SOMAIYA VIDYAVIHAR UNIVERSITY
K J Somaiya College of Engineering

Somaiya TRUST

Discrete Mathematics

Course Code	Course Title								
2UCC305	Discrete Mathematics								
	TH				P		TUT		Total
Teaching Scheme (Hrs./Week)	03				--		01*		04
Credits Assigned	03				--		01		04
Examination Scheme	Marks								
	CA			ESE	TW	O	P	P&O	Total
	T-1	T-2	IA						
	15	15	20	50	25	--	--	--	125

Course Outcomes

- CO1: Use various mathematical notations, apply various proof techniques to solve real world problems
- CO2: Learn and apply core ideas of Set Theory, Relations & Functions
- CO3: Use graphs and their types, to solve the practical examples
- CO4: Understand the use of Algebraic Structures and lattice, to solve the problems

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Recommended Book

- Kenneth H. **Rosen**. ”Discrete Mathematics and its Applications”, Tata McGraw-Hill.
- Bernard Kolman, Robert C. Busby ,’ Discrete Mathematical Structure’, Pearson ,6th Edition, 2017
- C. L. Liu, D. P. Mohapatra, “Elements of Discrete Mathematics” Tata McGrawHill.
- Douglas West ,’Graph Theory’, Pearson,2nd Edition, 2017

Discrete Structure

- Abstract mathematical structures that represent objects and the relationships between them.
- Examples are sets, permutations, relations, graphs, trees, and finite state machines.

Applications of Discrete Mathematics

Everyday applications of discrete mathematics

Computers run software and store files. The software and files are both stored as huge strings of 1s and 0s. Binary math is discrete mathematics.

Networks are, at base, discrete structures. The routers that run the internet are connected by long cables. People are connected to each other by social media ("following" on Twitter, "friending" on Facebook, etc.). The US highway system connects cities with roads.

Doing web searches in multiple languages at once, and returning a summary, uses linear algebra.

Google Maps uses discrete mathematics to determine fastest driving routes and times. There is a simpler version that works with small maps and technicalities involved in adapting to large maps.

Scheduling problems---like deciding which nurses should work which shifts, or which airline pilots should be flying which routes, or scheduling rooms for an event, or deciding timeslots for committee meetings, or which chemicals can be stored in which parts of a warehouse---are solved either using graph coloring or using combinatorial optimization, both parts of discrete mathematics. One example is scheduling games for a professional sports league.

An **analog clock** has gears inside, and the sizes/teeth needed for correct timekeeping are determined using discrete math.

Encryption and decryption are part of **cryptography**, which is part of discrete mathematics. For example, secure internet shopping uses public-key cryptography.

Wiring a computer network using the least amount of cable is a minimum-weight spanning tree problem.

Area codes: How do we know when we need more area codes to cover the phone numbers in a region? This is a basic combinatorics problem.

Designing **password criteria** is a counting problem: Is the space of passwords chosen large enough that a hacker can't break into accounts just by trying all the possibilities? How long do passwords need to be in order to resist such attacks? (find out here!)

Module 1:Set Theory

- Sets, Venn diagrams, Operations on Sets
- Laws of set theory, Power set and Products
- Partitions of sets, The Principle of Inclusion and Exclusion

Sets

- Set
  - collection of **distinct** objects of **same type** or class of objects.
  - **Unordered collection**
- Elements or Members
  - The objects of set
  - Number, alphabets, names etc.
- e.g. A = {1, 2, 3, 4, 5}
- A set is denoted by capital letters.

Set Formation

- **Roster form of a set**
  - defined by actually listing its members.
  - E.g. P = {a, b, c, d, e} Q={b, c, a, e, d} R={a, b, c, a, d, e, d, c}
  - Elipses (...) may be used to describe a set without listing all of the members when the pattern is clear.

    e.g. $S = \{a, b, c, d, \ldots\ldots, z\}$
  - Set of all vowels in the English alphabet:

    $V = \{\text{a, e, i, o, u}\}$

Represent the set in Roster form for the following:

- Set of consonants
- Set of all positive integers less than 10
- Set of all integers less than 0

Set Formation Continued….

