Vector Differenciation

S.No	Questions
	GRADIENT and DIRECTIONAL DERIVATIVES
1	Find $\nabla\phi$ if $\phi = 3x^2y - y^3z^2$ at $(1,-2,1)$
2	Find the directional derivative of $\phi = x^2y + y^2z + z^2x^2$ at $P(1, 2, 1)$ in the direction of the normal to the surface $x^2 + y^2 - z^2x = 1$ at $Q(1, 1, 1)$
3	Find the directional derivative of $\phi = 2x^3y - 3y^2z$ at $P(1, 2, -1)$ in the direction towards $Q(3, -1, 5)$. In what direction from P is the directional derivative maximum? Find the magnitude of maximum directional derivative.
4	Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at $A(1, -2, 1)$ in the direction of AB where B is $(2, 6, -1)$. Also find the maximum directional derivative of ϕ at $(1, -2, 1)$.
5	Find the directional derivative of $\phi = x^2y^2 + y^2z^2 + z^2x^2$ at $(1, 1, -2)$ in the direction of the tangent to the curve $x = e^{-t}, y = 2\sin t + 1, z = t - \cos t$ at $t = 0$
6	Find the directional derivative of $\phi = e^{2x}\cos yz$ at (0, 0, 0) in the direction of the tangent to the curve $x = a\sin t,\ y = a\cos t,\ z = at$ at $t = \pi/4$.
7	Find the directional derivative of $\phi = x^2 + y^2 + z^2$ in the direction of the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ at (1, 2, 3)
8	Find the directional derivative of $\phi = x^2y\cos z$ in the direction of the line $\overline{a} = 2i + 3j + 2k$ at (1, 2, $\pi/2$)
9	Find the acute angle between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$
10	Find the angle between the two surfaces $x^2 + y^2 + a\,z^2 = 6$ and $z = 4 - y^2 + b\,x\,y$ at $(1, 1, 2)$
11	Find the rate of change of $\phi = xy + yz + zx$ at $(1, -1, 2)$ in the direction of the normal to the surface $x^2 + y^2 = z + 4$.
12	In what direction is the directional derivative of $\phi = 2xz - y^2$ at $(1, 3, 2)$ maximum? Find its magnitude.
13	Find the rate of change of $\phi = x\,y^2 + y\,z^3$ at $(2,-1,1)$ in the direction of the normal to the surface $x\log z - y^2 + 4 = 0$ at $(-1, 2, 1)$
14	Find the angle between the normals to the surfaces $x^2y + 2xz = 4$ at $(2, -2, 3)$ and to $x^3 + y^3 + 3xyz = 3$ at $(1, 2, -1)$
15	Find the constants a and b such that the surfaces $ax^2 - 2byz = (a+4)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1,-1, 2)$.
16	Find the constants a, b if the angle between the surfaces $x^2 + axz + byz = 2$ & $x^2z + xy + y + 1 = z$ at (0, 1, 2) is $\cos^{-1}(1/\sqrt{3})$.
17	Find the constants a, b such that the surfaces $5x^2 - 2yz - 9x = 0$ & $ax^2y + bz^3 = 4$ cut orthogonally at (1,-1, 2)
18	If the directional derivative of $\phi = ax^2 + by + 2z$ at (1, 1, 1) is maximum in the direction of $i + j + k$, find a &b.
19	. Find the constants a and b such that the surface $ax^2 - bxy + xz = 10$ is orthogonal to the

