INVERSE LAPLACE TRANSFORM

FIND THE INVERSE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

1. $\dfrac{4s+12}{s^2+8s+12}$ [Ans: $e^{-4t}(4\cosh 2t-\sinh 2t)$]

2. $\dfrac{s}{s^2+2s+2}$ [Ans: $e^{-t}(\cos t-\sin t)$]

3. $\dfrac{s}{(2s+1)^2}$ [Ans: $e^{-1/2}(t-4)/16$]

4. $\dfrac{s+1}{s^2-4}$ [Ans: $\dfrac{1}{4}\left(3e^{2t}+e^{-2t}\right)$]

5. $\dfrac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\dfrac{3}{2}e^{-t}\sin 2t-2e^{t}\sin t$]

6. $\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\dfrac{1}{a^2-b^2}(a\sin at-b\sin bt)$]

7. $\dfrac{s}{(s^2+a^2)(s^2+b^2)}$ [Ans: $\dfrac{1}{b^2-a^2}(\cos at-\cos bt)$]

8. $\dfrac{5s^2+8s-1}{(s+3)(s^2+1)}$ [Ans: $2e^{-3t}+3\cos t-\sin t$]

9. $\dfrac{2s}{s^4+4}$ [Ans: $\sin t\sinh t$]

10. $\dfrac{1}{s^3+1}$ [Ans: $\dfrac{1}{3}e^{-t}-\dfrac{e^{t/2}}{3}\cos\left(\dfrac{\sqrt{3}}{2}t\right)+\dfrac{e^{t/2}}{\sqrt{3}}\sin\left(\dfrac{\sqrt{3}}{2}t\right)$]

11. $\dfrac{1}{s^3(s-1)}$ [Ans: $1-t+\dfrac{t^2}{2}-e^{-t}$]

12. $\dfrac{s}{(s+1)^2(s^2+1)}$ [Ans: $\dfrac{1}{2}\left[\sin t-te^{-t}\right]$]

13. $\dfrac{5s^2-15s-11}{(s+1)\,(s-2)^2}$ [Ans: $e^{-t}+4e^{2t}-7te^{2t}$]

14. $\dfrac{s}{(s^2+1)\,(s^2+4)(s^2+9)}$ [Ans: $\dfrac{1}{24}\cos t-\dfrac{1}{15}\cos 2t+\dfrac{1}{40}\cos 3t$]

15. $\dfrac{s^2}{(s+1)^3}$ [Ans: $e^{-t}\left(1-2t+t^2\right)$]

16. $\dfrac{3s-2}{s^{5/2}}-\dfrac{7}{3s+2}$

17. $\log\left(\dfrac{s+a}{s+b}\right)$ [Ans: $-\dfrac{1}{t}(e^{-at}-e^{-bt})$]

18. $2\tanh^{-1}s$ [Ans: $\dfrac{2}{t}\ \sinh t$]

19. $\tan^{-1}\left(\dfrac{2}{s^2}\right)$ [Ans: $2\sin t\sinh t$]

20. $\tan^{-1}\left(\dfrac{s+a}{b}\right)$ [Ans: $-\dfrac{1}{t}\ e^{-at}\sin bt$]

21. $\log\sqrt{\dfrac{s^2+1}{s^2}}$ [Ans: $\dfrac{1}{t}(1-\cos t)$]

22. $\cot^{-1}(s+1)$ [Ans: $\frac{1}{t}\, e^{-t}\sin t$]

23. $\log\left|s^2+4\right|$ [Ans: $-\frac{2}{t}\cos 2t$]

FIND THE INVERSE OF THE FOLLOWING USING CONVOLUTION THEOREM:

1. $\frac{s^2}{(s^2+a^2)^2}$ [Ans: $\frac{1}{2a}\left[\sin at + at\cos at\right]$]

2. $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ [Ans: $\frac{e^{-t}}{3}(\sin 2t+\sin t)$]

3. $\frac{(s+2)^2}{(s^2+4s+8)^2}$ [Ans: $\frac{e^{-2t}}{4}(2t\cos 2t+\sin 2t)$]

4. $\frac{1}{(s+3)(s^2+2s+2)}$ [Ans: $\frac{1}{5}\left[e^{-t}(2\sin t-\cos t)+e^{-3t}\right]$]

5. $\frac{1}{(s-2)^4(s+3)}$ [Ans: $\frac{e^{-3t}}{625}-e^{2t}\left[\frac{1}{625}-\frac{t}{125}+\frac{t^2}{50}-\frac{t^3}{30}\right]$]

6. $\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)$ [Ans: $\int_0^t -\frac{2}{u}(\cos u-1)\,du$]

7. $\frac{s^2+s}{(s^2+1)(s^2+2s+2)}$ [Ans: $\frac{1}{10}\left[e^{-t}(2\sin t-6\cos t)+(2\sin t+6\cos t)\right]$]

8. $\frac{s}{s^4+8s^2+16}$ [Ans: $\frac{1}{4}t\sin 2t$]

9. Find $\int_0^\infty \sin(tx^2)\,dx$ and hence find $\int_0^\infty \sin x^2\,dx$ [Ans: $\frac{1}{2}\sqrt{\frac{\pi}{2}}$]

10. Using Convolution theorem prove that

i) $$L^{-1}\left[\frac{1}{s}\log\left(a+\frac{b}{s^2}\right)\right]=\int_0^t \frac{2}{u}\left[1-\cos\left(\frac{b}{a}\right)u\right]du$$

ii) $$L^{-1}\left[\frac{1}{s}\log\left(\frac{s+1}{s+2}\right)\right]=\int_0^t \frac{e^{-2u}-e^{-u}}{u}\,du$$