- Set-Builder form of set
  - Set is defined by the **properties** which its elements must **satisfy**.
  - E.g. P = {x : x ϵ N, x is a multiple of 3}
  - Q = {x : x>1 and x<10 and x is an odd integer}
  - A predicate may be used: S = {x | P(x)}
    - Example: S = {x | Prime(x)}
  - O = {x ∈ Z^+ | x is odd and x < 10}
  - $A\ \ \{x \in \mathrm{N} : \exists n \in \mathrm{N}(x\ \ 2n)\}$.
    - we would read the set as, “the set of all *x* in the natural numbers, such that there exists some *n* in the natural numbers for which *x* is twice *n*.”

Standard Notations

- x Є A   x belongs to A or x is an element of set A.
- x ∉ A   x does not belongs to set A OR is not in
- ϕ = Empty set
- U =   Universal set
- N =   Natural Number set
- $\mathbf{N}$ = *natural numbers* = $\{0,1,2,3....\}$
- $\mathbf{Z}$ = *integers* = $\{...,-3,-2,-1,0,1,2,3,...\}$
- $\mathbf{Z}^{+}$ = *positive integers* = $\{1,2,3,.....\}$
- $\mathbf{R}$ = set of *real numbers*
- $\mathbf{R}^{+}$ = set of *positive real numbers*
- $\mathbf{C}$ = set of *complex numbers*.
- **Q** = set of rational numbers

Universal Set and Empty Set

- The *universal set* *U* is the set containing everything **currently under consideration**.
  - Sometimes implicit/explicit/Contents depend on the context.
- Empty set is the set with no elements. Symbolized Ø, but { } also used , but {Ø}???
- { } ??? {Ø}

"A null set is a subset of every set "

- P = {x | x is a real number and $x^2 = -1$} ???

Sets….

- Sets can be elements of sets.

  $\{\{1,2,3\},a, \{b,c\}\}$

  $\{\mathbf{N},\mathbf{Z},\mathbf{Q},\mathbf{R}\}$

- The empty set is different from a set containing the empty set.

  $\emptyset \neq \{ \emptyset \}$

Set Equality

- Sets are completely known when their members are all known.
- two sets A and B are equal if they have the same elements, and we write A = B.
- If A = {1, 2, 3} and B = {x | x is a positive integer and $x^2 < 12$} ???
- if A and B are sets, then A and B are equal if and only if

  $A \subseteq B$ and $B \subseteq A$ then $A=B$

  - **E.g.** {1,3,5} ?? {3, 5, 1}

    {1,5,3, 5,1, 3,1} ?? {1,3,5}
  - If A = {BASIC, PASCAL, ADA} and B = {{A,D,A}, BASIC, PASCAL} are the set equal?

Subset

- **Definition**: The set *A* is a *subset* of *B*, if and only if every element of *A* is also an element of *B*.
- If every element of A is also an element of B, that is, if whenever x ∈ A then x ∈ B, we say that A is a subset of B or that A is contained in B, and we write A ⊆ B.
- If A is not a subset of B, denoted as A ⊈ B.

Eg: If A={1,2,3}, B={1,2,3,4,5}, C=(3,2,1}

- ⊈ - not a subset

"Every set is a subset of itself" (If A is any set, then A ⊆ A)

$Z^+ \subseteq Z$

Proper Subsets

Definition: **If $A \subseteq B$, but $A \neq B$**, then we say A is a *proper subset* of B, denoted by $A \subset B$.

A= {x ,y } , B= {x, y ,z}

then is $A \subset B$?

A= {1,3}

B={1,2,3}

C={1,3,2}

A $\subset$ C ? , B $\subset$ C ?

