Relations, Diagraph

Module 3: Relations , Diagraph (Total Lectures: **09)**

3.1 Relations, Paths and Digraphs

3.2 Properties and types of binary relations

3.3 Manipulation of relations, Closures, Warshall's algorithm

3.4 Equivalence relations

Introduction

- A relation between elements of two sets is a subset of their Cartesian products (set of all ordered pairs)
- **Definition**: A binary <u>relation</u> from a set A to a set B is a subset $R \subseteq A \times B = \{ (a,b) \mid a \in A, b \in B \}$
- Relation versus function

  In a relation, each $a \in A$ can map to <u>multiple</u> elements in B

  Relations are more general than functions
- When $(a,b) \in R$, we say that a is <u>related</u> to b.

  Notation: a*R*b, a*R*b

Relations: Representation

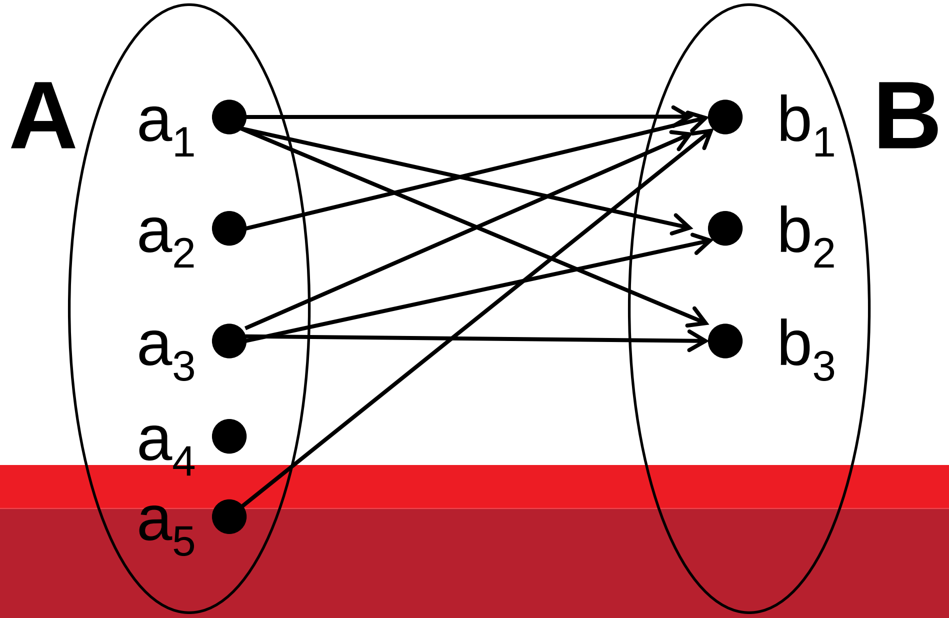
- To represent a relation, we can enumerate every element of R
- Example

  Let $A=\{a_1,a_2,a_3,a_4,a_5\}$ and $B=\{b_1,b_2,b_3\}$

  Let *R* be a relation from A to B defined as follows

$$R=\{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_2,b_1),(a_3,b_1),(a_3,b_2),(a_3,b_3),(a_5,b_1)\}$$

Graphically,



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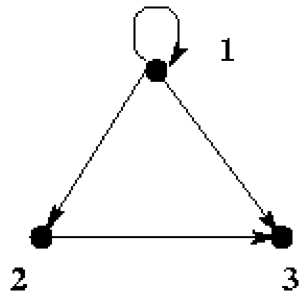
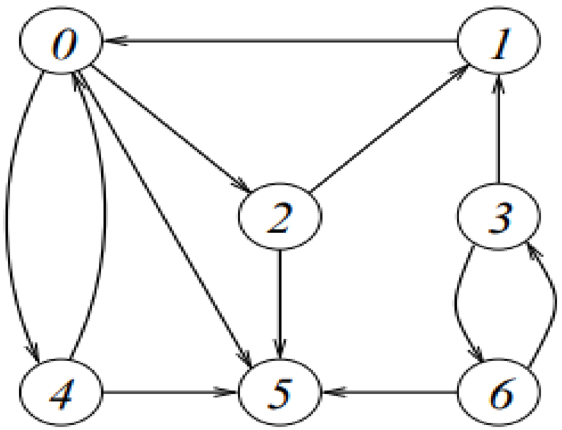
DIGRAPHS

A **digraph** (directed graph) is a diagram composed of points called **vertices** (nodes) and arrows called **edges** going from a vertex to a vertex.



Example :-A digraph with 3 vertices and 4 edges

Example: -V = {0, 1, 2, 3, 4, 5, 6 } ,E = {(0, 2),(0, 4),(0, 5),(1, 0), (2, 1),(2, 5),(3, 1),(3, 6), (4, 0),(4, 5),(6, 3),(6, 5) }



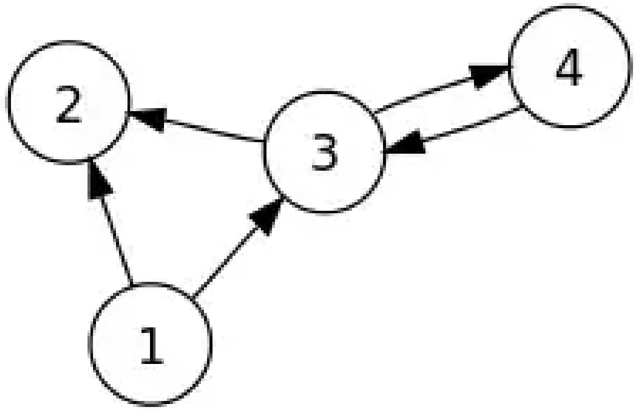
Matrix Representation ?

