# Logic

**Module 2: Logic** (Total Lectures: **04** )

**2.1** Propositions and logical operations, Truth tables
**2.2** Equivalence, Implications
**2.3** Laws of logic, Normal Forms
**2.4** Predicates and Quantifiers
**2.5** Mathematical Induction

# Logic

- Logic is the discipline that deals with the methods of reasoning.
- On an elementary level, logic provides rules and techniques for determining whether a given argument is valid.
- Logical reasoning is used in mathematics to prove theorems, in computer science to verify the correctness of programs and to prove theorems, and in our everyday lives to solve a multitude of problems.

# Propositions

- **Definition**: The value of a proposition is called its truth value; denoted by
  - ***T* or 1 if it is true** or
  - ***F* or 0 if it is false**
- Opinions, interrogative, and imperative are not propositions
- **Truth table (If p is proposition)**

| p |
|---|
| 0 |
| 1 |

# Propositions

- A statement or proposition is a declarative sentence that is either true or false, but not both.
- Example Which of the following are statements?
  - (a) The earth is round.
  - (b) $2 + 3 = 5$
  - (c) Do you speak English?
  - (d) $3 - x = 5$
  - (e) Take two aspirins.
  - (f) The temperature on the surface of the planet Venus is 800° F.
  - (g) The sun will come out tomorrow.

# Logical Connectives and Compound Statements

- In logic, the letters p, q, r, . . .denote propositional variables. variables that can be replaced by statements.
- e.g. p: The sun is shining today. q: It is cold.
- Connectives are used to create a compound proposition from two or more propositions
  - **Negation** (e.g., $\neg a$ or !a or $\bar{a}$ OR $\sim a$)
  - **AND** or logical **Conjunction** (denoted $\wedge$)
  - **OR** or logical **Disjunction** (denoted $\vee$)
  - **XOR** or exclusive or (denoted $\oplus$)
  - **Imply** on (denoted $\Rightarrow$ or $\rightarrow$)
  - Biconditional (denoted $\Leftrightarrow$ or $\leftrightarrow$) IFF
- giving the truth values of a compound statement in terms of its component parts, is called a truth table.

# Precedence of Logical Operators

- As in arithmetic, an ordering is imposed on the use of logical operators in compound propositions
- However, it is preferable to use parentheses to disambiguate operators and facilitate readability

$$\neg p \vee q \wedge \neg r \equiv (\neg p) \vee (q \wedge (\neg r))$$

- To avoid unnecessary parenthesis, the following precedences hold:
  1. Negation ($\neg$)
  2. Conjunction ($\wedge$)
  3. Disjunction ($\vee$)
  4. Implication ($\rightarrow$)
  5. Biconditional ($\leftrightarrow$)

# Logical Connective: Negation

- $\neg p$, the negation of a proposition $p$, is also a proposition
- Examples:
  - Today is not Monday
  - It is not the case that today is Monday, etc.
- **Truth table**

| **P** | $\neg p$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

# Logical Connective: Logical AND

- The logical connective And is true only when both of the propositions are true. It is also called a <u>conjunction</u>
- Examples
  - It is raining and it is warm
  - (2+3=5) and (1<2)
- Truth table

| p | q | p∧q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

# Logical OR



- The logical disjunction, or logical OR, is true if one or both of the propositions are true.
- Examples
  - It is raining or it is the second lecture
  - $(2+2=5) \lor (1<2)$
  - You may have cake or ice cream
- **Truth table**

| $p$ | $q$ | $p \lor q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

# Logical Connective: Exclusive OR

- The exclusive OR, or XOR, of two propositions is true when exactly one of the propositions is true and the other one is false
- Example
  - The circuit is either ON or OFF but not both
- Truth table

| p | q | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

# Logical Connective: Biconditional (1)

- **Definition:** The biconditional $p \leftrightarrow q$ is the proposition that is true when $p$ and $q$ have the same truth values. It is false otherwise.
- Note that it is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$
- **Truth table**

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# Logical Connective: Biconditional (2)

