Taylor Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

where $x \in (-1, 1]$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$x \in (-\pi/2, \pi/2)$

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \cdots$$

$x \in (0, \pi)$

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots$$

$x \in (-\pi/2, \pi/2)$

$$\operatorname{cosec} x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \cdots$$

$x \in (0, \pi)$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$$

$|x| < \pi/2$

$$\coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45}$$

$|x| \in (0, \pi)$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$x \in (-1, 1)$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$$

$x \in (-1, 1)$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{1\cdot 3}{2\cdot 4}x^2 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3 + \cdots \qquad x \in (-1, 1)$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{2\cdot 4}x^2 + \frac{1\cdot 3}{2\cdot 4\cdot 6}x^3 + \cdots \qquad x \in (-1, 1)$$

I Table of elementary integrals

$\int \sin x \, dx = -\cos x$

$\int \sinh x \, dx = \cosh x$

$\int \cos x \, dx = \sin x$

$\int \cosh x \, dx = \sinh x$

$\int \sec^2 x \, dx = \tan x$

$\int \operatorname{sech}^2 x \, dx = \tanh x$

$\int \operatorname{cosec}^2 x \, dx = -\cot x$

$\int \operatorname{cosech}^2 x \, dx = -\coth x$

$\int \sec x \tan x \, dx = \sec x$

$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$

$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x$

$\int \operatorname{cosech} x \coth x \, dx = -\operatorname{cosech} x$

$\int e^x dx = e^x$

$\int a^x dx = (a^x)/\log_e a$

$\int x^n dx = \dfrac{x^{n+1}}{n+1}, \ n \neq -1$

$\int \{f(x)\}^n f'(x) dx = \dfrac{\{f(x)\}^{n+1}}{n+1}, \ n \neq -1$

$\int \dfrac{1}{x} dx = \log x$

$\int \dfrac{f'(x)}{f(x)} dx = \log x$

$\int \tan x \, dx = \log \sec x = -\log \cos x$

$\int \cot x \, dx = \log \sin x$

$\int \operatorname{cosec} x \, dx = \log \tan(x/2) = \log(\operatorname{cosec} x - \cot x)$

$\int \sec x \, dx = \log \tan(\pi/4 + x/2) = \log(\sec x + \tan x)$

$\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$; $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\dfrac{x-a}{x+a}, \ x > a$; $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\dfrac{a+x}{a-x}, \ x < a$

$\int \dfrac{dx}{x(x^2-a^2)^{1/2}} = \dfrac{1}{a}\sec^{-1}\dfrac{x}{a}$ or $-\dfrac{1}{a}\operatorname{cosec}^{-1}\dfrac{x}{a}$

$\int \dfrac{dx}{(a^2-x^2)^{1/2}} = \sin^{-1}\dfrac{x}{a}$ or $-\cos^{-1}\dfrac{x}{a}$

$\int \dfrac{dx}{(a^2+x^2)^{1/2}} = \sinh^{-1}\dfrac{x}{a}$ or $\log\{x + (x^2+a^2)^{1/2}\}$

$\int \dfrac{dx}{(x^2-a^2)^{1/2}} = \cosh^{-1}\dfrac{x}{a}$ or $\log\{x + (x^2-a^2)^{1/2}\}$

LAPLACE TRANSFORM CHEATSHEET

(THANK ME LATER)

$\# \, 1[f(t)] = \phi(s) = \int_0^\infty f(t) e^{-st} dt$

for $t > 0$ & if integral exists

$L(e^{-at}) = \frac{1}{(s+a)}$ $\qquad$ $L(\sinh at) = \frac{a}{s^2-a^2}$

$L(1) = \frac{1}{s}$ $\qquad$ $L(\cosh at) = \frac{s}{s^2-a^2}$

$L(t^n) = \frac{\Gamma(n+1)}{s^{n+1}}$ $\qquad$ $L(\sin at) = \frac{a}{s^2+a^2}$

$L(\cos at) = \frac{s}{s^2+a^2}$

$\# \, L[k_1 f_1(t) + k_2 f_2(t)] = K_1 L[f_1(t)] + K_2 L[f_2(t)]$ [Linearity Property]

where k_1 & k_2 are constants

Change Of Scale

$\# \, L[f(t)] = \phi(s)$ then $L[f(at)] = \frac{1}{a} \phi\left[\frac{s}{a}\right]$

First Shifting Property.

$L[e^{-at} f(t)] = \phi(s+a)$ $\qquad$ $L[e^{at} f(t)] = \phi[s-a]$

If $L[f(t)] = \phi(s)$

Second Shifting.

$g(t) = \begin{cases} f(t-a) & \text{\& for } t > a \\ 0 & \text{\& for } t < a \end{cases}$ $\qquad$ Then $L[g(t)] = e^{-as} \phi(s)$

Multiplying By 't'

If $L[f(t)] = \phi(s)$ then: $L[t^n f(t)] = (-1)^{n} \frac{d^n}{ds^n}[\phi(s)]$

Dividing By 't'

If $1[f(t)] = \phi(s)$ then $L\left[\frac{1}{t} f(t)\right] = \int_s^\infty \phi(s)\, ds$

For Derivatives:

$$\star \quad L[f'(t)] = -f(0) + s\,L[f(t)]$$

$$\star \quad L[f''(t)] = s^2 L[f(t)] - s f(0) - f'(0)$$

$$\star \quad L[f'''(t)] = s^3 L[f(t)] - s^2 f(0) - s f'(0) - f''(0)$$

$$\star \quad L[f^{''(n\,\text{times})}(t)] = s^n L[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) \ldots f^{(n-1)}(0)$$