Cardinality of a Set

- A set A is called finite if it has n distinct elements, where n ∈ N. In this case, n is called the cardinality of A and is denoted by |A|
- Number of distinct elements
- **Examples**:

1. $|\emptyset| = 0$
2. Let S be the letters of the English alphabet. Then $|S| = 26$
3. $|\{1,2,3\}| = 3$
4. $|\{\emptyset\}| = 1$
5. The set of integers is infinite.

SUPERSET

- **SUPERSET**
  - If A is the subset of B then **B is the SUPERSET of A**
- **DISJOINT SET**
  - Two sets are said to be disjoint if they have **no elements in common**
- **POWER SET**
  - If A is a set, then the set of **all subsets of A** is called the **power set of A** and is denoted by **P(A)**.
  - If a set has *n* elements, then the cardinality of the power set is $\mathbf{2^n}$.

Example: If A = {a,b} then

$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$

EXERCISE TO SOLVE

1. Let A={1 , 2 ,3 }.Determine the power set of A

1. Let A={a , b , c , d }. Determine the power set of A

SET PROPERTIES

- Every Set A is a subset of the Universal set U

  $\emptyset \subseteq A \subseteq U$

- Every set A is a subset of itself

  $A \subseteq A$

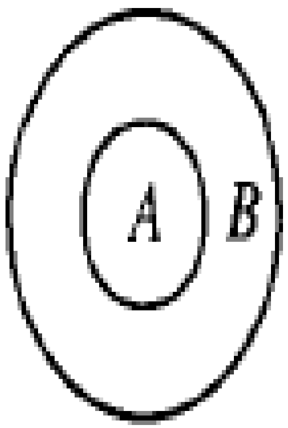
- Transitivity

  $A \subseteq B$, $B \subseteq C$, then $A \subseteq C$

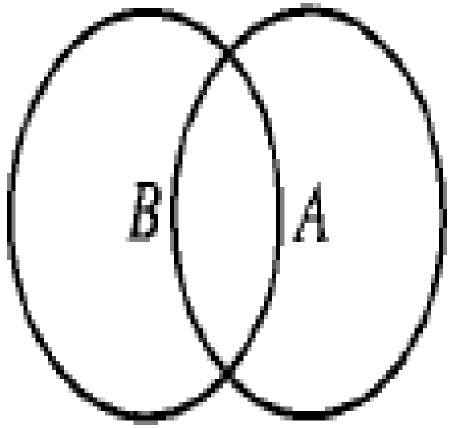
- If **$A \subseteq B$ and $B \subseteq A$ then $A=B$**; converse also holds true

Venn Diagrams

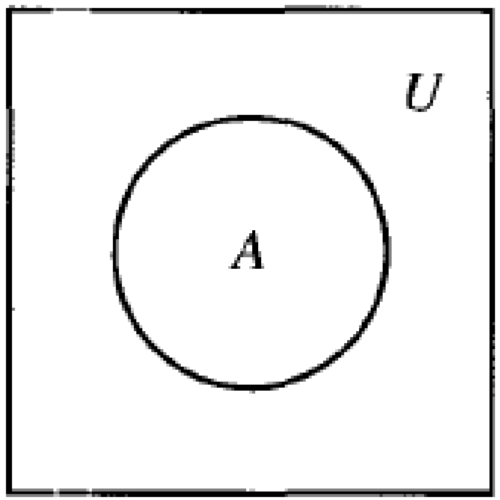
- Diagrams which are used to show relationships between sets, are called **Venn diagrams**