- The biconditional $p \leftrightarrow q$ can be equivalently read as
  - *p* if and only if *q*
  - *p* is a necessary and sufficient condition for *q*
  - if *p* then *q*, and conversely
  - *p* iff *q* (Note typo in textbook, page 9, line 3)
- Examples
  - $x>0$ if and only if $x^2$ is positive
  - The alarm goes off iff a burglar breaks in
  - You may have pudding iff you eat your meat

# Logical Connective: Implication (1)

- **Definition:** Let *p* and *q* be two propositions.
- The implication $p \rightarrow q$ is the proposition that is false when *p* is true and *q* is false and true otherwise
  - *p* is called the hypothesis/antecedent (If I give you 1 million $)
  - *q* is called the conclusion, consequent or conclusion(then you will become a millionaire)

**Note that this is logically equivalent to ¬p V q**

# Logical Connective: Implication

- The implication of $p \rightarrow q$ can be also read as
  - If *p* then *q*
  - *p* implies *q*
  - If *p*, *q*
  - *p* only if *q*
  - *q* if *p*
  - *q* when *p*
  - *q* whenever *p*
  - *q* follows from *p*
  - *p* is a sufficient condition for *q* (*p* is sufficient for *q*)
  - *q* is a necessary condition for *p* (*q* is necessary for *p*)

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Truth table

# Logical Connective: Implication

- Examples
  - If you buy you air ticket in advance, it is cheaper.
  - If $x$ is an integer, then $x^2 \geq 0$.
  - If it rains, the grass gets wet.
  - If 2+2=5, then all unicorns are pink.

# Converse, Inverse, Contrapositive

- Consider the proposition $p \to q$ (Conditional If….. then )
  - Its converse is the proposition $q \to p$
  - Its contrapositive is the proposition $\neg q \to \neg p$
  - Its inverse is the proposition $\neg p \to \neg q$

# Write logical/conditional propositions

1. There is an error in the program or the data is wrong
2. If Peter works hard then he will pass the exam
3. Farmers will face hardship if the dry spell continues

# Problems

Let p denote Peter is rich, q denote Peter is happy. Write symbolic form for the following

1. Peter is poor but happy
2. Peter is neither rich nor happy
3. Peter is either rich or unhappy
4. Peter is poor ,else he is rich and unhappy

# Problems to Solve

Let “ A ” be the proposition ‘High speed driving is dangerous ‘

and

“ B ” be the proposition ‘Rajesh was a wise man’

1. $A \wedge B$
2. $\sim A \wedge B$
3. $\sim ( A \wedge B)$
4. $(A \wedge B) \vee ( \sim A \wedge \sim B)$
5. $(A \wedge B) \wedge \sim (A \wedge B )$

# Write in symbolic form

Let p be ‘He is hardworking’ and let q be ‘He is successful’

1. He is hardworking and successful
2. He is hardworking but not successful
3. It is false that he not successful or hardworking
4. He is neither hardworking nor successful
5. He is hardworking  or he is lazy and successful
6. It is not true that he is lazy or not successful

1. Consider p:You stay in Mumbai; q: You stay in Taj

   Determine Converse, Contrapositive and Inverse for

“If you stay in Mumbai , you stay in Taj”

2. Write down the English sentences for converse and contrapositive

   of : “If 250 is divisible by 4 then 250 is an even number “

# Usefulness of Logic

- Logic is more precise than natural language
  - You may have cake or ice cream.
    - Can I have both?
  - If you buy your air ticket in advance, it is cheaper.
    - Are there or not cheap last-minute tickets?
- For this reason, logic is used for hardware and software <u>specification</u>
  - Given a set of logic statements,
  - One can decide whether or not they are <u>satisfiable</u> (i.e., consistent), although this is a costly process…

# Terminology: Tautology, Contradictions, Contingencies

- Definitions
  - A compound proposition that is always true, no matter what the truth values of the propositions that occur in it is called a <u>tautology</u>
  - A compound proposition that is always false is called a <u>contradiction</u>
  - A proposition that is neither a tautology nor a contradiction is a <u>contingency</u>
- Examples
  - A simple tautology is $p \vee \neg p$
  - A simple contradiction is $p \wedge \neg p$