Degree of Vertex in a Directed Graph

A directed graph, each vertex has an **in-degree** and an **out-degree**.

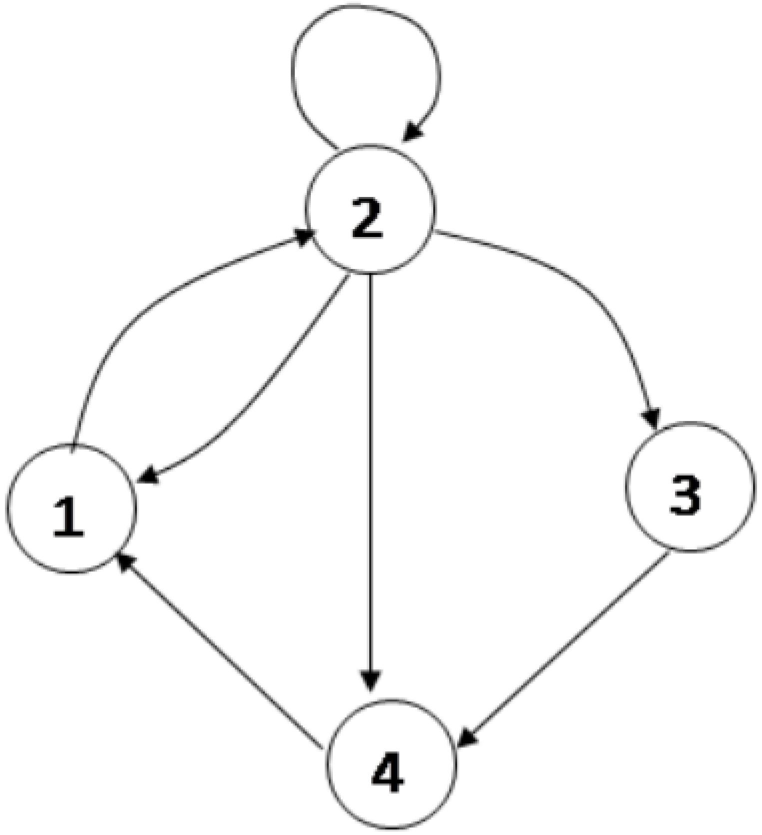
In-degree of a Graph-Number of edges which are coming into the vertex V.

Out-degree of a Graph-Number of edges which are going out from the vertex V



VERTEX	1	2	3	4
In Degree	0	2	2	1
Out-degree	2	0	2	1

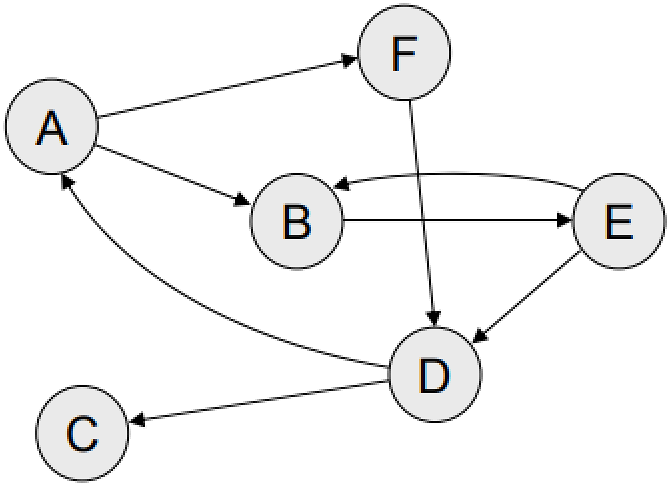
Find out in degree and out degree



VERTEX	1	2	3	4
In Degree	2	2	1	2
Out-degree	1	4	1	1

Problems

For the digraph shown let R be given by digraph shown. Find M_R and R



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Domain And Range of Relation

Domain of Relation:

Domain of relation set R is the set of elements in P which are related to some element in Q or it is the set of all first entries of the ordered pair in R.

It is denoted by DOM(R)

Range of Relation:

The range of relation R is the set of elements in Q which are related to some element in P or it is the set of all second entries of the ordered pair in R.

