5 - x^4 - 18x^3 \\ \qquad\qquad\quad \underline{-3x^5 \mp 9x^4 \pm 3x^3} \\ \qquad\qquad\qquad 8x^4 - 21x^3 \\ \qquad\qquad\qquad \underline{-8x^4 \mp 24x^3 \pm 8x^2} \\ \qquad\qquad\qquad\quad 3x^3 - 8x^2 \\ \qquad\qquad\qquad\quad \underline{-3x^3 \mp 9x^2 \pm 3x} \\ \qquad\qquad\qquad\qquad x^2 - 3x \end{array}$$

$\therefore x^6 - 18x^3 = (x^4 + 3x^3 + 8x^2 + 3x)(x^2 - 3x + 1) + x^2 - 3x$
$= 0 + x(x - 3) = -1$

108. (2) $a^2 + b^2 + c^2 = ab + bc + ca$
$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$
On multiplying by 2,
$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$
$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac = 0$
$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0 \Rightarrow$
$a - b = 0$
$\Rightarrow a = b$
$b - c = 0 \Rightarrow b = c$
$c - a = 0 \Rightarrow c = a$

$\therefore \dfrac{a + c}{b} = \dfrac{2a}{a} = 2$

109. (1) $ab + bc + ca = 0$
$\Rightarrow ab + ca = -bc$
$\therefore a^2 - bc = a^2 + ab + ac$
$= a(a + b + c)$
Similarly,
$b^2 - ac = b(a + b + c)$
$c^2 - ab = c(a + b + c)$

$$\therefore \frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}$$

$$= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)}$$

$$= \frac{1}{(a+b+c)}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= \frac{1}{a+b+c}\left(\frac{bc + ca + ab}{abc}\right)$$

$$= \frac{1}{a+b+c} \times \frac{0}{abc} = 0$$

110. (2) $3x + \dfrac{3}{x} = 1$

$\Rightarrow x + \dfrac{1}{x} = \dfrac{1}{3}$

On cubing both sides,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{1}{3} = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 1 = \frac{1}{27}$$

111. (1) $a^2 + 4b^2 + 4b - 4ab - 2a - 8$
$= a^2 + 4b^2 - 4ab - 2a + 4b - 8$
$= (a - 2b)^2 - 2(a - 2b) - 8$
Let $(a - 2b) = x$
$\therefore$ Expression $= x^2 - 2x - 8$
$= x^2 - 4x + 2x - 8$
$= x(x - 4) + 2(x - 4)$
$= (x - 4)(x + 2)$
$= (a - 2b - 4)(a - 2b + 2)$

112. (4) $\dfrac{1}{a^2 + ax + x^2} - \dfrac{1}{a^2 - ax + x^2} + \dfrac{2ax}{a^4 + a^2x^2 + x^4}$

$$= \frac{a^2 - ax + x^2 - a^2 - ax - x^2}{(a^2 + ax + x^2)(a^2 - ax + x^2)} + \frac{2ax}{a^4 + a^2x^2 + x^4}$$

$$= \frac{-2ax}{a^4 + a^2x^2 + x^4} + \frac{2ax}{a^4 + a^2x^2 + x^4} = 0$$

113. (2) $x = 11$ (Given)
$\therefore x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$
$= x^5 - (11 + 1)x^4 + (11 + 1)x^3 - (11 + 1)x^2 + (11 + 1)x - 1$
$= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$
When $x = 11$,
$= 11^5 - 11^5 - 11^4 + 11^4 + 11^3 - 11^3 - 11^2 + 11^2 + 11 - 1 = 10$

114. (3) Using Rule 8,
$p = 99$ (Given)
$\therefore p(p^2 + 3p + 3) = p^3 + 3p^2 + 3p$
$= p^3 + 3p^2 + 3p + 1 - 1$
$= (p + 1)^3 - 1 = (99 + 1)^3 - 1$
$= (100)^3 - 1 = 999999$

115. (3) According to equality relation $(x + 2)^2 = x^2 + 4x + 4$ is not an identity

116. (4) Expression

$$= \frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$$

$$= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

[Here, $a - b + b - c + c - a = 0$. If $x + y + z = 0$, $x^3 + y^3 + z^3 = 3xyz$]

117. (1) Using Rule 8,

$$\left(a + \frac{1}{a}\right)^2 = 3$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

On cubing both sides,

$$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

118. (1) Using Rule 1 and 8,

$$a + \frac{1}{a} = \sqrt{3}$$

On squaring both sides.

$$a^2 + \frac{1}{a^2} + 2 = 3$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 3 - 2 = 1$$

On cubing both sides,

$$\left(a^2 + \frac{1}{a^2}\right)^3 = 1^3$$

$$\Rightarrow a^6 + \frac{1}{a^6} + 3\left(a^2 + \frac{1}{a^2}\right) = 1$$

$$\Rightarrow a^6 + \frac{1}{a^6} = 1 - 3 = -2$$

$$\Rightarrow \frac{a^{12} + 1}{a^6} = -2$$

$\Rightarrow a^6 + 2a^6 + 1 = 0$

$\Rightarrow (a^6 + 1)^2 = 0$

$\Rightarrow a^6 + 1 = 0$

$\therefore$ Expression

$= a^{18} + a^{12} + a^6 + 1$

$= a^{12}(a^6 + 1) + (a^6 + 1) = 0$

119. (4) $x = 997$

$y = 998$

$z = 999$

$\therefore x - y = 997 - 998 = -1$

$y - z = 998 - 999 = -1$

$z - x = 999 - 997 = 2$

$\therefore x^2 + y^2 + z^2 - xy - yz - zx$

$= \frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx)$

$= \frac{1}{2}(x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2zx)$

$= \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}[(-1)^2 + (-1)^2 + (2)^2]$

$= \frac{1}{2}(1 + 1 + 4) = \frac{1}{2} \times 6 = 3$

120. (3) $x + \frac{1}{x} = 3$ (Given)

$$\text{Expression} = \frac{3x^2 - 4x + 3}{x^2 - x + 1}$$

$$= \frac{(3x^2 - 3x + 3) - x}{x^2 - x + 1}$$

$$= \frac{3(x^2 - x + 1)}{x^2 - x + 1} - \frac{x}{x^2 - x + 1}$$

$$= 3 - \frac{1}{x - 1 + \frac{1}{x}}$$

$$= 3 - \frac{1}{x + \frac{1}{x} - 1}$$

$$= 3 - \frac{1}{3 - 1} = 3 - \frac{1}{2}$$

$$= \frac{6 - 1}{2} = \frac{5}{2}$$

121. (4) Expression

$$= \frac{x^6 + x^4 + x^2 + 1}{x^3}$$

$$= \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$$

$$= \left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) + \left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)^3 - 2\left(x + \frac{1}{x}\right) \quad \text{---(i)}$$

Now, $x = 3 + 2\sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{1}{3 + 2\sqrt{2}} \times \frac{3 - 2\sqrt{2}}{3 - 2\sqrt{2}} = \frac{3 - 2\sqrt{2}}{9 - 8}$$

$= 3 - 2\sqrt{2}$

$\therefore x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$= 6$

$\therefore$ Expression $= (6)^3 - 2 \times 6$

$= 216 - 12 = 204$

122. (3) $x = p + \frac{1}{p}$

$y = p - \frac{1}{p}$

$\therefore x + y = p + \frac{1}{p} + p - \frac{1}{p} = 2p$

$x - y = p + \frac{1}{p} - p + \frac{1}{p} = \frac{2}{p}$

$\therefore x^4 - 2x^2y^2 + y^4 = (x^2 - y^2)^2$

$= \{(x + y)(x - y)\}^2$

$= (2p \times \frac{2}{p})^2 = 4^2 = 16$

123. (3) $a + b + c = 0$ (Given)

$\therefore a + b = -c$

$b + c = -a$

$c + a = -b$

$\therefore (a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$

$= (-c - c)^2 + (-a - a)^2 + (-b - b)^2$

$= (-2c)^2 + (-2a)^2 + (-2b)^2$

$= 4c^2 + 4a^2 + 4b^2 = 4(c^2 + a^2 + b^2)$

124. (2) Using Rule 8,
$p^3 + 3p^2 + 3p = 7$
$\Rightarrow p^3 + 3p^2 + 3p + 1 = 7 + 1 = 8$
$\Rightarrow (p + 1)^3 = (2)^3$
$\Rightarrow p + 1 = 2 \Rightarrow p = 2 - 1 = 1$
$\therefore p^2 + 2p = 1 + 2 \times 1 = 3$

125. (1) $x - y = 2015 - 2014 = 1$
$y - z = 2014 - 2013 = 1$
$z - x = 2013 - 2015 = -2$
$\therefore x^2 + y^2 + z^2 - xy - yz - zx$

$$= \frac{1}{2}\begin{pmatrix}2x^2 + 2y^2 + 2z^2 - \\ 2xy - 2yz - 2zx\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}x^2 + y^2 - 2xy + y^2 + z^2 - \\ 2yz + z^2 + x^2 - 2zx\end{pmatrix}$$

$$= \frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$$

$$= \frac{1}{2}(1 + 1 + 4) = \frac{1}{2} \times 6 = 3$$

126. (1) Expression

$$= \frac{(a+b)^3 - (a-b)^3}{(a+b)^2 + (a-b)^2}$$

$$= \frac{a^3 + 3a^2b + 3ab^2 + b^3 - (a^3 - 3a^2b + 3ab^2 - b^3)}{a^2 + b^2 + 2ab + a^2 + b^2 - 2ab}$$

$$= \frac{a^3 + 3a^2b + 3ab^2 + b^3 - a^3 + 3a^2b - 3ab^2 + b^3}{a^2 + b^2 + a^2 + b^2}$$

$$= \frac{6a^2b + 2b^3}{2(a^2 + b^2)} = \frac{2b(3a^2 + b^2)}{2(a^2 + b^2)}$$

$$= \frac{b(b^2 + b^2)}{\left(\frac{b^2}{3} + b^2\right)} = \frac{b \times 2b^2}{\frac{4b^2}{3}}$$

$$= \left(\frac{3 \times 2}{4}\right)b = \frac{3b}{2}$$

127. (1) $x + \frac{1}{x} = 2\frac{1}{12} = \frac{25}{12}$

On squaring both sides,

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{25}{12}\right)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = \frac{625}{144}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \frac{625}{144} - 2$$

$$= \frac{625 - 288}{144} = \frac{337}{144}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = \frac{337}{144}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{337}{144} - 2$$

$$= \frac{337 - 288}{144} = \frac{49}{144}$$

$$\Rightarrow x - \frac{1}{x} = \sqrt{\frac{49}{144}} = \frac{7}{12}$$

$$\therefore x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= \frac{337}{144} \times \frac{25}{12} \times \frac{7}{12} = \frac{58975}{20736}$$

128. (3) Expression

$$= \frac{4x^3 - x}{(2x+1)\,(6x-3)}$$

$$= \frac{x(4x^2 - 1)}{(2x+1) \times 3(2x-1)}$$

$$= \frac{x(2x+1)(2x-1)}{3(2x+1)(2x-1)}$$

$$= \frac{x}{3} = \frac{9999}{3} = 3333$$

129. (2) $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
$\Rightarrow 9 = 3(a^2 + b^2 - ab)$

$$\Rightarrow a^2 + b^2 - ab = \frac{9}{3} = 3$$

$\Rightarrow (a + b)^2 - 2ab - ab = 3$
$\Rightarrow 9 - 3ab = 3$
$\Rightarrow 3ab = 9 - 3 = 6$
$\Rightarrow ab = 2$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{3}{2}$$

130. (3) $t^2 - 4t + 1 = 0$
$\Rightarrow t^2 + 1 = 4t$

$$\Rightarrow \frac{t^2 + 1}{t} = 4$$

$$\Rightarrow t + \frac{1}{t} = 4$$

On cubing both sides,

$$\left(t + \frac{1}{t}\right)^3 = 4^3$$

$$t^3 + \frac{1}{t^3} + 3\left(t + \frac{1}{t}\right) = 64$$

$$\Rightarrow t^3 + \frac{1}{t^3} + 3 \times 4 = 64$$

$$\Rightarrow t^3 + \frac{1}{t^3} = 64 - 12 = 52$$

131. (4) $\sqrt[3]{a} + \sqrt[3]{b} - \sqrt[3]{c} = 0$

$$\therefore a + b - c = -3(abc)^{\frac{1}{3}}$$

On cubing both sides,
$(a + b - c)^3 = -27abc$
$\therefore (a + b - c)^3 + 27abc = 0$

132. (3) Using Rule 9,

$$27\,p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3.(3p)^2.\left(\frac{1}{6}\right) + 3 \times 3p \times \frac{1}{6} \times \frac{1}{6}$$

$$= \left(3p - \frac{1}{6}\right)^3 = \left(3 \times \frac{5}{18} - \frac{1}{6}\right)^3$$

$$= \left(\frac{5}{6} - \frac{1}{6}\right)^3 = \left(\frac{4}{6}\right)^3$$

$$= \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

133. (4) $x + \frac{1}{x} = 2$
$\Rightarrow x^2 + 1 = 2x$
$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$

$$\therefore x^{2013} + \frac{1}{x^{2014}} = 1 + 1 = 2$$

Aliter :
Using Rule 16,

Here, $x + \frac{1}{x} = 2$

$$x^{2013} + \frac{1}{x^{2014}} = 2$$

134. (4) Using Rule 21,
$a + b + c = 331 + 336 - 667 = 0$
$\therefore a^3 + b^3 + c^3 - 3\,abc = 0$

135. (3) Using Rule 21,
$a = 4.965$, $b = 2.343$,
$c = 2.622$

$a + (-b) + (-c) = 4.965 - 2.343 - 2.622 = 0$

$\therefore\ a^3 - b^3 - c^3 - 3abc = a^3 + (-b)^3 + (-c)^3 - 3abc = 0$

136. (4) $x + y + z = 0$

$\Rightarrow -x = y + z$

$\Rightarrow (-x)^2 = (y + z)^2$

$\Rightarrow x^2 = y^2 + z^2 + 2yz \quad ...(i)$

$\therefore$ Expression $= \dfrac{x^2+y^2+z^2}{x^2-yz}$

$= \dfrac{y^2+z^2+2yz+y^2+z^2}{y^2+z^2+2yz-yz}$

$= \dfrac{2y^2+2z^2+2yz}{y^2+z^2+yz}$

$= \dfrac{2(y^2+z^2+yz)}{y^2+z^2+yz} = 2$

137. (4) Using Rule 1 and 8,

$x + \dfrac{1}{x} = 0$

On squaring both sides,

$\left(x+\dfrac{1}{x}\right)^2 = 0$

$\Rightarrow x^2 + \dfrac{1}{x^2} + 2 = 0$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2 \; ..... \; (i)$

(not admissible)

On cubing $\left(x+\dfrac{1}{x}\right) = 0,$

$x^3 + \dfrac{1}{x^3} + 3 \times 0 = 0$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 0$

$\therefore \left(x^2+\dfrac{1}{x^2}\right)\left(x^3+\dfrac{1}{x^3}\right) = 0$

$\Rightarrow x^5 + \dfrac{1}{x^5} + x + \dfrac{1}{x} = 0$

$\Rightarrow x^5 + \dfrac{1}{x^5} = 0$

138. (1) $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

It is possible only when,

$a - b = 0 \Rightarrow a = b$

$b - c = 0 \Rightarrow b = c$

$c - a = 0 \Rightarrow c = a$

$\therefore\ a = b = c$

139. (1) $x^4 + \dfrac{1}{x^4} = 119$

$\Rightarrow \left(x^2+\dfrac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2+\dfrac{1}{x^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow x^2 + \dfrac{1}{x^2} = \sqrt{121} = 11$

Again,

$\left(x+\dfrac{1}{x}\right)^2 - 2 = 11$

$\Rightarrow \left(x+\dfrac{1}{x}\right)^2 = 11 + 2 = 13$

$\Rightarrow x + \dfrac{1}{x} = \pm\sqrt{13}$

On cubing both sides,

$x^3 + \dfrac{1}{x^3} + 3\left(x+\dfrac{1}{x}\right) = \pm 13\sqrt{13}$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3 \times (\pm\sqrt{13}) = \pm 13\sqrt{13}$

$\Rightarrow x^3 + \dfrac{1}{x^3}$

$= \pm\left(13\sqrt{13} - 3\sqrt{13}\right)$

$= \pm 10\sqrt{13}$

140. (4) Using Rule 8,

$x + \dfrac{1}{x} = \sqrt{3}$

On cubing both sides,

$x^3 + \dfrac{1}{x^3} + 3\left(x+\dfrac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 0$

$\therefore$ Expression $= x^{30} + x^{24} + x^{18} + x^{12} + x^6 + 1$

$= x^{24}(x^6+1) + x^{12}(x^6 + 1) + 1(x^6 + 1)$

$= (x^6 + 1)(x^{24} + x^{12} + 1)$

$= x^3\left(x^3+\dfrac{1}{x^3}\right)(x^{24} + x^{12} + 1)$

$= 0$

141. (3) Using Rule 8,

$m + n = -2$

On cubing both sides,

$(m + n)^3 = (-2)^3 = -8$

$\Rightarrow m^3 + n^3 + 3mn(m + n) = -8$

$\Rightarrow m^3 + n^3 - 6mn = -8$

142. (4) $u_n = \dfrac{1}{n} - \dfrac{1}{n+1}$

$\therefore\ u_1 = \dfrac{1}{1} - \dfrac{1}{1+1}$

$= 1 - \dfrac{1}{2};\ u_2 = \dfrac{1}{2} - \dfrac{1}{3}$

$u_3 = \dfrac{1}{3} - \dfrac{1}{4};\ u_4 = \dfrac{1}{4} - \dfrac{1}{5};$

$u_5 = \dfrac{1}{5} - \dfrac{1}{6}$

$\therefore\ u_1 + u_2 + u_3 + u_4 + u_5$

$= 1 - \dfrac{1}{2} + \dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{4} - \dfrac{1}{5} + \dfrac{1}{5} - \dfrac{1}{6}$

$= 1 - \dfrac{1}{6} = \dfrac{6-1}{6} = \dfrac{5}{6}$

143. (4) Using Rule 21,

$x + y + z = 5 + 6 - 11 = 0$

$\therefore\ x^3 + y^3 + z^3 = 3xyz$

$= 3 \times 5 \times 6 \times (-11) = -990$

144. (4) Using Rule 8,

$(p + m)^3 = p^3 + m^3 + 3pm(p + m)$

$\Rightarrow (6)^3 = 72 + 3pm \times 6$

$\Rightarrow 216 - 72 = 18pm$

$\Rightarrow 18pm = 144$

$\Rightarrow pm = 144 \div 18 = 8$

145. (2) According to the question,

$\dfrac{x+\dfrac{1}{x}}{2} = A$

$\Rightarrow x + \dfrac{1}{x} = 2A$

On cubing both sides,

$\left(x+\dfrac{1}{x}\right)^3 = (2A)^3 = 8A^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right) = 8A^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 2A = 8A^3$

$\Rightarrow x^3 + \frac{1}{x^3} = 8A^3 - 6A$

$\therefore$ Required average

$= \frac{x^3 + \frac{1}{x^3}}{2}$

$= \frac{8A^3 - 6A}{2}$

$= 4A^3 - 3A$

146. (4) $a = 2 + \sqrt{3}$

$\Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

$\therefore a + \frac{1}{a} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$\therefore$ Expression $= \frac{a^6 + a^4 + a^2 + 1}{a^3}$

$= a^3 + a + \frac{1}{a} + \frac{1}{a^3}$

$= a^3 + \frac{1}{a^3} + a + \frac{1}{a}$

$= \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right) + \left(a+\frac{1}{a}\right)$

$= \left(a+\frac{1}{a}\right)^3 - 2\left(a+\frac{1}{a}\right)$

$= (4)^3 - 2 \times 4 = 64 - 8 = 56$

147. (1) $x = \sqrt{5} + \sqrt{3}$

$x^2 = \left(\sqrt{5}+\sqrt{3}\right)^2$

$= 5 + 3 + 2\sqrt{15} = 8 + 2\sqrt{15}$

$y = \sqrt{5} - \sqrt{3}$

$\therefore y^2 = \left(\sqrt{5}-\sqrt{3}\right)^2 = 8 - 2\sqrt{15}$

$\therefore x^4 - y^4 = \left(x^2+y^2\right)(x+y)(x-y)$

$= \left(8+2\sqrt{15}+8-2\sqrt{15}\right)$

$\left(\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}\right)$

$\left(\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3}\right)$

$= 16 \times 2\sqrt{5} \times 2\sqrt{3} = 64\sqrt{15}$

148. (1) Using Rule 21,
If. $a + b + c = 0$, then
$a^3 + b^3 + c^3 = 3abc$
Here, $x - 1 + y - 2 + z - 3$
$= x + y + z - 6$
$= 6 - 6 = 0$
$\therefore (x-1)^3 + (y-2)^3 + (z-3)^3$
$= 3(x-1)(y-2)(z-3)$

149. (3) $p^4 = 119 - \frac{1}{p^4}$

$\Rightarrow p^4 + \frac{1}{p^4} = 119$

$\Rightarrow \left(p^2 + \frac{1}{p^2}\right)^2 - 2 = 119$

$\Rightarrow \left(p^2 + \frac{1}{p^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow p^2 + \frac{1}{p^2} = \sqrt{121} = 11$

Again, $\left(p - \frac{1}{p}\right)^2 + 2 = 11$

$\Rightarrow \left(p - \frac{1}{p}\right)^2 = 11 - 2 = 9$

$\Rightarrow p - \frac{1}{p} = \sqrt{9} = \pm 3$

On cubing both sides,

$\left(p - \frac{1}{p}\right)^3 = \pm 27$

$\Rightarrow p^3 - \frac{1}{p^3} - 3(p - q) = \pm 27$

$\Rightarrow p^3 - \frac{1}{p^3} - 3 \times (\pm 3) = \pm 27$

$\Rightarrow p^3 - \frac{1}{p^3} = \pm 27 \pm 9$

$\Rightarrow p^3 - \frac{1}{p^3} = \pm 36$

150. (2) $x + \frac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$
$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x-1)^2 = 0$
$\Rightarrow x - 1 = 0 \Rightarrow x = 1$

$\therefore x^7 + \frac{1}{x^5} = 1 + 1 = 2$

Aliter :
Using Rule 16,

Here, $x + \frac{1}{x} = 2$

$\Rightarrow x^7 + \frac{1}{x^5} = 2$

151. (2) Using Rule 22,
$x = 332,\ y = 333,\ z = 335$
$\therefore x + y + z = 332 + 333 + 335$
$= 1000$
$\therefore x^3 + y^3 + z^3 - 3xyz$

$= \frac{1}{2}(x + y + z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= \frac{1000}{2}[(332-333)^2 + (333-335)^2 + (335-332)^2]$
$= 500(1 + 4 + 9) = 500 \times 14$
$= 7000$

152. (1) Using Rule 8 and 9,
Expression $= m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$
$= m^3 - 3m^2 + 3m - 1 + n^3 + 3n^2 + 3n + 1$
$= (m-1)^3 + (n+1)^3$
$= (-4-1)^3 + (-2+1)^3$
$= (-5)^3 + (-1)^3$
$= -125 - 1 = -126$

153. (1) $x + \frac{1}{x} = 2$

$\Rightarrow \frac{x^2+1}{x} = 2 \Rightarrow x^2 + 1 = 2x$
$\Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x-1)^2 = 0$
$\Rightarrow x - 1 = 0 \Rightarrow x = 1$

$\therefore x^{12} + \frac{1}{x^{12}} = 1 + 1 = 2$

Aliter :
Using Rule 14,

Here, $x + \frac{1}{x} = 2$

$x^{12} + \frac{1}{x^{12}} = 2$

154. (3) $x^3 + y^3 = 72$
$= 64 + 8 = 4^3 + 2^3$
$\therefore x = 4, y = 2 \Rightarrow xy = 8$
$\therefore x - y = 4 - 2 = 2$