# Truth Tables

- Truth tables are used to show/define the relationships between the truth values of
  - the individual propositions and
  - the compound propositions based on them

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $p \oplus q$ | $p \Rightarrow q$ | $p \Leftrightarrow q$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 |

# Logical Equivalences: Example 1

- Are the propositions $(p \rightarrow q)$ and $(\neg p \vee q)$ logically equivalent?
- To find out, we construct the truth tables for each:

| $p$ | $q$ | $p \rightarrow q$ | $\neg p$ | $\neg p \vee q$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

The two columns in the truth table are identical, thus we conclude that

$$(p \rightarrow q) \equiv (\neg p \vee q)$$

# Constructing Truth Tables

Construct the truth table for the following compound proposition ~ p ∧ ( p →q)

$\sim P \wedge (P \to Q)$ .

| P | Q | $\sim P$ | $P \to Q$ | $\sim P \wedge (P \to Q)$ |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

. Show that $(P \to Q) \vee (Q \to P)$ is a tautology.

| P | Q | $P \to Q$ | $Q \to P$ | $(P \to Q) \vee (Q \to P)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

Prove $(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$ is a contradiction

Prove $(A \vee B) \wedge (\neg A)$ a contingency

Construct a truth table for $(P \to Q) \wedge (Q \to R)$ .

| P | Q | R | $P \to Q$ | $Q \to R$ | $(P \to Q) \wedge (Q \to R)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | F | T | F |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

Show that $(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

| $p$ | $q$ | $r$ | $p \to r$ | $q \to r$ | $(p \to r) \lor (q \to r)$ | $p \land q$ | $(p \land q) \to r$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

# Quantifiers

**Predicates**- An assertion that contains more than one variable i.e

- Value is predicated after assigning truth values to its variables

Eg : x is a city in India P(x)

x is the father of y P(x , y )

x + y > = z is denoted by P(x, y, z)

- The variable of predicates is quantified by quantifiers.
  - Two types of quantifier in predicate logic

  **Universal Quantifier and Existential Quantifier**

Universal **quantifier** "**for all, for every, for each** " is **"∀"**,

Existential **quantifier**" **there exists** " is "∃"

$\forall\ a\ \exists\ b\ P(x, y)$

where $P(a, b)$ denotes $a + b = 0$

$\forall\ a\ \forall\ b\ \forall\ c\ P(a, b, c)$

where $P(a, b, c)$ denotes $a + (b + c) = (a + b) + c$

**Note** – $\forall\ a\ \exists\ b\ P(x, y) \neq \exists\ a\ \forall\ b\ P(x, y)$

# Write in symbolic form using Quantifiers

For all x , x < 5 or x > = 5

There exists an x such that x < 4

For every number x there is a number y such that ‘y=x+1’

There is a number y such that ,for every number x, ‘y=x+1’

# Write English sentences for the following

1. **$\forall x \exists y R(x, y)$**
2. **$\exists x \forall y R(x, y)$**
3. **$\forall x (\sim Q(x))$**
4. **$\exists x (\sim P(x))$**
5. **$\forall x P(x)$** where

P(x) : x is even

Q(x) : x is prime nos

R(x, y): x + y is even

Let $Q(x, y, z)$ be the statement " $x + y = z$ " where the domain of all variables consists of **real numbers** and x, y and z are assigned values

→Suppose that x and y are assigned values , there exists is a real number z that $x + y = z$ is true

$$\forall x \forall y \exists z \, Q(x, y, z)$$

# Disjunctive Normal Form

- A disjunction of conjunctions where every variable or its negation is represented once in each conjunction (a *minterm*)
  - each minterm appears only once

Example: DNF of $p \oplus q$ is

$(p \wedge \neg q) \vee (\neg p \wedge q)$.

| p | q | $p \oplus q$ | $(p \wedge \neg q) \vee (\neg p \wedge q)$ |
|---|---|---|---|
| T | T | F | F |
| T | F | T | T |
| F | T | T | T |
| F | F | F | F |

# Method to construct DNF

- Construct a truth table for the proposition.
- Use the rows of the truth table where the proposition is True to construct minterms
  - If the variable is true, use the propositional variable in the minterm
  - If a variable is false, use the negation of the variable in the minterm
- Connect the minterms with $\vee$'s.