It is denoted by RAN(R)

For example: Let A={1,2,3,4} , B={a,b,c,d} and

R={ (1,a}, (1,b), (1,c), (2,b), (2,c), (2,d) } Then

DOM(R) = {1, 2}

RAN(R) = { a, b, c, d }

Problems

Draw the graphical representation of relation ' less than ' on { 1 , 2 , 3 , 4 }

R= { (1 , 2) , (1 , 3) , (1 , 4) , (2 , 3), (2 , 4), (3 , 4) }

A = { 2 , 3 , 4 , 5 } ,

R = { (2 , 3) , (3 , 2) , (3 , 4) , (3 , 5) , (4 , 3) , (4 , 4) , (4 , 5) } Draw Digraph

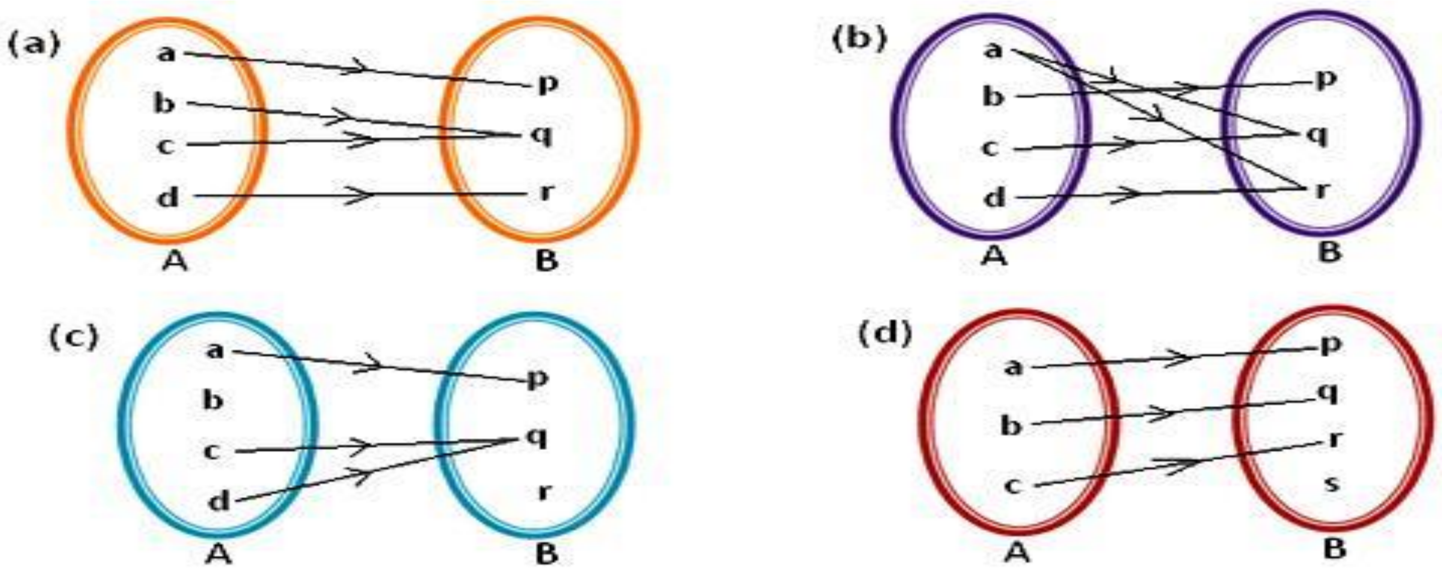
→Domain, Range of Relation R

Ex : A={ a , b , c , d } , B = { 1, 2 ,3 }

R= { (a ,1), (a , 2), (b , 1), (c ,2), (d, 1)}

Dom(R)={a , b , c , d }

Ran (R)={ 1 , 2 }



Problems

1. Let A = { 1 , 2 , 3 , 4 , 8 }, B = { 1 , 4 , 6 , 9 }.

Let "a R b " iff (a/b) a divides b.

Find the relation R ,draw digraph and also write M_R

R={(1,1),(1,4),(1,6),(1,9),(2,4),(2,6),(3,6),(3,9),(4,4)}

2. Let A = { 1 ,2 , 3 , 4 , 8 } = B

a R b iff a + b < = 9; Find the relation R ,draw digraph and also write M_R

R={(1,1),(1,2),(1,3),(1,4),(1,8),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4),(8,1)}

Complement of a Relation

Relation R from A to B

Complement of relation R is denoted by $\overline{R}$

$\overline{R}$: relation form A to B such that

$$\overline{R} = \{ (a.b) : (a,b) \notin R\}$$

Example: R is relation from X to Y

X={1,2,3}, Y={8,9} and R={(1,8), (2,8), (1,9), (3,9)}

X x Y = {(1,8), (1,9), (2,8), (2,9),(3,8), (3,9) }

$\overline{R}$ ={ (2,9), (3,8) }

Inverse of a Relation

R relation from A to B, then inverse of relation R is a relation from B to A sucht that $bR^{-1}a$ iff aRb i. e.

$$R^{-1} = \{ (b,a) : (a,b) \in R\}$$

Example: Consider the relation '<=' on the set A ={2, 3, 4, 5} Determine its inverse set.