For Integrals

$$\star \quad L\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\phi(s) \qquad \left[\phi(s) = L[f(u)]\right]$$

$$\star \quad L\left[\int_0^t \int_0^t (n\,\text{times}) \int_0^t f(u)(du)^n\right] = \frac{1}{s^n} L[f(u)]$$

Inverse Laplace Identities

Function	Laplace Inverse
$\frac{1}{s-a}$	e^a
$\frac{1}{s^2+a^2}$	$\frac{1}{a} \sin at$
$\frac{1}{s^2-a^2}$	$\frac{1}{a} \sinh at$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{s}{s^2-a^2}$	$\cosh at$
$\frac{1}{s^{-n}}$	$\frac{t^{n-1}}{(n-1)!}$

First Shifting Theorem (Inverse)

If $f(t) = L^{-1}[\phi(s)]$ then

$$L^{-1}[\phi(s+a)] = e^{-at} f(t)$$

$$L[\phi(S-a)] = e^{at} f(t)$$

Convolution Theorem

for $f_1(t)$ & $f_2(t)$ - functions.

$\int_0^t f_1(u) f_2(t-u)\, du$ is called the convulution of $f_1(t)$ & $f_2(t)$ and denoted by $f_1(t) \cdot f_2(t)$

Differential Of $\phi(s)$

$$L^{-1}[\phi'(s)] = -t f(t) = -t\, L^{-1}[\phi(s)]$$

Integral Of $\phi(s)$

$$L^{-1}\left[\frac{1}{s}\phi(s)\right] = \int_0^t L^{-1}[\phi(s)]\, ds$$

for $f(t)$ periodic with period 'a'

$$L[f(t)] = \frac{1}{1-e^{-as}} \int_0^a e^{-st} f(t)\, dt$$

Heaviside Unit Step Function

$$H(t-a) = \begin{cases} 1 ; & t > a \\ 0 ; & t < a \end{cases}$$

Laplace Transform Of Heaviside Functions.

$$L[f(t) H(t-a)] = e^{-as} L[f(t+a)]$$

$$L[f(t-a) H(t-a)] = e^{-as} L[F(t)]$$

$$L^{-1}[e^{-as} \phi(s)] = f(t-a) H(t-a)$$

Z - TRANSFORM

For sequence $f(k)$

$$Z[\{f(k)\}] = F(z) = \sum_{k=-\infty}^{\infty} f(k)\, z^{-k} = \sum_{k=-\infty}^{\infty} \frac{f(k)}{z^{-k}}$$

[Only for convergent series]

$z \rightarrow$ complex number

$F(z) \rightarrow$ Z-Transform for $\{f(k)\}$

$Z \rightarrow$ Z-Transform operator

Using convolution Theorem

$$Z\{f(k) * g(k)\} = F(z) \cdot G(z) = \frac{z^2}{(z-1)(z-2)}$$

Fourier Series Cheatsheet

$$a_0 = \frac{1}{2l}\int_a^{a+2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_a^{a+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$b_n = \frac{1}{l}\int_a^{a+2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$f(x)$ is periodic with period $2l$

$$a_0 = \frac{1}{2\pi}\int_a^{a+2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_a^{a+2\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_a^{a+2\pi} f(x)\sin(nx)\,dx$$

$f(x)$ is periodic with period 2π

Half-Range Cosine Series

$a_0 = \frac{1}{l}\int_0^l f(x)\,dx \qquad a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx \qquad b_n = 0$

Half-Range Sine Series

$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

Recursive Integration By Parts (For Fourier Series)

$$\int uv\,dx = u\int v\,dx - u'\iint v\,dx^2 + u''\iiint v\,dx^3 \dots$$

VECTOR CALCULUS

$$\nabla \phi = i \frac{\partial \phi(P)}{\partial x} + j \frac{\partial \phi(P)}{\partial y} + k \frac{\partial \phi(P)}{\partial z}$$

Some things based on convention

$r = \sqrt{x^2 + y^2 + z^2}$

$\nabla f(r) = \frac{f'(r)}{r} \bar{r}$

Directional Derivative of F

$(\nabla F)_{(a,b,c)} \cdot (x\hat{i} + y\hat{j} + z\hat{k})$

θ : angle between G & F then;

$$\cos\theta = \frac{(\nabla F)_{(a,b,c)} (\nabla G)_{(a,b,c)}}{|\nabla F| \, |\nabla G|}$$

Two methods Divergence & Curl

$$\text{div } F = \nabla F = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] \cdot F$$

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Where F is a vector function

When div F = 0, F → Solenoidal Function

When curl F = 0, F → Irrotational Function

★ Gradient of any function is an irrotational function

Line Integral

$$\int_C \bar{F} \, d\bar{r} = \int_{x_1}^{x_2} f_1(x)dx + \int_{y_1}^{y_2} f_2(y)dy + \int_{z_1}^{z_2} f_3(z)dz$$

Function is conservative if curl $F = 0$

Hence $F = -\nabla\phi$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$$

(Integrate to get ϕ)

Green's Theorem

$$\oint_C f_1\,dx + f_2\,dy = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)dx\,dy$$

Stokes' Law

$$\oint_C \bar{F}\,d\bar{r} = \iint_R \text{curl}\,F\,\hat{n}\,ds$$ $\hat{n}$ - normal to surface