

Vector Integration

S.No	Questions
	Line Integral
1	Find the work done of the moving partical in the force filed $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the line (0, 0,0) to (2,1,3)
2	Evaluate $\int_A^B (3xy dx - y^2 dy)$ along the parabola $y = 2x^2$ from $A(0,0)$ to $B(1,2)$. What is the integral if the path is a straight line joining A to B ?
3	Find the work done in moving a particle in the force field $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.
4	Show that $\vec{F} = (2xy z^2)\hat{i} + (x^2 z^2 + z \cos yz)\hat{j} + (2x^2 yz + y \cos yz)\hat{k}$ is conservative. Find scalar potential ϕ such that $\vec{F} = \nabla\phi$ and hence, find the work done by \vec{F} in displacing a particle from (0,0,1) to (1, $\pi/4$,2) along the straight line AB .
5	Prove that $\vec{F} = (2xy + z)\hat{i} + (x^2 + 2yz^3)\hat{j} + (3y^2 z^2 + x)\hat{k}$ is irrotational. Find its scalar potential ϕ and evaluate $\int_{(1,2,0)}^{(2,2,1)} \vec{F} \cdot d\vec{r}$ along the straight line .
6	If $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ be the force field, find the work done in moving a particle from (1,2,-4) to (3,3,2).
7	Prove that $\vec{F} = 2xye^z\hat{i} + x^2e^z\hat{j} + x^2ye^z\hat{k}$ is irrotational. Find scalar potential ϕ such that $\vec{F} = \nabla\phi$ and hence, find the work done by \vec{F} in displacing a particle from (2,1,1) to (2,0,1) .
8	Prove that $\vec{F} = 3x^2y\hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)\hat{k}$ is irrotational. Find scalar potential ϕ such that $\vec{F} = \nabla\phi$ and hence, find the work done by \vec{F} in displacing a particle from (0,0,0) to (1,1,1) .
9	A vector field is given by $\vec{F} = (x^2 - y^2 + x)\hat{i} + (2xy + y)\hat{j}$ is irrotational. Find its scalar potential and also find the line integral from (1,2) to (2,1)
10	Prove that $\vec{F} = (3x^2yz - 3y)\hat{i} + (x^3z - 3x)\hat{j} + (x^3y + 2z)\hat{k}$ is irrotational. Find scalar potential ϕ such that $\vec{F} = \nabla\phi$ and hence, find the work done by \vec{F} in displacing a particle from (0,0,0) to (1,1,1) .
11	Find scalar potential of $\vec{F} = (6xy^2 - 2z^3)\hat{i} + (6x^2y + 2yz)\hat{j} + (y^2 - 6z^2x)\hat{k}$ if exists. Also find the work done by \vec{F} in displacing a particle from (1,0,2) to (0,1,1) .
12	Evaluate $\int_c \vec{F} \cdot d\vec{r}$ along the arc of the curve $\vec{r} = (e^t \cos t)\hat{i} + (e^t \sin t)\hat{j}$ from (1,0) to ($e^{2\pi}$, 0) where $\vec{F} = \frac{xi+yj}{(x^2+y^2)^{3/2}}$.
13	Find the constants a, b, c such that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is a conservative field . Find its scalar potential and work done in moving a particle from (1,2,0) to (11,-1).
14	Prove that $\int_{(1,2)}^{(3,4)} (6xy^2 - y^3)dx + (6x^2y - 3xy^2)dy$ is independent of the path joining the points (1,2) and (3,4) and hence evaluate it.
15	Find the constant a so that $\vec{F} = (axy - z^3)\hat{i} + (a - 2)x^2\hat{j} + (1 - a)xz^2\hat{k}$ is a conservative field . Find its scalar potential and work done in moving a particle from (1,2,-3) to (1,-4,2).
	GREEN'S Theorem: State GREEN'S Theorem and hence evaluate the following integrals