155. (1) Using Rule 8,
$x^3 + 27x^2 + 243x + 631$
$= x^3 + 3 . x^2 \times 9 + 3x . 9^2 + 9^3 - 9^3 + 631$
$= (x + 9)^3 - 729 + 631$
$= (2 + 9)^3 - 98$
$= 11^3 - 98 = 1331 - 98 = 1233$

156. (2) $\frac{x^{24}+1}{x^{12}} = 7$

$\Rightarrow \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}} = 7$

$\Rightarrow x^{12} + \frac{1}{x^{12}} = 7$

$\therefore \frac{x^{72}+1}{x^{36}} = \frac{x^{72}}{x^{36}} + \frac{1}{x^{36}}$

$= x^{36} + \frac{1}{x^{36}}$

$= \left(x^{12} + \frac{1}{x^{12}}\right)^3 - 3 \times x^{12} \times \frac{1}{x^{12}} \left(x^{12} + \frac{1}{x^{12}}\right)$

[$\because a^3 + b^3 = (a + b)^3 - 3ab (a + b)$]
$= 7^3 - 3 \times 7 = 343 - 21 = 322$

157. (2) $x^8 - 1 = (x^4)^2 - 1^2$
$= (x^4 + 1) (x^4 - 1)$
$= (x^4 + 1) (x^2 + 1) (x^2 - 1)$
$= (x^4 + 1) (x^2 + 1) (x + 1) (x - 1)$
[$\because a^2 - b^2 = (a + b) (a - b)$]
$x^4 + 2x^3 - 2x - 1$
$= (x^4 - 1) + 2x^3 - 2x$
$= (x^2 + 1) (x^2 - 1) + 2x (x^2 - 1)$
$= (x^2 + 1 + 2x) (x^2 - 1)$
$= (x + 1)^2 (x + 1) (x - 1)$
$\therefore$ H.C.F $= (x + 1) (x - 1)$
$= x^2 - 1$

158. (1) $x^2 + y^2 + z^2 = 2(x + z - 1)$
$\Rightarrow x^2 + y^2 + z^2 = 2x + 2z - 2$
$\Rightarrow x^2 - 2x + y^2 + z^2 - 2z + 2 = 0$
$\Rightarrow x^2 - 2x + 1 + y^2 + z^2 - 2z + 1 = 0$
$\Rightarrow (x - 1)^2 + y^2 + (z - 1)^2 = 0$
[$\because a^2 + b^2 + c^2 = 0 \Rightarrow a = 0, b = 0, c = 0$]
$\therefore x - 1 = 0 \Rightarrow x = 1$
$y = 0$
$z - 1 = 0 \Rightarrow z = 1$
$\therefore x^3 + y^3 + z^3 = 1 + 0 + 1 = 2$

159. (2) $x^2 + x = 5$ (Given)
Let, $x + 3 = a$

$\therefore \frac{1}{x+3} = \frac{1}{a}$

Now,

$a + \frac{1}{a} = (x + 3) + \frac{1}{(x+3)}$

$= \frac{(x+3)^2+1}{x+3}$

$= \frac{x^2+6x+9+1}{x+3}$

$= \frac{x^2+6x+10}{x+3}$

$= \frac{x^2+x+5x+10}{x+3}$

$= \frac{5+5x+10}{x+3}$

$= \frac{5x+15}{x+3} = \frac{5(x+3)}{x+3} = 5$

$\therefore a^3 + \frac{1}{a^3}$

$= \left(a+\frac{1}{a}\right)^3 - 3a \times \frac{1}{a}\left(a+\frac{1}{a}\right)$

$= (5)^3 - 3 \times 5 = 125 - 15 = 110$

160. (2) Using Rule 22,
$x = z = 225, y = 226$
$\therefore x + y + z = 225 + 226 + 225 = 676$
$\therefore x^3 + y^3 + z^3 - 3xyz$

$= \frac{1}{2} (x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2} \times 676 [(225 - 226)^2 + (226 - 225)^2 + (225 - 225)^2]$

$= \frac{1}{2} \times 676 \times (1 + 1) = 676$

161. (3) $4a - \frac{4}{a} = -3$

On dividing by 4,

$\Rightarrow a - \frac{1}{a} = \frac{-3}{4}$

$\therefore a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3a \times \frac{1}{a}\left(a - \frac{1}{a}\right)$

$= \left(\frac{-3}{4}\right)^3 + 3 \times \frac{-3}{4}$

$= -\frac{27}{64} - \frac{9}{4} = \frac{-27-144}{64}$

$= \frac{-171}{64}$

$\therefore a^3 - \frac{1}{a^3} + 3 = \frac{-171}{64} + 3$

$= \frac{-171+192}{64} = \frac{21}{64}$

162. (2) Expression $= 2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$
$= 4b^2c^2 - (2b^2c^2 - 2c^2a^2 - 2a^2b^2 + a^4 + b^4 + c^4)$
$= (2bc)^2 - (a^2 - b^2 - c^2)^2$
$= (2bc + a^2 - b^2 - c^2) (2bc - a^2 + b^2 + c^2)$
$= (a^2 - (b^2 + c^2 - 2bc)) (b^2 + c^2 + 2bc - a^2)$
$= (a^2 - (b - c)^2) ((b + c)^2 - a^2)$
$= (a - b + c) (a + b - c) (a + b + c) (b + c - a)$
If $a + b - c = 0$,
$\therefore$ Expression = 0.

163. (1) $\frac{p^2}{q^2} + \frac{q^2}{p^2} = 1$

$\Rightarrow \frac{p^4+q^4}{p^2 q^2} = 1 \Rightarrow p^4 + q^4 = p^2 q^2$

$\Rightarrow p^4 + q^4 - p^2 q^2 = 0$ ...... (i)
$\therefore p^6 + q^6 = (p^2)^3 + (q^2)^3$
$= (p^2 + q^2) (p^4 + q^4 - p^2 q^2)$

$\left[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)\right]$

$= (p^2 + q^2) \times 0 = 0$

164. (1) $m + 1 = \sqrt{n} + 3$ (Given)

$\Rightarrow m + 1 - 3 = \sqrt{n}$

$\Rightarrow m - 2 = \sqrt{n}$

On cubing both sides,

$(m - 2)^3 = \left(\sqrt{n}\right)^3$

$\Rightarrow m^3 - 3m^2 \times 2 + 3m(2)^2 - 2^3 = n\sqrt{n}$

[$\because (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$\Rightarrow m^3 - 6m^2 + 12m - 8 = n\sqrt{n}$

$\Rightarrow \dfrac{m^3 - 6m^2 + 12m - 8}{\sqrt{n}} = n$

$\Rightarrow \dfrac{m^3 - 6m^2 + 12m - 8}{\sqrt{n}} - n = 0$

$\Rightarrow \dfrac{1}{2}\left[\dfrac{m^3 - 6m^2 + 12m - 8}{\sqrt{n}} - n\right] = 0$

165. (3) $\dfrac{3x - 2y}{2x + 3y} = \dfrac{5}{6}$

$\Rightarrow 18x - 12y = 10x + 15y$
$\Rightarrow 18x - 10x = 12y + 15y$
$\Rightarrow 8x = 27y$

$\Rightarrow \dfrac{x}{y} = \dfrac{27}{8}$

On taking cube root of both sides,

$\dfrac{\sqrt[3]{x}}{\sqrt[3]{y}} = \sqrt[3]{\dfrac{27}{8}} = \dfrac{3}{2}$

By componendo and dividendo,

$\dfrac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}} = \dfrac{3+2}{3-2} = \dfrac{5}{1}$

On squaring both sides,

$\left(\dfrac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}}\right)^2 = 5 \times 5 = 25$

166. (4) $a - \dfrac{1}{(a-3)} = 5$

$\Rightarrow (a - 3) - \dfrac{1}{(a-3)} = 2$

On cubing both sides,

$\left\{(a-3) - \dfrac{1}{(a-3)}\right\}^3 = 8$

$\Rightarrow (a - 3)^3 - \left(\dfrac{1}{a-3}\right)^3 - 3 \times (a - 3)$

$\left(\dfrac{1}{a-3}\right)\left((a-3) - \dfrac{1}{(a-3)}\right) = 8$

$[\because (a - b)^3 = a^3 - b^3 - 3ab\,(a-b)]$

$\Rightarrow (a - 3)^3 - \left(\dfrac{1}{a-3}\right)^3 - 3 \times 2 = 8$

$\Rightarrow (a - 3)^3 - \left(\dfrac{1}{a-3}\right)^3 = 8 + 6$

$= 14$

167. (*) $\left(\dfrac{p^{-1}q^2}{p^3q^{-2}}\right)^{\frac{1}{3}} \div \left(\dfrac{p^6q^{-3}}{p^{-2}q^3}\right)^{\frac{1}{3}}$

$= p^a q^b$

$\Rightarrow \left(p^{-1-3}\ q^{2+2}\right)^{\frac{1}{3}} \div \left(p^{6+2}\ q^{-3-3}\right)^{\frac{1}{3}}$

$= p^a q^b$

$\Rightarrow \left(p^{-4}q^4\right)^{\frac{1}{3}} \div \left(p^8q^{-6}\right)^{\frac{1}{3}} = p^a q^b$

$\Rightarrow \dfrac{p^{-\frac{4}{3}}q^{\frac{4}{3}}}{p^{\frac{8}{3}}q^{\frac{-6}{3}}} = p^a q^b$

$\Rightarrow p^{\frac{-4}{3} - \frac{8}{3}}\ q^{\frac{4}{3} + \frac{6}{3}} = p^a q^b$

$\Rightarrow p^{-4}\ q^{\frac{10}{3}} = p^a q^b$

$\Rightarrow a = -4,\ b = \dfrac{10}{3}$

$\therefore\ a + b = -4 + \dfrac{10}{3} = \dfrac{-2}{3}$

168. (3) $a + b = 1$ (Given)

Expression $= a^3 + b^3 - ab - (a^2 - b^2)^2$

$= (a + b)\,(a^2 - ab + b^2) - ab - (a^2 - b^2)^2$
$= (a^2 - ab + b^2) - ab - (a + b)^2\,(a - b)^2$
$= a^2 - ab + b^2 - ab - (a^2 - 2ab + b^2)$
$= a^2 - 2ab + b^2 - a^2 + 2ab - b^2 = 0$

169. (1) $x = a^{\frac{1}{2}} + a^{\frac{-1}{2}}$

$y = a^{\frac{1}{2}} - a^{\frac{-1}{2}}$

$\therefore\ x^2 - y^2 = 4\,a^{\frac{1}{2}}\,.\,a^{\frac{-1}{2}} = 4$

$[\because (a + b)^2 - (a - b)^2 = 4ab]$

Again, $y^2 - x^2 = -4\,.\,a^{\frac{1}{2}}\,.\,a^{\frac{-1}{2}} = -4$

Expression

$= (x^4 - x^2y^2 - 1) + (y^4 - x^2y^2 + 1)$
$x^2\,(x^2 - y^2) - 1 + y^2\,(y^2 - x^2) + 1$
$= 4x^2 - 1 - 4y^2 + 1$
$= 4\,(x^2 - y^2) = 4 \times 4 = 16$

170. (2) $x^2 + y^2 + z^2 = xy + yz + zx$

$\Rightarrow x^2 + y^2 + z^2 - xy - yz - zx = 0$
$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = 0$
$\Rightarrow x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2zx = 0$
$\Rightarrow (x - y)^2 + (y - z)^2 + (z - x)^2 = 0$
$\therefore\ x - y = 0 \Rightarrow x = y$
$y - z = 0 \Rightarrow y = z$
$z - x = 0 \Rightarrow z = x$
$\therefore\ x = y = z$

[If $a^2 + b^2 + c^2 = 0$, then $a = 0$, $b = 0$, $c = 0$]

$\therefore$ Expression

$= \dfrac{3x^4 + 7y^4 + 5z^4}{5x^2y^2 + 7y^2z^2 + 3z^2x^2}$

$= \dfrac{3x^4 + 7x^4 + 5x^4}{5x^4 + 7x^4 + 3x^4}$

$= \dfrac{15x^4}{15x^4} = 1$

171. (1) $x - \sqrt{3} - \sqrt{2} = 0$

$\Rightarrow x = \sqrt{3} + \sqrt{2}$

Again,

$y - \sqrt{3} + \sqrt{2} = 0$

$\Rightarrow y = \sqrt{3} - \sqrt{2}$

$\therefore\ x - y = \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2}$

$= 2\sqrt{2}$

and $xy = \left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right)$

$= 3 - 2 = 1$

$\therefore$ Expression

$= x^3 - 20\sqrt{2} - y^3 - 2\sqrt{2}$

$= x^3 - y^3 - 22\sqrt{2}$

$= (x - y)^3 + 3xy\,(x - y) - 22\sqrt{2}$

$= \left(2\sqrt{2}\right)^3 + 3\left(2\sqrt{2}\right) - 22\sqrt{2}$

$= 16\sqrt{2} + 6\sqrt{2} - 22\sqrt{2} = 0$

172. (2) $p^3 - q^3 = (p - q)\,\{(p - q)^2 - x\,pq\}$

$\Rightarrow (p - q)\,(p^2 + q^2 + pq) = (p - q)\,(p^2 + q^2 - 2pq - x\,pq)$
$\Rightarrow (p^2 + q^2 + pq) = p^2 + q^2 - (2 + x)\,pq.$
$\therefore\ -(2 + x) = 1$
$\Rightarrow x = -2 - 1 = -3$

173. (1) $x + y + z = 6$

$xy + yz + zx = 10$
$\therefore\ (x + y + z)^2 = 36$
$\Rightarrow x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 36$
$\Rightarrow x^2 + y^2 + z^2 + 2 \times 10 = 36$
$\Rightarrow x^2 + y^2 + z^2 = 36 - 20 = 16$
$\therefore\ x^3 + y^3 + z^3 - 3xyz = (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$
$= 6\,(16 - 10)$
$= 6 \times 6 = 36$

174. (3) $x - \frac{1}{x} = 2$

On cubing both sides,

$\left(x - \frac{1}{x}\right)^3 = 2^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 8$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 2 = 8$

$\Rightarrow x^3 - \frac{1}{x^3} = 8 + 6 = 14$

175. (3) $a^2 + a + 1 = 0$

$\Rightarrow \frac{a^2 + a + 1}{a} = 0$

$\Rightarrow a + 1 + \frac{1}{a} = 0$ ....(i)

Expression $= a^5 + a^4 + 1$

$= a^4 (a + 1) + 1$

$= a^4 \left(-\frac{1}{a}\right) + 1$

$= -a^3 + 1 = 1 - a^3$

$= (1 - a)(1 + a + a^2)$

$= (1 - a) \times 0 = 0$

176. (4) $x = a(b - c)$

$\Rightarrow \frac{x}{a} = b - c$

Similarly, $y = b(c - a)$

$\Rightarrow \frac{y}{b} = c - a$

and, $\frac{z}{c} = a - b$

$\therefore \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = b - c + c - a + a - b = 0$

$\therefore \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c} = \frac{3xyz}{abc}$

[If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$]

177. (3) $x = y = z$

$\therefore$ Expression $= \frac{(x + y + z)^2}{x^2 + y^2 + z^2}$

$= \frac{(x + x + x)^2}{x^2 + x^2 + x^2}$

$= \frac{9x^2}{3x^2} = 3$

178. (3) Expression

$= \frac{\frac{3}{15}a^5b^6c^3 \times \frac{5}{9}ab^5c^4}{\frac{10}{27}a^2bc^3}$

$= \left(\frac{3}{15} \times \frac{5}{9} \times \frac{27}{10}\right)\left(\frac{a^6b^{11}c^7}{a^2bc^3}\right)$

$= \frac{3}{10} a^{6-2} b^{11-1} c^{7-3}$

$= \frac{3}{10} a^4 b^{10} c^4$

$\begin{bmatrix} \because a^m \times a^n = a^{m+n} \\ a^m \div a^n = a^{m-n} \end{bmatrix}$

179. (4) $(2a - 1)^2 + (4b - 3)^2 + (4c + 5)^2 = 0$

$\therefore 2a - 1 = 0 \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$

$4b - 3 = 0 \Rightarrow 4b = 3 \Rightarrow b = \frac{3}{4}$

$4c + 5 = 0 \Rightarrow 4c = -5 \Rightarrow c = \frac{-5}{4}$

[If $x^2 + y^2 + z^2 = 0$, $x = 0$, $y = 0$, $z = 0$]

$\therefore a + b + c = \frac{1}{2} + \frac{3}{4} - \frac{5}{4}$

$= \frac{6 + 9 - 15}{12} = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$

$\therefore$ Required answer $= 0$

180. (4) Using Rule 1 and 8,

$x + \frac{1}{x} = 3$

On squaring both sides,

$x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2}$

$= 9 - 2 = 7$ ... (i)

Again,

$\left(x + \frac{1}{x}\right)^3 = 3^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$\therefore x^3 + \frac{1}{x^3} = 27 - 9 = 18$...(ii)

$\therefore \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right)$

$= 18 \times 7 = 126$

$\Rightarrow x^5 + x + \frac{1}{x^5} + \frac{1}{x} = 126$

$\Rightarrow x^5 + \frac{1}{x^5} = 126 - 3 = 123$

181. (2) $2x + \frac{2}{x} = 3$

On dividing by 2,

$x + \frac{1}{x} = \frac{3}{2}$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + \frac{3 \times 3}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$

$= \frac{27 - 36}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{-9}{8}$

$\therefore x^3 + \frac{1}{x^3} + 2$

$= 2 - \frac{9}{8} = \frac{16 - 9}{8} = \frac{7}{8}$

182. (2) $x = \sqrt[3]{x^2 + 11} - 2$

$\Rightarrow x + 2 = \sqrt[3]{x^2 + 11}$

On cubing both sides,

$(x + 2)^3 = x^2 + 11$

$\Rightarrow x^3 + 2^3 + 3x^2 \times 2 + 3x \times 2^2$
$= x^2 + 11$
$\Rightarrow x^3 + 8 + 6x^2 + 12x = x^2 + 11$
$\Rightarrow x^3 + 5x^2 + 12x = 11 - 8 = 3$

183. (4) If $a^2 + b^2 + c^2 = 0$ then, $a = 0$, $b = 0$ and $c = 0$
$\therefore (x-3)^2 + (y-4)^2 + (z-5)^2 = 0$
$\therefore x - 3 = 0 \Rightarrow x = 3$
$y - 4 = 0 \Rightarrow y = 4$
$z - 5 = 0 \Rightarrow z = 5$
$\therefore x + y + z = 3 + 4 + 5 = 12$

184. (3) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$\therefore (x-4)(x^2 + 4x + 4^2)$
$= x^3 - 4^3 = x^3 - 64$
$\Rightarrow x^3 - p = x^3 - 64$
$\Rightarrow p = 64$

185. (2) Expression

$$= \left(1 - \frac{2xy}{x^2+y^2}\right) \div \left(\frac{x^3-y^3}{x-y} - 3xy\right)$$

$$= \left(\frac{x^2+y^2-2xy}{x^2+y^2}\right) \div \left(\frac{(x-y)(x^2+xy+y^2)}{x-y} - 3xy\right)$$

$$= \frac{(x-y)^2}{x^2+y^2} \div (x^2 + xy + y^2 - 3xy)$$

$$= \frac{(x-y)^2}{x^2+y^2} \div (x^2 - 2xy + y^2)$$

$$= \frac{(x-y)^2}{x^2+y^2} \div (x-y)^2 = \frac{1}{x^2+y^2}$$

186. (1) $\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)}$

$$+ \frac{1}{(c+a)(a+b)}$$

$$= \frac{c+a+a+b+b+c}{(a+b)(b+c)(c+a)}$$

$$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)} = 0$$

187. (1) $x^2 + y^2 + 2x + 1 = 0$
$\Rightarrow x^2 + 2x + 1 + y^2 = 0$
$\Rightarrow (x+1)^2 + y^2 = 0$
$\therefore x + 1 = 0$
$\Rightarrow x = -1$
$y = 0$
$\therefore x^{31} + y^{35} = (-1)^{35} + 0 = -1$

188. (3) $\left(x - \frac{1}{x}\right)^2 = 3$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 3$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 5$$

On cubing both sides,

$$\left(x^2 + \frac{1}{x^2}\right)^3 = (5)^3$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = 125$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3 \times 5 = 125$$

$$\Rightarrow x^6 + \frac{1}{x^6} = 125 - 15 = 110$$

189. (3) $(a + b + c)^2$
$= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
$\therefore (x^2 + x + 3)^2$
$= x^4 + x^2 + 9 + 2x^3 + 6x + 6x^2$
$= x^4 + 2x^3 + 7x^2 + 6x + 9$
On comparing with $x^4 + 2x^3 + ax^2 + bx + 9$
$a = 7,\ b = 6$

190. (3) $(ax + by + cz)^2$
$= (a^2 + b^2 + c^2)(x^2 + y^2 + z^2)$
$= 400$
$\Rightarrow a^2x^2 + b^2y^2 + c^2z^2 + 2abxy + 2bcyz + 2acxz$
$= a^2x^2 + a^2y^2 + a^2z^2 + b^2x^2 + b^2y^2 + b^2z^2 + c^2x^2 + c^2y^2 + c^2z^2$
$\Rightarrow a^2y^2 + a^2z^2 + b^2x^2 + b^2z^2 + c^2x^2 + c^2y^2$
$= 2abxy + 2bcyz + 2acxz$
$\Rightarrow a^2y^2 - 2abxy + b^2x^2 + a^2z^2 + c^2x^2 - 2acxz + b^2z^2 + c^2y^2 - 2bcyz = 0$
$\Rightarrow (ay - bx)^2 + (az - cx)^2 + (bz - cy)^2 = 0$

$$\Rightarrow ay - bx = 0 \Rightarrow ay = bx \Rightarrow \frac{a}{b} = \frac{x}{y}$$

$$az - cx = 0 \Rightarrow az = cx \Rightarrow \frac{a}{c} = \frac{x}{z}$$

$\therefore a = kx$; $b = ky$; $c = kz$
$\therefore a^2 + b^2 + c^2 = 16$
$\Rightarrow k^2(x^2 + y^2 + z^2) = 16$
$\Rightarrow k^2 \times 25 = 16$

$$\Rightarrow k^2 = \frac{16}{25} \Rightarrow k = \frac{4}{5}$$

$$\therefore \frac{a+b+c}{x+y+z} = k = \frac{4}{5}$$

191. (2) Of the given options,
$x = -(a^2 + b^2 + c^2)$

$$\therefore \frac{x+a^2+2c^2}{b+c}$$

$$= \frac{-a^2-b^2-c^2+a^2+2c^2}{b+c}$$

$$= \frac{c^2-b^2}{b+c} = c - b$$

$$\frac{x+b^2+2a^2}{c+a}$$

$$= \frac{-a^2-b^2-c^2-b^2+2a^2}{c+a}$$

$$= \frac{a^2-c^2}{c+a} = a - c$$

$$\frac{x+c^2+2b^2}{a+b}$$

$$= \frac{-a^2-b^2-c^2+c^2+2b^2}{a+b}$$

$$= \frac{b^2-a^2}{a+b} = b - a$$

$\therefore c - b + a - c + b - a = 0$

192. (1) $a^3 - b^3 = 117$; $a - b = 3$
$\Rightarrow (a-b)(a^2 + b^2 + ab) = 117$
$\Rightarrow 3 \times (a^2 + b^2 + ab) = 117$

$$\Rightarrow a^2 + b^2 + ab = \frac{117}{3} = 39$$

$\Rightarrow (a-b)^2 + 3ab = 39$
$\Rightarrow 3^2 + 3ab = 39$
$\Rightarrow 3ab = 39 - 9 = 30$

$$\Rightarrow ab = \frac{30}{3} = 10$$

$\therefore (a+b)^2 = (a-b)^2 + 4ab$
$= 9 + 4 \times 10 = 49$
$\therefore a + b = \sqrt{49} = \pm 7$