# How to find the DNF of $(p \vee q) \rightarrow \neg r$

$$(p \vee q) \rightarrow \neg r \iff$$
$$(p \wedge q \wedge \neg r) \vee$$
$$(p \wedge \neg q \wedge \neg r) \vee$$
$$(\neg p \wedge q \wedge \neg r) \vee$$
$$(\neg p \wedge \neg q \wedge r) \vee$$
$$(\neg p \wedge \neg q \wedge \neg r)$$

| P | q | r | $(p \vee q)$ | $\neg r$ | $(p \vee q) \rightarrow \neg r$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | T | F | T | T | T |
| T | F | T | T | F | F |
| T | F | F | T | T | T |
| F | T | T | T | F | F |
| F | T | F | T | T | T |
| F | F | T | F | F | T |
| F | F | F | F | T | T |

# Conjunctive Normal Form$(p \Leftrightarrow q) \Rightarrow (\neg p \land r)$

Truth table:

| $p$ | $q$ | $r$ | $p \Leftrightarrow q$ | $\neg p \land r$ | $(p \Leftrightarrow q) \Rightarrow (\neg p \land r)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 |

Solution: (full) conjunctive normal form (FCNF) of the formula is
$(p \lor q \lor r) \land (\neg p \lor \neg q \lor r) \land (\neg p \lor \neg q \lor \neg r)$

Find DNF of $(\sim p \rightarrow r) \wedge (p \Leftrightarrow q)$

Find CNF of $(p \Rightarrow q) \Rightarrow r$

# Laws of Logic

Commutative laws

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

Associative laws

$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$

Inverse laws

$p \wedge \neg p \Leftrightarrow F$

$p \vee \neg p \Leftrightarrow T$

Distributive laws

$p \land (q \lor r) \Leftrightarrow (p \land q) \lor (p \land r)$

$p \lor (q \land r) \Leftrightarrow (p \lor q) \land (p \lor r)$

Idempotent laws

$p \land p \Leftrightarrow p$

$p \lor p \Leftrightarrow p$

Identity laws

$p \land T \Leftrightarrow p$

$p \lor F \Leftrightarrow p$

Domination laws

$p \land F \Leftrightarrow F$

$p \lor T \Leftrightarrow T$

Absorption law

$p \land (p \lor q) \Leftrightarrow p$

$p \lor (p \land q) \Leftrightarrow p$

De Morgan Law:

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

# Table of Logical Equivalences

| | | |
|---|---|---|
| Commutative | $p \wedge q \iff q \wedge p$ | $p \vee q \iff q \vee p$ |
| Associative | $(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$ | $(p \vee q) \vee r \iff p \vee (q \vee r)$ |
| Distributive | $p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$ |
| Identity | $p \wedge T \iff p$ | $p \vee F \iff p$ |
| Negation | $p \vee \sim p \iff T$ | $p \wedge \sim p \iff F$ |
| Double Negative | $\sim (\sim p) \iff p$ | |
| Idempotent | $p \wedge p \iff p$ | $p \vee p \iff p$ |
| Universal Bound | $p \vee T \iff T$ | $p \wedge F \iff F$ |
| De Morgan's | $\sim (p \wedge q) \iff (\sim p) \vee (\sim q)$ | $\sim (p \vee q) \iff (\sim p) \wedge (\sim q)$ |
| Absorption | $p \vee (p \wedge q) \iff p$ | $p \wedge (p \vee q) \iff p$ |
| Conditional | $(p \implies q) \iff (\sim p \vee q)$ | $\sim (p \implies q) \iff (p \wedge \sim q)$ |