16	$\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the boundary of the surface enclosed by the lines $x = 0, y = 0, x = 2, y = 2$.
17	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2\vec{i} - xy\vec{j}$ & 'c' is the triangle having vertices (0,2),(2,0),(4,2).
18	$\int_c \left(\frac{1}{y}dx + \frac{1}{x}dy \right)$ where 'c' is the boundary of the region defined by $x = 1, x = 4, y = 1, \& y = \sqrt{x}$.
19	$\int_c (x^2 - y)dx + (2y^2 + x)dy$ around the boundary of the region defined by $y = 4, \& y = x^2$.
20	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = -xy(x\vec{i} - y\vec{j})$ and c is $r = a(1 + \cos\theta)$
21	The work done by $\vec{F} = (4x - 2y)\vec{i} + (2x - 4y)\vec{j}$ in moving a particle once counter clockwise around the circle $(x - 2)^2 + (y - 2)^2 = 4$
22	$\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ around the boundary in the xy plane enclosed by the x-axis and the semi circle $y = \sqrt{1 - x^2}$.
23	$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the region bounded by $y = \sqrt{x}$ & $y = x$
24	$\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the region bounded by $y = \sqrt{x}$ & $y = x^2$
25	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - xy)\vec{i} + (x^2 - y^2)\vec{j}$ and c is the region bounded by $x^2 = 2y$ & $x = y$
26	$\int_c [(x^2 + y^2)\vec{i} + (x^2 - y^2)\vec{j}] \cdot d\vec{r}$ where 'c' is the boundary of the region enclosed by circles $x^2 + y^2 = 4, x^2 + y^2 = 16$
27	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2\vec{i} + xy\vec{j}$ and C is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$.
28	$\int_c (xydx + xy^2dy)$ where C is the square in the xy-plane with vertices (1, 0), (0, 1), (-1, 0), and (0, -1)
	STOKE'S Theorem: State STOKE'S Theorem and hence evaluate the following integrals
29	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ and C is the boundary of the surface $x^2 + y^2 = 1 - z, z > 0$.
30	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2zk\vec{k}$ and C is the boundary of the hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane.
31	$\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = -xy\vec{i} + 2yz\vec{j} + y^2\vec{k}$ and C is the boundary of the sphere $x^2 + y^2 + z^2 = a^2, z = 0$.

32	$\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = (x+y)i + (y+z)j - xk$ and s is the surface of the plane $2x + y + z = 2$ in the first quadrant.
33	Evaluate $\iint_s (\nabla \times \bar{F}) \cdot d\bar{s}$ where $\bar{F} = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$ and s is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $z = 9$ and open at the other end.
34	$\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = (x^2 + y^2)i + 4xyj$ and c is the boundary of the region bounded by the parabola $y^2 = 4x$ and line $x = 4$.
35	$\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = (x+y)i + (2x-z)j + (y+z)k$ and c is the boundary of the triangle cutoff by the plane $x + 2y + 3z = 6$ on the coordinate axes.
36	$\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = z^2i + x^2j + y^2k$ and C is the curved surface of the hemisphere $x^2 + y^2 + z^2 = 100, z \geq 0$
37	Work done in moving a particle once around the perimeter of the triangle with vertices $(2,0,0)$, $(0,3,0)$ and $(0,0,6)$ under the force $\bar{F} = (x+y)i + (2x-z)j + (y+z)k$
38	$\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = zi + xj + yk$ and C is the boundary of the hemisphere $z = \sqrt{1 - x^2 - y^2}$ in the xy plane
	GAUSS'S DIVERGENCE Theorem: State GAUSS'S DIVERGENCE Theorem and hence evaluate the following
39	$\iint_S \bar{N} \cdot \bar{F} ds$ where $\bar{F} = 4xi - 2y^2j + z^2k$ and S is the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.
40	$\iint_S \bar{N} \cdot \bar{F} ds$ where $\bar{F} = (2x + 3z^2)i - (xz^2 + y)j + (y^2 + 2z)k$ and S is the surface of the sphere with centre $(3, -14, -17)$ and radius 3.
41	$\iint_S \bar{N} \cdot \bar{F} ds$ where $\bar{F} = 4xzi - y^2j + yzk$ and S is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.
42	$\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = xi + yj + zk$ and S is the triangle $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.
43	$\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = x^3i + y^3j + z^3k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$.
44	$\iint_S \bar{F} \cdot d\bar{s}$ where $\bar{F} = xi + yj + zk$ and S is the cylindrical surface bounded by $x^2 + y^2 = a^2, z = 0, z = h$
45	$\iint_S \bar{N} \cdot \bar{F} ds$ where $\bar{F} = 2x^2yi - y^2j + 4xz^2k$ and S is the region of first octant bounded by $y^2 + z^2 = 9, x = 2$
46	By expressing $\iint_S (y^2z^2i + z^2x^2j + x^2y^2k) \cdot d\bar{s}$ as volume integral and evaluate over the part of $x^2 + y^2 + z^2 = 1$ lying above the xy plane.
47	$\iiint_v \nabla \cdot \bar{F} dv$ where $\bar{F} = 4xi - 2y^2j + z^2k$ over the cylindrical region $x^2 + y^2 = a^2, z = 0, z = b$.