193. (1) $a + \frac{1}{a} = -2$
$\Rightarrow a^2 + 1 = -2a$
$\Rightarrow a^2 + 2a + 1 = 0$
$\Rightarrow (a+1)^2 = 0$
$\Rightarrow a + 1 = 0$
$\Rightarrow a = -1$
$\therefore (a)^{1000} + (a)^{-1000}$
$= (-1)^{1000} + (-1)^{-1000}$
$= 1 + 1 = 2$

194. (3) $a^2 = b + c$
$\Rightarrow a^2 + a = a + b + c$
$\Rightarrow a(a+1) = a + b + c$

$$\Rightarrow \frac{1}{a+1} = \frac{a}{a+b+c}$$

Similarly,
$b^2 = a + c$

$$\Rightarrow \frac{1}{b+1} = \frac{b}{a+b+c}$$

and
$c^2 = b + a$

$$\Rightarrow \frac{1}{c+1} = \frac{c}{a+b+c}$$

$\therefore \frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}$

$= \frac{a}{a+b+c} + \frac{b}{a+b+c} +$

$\frac{c}{a+b+c} = \frac{a+b+c}{a+b+c} = 1$

195. (3) $8a + 4b + 6c + 2d = -4$
$2a + 6b + 4c + 8d = 4$
On adding,
$10a + 10b + 10c + 10d = 0$
$\Rightarrow a + b + c + d = 0$
$\Rightarrow a + d = -(b + c)$

$\Rightarrow \frac{a+d}{b+c} = -1$

196. (2) $\frac{x}{(b-c)(b+c-2a)}$

$= \frac{y}{(c-a)(c+a-2b)}$

$= \frac{z}{(a-b)(a+b-2c)} = k$

$\therefore x = k(b-c)(b+c-2a)$
$= k(b^2 - c^2 - 2a(b-c))$
$y = k(c-a)(c+a-2b)$
$= k(c^2 - a^2 - 2b(c-a))$
$z = k(a-b)(a+b-2c)$
$= k(a^2 - b^2 - 2c(a-b))$
$\therefore x + y + z = k(b^2 - c^2 + c^2 - a^2 + a^2 - b^2) - 2\{a(b-c) + b(c-a) + c(a-b)\}$
$= 0 - 2(ab - ac + bc - ab + ac - bc)$
$= 0$

197. (3) $a + \frac{1}{a} = 3$

On cubing both sides,

$\left(a + \frac{1}{a}\right)^3 = 3^3 = 27$

$a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right)$

$= 27$

$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times 3 = 27$

$\Rightarrow a^3 + \frac{1}{a^3} = 27 - 9 = 18$

$\therefore a^3 + 1\frac{1}{a^3} = a^3 + \frac{1}{a^3} + 1$

$= 18 + 1 = 19$

198. (2) $c + \frac{1}{c} = 3$

$\Rightarrow c - 3 = -\frac{1}{c}$

$\therefore (c-3)^7 + \frac{1}{c^7} = \left(-\frac{1}{c}\right)^7 + \frac{1}{c^7}$

$= -\frac{1}{c^7} + \frac{1}{c^7} = 0$

199. (1) $x = \sqrt[3]{7} + 3$

$\Rightarrow x - 3 = \sqrt[3]{7}$

On cubing both sides

$(x-3)^3 = \left(\sqrt[3]{7}\right)^3$

$\Rightarrow x^3 - 3.x^2.3 + 3.x(3)^2 - (3)^3 = 7$
$\Rightarrow x^3 - 9x^2 + 27x - 27 = 7$
$\Rightarrow x^3 - 9x^2 + 27x - 34 = 0$

200. (2) $p(x+y)^2 = 5$

$\Rightarrow (x+y)^2 = \frac{5}{p}$

$q(x-y)^2 = 3 \Rightarrow (x-y)^2 = \frac{3}{q}$

$\therefore (x+y)^2 - (x-y)^2 = \frac{5}{p} - \frac{3}{q}$

$\Rightarrow 4xy = \frac{5}{p} - \frac{3}{q} = \frac{5q-3p}{pq}$

$\therefore p^2(x+y)^2 + 4pq\,xy - q^2(x-y)^2$

$= p^2.\frac{5}{p} + pq.\frac{(5q-3p)}{pq} - q^2.\frac{3}{q}$

$= 5p + 5q - 3p - 3q$
$= 2p + 2q$

201. (4) $x + \frac{1}{x} = -2$

$\Rightarrow x^2 + 1 = -2x$
$\Rightarrow x^2 + 2x + 1 = 0$
$\Rightarrow (x+1)^2 = 0$
$\Rightarrow x = -1$
$\therefore x^p + x^q$
$= (-1)^p + (-1)^q$
$= 1 - 1 = 0$

202. (4)
$(2a-3)^2 + (3b+4)^2 + (6c+1)^2 = 0$

$\therefore 2a - 3 = 0 \Rightarrow a = \frac{3}{2}$,

$3b + 4 = 0 \Rightarrow b = \frac{-4}{3}$

$6c + 1 = 0 \Rightarrow c = -\frac{1}{6}$

$\therefore a + b + c = \frac{3}{2} - \frac{4}{3} - \frac{1}{6}$

$= \frac{9-8-1}{6} = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$

$\therefore \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2} + 3$

$= 0 + 3 = 3$

203. (1) $a + b + c = 1$

$ab + bc + ca = -1$
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow 1 = a^2 + b^2 + c^2 + 2(-1)$
$\Rightarrow a^2 + b^2 + c^2 = 3$
$\therefore a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 1(3+1) = 4$
$\therefore a^3 + b^3 + c^3 = 3abc + 4$
$= -3 + 4 = 1$

204. (1) $3x^2 + 5x + 3 = 0$
$\Rightarrow 3x^2 + 3 = -5x$

$\Rightarrow \frac{3x^2+3}{x} = -5$

$\Rightarrow 3x + \frac{3}{x} = -5$

$\Rightarrow x + \frac{1}{x} = -\frac{5}{3}$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = \left(\frac{-5}{3}\right)^3 = \frac{-125}{27}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$= \frac{-125}{27}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{-5}{3} = \frac{-125}{27}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{-125}{27} + 5$

$= \frac{-125+135}{27} = \frac{10}{27}$

205. (3) $x + y + z = 9$
$x^2 + y^2 + z^2 = 31$
$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow$ 81 = 31 + 2 $(xy + yz + zx)$
$\Rightarrow$ 2 $(xy + yz + zx)$
= 81 – 31
= 50
$\Rightarrow$ $xy + yz + zx = 25$
$\therefore$ $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
= 9 (31 – 25)
= 9 × 6 = 54

206. (3) $x^2 - y^2 + y^2 - z^2 + z^2 - x^2 = 0$

$$\therefore \left(x^2 - y^2\right)^3 + \left(y^2 - z^2\right)^3 + \left(z^2 - x^2\right)^3$$

$$= 3\left(x^2 - y^2\right)\left(y^2 - z^2\right)\left(z^2 - x^2\right)$$

[If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$]

Similarly,

$x - y + y - z + z - x = 0$

$\therefore (x - y)^3 + (y - z)^3 + (z - x)^3$

$= 3(x - y)(y - z)(z - x)$

$$\therefore \frac{\left(x^2 - y^2\right)^3 + \left(y^2 - z^2\right)^3 + \left(z^2 - x^2\right)^3}{(x - y)^3 + (y - z)^3 + (z - x)^3}$$

$$= \frac{3\left(x^2 - y^2\right)\left(y^2 - z^2\right)\left(z^2 - x^2\right)}{3(x - y)(y - z)(z - x)}$$

$= (x + y)(y + z)(z + x)$

207. (3) $\dfrac{x^3 + 3y^2x}{y^3 + 3x^2y} = \dfrac{35}{19}$

By componendo and dividendo,

$$\frac{x^3 + 3y^2x + y^3 + 3x^2y}{x^3 + 3y^2x - y^3 - 3x^2y}$$

$$= \frac{35 + 19}{35 - 19} = \frac{54}{16}$$

$$\Rightarrow \frac{(x + y)^3}{(x - y)^3} = \frac{27}{8} = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \frac{x + y}{x - y} = \frac{3}{2}$$

By componendo and dividendo again

$$\frac{x + y + x - y}{x + y - x + y} = \frac{3 + 2}{3 - 2} \Rightarrow \frac{x}{y} = 5$$

208. (3) $(a - b)^3 = 2^3$
$\Rightarrow a^3 - b^3 - 3ab(a - b) = 8$
$\Rightarrow 26 - 3ab \times 2 = 8$
$\Rightarrow 6ab = 26 - 8 = 18$

$$\Rightarrow ab = \frac{18}{6} = 3$$

$\Rightarrow (a + b)^2 = (a - b)^2 + 4ab$
$= (2)^2 + 4 \times 3 = 4 + 12 = 16$

209. (3) If $a + b + c = 0$, then $a^3 + b^3 + c^3 - 3abc = 0$
Here, $x - 4 + y - 2 + z - 3$
$= x + y + z - 9 = 9 - 9 = 0$
$\therefore (x - 4)^3 + (y - 2)^3 + (z - 3)^3 - 3(x - 4)(y - 2)(z - 3) = 0$

210. (4) $27a^3 - 54a^2b + 36ab^2 - 8b^3$
$= (3a)^3 - 3(3a)^2(2b) + 3 \times 3a \times (2b)^2 - (2b)^3$
$= (3a - 2b)^3$
$= (3 \times 2 - 2(-3))^3 = (6 + 6)^3$
$= (12)^3 = 1728$

211. (2) $a^3 + \dfrac{1}{a^3} = 2$

$\Rightarrow a^6 + 1 = 2a^3$
$\Rightarrow a^6 - 2a^3 + 1 = 0$
$\Rightarrow (a^3 - 1)^2 = 0$
$\Rightarrow a^3 - 1 = 0$
$\Rightarrow a^3 = 1 \Rightarrow a = 1$

$$\therefore \frac{a^2 + 1}{a} = 1 + 1 = 2$$

212. (3) $pq(p + q) = 1$

$$\Rightarrow p + q = \frac{1}{pq}$$

On cubing both sides,

$$(p + q)^3 = \frac{1}{p^3q^3}$$

$$\Rightarrow p^3 + q^3 + 3pq(p + q) = \frac{1}{p^3q^3}$$

$$\Rightarrow \frac{1}{p^3q^3} - p^3 - q^3$$

$= 3pq(p + q) = 3 \times 1 = 3$

213. (3) $x + \dfrac{1}{x} = \sqrt{3}$

On cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3 = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

214. (1) $\dfrac{a}{b} + \dfrac{b}{a} = 1 \Rightarrow \dfrac{a^2 + b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$
$\Rightarrow a^2 - ab + b^2$
$= 0$
$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2) = 0$

215. (3) $l^2 + m^2 + n^2 = 31$;
$l + m + n = 9$
On squaring both sides,
$(l + m + n)^2 = 81$
$\Rightarrow l^2 + m^2 + n^2 + 2(lm + mn + nl)$
$= 81$
$\Rightarrow 31 + 2(lm + mn + nl) = 81$
$\Rightarrow 2(lm + mn + nl) = 81 - 31$
$= 50$

$$\Rightarrow lm + mn + nl = \frac{50}{2} = 25$$

216. (1) $\left(x + \dfrac{1}{x}\right)^2 = 3$

$$\therefore x + \frac{1}{x} = \sqrt{3}$$

On cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

217. (4) $x = \dfrac{3}{2}$ (Given)

$\therefore 27x^3 - 54x^2 + 36x - 11$
$= (3x)^3 - 3 \times (3x)^2 \times 2 + 3 \times 3x(2)^2 - (2)^3 - 3$
$= (3x - 2)^3 - 3$
$[\because (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$

$$= \left(\frac{3 \times 3}{2} - 2\right)^3 - 3$$

$$= \left(\frac{9}{2} - 2\right)^3 - 3$$

$$= \left(\frac{9 - 4}{2}\right)^3 - 3$$

$$= \left(\frac{5}{2}\right)^3 - 3 = \frac{125}{8} - 3$$

$$= \frac{125 - 24}{8} = \frac{101}{8} = 12\frac{5}{8}$$

218. (2) Given,
$a + b + c = 6$ and $ab + bc + ca = 11$
$\therefore\ bc(b + c) + ca(c + a) + ab(a + b) + 3abc$
$= bc(b + c) + abc + ca(c + a) + abc + ab(a + b) + abc$
$= bc(a + b + c) + ca(a + b + c) + ab(a + b + c)$
$= (a + b + c)(bc + ca + ab)$
$= 6 \times 11 = 66$

219. (3) $\left(a + \frac{1}{a}\right)^2 = 3$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

On cubing both sides,

$\left(a + \frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

$\therefore a^6 - \frac{1}{a^6}$

$= \left(a^3 + \frac{1}{a^3}\right)\left(a^3 - \frac{1}{a^3}\right) = 0$

220. (2) $m^3 + n^3 + 3mn$
$= m^3 + n^3 + 3mn(m + n)$
$[\because m + n = 1]$
$= (m + n)^3 = 1$

221. (4) $x^4 + \frac{1}{x^4} = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 119$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 119 + 2 = 121$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 11^2$

$\Rightarrow x^2 + \frac{1}{x^2} = 11$

$\therefore \left(x - \frac{1}{x}\right)^2 + 2 = 11$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 11 - 2 = 9 = 3^2$

$\Rightarrow x - \frac{1}{x} = 3$

222. (4) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$\therefore x^3 + \frac{1}{x^3} = 110$

$\Rightarrow \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 110$

$= 125 - 15$

$\Rightarrow \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$= (5)^3 - 3 \times 5$

$\Rightarrow x + \frac{1}{x} = 5$

223. (3) Given,
$x^2 + y^2 + z^2 = 14$
$xy + yz + zx = 11$
$\therefore (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
$= 14 + 2 \times 11$
$= 14 + 22 = 36$

224. (3) $x = \sqrt[3]{28}$

$\therefore x^3 = \left(\sqrt[3]{28}\right)^3 = 28$

Again, $y = \sqrt[3]{27}$

$\therefore y^3 = \left(\sqrt[3]{27}\right)^3 = 27$

$\therefore$ Expression

$= (x + y) - \frac{1}{x^2 + xy + y^2}$

$= (x + y) - \frac{(x - y)}{(x - y)\left(x^2 + xy + y^2\right)}$

$= (x + y) - \frac{(x - y)}{x^3 - y^3}$

$= (x + y) - \frac{(x - y)}{28 - 27}$

$= x + y - x + y$

$= 2y = 2 \times \sqrt[3]{27} = 2 \times 3 = 6$

225. (3) $x = 12$ and $y = 4$

$\therefore (x + y)^{\frac{x}{y}} = (12 + 4)^{\frac{12}{4}} = (16)^3$

$= 16 \times 16 \times 16 = 4096$

226. (4) $2x + \frac{2}{x} = 3$

On dividing by 2,

$x + \frac{1}{x} = \frac{3}{2}$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} + \frac{9}{2} = \frac{27}{8}$

$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$

$= \frac{27 - 36}{8} = \frac{9}{8}$

$\therefore x^3 + \frac{1}{x^3} + 2$

$= 2 - \frac{9}{8} = \frac{16 - 9}{8} = \frac{7}{8}$

227. (3) $a + b = 3$
On cubing both sides,
$(a + b)^3 = 3^3$
$\Rightarrow a^3 + b^3 + 3ab(a + b) = 27$
$\Rightarrow a^3 + b^3 + 3ab \times 3 = 27$
$\Rightarrow a^3 + b^3 + 9ab - 27 = 0$

228. (4) $x + \frac{1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x \Rightarrow x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0 \Rightarrow x - 1 = 0$
$\Rightarrow x = 1$

$\therefore x^2 + \frac{2}{x^6} = 1 + \frac{2}{1} = 1 + 2 = 3$

229. (2) $\frac{a}{b} + \frac{b}{a} = 1$

$\Rightarrow \frac{a^2 + b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$
$\Rightarrow a^2 - ab + b^2 = 0$
$\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2) = 0$

230. (2) $a^3 - b^3$
$= (a - b)^3 + 3ab(a - b)$
$\Rightarrow 61 = 1 + 3ab \times 1$
$\Rightarrow 61 - 1 = 3ab = 60$

$\Rightarrow ab = \frac{60}{3} = 20$

231. (1) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (a - b)\{(a + b)^2 - ab\}$
On comparing with
$p^3 - q^3 = (p - q)\{(p + q)^2 - x\,pq\}$, $x = 1$

232. (1) $a^2 = by + cz$

$\Rightarrow a^2 + ax = ax + by + cz$

$\Rightarrow a\,(a + x) = ax + by + cz$

$\Rightarrow \dfrac{1}{a+x} = \dfrac{a}{ax+by+cz}$

Similarly,

$b^2 = cz + ax$

$\Rightarrow b^2 + by = by + cz + ax$

$\Rightarrow b\,(b + y) = ax + by + cz$

$\Rightarrow \dfrac{1}{b+y} = \dfrac{b}{ax+by+cz}$

$c^2 = ax + by$

$\Rightarrow c^2 + cz = ax + by + cz$

$\Rightarrow c\,(c + z) = ax + by + cz)$

$\Rightarrow \dfrac{1}{c+z} = \dfrac{c}{ax+by+cz}$

$\therefore \dfrac{x}{a+x} + \dfrac{y}{b+y} + \dfrac{z}{c+z}$

$= \dfrac{ax}{ax+by+cz} + \dfrac{by}{ax+by+cz} + \dfrac{cz}{ax+by+cz}$

$= \dfrac{ax+by+cz}{ax+by+cz} = 1$

233. (3) $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
$= (a - b)\,((a - b)^2 + 3ab)$

$\therefore$ On comparing with
$p^3 - q^3 = (p - q)\,((p - q)^2 + x\,pq)$ $x = 3$

234. (3) $\left(a + \dfrac{1}{a}\right)^2 = 3$

$\Rightarrow a + \dfrac{1}{a} = \sqrt{3}$

On cubing both sides,

$\left(a + \dfrac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 3\left(a + \dfrac{1}{a}\right) = 3\sqrt{3}$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \dfrac{1}{a^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

$\Rightarrow \dfrac{a^6+1}{a^3} = 0$

$\Rightarrow a^6 + 1 = 0$

$\therefore a^{18} + a^{12} + a^6 + 1$
$= a^{12}\,(a^6 + 1) + 1\,(a^6 + 1)$
$= (a^6 + 1)\,(a^{12} + 1) = 0$

235. (1) $x + 5 + \dfrac{1}{x+1} = 6$

$\Rightarrow (x + 1) + \dfrac{1}{(x+1)} = 6 - 4 = 2$

On cubing both sides,

$\left\{(x+1) + \dfrac{1}{(x+1)}\right\}^3 = 8$

$\Rightarrow (x + 1)^3 + \dfrac{1}{(x+1)^3} + 3$

$\left\{(x+1) + \dfrac{1}{(x+1)}\right\} = 8$

$\Rightarrow (x + 1)^3 + \dfrac{1}{(x+1)^3} + 3 \times 2$
$= 8$

$\Rightarrow (x + 1)^3 + \dfrac{1}{(x+1)^3} = 8 - 6$
$= 2$

236. (2) $a + b + c = 15$,

$\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{71}{abc}$

$\Rightarrow \dfrac{bc+ac+ab}{abc} = \dfrac{71}{abc}$

$\Rightarrow ab + bc + ca = 71$

$\therefore a^3 + b^3 + c^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ac)$
$= (a + b + c)\,\{(a + b + c)^2 - 3\,(ab + bc + ac)\}$
$= 15\,(15^2 - 3 \times 71)$
$= 15\,(225 - 213) = 15 \times 12$
$= 180$

237. (4) $p^4 + q^4 = (p^2)^2 + (q^2)^2$
$= (p^2 + q^2)^2 - 2p^2q^2$

$= (p^2 + q^2)^2 - \left(\sqrt{2}\,pq\right)^2$

$= (p^2 + q^2 + \sqrt{2}\,pq)\,(p^2 + q^2 - \sqrt{2}\,pq)$

Clearly, $k = \sqrt{2}$

238. (4) $x^4 + 64 = (x^2)^2 + (8)^2$
$= (x^2 + 8)^2 - 2 \times 8x^2$
$[\because a^2 + b^2 = (a + b)^2 - 2ab]$
$= (x^2 + 8)^2 - (4x)^2$
$= (x^2 + 4x + 8)\,(x^2 - 4x + 8)$

239. (4) $a^4 + b^4 - a^3 - b^3 - 2a^2b^2 + ab$
$= a^4 + b^4 - 2a^2b^2 - a^3 - b^3 + ab$
$= (a^2 - b^2)^2 - (a^3 + b^3) + ab$
$= (a + b)^2\,(a - b)^2 - (a + b)\,(a^2 - ab + b^2) + ab$
$= (a - b)^2 - a^2 + ab - b^2 + ab$
$[\because a + b = 1]$
$= (a - b)^2 - (a - b)^2 = 0$

240. (3) $2a^3 + 2b^3 + 2c^3 - 6abc$
$= 2\,(a^3 + b^3 + c^3 - 3abc)$

$= 2\,(a + b + c) \times \dfrac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$
$= (299 + 298 + 297)\,\{(299 - 298)^2 + (298 - 297)^2 + (297 - 299)^2\}$
$= 894 \times (1 + 1 + 4)$
$= 894 \times 6 = 5364$

241. (1) $x + \dfrac{1}{x} = \sqrt{3}$

On cubing both sides,

$\left(x + \dfrac{1}{x}\right)^3 = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3\left(x + \dfrac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

$\Rightarrow x^6 + 1 = 0$

$\therefore x^{18} + x^{12} + x^6 + 1$
$= x^{12}\,(x^6 + 1) + 1\,(x^6 + 1)$
$= (x^6 + 1)\,(x^{12} + 1) = 0$

242. (4) $x = 1 + \sqrt{2} + \sqrt{3}$

$\Rightarrow x - 1 = \sqrt{3} + \sqrt{2}$

On squaring both sides,

$x^2 - 2x + 1 = 3 + 2 + 2\sqrt{6}$

$\Rightarrow x^2 - 2x + 1 - 5 = 2\sqrt{6}$

$\Rightarrow x^2 - 2x - 4 = 2\sqrt{6}$

On squaring again,

$(x^2 - 2x - 4)^2 = \left(2\sqrt{6}\right)^2$

$\Rightarrow x^4 + 4x^2 + 16 - 4x^3 + 16x - 8x^2 = 24$

$\Rightarrow x^4 - 4x^3 - 4x^2 + 16x - 8 = 0$

$\Rightarrow 2x^4 - 8x^3 - 8x^2 + 32x - 16 = 0$

$\therefore 2x^4 - 8x^3 - 5x^2 + 26x - 28$
$= 2x^4 - 8x^3 - 8x^2 + 32x - 16 + 3x^2 - 6x - 12$
$= 0 + 3\,(x^2 - 2x - 4) = 3 \times 2\sqrt{6}$
$= 6\sqrt{6}$

243. (2) $x + y = 1 + xy$ (given)

$\therefore x^3 + y^3 - x^3y^3$
$= (x + y)^3 - 3xy\,(x + y) - x^3y^3$
$= (1 + xy)^3 - 3xy\,(1 + xy) - x^3y^3$
$= 1 + x^3y^3 + 3xy + 3x^2y^2 - 3xy - 3x^2y^2 - x^3y^3 = 1$

244. (4) $p = 3 + \dfrac{1}{p}$ (Given)

