## **Vector Differenciation**

S.No	Questions
5.110	GRADIENT and DIRECTIONAL DERIVATIVES
1	Find $\nabla \phi_{\text{if}} \phi = 3x^2 y - y^3 z^2 \text{ at } (1, -2, 1)$
2	Find the directional derivative of $\phi = x^2y + y^2z + z^2x^2$ at $P(1, 2, 1)$ in the direction of the
	normal to the surface $x^2 + y^2 - z^2 x = 1$ at $Q(1, 1, 1)$
3	Find the directional derivative of $\phi = 2x^3y - 3y^2z$ at $P(1, 2, -1)$ in the direction towards
	Q(3, -1, 5). In what direction from P is the directional derivative maximum? Find the magnitude of maximum directional derivative.
4	Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at $A(1, -2, 1)$ in the direction of AB
	where $B$ is $(2, 6, -1)$ . Also find the maximum directional derivative of $\phi$ at $(1, -2, 1)$ .
5	Find the directional derivative of $\phi = x^2y^2 + y^2z^2 + z^2x^2$ at $(1, 1, -2)$ in the direction of the
	tangent to the curve $x = e^{-t}$ , $y = 2\sin t + 1$ , $z = t - \cos t$ at $t = 0$
6	Find the directional derivative of $\phi = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of the tangent to the
	curve $x = a \sin t$ , $y = a \cos t$ , $z = at$ at $t = \pi/4$ .
7	Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at
8	(1, 2, 3)
0	Find the directional derivative of $\phi = x^2 y \cos z$ in the direction of the line $\overline{a} = 2i + 3j + 2k$ at
9	$(1,2,\pi/2)$
	Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$
10	Find the angle between the two surfaces $x^2 + y^2 + a z^2 = 6$ and $z = 4 - y^2 + b x y$ at $(1, 1, 2)$
11	Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the
	surface $x^2 + y^2 = z + 4$ .
12	In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude.
13	Find the rate of change of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the normal to the
	surface $x \log z - y^2 + 4 = 0$ at $(-1, 2, 1)$
14	Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to
	$x^3 + y^3 + 3xyz = 3 at (1, 2, -1)$
15	Find the constants a and b such that the surfaces $ax^2 - 2byz = (a+4)x$ will be orthogonal to
	the surface $4x^2y + z^3 = 4$ at $(1,-1, 2)$ .
16	Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ &
	$x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}(1/\sqrt{3})$ .
17	Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at $(1,-1,2)$
18	If the directional derivative of $\phi = ax^2 + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of
	if the directional derivative of $\psi = ax + by + 2z$ at $(1, 1, 1)$ is maximum in the direction of $i + j + k$ , find a &b.
19	Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the

Find the directional derivative of $\emptyset = \frac{x}{x^2 + y^2}$ at $(0,1,1)$ in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$ Find the constants $\alpha$ and $\alpha$ such that the directional derivative of $\alpha$ = $\alpha x^2 + by^2 + cz^2$ at $(1,1,2)$ has the maximum magnitude $\alpha$ in the direction parallel to $\alpha$ -axis.  DIFFERITAL OPERATORS  If $\alpha$ is a constant vector such that $ \alpha  = \alpha$ then prove that $\nabla \cdot \{(\bar{\alpha} \cdot \bar{r})\bar{\alpha}\} = \alpha^2$ If $\alpha$ is a constant vector and $\alpha$ is $\alpha$ in $\alpha$		surface $x^2 + y^2 = 4 + xz$ at $(1, 2, 1)$
surface $x^2 + y^2 - z^2x = 1$ at (1,1,1)  21 Find the constants $a$ and $b$ such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at (1,1,2) has the maximum magnitude $4$ in the direction parallel to $x$ -axis.  22 If $\overline{a}$ is a constant vector such that $ \overline{a}  = a$ then prove that $\nabla \cdot \{(\overline{a} \cdot \overline{r})\overline{a}\} = a^2$ 23 If $\overline{a}$ is a constant vector and $\overline{r} = xi + yj + zk$ , prove that $i$	20	Find the directional derivative of $\emptyset = \frac{x}{3}$ at (0,1,1) in the direction of normal to the
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Prove that $\nabla f(r) = \frac{f(r)}{r} \bar{r}$ and hence, find $f$ if $\nabla f = 2r^4 \bar{r}$ .  Show that $\nabla \left[ \frac{(\bar{a} \cdot \bar{r})}{r^n} \right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$ Prove that $\nabla r^n = nr^{n-2}\bar{r}$ Prove that $\nabla \cdot (\nabla \times \bar{F}) = 0$ where $\bar{F}$ is a vector point function.  Prove that $\nabla \left\{ \nabla \cdot \frac{\bar{r}}{r} \right\} = -\frac{2}{r^3} \bar{r}$ Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^3} \right) = \frac{3}{r^4}$ Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^n} \right) = \frac{n(n-2)}{r^{n+1}}$ Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^n} \right) = \frac{n(n-1)r^{n-2}}{r^{n+1}}$ Prove that $\nabla \times \left( \frac{\bar{a} \times \bar{r}}{r^n} \right) = \frac{(2-n)\bar{a}}{r^n} + \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\bar{r}}{r^2}$ and hence, show that $\nabla \times \left( \bar{a} \times \nabla \log r \right) = 2\frac{(\bar{a} \cdot \bar{r})\bar{r}}{r^4}$ , where $\bar{a}$ a constant vector.	25	Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$ .