$A \subseteq B$



$A \not\subseteq B$

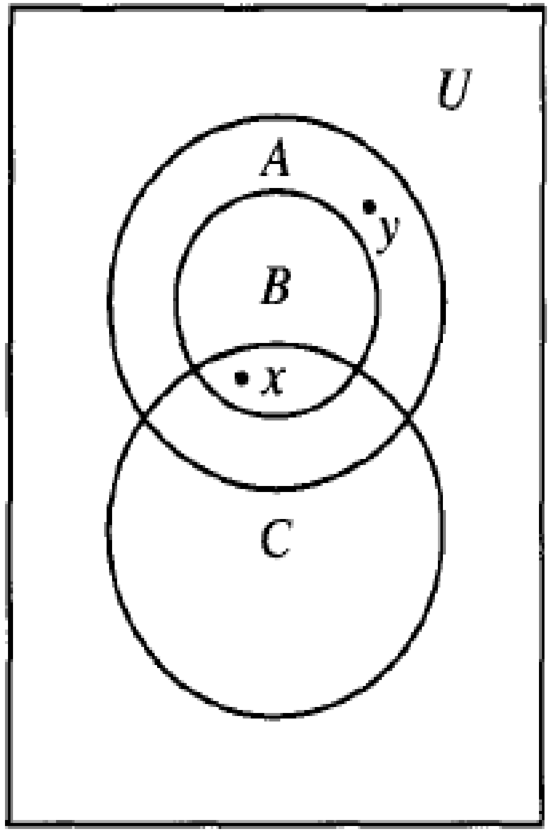


Example on Venn Diagrams

Use the Venn diagram to identify each of the following as true or false.

(a) $A \subseteq B$ (b) $B \subseteq A$ (c) $C \subseteq B$

(d) $x \in B$ (e) $x \in A$ (f) $y \in B$



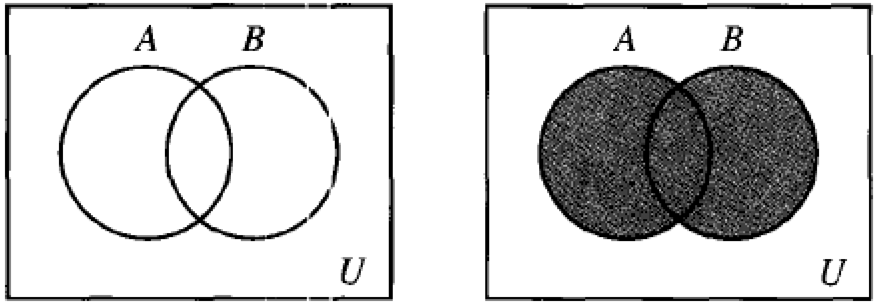
SET operations

- **Definition**: Let *A* and *B* be sets. The *union* of the sets *A* and *B*, denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

- Symbol : V (OR)
- **Example**: What is $\{1,2,3\} \cup \{3, 4, 5\}$?

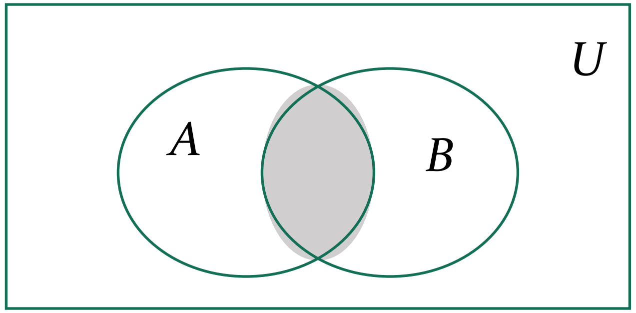
Solution: $\{1,2,3,4,5\}$



(a) (b) $A \cup B$

The Cardinality of the Union of Two Sets

- Inclusion-Exclusion
- $|A \cup B| = |A| + |B| - |A \cap B|$



Venn Diagram for A, B, $A \cap B$, $A \cup B$

- **Example**: Let A be the math majors in your class and B be the CS majors. To count the number of students who are either math majors or CS majors, add the number of math majors and the number of CS majors, and subtract the number of joint CS/math majors.