$\therefore p - \dfrac{1}{p} = 3$

On squaring both sides,

$\left(p - \dfrac{1}{p}\right)^2 = (3)^2 = 9$

$\Rightarrow p^2 + \dfrac{1}{p^2} - 2 = 9$

$\Rightarrow p^2 + \dfrac{1}{p^2} = 9 + 2 = 11$

On squaring again,

$$\left(p^2+\frac{1}{p^2}\right)^2 = (11)^2$$

$$\Rightarrow p^4+\frac{1}{p^4}+2 = 121$$

$$\Rightarrow p^4+\frac{1}{p^4} = 121 - 2 = 119$$

245. (3) $x^4 + x^2y^2 + y^4 = 6$

$\Rightarrow (x^2 - xy + y^2)(x^2 + xy + y^2) = 6$

$\Rightarrow 2 \times (x^2 + xy + y^2) = 6$

$\Rightarrow x^2 + xy + y^2 = \frac{6}{2} = 3$

246. (1) Given, $\left(a+\frac{1}{a}\right)^2 = 3$

$$\Rightarrow a+\frac{1}{a} = \sqrt{3}$$

On cubing both sides,

$$\left(a+\frac{1}{a}\right)^3 = \left(\sqrt{3}\right)^3$$

$$\Rightarrow a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3+\frac{1}{a^3}+3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow a^3+\frac{1}{a^3} = 3\sqrt{3}-3\sqrt{3} = 0$$

247. (4)

$$\frac{a^2+b^2}{c^2} = \frac{b^2+c^2}{a^2} = \frac{c^2+a^2}{b^2} = \frac{1}{k}$$

$\Rightarrow c^2 = k(a^2 + b^2)$;

$a^2 = k(b^2 + c^2)$;

$b^2 = k(c^2 + a^2)$

$\therefore a^2 + b^2 + c^2 = k(b^2 + c^2 + c^2 + a^2 + a^2 + b^2)$

$\Rightarrow a^2 + b^2 + c^2 = 2k(a^2 + b^2 + c^2)$

$\Rightarrow 2k = 1$

$\Rightarrow k = \frac{1}{2}$

248. (2) $\because 2x + \frac{2}{9x} = 4$

On dividing both sides by 2,

$$x+\frac{1}{9x} = 2$$

On multiplying both sides by 3,

$$3x+\frac{1}{3x} = 6$$

On cubing both sides,

$$\left(3x+\frac{1}{3x}\right)^3 = 6^3$$

$$\therefore 27x^3 + \frac{1}{27x^3}+3\times 3x\times\frac{1}{3x}\left(3x+\frac{1}{3x}\right) = 216$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} + 3 \times 6 = 216$$

$$\Rightarrow 27x^3 + \frac{1}{27x^3} = 216 - 18$$

$= 198$

249. (3) $xy(x + y) = m$ (Given)

$\therefore x^3 + y^3 + 3m$

$= x^3 + y^3 + 3xy(x + y)$

$$= (x+y)^3 = \left(\frac{m}{xy}\right)^3 = \frac{m^3}{x^3y^3}$$

250. (4) Given,

$$p+\frac{1}{p+2} = 1$$

$$\Rightarrow (p + 2) + \frac{1}{p+2} = 2 + 1 = 3$$

On cubing both sides,

$$(p + 2)^3 + \frac{1}{(p+2)^3} + 3(p + 2)\times\frac{1}{(p+2)}\left(p+2+\frac{1}{p+2}\right) = 27$$

$$\Rightarrow (p + 2)^3 + \frac{1}{(p+2)^3} + 3 \times 3 = 27$$

$$\Rightarrow (p + 2)^3 + \frac{1}{(p+2)^3} = 27 - 9 = 18$$

$$\therefore (p + 2)^3 + \frac{1}{(p+2)^3} - 3$$

$= 18 - 3 = 15$

251. (1) $x^3+\frac{1}{x^3} = 0$

$$\Rightarrow \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) = 0$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^3 = 3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 3$$

On squaring both sides,

$$\left(x+\frac{1}{x}\right)^4 = 3^2 = 9$$

252. (2) $2x-\frac{2}{x} = 1$

On dividing both sides by 2,

$$x-\frac{1}{x} = \frac{1}{2}$$

On cubing both sides,

$$\left(x-\frac{1}{x}\right)^3 = \frac{1}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right) = \frac{1}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\times\frac{1}{2} = \frac{1}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3} = \frac{3}{2}+\frac{1}{8}$$

$$\Rightarrow x^3-\frac{1}{x^3} = \frac{12+1}{8} = \frac{13}{8}$$

253. (2) $4b^2c^2 - (b^2 + c^2 - a^2)^2$

$= (2bc)^2 - (b^2 + c^2 - a^2)^2$

$= (2bc + b^2 + c^2 - a^2)(2bc - b^2 - c^2 + a^2)$

$= \{(b + c)^2 - a^2\}\{a^2 - (b^2 + c^2 - 2bc)\}$

$= (b + c + a)(b + c - a)\{a^2 - (b-c)^2\}$

$= (b + c + a)(b + c - a)(a + b - c)(a - b + c)$

$\therefore$ Required sum

$= b + c + a + b + c - a + a + b - c + a - b + c$

$= 2(a + b + c)$

254. (3) $(4a - 3)^2 = 0 \Rightarrow 4a - 3 = 0$

$$\Rightarrow 4a = 3 \Rightarrow a = \frac{3}{4}$$

$\therefore 64a^3 - 48a^2 + 12a + 13$

$$= 64 \times \left(\frac{3}{4}\right)^3 - 48 \times \left(\frac{3}{4}\right)^2 + 12 \times \frac{3}{4} + 13$$

$$= 64 \times \frac{27}{64} - \frac{48\times 9}{16} + 9 + 13$$

$= 27 - 27 + 22 = 22$

255. (3) $a = 101$ (Given)

$\therefore a(a^2 - 3a + 3)$

$= a^3 - 3a^2 + 3a - 1 + 1$

$= (a - 1)^3 + 1 = (100)^3 + 1$

$= 1000001$

256. (4) $x + \frac{1}{x} = -2$

$$\Rightarrow \frac{x^2+1}{x} = -2$$

$\Rightarrow x^2 + 1 = -2x$

$\Rightarrow x^2 + 2x + 1 = 0$
$\Rightarrow (x + 1)^2 = 0$
$\Rightarrow x + 1 = 0 \quad \Rightarrow x = -1$

$\therefore\ x^7 + \dfrac{1}{x^7} = (-1)^7 + \dfrac{1}{(-1)^7}$
$= -1 -1 = -2$

257. (1) $a^2 + b^2 + c^2 = 14$ ..... (i)
$a + b + c = 6$
$\therefore\ (a + b + c)^2 = 6^2 = 36$
$\Rightarrow\ a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$
$\Rightarrow 14 + 2(ab + bc + ca) = 36$
$\Rightarrow 2(ab + bc + ca) = 36 - 14 = 22$

$\Rightarrow ab + bc + ca = \dfrac{22}{2} = 11$

258. (2) $\dfrac{a}{b} + \dfrac{b}{a} = 1$

$\Rightarrow \dfrac{a^2+b^2}{ab} = 1$
$\Rightarrow a^2 + b^2 = ab$
$\Rightarrow a^2 - ab + b^2 = 0$
$\therefore\ a^3 + b^3$
$= (a + b)(a^2 - ab + b^2) = 0$

259. (4) $a + b = 5$
$\Rightarrow a - 3 = 2 - b$
$\Rightarrow (a - 3)^7 = (2 - b)^7$
$\Rightarrow (a - 3)^7 = -(b - 2)^7$
$\Rightarrow (a - 3)^7 + (b - 2)^7 = 0$

260. (3) $x^2 - 2x + 1 = 0$
$\Rightarrow (x - 1)^2 = 0$
$\Rightarrow x - 1 = 0 \Rightarrow x = 1$

$\therefore\ x^4 + \dfrac{1}{x^4} = 1 + 1 = 2$

261. (3) $a^2 + b^2 + c^2 = 83$
$a + b + c = 15$
$\therefore\ (a + b + c)^2$
$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\Rightarrow (15)^2 = 83 + 2(ab + bc + ca)$
$\Rightarrow 225 - 83 = 2(ab + bc + ca)$
$\Rightarrow 142 = 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = \dfrac{142}{2} = 71$

262. (4) $m - n = 2;\ mn = 15$
$\therefore\ (m + n)^2 = (m - n)^2 + 4mn$
$= 4 + 4 \times 15 = 64$
$\Rightarrow m + n = \sqrt{64} = 8$
$\therefore\ m + n + m - n = 8 + 2 = 10$
$\Rightarrow 2m = 10 \Rightarrow m = 5$
$\therefore\ m + n = 8$
$\Rightarrow 5 + n = 8$
$\Rightarrow n = 8 - 5 = 3$
$\therefore\ (m^2 - n^2)(m^3 - n^3)$
$= (5^2 - 3^2)(5^3 - 3^3)$
$= (25 - 9)(125 - 27)$
$= 16 \times 98 = 1568$

263. (4) Given, $xy + yz + zx = 1$

$\therefore$ Expression $= \dfrac{1+y^2}{(x+y)(y+z)}$

$= \dfrac{1+y^2}{xy+xz+y^2+yz} = \dfrac{1+y^2}{1+y^2}$
$= 1$

264. (2) $x^2 - 4x + 1 = 0$
$\Rightarrow x^2 + 1 = 4x$

$\Rightarrow \dfrac{x^2+1}{x} = \dfrac{4x}{x}$

$\Rightarrow x + \dfrac{1}{x} = 4$

On cubing both sides,

$\left(x + \dfrac{1}{x}\right)^3 = 64$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3\left(x + \dfrac{1}{x}\right) = 64$

$\Rightarrow x^3 + \dfrac{1}{x^3} + 3 \times 4 = 64$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 64 - 12 = 52$

$\Rightarrow \dfrac{x^6+1}{x^3} = 52$

265. (3) $x = a + \dfrac{1}{a};\ y = a - \dfrac{1}{a}$

$\therefore\ x^2 - y^2 = \left(a + \dfrac{1}{a}\right)^2 - \left(a - \dfrac{1}{a}\right)^2$

$= 4a \times \dfrac{1}{a} = 4$
$[\because (a + b)^2 - (a - b)^2 = 4ab]$
$\therefore\ x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2$
$= 4^2 = 16$

266. (2) $a^3 - b^3 = 56$
$\Rightarrow (a - b)^3 + 3ab(a - b) = 56$
$\Rightarrow (2)^3 + 3ab \times 2 = 56$
$\Rightarrow 6ab = 56 - 8 = 48$

$\Rightarrow ab = \dfrac{48}{6} = 8$

$\therefore\ a^2 + b^2 = (a - b)^2 + 2ab$
$= 2^2 + 2 \times 8 = 4 + 16 = 20$

267. (2) $x + y + z = 1$ ..... (i)
Again,

$\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = \dfrac{yz+zx+xy}{xyz} = 1$

$\Rightarrow xy + yz + zx = xyz = -1$ .. (ii)
$\therefore\ (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
$\Rightarrow 1 = x^2 + y^2 + z^2 - 2$
$\Rightarrow x^2 + y^2 + z^2 = 2 + 1 = 3$.(iii)
$\therefore\ x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= 1(3 + 1) = 4$
$\Rightarrow x^3 + y^3 + z^3 + 3 = 4$
$\Rightarrow x^3 + y^3 + z^3 = 4 - 3 = 1$

268. (2) $\dfrac{a^2+1}{a} = 3$

$\Rightarrow a + \dfrac{1}{a} = 3$

On cubing both sides,

$\left(a + \dfrac{1}{a}\right)^3 = 3^3$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 3\left(a + \dfrac{1}{a}\right) = 27$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 3 \times 3 = 27$

$\Rightarrow \dfrac{a^6+1}{a^3} = 27 - 9 = 18$

269. (2) If c be the third proportional between a and b, then

$\dfrac{a}{b} = \dfrac{b}{c}$

$\Rightarrow c = \dfrac{b^2}{a} = \dfrac{\left\{\left(x^2-y^2\right)^2\right\}^2}{(x-y)^2}$

$= \dfrac{\{(x+y)(x-y)\}^4}{(x-y)^2}$
$= (x + y)^4 (x - y)^2$

270. (3) If $a^2 + b^2 + c^2 = 0$
$\Rightarrow a = 0,\ b = 0,\ c = 0$
$\therefore\ (x - 5)^2 + (y - 2)^2 + (z - 9)^2 = 0$
$\therefore\ x - 5 = 0 \Rightarrow x = 5$
$y - 2 = 0 \Rightarrow y = 2$
$z - 9 = 0 \Rightarrow z = 9$
$\therefore\ x + y - z = 5 + 2 - 9 = -2$

271. (3) $x + \dfrac{1}{x} = 3$

On squaring both sides,

$\left(x + \dfrac{1}{x}\right)^2 = 9$

$\Rightarrow x^2 + \dfrac{1}{x^2} + 2 = 9$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 9 - 2 = 7$

On squaring again,

$\left(x^2 + \dfrac{1}{x^2}\right)^2 = 49$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 49$

$\Rightarrow x^4 + \frac{1}{x^4} = 49 - 2 = 47$

On squaring again,

$(x^4)^2 + \left(\frac{1}{x^4}\right)^2 + 2 = 47^2 = 2209$

$\Rightarrow x^8 + \frac{1}{x^8} = 2209 - 2 = 2207$

272. (4) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\{(x - y)^2 + (y - z)^2 + (z - x)^2\}$

$\therefore \frac{x^3 + y^3 + z^3 - 3xyz}{x - y + z}$

$= \frac{\frac{1}{2}(x+y+z)\left\{(x-y)^2 + (y-z)^2 + (z-x)^2\right\}}{x - y + z}$

$= \frac{\frac{1}{2}(999 + 1000 + 1001)\left\{\begin{matrix}(999-1000)^2 + \\ (1000-10001)^2 + (1001-999)^2\end{matrix}\right\}}{(999 - 1000 + 1001)}$

$= \frac{3000}{2 \times 1000} \times (1 + 1 + 4)$

$= \frac{18}{2} = 9$

273. (3) If $a + b + c = 0$ then, $a^3 + b^3 + c^3 = 3abc$

$\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

274. (4) $\frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$

$\Rightarrow \frac{q+p}{pq} = \frac{1}{p+q}$

$\Rightarrow (p + q)^2 = pq$

$\Rightarrow p^2 + 2pq + q^2 = pq$

$\Rightarrow p^2 + pq + q^2 = 0$

$\therefore p^3 - q^3$

$= (p - q)(p^2 + pq + q^2) = 0$

275. (3) $x^2 - y^2 + 10xz + 10yz$

$\Rightarrow (x + y)(x - y) + 10z(x + y)$

$= (x + y)(x - y + 10z)$

$= (93 + 93)(93 - 93 + 10 \times 94)$

$= 186 \times 940 = 174840$

276. (2) $x^3 + y^3 + z^3 - 3xyz$

$= \frac{1}{2}(x + y + z)\left\{(x-y)^2 + (y-z)^2 + (z-x)^2\right\}$

$= \frac{1}{2}(222 + 223 + 225)\left\{(222-223)^2 + (223-225)^2 + (225-222)^2\right\}$

$= \frac{1}{2} \times 670\,(1 + 4 + 9)$

$= \frac{670 \times 14}{2} = 4690$

277. (2) $\frac{a}{b} + \frac{b}{a} = 1$

$\Rightarrow \frac{a^2 + b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$

$\Rightarrow a^2 - \text{ab} + b^2 = 0$

$\therefore a^3 + b^3 - 2 = (a + b)(a^2 - ab + b^2) - 2$

$= -2$

278. (3) $x + \frac{1}{x} = \sqrt{3}$

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = \left(\sqrt{3}\right)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

279. (1) It is given,

$a + b = 3$

$\therefore a^3 + b^3 + 9ab$

$= a^3 + b^3 + 3ab \times 3$

$= a^3 + b^3 + 3ab\,(a + b)$

$= (a + b)^3 = (3)^3 = 27$

280. (2) $6x^2 - 12x + 1 = 0$

$\Rightarrow 6x^2 + 1 = 12x$

$\Rightarrow \frac{6x^2 + 1}{2x} = \frac{12x}{2x}$

$\Rightarrow 3x + \frac{1}{2x} = 6$

On cubing both sides,

$\left(3x + \frac{1}{2x}\right)^3 = (6)^3$

$\Rightarrow (3x)^3 + \left(\frac{1}{2x}\right)^3 = 3 \times 3x \times \frac{1}{2x}\left(3x + \frac{1}{2x}\right) = 216$

$\Rightarrow 27x^3 + \frac{1}{8x^3} + \frac{9}{2} \times 6 = 216$

$\Rightarrow 27x^3 + \frac{1}{8x^3} = 216 - 27 = 189$

281. (1) $x^2 + \frac{1}{x^2} = 98$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 = 98$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 98 + 2 = 100$

$\Rightarrow x + \frac{1}{x} = \sqrt{100} = 10$ ...(i)

On cubing both sides,

$\left(x + \frac{1}{x}\right)^3 = (10)^3 = 1000$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 1000$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 10 = 1000$

$\Rightarrow x^3 + \frac{1}{x^3} = 1000 - 30 = 970$

282. (2) $x = y + z \Rightarrow x - y - z = 0$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

$\therefore (x)^3 + (-y)^3 + (-z)^3$

$= 3x(-y)(-z) = 3xyz$

283. (2) $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$

$= x^5 - (11 + 1)x^4 + (11 + 1)x^3 - (11 + 1)x^2 + (11 + 1)x - 1$

$= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$

$= x - 1 = 11 - 1 = 10$ [$\because x = 11$]

284. (1) $a^3 - 7a - 6 = 0$

When $a = -1$

$f(a) = -1 + 7 - 6 = 0$

$\therefore (a + 1)$ is a factor.

$$
\begin{array}{l}
a+1)\ a^3 - 7a - 6\ (a^2 - a - 6 \\
\underline{\ \ a^3 + a^2} \\
\quad -a^2 - 7a \\
\quad \underline{-a^2 - a} \\
\quad\quad\ \ + \quad + \\
\quad\quad -6a - 6 \\
\quad\quad \underline{-6a - 6} \\
\quad\quad\quad \times
\end{array}
$$

$\therefore a^2 - a - 6 = a^2 - 3a + 2a - 6$

$= a\,(a - 3) + 2\,(a - 3)$

$= (a - 3)(a + 2)$

$\therefore x + y + z$

$= a + 1 + a - 3 + a + 2 = 3a$

TYPE-III

1. (1) $2^x . 2^y = 8$

$\Rightarrow 2^{x+y} = 2^3$

$\Rightarrow x + y = 3$ ...(i)

$9^x . 3^y = 3^4$

$\Rightarrow 3^{2x} . 3^y = 3^4$

$\Rightarrow 2x + y = 4$ ...(ii)

By equation (ii) – (i),

$x = 1$

From equation (i),

$1 + y = 3$

$\Rightarrow y = 3 - 1 = 2$

Method 2 :

You can check through options also.

$\Rightarrow y = 2$

$\Rightarrow (1, 2)$

2. (2) $2x + y = 5$ ...(i)

$x + 2y = 4$ ...(ii)

By equation (i) × 2 – equation (ii), we have

$4x + 2y = 10$

$x + 2y = 4$

– – –

$3x = 6$

$\Rightarrow x = 2$

From equation (i),

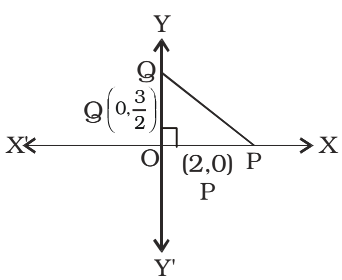
$2 \times 2 + y = 5$

$\Rightarrow y = 5 - 4 = 1$

$\therefore$ Point of intersection = (2, 1)

3. (2) OP = 2

$OQ = \frac{3}{2}$



$\therefore PQ = \sqrt{OP^2 + OQ^2}$

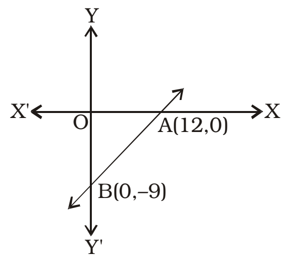
$= \sqrt{2^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{4 + \frac{9}{4}}$

$= \sqrt{\frac{16+9}{4}} = \sqrt{\frac{25}{4}}$

$= \frac{5}{2} = 2.5$ cm

4. (1)



Putting $x = 0$ in $9x - 12y$

$= 108$,

$0 - 12y = 108$

$y = -9$

Putting $y = 0$ in $9x - 12y = 108$

$9x - 0 = 108$

$\Rightarrow x = 12$

$\therefore$ OA = 12, OB = 9

$\therefore AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{12^2 + 9^2}$

$= \sqrt{144 + 81}$

$= \sqrt{225}$

$\therefore$ AB = 15 units

5. (4) At x-axis, y-co-ordinate = 0

$\therefore$ Putting $y = 0$ in $7x - 3y = 2$,

$7x - 3 \times 0 = 2$

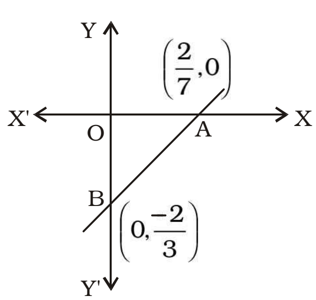
$\Rightarrow 7x = 2$

$\Rightarrow x = \frac{2}{7}$

Similarly, putting $x = 0$ in

$7x - 3y = 2$,

$y = -\frac{2}{3}$



6. (3) Putting $x = 2$ in the equation $2x + y = 6$,

$2 \times 2 + y = 6$

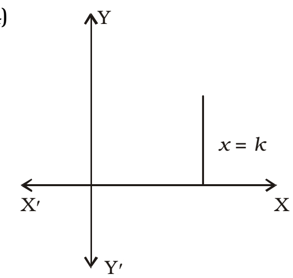
$\Rightarrow y = 6 - 4 = 2$

$\therefore$ Required point = (2, 2)

7. (3) Putting $x = 0$ in equation $2x + 3y = 0$, we get $y = 0$

Hence, this straight line passes through the origin.

8. (4)



Hence, the graph of the equation will be a line parallel to y – axis i.e. $x = k$.

9. (2) At y-axis, $x = 0$

$\therefore$ Putting $x = 0$

in $2x - 3y = 6$,

$0 - 3y = 6 \Rightarrow y = -2$

$\therefore$ Co-ordinates of point of intersection = (0, – 2)

10. (2) Putting $x = 9$ in the equation $25x + 75y = 225$,

$\Rightarrow 25 \times 9 + 75y = 225$

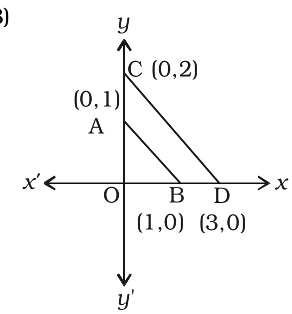
$\Rightarrow 75y = 225 - 225 = 0$

$\Rightarrow y = 0$

$\therefore$ Point of intersection

= (9, 0)

11. (3)



$x = 0$ is the equation of y-axis.

$y = 0$ is the equation of x-axis.

Putting $x = 0$ in $x + y = 1$, $y = 1$

Putting $y = 0$ in $x + y = 1$, $x = 1$

Putting $x = 0$ in $2x + 3y = 6$

$3y = 6 \Rightarrow y = 2$

Putting $y = 0$ in $2x + 3y = 6$

$2x = 6 \Rightarrow x = 3$

$\therefore$ OB = 1; OA = 1

OD = 3; OC = 2

$\therefore$ Required area = ΔOCD – ΔOAB

$= \frac{1}{2} \times 3 \times 2 - \frac{1}{2} \times 1 \times 1$

$= 3 - \frac{1}{2} = 2\frac{1}{2}$ sq. units

12. (2) When a straight line cuts x-axis, the coordinates of point of intersection $= (x, 0)$, i.e., $y = 0$.

$\therefore$ Putting $y = 0$ in $4x - 5y = 20$

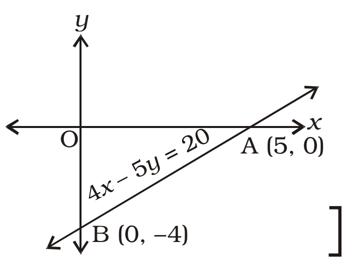
$4x = 20 \Rightarrow x = 5$

$\therefore$ Point of intersection $= (5, 0)$

[**Note :** Putting $y = 0$ in $4x - 5y = 20$, point of intersection on x-axis $= (5, 0)$

Putting $x = 0$ in $4x - 5y = 20$, point of intersection on y-axis $= (0, -4)$.