# $(p \wedge \neg q) \vee q \Leftrightarrow p \vee q$

| | |
|---|---|
| $(p \wedge \neg q) \vee q$ | Left-Hand Statement |
| $\Leftrightarrow q \vee (p \wedge \neg q)$ | Commutative |
| $\Leftrightarrow (q \vee p) \wedge (q \vee \neg q)$ | Distributive |
| $\Leftrightarrow (q \vee p) \wedge T$ | Negation |
| $\Leftrightarrow q \vee p$ | Identity |
| $\Leftrightarrow p \vee q$ | Commutative |

# Prove: $p \rightarrow p \vee q$ is a tautology

| | |
|---|---|
| $p \rightarrow p \vee q$ | |
| $\Leftrightarrow \neg p \vee (p \vee q)$ | Implication Equivalence |
| $\Leftrightarrow (\neg p \vee p) \vee q$ | Associative |
| $\Leftrightarrow (p \vee \neg p) \vee q$ | Commutative |
| $\Leftrightarrow T \vee q$ | Negation |
| $\Leftrightarrow q \vee T$ | Commutative |
| $\Leftrightarrow T$ | Domination |

Use Logical Equivalences to prove that $[(p \land \neg(\neg p \lor q)) \lor (p \land q)] \rightarrow p$ is a tautology.

Proof: $[(p \land \neg(\neg p \lor q)) \lor (p \land q)] \rightarrow p$

| | |
|---|---|
| $\equiv [(p \land (\neg(\neg p) \land \neg q)) \lor (p \land q)] \rightarrow p$ | De Morgan's law |
| $\equiv [(p \land (p \land \neg q)) \lor (p \land q)] \rightarrow p$ | Double Negation law |
| $\equiv [((p \land p) \land \neg q) \lor (p \land q)] \rightarrow p$ | Associative law |
| $\equiv [(p \land \neg q) \lor (p \land q)] \rightarrow p$ | Idempotent law |
| $\equiv [p \land (\neg q \lor q)] \rightarrow p$ | Distributive law |
| $\equiv [p \land (q \lor \neg q)] \rightarrow p$ | Commutative law |
| $\equiv [p \land T] \rightarrow p$ | Negation law |
| $\equiv p \rightarrow p$ | Identity law |
| $\equiv \neg p \lor p$ | Equivalence of Implication |
| $\equiv p \lor \neg p$ | Commutative law |
| $\equiv T$ | Negation law |

# Problem to solve

1. Obtain CNF $(p \wedge q) \vee (p \wedge \neg q)$

2. Obtain DNF $\sim(p \rightarrow (q \wedge r))$

1. CNF Solution: $(p \wedge q) \vee (p \wedge \neg q)$

$\equiv ((p \wedge q) \vee p) \wedge ((p \wedge q) \vee \neg q)$

$\equiv ((p \vee p) \wedge (q \vee p)) \wedge ((p \vee \neg q) \wedge (q \vee \neg q))$

2. DNF solution: $(p \wedge \sim q) \vee (p \wedge \sim r)$

$\sim[\sim p \vee (q \wedge r)]$

$\sim(\sim p) \wedge \sim(q \wedge r)$

$p \wedge (\sim q \vee \sim r)$

$(p \wedge \sim q) \vee (p \wedge \sim r)$

# Use MATHEMATICAL INDUCTION to prove that $1 + 2 + 3 + \ldots + n = n(n + 1) / 2$ for all positive integers n.

Let the statement P (n) be $1 + 2 + 3 + \ldots + n = n(n + 1) / 2$

STEP 1. Basis: We first show that p (1) is true.
Left Side = 1
Right Side = $1(1 + 1) / 2 = 1$

Both sides of the statement are equal hence p (1) is true.

STEP 2-Inductive step : We now assume that p (k) is true
$1 + 2 + 3 + \ldots + k = k(k + 1) / 2$

STEP 3:Inductive hypothesis , Show that p (k + 1) is true by adding k + 1 to both sides of the above statement

$1 + 2 + 3 + \ldots + k + (k + 1) = k(k + 1) / 2 + (k + 1)$
$= (k + 1)(k / 2 + 1)$
$= (k + 1)(k + 2) / 2$

The last statement may be written as
$1 + 2 + 3 + \ldots + k + (k + 1) = (k + 1)(k + 2) / 2$ Which is the statement p(k + 1).