R = {(2,2), (2,3), (2,4), (2,5), (3,3), (3,4), (3,5), (4,4), (4,5), (5,5)}

R^{-1} = {(2,2), (3,2), (4,2), (5,2), (3,3), (4,3), (5,3), (4,4), (5,4), (5,5)}

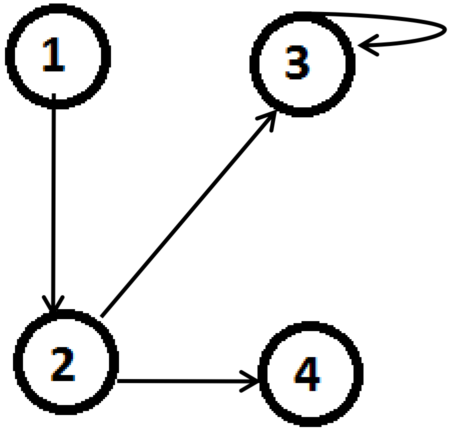
Path in Relations

- A path of length n in R from a to b, is a finite sequence a,Y_1,Y_2,Y_3……, Y_{n-1}, b which begins with a and ends with b such that
  $aRY_1RY_2RY_3……, Y_{n-1}R\ b$
- A path which has length n must have n+1 elements of A. The elements may be distinct or same.
- A path begins and end with same vertex is called **Cycle**.
- Path length is number of edges in the path.

PATHS

R = { (1 , 2) ,(2 , 3) , (2 , 4), (3 , 3) } is a relation on A = {1,2,3,4}

$R^1 = R = \{(1,2),(2,3),(2,4), (3,3)\}$



$R^2 = \{(1,3), (1,4), (2,3), (3,3)\}$

1 R^2 3 Since 1 R 2 and 2 R 3

1 R^2 4 Since 1 R 2 and 2 R 4 ...

$R^3 = \{ (1 , 3) , (2 , 3), (3 , 3) \}$

$R^4 = \{ (1 , 3) , (2 , 3), (3 , 3)\}$

PROBLEMS

1. Let A = { 1 , 2 , 3 , 4 , 5 } and R be relation defined by a R b iff a < b compute R , R^2, R^3 Draw digraph of R , R^2 and R^3

R = (1 , 2) , (1 , 3) , (1 , 4) , (1 , 5) , (2 , 3) , (2 , 4) , (2 , 5) , (3 , 4) , (3 ,5) , (4 , 5) }

R^2 = { (1 , 3) , (1 , 4) , (1 , 5) , (2 , 4) , (2 , 5) , (3 , 5) }

R^3 ={ (1 ,4) , (1, 5), (2 ,5)}

2. Consider *R* = { (1 , 1) , (2 , 1) , (3 , 2) , (4 , 3) }

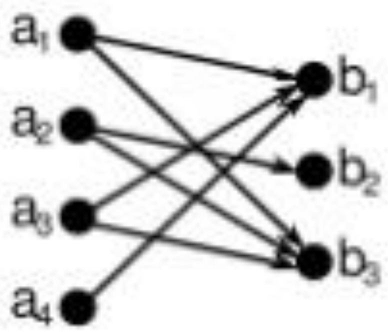
Compute R^2 R^3 R^4

3. Let A = { a , b , c , d , e } , R ={ (a , a) , (a , b) , (b , c) , (c , e) , (c , d) , (d , e) }

Draw digraph of R , M_R , Compute R^∞

Representation of Relation (R) using Matrix(M_R)

$$A = \{a_1, a_2, a_3, a_4\}, B = \{b_1, b_2, b_3\}$$
$$\alpha = \{(a_1, b_1), (a_1, b_3), (a_2, b_2), (a_2, b_3), (a_3, b_1), (a_3, b_3), (a_4, b_1)\}$$



	b_1	b_2	b_3
a_1	1	0	1
a_2	0	1	1
a_3	1	0	1
a_4	1	0	0

$$M_\alpha = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

Computation of M_R^2 using M_R

boolean matrix multiplication (BMM) problem as computing

$$\mathbf{C} = \mathbf{A} \odot \mathbf{B} \in \{0,1\}^{n \times n}$$

where for all i, j ∈ [n] we have

$$\mathbf{C}_{ij} = \vee_{k \in [n]} (\mathbf{A}_{ik} \wedge \mathbf{B}_{kj}) = \begin{cases} 1 & \exists k \in [n] \text{ with } \mathbf{A}_{ik} = \mathbf{B}_{kj} = 1 \\ 0 & \text{otherwise} \end{cases}$$

Properties/Types of Relations

- Reflexive
- Symmetric
- Transitive
- Antisymmetric
- Asymmetric

Properties: Reflexivity

- A relation R on a set A is reflexive if (a,a) ∈ R for all a ∈ A. Thus R is not reflexive if there exists a ∈ A such that (a, a)∉ R.
- **Definition**: A relation *R* on a set A is called reflexive iff

$$\forall a \in A \ (a, a) \in R$$

  - Eg: A = { 1 , 2 , 3 } ,

    R = { (1 , 1) , (2 , 2) , (3 , 3) }

    R2={(1,2), (1,1), (1,3), (2,2), (3,2), (3,3)}

  - Irreflexive ?

  Assume the relation R on A= { 1 , 2 , 3 , 4 } Is R1/R2 irreflexive?

  R1 = { (1 , 2) , (1 , 3) , (1 , 4) , (2 , 1) , (2 , 3) , (2 , 4) , (3 , 1) , (3 , 2) , (3 , 4) , (4 , 1) , (4 , 2) , (4 , 3) }

  R2= { (1 , 2) , (2 , 2) , (3 , 3) }

1. Example: The following relations on the integers are reflexive:

   $R1 = \{(a,b) \mid a \leq b\}$,

   $R3 = \{(a,b) \mid a = b \text{ or } a = -b\}$,

   $R4 = \{(a,b) \mid a = b\}$.