Look at the graph of the equation:



]

13. (2) $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

and $3y - 9 = 0 \Rightarrow y = 3$

$\therefore \left(\frac{-1}{2}, 3\right)$

14. (1) $ax + by + c = 0$

When $c = 0$,

$ax + by = 0$

$by = -ax \Rightarrow y = -\frac{a}{b}x$

When $x = 0$, $y = 0$ i.e. this line passes through the origin $(0, 0)$.

15. (2) Check through options

$y = 4x$,

When, $x = 1$, $y = 4$

16. (1) $3x + 2y = 18$ ...(i)

$3y - 2x = 1$ ...(ii)

By equation (i) $\times 2$ + (ii) $\times 3$ gives,

$$\begin{array}{r} 6x + 4y = 36 \\ -6x + 9y = 3 \\ \hline 13y = 39 \end{array}$$

$\Rightarrow y = 3$

Putting $y = 3$ in (ii)

$3(3) - 2x = 1 \Rightarrow x = 4$

$\therefore (p, q) = (4, 3)$

and hence, $p + q = 7$

17. (3) On putting $y = -x$ in the equation $5y + 7x = 24$,

$-5x + 7x = 24$

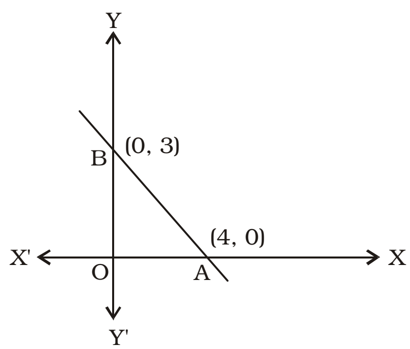
$\Rightarrow 2x = 24 \Rightarrow x = 12$

& $y = -12$

$\therefore m = x = 12$, $n = y = -12$

$\Rightarrow m + n = 12 - 12 = 0$

18. (3)



x – axis $\Rightarrow y = 0$, putting in equation $3x + 4y = 12$

$3x = 12 \Rightarrow x = 4$

$\Rightarrow$ Co-ordinates of point of intersection on x-axis $= (4, 0)$

Putting $x = 0$ in the equation $3x + 4y = 12$

$4y = 12 \Rightarrow y = 3$

$\therefore$ Co-ordinates of point of intersection on y – axis $= (0, 3)$

$\therefore$ OA = 4

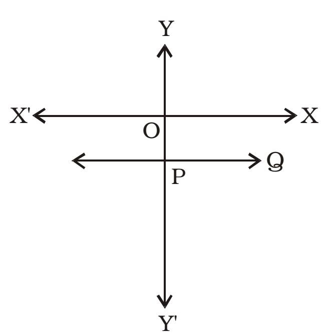
OB = 3

$\therefore$ Area of Δ OAB

$= \frac{1}{2} \times \text{OA} \times \text{OB} = \frac{1}{2} \times 4 \times 3$

= 6 sq. units

19. (3)



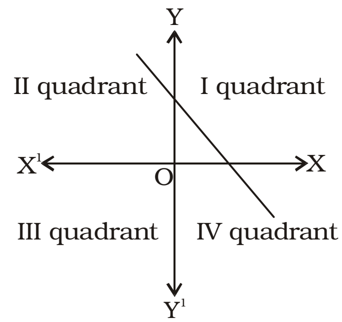
Equation of a straight line parallel to x-axis : $y = a$

Here, $a = -3$

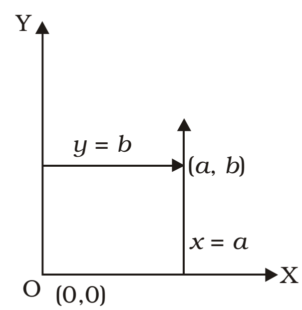
$\therefore$ Equation is : $y = -3$

20. (2) Putting $y = 0$ in $4x + 3y = 12$, we get $x = 3$

Putting $x = 0$ in $4x + 3y = 12$, we get, $y = 4$



21. (1) Point of intersection $= (a, b)$



22. (4) $x = 4$, a straight line parallel to y – axis.

$y = 3$, a straight line parallel to x – axis.

Putting $x = 0$ in $3x + 4y = 12$,

$3 \times 0 + 4y = 12$,

$\Rightarrow 4y = 12 \Rightarrow y = \frac{12}{3} = 4$

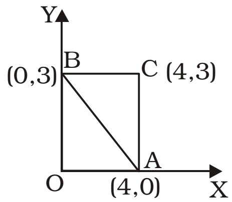
$\therefore$ Point of intersection on y – axis $= (0, 4)$

Again, putting $y = 0$ in $3x + 4y = 12$,

$3x + 4 \times 0 = 12$

$\Rightarrow 3x = 12 \Rightarrow x = \frac{12}{3} = 4$

$\therefore$ Point of intersection on x - axis $= (4, 0)$



Area of □OACB = OA × OB
= 4 × 3 = 12 sq. units

Area of Δ OAB = $\frac{1}{2}$ × OA × OB

= $\frac{1}{2}$ × 4 × 3 = 6 sq. units

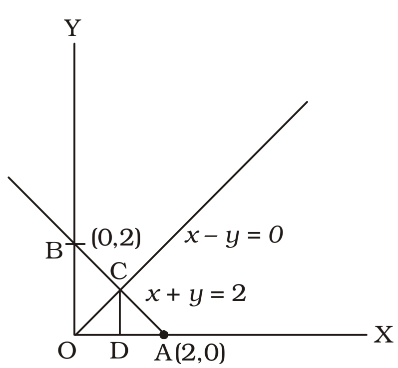
∴ Area of Δ ABC = 12 – 6
= 6 sq. units

23. (3) $3x + 4y = 10$ ---(i)
$-x + 2y = 0$
$\Rightarrow x = 2y$
∴ From equation (i),
$3 \times 2y + 4y = 10 \Rightarrow 10y = 10$

$\Rightarrow y = \frac{10}{10} = 1$

∴ $x = 2$
∴ $(a, b) = (2, 1)$
∴ $a + b = 2 + 1 = 3$

24. (1) On putting $x = 0$ in
$x + y = 2$,
$0 + y = 2 \Rightarrow y = 2$
∴ Point of intersection on y-axis = (0, 2)
Again, putting $y = 0$ in $x + y = 2$,
$x = 2$
∴ Point of intersection on x-axis = (2, 0)
$x - y = 0$ will pass through origin and be equally inclined to axes.



On putting $x = y$ in $x + y = 2$,
$2y = 2 \Rightarrow y = 1$
∴ CD = 1
OA = 2

Area of ΔOAC = $\frac{1}{2} \times OA \times CD$

$= \frac{1}{2} \times 2 \times 1 = 1$ sq. unit

25. (2) $2\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 7 = 0$

$$\Rightarrow 2\left\{\left(x - \frac{1}{x}\right)^2 + 2\right\} - \left(x - \frac{1}{x}\right) - 7 = 0$$

$$\Rightarrow 2\left(x - \frac{1}{x}\right)^2 + 4 - \left(x - \frac{1}{x}\right) - 7 = 0$$

$$\Rightarrow 2\left(x - \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right) - 3 = 0$$

If $x - \frac{1}{x} = y$, then

$2y^2 - y - 3 = 0$
$\Rightarrow 2y^2 - 3y + 2y - 3 = 0$
$\Rightarrow y(2y - 3) + 1(2y - 3) = 0$
$\Rightarrow (y + 1)(2y - 3) = 0$

$\Rightarrow y = -1$ or $\frac{3}{2}$

when $y = -1$

$\Rightarrow x - \frac{1}{x} = -1$

$\Rightarrow x^2 + x + = 0$
The value of x will not be real.
Again,

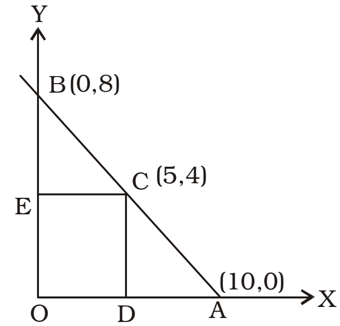
$x - \frac{1}{x} = \frac{3}{2}$

$\Rightarrow \frac{x^2 - 1}{x} = \frac{3}{2}$

$\Rightarrow 2x^2 - 2 = 3x$
$\Rightarrow 2x^2 - 3x - 2 = 0$
$\Rightarrow 2x^2 - 4x + x - 2 = 0$
$\Rightarrow 2x(x - 2) + 1(x - 2) = 0$
$\Rightarrow (2x + 1)(x - 2) = 0$

$\Rightarrow x = -\frac{1}{2}$ or 2

26. (2)



Putting $x = 0$ in $4x + 5y = 40$,
$4 \times 0 + 5y = 40 \Rightarrow 5y = 40$

$\Rightarrow y = \frac{40}{5} = 8$

∴ Point of intersection on y-axis = (0, 8)
Again, putting y
= 0 in $4x + 5y = 40$,
$4x + 5 \times 0 = 40 \Rightarrow 4x = 40$

$\Rightarrow x = \frac{40}{4} = 10$

∴ Point of intersection on x-axis = (10, 0)
OA = 10 units
OD = 5 units = EC
∴ DA = 10 – 5 = 5 units
Again, OB = 8 units
OE = 4 units
BE = 8 – 4 = 4 units
∴ Area of ΔADC

$= \frac{1}{2} \times DA \times DC$

$= \frac{1}{2} \times 5 \times 4 = 10$ sq. units

Area of ΔBEC = $\frac{1}{2} \times EC \times BE$

$= \frac{1}{2} \times 5 \times 4 = 10$ sq. units

∴ Required area = 10 + 10
= 20 sq. units.

27. (4) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will be coincident if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{k}{3} = \frac{2}{1} = \frac{2}{1}$

$\Rightarrow k = 3 \times 2 = 6$

The system of equations has infinite solutions.

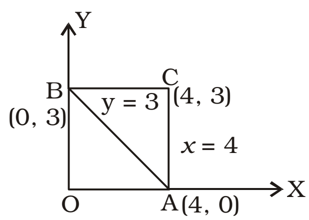
28. (3) On putting $x = 0$ in the equation $3x + 4y = 12$,

$4y = 12, \Rightarrow y = 3$

Again on putting $y = 0$,

$3x = 12 \Rightarrow x = 4$

x	0	4
y	3	0



Area of Δ ABC $= \frac{1}{2} \times AC \times BC$

$= \frac{1}{2} \times 3 \times 4 = 6$ square units

29. (3) Abscissa $= k$,

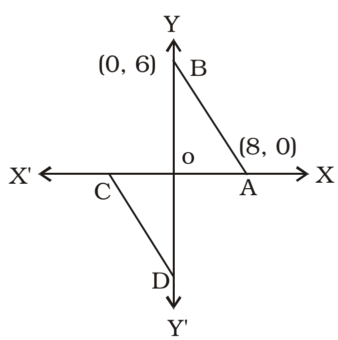
Ordinate $= 2k - 1$

According to the question,

$k = 2k - 1$

$\Rightarrow 2k - k = 1 \Rightarrow k = 1$

30. (2)



On putting $x = 0$ in the equation $3x + 4y = 24$,

$4y = 24 \Rightarrow y = \frac{24}{4} = 6$

$\therefore$ Co-ordinates of B = (0, 6)

Again, putting y = 0 in the equation $3x + 4y = 24$,

$3x = 24 \Rightarrow x = 8$

$\therefore$ Co-ordinates of A = (8,0)

Similarly, for $x + y = -4$

Co-ordinates of C = (–4, 0)

Co-ordinates of D = (0, –4)

$\therefore$ Area of Δ OAB

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 8 \times 6 = 24$ sq. units

Area of Δ OCD

$= \frac{1}{2} \times OC \times OD$

$= \frac{1}{2} \times 4 \times 4 = 8$ sq. units

Clearly,

$\Delta OCD \equiv \frac{1}{3} \Delta OAB.$

31. (4) Putting $y = 0$ in the equation $239x - 239y + 5 = 0$

$\Rightarrow x = \frac{-5}{239}$

$\therefore$ Co-ordinates of A

$= \left(\frac{-5}{239}, 0\right)$

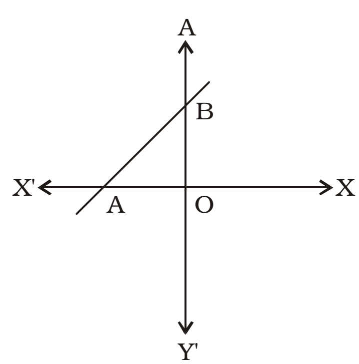
Again putting $x = 0$ in the equation $239x - 239y + 5 = 0$,

$-239y = -5$

$\Rightarrow y = \frac{5}{239}$

$\therefore$ Co-ordinates of B

$= \left(0, \frac{5}{239}\right)$

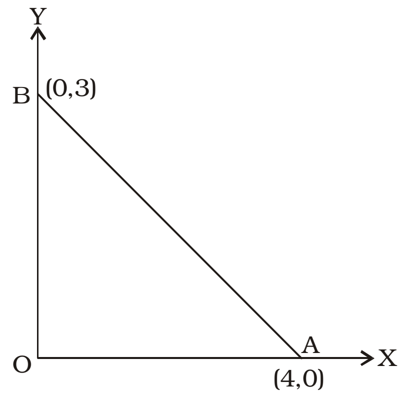


$\therefore$ OA = OB $= \frac{5}{239}$

$\therefore \angle OAB = \angle OBA = 45°$

because $\angle AOB = 90°$

32. (1)



Putting $x = 0$ in $3x + 4y = 12$

$3 \times 0 + 4y = 12$

$\Rightarrow y = \frac{12}{4} = 3$

$\therefore$ Point of intersection on y – axis = (0, 3)

Again, putting $y = 0$ in $3x + 4y = 12$

$3x + 4 \times 0 = 12$

$\Rightarrow 3x = 12 \Rightarrow x = 4$

$\therefore$ Point of intersection on x – axis = (4,0)

$\therefore$ OA = 4 and OB = 3

$\therefore AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{4^2 + 3^2} = \sqrt{16 + 9}$

$= \sqrt{25} = 5$ units

33. (3) For pair of equations,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$, there is no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \frac{2}{6} = \frac{-k}{-12}$

$\Rightarrow \frac{1}{3} = \frac{k}{12} \Rightarrow k = \frac{12}{3} = 4$

34. (1) Co-ordinates of origin = (0, 0). These co-ordinates satisfy the equation $2x - 3y = 0$

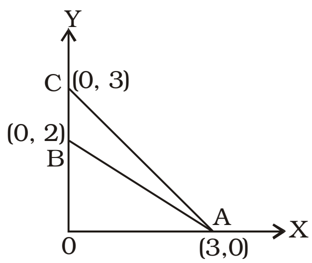
35. (2) Vertices of parallel to piped = $v = 8$

Edges = $e = 12$

Surfaces = $f = 6$

$\therefore\ v - e + f = 8 - 12 + 6 = 2$

36. (3)



$x = 0 \Rightarrow$ Equation of y – axis

Putting $x = 0$ in $2x + 3y = 6$

$0 + 3y = 6 \Rightarrow y = 2$

$\therefore$ Co-ordinates of point of intersection on y – axis

= (0, 2)

Again, putting $y = 0$, $x = 3$

$\therefore$ Point of intersection on x– axis = (3, 0)

In $x + y = 3$

Putting $x = 0$, $y = 3$

and on putting $y = 0$, $x = 3$

$\therefore$ Required area

= Δ OAC – Δ OAB

$= \frac{1}{2} \times 3 \times 3 - \frac{1}{2} \times 3 \times 2$

$= \frac{9}{2} - \frac{6}{2} = \frac{3}{2}$

$= 1\frac{1}{2}$ sq. units

37. (2) $5x + 9y = 5$

On cubing both sides,

$(5x)^3 + (9y)^3 + 3 \times 5x \times 9y\ (5x + 9y) = (5)^3$

$[\because\ (a + b)^3 = a^3 + b^3 + 3ab\ (a + b)]$

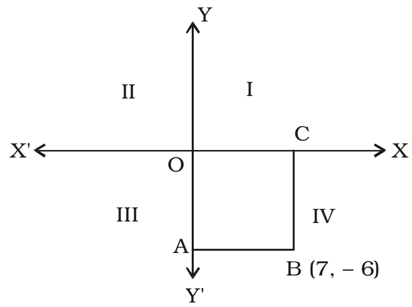
$\Rightarrow 125x^3 + 729y^3 + 135xy \times 5 = 125$

$\Rightarrow 120 + 135 \times 5xy = 125$

$\Rightarrow 135 \times 5xy = 125 - 120 = 5$

$\Rightarrow xy = \frac{5}{135 \times 5} = \frac{1}{135}$

38. (1)



39. (1) $y = 3x$, passes through the origin (0, 0).

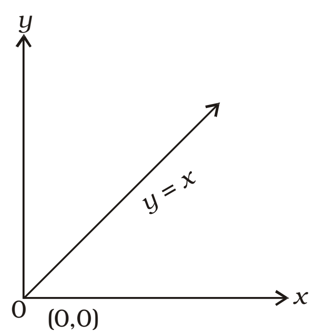
40. (4) Solution of $2x + 3y = k$

= (2, 0)

$\therefore\ 2 \times 2 + 3 \times 0 = k$

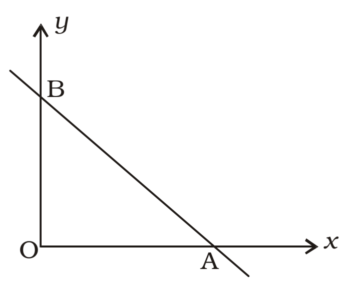
$\Rightarrow\ k = 4$

41.(2)



Point (1, 1) satisfies the equation $y = x$.

42. (2)



Putting $y = 0$ in $9x + 4y = 36$

$9x = 36 \Rightarrow x = 4$

$\therefore$ Co-ordinates of point A

= (4, 0)

i.e OA = 4 units

Putting $x = 0$ in $9x + 4y = 36$

$4y = 36 \Rightarrow y = 9$

$\therefore$ Co-ordinates of point B

= (0, 9)

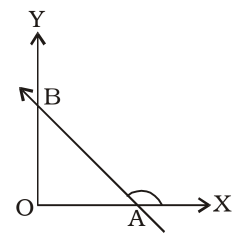
i.e. OB = 9 units

$\therefore$ Area of Δ OAB

$= \frac{1}{2} \times \text{OA} \times \text{OB}$

$= \frac{1}{2} \times 4 \times 9 = 18$ sq. unit

43. (2)

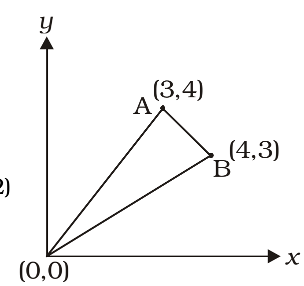


Slope = tan XAB

$\because\ 90° < \angle XAB < 180°$

$\therefore$ The slope will be negative because $\tan\theta$ is negative in second quadrant.

44. (2)



$(x_1, y_1) = 0, 0, (x_2, y_2) = (3, 4)$, $(x_3, y_3) = (4, 3)$

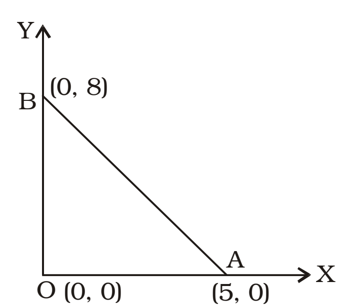
Area of Δ OAB

$= \left|\frac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{2}\right|$

$= \left|\frac{0(4-3) + 3(3-0) + 4(0-4)}{2}\right|$

$= \left|\frac{9-16}{2}\right| = \frac{7}{2}$ sq. units

45. (3)



Clearly, OA = 5 units

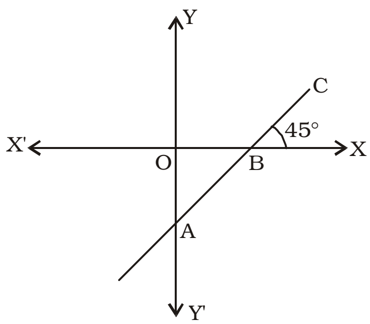
OB = 8 units

$\therefore$ Area of ΔOAB

$= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 5 \times 8 = 20$ sq. units

46. (1)



Slope of straight line

$= m = \tan\theta = \tan 45° = 1$

Intercept on Y–axis $= c = \frac{-3}{4}$

$\therefore$ The required equation is :

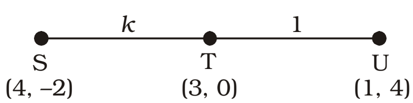
$y = mx + c$

$\Rightarrow y = 1.x - \frac{3}{4}$

$\Rightarrow 4y = 4x - 3$

$\Rightarrow 4x - 4y = 3$

47. (2)



Let point T divide line segment SU in the ratio $k : 1$.

If the co-ordinates of point T be (x, y) and that of points S an U be (x_1, y_1) and (x_2, y_2) respectively, then

$x = \frac{kx_2 + x_1}{k+1}$; $y = \frac{ky_2 + y_1}{k+1}$

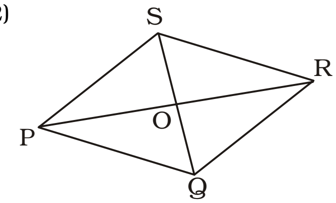
$\therefore 3 = \frac{k \times 1 + 1 \times 4}{k+1}$

$\Rightarrow 3k + 3 = k + 4$

$\Rightarrow 3k - k = 4 - 3 \Rightarrow 2k = 1$

$\Rightarrow k = \frac{1}{2} = 1 : 2$

48. (2)



The diagonals of a rhombus bisect each other at right angles.