FIND THE LAPLACE TRANSFORM OF PERIODIC FUNCTION:

1. $f(t)=K\frac{t}{T}\ for\ 0<t<T\ and\ f(t)=f(t+T)$ [Ans: $K\left[\frac{1}{Ts^2}-\frac{e^{-st}}{s(1-e^{-st})}\right]$]

2. $f(t)=1,\ for\ 0\le t<a\ and\ f(t)=-1,\ a<t<2a\ and\ f(t)$ is periodic with period 2a.
[Ans: $\frac{1}{s}\tanh\left(\frac{as}{2}\right)$]

3. $f(t)=|\sin pt|,\ t\ge 0$ [Ans: $\frac{p}{s^2+p^2}\cdot\coth\left(\frac{\pi s}{2p}\right)$]

4. $f(t)=t,\ for\ 0<t<1\ and\ f(t)=0, 1<t<2\ and\ f(t+2)=f(t)\ for\ t>0$
[Ans: $\frac{1}{s^2(1-e^{-2s})}\left(1-e^{-s}-se^{-s}\right)$]

5. $f(t)=\frac{t}{a}, 0<t\le a;\ f(t)=\frac{1}{a}(2a-t), a<t<2a\ \ and\ f(t)=f(t+2a)$
[Ans: $\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$]

HEAVISIDE'S UNIT-STEP FUNCTION FIND THE LAPLACE TRANSFORM OF FOLLOWING FUNCTIONS:

1. $t^2\, H(t-3)$ [Ans: $e^{-3s}\left[\frac{9}{s}+\frac{6}{s^2}+\frac{2}{s^3}\right]$]

2. $\sin t\cdot H\left(t-\frac{\pi}{2}\right)-H\left(t-\frac{3\pi}{2}\right)$ [Ans: $e^{-\pi s/2}\cdot\frac{s}{s^2+1}-e^{-3\pi s/2}\cdot\frac{1}{s}$]

3. $\left(1+2t-3t^2+4t^3\right)H(t-2)$ [Ans: $e^{-2s}\left[\frac{25}{s}+\frac{38}{s^2}+\frac{42}{s^3}+\frac{24}{s^4}\right]$]

4. Using Laplace transform evaluate

$$\int_0^\infty e^{-t}\left(1+2t-3t^2+4t^3\right)H(t-2)\,dt$$ [Ans: $\frac{e^{-2}}{129}$]

FIND THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\frac{e^{-as}}{(s+b)^{5/2}}$ [Ans: $\frac{4}{3\sqrt{\pi}}\cdot e^{b(t-a)}\cdot(t-a)^{3/2}\cdot H(t-a)$]

2. $\frac{(s+1)e^{-s}}{s^2+s+1}$ [Ans: $e^{-t/2}\left[\cos\left(\sqrt{3}\,(t-1)/2\right)+\frac{1}{\sqrt{3}}\sin\left(\sqrt{3}\,(t-1)/2\right)\right]\cdot H(t-1)$]

3. $\frac{e^{-\pi s}}{s^2-2s+2}$ [Ans: $e^{(t-\pi)}\cdot\sin(t-\pi)\cdot H(t-\pi)$]

4. $e^{-s}\left(\frac{1-\sqrt{s}}{s^2}\right)^2$ [Ans: $\left[\frac{(t-1)^3}{6}-\frac{16}{15\sqrt{\pi}}(t-1)^{5/2}+\frac{(t-1)^2}{2}\right]\cdot H(t-1)$]

USING LAPLACE TRANSFORM SOLVE THE FOLLOWING DIFFERENTIAL EQUATIONS WITH THE GIVEN CONDITION:

1. $\left(D^2-4\right)y=3e^t,\quad y(0)=0,\ y'(0)=3$ [Ans: $y=-e^t+\frac{3}{2}e^{2t}-\frac{1}{2}e^{-2t}$]

2. $\left(D^2+D\right)y=t^2+2t,\quad y(0)=4,\ y'(0)=-2$ [Ans: $y=2+2e^{-t}+\frac{t^3}{3}$]

3. $\left(D^2+2D+1\right)y=3te^{-t},\quad y(0)=4,\ y'(0)=-2$ [Ans: $y=e^{-t}\left(4+6t+\frac{t^3}{2}\right)$]

4. $\left(D^2-2D-8\right)y=4,\quad y(0)=0,\ y'(0)=1$ [Ans: $y=-\frac{1}{2}+\frac{1}{6}e^{-2t}+\frac{1}{3}e^{4t}$]

5. $\frac{d^2y}{dt^2}+4y=H(t-2)$ *with conditions* $y(0)=0,\ y'(0)=1$

[Ans: $y=\frac{1}{2}\sin 2t+\frac{1}{4}H(t-2)-\frac{1}{4}\cos 2(t-2)\,H(t-2)$]

6. $\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t$, *given that* $y(0)=1$ [Ans: $y=e^{-t}-\frac{3}{2}t\,e^{-t}+\frac{1}{2}\sin t$]

7. $\frac{d^2y}{dt^2}+9y=18t$ *with conditions* $y(0)=0,\ y(\pi/2)=0$ [Ans: $y=2t+\pi\sin 3t$]

8. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$, *where* $y(0)=0,\ y'(0)=4$ [Ans: e^x-e^{-3x}]

9. $\frac{d^2y}{dt^2}+4y=f(t)$ *with conditions* $y(0)=0,\ y'(0)=1$ *and* $f(t)=1$, *when* $0<t<1$

$=0$, *when* $t>1$

[Ans: $y=\frac{1}{2}\sin 2t+\frac{1}{4}(1-\cos 2t)-\frac{1}{4}\{1-\cos(t-1)\}\,H(t-1)$]