2. The following relations are not reflexive:

   $R2 = \{(a,b) \mid a > b\}$ (note that $3 \ngtr 3$),

   $R5 = \{(a,b) \mid a = b + 1\}$ (note that $3 \neq 3 + 1$),

   $R6 = \{(a,b) \mid a + b \leq 3\}$ (note that $4 + 4 \nleq 3$).

Problems to solve

1. Determine the relations on Set {1,2,3,4} is reflexive or not

a. {(2,2),(2,3),(2,4),(3,2),(3,3), (3,4)}

b. {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)}

c. {(2,4), (4,2)}

d. {(1,2), (2,3), (3,4)}

e. {(1,1),(2,2),(3,3),(4,4)}

f. {(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)}

Answer: b, e

Properties: Symmetry

- **Definitions**:
  - A relation R on a set A is called **symmetric** if whenever a R b and b R a i.e

$$\forall\, a, b \in A \;\; ((b, a) \in R \Leftrightarrow (a, b) \in R)$$

- Thus R is **not symmetric** if there exists a, b ∈ A such that

$$(a, b) \in R \text{ but } (b, a) \notin R.$$

Eg 1 : A = { 1 , 2, 3} , Is R symmetric ?

R = { (1 , 2) , (2 , 1) , (2 , 3), (3 , 2 , (1, 1)) }

Eg 2 : A = { 1 , 2 , 3 , 4 } , Is R symmetric ?

R= { (1 , 2) , (1 , 3) , (1 , 4) , (2 , 1) , (2 , 3) , (2 , 4) , (3 , 1) , (3 , 2), (3 ,4) , (4 , 1) , (4 , 2) , (4 , 3) }

Asymmetric relation: Asymmetric relation is opposite of symmetric relation.

A relation R on a set A is called asymmetric if no (b,a) Є R when (a,b) ЄR

AntiSymmetric Relation: A relation R on a set A is called antisymmetric if (a,b) Є R and (b,a) Є R if a = b is called antisymmetric. i.e. **UNLESS there exists (a , b)∈ R and (b , a) ∈ R, AND a ≠ b**

Eg : A = { 1 , 2 , 3 , 4 } and R = { (1 , 2) , (2 , 2) , (3 , 3) }

Is R anti-symmetric?

Answer: Yes. It is anti-symmetric

Note: Not symmetric ≠ antisymmetric .

Symmetry versus Anti-symmetry

- In a symmetric relation a*R*b ⇔ b*R*a
- In an antisymmetric relation, if we have a*R*b and b*R*a hold only when a=b
- An antisymmetric relation is not necessarily a reflexive relation
- A relation that is not symmetric is not necessarily asymmetric

Properties: Transitivity

- **Definition**: A relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all a,b,c $\in$ A

$$\forall a,b,c \in A\ ((aRb) \wedge (bRc)) \Rightarrow aRc$$

Example Consider Set A = { 1 , 2 , 3 }

R={ (1 , 2) , (2 , 3) , (1 , 3) } on set A is transitive.

R1 = {(1, 1), (1, 2), (2, 3), (1, 3)}

R2 = {(1, 1), (1, 2), (2,2), (2,3)}

R3 = {(1, 1), (1, 2), (1,3), (3,3)} Determine which relation is transitive.

Special cases

1) Let A = { 1 , 2 , 3 , 4 }

R= { (1 , 2) , (1 , 3) , (4 , 2) }

Is R transitive?

YES

2) R = { }

3)A relation that is symmetric and anti-symmetric

R = {(1,1),(2,2)} on the set A = {1,2,3}

Properties of Relations

State whether R satisfies property of reflexive , irreflexive , symmetry, asymmetry , antisymmetry , transitivity for A={1,2,3,4}

1. R={(1,1),(1,2),(2,1),(2,2),(3,3),(4,3),(3,4),(4,4)}
2. R= {(1,3),(1,1),(3,1),(1,2),(3,3),(4,4)}   3. R={(1,1),(2,2),(3,3),(4,4)}
4. R={(1,1),(1,2),(2,1),(2,2),(3,3),(3,4)}   5. R={(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)}

Let A={4,5,6,7}

i. R1={(4,4),(5,5),(6,6),(7,7)}   ii. R2={(4,4),(5,5)}   iii. R3=ϕ

iv. R4={(4,5),(5,4),(7,6),(6,7)}

EQUIVALENCE RELATION

A relation is an **Equivalence Relation** if it is reflexive, symmetric, and transitive.

Let A = { a , b , c } and

R= { (a , a), (b , b), (b , c), (c , b),(c , c) }

is an equivalence relation since it is **reflexive, symmetric, and transitive.**

1. Determine whether R is an Equivalence relation

1) R = { (1 , 1) , (2 , 2) , (3 , 3) , (1 , 2) , (2 , 1) , (2 , 3) , (3 , 2) , (1 , 3) , (3 , 1) }

on set A = { 1 , 2 , 3 }

2) A= { 1 , 2 , 3 , 4 }, R = { (1 , 1) , (1 , 2) , (2 , 1) , (2 , 2) , (3 , 4) , (4 , 3) , (3 , 3) , (4 , 4) }

3) Let A= {a ,b , c ,d }

R= { (a , a) , (b , a) , (b , b) , (c , c) , (d , d) , (d , c) }

Equivalence Class and Partitions

- Let A = { 1 , 2 , 3 , 4 } and consider the partition

  P = { { 1 , 2 , 3 } , { 4} } of A.