$\therefore$ Co-ordinates of point 'O'

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{4-2}{2}, \frac{2+0}{2}\right) = (1, 1)$

Slope of straight line PR

$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{-2-4}$

$= \frac{-2}{-6} = \frac{1}{3}$

$\because PR \perp QS$

$\therefore$ Slope of QS $= -\frac{1}{\frac{1}{3}} = -3$

$[\because m_1 m_2 = -1]$

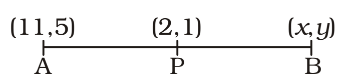
$\therefore$ Equation of straight line QS passing through point (1, 1) :

$y - 1 = -3(x - 1)$

$\Rightarrow y - 1 = -3x + 3$

$\Rightarrow 3x + y = 4$

49. (1)



Co-ordinates of the mid-point of line segment

$= \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$\therefore \frac{11 + x}{2} = 2 \Rightarrow 11 + x = 4$

$\Rightarrow x = 4 - 11 = -7$

and $\frac{5 + y}{2} = 1$

$\Rightarrow y + 5 = 2$

$\Rightarrow y = 2 - 5 = -3$

$\therefore$ Co-ordinates of B $\Rightarrow$ (–7, –(3)

TYPE-IV

1. (1) $\frac{a}{b} = \frac{2}{3} = \frac{8}{12}$

$\frac{b}{c} = \frac{4}{5} = \frac{12}{15}$ [Making B equal]

$\therefore$ Required ratio

$= \frac{8+12}{12+15} = \frac{20}{27}$

2. (2) a : b = 2 : 3

b : c = 4 : 5

or a : b = 8 : 12

b : c = 12 : 15

$\therefore$ a : b : c = 8 : 12 : 15

$\therefore a^2 : b^2 : bc$

$= 8^2 : 12^2 : 15 \times 12$

= 64 : 144 : 180

= 16 : 36 : 45

3. (3) $A : B = \frac{1}{2} : \frac{3}{8}$

$= \frac{8}{2} : \frac{8 \times 3}{8} = 4 : 3$

$B : C = \frac{1}{3} : \frac{5}{9}$

$= \frac{9}{3} : \frac{9 \times 5}{9} = 3 : 5$

$C : D = \frac{5}{6} : \frac{3}{4}$

$= \frac{5 \times 6}{6} : \frac{3 \times 6}{4} = 5 : \frac{9}{2}$

$\therefore A : B : C : D = 4 : 3 : 5 : \frac{9}{2}$

= 8 : 6 : 10 : 9

4. (3) Here $\frac{x}{y} = \frac{3}{2}$

$\therefore \frac{x^2}{y^2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Now, $\frac{2x^2 + 3y^2}{3x^2 - 2y^2} = \frac{2\left(\frac{x^2}{y^2}\right) + 3}{3\left(\frac{x^2}{y^2}\right) - 2}$

[On dividing N^r and D^r by y^2]

$$=\frac{\left(2\times\frac{9}{4}\right)+3}{\left(3\times\frac{9}{4}\right)-2}=\frac{\frac{9}{2}+3}{\frac{27}{4}-2}$$

$$=\frac{\frac{9+6}{2}}{\frac{27-8}{4}}=\frac{15}{2}\times\frac{4}{19}$$

$$=\frac{30}{19}=30:19$$

5. (3) A : B : C = 2 : 3 : 4

$\therefore \frac{A}{B}=\frac{2}{3}, \frac{B}{C}=\frac{3}{4}, \frac{C}{A}=\frac{4}{2}$

Now,

$$\frac{A}{B}:\frac{B}{C}:\frac{C}{A}=\frac{2}{3}:\frac{3}{4}:\frac{4}{2}$$

$$=\frac{2}{3}:\frac{3}{4}:\frac{2}{1}$$

$$=\frac{2}{3}\times 12:\frac{3}{4}\times 12:2\times 12$$

$\because$ [LCM of 3, 4, 1 = 12]

= 8 : 9 : 24

6. (4) C : D = 5 : 6

$\Rightarrow$ D : C = 6 : 5,

C : B = 4 : 3 and B : A = 2 : 1

$\therefore$ D : C : B : A

= 6 × 4 × 2 : 5 × 4 × 2 : 5 × 3 × 2 : 5 × 3 × 1

= 48 : 40 : 30 : 15

7. (3) $\frac{2a-5b}{3a+6b}=\frac{4}{7}$

$\Rightarrow 14a - 35b = 12a + 24b$

$\Rightarrow 2a = 59b$

$\Rightarrow \frac{a}{b}=\frac{59}{2}=59:2$

8. (1) a : b = 7 : 9

b : c = 3 : 5 = 9 : 15

$\therefore$ a : b : c = 7 : 9 : 15

9. (1) $\frac{x}{y}=\frac{7}{3}$ (Given)

Now,

$$\frac{xy+y^2}{x^2-y^2}=\frac{y(x+y)}{(x+y)(x-y)}$$

$$=\frac{y}{x-y}=\frac{1}{\frac{x}{y}-1}=\frac{1}{\frac{7}{3}-1}=\frac{1}{\frac{7-3}{3}}=\frac{3}{4}$$

10. (4) $\frac{3a+5b}{3a-5b}=\frac{5}{1}$

By componendo and dividendo,

$$\frac{3a+5b+3a-5b}{3a+5b-3a+5b}=\frac{5+1}{5-1}$$

$$\Rightarrow \frac{6a}{10b}=\frac{6}{4}\Rightarrow \frac{a}{b}=\frac{6}{4}\times\frac{10}{6}=\frac{5}{2}$$

$\Rightarrow 5 : 2 = a : b$

11. (2) $\frac{p}{q}=\frac{r}{s}=\frac{t}{u}=\frac{2}{3}$

$\Rightarrow \frac{p}{2}=\frac{q}{3}=k$

$\Rightarrow p = 2k,\ q = 3k$

Similarly, $r = 2k,\ s = 3k$,

$t = 2k,\ u = 3k$

Now, $\frac{mp+nr+ot}{mq+ns+ou}$

$$=\frac{m.2k+n.2k+o.2k}{m.3k+n.3k+o.3k}$$

$$=\frac{2k(m+n+o)}{3k(m+n+o)}=\frac{2}{3}\text{ or }2:3$$

12. (3) $\frac{x}{y}=\frac{3}{4}\Rightarrow\frac{7x}{3y}=\frac{7}{3}\times\frac{3}{4}=\frac{7}{4}$

By componendo and dividendo,

$$\frac{7x+3y}{7x-3y}=\frac{7+4}{7-4}=\frac{11}{3}\text{ or }11:3$$

13. (3) $\frac{a}{y-z}=\frac{b}{z-x}=\frac{c}{x-y}=k$

$\Rightarrow a = k(y–z);\ b = k(z–x);$

$c = k(x–y)$

$\therefore ax + by + cz = k(xy - xz + yz - xy + xz - yz) = 0$

14. (2) $\frac{50}{100}(p-q)=\frac{30}{100}(p+q)$

$\Rightarrow 5(p - q) = 3(p + q)$

$\Rightarrow 5p - 5q = 3p + 3q$

$\Rightarrow 2p = 8q$

$\Rightarrow p = 4q$

$\therefore p : q = 4 : 1$

15. (3) $\frac{x}{y}=2\Rightarrow x = 2y$

$\Rightarrow x - 2y = 0$ ...(i)

$\therefore 5x^2 - 13xy + 6y^2$

$= 5x^2 - 10xy - 3xy + 6y^2$

$= 5x(x - 2y) - 3y(x - 2y)$

$= (x - 2y)(5x - 3y)$

$= 0 \times (5x - 3y) = 0$ [Using (i)]

16. (1) $y : x = 4 : 15$

$\Rightarrow x : y = 15 : 4$

By componendo and dividendo,

$$\frac{x-y}{x+y}=\frac{15-4}{15+4}=\frac{11}{19}$$

17. (3) $\frac{x}{y}=\frac{3}{4}$ (Given)

$$\therefore \frac{5x-2y}{7x+2y}=\frac{5\frac{x}{y}-2}{7\frac{x}{y}+2}$$

$$=\frac{5\times\frac{3}{4}-2}{7\times\frac{3}{4}+2}=\frac{\frac{15-8}{4}}{\frac{21+8}{4}}=\frac{7}{29}$$

18. (3) $x^2 + 9y^2 = 6xy$

$\Rightarrow x^2 - 6xy + 9y^2 = 0$

$\Rightarrow x^2 - 2.x.3y + (3y)^2 = 0$

$\Rightarrow (x - 3y)^2 = 0$

$\Rightarrow x - 3y = 0$

$\Rightarrow x = 3y$

$\Rightarrow x : y = 3 : 1$

19. (1) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow (4\sqrt{3})^2 = 16 + 2(ab + bc + ca)$

$\Rightarrow 48 = 16 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = 48 - 16 = 32$

$\Rightarrow ab + bc + ca = 16$

$\therefore a = b = c = \frac{4\sqrt{3}}{3}=\frac{4}{\sqrt{3}}$

$\therefore a : b : c = 1 : 1 : 1$

20. (2) $\frac{3x}{2y}=\frac{21}{22}$

$$\Rightarrow \frac{x}{y}=\frac{21}{22}\times\frac{2}{3}=\frac{7}{11}$$

$\Rightarrow \frac{x}{7} = \frac{y}{11} = k$

$\therefore 4x + 5y = 83$

$\Rightarrow 4 \times 7k + 5 \times 11k = 83$

$\Rightarrow 28k + 55k = 83$

$\Rightarrow 83k = 83 \Rightarrow k = 1$

$\therefore x = 7,\ y = 11$

$\therefore y - x = 11 - 7 = 4$

21. (4) $\frac{x}{xa + yb + zc} = \frac{y}{ya + zb + xc}$

$= \frac{z}{za + xb + yc}$

$= \frac{x + y + z}{xa + yb + zc + ya + zb + xc + za + xb + yc}$

$= \frac{x + y + z}{xa + ya + za + yb + ya + yc + zc + zb + za}$

$= \frac{x + y + z}{a(x + y + z) + b(x + y + z) + c(x + y + z)}$

$= \frac{x + y + z}{(a + b + c)(x + y + z)}$

$= \frac{1}{a + b + c}$

22. (1) $\frac{x}{y} = \frac{3}{2}$

By componendo and dividend,

$\frac{x + y}{x - y} = \frac{3 + 2}{3 - 2}$

$\Rightarrow \frac{x + y}{x - y} = \frac{5}{1} = 5 : 1$

23. (2) $a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$\Rightarrow (a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ca) = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

[If $x^2 + y^2 + z^2 = 0$ then, $x = 0$, $y = 0$, $z = 0$]

$\therefore a - b = 0 \Rightarrow a = b$

$b - c = 0 \Rightarrow b = c$

$c - a = 0 \Rightarrow c = a$

$\therefore a = b = c$

$\therefore a : b : c = 1 : 1 : 1$

24. (3) $a^2 + 13b^2 + c^2 - 4ab - 6bc = 0$

$\Rightarrow a^2 - 4ab + 4b^2 + 9b^2 + c^2 - 6bc = 0$

$\Rightarrow a^2 - 4ab + 4b^2 + c^2 - 6bc + 9b^2 = 0$

$\Rightarrow (a - 2b)^2 + (c - 3b)^2 = 0$

$\Rightarrow a - 2b = 0$ and $c - 3b = 0$

$\Rightarrow a = 2b$ and $c = 3b$

$\Rightarrow \frac{a}{b} = \frac{2}{1}$ and $\frac{b}{c} = \frac{1}{3}$

$\therefore a : b : c = 2 : 1 : 3$

25. (2) If $a^2 + b^2 = 0$

$\Rightarrow a = 0$ and $b = 0$

$\therefore (2x - y)^2 + (3y - 2z)^2 = 0$

$\therefore 2x - y = 0 \Rightarrow 2x = y$

$\Rightarrow x : y = 1 : 2$

and, $3y - 2z = 0 \Rightarrow 3y = 2z$

$\Rightarrow y : z = 2 : 3$

$\therefore x : y : z = 1 : 2 : 3$

TYPE-V

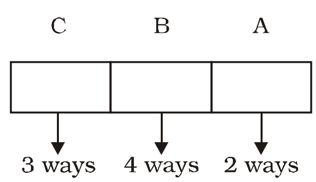
1. (3) Required number of ways = $^5P_3 = 5 \times 4 \times 3 = 60$

2. (4) The unit's place will be occupied by 2 or 6 in three digit even numbers.

The remaining two places can be occupied by selecting from remaining four digits in $^4P_2$ ways

$^4P_2 = \frac{4!}{(4-2)!} = \frac{4 \times 3 \times 2}{2} = 12$

$\therefore$ Total number of even three digit numbers $= 2 \times 12 = 24$



Total ways $= 3 \times 4 \times 2 = 24$ ways.

{$\because$ Total available digits are 1, 2, 5, 6, 9.

Even digits = 2 and 6.

$\Rightarrow$ A can either be filled by 2 or 6 i.e. 2 ways.

B can either be filled by 4 ways [$\because$ Total – digit used at A i.e. 5 – 1]}

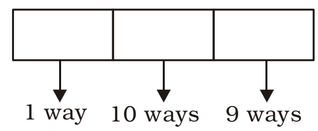
and C can either be filled by 3 ways [Total – digit used at A – digit used at B i.e. 5 – 1 – 1]

3. (1) It is to be noted that when two persons shake hands it is counted as one hand shake not two. So this is a problem on combination. The total number of hand shakes is

= The number of ways of selecting 2 persons out of 10 persons

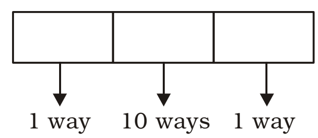
$= {}^{10}C_2 = \frac{10 \times 9}{1 \times 2} = 45$

4. (3) When 3 lies at hundreds place



$\therefore$ Total integers $= 10 \times 9 = 90$

When 3 lies at units place



Total integers = 10

When 3 lies at unit's and hundred's place

Total integers = 10

$\therefore$ Total integers

$= 90 + 10 + 10 = 110$

TYPE-VI

1. (3) $\left[-\frac{1}{4}\right] + \left[4\frac{1}{4}\right] + [3]$

$= -1 + 4 + 3 = 6$

2. (3) $a \oplus b = 2a$ if $a > b$

$= a + b$ if $a < b$

$= a^2$ if $a = b$

$\therefore \frac{(5 \oplus 7) + (4 \oplus 4)}{3(5 \oplus 5) - (15 \oplus 11) - 3}$

$= \frac{(5 + 7) + 4^2}{3 \times 5^2 - 2 \times 15 - 3}$

$= \frac{12 + 16}{75 - 30 - 3} = \frac{28}{42} = \frac{2}{3}$

3. (3) Given $a \circledast b = a + b$ when $a > 0$, $b > 0$

$a \circledast b = \sqrt{a^2+b^2}$; for other values of a and b

Expression,

$$= \frac{8 \circledast (7-13)-(3 \circledast 1)}{(3-6) \circledast (9-5)}$$

$$= \frac{8 \circledast (-6)-(3+1)}{(-3) \circledast (-4)}$$

$$= \frac{\sqrt{(8)^2+(-6)^2}-4}{\sqrt{(-3)^2+(-4)^2}}$$

$$= \frac{\sqrt{64+36}-4}{\sqrt{9+16}}$$

$$= \frac{\sqrt{100}-4}{\sqrt{25}} = \frac{10-4}{5} = \frac{6}{5}$$

4. (2) $(a-b)^2 = a^2 - 2ab + b^2$

$x^4 - 2x^2 + k = (x^2)^2 - 2.x^2.1 + k$

$\therefore\ k = (1)^2 = 1$

5. (3) $x = \sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}$

Cubing both sides,

$$x^3 = \left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)^3 + \left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3$$

$$+ 3\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)\left(\sqrt[3]{a+\sqrt{a^2+b^3}} + \sqrt[3]{a-\sqrt{a^2+b^3}}\right)$$

$$= a+\sqrt{a^2+b^3} + a - \sqrt{a^2+b^3} + 3\left(a+\sqrt{a^2+b^3} \times a-\sqrt{a^2+b^3}\right)^{\frac{1}{3}} x$$

$$= 2a + 3\left(a^2 - a^2 - b^3\right)^{\frac{1}{3}} x$$

$= 2a + (-3bx)$

$\therefore\ x^3 + 3bx = 2a$

6. (3) Let $\frac{1}{3} = a, \frac{1}{4} = b$ and $\frac{1}{5} = c$

$\therefore$ Expression

$$= \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-ac-bc}$$

$$= \frac{(a+b+c)\left(a^2+b^2+c^2-ab-ac-bc\right)}{a^2+b^2+c^2-ab-ac-bc} = a+b+c$$

$$= \frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{20+15+12}{60} = \frac{47}{60}$$

7. (4) $x^m \times x^n = 1$

$\Rightarrow\ x^{m+n} = x^0$

$\Rightarrow\ m + n = 0$

$\Rightarrow\ m = -n$

8. (2) $(a + b)^2 = a^2 + 2ab + b^2$

$\therefore\ 4x^2 + 8x + 4 = (2x)^2 + 2 \times 2x \times 2 + (2)^2 = (2x + 2)^2$

$\therefore$ Required number = 4

9. (2) $x + \frac{1}{x} = 2N$

$\therefore$ Mean of x^2 and $\frac{1}{x^2}$

$$= \frac{x^2+\frac{1}{x^2}}{2}$$

$$= \frac{\left(x+\frac{1}{x}\right)^2-2}{2} = \frac{(2N)^2-2}{2}$$

$$= \frac{4N^2-2}{2} = 2N^2 - 1$$

10. (4) $3a^2 + 3b^2 + 3c^2 = (a + b + c)^2$

$\Rightarrow\ 3a^2 + 3b^2 + 3c^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$\Rightarrow\ 3a^2 + 3b^2 + 3c^2 - a^2 - b^2 - c^2 - 2ab - 2bc - 2ac = 0$

$\Rightarrow\ 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac = 0$

$\Rightarrow\ a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + a^2 + c^2 - 2ac = 0$

$\Rightarrow\ (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$\Rightarrow\ a - b = 0 \Rightarrow a = b$

$b - c = 0 \Rightarrow b = c$

$c - a = 0 \Rightarrow c = a$

$\therefore\ a = b = c$

11. (4) $15! = 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Number of 5's = 3

Number of 2's = More than 3

$\therefore$ Number of zeroes in the product = 3

$\therefore$ Unit's digit in $\frac{15!}{100} = 0$

12. (3) Let three numbers in A.P. be $a - d$, a and $a + d$ respectively.

According to the question,

$a - d + a + a + d = 30$

$\Rightarrow\ 3a = 30 \Rightarrow a = \frac{30}{3} = 10$

Again, $a(a - d)(a + d) = 910$

$\Rightarrow\ 10(10 - d)(10 + d) = 910$

$\Rightarrow\ 100 - d^2 = 91$

$\Rightarrow\ d^2 = 100 - 91 = 9$

$\Rightarrow\ d = \sqrt{9} = 3$

$\therefore$ Largest number = $a + d$

= 10 + 3 = 13

13. (2) $U_n = \frac{1}{n} - \frac{1}{n+1}$

$\therefore\ U_1 = \frac{1}{1} - \frac{1}{2}$

$U_2 = \frac{1}{2} - \frac{1}{3}$

$U_3 = \frac{1}{3} - \frac{1}{4}$

$U_4 = \frac{1}{4} - \frac{1}{5}$

$U_5 = \frac{1}{5} - \frac{1}{6}$

$\therefore\ U_1 + U_2 + U_3 + U_4 + U_5$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$$

$$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

❑❑❑

TEST YOURSELF

1. If $x^2 - 3x + 1 = 0$, find the value of $x^3 + \frac{1}{x^3}$.

(1) 18 (2) 16
(3) 27 (4) 23

2. If $a^2 + b^2 + c^2 = 2(2a - 3b - 5c) - 38$, find the value of $(a - b - c)$.

(1) 9 (2) 10
(3) 11 (4) 12

3. If $a + b + c = 0$, then

$$\frac{2a^2}{b^2+c^2-a^2} + \frac{2b^2}{c^2+a^2-b^2} + \frac{2c^2}{a^2+b^2-c^2} + 3 = ?$$

(1) 3 (2) – 4
(3) 0 (4) – 3

4. What are the factors of the following expression ?

$$a^2 + \frac{1}{a^2} - 13a + \frac{13}{a} + 34 :$$

(1) $\left(a - \frac{1}{a} + 4\right)\left(a - \frac{1}{a} - 9\right)$

(2) $\left(a - \frac{1}{a} - 4\right)\left(a - \frac{1}{a} + 9\right)$

(3) $\left(a + \frac{1}{a} - 4\right)\left(a - \frac{1}{a} + 9\right)$

(4) $\left(a + \frac{1}{a} - 4\right)\left(a + \frac{1}{a} + 9\right)$

5. If $2x - \frac{1}{3x} = 5$, find the value of $\left(27x^3 - \frac{1}{8x^3}\right)$.

(1) $\frac{3645}{8}$ (2) $\frac{3465}{8}$
(3) $\frac{3645}{4}$ (4) 459

6. Resolve into factors :
$(x - 1)(x + 1)(x + 3)(x + 5) + 7$

(1) $(x + 2 + \sqrt{2})(x + 2 - \sqrt{2})$
$(x + 2 + 2\sqrt{2})(x + 2 - 2\sqrt{2})$

(2) $(x - 2 + \sqrt{2})(x - 2 - \sqrt{2})$
$(x + 2 + 2\sqrt{2})(x + 2 - 2\sqrt{2})$

(3) $(x - 2 - \sqrt{2})(x + 2 + \sqrt{2})$
$(x - 2 - 2\sqrt{2})(x - 2 - 2\sqrt{2})$

(4) None of these

7. If $a + b + c = 0$, then

$$\frac{bc}{bc - a^2} + \frac{ca}{ca - b^2} + \frac{ab}{ab - c^2} = ?$$

(1) 1 (2) –1
(3) 0 (4) 2

8. If $a = 2 + \sqrt{3}$, find the value of $\frac{a^3}{a^6 + 3a^3 + 1}$.

(1) 55 (2) $\frac{1}{55}$
(3) $\frac{1}{40}$ (4) $\frac{3}{55}$

9. p and q are positive numbers satisfying $3p + 2pq = 4$ and $5q + pq = 3$. Find the value of p.

(1) 1 or $-\frac{9}{5}$ (2) $\frac{1}{2}$ or $-\frac{20}{3}$
(3) 1 or $-\frac{20}{3}$ (4) $\frac{1}{2}$ or $-\frac{9}{5}$

10. If $a + b + c = 5$, $ab + bc + ca = 7$ and $abc = 3$, find the value of

$$\left(\frac{a}{b} + \frac{b}{a}\right) + \left(\frac{b}{c} + \frac{c}{b}\right) + \left(\frac{c}{a} + \frac{a}{c}\right)$$

(1) $8\frac{2}{3}$ (2) $7\frac{2}{3}$
(3) $9\frac{2}{3}$ (4) $8\frac{1}{3}$

11. If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$ then

$$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = ?$$

(1) 1 (2) –1
(3) 0 (4) 2

12. If $a = 7 + 4\sqrt{3}$, find the value of

$$\frac{3a^6 + 2a^4 + 4a^3 + 2a^2 + 3}{a^4 + a^3 + a^2}.$$

(1) $\frac{8138}{17}$ (2) $\frac{8138}{15}$
(3) $\frac{8238}{15}$ (4) $\frac{8338}{15}$

13. If $x\ \frac{x - bc}{b + c} + \frac{x - ca}{c + a} + \frac{x - ab}{a + b}$ $= a+b+c$ what is the value of x?

(1) ab (2) $ab + bc$
(3) $bc + ca$ (4) $ab + bc + ca$

14. If $x^4 + \frac{1}{x^4} = 47$, what will be the value of $x^3 + \frac{1}{x^3}$?

(1) 18 (2) 17
(3) 19 (4) 20

15. If $x = \frac{1}{2 - \sqrt{3}}$, what will be the value of $x^3 - 2x^2 - 7x + 5$?

(1) 0 (2) 2
(3) 3 (4) 4

16. If $\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a + b\sqrt{3}$, the values of a and b respectively are :

(1) $a = -11$, $b = 6$
(2) $a = 11$, $b = -6$
(3) $a = 6$, $b = 11$
(4) $a = -6$, $b = -11$

17. If $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$; $a + b \neq 0$, then $x = ?$

(1) $x = -a$ (2) $x = -b$
(3) $x = -a$ or $-b$ (4) $x = a$ or b

18. For what value of 'a', the polynomial $2x^3 + ax^2 + 11x + a + 3$, is exactly divisible by $(2x - 1)$?

(1) 7 (2) –7
(3) 5 (4) –5

19. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$, then $a^3 + b^3 + c^3 - 3abc = ?$

(1) 160 (2) 175

(3) 180 (4) 100

20. What will be the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ if $a + b + c = 3x$?

(1) 1 (2) 3

(3) 0 (4) 5

21. If $p = 2 - a$, then $a^3 + 6ap + p^3 - 8 = ?$

(1) 0 (2) 8

(3) 6 (4) 5

22. If $4x^2 + 4y^2 + 4z^2 = 12x + 12y - 18$ then $x + y + z = ?$

(1) 3 (2) 4

(3) $\frac{3}{2}$ (4) 2

23. For what value of k, the system of equations

$5x + 2y = k$

$10x + 4y = 3$ has infinite solutions ?

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{5}{2}$ (4) 2

24. ₹120 is divided between x, y and z, so that x's share is ₹20 more than y's and ₹20 less than z's. What is y's share?

(1) ₹25 (2) ₹20

(3) ₹30 (4) None of these

25. The value of x, if the slope of the line joining $(-8, 11)$, $(2, x)$ is $\left(\frac{-4}{3}\right)$ will be

(1) $\frac{-7}{3}$ (2) $\frac{7}{3}$

(3) $\frac{1}{3}$ (4) $\frac{5}{3}$

26. Find the equation of the line which passes through the point (2, 2) and makes an angle of 45° with x-axis ?

(1) $x + y = 2$ (2) $x - y = 0$

(3) $2x + y = 3$ (4) $x - 2y = 4$

27. If the vertices of a quadrilateral are A(– 2, 6), B(1, 2), C(10, 4) and D(7, 8). Then the equation of diagonal AC will be ?

