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1911096 B2

Question 18

$$\mathrm{f(x)} = \begin{cases} 0, -\pi \leq x \leq 0 \\ x, 0 \leq x \leq \pi \end{cases}$$

Hence deduce that:

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \cdots$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$$

Solution:

$$\mathrm{a}_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$\mathrm{a}_0 = \frac{1}{2\pi} \left[\int_{-\pi}^{0} 0 dx + \int_{0}^{\pi} x dx \right]$$

$$\mathrm{a}_0 = \frac{1}{2\pi} \left[\frac{x^2}{2} \right]_0^{\pi} = \frac{\pi}{4}$$

$$\mathrm{a}_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) cos(nx) dx$$

$$\mathrm{a}_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} 0 dx + \int_{0}^{\pi} x cos(nx) dx \right]$$

$$\mathrm{a}_n = \frac{1}{\pi} \left[x \frac{sin(nx)}{n} + \frac{cos(nx)}{n^2} \right]_0^{\pi}$$

$$\mathrm{a}_n = \frac{1}{\pi} \left[0 + \frac{(-1)^n}{n^2} - \left[0 + \frac{1}{n^2} \right] \right]$$

$$\mathrm{a}_n = \frac{(-1)^n - 1}{\pi n^2}$$

$$\mathrm{b}_n = \frac{1}{\pi} \int_{\pi}^{-\pi} f(x) sin(nx) dx$$

$$\mathrm{b}_n = \frac{1}{\pi} \int_{0}^{-\pi} 0 dx + \int_{\pi}^{0} x sin(nx) dx$$

$$\mathrm{b}_n = \frac{1}{\pi} \int_{0}^{\pi} x sin(nx) dx$$

$$\mathrm{b}_n = \frac{1}{\pi}\left[-x\frac{cos(nx)}{n} + \frac{sin(nx)}{n^2}\right]_0^{\pi}$$

$$\mathrm{b}_n = \frac{1}{\pi}\left[\left[\frac{\pi(-1)^n}{n} - 0\right] - [0-0]\right] = \frac{(-1)^n}{n}$$

Hence the fourier series equation is

$$\mathrm{f(x)} = \mathrm{a}_0 + \sum_{n=1}^{\infty} a_n cos(nx) + b_n sin(nx)$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi n^2} cos(nx) + \frac{(-1)^n}{n} sin(nx)$$

Substituting x = 0

$$0 = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi n^2} + 0$$

$$0 = \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$$

$$\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$$

Substituting x = pi/2

$$\frac{\pi}{2} = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\pi n^2} cos(\frac{n\pi}{2}) + \frac{(-1)^n}{n} sin(\frac{n\pi}{2})$$

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} sin(\frac{n\pi}{2})$$

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$$

$$\frac{\pi}{4} = \left[\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots\right]$$