Find the equivalence relation R on A determined by P

" Each element in a block is related to every other element in the same block and only to those elements "

R= {(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(4,4)}

Problems

Find the equivalence relation on A by P and construct its **digraph**

1) Let A ={ a , b , c , d } and P = {{a , b } , { c }, { d } }

2) Let A={1,2,3,4,5} and P={{ 1 ,2 },{ 3 },{ 4, 5}}

R={(1,1),(1,2),(2,1),(2,2),(3,3),(4,4),(4,5),(5,4),(5,5)}

3) If {{1,3,5},{2,4}} is a partition on the set A={1,2,3,4,5},determine the corresponding equivalence relation

R={(1,1),(3,3),(5,5),(1,3),(1,5),(3,5),(3,1),(5,1),(5,3),(2,2),(4,4),(2,4),(4,2)}

Equivalence Class

Let A ={1,2,3,4,5,6} and let R be the equivalence relation on A defined by

R={(1,1),(1,5),(2,2),(2,3),(2,6),(3,2),(3,3),(3,6),(4,4),(5,1)

,(5,5),(6,2),(6,3),(6,6)}

Find the equivalence classes of R and find the partition of A induced by R

$R=\{(1,1),(1,5),(2,2),(2,3),(2,6),(3,2),(3,3),(3,6),(4,4),(5,1),(5,5),(6,2),(6,3),(6,6)\}$

Equivalence Classes:

$R(1)=\{1,5\}$ $R(2)=\{2,3,6\}$

$R(3)=\{2,3,6\}$ $R(4)=\{4\}$

$R(5)=\{1,5\}$ $R(6)=\{2,3,6\}$

Therefore, the partition of A induced by R i.e

$A|R=\{\{1,5\},\{2,3,6\},\{4\}\}$

Rank R (Number of distinct equivalence classes)

= 3

Problems to solve

1. Let A={1,2,3} and let R={(1,1),(2,2),(1,3),(3,1),(3,3)}.

   Find A|R.

2. Let A ={1,2,3,4},and let

   R={(1,1),(1,2),(2,1),(2,2),(3,4),(4,3),(3,3),(4,4)}

   Determine A|R.

3. Let A ={1,2,3,4},and let

   R={(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),(2,3),(3,2),(3,3),(4,4)} Show that R is an equivalence relation and determine the equivalence classes and hence find the rank of R

Combining Relations

- Relations are simply… sets (of ordered pairs); subsets of the Cartesian product of two sets
- Therefore, in order to combine relations to create new relations, it makes sense to use the usual set operations
  - Compliment R
  - Intersection ($R_1 \cap R_2$)
  - Union ($R_1 \cup R_2$)
  - Set difference ($R_1 \setminus R_2$)
  - Inverse R^{-1}

Example: Let $A = \{1, 2, 3\}$ and $B = \{u, v\}$ and

$R1 = \{(1, u), (2, u), (2, v), (3, u)\}$ and

$R2 = \{(1, v), (3, u), (3, v)\}$

$R1 \cup R2 =$

$\{(1, u), (1, v), (2, u), (2, v), (3, u), (3, v)\}$

$R1 \cap R2 =$

$\{(3, u)\}$

$R1 - R2 =$

$\{(1, u), (2, u), (2, v)\}$

$R2 - R1 =$

$\{(1, v), (3, v)\}$

Let A={ 1 , 2 , 3 , 4 } and B={ a , b , c } and let

R ={(1,a),(1,b),(2,b),(2,c),(3,b),(4,a)} and S={(1,b),(2,c),(3,b),(4,b)}

Compute $R \cap S$, $R \cup S$, R^{-1}

Combining Relations: Example

- Let
  - A={1,2,3,4}
  - B={1,2,3,4}
  - R_1={(1,2),(1,3),(1,4),(2,2),(3,4),(4,1),(4,2)}
  - R_2={(1,1),(1,2),(1,3),(2,3)}
- Let
  - $R_1 \cup R_2$=
  - $R_1 \cap R_2$ =
  - $R_1 - R_2$ =
  - $R_2 - R_1$ =

Composite relation

- **Definition**: Let R_1 be a relation from the set A to B and R_2 be a relation from B to C, i.e.

$$R_1 \subseteq A \times B \text{ and } R_2 \subseteq B \times C$$

the composite of R_1 and R_2 is the relation consisting of ordered pairs (a,c) where $a \in A$, $c \in C$ and for which there exists an element $b \in B$ such that $(a,b) \in R_1$ and $(b,c) \in R_2$. We denote the composite of R_1 and R_2 by

$$R_2 \circ R_1$$

Ex: Let $A = \{1, 2, 3\}$, $B = \{0, 1, 2\}$ and $C = \{a, b\}$

$R = \{(1, 0), (1, 2), (3, 1), (3, 2)\}$

$S = \{(0, b), (1, a), (2, b)\}$

$S \circ R = ?$

$\{(1, b), (3, a), (3, b)\}$

Since $(1,0) \in R$ and $(0,b) \in S$, $\therefore$ $(1,b) \in S \circ R$

Since $(1,2) \in R$ and $(2,b) \in S$, $\therefore$ $(1,b) \in S \circ R$

Since $(3,1) \in R$ and $(1,a) \in S$, $\therefore$ $(3,a) \in S \circ R$

Since $(3,2) \in R$ and $(2,b) \in S$, $\therefore$ $(3,b) \in S \circ R$

Problems

1. Let A={1,2,3} and let

   R={(1,1),(1,3),(2,1),(2,2),(2,3),(3,2)} and

   S={(1,1),(2,2),(2,3),(3,1),(3,3)}.