(1) $x + 6y = 34$

(2) $x - 6y = 34$

(3) $x + 2y = 1$

(4) $x - y = 13$

28. What is the slope between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$

(1) 1 (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$ (4) $\sqrt{3}$

29. The distance of the point (3, – 1) from the line $12x - 5y - 7 = 0$ will be ?

(1) $\frac{1}{13}$ units (2) $\frac{43}{13}$ units

(3) $\frac{34}{13}$ units (4) $\frac{1}{5}$ units

30. What is the equation of a line, which passes through the points (–1, 1) and (2, –4).

(1) $5x + 3y + 2 = 0$

(2) $-5x + 3y + 4 = 0$

(3) $3x + 5y + 6 = 0$

(4) $5x + 3y + 3 = 0$

31. What is the equation of line which has y-intercept 2 and is inclined at 60° to the x-axis. (1) $y = -\sqrt{3}x + 2$

(2) $y = x - \sqrt{3}$

(3) $y = x + 2\sqrt{3}$

(4) $y = \sqrt{3}x + 2$

32. What will be the point on the x-axis, which is eduidistant from the points (7, 6) and (–3, 4)

(1) (6, 0) (2) (–2, 0)

(3) (4, 0) (4) (3, 0)

33. Equation of a line is taken as $3x - 4y + 5 = 0$. Its slope and intercept on y-axis

(1) $\left(\frac{3}{4}, \frac{-5}{4}\right)$ (2) $\left(\frac{-3}{4}, \frac{-5}{4}\right)$

(3) $\left(\frac{3}{4}, \frac{5}{4}\right)$ (4) $\left(\frac{5}{4}, \frac{3}{4}\right)$

34. Equation of line $\sqrt{3}x + y - 8 = 0$ can be represented in normal form as

(1) $\frac{\sqrt{3}x}{2} + \frac{y}{2} - \frac{8}{2} = 0$

(2) $\sqrt{3}x + y - 4 = 0$

(3) $\sqrt{3}x - y - 8 = 0$

(4) $\frac{\sqrt{3}x}{2} - \frac{y}{2} - \frac{8}{2} = 0$

35. What will be the angle between the lines $y - x - 7 = 0$ and $\sqrt{3}y - x + 6 = 0$?

(1) $\theta = \tan^{-1}\left(2 + \sqrt{3}\right)$

(2) $\theta = \tan^{-1}\left(2 - \sqrt{3}\right)$

(3) $\theta = \tan^{-1}\left(1 + \sqrt{3}\right)$

(4) $\theta = \tan^{-1}\left(1 - \sqrt{3}\right)$

36. Equation of line $3x + 2y - 5 = 0$ can be written in intercept form as

(1) $\frac{x}{\frac{5}{3}} + \frac{y}{\frac{5}{2}} = 1$ (2) $\frac{x}{5} + \frac{y}{3} = 1$

(3) $\frac{x}{2} + \frac{y}{5} = 1$ (4) $\frac{x}{\frac{5}{3}} - \frac{y}{\frac{5}{2}} = 1$

37. What is the distance between the parllel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$?

(1) 2 units (2) 5 units

(3) 6 units (4) $\frac{65}{17}$ units

38. Find the ratio in which the line segment joining the points (1, 2) and (4, 6) is divided by point (2, 0).

(1) 1 : 2 (2) 2 : 1

(3) 1 : 4 (4) 2 : 3

39. What is the distance of the point (2, 3) from the line $2x + 3y + 4 = 0$?

(1) $\frac{15}{\sqrt{13}}$ (2) $\frac{16}{\sqrt{13}}$

(3) $\frac{17}{\sqrt{13}}$ (4) $\frac{8}{\sqrt{13}}$

40. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?

(1) 2 : 1 (2) 1 : 3

(3) 1 : 2 (4) 1 : 4

41. For what value of x the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear?
(1) $x = 2$ (2) $x = -1$
(3) $x = 4$ (4) $x = 1$

42. What is the equation of line, which makes intercepts –5 and 2 on the x and y-axis respectively
(1) $2x - 5y = 10$
(2) $-2x + 5y = -10$
(3) $2x - 5y = -10$
(4) $5x - 2y = 10$

43. For what value of k, the following pair of lines $-kx + 2y + 3 = 0$ and $2x + 4y + 7 = 0$ are perpendicular?
(1) $k = 2$ (2) $k = 4$
(3) $k = -1$ (4) $k = 3$

44. What will be the equation of line which passes through the point $(-2, 3)$ and parallel to any other line $3x - 4y + 2 = 0$
(1) $3x - 4y + 18 = 0$
(2) $-3x + 4y + 12 = 0$
(3) $x - 3y + 10 = 0$
(4) $2x + 3y + 6 = 0$

45. What will be the equation of a line passing through the point $(-4, 3)$ and having slope $\frac{1}{2}$?
(1) $x - 2y + 5 = 0$
(2) $-x + 2y + 10 = 0$
(3) $x - 2y + 6 = 0$
(4) $x - 2y + 10 = 0$

46. Find the co-ordinates of the mid point of a line segment joining the points (2, 4) and (6, 8) ?
(1) (2, 6) (2) (4, 6)
(3) (6, 4) (4) (–4, –6)

47. For what value of k, the line $kx + 3y + 6 = 0$, will pass through the point (2, 4).
(1) $k = -8$ (2) $k = 7$
(3) $k = -9$ (4) $k = 6$

48. In what ratio the line segment joining the points (2, 3) and (4, 6) is divided by y-axis?
(1) Internally 1 : 2
(2) Externally 2 : 3
(3) Externally 2 : 1
(4) Externally 1 : 2

49. What is the equation of line passes through the point (3, 2) and make an angle of 45° with the line $x - 2y = 3$?
(1) $3x - y - 7 = 0$
(2) $3x + y = 7$
(3) $x - 3y = 7$
(4) $-3x + y = 6$

50. A point R(h, k) divides a line segment between the axis in the ratio 1 : 2 what will be the equation of line ?
(1) $\frac{x}{h} - \frac{2y}{k} = 4$
(2) $\frac{x}{h} + \frac{2y}{k} = 3$
(3) $\frac{x}{k} + \frac{y}{h} = 1$
(4) $\frac{x}{2k} + \frac{y}{h} = 1$

51. What will be the equation of line for which $p = 3$ and $\alpha = 120°$?
(1) $x - \sqrt{3}y = 6$
(2) $\sqrt{3}x + y = 6$
(3) $-x + \sqrt{3}y = 6$
(4) $x - \sqrt{3}y = 5$

52. If the points (h, o), (a, b) and (o, k) lie on a line, then?
(1) $\frac{a}{h} + \frac{b}{k} = 1$ (2) $\frac{a}{k} + \frac{b}{h} = 1$
(3) $\frac{h}{a} + \frac{k}{b} = 1$ (4) $\frac{a}{h} - \frac{b}{k} = 1$

53. What is the equation of line parallel to $2x + 3y + 4 = 0$ and passing through the point $(-4, -5)$?
(1) $2x + 5y - 23 = 0$
(2) $-x + 5y = 20$
(3) $2x - 3y - 30 = 0$
(4) $2x + 3y + 23 = 0$

54. Find the equation of a line which passes through the point of intersection of lines $x + 2y = 5$ and $x - 3y = 7$ and also passes through the point $(0, -1)$
(1) $3x - 29y + 1 = 0$
(2) $3x - 29y - 29 = 0$
(3) $3x + 4y - 6 = 0$
(4) $-3x + 29y + 7 = 0$

55. If the angle between two lines is $\frac{\pi}{4}$ and the slope of one of the lines is $\frac{1}{2}$, then the slope of other line will be
(1) $m = 1$ (2) $m = 2$
(3) $m = 3$ (4) $m = 4$

56. What point on the x-axis are at a distance of 4 units from the line $3x - 4y - 5 = 0$
(1) $\left(\frac{1}{3}, 0\right)$ (2) $\left(0, \frac{25}{3}\right)$
(3) (5, 1) (4) $\left(\frac{25}{3}, 0\right)$

57. What is the equation of a line perpendicular to the line $x - 7y + 5 = 0$ and having x-intercept 3?
(1) $x + 7y = 21$ (2) $7x + y = 21$
(3) $x + 2y = 10$ (4) $-x + y = 15$

58. For what value of k the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is parallel to x-axis ?
(1) $k = \pm 1$ (2) $k = \pm 4$
(3) $k = \pm 6$ (4) $k = \pm 2$

59. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?
(1) 5 : 13 (2) 5 : 2
(3) 1 : 3 (4) 4 : 7

60. The distance between the lines $y = mx + c_1$ and $y = mx + c_2$ is
(1) $\frac{c_1 - c_2}{\sqrt{m^2 + 1}}$ (2) $\left|\frac{c_1 - c_2}{\sqrt{1 + m^2}}\right|$
(3) $\frac{c_2 - c_1}{\sqrt{1 + m^2}}$ (4) 0

61. A point equidistant from the lines $4x + 3y + 10 = 0$, $5x - 12y + 26 = 0$ and $7x + 24y - 50 = 0$ is
(1) (1, –1) (2) (1, 1)
(3) (0, 0) (4) (0, 1)

62. If the line $\frac{x}{a}+\frac{y}{b}=1$ passes through the points (2, –3) and (4, –5) then (a, b) is

(1) (1, 1) (2) (–1, 1)
(3) (1, –1) (4) (–1, –1)

63. What will be the co-ordinates of centroid of a triangle whose vertices are A(1, 2), B (2, 4) and C(6, 2).

(1) (3, 1) (2) $\left(3, \frac{8}{3}\right)$
(3) $\left(\frac{8}{3}, 3\right)$ (4) (1, 5)

64. Slope of a line which cuts off intercepts of equal lengths on the axis is

(1) 1 (2) 2
(3) –1 (4) 3

SHORT ANSWERS

1. (1)	**2.** (2)	**3.** (3)	**4.** (2)
5. (1)	**6.** (1)	**7.** (1)	**8.** (2)
9. (3)	**10.** (1)	**11.** (3)	**12.** (2)
13. (4)	**14.** (1)	**15.** (3)	**16.** (2)
17. (3)	**18.** (2)	**19.** (3)	**20.** (3)
21. (1)	**22.** (1)	**23.** (1)	**24.** (2)
25. (1)	**26.** (2)	**27.** (1)	**28.** (2)
29. (3)	**30.** (1)	**31.** (4)	**32.** (4)
33. (3)	**34.** (1)	**35.** (2)	**36.** (1)
37. (4)	**38.** (1)	**39.** (3)	**40.** (3)
41. (4)	**42.** (3)	**43.** (2)	**44.** (1)
45. (4)	**46.** (2)	**47.** (3)	**48.** (4)
49. (1)	**50.** (2)	**51.** (3)	**52.** (1)
53. (4)	**54.** (2)	**55.** (3)	**56.** (4)
57. (2)	**58.** (4)	**59.** (1)	**60.** (2)
61. (3)	**62.** (4)	**63.** (2)	**64.** (3)

EXPLANATIONS

1. (1) $x^2 - 3x + 1 = 0$
$\Rightarrow x^2 + 1 = 3x$

$$\Rightarrow \frac{x^2+1}{x} = 3$$

$$\Rightarrow x + \frac{1}{x} = 3$$

On cubing both sides,

$$\left(x+\frac{1}{x}\right)^3 = 27$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = 27$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\times 3 = 27$$

$$\Rightarrow x^3+\frac{1}{x^3} = 27-9 = 18$$

2. (2) $a^2 + b^2 + c^2 = 4a - 6b - 10c - 38$
$\Rightarrow a^2+b^2+c^2-4a+6b+10c+38 = 0$
$\Rightarrow a^2 - 4a + 4 + b^2 + 6b + 9 + c^2 + 10c + 25 = 0$
$\Rightarrow (a-2)^2 + (b+3)^2 + (c+5)^2 = 0$
$\therefore a - 2 = 0 \Rightarrow a = 2$
$b + 3 = 0 \Rightarrow b = -3$
$c + 5 = 0 \Rightarrow c = -5$
$\therefore a - b - c = 2 + 3 + 5 = 10$

3. (3) If $a + b + c = 0$
$a + b = -c$
On squaring both sides,
$\Rightarrow a^2 + b^2 + 2ab = c^2$
Similarly, $a^2 = b^2 + c^2 + 2ac$
$b^2 = a^2 + c^2 + 2ac$

$$\therefore \text{Expression} = \frac{2a^2}{b^2+c^2-a^2}+\frac{2b^2}{c^2+a^2-b^2}+\frac{2c^2}{a^2+b^2-c^2}+3$$

$$\text{L.H.S.} = \frac{2a^2}{b^2+c^2-a^2}+1+\frac{2b^2}{c^2+a^2-b^2}+1+\frac{2c^2}{a^2+b^2-c^2}+1$$

$$= \frac{a^2+b^2+c^2}{b^2+c^2-a^2}+\frac{a^2+b^2+c^2}{c^2+a^2-b^2}+\frac{a^2+b^2+c^2}{a^2+b^2-c^2} = (a^2+b^2+c^2)$$

$$\left[\frac{1}{b^2+c^2-b^2-c^2-2bc}+\frac{1}{c^2+a^2-c^2-a^2-2ac}+\frac{1}{a^2+b^2-a^2-b^2-2ab}\right]$$

$$= (a^2+b^2+c^2)\left(\frac{1}{-2bc}-\frac{1}{2ac}-\frac{1}{2ab}\right) = (a^2+b^2+c^2)\left(\frac{-a-b-c}{2abc}\right) = 0$$

4. (2) $a^2+\frac{1}{a^2}-13\left(a-\frac{1}{a}\right)+34$

$$= \left(a-\frac{1}{a}\right)^2+2-13\left(a-\frac{1}{a}\right)+34$$

$$= \left(a-\frac{1}{a}\right)^2-13\left(a-\frac{1}{a}\right)+36$$

Let $\left(a-\frac{1}{a}\right) = x$
$\therefore$ Expression $= x^2 - 13x + 36$
$= x^2 - 9x - 4x + 36 = x(x-9) - 4(x-9)$
$= (x-4)(x-9)$

$$= \left(a-\frac{1}{a}-4\right)\left(a-\frac{1}{a}-9\right)$$

5. (1) $2x-\frac{1}{3x} = 5$

On multiplying both sides by $\frac{3}{2}$,

$$3x-\frac{1}{2x} = \frac{15}{2}$$

On cubing both sides,

$$27x^3-\frac{1}{8x^3}-3.3x.\frac{1}{2x}\left(3x-\frac{1}{2x}\right) = \frac{3375}{8}$$

$$\Rightarrow 27x^3-\frac{1}{8x^3}-\frac{9}{2}\times\frac{15}{2} = \frac{3375}{8}$$

$$\Rightarrow 27x^3-\frac{1}{8x^3} = \frac{3375}{8}+\frac{135}{4}$$

$$= \frac{3375+270}{8} = \frac{3645}{8}$$

6. (1) $(x-1)(x+5)(x+1)(x+3)+7$
$= (x^2+5x-x-5)(x^2+3x+x+3)+7$
$= (x^2+4x-5)(x^2+3x+x+3)+7$
Putting $x^2+4x = y$, we have,
Expression $= (y-5)(y+3)+7$
$= y^2 - 5y + 3y - 15 + 7$
$= y^2 - 2y - 8$
$= y^2 - 4y + 2y - 8$
$= y(y-4) + 2(y-4)$

= (y + 2) (y – 4)
Now,
$y + 2 = x^2 + 4x + 2$
$= x^2 + 4x + 4 - 2$

$= (x+2)^2 - \left(\sqrt{2}\right)^2$

$= \left(x+2+\sqrt{2}\right)\left(x+2-\sqrt{2}\right)$

Again, $y - 4$
$= x^2 + 4x - 4$
$= x^2 + 4x + 4 - 8$

$= (x + 2)^2 - (2\sqrt{2})^2$

$= \left(x+2+2\sqrt{2}\right)\left(x+2-2\sqrt{2}\right)$

$\therefore$ Factorisation is

$= \left(x+2+\sqrt{2}\right)\left(x+2-\sqrt{2}\right)$

$\left(x+2+2\sqrt{2}\right)\left(x+2-2\sqrt{2}\right)$

7. (1) $a + b + c = 0$
$\Rightarrow a = -b - c$
$\Rightarrow a^2 = -ab - ac$
$\therefore bc - a^2 = bc + ab + ac$
Similarly,
$ca - b^2 = ca + ab + bc$
$ab - c^2 = ab + bc + ca$

$\therefore \frac{bc}{bc-a^2} + \frac{ca}{ca-b^2} + \frac{ab}{ab-c^2}$

$= \frac{bc}{ab+bc+ca} + \frac{ca}{ab+bc+ca} + \frac{ab}{ab+bc+ca}$

$= \frac{ab+bc+ca}{ab+bc+ca} = 1$

8. (2) $a = 2+\sqrt{3}$

$\therefore \frac{1}{a} = \frac{1}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} = \frac{2-\sqrt{3}}{4-3}$

$= 2-\sqrt{3}$
Now,

$\frac{a^3}{a^6+3a^3+1} = \frac{1}{a^3+3+\frac{1}{a^3}}$

[Dividing numerator and denominator by a^3]

$= \frac{1}{a^3+\frac{1}{a^3}+3}$

$= \frac{1}{\left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right)+3}$

$= \frac{1}{(4)^3 - 3(4) + 3}$

$= \frac{1}{64-12+3} = \frac{1}{55}$

9. (3) $3p + 2pq = 4$
$\Rightarrow p(3 + 2q) = 4$

$\Rightarrow p = \frac{4}{3+2q}$ .....(i)

Now, putting the value of p in $5q + pq = 3$, we get

$5q + \frac{4}{3+2q}(q) = 3$

$\Rightarrow \frac{15q+10q^2+4q}{3+2q} = 3$

$\Rightarrow 19q + 10q^2 = 9 + 6q$
$\Rightarrow 10q^2 + 13q - 9 = 0$
$\Rightarrow 10q^2 + 18q - 5q - 9 = 0$
$\Rightarrow 2q(5q + 9) - 1(5q + 9) = 0$
$\Rightarrow (2q - 1)(5q + 9) = 0$

$\Rightarrow q = \frac{1}{2}$ or $-\frac{9}{5}$

Putting $q = \frac{1}{2}$ in (i),

$p = \frac{4}{3+2\times\frac{1}{2}} = 1$

Putting $q = -\frac{9}{5}$

$p = \frac{4}{3+2.\left(-\frac{9}{5}\right)} = \frac{4\times5}{15-18}$

$= -\frac{20}{3}$

10. (1) $a + b + c = 5;$
$ab + bc + ca = 7$
$abc = 3$
$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$
$= 25 - 2 \times 7 = 11.$
Clearly, $a = b = 1,\ c = 3$
$a = c = 1,\ b = 3$
$b = c = 1,\ a = 3$

$\therefore \left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)$

$= (1+1)+\left(\frac{1}{3}+3\right)+\left(3+\frac{1}{3}\right)$

$= 8+\frac{1}{3}+\frac{1}{3} = 8\frac{2}{3}$

11. (3) $\frac{a}{b+c} = 1 - \frac{b}{c+a} - \frac{c}{a+b}$

$\therefore \frac{a^2}{b+c} = a - \frac{ab}{c+a} - \frac{ac}{a+b}$

$\frac{b}{a+c} = 1 - \frac{a}{b+c} - \frac{c}{a+b}$

$\therefore \frac{b^2}{a+c} = b - \frac{ab}{b+c} - \frac{bc}{a+b}$

$\frac{a}{a+b} = 1 - \frac{a}{b+c} - \frac{b}{c+a}$

$\therefore \frac{c^2}{a+c} = c - \frac{ac}{b+c} - \frac{bc}{c+a}$

$\therefore \frac{a^2}{b+c} + \frac{b^2}{a+c} + \frac{c^2}{a+b}$

$= a + b + c - \left(\frac{ab}{c+a}+\frac{bc}{c+a}\right)$

$-\left(\frac{ac}{a+b}+\frac{bc}{a+b}\right)-\left(\frac{ab}{b+c}+\frac{ac}{b+c}\right)$

$= a + b + c - b\left(\frac{a+c}{c+a}\right) - c\left(\frac{a+b}{a+b}\right)$

$- a\left(\frac{b+c}{b+c}\right)$

$= a + b + c - b - c - a = 0$

12. (2) $a = 7+4\sqrt{3}$

$\therefore \frac{1}{a} = \frac{1}{7+4\sqrt{3}}$

$= \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{7-4\sqrt{3}}{49-48} = 7-4\sqrt{3}$

Expression

$= \frac{3a^6+2a^4+4a^3+2a^2+3}{a^4+a^3+a^2}$

$= \frac{3a^3+2a+4+\frac{2}{a}+\frac{3}{a^3}}{a+1+\frac{1}{a}}$

[Dividing numerator and denominator by a^3]

$$= \frac{3\left(a^3+\frac{1}{a^3}\right)+2\left(a+\frac{1}{a}\right)+4}{\left(a+\frac{1}{a}\right)+1}$$

$$= \frac{3\left(\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)\right)+2\left(a+\frac{1}{a}\right)+4}{\left(a+\frac{1}{a}\right)+1}$$

$$= \frac{3\left((14)^3-3\times14\right)+2\times14+4}{14+1}$$

$$= \frac{3\times2702+28+4}{15} = \frac{8138}{15}$$

13. (4) $\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}$

$= a + b + c$

$\Rightarrow \frac{x-bc}{b+c}-a+\frac{x-ca}{c+a}-b+\frac{x-ab}{a+b}-c=0$

$\Rightarrow \frac{x-bc-ab-ac}{b+c}+\frac{x-ca-bc-ab}{c+a}$

$+ \frac{x-ab-ac-bc}{a+b} = 0$

$\Rightarrow x - bc - ab - ac = 0$

$\Rightarrow x = ab + bc + ac$

14. (1) $x^4 + \frac{1}{x^4} = 47$

$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right)^2 = 47$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 - 2 = 47$

[$\because$ $a^2 + b^2 = (a + b)^2 - 2ab$]

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = 47 + 2 = 49$

$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{49} = 7$

Again, $\left(x+\frac{1}{x}\right)^2 - 2 = 7$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 7 + 2 = 9$

$\Rightarrow x + \frac{1}{x} = \sqrt{9} = 3$

On cubing both sides,

$\left(x+\frac{1}{x}\right)^3 = 3^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$

$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 = 18$

15. (3) $x = \frac{1}{2-\sqrt{3}}$

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2} = \frac{2+\sqrt{3}}{4-3}$

$= 2 + \sqrt{3}$

$\Rightarrow x - 2 = \sqrt{3}$

On squaring both sides,

$\Rightarrow (x - 2)^2 = \left(\sqrt{3}\right)^2$

$\Rightarrow x^2 - 4x + 4 = 3$

$\Rightarrow x^2 - 4x + 1 = 0$

$$\begin{array}{r|l} x^2-4x+1 & x^3-2x^2-7x+5 \;(x+2 \\ & \underline{\overset{-}{}x^3 \overset{+}{-} 4x^2 \overset{-}{+} x} \\ & 2x^2-8x+5 \\ & \underline{\overset{-}{}2x^2 \overset{+}{-} 8x \overset{-}{+} 2} \\ & 3 \end{array}$$

$\therefore$ $x^3 - 2x^2 - 7x + 5 = (x^2 - 4x + 1)(x + 2) + 3 = 0 + 3 = 3$

16. (2) Expression $= \frac{5+2\sqrt{3}}{7+4\sqrt{3}}$

$= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

Rationlising the denominator

$= \frac{5\times7-5\times4\sqrt{3}+2\sqrt{3}\times7-2\sqrt{3}\times4\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2}$

$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$

$= 11 - 6\sqrt{3}$

$\therefore$ $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$

$\Rightarrow a + b\sqrt{3} = 11 - 6\sqrt{3}$

$\Rightarrow a = 11,\ b = -6$

17. (3) $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{x-(a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow -ab(a + b) = (a+b)\,x\,(a+b+x)$

$\Rightarrow (a + b)\{x(a + b + x) + ab\} = 0$

$\Rightarrow x(a + b + x) + ab = 0$

[$\because$ $a + b \neq 0$]

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow x(x + a) + b(x + a) = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x = -a$ or, $-b$

18. (2) Let, $P(x) = 2x^3 + ax^2 + 11x + a + 3$

$(2x - 1)$ is its factor.