	surface $x^2 + y^2 = 4 + xz$ at (1, 2, 1)
20	Find the directional derivative of $\emptyset = \frac{x}{x^2+y^2}$ at (0,1,1) in the direction of normal to the surface $x^2 + y^2 - z^2x = 1$ at (1,1,1)
21	Find the constants a and b such that the directional derivative of $\emptyset = ax^2 + by^2 + cz^2$ at (1,1,2) has the maximum magnitude 4 in the direction parallel to x-axis.
	DIFFERENTIAL OPERATORS
22	If $\bar{a}$ is a constant vector such that $\lvert \bar{a} \rvert = a$ then prove that $\nabla \cdot \left\{ (\bar{a} \cdot \bar{r}) \bar{a} \right\} = a^2$
23	If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$, prove that i) $div\,(\bar{a} \times \bar{r}) = 0$ ii) $div\,(\bar{a} \cdot \bar{r})\bar{a} = a^2$ iii) $div\,(\bar{a} \times \bar{r} \times \bar{a}) = 2a^2$ iv) $curl\,(\bar{a} \times \bar{r}) = 2\bar{a}$
24	If $\phi = x^3 + y^3 + z^3 - 3xyz$, find (i) $\bar{r} \cdot \nabla\phi$, (ii) $\nabla \cdot \bar{F}, \nabla \times \bar{F}$ $where$ $\bar{F} = \nabla\phi$
25	Prove that $\nabla\left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$.
26	Prove that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$ and hence, find f if $\nabla f = 2r^4\bar{r}$.
27	Show that $\nabla\left[\frac{(\bar{a} \cdot \bar{r})}{r^n}\right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$
28	Prove that $\nabla r^n = n r^{n-2}\bar{r}$
29	Prove that $\nabla \cdot (\nabla \times \bar{F}) = 0$ $where$ $\bar{F}$ is a vector point function.
30	Prove that $\nabla\left\{\nabla \cdot \frac{\bar{r}}{r}\right\} = -\frac{2}{r^3}\bar{r}$
31	Prove that $\nabla \cdot \left(r\,\nabla\frac{1}{r^3}\right) = \frac{3}{r^4}$
32	Prove that $\nabla \cdot \left(r\,\nabla\frac{1}{r^n}\right) = \frac{n(n-2)}{r^{n+1}}$
33	Prove that $div\ grad\ r^n = n(n+1)r^{n-2}$
34	Prove that $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n}\right) = \frac{(2-n)\bar{a}}{r^n} + \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$
35	Prove that $\nabla \log r = \frac{\bar{r}}{r^2}$ and hence, show that $\nabla \times (\bar{a} \times \nabla \log r) = 2\frac{(\bar{a} \cdot \bar{r})\bar{r}}{r^4}$, where $\bar{a}$ is a constant vector.
	DIVERGENCE AND CURL
36	$div\,\bar{F}$ and $curl\,\bar{F}$ $where$ $\bar{F} = \frac{xi - yj}{x^2 + y^2}$ Find
37	If $\bar{A} = \nabla(xy + yz + zx)$, $find$ $\nabla \cdot \bar{A}$ and $\nabla \times \bar{A}$
38	If $\bar{F} = (\bar{a} \cdot \bar{r})\bar{r}$ $where$ $\bar{a}$ is constant vector, find $curl\,\bar{F}$ and P.T. it is perpendicular to $\bar{a}$.
39	Prove that $\bar{F} = \frac{\bar{r}}{r^3}$ is both irrotational and solenoidal.
40	A vector field $\bar{F}$ is given by

	$\overline{F} = (y\sin z - \sin x)i + (x\sin z + 2yz)j + (xy\cos z + y^2)k$ Prove that it is irrotational and hence, find its scalar potential.
41	A vector field is given by $\overline{F} = (x^2 + xy^2)i + (y^2 + x^2 y)j$. Show that $\overline{F}$ irrotational and find its scalar potential.
42	If $\nabla\phi = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k,\ find\ \phi\ where\ \phi(1,0,1) =$
43	Find the value of n for which the vector $r^n \overline{r}$ is solenoidal, where $\overline{r} = xi + yj + zk$
44	Prove that $\nabla \cdot \left\{ \frac{f(r)}{r} \overline{r} \right\} = \frac{1}{r^2} \frac{d}{dr}\left[r^2 f(r)\right]$ hence or otherwise prove that $div(r^n \overline{r}) = (n+3)r^n$
45	Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal & irrotational.
46	If $\overline{r}$ is the position vector of point (x, y, z) and r is the modulus of $\overline{r}$, then prove that $r^n \overline{r}$ is an irrotational vector for any value of n but solenoidal only if $n = -3$.
47	If $\bar{f} = (x + y + 1)i + j - (x + y)k$, prove that $\bar{f} \cdot curl\ \bar{f} = 0$
48	Define irrotational field and hence check whether the vector field $\overline{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$ is irrotational.
	DIFFERENTIAL OPERATORS of Higher Order
49	With usual notation, prove that $\nabla^2\left[\nabla \cdot \frac{\bar{r}}{r^2}\right] = \frac{2}{r^4}$
50	Show that $\nabla^4 r^2 \log r = \frac{6}{r^2}$
51	Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$
52	Prove that $\nabla^2(r^2 \log r) = 5 + 6\log r$