Mugdha Kurkure
1911096 B2

Q11: Find Z-Transform and Region of Convergence of:

$f(k) = \frac{a^k}{k}$ **where k > 1, a > 0**

Solution

Let us assume
f(k) = 0 for k < 1

By using the definition of Z-Transform:

$$Z\left\{k\right\} = \sum_{k=-\infty}^{0} 0.z^{-k} + \sum_{k=1}^{\infty} \frac{a^k}{k}.z^{-k}$$

$$Z\left\{k\right\} = \sum_{k=1}^{\infty} \frac{a^k}{k}.z^{-k}$$

$$F\left(z\right) = \frac{a}{z} + \frac{a^2}{2z^2} + \frac{a^3}{3z^3} + \frac{a^4}{4z^4} + \frac{a^5}{5z^5} + \cdots$$

$$F\left(z\right) = -\left[-\frac{a}{z} - \frac{a^2}{2z^2} - \frac{a^3}{3z^3} - \frac{a^4}{4z^4} - \frac{a^5}{5z^5} \cdots\right]$$

This given expression inside the brackets is equal to $log_e\left(1 - \frac{a}{z}\right)$
Hence,

$$Z\left\{k\right\} = F(k) = -log_e\left(1 - \frac{a}{z}\right)$$

Hence this series will be convergent only when:

$$\left|\frac{a}{z}\right| < 1$$

$$|a| < |z|$$

Since a > 0, ROC is:

$$|z| > a$$