   Find M_{SoR}

SoR={(1,1),(1,3),(2,1),(2,2),(2,3),(3,2),(3,3)}

2. Let A={1,2,3,4}

   R={(1,1),(1,2),(2,3),(2,4),(3,4),(4,1),(4,2)}

   S={(3,1),(4,4),(2,3),(2,4),(1,1),(1,4)}

Compute SoR,RoS,RoR,SoS

SoR={(1,1),(1,3),(2,1),(2,4),(3,4),(4,1),(4,4),(1,4)}

RoS={(3,1),(3,2),(4,1),(4,2),(2,4),(2,1),(2,2),(1,1),(1,2)}

RoR

SoS

Warshall's Algorithm

1. Transitive Closure and Warshall's Algorithm

- Main idea: a path exists between two vertices i, j, iff
  - there is an edge from i to j; or
  - there is a path from i to j going through vertex 1; or
  - there is a path from i to j going through vertex 1 and/or 2; or
  - …
  - there is a path from i to j going through vertex 1, 2, … and/or k; or
  - ...
  - there is a path from i to j going through any of the other vertices

Warshall's Algorithm

1. Suppose W_k =[t_{ij}] and w_{k-1}=[s_{ij}]. If t_{ij}=1 then there must be a path from a_i to a_j whose interior vertices come from {a_1,a_2,…..a_k}.
2. If the vertex ak is not interior vertex of this path, them all interior vertices must actually come from set {a_1,a_2…..,a_{k-1}}, so s_{ij}=1

   Thus, t_{ij} = 1 if and only if either

   a. s_{ij} = 1 or

   b. s_{ik} = 1 and s_{kj} = 1.

3. If w_{k-1} has 1 in position I,j then , by (a), So will W_k. By (b), a new 1 can be added in position I ,j of W_k iff column k of w_{k-1} has 1 in position i and row of w_{k-1} has a 1 in position j.

Warshall's Algorithm

1. Steps for computing W_k from W_{k-1}
   1. First transfer to W_k all 1's in W_{k-1} .
   2. List the locations $p_1,p_2,\ldots\ldots$ in column k of W_{k-1}, where the entry is 1, and the location $q_1,q_2,\ldots.$ In row k of W_{k-1}, where the entry is 1.
   3. Put 1's in all the positions p_i,q_j of W_k (if they are not already there)

Example: Let A = {1,2,3,4} and let R is relation on Set A

R= { (1,2),(2,3), (3,4),(2,1) }. Find the transitive closure of R.

$R = \{ (1,2),(2,3), (3,4),(2,1) \}$

$$W_0 = M_R = \begin{bmatrix} \boxed{0} & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And n =4

To find W_1, k=1 W_0 has 1's in location **2** of column 1 and location **2** of row 1. Thus W1 is just w0 with new 1 in position 2,2.

$$W_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & \mathbf{1} & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R = \{ (1,2),(2,3), (3,4),(2,1) \}$

$$W_1 = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To find W_2, k=2 W_1 has 1's in location **1,2** of column 2 and location **1, 2, 3** of row 2. Thus to obtain W_2, we must put 1's in the position :

1,1; 1,2; 1,3; 2,1; 2,2; and 2,3 of W_1 .

$$W_2 = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 1 & \mathbf{1} & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R= { (1,2),(2,3), (3,4),(2,1) }

$$W_2 = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To find W_3, k=3 W_2 has 1's in location **1,2** of column 3 and location **4** of row 3.

Thus to obtain W_3, we must put 1's in the position : 1,4; 2,4 of W_2 .

$$W_3 = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 1 & \mathbf{1} & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

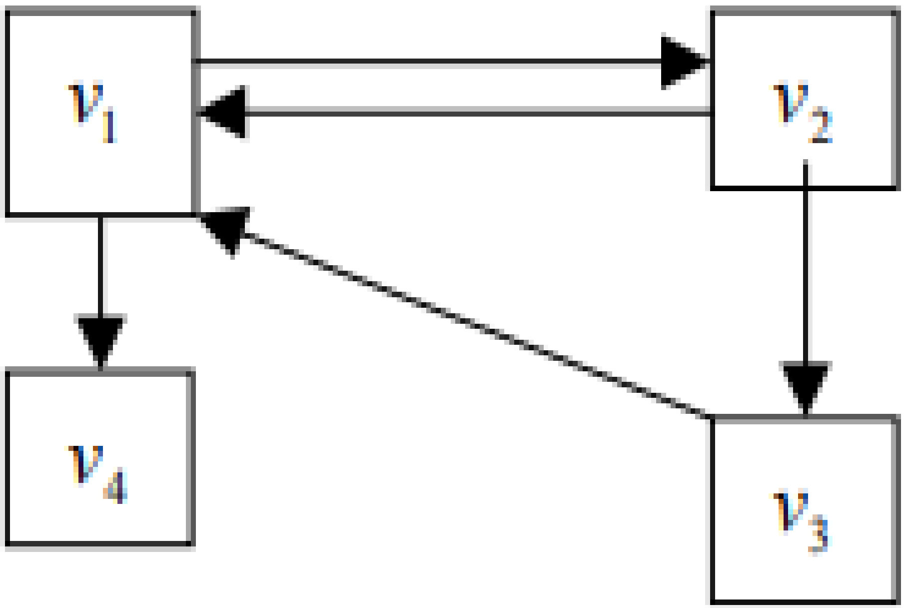
To find W_4, k=4 W_3 has 1's in location **1,2,3** of column 4 and NO ONE's in row 4.