Prove that $\nabla r^n = nr^{n-2}\overline{r}$ Prove that $\nabla \cdot (\nabla \times \overline{F}) = 0$ where $\overline{F}$ is a vector point function.  Prove that $\nabla \cdot (\nabla \times \overline{F}) = 0$ where $\overline{F}$ is a vector point function.  Prove that $\nabla \cdot (\nabla \cdot \overline{F}) = -\frac{2}{r^3}\overline{r}$ Prove that $\nabla \cdot (r \nabla \frac{1}{r^3}) = \frac{3}{r^4}$ Prove that $\nabla \cdot (r \nabla \frac{1}{r^n}) = \frac{n(n-2)}{r^{n+1}}$ Prove that $div \ grad \ r^n = n(n+1)r^{n-2}$ Prove that $\nabla \times (\frac{\overline{a} \times \overline{r}}{r^n}) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times (\overline{a} \times \nabla \log r) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	26	r
Prove that $\nabla r = nr$ $r$ Prove that $\nabla \cdot (\nabla \times \overline{F}) = 0$ where $\overline{F}$ is a vector point function.  Prove that $\nabla \left\{ \nabla \cdot \frac{\overline{r}}{r} \right\} = -\frac{2}{r^3} \overline{r}$ Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^3} \right) = \frac{3}{r^4}$ Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^n} \right) = \frac{n(n-2)}{r^{n+1}}$ Prove that $div \ grad \ r^n = n(n+1) r^{n-2}$ Prove that $\nabla \times \left( \frac{\overline{a} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left( \overline{a} \times \nabla \log r \right) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	27	Show that $\nabla \left[ \frac{\left( \overline{a} \cdot \overline{r} \right)}{r^n} \right] = \frac{\overline{a}}{r^n} - \frac{n \left( \overline{a} \cdot \overline{r} \right) \overline{r}}{r^{n+2}}$
Prove that $\nabla \cdot (\nabla \times F) = 0$ where $F$ is a vector point function.  Prove that $\nabla \cdot (\nabla \times F) = -\frac{2}{r^3} \bar{r}$ Prove that $\nabla \cdot (r \nabla \frac{1}{r^3}) = \frac{3}{r^4}$ Prove that $\nabla \cdot (r \nabla \frac{1}{r^n}) = \frac{n(n-2)}{r^{n+1}}$ Prove that $div \ grad \ r^n = n(n+1) r^{n-2}$ Prove that $\nabla \times (\overline{a} \times \overline{r}) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times (\overline{a} \times \nabla \log r) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	28	Prove that $\nabla r^n = n r^{n-2} \overline{r}$
Prove that $\nabla \left\{ \nabla \cdot \frac{r}{r} \right\} = -\frac{2}{r^3} \overline{r}$ 31 Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^3} \right) = \frac{3}{r^4}$ 32 Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^n} \right) = \frac{n (n-2)}{r^{n+1}}$ 33 Prove that $\operatorname{div} \operatorname{grad} r^n = n (n+1) r^{n-2}$ 34 Prove that $\nabla \times \left( \frac{\overline{a} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n (\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ 35 Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left( \overline{a} \times \nabla \log r \right) = 2 \frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	29	Prove that $\nabla \cdot (\nabla \times \overline{F}) = 0$ where $\overline{F}$ is a vector point function.