# Exercises- Prove each of the following by Mathematical Induction

For n positive integers n, solve the following

→ $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

→ $1^2 + 2^2 + \ldots + n^2 = (n)(n + 1)(2n + 1)/6$

→ $1.2 + 2.3 + 3.4 + \ldots + n(n+1) = [n(n+1)(n+2)]/3$

→ $1^3 + 2^3 + 3^3 + \ldots + n^3 = n^2 (n + 1)^2 / 4$

# Prove that $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

Let the statement P(n) be $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

STEP 1. Basis: We first show that p (1) is true.

P(n) for n=1 is

| | |
|---|---|
| LHS = 4*1-3 | RHS = 1(2*1-1) |
| =4-3 | = 2-1 |
| =1 | =1 |

LHS=RHS ------------------(I)

Hence , p(1) is true

# Prove that $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

STEP 2-Inductive step : We now assume that p (k) is true

P(k) for n=k is

$1+5+9+\ldots+(4k-3) = k(2k-1)$ ------------(II)

STEP 3:Inductive hypothesis , Show that p (k + 1) is true by adding (4(k+1)-3) to both sides of the above statement P(k).

$$1+5+9+(4k-3)+[4(k+1)-3] = k(2k-1)+ [4(k+1)-3]$$
$$= 2k^2 - k + 4k + 1$$
$$= (k+1)(2k+1)$$
$$= (k+1)\ [2(k+1)-1] \text{--------------(III)}$$

As P(k) is true , hence P(k+1) is also true.

From (I), (II), and (III) we conclude that,

$1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$

# Prove that $1^2 + 2^2 + \ldots + n^2 = (n)(n+1)(2n+1)/6$

let $P(n)$ be $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$

Step I – Basis Step. Let $n = 1$

$P(1)$ be $1^2 = 1 \cdot (1+1)(2 \times 1 + 1)/6$

$1 = 1 \times 2 \times 3/6$

$1 = 6/6$

$1 = 1$ ---- (I)

Hence $P(1)$ is true

Step II – Induction step

assume $P(k)$ is true.

$P(k)$ be

$1^2 + 2^2 + \cdots + k^2 = k(k+1)(2k+1)/6$ ---- (II)

Step III : Induction hypothesis,

show that $P(k+1)$ is true by adding $(k+1)^2$ to both sides of statement $P(k)$

$\therefore \quad 1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = k(k+1)(2k+1)/6 + (k+1)^2$

$= \dfrac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$= \dfrac{(k+1)}{6}\left[k(2k+1) + 6(k+1)\right]$

$= \dfrac{(k+1)}{6}\left[2k^2 + k + 6k + 6\right]$

$= \dfrac{(k+1)}{6}(2k^2 + 7k + 6)$

$= \dfrac{(k+1)(k+2)(2k+3)}{6}$ ------ (III)

As $P(k)$ is true, hence $P(k+1)$ is true.

From, (I, II & III) we conclude that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$

# Prove that for any positive integer number n , $n^3 + 2n$ is divisible by 3

Statement P (n) is defined by $n^3 + 2n$ is divisible by 3

STEP 1: We first show that p (1) is true. Let n = 1 and calculate $n^3 + 2n$

$1^3 + 2(1) = 3$

3 is divisible by 3

hence p (1) is true.

STEP 2: We now assume that p (k) is true

$k^3 + 2k$ is divisible by 3

is equivalent to

$k^3 + 2k = 3M$ , where M is a positive integer.

We now consider the algebraic expression $(k + 1)^3 + 2(k + 1)$; expand it and group like terms

$(k + 1)^3 + 2(k + 1) = k^3 + 3k^2 + 5k + 3$

$= [k^3 + 2k] + [3k^2 + 3k + 3]$

$= 3M + 3[k^2 + k + 1] = 3[M + k^2 + k + 1]$

Hence $(k + 1)^3 + 2(k + 1)$ is also divisible by 3 and therefore statement P(k + 1) is true.

ANY ????