Thus to obtain W_4 same as W_3 .

$$W_4 = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 1 & \mathbf{1} & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Transitive Closure

Find the transitive closure for the following



Problem solving

Compute the Warshall's Algorithm transitive closure of

- R={(a,b),(b,c),(c,d),(b,a)} on set A={a,b,c,d}

$$R_t = \{(a, a), (a, b), (a, c), (a, d), (2, a), (2, b), (2, c), (2, d), (3, d)\}.$$

- R = {(1,1),(1,2),(1,4),(2,2),(2,3),(3,1),(3,4),(4,1),(4,4)} on the set A={1,2,3,4}

$$R_t = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$$

Computer transitive closure using Warshall’s algorithm where $A=\{a_1,a_2,a_3,a_4,a_5\}$ and R be a relation on A whose matrix is

$$M_R=W_0=\begin{matrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{matrix}$$

POSETS(Partially Ordered Sets)

A relation R on a set A is called **partial order if R** is

REFLEXIVE,

ANTISYMMETRIC AND

TRANSITIVE

The set A together with the partial order R is called a POSET (A, R) or A

HASSE DIAGRAM

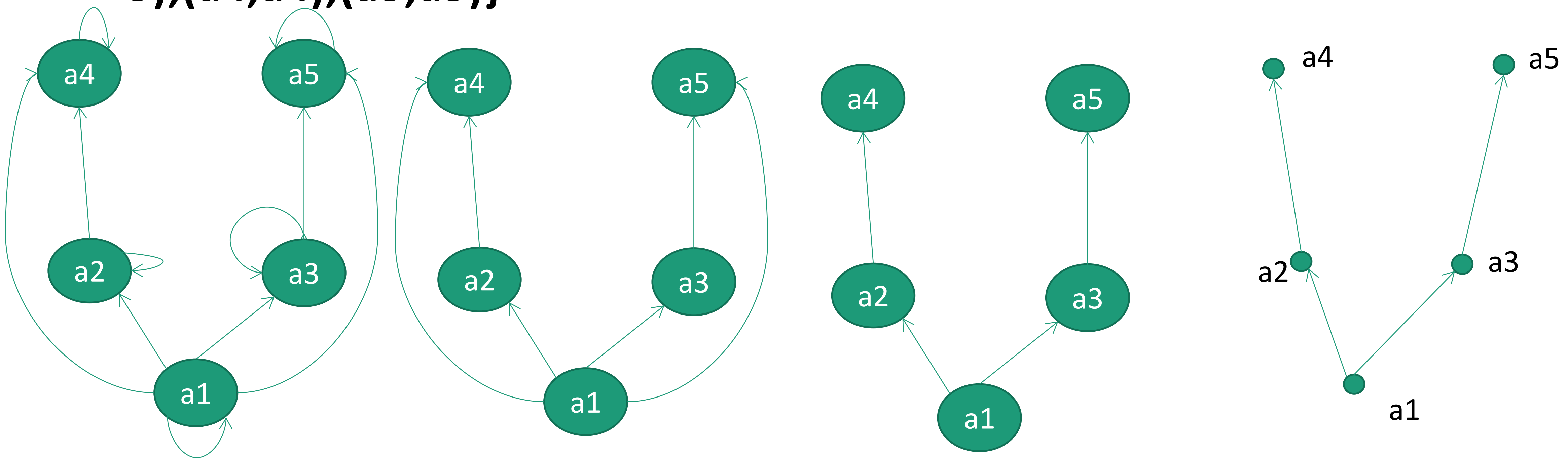
Mathematical diagram used to represent a finite partially ordered set

STEPS:

1. Draw the digraph of the given relation
2. Delete all cycles from the graph
3. Eliminate all edges that are implied by transitive relations
4. Draw the digraph of a partial order with all edges pointing upward, so that arrows may be omitted from the edges.
5. Replace the circles representing the vertices by dots. The resulting diagram of a partial order is called the Hasse diagram of the partial order of the poset.

Hasse Diagram for POSET

- R={(a1,a1),(a1,a2),(a1,a3),(a1,a4),(a1,a5),(a2,a2),a2,a4),(a3,a3),(a3,a5),(a4,a4),(a5,a5)}



STEP 1 STEP 2 STEP 3,4 STEP 5

ANY ????