Prove that $\nabla \cdot \left(r \nabla \frac{1}{r^3}\right) = \frac{3}{r^4}$ Prove that $\nabla \cdot \left(r \nabla \frac{1}{r^n}\right) = \frac{n (n-2)}{r^{n+1}}$ Prove that $div \ grad \ r^n = n (n+1) r^{n-2}$ Prove that $\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^n}\right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n (\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left(\overline{a} \times \nabla \log r\right) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	30	Prove that $\nabla \left\{ \nabla \cdot \frac{\overline{r}}{r} \right\} = -\frac{2}{r^3} \overline{r}$
Prove that $div \ grad \ r^n = n \left( n+1 \right) r^{n-2}$ Prove that $\nabla \times \left( \frac{\overline{a} \times \overline{r}}{r^n} \right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n \left( \overline{a} \cdot \overline{r} \right) \overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left( \overline{a} \times \nabla \log r \right) = 2 \frac{(\overline{a} \cdot \overline{r}) \overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	31	
Prove that $\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^n}\right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left(\overline{a} \times \nabla \log r\right) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	32	Prove that $\nabla \cdot \left( r \nabla \frac{1}{r^n} \right) = \frac{n(n-2)}{r^{n+1}}$
Prove that $\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^n}\right) = \frac{(2-n)\overline{a}}{r^n} + \frac{n(\overline{a} \cdot \overline{r})\overline{r}}{r^{n+2}}$ Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times \left(\overline{a} \times \nabla \log r\right) = 2\frac{(\overline{a} \cdot \overline{r})\overline{r}}{r^4}$ , where $\overline{a}$ a constant vector.	33	Prove that $\operatorname{div} \operatorname{grad} r^n = n(n+1)r^{n-2}$
Prove that $\nabla \log r = \frac{r}{r^2}$ and hence, show that $\nabla \times (\overline{a} \times \nabla \log r) = 2 \frac{\langle a \times r \rangle r}{r^4}$ , where $\overline{a}$ a constant vector.	34	
	35	Prove that $\nabla \log r = \frac{\overline{r}}{r^2}$ and hence, show that $\nabla \times (\overline{a} \times \nabla \log r) = 2 \frac{(\overline{a} \cdot \overline{r}) \overline{r}}{r^4}$ , where $\overline{a}$ is
DIVERGENCE AND CURL		
		DIVEKGENUE AND UUKL
36 $xi-yi$	36	$v_i - v_i$
div F and curl F where $F = \frac{x^2 + y^2}{x^2 + y^2}$		div $\overline{F}$ and curl $\overline{F}$ where $\overline{F} = \frac{xi - yj}{x^2 + v^2}$
Find  37  If $\overline{A} = \nabla(xy + yz + zy)$ find $\nabla \overline{A}$ and $\nabla \sqrt{A}$	37	Find $A = \nabla (xx + xx + xx)$ find $\nabla A = x A \nabla x A$
If $A = V(xy + yz + zx)$ , find $V \cdot A$ and $V \times A$ $\frac{38}{38}  \text{If } \overline{F} = (\overline{a} \cdot \overline{x}) \overline{x} \text{ where } \overline{a} \text{ is constant units} F \text{ and } \overline{F} \text{ and } \overline{F} \text{ is a constant units} \overline{a}$		If $A = v(xy + yz + zx)$ , find $v \cdot A$ and $v \times A$
		If $\overline{F} = (\overline{a} \cdot \overline{r}) \overline{r}$ where $\overline{a}$ is constant vector, find $\operatorname{curl} \overline{F}$ and P.T. it is perpendicular to $\overline{a}$ .
Prove that $\overline{F} = \frac{\overline{r}}{r^3}$ is both irrotational and solenoidal.		Prove that $\overline{F} = \frac{r}{r^3}$ is both irrotational and solenoidal.
40 A vector field $\overline{F}$ is given by	40	A vector field $\overline{F}$ is given by

	$\overline{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ Prove that it is
	irrotational and hence, find its scalar potential.
41	A vector field is given by $\overline{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$ . Show that $\overline{F}$ irrotational and find
	its scalar potential.
42	If
	$\nabla \phi = (y^2 - 2 xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k, \text{ find } \phi \text{ where } \phi(1, 0, 1) = 0$
43	Find the value of n for which the vector $r^n \bar{r}$ is solenoidal, where $\bar{r} = xi + yj + zk$
44	Prove that $\nabla \cdot \left\{ \frac{f(r)}{r} \bar{r} \right\} = \frac{1}{r^2} \frac{d}{dr} [r^2 f(r)]$ hence or otherwise prove that
	$div\left(r^n\;\bar{r}\right) = (n+3)r^n$
45	Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both
	solenoidal & irrotational.
46	If $\overline{r}$ is the position vector of point $(x, y, z)$ and $r$ is the modulus of $\overline{r}$ , then prove that $r^n \overline{r}$ is
	an irrotational vector for any value of n but solenoidal only if $n = -3$ .
47	If $\bar{f} = (x+y+1)i + j - (x+y)k$ , prove that $\bar{f} \cdot curl  \bar{f} = 0$
48	Define irrotational field and hence check whether the vector field $\overline{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$ is irrotational.
	DIFFERENTIAL OPERATORS of Higher Order
49	With usual notation, prove that $\nabla^2 \left[ \nabla \cdot \frac{\bar{r}}{r^2} \right] = \frac{2}{r^4}$
50	Show that $\nabla^4 r^2 \log r = \frac{6}{r^2}$
51	,
	Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$
52	Prove that $\nabla^2(r^2 \log r) = 5 + 6 \log r$