Intersection

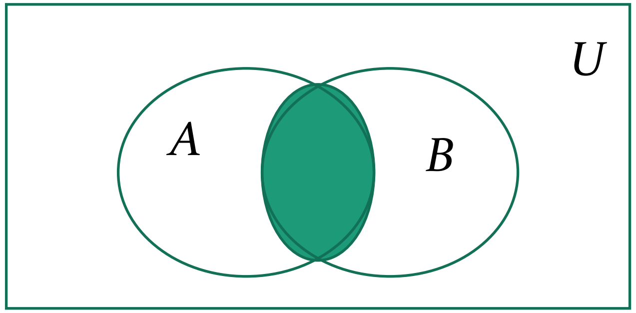
- The *intersection* of sets *A* and *B*, denoted by **$A \cap B$**, is
- Symbol: ∧ (AND) $\{x | x \in A \wedge x \in B\}$
- Note if the intersection is empty, then *A* and *B* are said to be *disjoint*.
- **Example**: What is? $\{1,2,3\} \cap \{3,4,5\}$?

  Solution: $\{3\}$
- **Example:** What is?

  $\{1,2,3\} \cap \{4,5,6\}$?

  Solution: ∅

Venn Diagram for $A \cap B$



Examples

Example 1. Let $A = \{a, b, c, e, f\}$ and $B = \{b, d, r, s\}$. Find $A \cup B$.

Example 2. Let $A = \{a, b, c, e, f\}$, $B = \{b, e, f, r, s\}$, and $C = \{a, t, u, v\}$. Find $A \cap B$, $A \cap C$, and $B \cap C$.

1. Solution: {a, b, c, d, e, f, r, s}
2. Solution : {b,e,f} , {a}, {}

Complement of Set

Definition: If A is a set, then the complement of the A (with respect to U), denoted by $\bar{A}$ is the set U - A

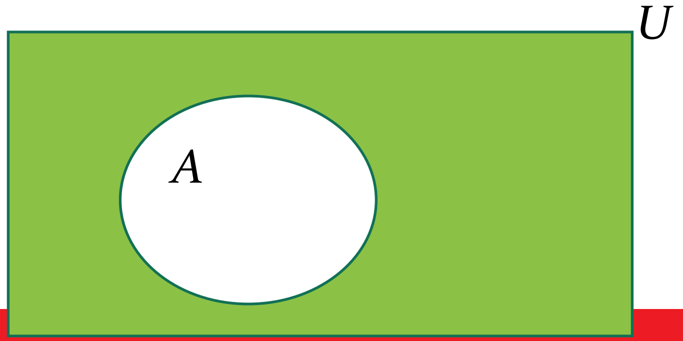
$$\bar{A} = \{x \in U \mid x \notin A\}$$

(The complement of A is sometimes denoted by A^c .)

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

Venn Diagram for Complement

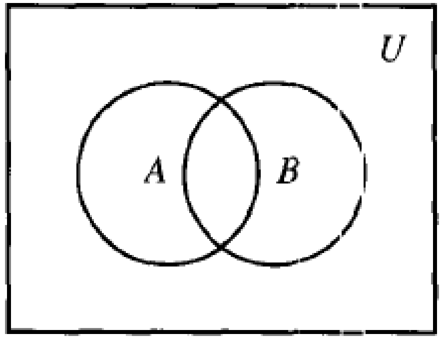


Difference

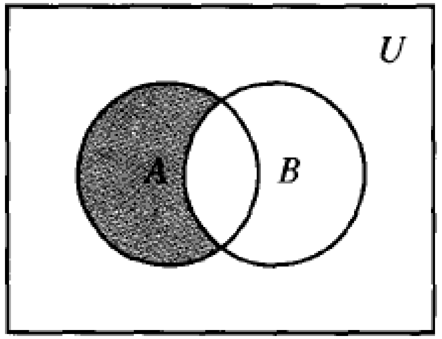
- If A and B are two sets, we define the complement of B with respect to A as the set of all elements that belong to A but not to B, and we denote it by A — B.
- $A - B$ *also denoted by* $A \backslash B$
- $A - B = \{x \mid x \in \mathrm{A} \wedge x \notin B\}$ ($\wedge$ - AND)
- Let A = {a, b, c} and B = {b, c, d, e}. Find A — B and B—A