$\therefore$ $P\left(\frac{1}{2}\right) = 0$

$\Rightarrow 2 \times \left(\frac{1}{2}\right)^3 + a \times \left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} + a + 3 = 0$

$\Rightarrow \frac{1}{4} + \frac{a}{4} + \frac{11}{2} + a + 3 = 0$

$\Rightarrow \frac{1+a+22+4a+12}{4} = 0$

$\Rightarrow \frac{5a+35}{4} = 0$

$\Rightarrow 5a + 35 = 0 \Rightarrow 5a = -35$

$\Rightarrow a = -7$

19. (3) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow 15^2 = 83 + 2(ab + bc + ca)$

$\Rightarrow 225 = 83 + 2(ab + bc + ca)$

$\Rightarrow 142 = 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = \frac{142}{2} = 71$

$\therefore$ $a^3 + b^3 + c^3 - 3abc = 15 \times (83 - 71) = 15 \times 12 = 180$

20. (3) $x - a + x - b + x - c$

$= 3x - (a + b + c) = 0$

$\therefore$ $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c) = 0$

[$\because$ $a^3 + b^3 + c^3 - 3abc = 0$ when $a + b + c = 0$

21. (1) $p = 2 - a \Rightarrow a + p - 2 = 0$

$\therefore\ a^3 + 6ap + p^3 - 8$

$= a^3 + p^3 + (-2)^3 - 3ap(-2)$

$= (a + p - 2)\{a^2 + p^2 + (-2)^2 - ap - p(-2) - a(-2)\}$

$= (a + p - 2)(a^2 + p^2 + 4 - ap + 2p + 2a) = 0$

22. (1) $4x^2 + 4y^2 + 4z^2 - 12x - 12y + 18 = 0$

$\Rightarrow\ (2x)^2 - 2 \times 2x \times 3 + 9 + (2y)^2 - 2 \times 2y \times 3 + 9 + 4z^2 = 0$

$\Rightarrow\ (2x - 3)^2 + (2y - 3)^2 + 4z^2 = 0$

$\Rightarrow\ 2x - 3 = 0$

$\Rightarrow x = \dfrac{3}{2}$;

$2y - 3 = 0$

$\Rightarrow y = \dfrac{3}{2}$, $z = 0$

$\therefore\ x + y + z = \dfrac{3}{2} + \dfrac{3}{2} + 0$

$= \dfrac{6}{2} = 3$

[If $x^2 + y^2 + z^2 = 0 \Rightarrow x = 0,\ y = 0,\ z = 0$]

23. (1) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have infinite solutions if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

$\Rightarrow\ \dfrac{5}{10} = \dfrac{2}{4} = \dfrac{-k}{-3}$

$\Rightarrow\ \dfrac{1}{2} = \dfrac{k}{3} \Rightarrow k = \dfrac{3}{2}$

24. (2) $x = \text{Rs. }(y + 20)$

$z = \text{Rs. }(y + 40)$

$\therefore\ y + y + 20 + y + 40 = 120$

$\Rightarrow 3y = 60$

$\Rightarrow y = \text{Rs. } 20$

25. (1) We know that

Slope of a line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{-4}{3} = \dfrac{x - 11}{2 + 8}$

$\Rightarrow \dfrac{-4}{3} = \dfrac{x - 11}{10}$

$\Rightarrow -40 = 3x - 33$

$\Rightarrow -40 + 33 = 3x$

$\Rightarrow -7 = 3x$

$\Rightarrow x = \dfrac{-7}{3}$

26. (2) Let the equation of line be

$y - y_1 = m(x - x_1)$

As it passes through (2, 2) and having slope $m = \tan 45° = 1$

$\Rightarrow y - 2 = 1(x - 2)$

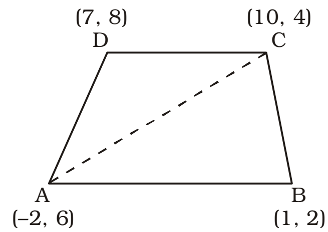
$y - x = 0$

or

$x - y = 0$

27. (1) We know that, equation of line is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



Equation of diagonal AC, will be

$$\frac{y - 6}{4 - 6} = \frac{x + 2}{10 + 2}$$

$$\frac{y - 6}{-2} = \frac{x + 2}{12}$$

$12y - 72 = -2x - 4$

$2x + 12y = -4 + 72$

$2x + 12y = 68$

$\Rightarrow x + 6y = 34$

28. (2) We know that,

Angle between two lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Here,

$m_1 = \sqrt{3}$ and

$m_2 = \dfrac{1}{\sqrt{3}}$

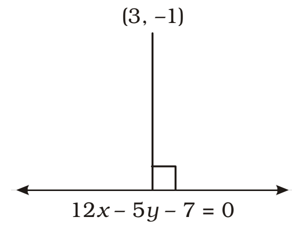
$$\tan\theta = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\,\frac{1}{\sqrt{3}}}\right|$$

$$= \left|\frac{3 - 1}{2\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$\therefore$ Slope $= \dfrac{1}{\sqrt{3}}$

29. (3) Let required distance = d



$$\Rightarrow d_{(3,\,-1)} = \left|\frac{12x - 5y - 7}{\sqrt{12^2 + 5^2}}\right|$$

$$= \left|\frac{12 \times 3 - 5 \times -1 - 7}{\sqrt{169}}\right|$$

$$= \left|\frac{36 + 5 - 7}{13}\right|$$

$$= \left|\frac{34}{13}\right|$$

$= \dfrac{34}{13}$ units

30. (1) Equation of required line be

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 1}{-4 - 1} = \frac{x + 1}{2 + 1}$$

$$\frac{y - 1}{-5} = \frac{x + 1}{3}$$

$3y - 3 = -5x - 5$

$5x + 3y + 2 = 0$

31. (4) Here,

$c = 2$

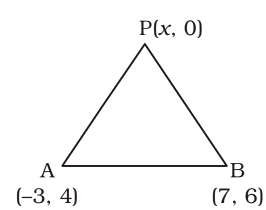
$m = \tan 60° = \sqrt{3}$

Equation of line be,

$y = mx + c$

$\Rightarrow y = \sqrt{3}x + 2.$

32. (4) Let the point on x-axis be P(x, 0)



A.T.Q

PA = PB

$\Rightarrow PA^2 = PB^2$

$(x + 3)^2 + (4 - 0)^2 = (x - 7)^2 + (6 - 0)^2$

$x^2 + 9 + 6x + 16 = x^2 + 49 - 14x + 36$

$x^2 + 6x + 25 = x^2 - 14x + 85$

$20x = 60$

$x = 3$

$\therefore$ Point P is (3, 0)

33. (3) Here,

$3x - 4y + 5 = 0$

$\Rightarrow -4y = -3x - 5$

$$y = \frac{-3}{-4}x\ \frac{-5}{-4}$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

Compare it with, $y = mx + c$ we get

$$m = \frac{3}{4} \quad \text{and} \quad c = \frac{5}{4}$$

34. (1) Equation of line in normal form can be written as

$$\frac{ax}{\sqrt{a^2 + b^2}} + \frac{by}{\sqrt{a^2 + b^2}} + \frac{c}{\sqrt{a^2 + b^2}} = 0$$

$$\Rightarrow \frac{\sqrt{3}x}{\sqrt{3}^2 + 1^2} + \frac{y}{\sqrt{\sqrt{3}^2 + 1^1}} - \frac{8}{\sqrt{\sqrt{3}^2 + 1^2}} = 0$$

$$\frac{\sqrt{3}x}{\sqrt{4}} + \frac{y}{\sqrt{4}} - \frac{8}{\sqrt{4}} = 0$$

$$\Rightarrow \frac{\sqrt{3}x}{2} + \frac{y}{2} - \frac{8}{2} = 0$$

35. (2) We know that angle between the lines is

$$\tan\theta = \left|\frac{m_1 + m_2}{1 + m_1 m_2}\right|$$

Here, Equation of line is

$y - x - 7 = 0$

$\Rightarrow m_1 = 1$

similarly,

$$m_2 = \frac{1}{\sqrt{3}}$$

Now,

$$\tan\theta = \left|\frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\cdot\frac{1}{\sqrt{3}}}\right|$$

$$= \left|\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}\right|$$

$$= \left|\frac{\sqrt{3} - 1}{\sqrt{3} + 1}\right|$$

$$= \left|\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}\right|$$

$$= \left|\frac{(\sqrt{3} - 1)^2}{\sqrt{3}^2 - 1^2}\right|$$

$$= \left|\frac{\sqrt{3}^2 + 1^2 - 2\sqrt{3}}{3 - 1}\right|$$

$$= \left|\frac{4 - 2\sqrt{3}}{2}\right|$$

$$\tan\theta = (2 - \sqrt{3})$$

$$\theta = \tan^{-1}(2 - \sqrt{3})$$

36. (1) When a line cuts an intercept of a and b x-axis. Its equation will be $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow 3x + 2y - 5 = 0$

$\Rightarrow 3x + 2y = 5$

Dividing by 5 on both sides, we get

$$\frac{3}{5}x + \frac{2}{5}y = 1$$

$$\Rightarrow \frac{x}{\frac{5}{3}} + \frac{y}{\frac{5}{2}} = 1 \text{ (Desired Result)}$$

37. (4) Distance between two parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$ be d

$$\Rightarrow d = \left|\frac{15x + 8y - 34}{\sqrt{15^2 + 8^2}}\right|$$

We know that from second equation

$15x + 8y = -31$

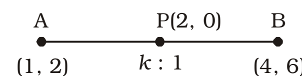
$$\Rightarrow d = \left|\frac{-31 - 34}{\sqrt{225 + 64}}\right|$$

$$= \left|\frac{-65}{\sqrt{289}}\right|$$

$$= \left|\frac{-65}{17}\right|$$

$$d = \frac{65}{17} \text{ units}$$

38. (1) Let the ratio be $k : 1$



Using internal section formula

$$2 = \frac{4 \times k + 1 \times 1}{k + 1}$$

$\Rightarrow 2(k + 1) = 4k + 1$

$2k + 2 = 4k + 1$

$-2k = -1$

$$k = \frac{1}{2}$$

and

$$0 = \frac{6 \times k + 1 \times 2}{k + 1}$$

$0\ (k + 1) = 6k + 2$

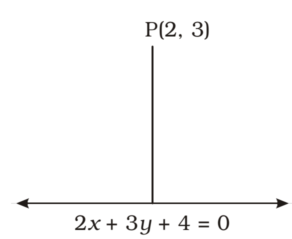
$6k = -2$

$$k = \frac{-1}{3}$$

The value of k is not negative.

$\therefore$ Ratio will be 1 : 2

39. (3) Let the distance be d



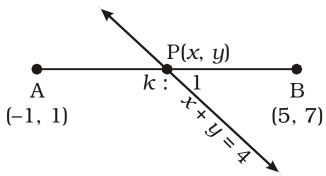
$$\Rightarrow d = \left|\frac{2x + 3y + 4}{\sqrt{2^2 + 3^2}}\right|$$

$$d_{(2,\,3)} = \left|\frac{2 \times 2 + 3 \times 3 + 4}{\sqrt{13}}\right|$$

$$d_{(2, 3)} = \left|\frac{17}{\sqrt{13}}\right|$$

$$\Rightarrow d_{(2, 3)} = \frac{17}{\sqrt{13}} \text{ units}$$

40. (3) Let the ratio $k : 1$



= Using section formula,

$$x = \frac{5 \times k + 1 \times -1}{k + 1}$$

$$\Rightarrow x = \frac{5k - 1}{k + 1}$$

$$y = \frac{7 \times k + 1 \times 1}{k + 1}$$

$$y = \frac{7k + 1}{k + 1}$$

Putting the value of x and y in the equation of line, we get

$$\frac{5k - 1}{k + 1} + \frac{7k + 1}{k + 1} = 4$$

$12k = 4(k + 1)$

$\Rightarrow 12k = 4k + 4$

$8k = 4$

$$k = \frac{1}{2}$$

$\therefore$ Ratio is $1 : 2$.

41. (4) When three points are collinear then area of triangle is zero.

$$\Rightarrow ar\,\Delta = \frac{1}{2}\begin{vmatrix} x & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[x\ (1 - 5) + 1(2 - 4) + 1(10 - 4)]$$

$\Rightarrow -4x - 2 + 6 = 0$

$\Rightarrow -4x + 4 = 0$

$\Rightarrow x = 1$

42. (3) Here,

$a = -5,\ b = 2$

$\therefore$ Equation of line will be

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \frac{x}{-5} + \frac{y}{2} = 1$$

$\Rightarrow 2x - 5y = -10$

43. (2) When two lines are perpendicular than the product of their slopes is –1.

i.e $m_1 \times m_2 = -1$

For equation

$-kx + 2y + 3 = 0$

$$m_1 = \frac{k}{2}$$

For equation

$2x + 4y + 7 = 0$

$$m_2 = -\frac{2}{4}$$

$$m_2 = -\frac{1}{2}$$

As lines are perpendicular

$m_1 \times m_2 = -1$

$$\frac{k}{2} \times -\frac{1}{2} = -1$$

$$\boxed{k = 4}$$

44. (1) When two lines are parallel then their slopes are equal.

i.e. $m_1 = m_2$

Here,

$m_1 = m$

$$m_2 = \frac{-3}{-4}$$

[From equation $3x - 4y + 2 = 0$]

$$m_2 = \frac{3}{4}$$

As lines are parallel.

$\therefore m_1 = m_2$

$$\Rightarrow m = \frac{3}{4}$$

Let the equation of line be

$y - y_1 = m(x - x_1)$

As line passes through (–2, 3)

$\therefore$ Equation of line be

$$(y - 3) = \frac{3}{4}(x + 2)$$

$4y - 12 = 3x + 6$

$3x - 4y + 18 = 0$

45. (4) Let the equation of line be $y - y_1 = m(x - x_1)$

Here, $m = \dfrac{1}{2}$

and $x_1 = -4,\ y_1 = 3$

$\Rightarrow$ Equation of line be

$$y - 3 = \frac{1}{2}(x + 4)$$

$2y - 6 = x + 4$

$x - 2y + 10 = 0$

46. (2) We know that co-ordinates of

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\Rightarrow x = \frac{2 + 6}{2} = 4$$

$$y = \frac{4 + 8}{2} = 6$$

47. (3) As the line $kx + 3y + 6 = 0$ passes through (2, 4)

$\therefore k \times 2 + 3 \times 4 + 6 = 0$

$2k + 12 + 6 = 0$

$2k + 18 = 0$

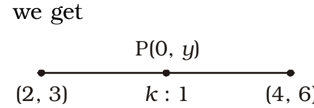
$2k = -18$

$k = -9$

48. (4) Let the co-ordinates of point be $(0, y)$ because on y-axis, x is zero.

Let the ratio be $k : 1$

Using internal section formula, we get



$$0 = \frac{4k + 2}{k + 1}$$

$$k = \frac{-2}{4}$$

$$k = \frac{-1}{2}$$

–ve sign shows that point divides the line segment externally.

49. (1) Let the slope of line be m

Here,

$\theta = 45°$

$$m_2 = \frac{-1}{-2} = \frac{1}{2}$$

We know that,

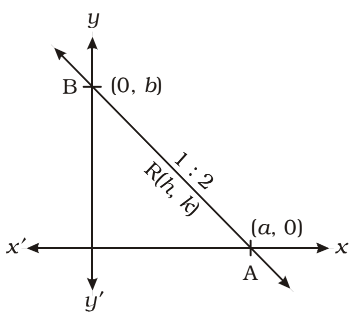
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 45° = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$$

$$1 = \frac{2m - 1}{2 + m}$$

$\Rightarrow 2 + m = 2m - 1$
$m = 3$
$\therefore$ Equation of line be
$y - y_1 = m(x - x_1)$
$y - 2 = 3(x - 3)$
$y - 2 = 3x - 9$
$3x - y - 7 = 0$

50. (2) Using internal section formula, we have



$$h = \frac{1 \times a + 2 \times 0}{1 + 2}$$

$\Rightarrow a = 3h$

Similarly,

$$k = \frac{1 \times 0 + 2 \times b}{1 + 2}$$

$$k = \frac{2b}{3}$$

$$b = \frac{3k}{2}$$

$\therefore$ Equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \frac{x}{3h} + \frac{2y}{3k} = 1$$

$$\Rightarrow \frac{x}{h} + \frac{2y}{k} = 3$$

51. (3) Here,

$p = 3$ and $\alpha = 120°$

We know that equation of line is
$x \cos\alpha + y \sin\alpha = p$
$\Rightarrow x\cos 120° + y \sin 120° = 3$
$\Rightarrow x\cos(180° - 60°) + y\sin(180° - 60°) = 3$
$\Rightarrow -x\cos 60° + y\sin 60° = 3$

$\because \cos(180° - \theta) = -\cos\theta$
$\sin(180° - \theta) = \sin\theta$

$$\Rightarrow -\frac{x}{2} + y\frac{\sqrt{3}}{2} = 3$$

$\Rightarrow -x + \sqrt{3}y = 6$

52. (1) We know that when three points are collinear than area of triangle is zero.

$$ar\Delta = \frac{1}{2}\begin{vmatrix} h & 0 & 1 \\ a & b & 1 \\ 0 & k & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}[h(b - k) + 1(ak)] = 0$$

$\Rightarrow bh - hk + ak = 0$
$ak + bh = hk$

Dividing both sides by hk, we get

$$\frac{ak}{hk} + \frac{bh}{hk} = 1$$

$$\frac{a}{h} + \frac{b}{k} = 1$$

53. (4) Let the slope of required line be m.

Also,

$$m_1 = \frac{-2}{3}$$

$m_1 = m_2$

($\because$ lines are parallel)

$$m = \frac{-2}{3}$$

Equation of line be
$y - y_1 = m(x - x_1)$

$$\Rightarrow (y + 5) = \frac{-2}{3}(x + 4)$$

$3y + 15 = -2x - 8$
$2x + 3y + 23 = 0$

54. (2) Let the equation of line be
$(x + 2y - 5) + \lambda(x - 3y - 7) = 0$
As it passes through (0, –1)
$\therefore 0 - 2 - 5 + \lambda(0 + 3 - 7) = 0$
$-7 - 4\lambda = 0$

$$\lambda = \frac{-7}{4}$$

$\therefore$ Equation of line is

$$(x + 2y - 5)\frac{-7}{4}(x - 3y - 7) = 0$$

$\Rightarrow 4x + 8y - 20 - 7x + 21y + 49 = 0$
$-3x + 29y + 29 = 0$
$3x - 29y - 29 = 0$

55. (3) Here,

$$\theta = \frac{\pi}{4}$$

$m_1 = m$

$$m_2 = \frac{1}{2}$$

We know that

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

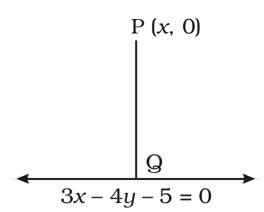
$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$$

$$\Rightarrow 1 = \frac{2m - 1}{2 + m}$$

$\Rightarrow 2 + m = 2m - 1$

$\Rightarrow \boxed{m = 3}$

56. (4) Let the co-ordinates of point p be $(x, 0)$



Also,
$PQ = 4$

$$\Rightarrow PQ = \left|\frac{3x - 4y - 5}{\sqrt{3^2 + 4^2}}\right|$$

$$PQ = \left|\frac{3x - 5}{5}\right|$$

$$4 = \frac{3x - 5}{5}$$

$3x - 5 = 20$

$3x = 25$

$$x = \frac{25}{3}$$

$\therefore$ Co-ordinates are $\left(\frac{25}{3}, 0\right)$

57. (2) Let the slope of line be m

Here,

$$m_1 = \frac{-1}{-7} = \frac{1}{7}$$

As lines are perpendicular,

$\therefore\ m_1 \times m_2 = -1$

$$m \times \frac{1}{7} = -1$$

$m = -7$

$\therefore$ Equation of line is

$(y - y_1) = m(x - x_1)$

$(y - 0) = -7(x - 3)$

$y = -7x + 21$

$\Rightarrow 7x + y = 21$

58. (4) We know that when a line is parallel to x-axis then

Slope = 0

$$\frac{(4 - k^2)}{k - 3} = 0$$

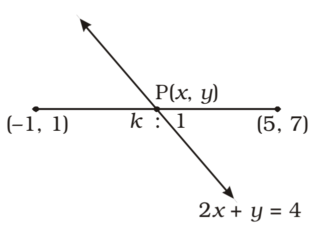
$4 - k^2 = 0$

$k^2 = 4$

$k = \sqrt{4}$

$k = \pm 2$

59. (1) Let the ratio be $k : 1$



Using internal section formula,

$$x = \frac{5k - 1}{k + 1}$$

$$y = \frac{7k + 1}{k + 1}$$

Putting the value of x and y in given equation $2x + y = 4$

$$2\left(\frac{5k - 1}{k + 1}\right) + \frac{7k + 1}{k + 1} = 4$$

$10k - 2 + 7k + 1 = 4(k + 1)$

$17k - 1 = 4k + 4$

$13k = 5$

$$k = \frac{5}{13}$$

$\therefore$ The ratio is $5 : 13$

60. (2) Let the distance between the lines be d

$$\Rightarrow d = \left|\frac{y - mx - c_1}{\sqrt{1 + m^2}}\right|$$

Also we know that

$y - mx = c_2$

$$\Rightarrow d = \left|\frac{c_2 - c_1}{\sqrt{1 + m^2}}\right|$$

$$= \left|\frac{c_1 - c_2}{\sqrt{1 + m^2}}\right|$$

61. (3) Here, it is clear that distance of the given lines from (0, 0) is equal.

$$d_1 = \left|\frac{4 \times 0 + 3 \times 0 + 10}{\sqrt{4^2 + 3^2}}\right|$$

$$= \left|\frac{10}{5}\right|$$

= 2 units

$$d_2 = \left|\frac{5 \times 0 - 12 \times 0 + 26}{\sqrt{5^2 + 12^2}}\right|$$

$$= \left|\frac{26}{13}\right|$$

= 2 units

$$d_3 = \left|\frac{7 \times 0 + 24 \times 0 - 50}{\sqrt{7^2 + 24^2}}\right|$$

$$d_3 = \left|\frac{-50}{\sqrt{625}}\right|$$

$$= \left|\frac{50}{25}\right|$$

$d_3 = 2$

62. (4) The line $\frac{x}{a} + \frac{y}{b} = 1$, passes through (2, –3)

$\therefore \frac{2}{a} - \frac{3}{b} = 1$ 1

Similarly, The line passes through (4, –5)

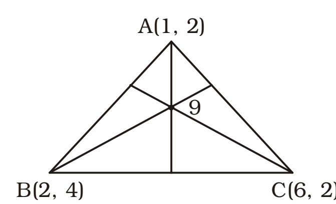
$\frac{4}{a} - \frac{5}{b} = 1$ 2

Point (–1, –1) satisfies both the equation.

63. (2) Co-ordinates of centroid

$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$



$$\Rightarrow x = \frac{1 + 2 + 6}{3}$$

$x = 3$

$$y = \frac{2 + 4 + 2}{3}$$

$$y = \frac{8}{3}$$

Co-ordinates of Centroid

$$= \left(3, \frac{8}{3}\right)$$

64. (3) As the lines have equal intercepts.

$\therefore$ Equation of line is

$x + y = a$

$\therefore$ Slope = –1