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(v) $J_0(t) = \sum_{r=0}^{\infty} \frac{-1}{(r!)^2}\left(\frac{t}{2}\right)^{2r}$ find $L[J_0(t)]$

→ Expand sigma
→ Apply Laplace transform (easy cause all are t^n form)
→ simplify.

Particular Integrals

- When value of 's' is given:

(i) $\int_0^{\infty} e^{-2t} \sin^3 t \, dt$ here $s=2$

$\Rightarrow L(\sin^3 t)$

$\Rightarrow L\left[\frac{3}{4}\sin t - \frac{1}{4}\sin 3t\right]$

$\Rightarrow \frac{3}{4}\left[\frac{1}{s^2+1}\right] - \frac{1}{4}\,\frac{3}{s^2+9}$

$\Rightarrow \frac{3}{4}\times\frac{1}{5} - \frac{1}{4}\times\frac{3}{13} \Rightarrow \frac{3}{4}\left[\frac{1}{5}-\frac{1}{13}\right] \Rightarrow \frac{3}{4}\times\frac{8}{65} \Rightarrow \frac{6}{65} = 0.0923$

(ii) $\int_0^{\infty} e^{-2t} \cosh^5 t \, dt$

$\Rightarrow L(\cosh^5 t)$

$\Rightarrow L\left[\left(\frac{e^{-x}+e^{x}}{2}\right)^5\right]$

$\Rightarrow L\left[\frac{1}{16}(\cosh 5t + 5\cosh 3t + 10\cosh t)\right]$

$\Rightarrow \frac{1}{16}\left[\frac{s}{s^2-25} + 5\,\frac{s}{s^2-9} + 10\,\frac{s}{s^2-1}\right]$ now substitute $s=2$

$\Rightarrow \frac{1}{16}\left[\frac{2}{-21} + \frac{\cancel{10}^{-2}}{\cancel{-5}} + \frac{20}{3}\right] \Rightarrow \frac{1}{8}\left[\frac{-1}{21} - 1 + \frac{10}{3}\right] = 0.2857$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx \qquad b_n = 0$$

Half-Range Sine Series

$$a_0 = 0 \qquad a_n = 0 \qquad b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

Formula for sine is same as even functions & for cosine is same as odd functions

Q) Find Half-Range Cosine for $f(x) = x(\pi - x)$ for $(0, \pi)$

Hence, show

(i) $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$ ii) $\sum_1^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ iii) $\sum_1^\infty \frac{1}{n^4} = \frac{\pi^4}{90}$

$f(x)$ is even hence

$$f(x) = a_0 + \sum a_n \cos nx$$

$$a_0 = \frac{1}{\pi} \int_0^\pi f(x)\, dx = \frac{1}{\pi} \int_0^\pi (\pi x - x^2)\, dx = \frac{1}{\pi}\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^\pi$$

$$a_0 = \frac{\pi^2}{6}$$

$$a_n = \frac{2}{\pi} \int_0^\pi (\pi x - x^2) \cos n\pi\, dx$$

$$= \frac{2}{\pi}\left[(\pi x - x^2)\cancel{\frac{\sin nx}{n}}^{\,0} - (\pi - 2x)\left[\frac{-\cos nx}{n^2}\right] + (-2)\left[\cancel{\frac{-\sin nx}{n^3}}^{\,0}\right]\right]_0^\pi$$

$$= \frac{-2}{} \left[\frac{1 + \cos n\pi}{n^2}\right]$$

$$= \begin{cases} 0 & , \quad n = \text{even} \\ -4/n^2 & , \quad n = \text{odd.} \end{cases}$$

$$x(\pi - x) = \frac{\pi^2}{6} - 4\left[\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} \cdots\cdots\right]$$

$$x(\pi - x) = \frac{\pi^2}{6} - \left[\frac{1}{1^2}\cos 2x + \frac{1}{2^2}\cos 4x + \frac{1}{3^2}\cos 6x \cdots\right]$$

$$0 = \frac{\pi^2}{6} - \left[\frac{1}{1^2} + \frac{1}{2^2} \cdots\cdots\right] \qquad \{x = 0\}$$

$$\therefore \frac{\pi^2}{6} = \sum_1^\infty \frac{1}{n^2} \qquad \text{(i) proved.}$$

$x = \pi/2$

$$\left(\frac{\pi}{2}\right)\left(\pi - \frac{\pi}{2}\right) = \frac{\pi^2}{6} - \left[\frac{1}{1^2}(-1) + \frac{1}{2^2} - \frac{1}{3^2} \cdots\cdots\right]$$

$$\frac{\pi^2}{4} = \frac{\pi^2}{6} + \left[\sum_1^\infty \frac{(-1)^{n+1}}{n^2}\right]$$

$$\sum_1^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{4} - \frac{\pi^2}{6} = \frac{\pi^2}{12}$$

Using Parseval Identity

$$\frac{1}{\pi}\int_0^\pi [f(x)]^2 dx = \frac{1}{2}\left[2a_0^2 + a_1^2 + a_2^2\right]\cdots\cdots$$

$$\therefore \frac{1}{\pi}\int_0^\pi x^2(\pi - x)^2 dx$$

$$\frac{1}{\pi}\int_0^\pi (\pi^2 x^2 - 2\pi x^3 + x^4)dx$$

$$\frac{1}{\pi}\left[\frac{\pi^5}{3} - \frac{\pi^5}{2} + \frac{\pi^5}{5}\right]$$

$$\pi^4\left[\frac{10 - 15 + 6}{30}\right] = \frac{\pi^4}{30} = \text{LHS}$$

$$\frac{\pi^4}{30} = \frac{1}{2}\left[\frac{2\pi^4}{36} + 0 - \frac{4}{(2)^2} - \frac{4}{(4)^2} - \frac{4}{(6)^2}\cdots\right]$$

$$\frac{\pi^4}{30} = \frac{1}{2}\left[\frac{\pi^4}{18} + \sum_{n=1}^\infty \left[\frac{-4}{(2n)^2}\right]^2\right]$$

$$\frac{\pi^4}{30} - \frac{\pi^4}{36} = \frac{1}{2}\sum_{n=1}^\infty \frac{\cancel{16}}{\cancel{16}n^4}$$

$$\frac{\pi^4}{180} = \frac{1}{2}\sum_{n=1}^\infty \frac{1}{n^4}$$

$$\frac{\pi^4}{90} = \sum_{n=1}^\infty \frac{1}{n^4}$$