Solution : A — B = {a} and B—A ={d, e}

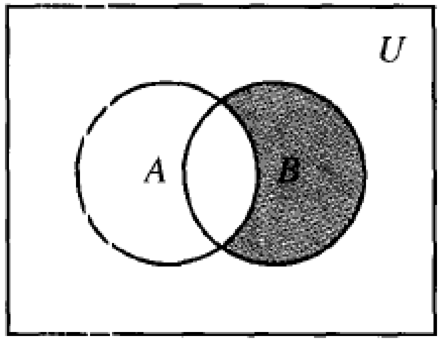
Venn Diagram representation for Set Difference



(a)



(b) A - B



(c) B - A

Symmetric Difference

- The *symmetric difference* of **A** and **B**, denoted by $A \oplus B$ is the set.
- the set of all elements that belong to A or to B, but not to both A and B
- $A \oplus B = (A \cup B) - (A \cap B)$ *OR* $(A - B) \cup (B - A)$

Example:

1. $U = \{0,1,2,3,4,5,6,7,8,9,10\}$

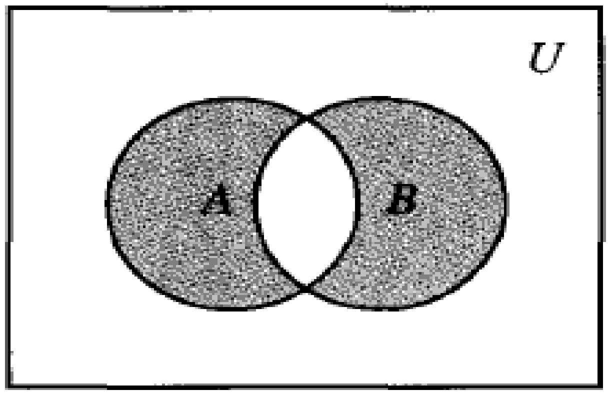
$A = \{1,2,3,4,5\}$ $B = \{4,5,6,7,8\}$

What is $A \oplus B$

- **Solution**: $\{1,2,3,6,7,8\}$

2. Let A = {a, b, c, d} and B = {a, c, e, f, g}.

- $A \oplus B$ = {b, d, e, f, g}



Venn Diagram for $A \oplus B$

Example

- **Example**: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

1. $A \cup B$

   Solution: $\{1,2,3,4,5,6,7,8\}$

2. $A \cap B$

   Solution: $\{4,5\}$

3. $\bar{A}$

   Solution: $\{0,6,7,8,9,10\}$

4. B^c

   Solution: $\{0,1,2,3,9,10\}$

5. $A - B$

   Solution: $\{1,2,3\}$

6. $B - A$

   Solution: $\{6,7,8\}$

Cartesian Product

Definition: The *Cartesian Product* of two sets *A* and *B*, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$ (first element from A and second element from B)

$$A \times B = \{(a,b) | a \in A \wedge b \in B\}$$

Example:

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

LAWS OF SET THEORY

Commutative Properties

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$

Associative Properties

3. $A \cup (B \cup C) = (A \cup B) \cup C$
4. $A \cap (B \cap C) = (A \cap B) \cap C$

Distributive Properties

5. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Idempotent Properties

7. $A \cup A = A$
8. $A \cap A = A$

LAWS OF SET THEORY

Properties of the Complement

9. $\overline{(\overline{A})} = A$
10. $A \cup \overline{A} = U$
11. $A \cap \overline{A} = \varnothing$
12. $\overline{\varnothing} = U$
13. $\overline{U} = \{\,\}$
14. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
15. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

De Morgan's laws.

LAWS OF SET THEORY

Properties of a Universal Set

16. $A \cup U = U$
17. $A \cap U = A$

Properties of the Empty Set

18. $A \cup \varnothing = A$ or $A \cup \{\,\} = A$
19. $A \cap \varnothing = \varnothing$ or $A \cap \{\,\} = \{\,\}$

- Absorption laws

$$A \cup (A \cap B) = A \qquad A \cap (A \cup B) = A$$

- Properties of complement law

$$A \cup \overline{A} = U \qquad A \cap \overline{A} = \emptyset$$

Membership Table

- Example: Construct a membership table to show that the distributive law holds.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

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Partition of Set

- If A is a set, a partition of A is any set of non empty subset A_1, A_2, A_3..... Of A such that

$$A_1 \cup A_2 \cup A_3 .... = A$$

&

$$A_i \cap A_j = \emptyset$$ for i = j (subsets are mutually disjoint)

Example A = { a , b , c}

Verify whether { { a } , { b , c } } is a partition of A or not

1. S={1,2,3,4,5,6,7,8,9}.Determine whether each is a partition or not
   (i) {{1,3,5},{2,6},{4,8,9}}
   (ii) {{1,3,5},{2,4,6,8},{5,7,9}}
   (iii) {{1,3,5},{2,4,6,8},{7,9}}
2. Let A={a,b,c,d,e,f,g,h }.Consider the following subsets of A

   A1={a,b,c,d} A3={a,c,e,g}

   A2={a,c,e,g,h} A4={b,d} A5={f,h}

   Determine whether following is a partition of A or not. Justify

   (i) {A1,A2} (ii) {A1, A5} (iii) {A3,A4,A5}

THEROEMS

Addition principle

$|A \cup B| = |A| + |B|$

$|A-B| = |A| - |A \cap B|$

PRINCIPLE OF INCLUSION EXCLUSION

$|A \cup B| = |A| + |B| - |A \cap B|$

MUTUAL INCLUSION EXCLUSION PRINCIPLE

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

EXERCISE TO SOLVE

- A company must hire 25 programmers to handle programming jobs & 40 programmers for application programming job of which 10 will be expected to do both kinds of roles. How many programmers must be hired?
- Solution: 55

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EXERCISE TO SOLVE

In a survey of 260 college students , the following data were obtained:

64 had taken a maths course

94 had taken a CS course

58 had taken a Business course

28 had taken both M and B course

26 had taken both M and CS course

22 had taken both CS and B course

14 had taken all three types of courses.

(i) How many students surveyed who had taken none of the three types of courses?

(ii)Of the students surveyed how many had taken only CS course?

EXERCISE TO SOLVE

Suppose that 100 of 120 students at a college take at least one of the languages French , German and Russian . Also suppose

65 study French

20 study French and German

45 study German

25 study French and Russian

42 study Russian

15 study German and Russian

(i) How many students study all three languages ?

(ii)Fill in the correct regions in Venn Diagram

(iii)Hence find out the number of students ‘k’ who study

* Exactly 1 language   *Exactly 2 languages

EXERCISE TO SOLVE

1) How many integers between 1 and 60 are not divisible by 2,not by 3,nor by 5 ?

2) How many integers between 1 and 300 are divisible by 3,5, or 7 and are not divisible by 3 nor 5 nor 7 ?

Summary

$\{,\}$	We use these braces to enclose the elements of a set. So $\{1, 2, 3\}$ is the set containing 1, 2, and 3.
$:$	$\{x : x > 2\}$ is the set of all x such that x is greater than 2.
$\in$	$2 \in \{1, 2, 3\}$ asserts that 2 is an element of the set $\{1, 2, 3\}$.
$\notin$	$4 \notin \{1, 2, 3\}$ because 4 is not an element of the set $\{1, 2, 3\}$.
$\subseteq$	$A \subseteq B$ asserts that A is a subset of B: every element of A is also an element of B.
$\subset$	$A \subset B$ asserts that A is a proper subset of B: every element of A is also an element of B, but $A \neq B$.
$\cap$	$A \cap B$ is the intersection of A and B: the set containing all elements which are elements of both A and B.
$\cup$	$A \cup B$ is the union of A and B: is the set containing all elements which are elements of A or B or both.
$\times$	$A \times B$ is the Cartesian product of A and B: the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$.
$\setminus$	$A \setminus B$ is A set-minus B: the set containing all elements of A which are not elements of B.
$\overline{A}$	The complement of A is the set of everything which is not an element of A.
$\lvert A \rvert$	The cardinality (or size) of A is the number of elements in A.